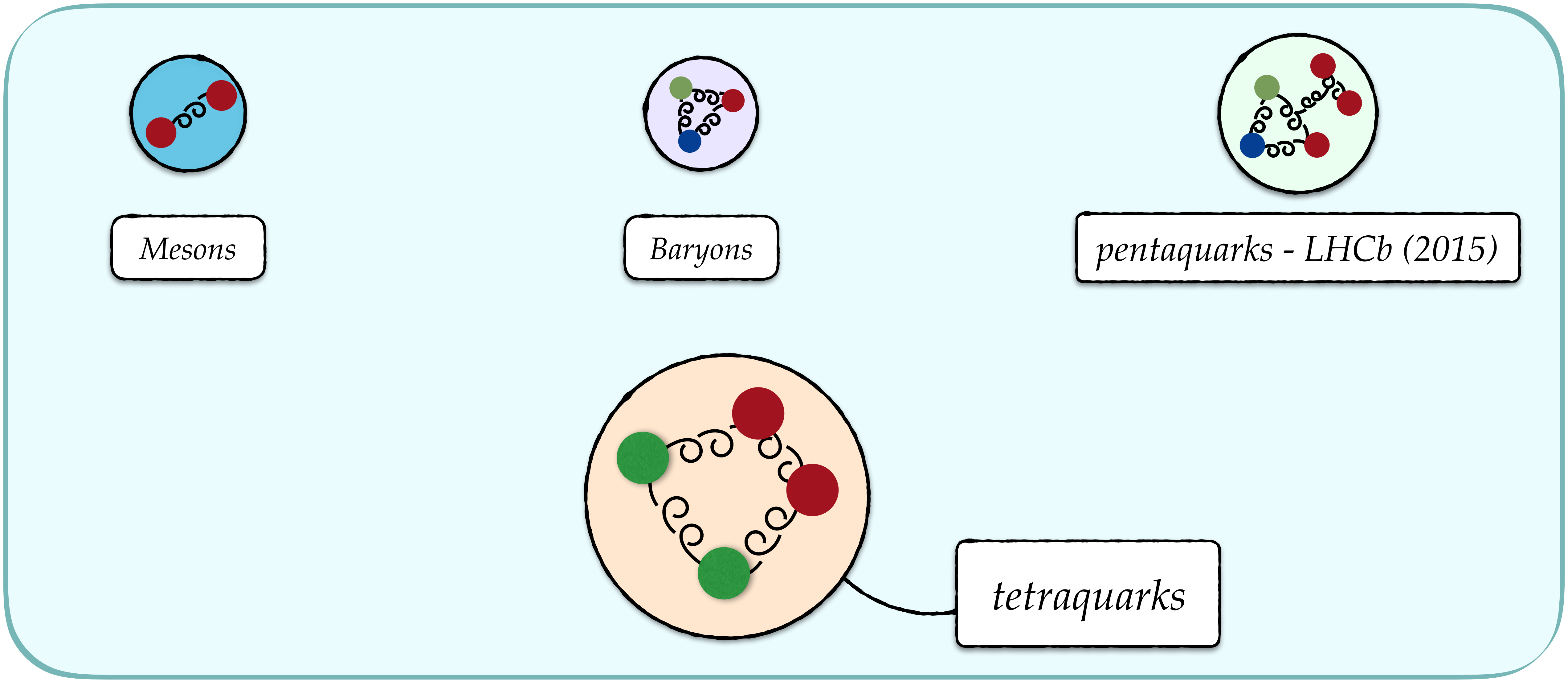
Searching for Beauty-fully Bound Tetraquarks Using Lattice NRQCD

Ciaran Hughes, Estia Eichten, Christine Davies



EHS, 2017

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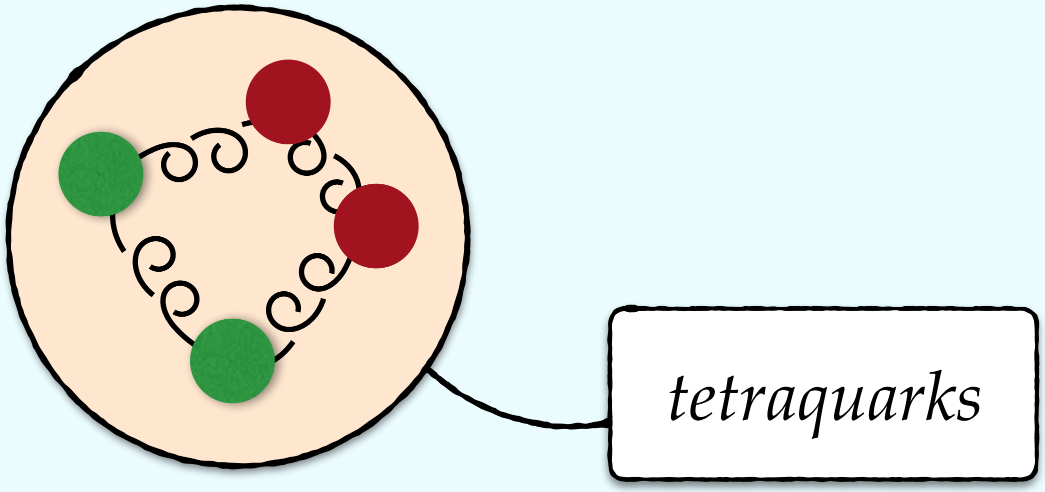
- This talk will be a bigger picture sketch of results from arxiv: 1710.03236
- For more details, please contact me (chughes@fnal.gov)!



