Unquenching the quark model

. . . without spoiling its best results

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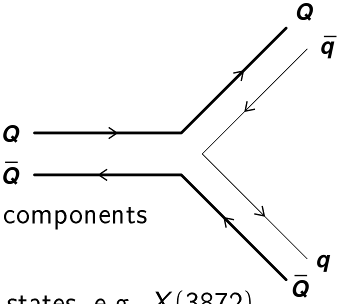
Unquenching the quark model

OZI-favoured pair creation implies physical states are admixtures of

- bare (valence) states $Q\overline{Q}$, and
- meson-meson continua $(Q\overline{q})(q\overline{Q})$



For states near threshold meson-meson components

- are almost unavoidable, and
- explain unusual properties of some states, e.g. $X(3872)$.

Even far from threshold, meson-meson components are present, and result in large mass shifts ...

Unquenching the quark model

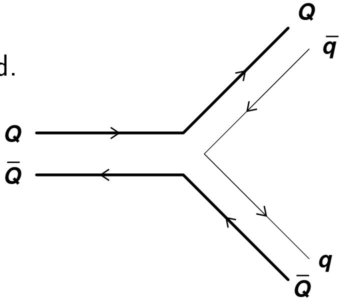
- Eichten *et al.* '76, '78, '06
- Ono & Törnqvist '84
- Törnqvist '85
- Kalashnikova & Yufryakov '94
- Shmatikov '99
- Kalashnikova '05
- Pennington and Wilson '07
- Barnes & Swanson '08
- Close & Thomas '09
- Li, Meng & Chao '09
- Yang, Li, Chen & Deng '11
- Liu & Ding '12
- Ferretti & Santopinto '13
- Lu, Anwar & Zou '16

Unquenching the quark model

Most models involve the operator $\chi \cdot \mathbf{O}$, where

- χ creates a spin triplet $q\bar{q}$ pair
- $\mathbf{O}$ is the spatial part

and the spins of Q and $\overline{Q}$ are conserved.

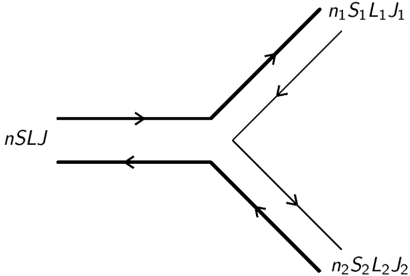


This includes:

- ${}^3P_0$ models
- flux tube models (${}^3P_0$ and ${}^3S_1$)
- the Cornell model (Lorentz vector)
- microscopic models in the H.Q. limit (Lorentz scalar + vector)
- pseudoscalar-meson emission models

[TJB'14]

Unquenching the quark model



Total spin j and partial wave l,

$$M_{jl}\begin{bmatrix} n & S & L & J \\ n_1 & S_1 & L_1 & J_1 \\ n_2 & S_2 & L_2 & J_2 \end{bmatrix} = \xi_{jl}\begin{bmatrix} S & L & J \\ S_1 & L_1 & J_1 \\ S_2 & L_2 & J_2 \end{bmatrix} \cdot \boldsymbol{A}_l\begin{bmatrix} n & L \\ n_1 & L_1 \\ n_2 & L_2 \end{bmatrix}$$

- $\boldsymbol{A}$ contains the spatial part $\mathbf{O}$ (model-dependent), and
- ξ contains the spin part χ (model-independent).

Orthogonality of the ξ coefficients gives a "closure" relation

$$\sum_{\substack{S_1 J_1 \\ S_2 J_2 \\ j}} M_{jl}\begin{bmatrix} \widehat{n} & \widehat{S} & \widehat{L} & J \\ n_1 & S_1 & L_1 & J_1 \\ n_2 & S_2 & L_2 & J_2 \end{bmatrix}^* M_{jl}\begin{bmatrix} n & S & L & J \\ n_1 & S_1 & L_1 & J_1 \\ n_2 & S_2 & L_2 & J_2 \end{bmatrix} = \delta_{\widehat{S}S}\delta_{\widehat{L}L}\boldsymbol{A}_l^*\begin{bmatrix} \widehat{n} & \widehat{L} \\ n_1 & L_1 \\ n_2 & L_2 \end{bmatrix} \cdot \boldsymbol{A}_l\begin{bmatrix} n & L \\ n_1 & L_1 \\ n_2 & L_2 \end{bmatrix}$$

Coupled channels

Discrete valence states: $H_0 \big| k \big\rangle = M_k \big| k \big\rangle$

Two-hadron continua: $H_0 \big| \lambda p \big\rangle = E_\lambda(p) \big| \lambda p \big\rangle$

$H = H_0 + V$ couples the sectors, and physical states

$$\big| i \big) = \sum_k \big| k \big\rangle \big\langle k \big| i \big) + \int dp p^2 \sum_\lambda \big| \lambda p \big\rangle \big\langle \lambda p \big| i \big)$$

with mass E_i (below threshold) are solutions of

$$\left[\begin{pmatrix} M_1 & 0 & \cdots \\ 0 & M_2 & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix} - \begin{pmatrix} \big\langle 1 \big| \Omega(E_i) \big| 1 \big\rangle & \big\langle 1 \big| \Omega(E_i) \big| 2 \big\rangle & \cdots \\ \big\langle 2 \big| \Omega(E_i) \big| 1 \big\rangle & \big\langle 2 \big| \Omega(E_i) \big| 2 \big\rangle & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix} \right] \begin{pmatrix} \big\langle 1 \big| i \big) \\ \big\langle 2 \big| i \big) \\ \vdots \end{pmatrix} = E_i \begin{pmatrix} \big\langle 1 \big| i \big) \\ \big\langle 2 \big| i \big) \\ \vdots \end{pmatrix}$$

with

$$\big\langle j \big| \Omega(E_i) \big| k \big\rangle = \int dp p^2 \sum_\lambda \frac{\big\langle j \big| V^\dagger | \lambda p \big\rangle \big\langle \lambda p \big| V \big| k \big\rangle}{E_\lambda(p) - E_i}$$

Coupled channels

$$\sum_{\lambda} \langle j | V^{\dagger} | \lambda p \rangle \langle \lambda p | V | k \rangle = \delta_{jk} |A(p)|^2$$

With degenerate continua,

$$\begin{aligned}
\langle j | \Omega(E_i) | k \rangle &= \int dp p^2 \sum_{\lambda} \frac{\langle j | V^{\dagger} | \lambda p \rangle \langle \lambda p | V | k \rangle}{E_{\lambda}(p) - E_i} \\
&\approx \int dp p^2 \frac{\sum_{\lambda} \langle j | V^{\dagger} | \lambda p \rangle \langle \lambda p | V | k \rangle}{E(p) - E_i} \\
&= \delta_{jk} \int dp p^2 \frac{|A(p)|^2}{E(p) - E_i} \\
&= \delta_{jk} \langle \Omega(E_i) \rangle
\end{aligned}$$

there is no mixing among valence states

$$\left[\begin{pmatrix} M_1 & 0 & \cdots \\ 0 & M_2 & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix} - \begin{pmatrix} \langle \Omega(E_i) \rangle & 0 & \cdots \\ 0 & \langle \Omega(E_i) \rangle & \cdots \\ \vdots & \vdots & \ddots \end{pmatrix} \right] = E_i \begin{pmatrix} \langle 1 | i \rangle \\ \langle 2 | i \rangle \\ \vdots \end{pmatrix}$$

Coupled channels

$$E_i = M_i - \langle \Omega(E_i) \rangle, \qquad Z_i = \left| \langle i | i) \right|^2 = \frac{1}{1 + \langle \Omega'(E_i) \rangle}$$

Degenerate valence states ($M_i = M$) imply degenerate physical states ($E_i = E$), with equal valence components

$$E = M - \langle \Omega(E) \rangle, \qquad Z = \frac{1}{1 + \langle \Omega'(E) \rangle}$$

Otherwise

$$E_i \approx M_i - \langle \Omega(E) \rangle - (E_i - E) \langle \Omega'(E) \rangle$$

Eliminating $\langle \Omega(E) \rangle$ gives

$$E_i - E = \frac{M_i - M}{1 + \langle \Omega'(E) \rangle} = Z(M_i - M)$$

[TJB '15]