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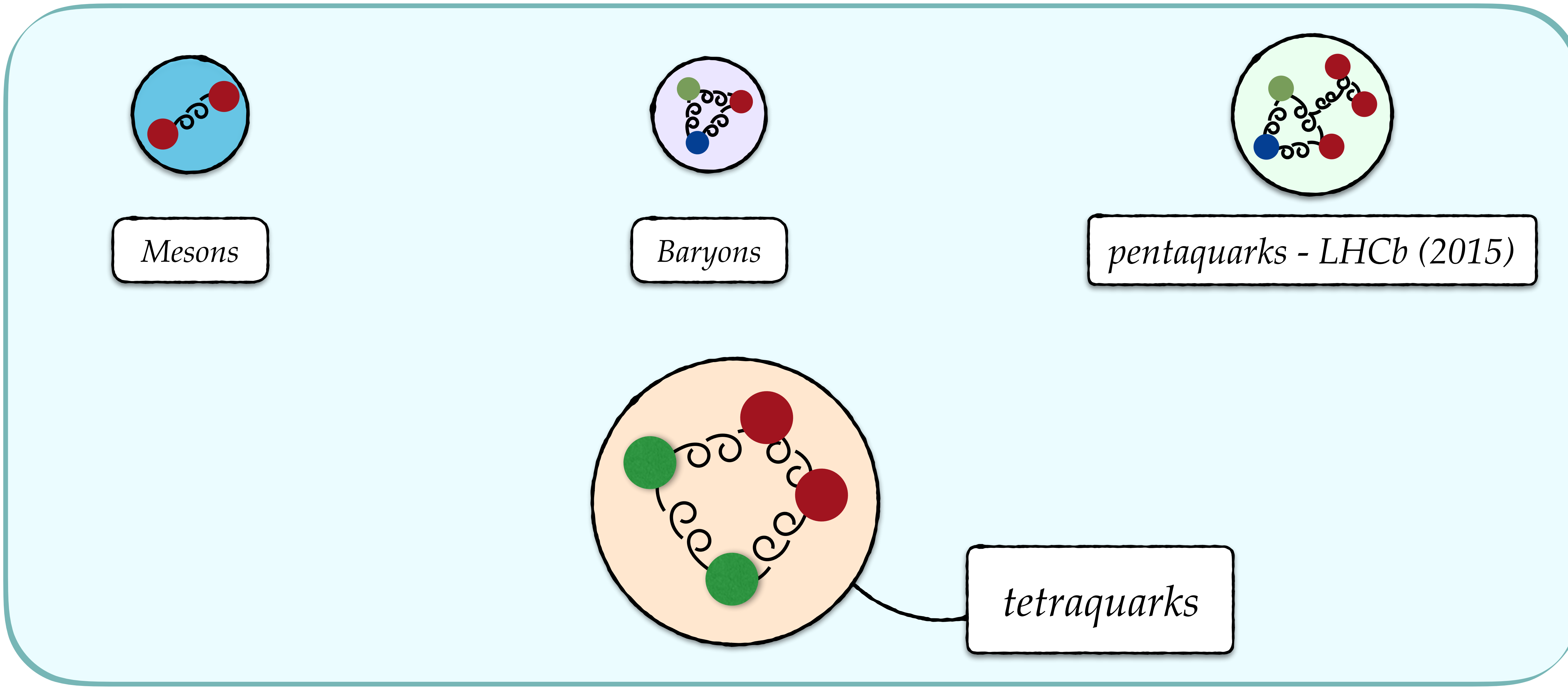

Quantum Chromodynamics

"The fundamental theory of the strong nuclear force"

QCD - For The Particle Physicists

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TABLE 9: As in Table 4, but for new *unconventional* states in the $c\bar{c}$ and $b\bar{b}$ regions, ordered by mass. For $X(3872)$, the values given are based only upon decays to $\pi^+\pi^- J/\psi$. $X(3945)$ and $Y(3940)$ have been subsumed under $X(3915)$ due to compatible properties. The state known as $Z(3930)$ appears as the $\chi_{c2}(2P)$ in Table 4. See also the reviews in [81–84]

State	m (MeV)	Γ (MeV)	J^{PC}	Process (mode)	Experiment (#σ)	Year	Status
$X(3872)$	3871.52±0.20	1.3±0.6	$1^{++}/2^{-+}$	$B \to K(\pi^+\pi^- J/\psi)$	Belle [85, 86] (12.8), *BABAR* [87] (8.6)	2003	OK
		(<2.2)		$p\bar{p} \to (\pi^+\pi^- J/\psi) + ...$	CDF [88–90] (np), DØ [91] (5.2)		
				$B \to K(\omega J/\psi)$	Belle [92] (4.3), *BABAR* [93] (4.0)		
				$B \to K(D^{*0}\bar{D}^0)$	Belle [94, 95] (6.4), *BABAR* [96] (4.9)		
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				$B \to K(\gamma\psi(2S))$	*BABAR* [98] (3.5), Belle [99] (0.4)		
$X(3915)$	3915.6 ± 3.1	28±10	$0/2^{?+}$	$B \to K(\omega J/\psi)$	Belle [100] (8.1), *BABAR* [101] (19)	2004	OK
				$e^+e^- \to e^+e^-(\omega J/\psi)$	Belle [102] (7.7)		
$X(3940)$	3942^{+9}_{-8}	37^{+27}_{-17}	$?^{?+}$	$e^+e^- \to J/\psi(D\bar{D}^*)$	Belle [103] (6.0)	2007	NC!
				$e^+e^- \to J/\psi\ (...)$	Belle [54] (5.0)		
$G(3900)$	3943 ± 21	52±11	1^{--}	$e^+e^- \to \gamma(D\bar{D})$	*BABAR* [27] (np), Belle [21] (np)	2007	OK
$Y(4008)$	4008^{+121}_{-49}	226±97	1^{--}	$e^+e^- \to \gamma(\pi^+\pi^- J/\psi)$	Belle [104] (7.4)	2007	NC!
$Z_1(4050)^+$	4051^{+24}_{-43}	82^{+51}_{-55}	?	$B \to K(\pi^+\chi_{c1}(1P))$	Belle [105] (5.0)	2008	NC!
$Y(4140)$	4143.4 ± 3.0	15^{+11}_{-7}	$?^{?+}$	$B \to K(\phi J/\psi)$	CDF [106, 107] (5.0)	2009	NC!
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$Y(4274)$	$4274.4^{+8.4}_{-6.7}$	32^{+22}_{-15}	$?^{?+}$	$B \to K(\phi J/\psi)$	CDF [107] (3.1)	2010	NC!
$X(4350)$	$4350.6^{+4.6}_{-5.1}$	$13.3^{+18.4}_{-10.0}$	$0,2^{++}$	$e^+e^- \to e^+e^-(\phi J/\psi)$	Belle [112] (3.2)	2009	NC!
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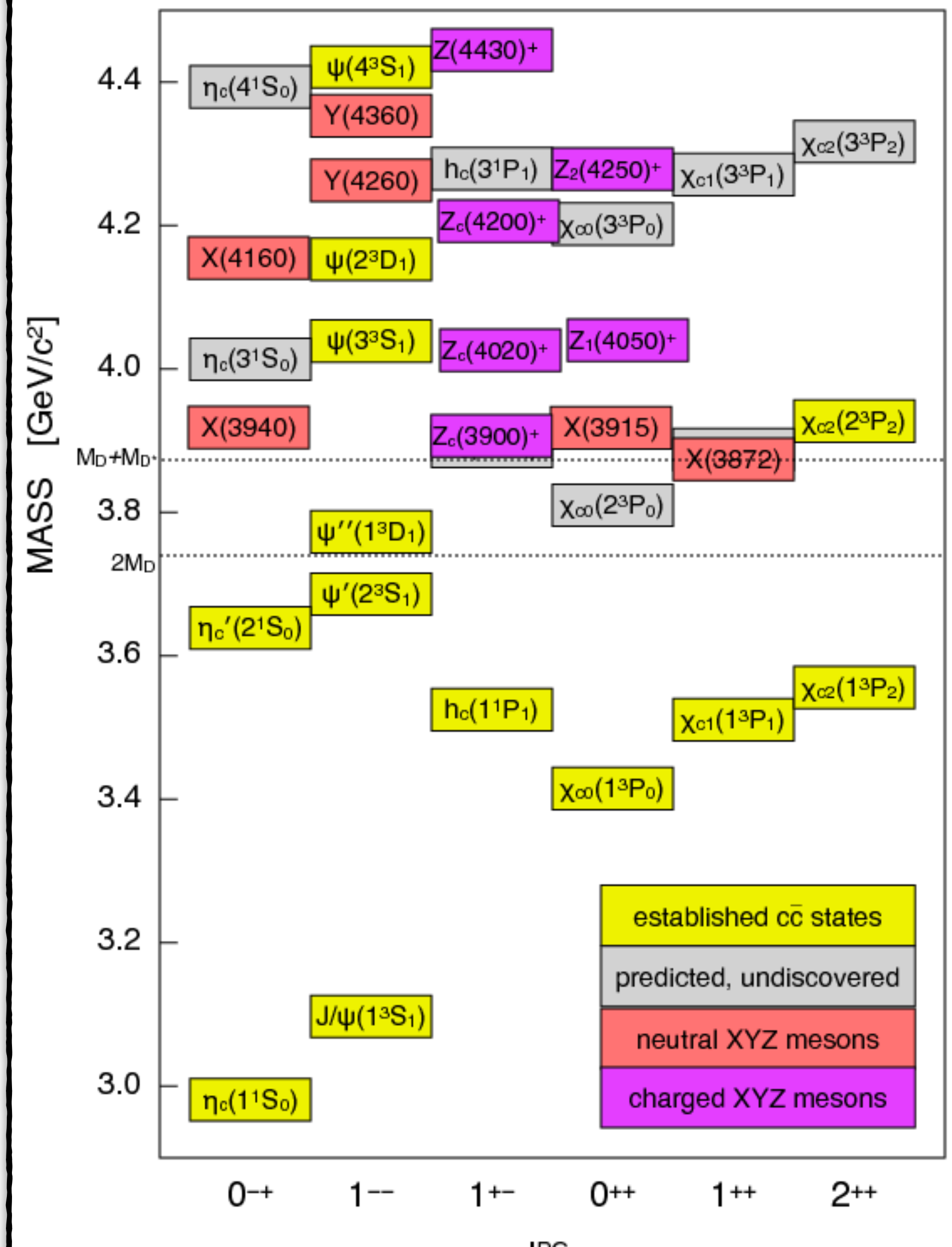
Quarkonium Working Group - 2010