Main results

Applying to families of valence mesons with common n and L, summing over *all* meson-meson continua: [TJB '15]

Degenerate valence states, degenerate continua

- $E_{nSLJ} = E_{nL}$ (*)
- $Z_{nSLJ} = Z_{nL}$
- ...configuration mixing independent of S and J

Split valence states, degenerate continua

- S and L are good quantum numbers (*)
- $E_{nSLJ} - E_{nL} = Z_{nL}(M_{nSLJ} - M_{nL})$

(* generalisation of Barnes & Swanson from pert. theory → full coupled channel problem, and from $^3P_0$ model → all models)

Main results

Split valence states, split continua ($L_1 = L_2 = 0$)

$$m(^1\mathrm{S}_0) = m - \frac{3\delta}{4}$$
$$m(^3\mathrm{S}_1) = m + \frac{\delta}{4}$$

Using ξ coefficients, obtain $\mathcal{O}(\delta/m)$ corrections to

$$E_{nSLJ} - E_{nL} = Z_{nL}(M_{nSLJ} - M_{nL})$$

$L = 0$ mesons: no corrections!

$L \geqslant 1$ mesons: corrections are

- small due to cancellations, and
- proportional to $\langle \mathbf{L} \cdot \mathbf{S} \rangle$.

Hyperfine splitting ($L = 0$)

	$\langle \Omega \rangle_{nS}$	δM	δE	$\delta E^{pred.}$
$c\bar{c}$				
1S [K]	174	129	117	
2S [K]	212	64	48	
$b\bar{b}$				
1S [LD]	57.41	71.39	68.50*	
2S [LD]	67.58	23.12	21.30	
3S [LD]	67.74	15.73	14.00	

[K=Kalashnikova, LD=Liu&Ding]

$\langle \Omega \rangle_{nS}$ = spin-averaged mass shifts

δM = bare hyperfine splitting

δE = physical hyperfine splitting

$\delta E^{pred.}$ = predicted hyperfine splitting, from the formula

Hyperfine splitting ($L = 0$)

	$\langle \Omega \rangle_{nS}$	δM	δE	$\delta E^{pred.}$
$c\bar{c}$				
1S [K]	174	129	117	116.4
2S [K]	212	64	48	48.4
$b\bar{b}$				
1S [LD]	57.41	71.39	68.50*	68.44
2S [LD]	67.58	23.12	21.30	21.36
3S [LD]	67.74	15.73	14.00	14.06

[K=Kalashnikova, LD=Liu&Ding]

$\langle \Omega \rangle_{nS}$ = spin-averaged mass shifts

δM = bare hyperfine splitting

δE = physical hyperfine splitting

$\delta E^{pred.}$ = predicted hyperfine splitting, from the formula

Hyperfine splitting ($L = 0$)

Quenched quark model:

$$M_{n^3S_1} - M_{n^1S_0} = \frac{8\alpha_s}{9m^2}|R_{nS}(0)|^2, \quad \Gamma^{e^+e^-}_{n^3S_1} = \frac{e_q^2\alpha^2}{m^2}|R_{nS}(0)|^2\left(1 - \frac{16\alpha_s}{3\pi}\right)$$

Model-independent relation:

$$\frac{M_{2^3S_1} - M_{2^1S_0}}{M_{1^3S_1} - M_{1^1S_0}} = \frac{\Gamma^{e^+e^-}_{2^3S_1}}{\Gamma^{e^+e^-}_{1^3S_1}}$$

Experimental data:

$$\frac{M_{\psi'} - M_{\eta_c'}}{M_{J/\psi} - M_{\eta_c}} = 0.407 \pm 0.015 \qquad \frac{M_{\Upsilon'} - M_{\eta_b'}}{M_{\Upsilon} - M_{\eta_b}} = 0.42 \pm 0.09$$

$$\frac{\Gamma^{e^+e^-}_{\psi'}}{\Gamma^{e^+e^-}_{J/\psi}} = 0.423 \pm 0.018 \qquad \frac{\Gamma^{e^+e^-}_{\Upsilon'}}{\Gamma^{e^+e^-}_{\Upsilon}} = 0.457 \pm 0.014$$

Can this be spoiled by unquenching?