S. Olsen - 2015

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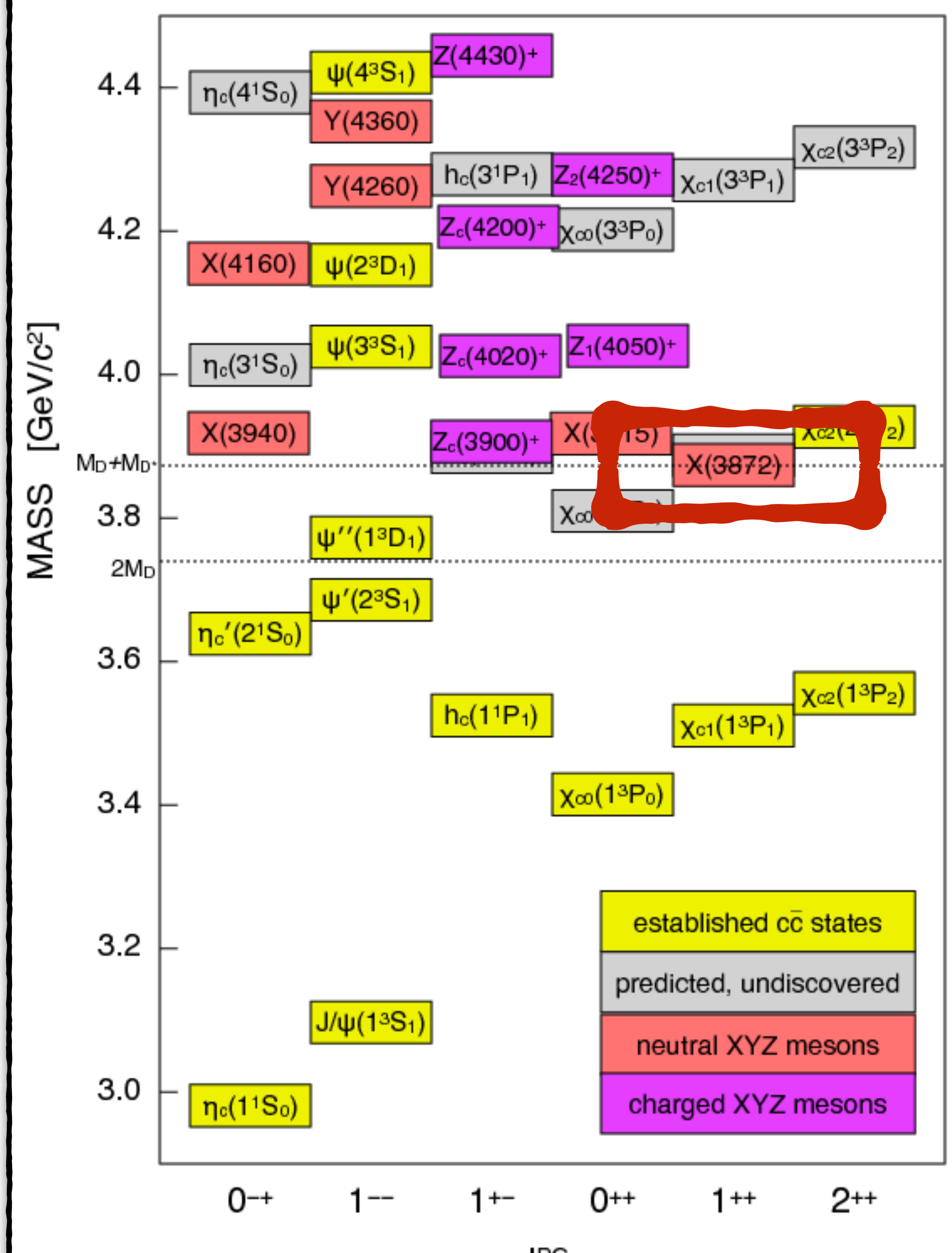

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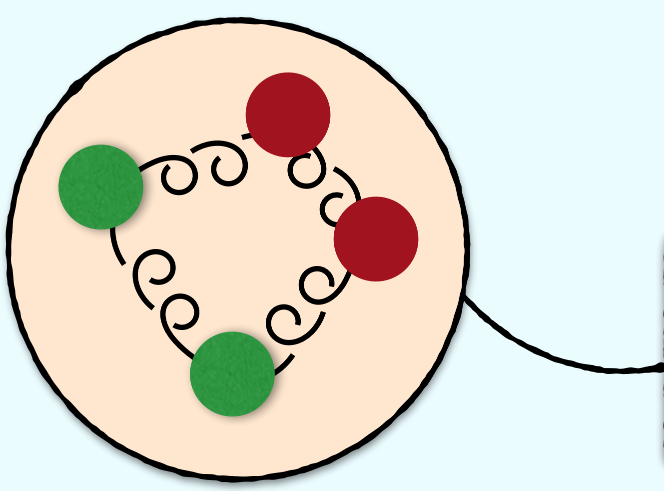
Simplest Extended System



$2q2\bar{q}$
tetraquarks

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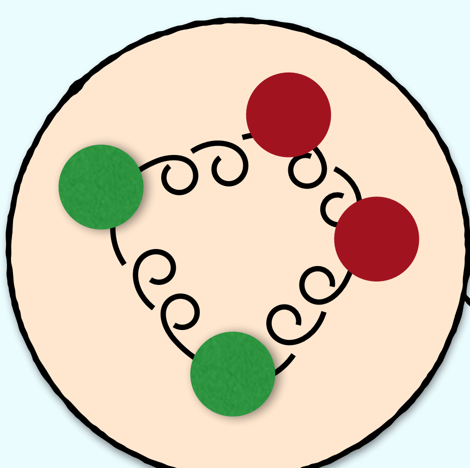

$\mathbf{2q2\bar{q}}$ *tetraquarks* → *Take* $\mathbf{m_q \gg \Lambda_{QCD}}$

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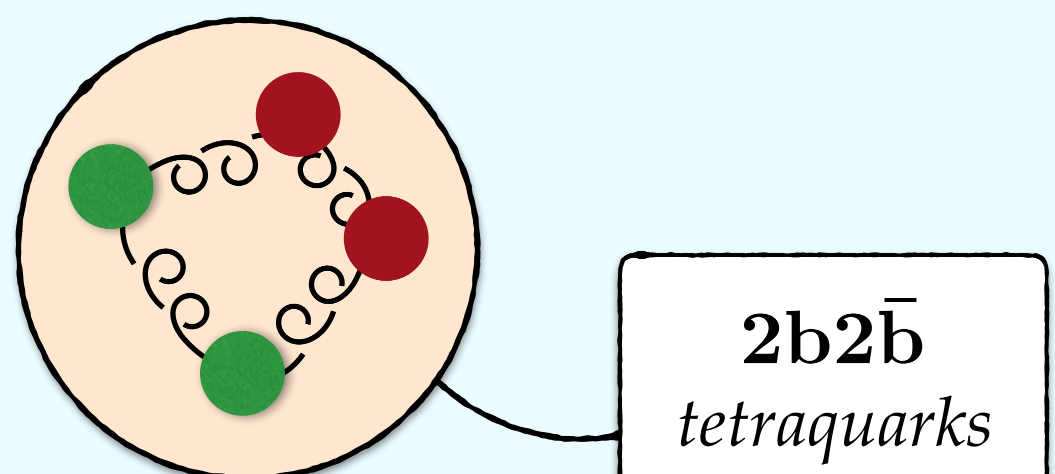