Hyperfine splitting ($L = 0$)

The physical mass splitting is scaled downwards, [TJB '13]

$$E_{n^3\mathrm{S}_1} - E_{n^1\mathrm{S}_0} = Z_{n\mathrm{S}}(M_{n^3\mathrm{S}_1} - M_{n^1\mathrm{S}_0})$$

but so is the wavefunction at the origin,

$$\Gamma^{e^+e^-}_{n^3\mathrm{S}_1} \to Z_{n^3\mathrm{S}_1}\Gamma^{e^+e^-}_{n^3S_1}$$

To a very good approximation $Z_{n^3\mathrm{S}_1} \approx Z_{nS}$, so the relation survives:

$$\frac{E_{2^3\mathrm{S}_1} - E_{2^1\mathrm{S}_0}}{E_{1^3\mathrm{S}_1} - E_{1^1\mathrm{S}_0}} = \frac{\Gamma^{e^+e^-}_{2^3S_1}}{\Gamma^{e^+e^-}_{1^3S_1}}$$

Predictions for $\eta_b(3S)$:

$E_{\Upsilon(3\mathrm{S})} - E_{\eta_b(3\mathrm{S})} = 20.6 \pm 1.7$ MeV, $E_{\eta_b(3\mathrm{S})} = 10334.6 \pm 2.2$ MeV

Hyperfine splitting ($L \neq 0$)

The quenched quark model result,

$$\frac{1}{9}\left(M_{^3P_0} + 3M_{^3P_1} + 5M_{^3P_2}\right) - M_{^1P_1} = 0$$

is well-satisfied by charmonia and bottomonia:

$$\overline{M}_{\chi_c(1P)} - M_{h_c(1P)} = -0.08 \pm 0.22 \text{ MeV}$$
$$\overline{M}_{\chi_b(1P)} - M_{h_b(1P)} = 0.57 \pm 1.08 \pm 0.31 \text{ MeV}$$
$$\overline{M}_{\chi_b(2P)} - M_{h_b(2P)} = 0.44 \pm 1.44 \pm 0.5 \text{ MeV}$$

Can this be spoiled by unquenching?

Hyperfine splitting ($L \neq 0$) [TJB '11]

	$\langle\Omega\rangle_{^3P_0}$	$\langle\Omega\rangle_{^3P_1}$	$\langle\Omega\rangle_{^3P_2}$	$\langle\Omega\rangle_{^1P_1}$	Induced
$c\bar{c}$					
1P [BS]	459	496	521	504	
1P [K]	198	215	228	219	
1P [LMC]	35	38	63	52	
1P [YLCD]	131	152	175	162	
1P [OT]	173	180	185	182	
$b\bar{b}$					
1P [OT]	43	44	45	44	
2P [OT]	55	56	58	57	
1P [FS]	108	114	117	115	
2P [FS]	137	144	149	146	
1P [LD]	80.777	84.823	87.388	85.785	
2P [LD]	73.578	77.608	80.146	78.522	

[BS=Barnes & Swanson, K=Kalashnikova, LMC=Li, Meg & Chao, YLCD=Yang, Li, Chen & Deng, OT=Ono& Tornqvist, LD=Liu &Ding, FS=Ferretti & Santopinto]

Hyperfine splitting ($L \neq 0$) [TJB '11]

	$\langle \Omega \rangle_{^3P_0}$	$\langle \Omega \rangle_{^3P_1}$	$\langle \Omega \rangle_{^3P_2}$	$\langle \Omega \rangle_{^1P_1}$	Induced
$c\bar{c}$					
1P [BS]	459	496	521	504	−1.8
1P [K]	198	215	228	219	−1.3
1P [LMC]	35	38	63	52	−2.9
1P [YLCD]	131	152	175	162	−0.4
1P [OT]	173	180	185	182	−0.0
$b\bar{b}$					
1P [OT]	43	44	45	44	−0.4
2P [OT]	55	56	58	57	−0.0
1P [FS]	108	114	117	115	−0.0
2P [FS]	137	144	149	146	−0.0
1P [LD]	80.777	84.823	87.388	85.785	−0.013
2P [LD]	73.578	77.608	80.146	78.522	−0.048

[BS=Barnes & Swanson, K=Kalashnikova, LMC=Li, Meg & Chao, YLCD=Yang, Li, Chen & Deng, OT=Ono& Tornqvist, LD=Liu &Ding, FS=Ferretti & Santopinto]

Spin-orbit and tensor splittings ($L \neq 0$)

Spin-orbit (LS) and tensor (T) splittings are also scaled by Z_{nL}.