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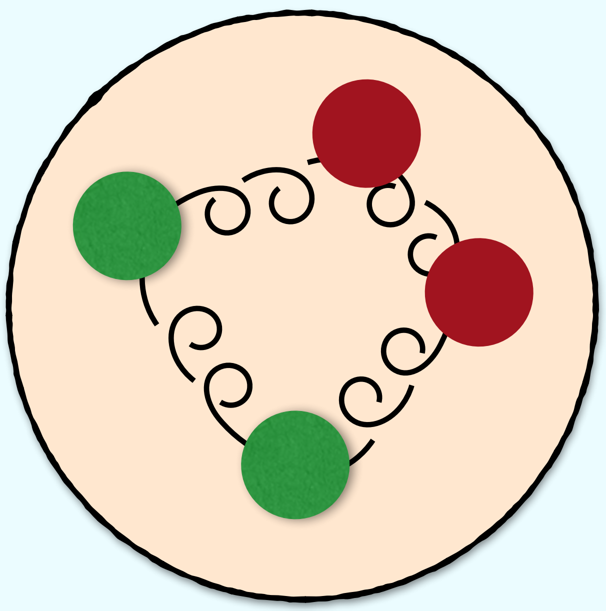
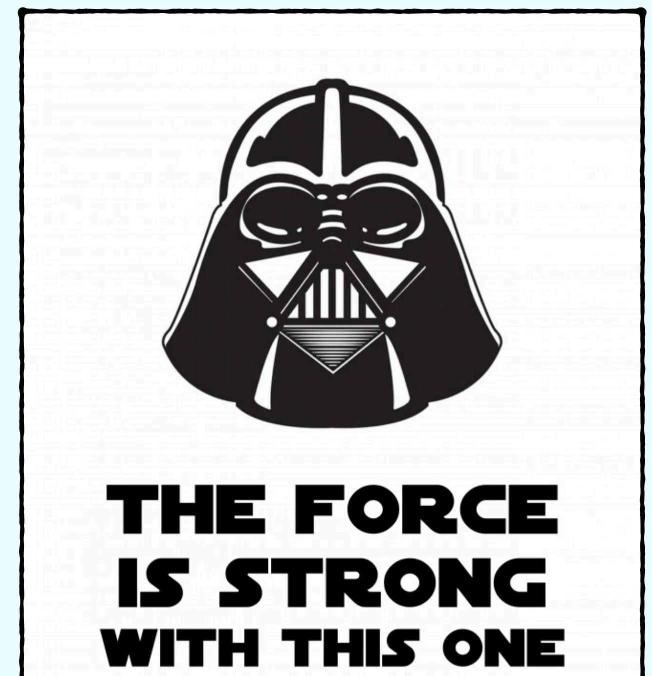
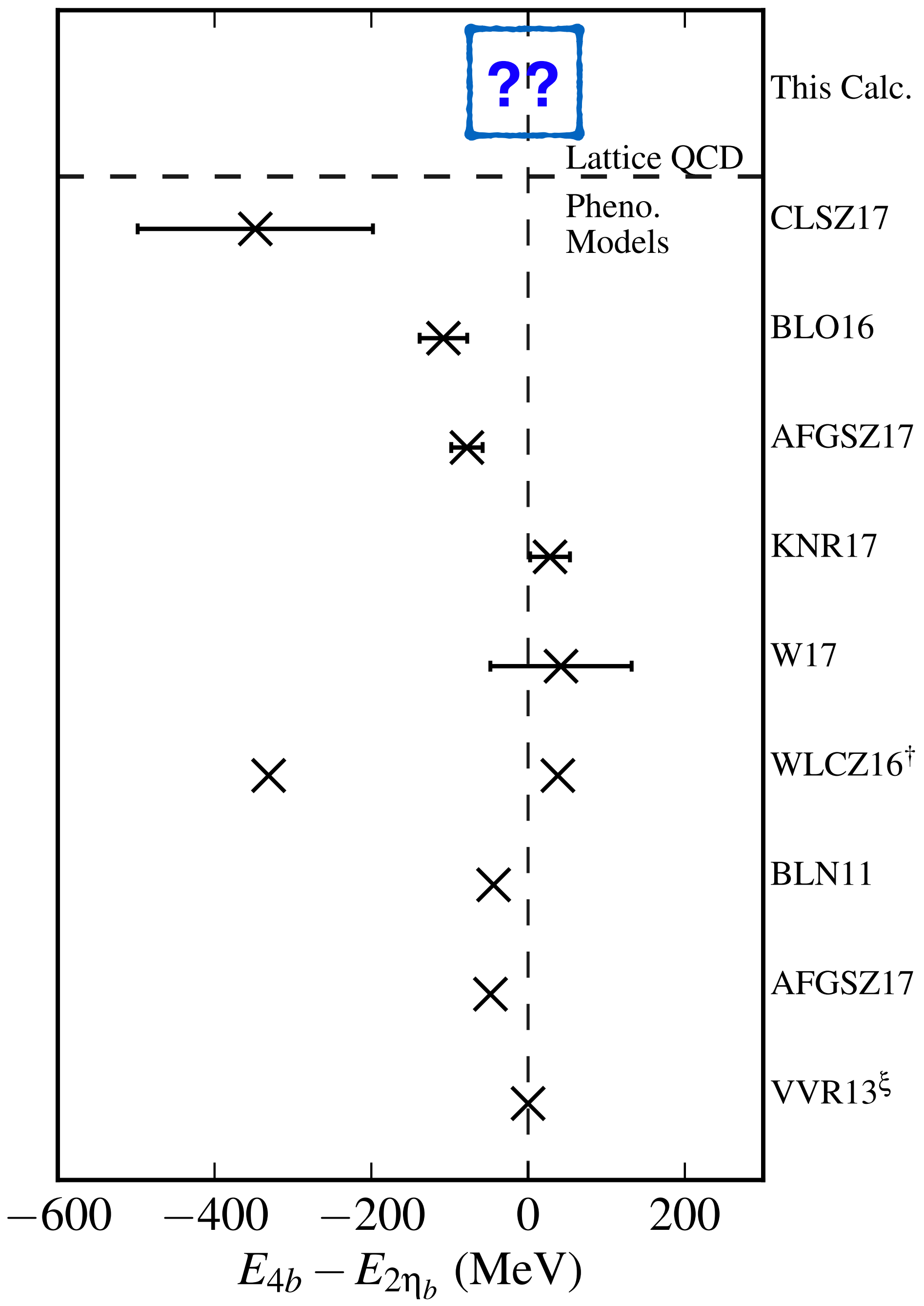


tetraquarks



But is it strong enough to bind $2b2\bar{b}$ into a stable state below the $2\eta_b$ threshold.

Model Predictions for 0^{++} $\mathbf{2b2\bar{b}}$ tetraquark



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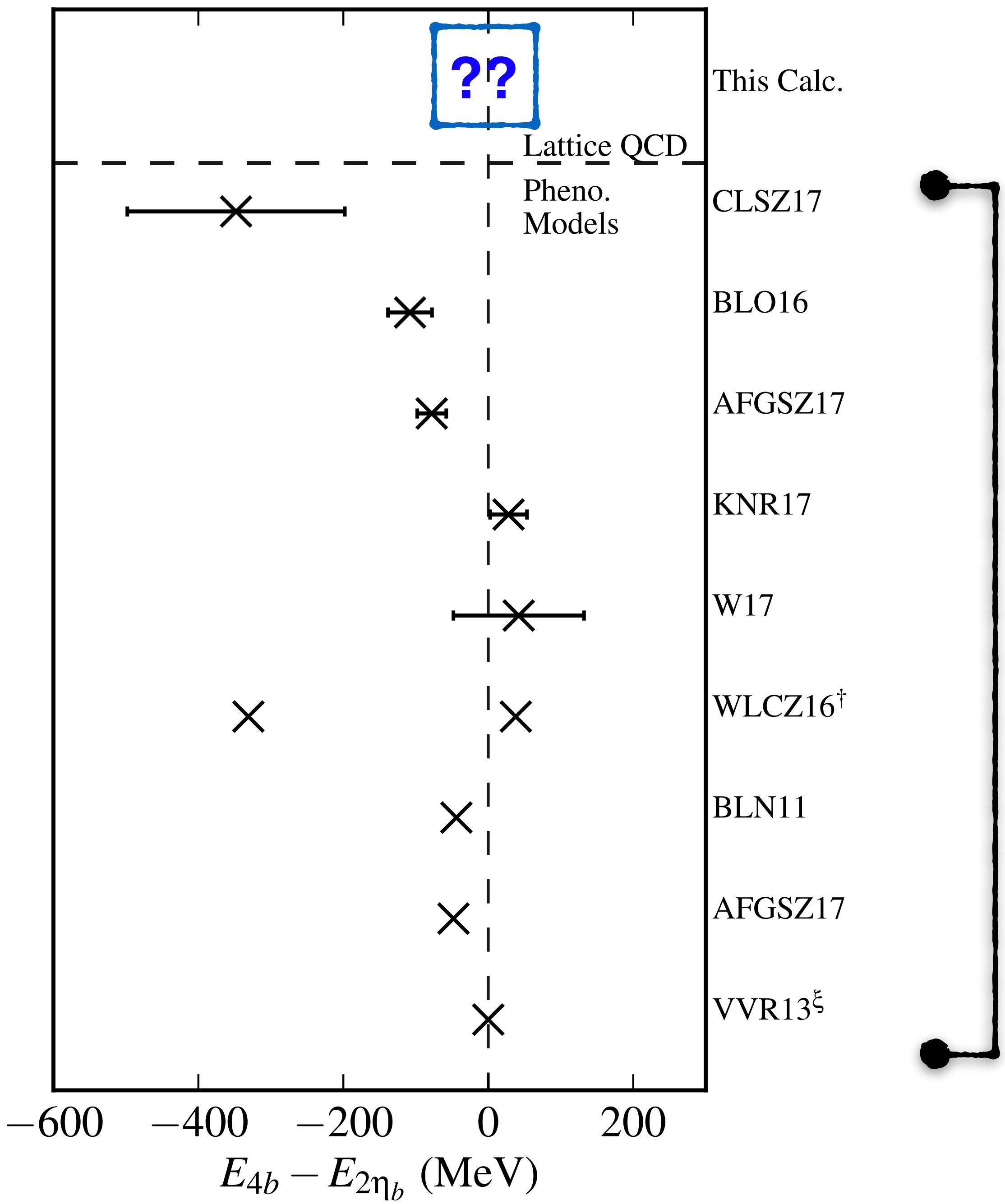

- Results Very Model Dependent!!
  - Not from first-principles
  - Inconclusive whether tetraquark bound or not?

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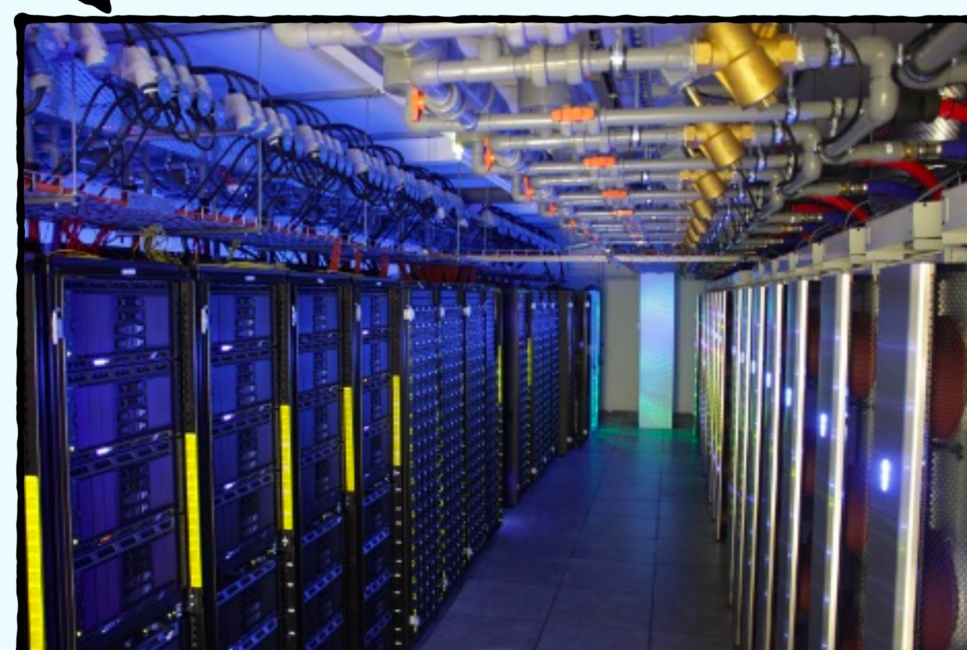