	$\langle\Omega\rangle$	δM_{LS}	δE_{LS}	$\delta E_{LS}^{pred.}$	δM_T	δE_T	$\delta E_T^{pred.}$
$c\bar{c}$							
1P [K]	220	43	34.8		−23.3	−20.4	
1P [OT]	182	35.8	32.5		−22.1	−20.8	
$b\bar{b}$							
1P [LD]	85.84	16.31	14.75		−6.05	−5.58	
2P [LD]	78.92	12.99	11.46		−4.78	−4.35	
3P [LD]	80.15	11.75	9.86		−4.23	−3.85	
1D [LD]	97.96	2.44	2.07		−1.26	−1.17	
2D [LD]	89.36	2.36	2.03		−1.17	−1.16	

Spin-orbit and tensor splittings ($L \neq 0$)

Spin-orbit (LS) and tensor (T) splittings are also scaled by Z_{nL}.

	$\langle\Omega\rangle$	δM_{LS}	δE_{LS}	$\delta E_{LS}^{pred.}$	δM_T	δE_T	$\delta E_T^{pred.}$
$c\bar{c}$							
1P [K]	220	43	34.8	35.1	−23.3	−20.4	−19.0
1P [OT]	182	35.8	32.5	33.2	−22.1	−20.8	−20.5
$b\bar{b}$							
1P [LD]	85.84	16.31	14.75	14.84	−6.05	−5.58	−5.50
2P [LD]	78.92	12.99	11.46	11.49	−4.78	−4.35	−4.23
3P [LD]	80.15	11.75	9.86	9.76	−4.23	−3.85	−3.51
1D [LD]	97.96	2.44	2.07	2.12	−1.26	−1.17	−1.10
2D [LD]	89.36	2.36	2.03	1.97	−1.17	−1.16	−0.98

Valence component inequalities

Expanding $\Omega'(E_{nSLJ})$ in a similar way to $\Omega(E_{nSLJ})$, leads to inequalities among the Z-factors, e.g.

$$Z_{^1\mathrm{S}_0} > Z_{^3\mathrm{S}_1}$$

$$Z_{^3\mathrm{P}_0} > Z_{^3\mathrm{P}_1} > Z_{^3\mathrm{P}_2}$$

Relations among leptonic widths, arising from cancelling $\left|R_{nL}(0)\right|^2$ factors, are modified, e.g.

$$\frac{\Gamma_{^1\mathrm{S}_0\to\gamma\gamma}}{\Gamma_{^3\mathrm{S}_1\to e^+e^-}} > \frac{4}{3}\left(1+1.96\frac{\alpha_s}{\pi}\right)$$

$$\frac{\Gamma_{^3\mathrm{P}_2\to\gamma\gamma}}{\Gamma_{^3\mathrm{P}_0\to\gamma\gamma}} < \frac{4}{15}\left(1-5.51\frac{\alpha_s}{\pi}\right)$$

Exotic mesons imply the need to unquench the quark model

Everything follows from $\chi \cdot \mathbf{O}$.

With no spin splittings, previous results in pert. theory generalise to

- full coupled-channel problem,
- all models,
- configuration mixing,
- ... hybrids.

Including splittings, the formula $E_{nSLJ} - E_{nL} = Z_{nL}(M_{nSLJ} - M_{nL})$

- implies unquenching decreases splittings
- agrees with model calculations (and explains observations),
- protects e^+e^- width formulae for $L = 0$,
- protects zero hyperfine for $L \neq 0$,
- might be testable on lattice QCD,

...and is useful for calculations.