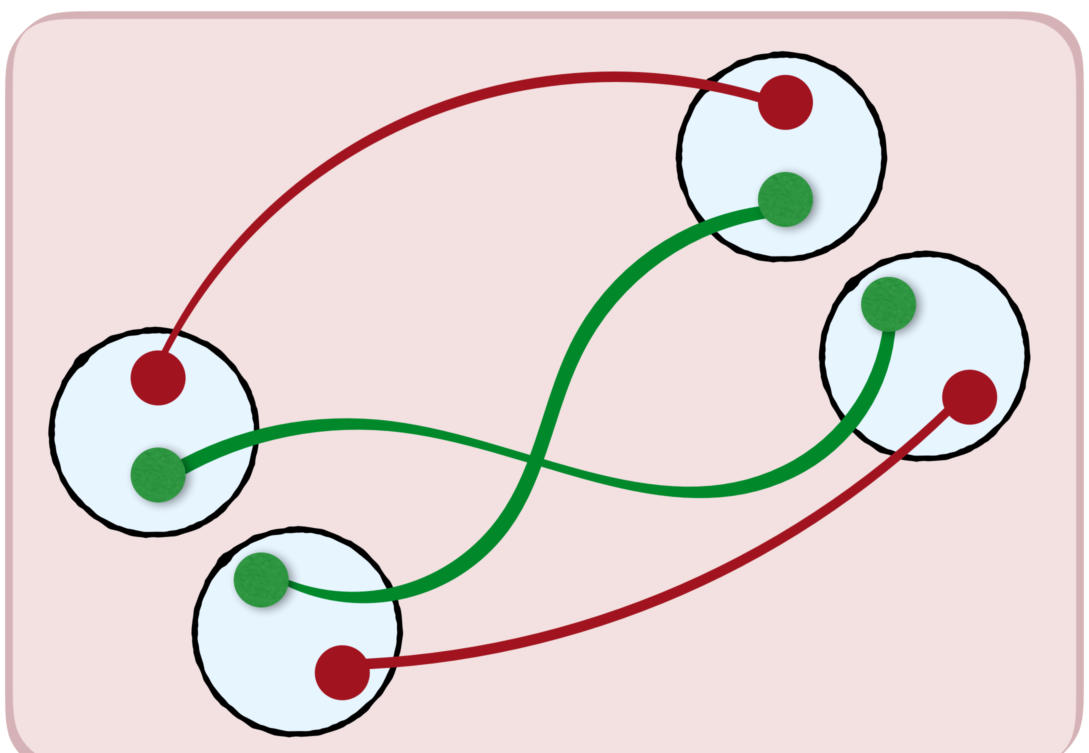

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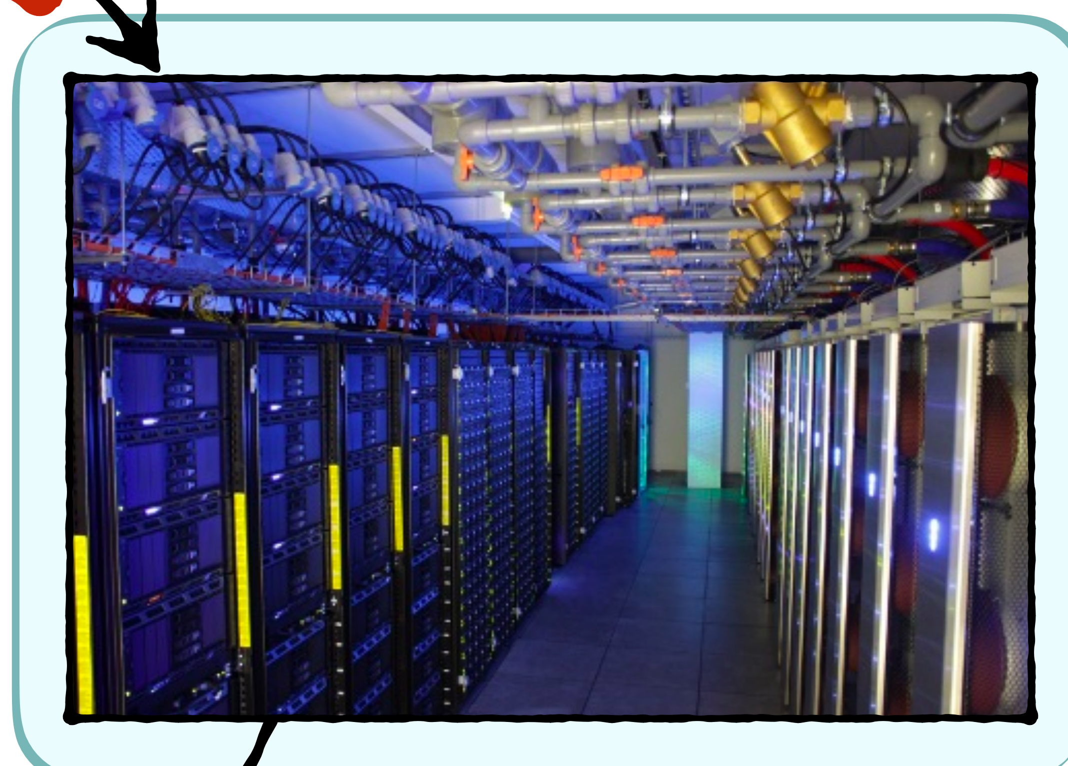

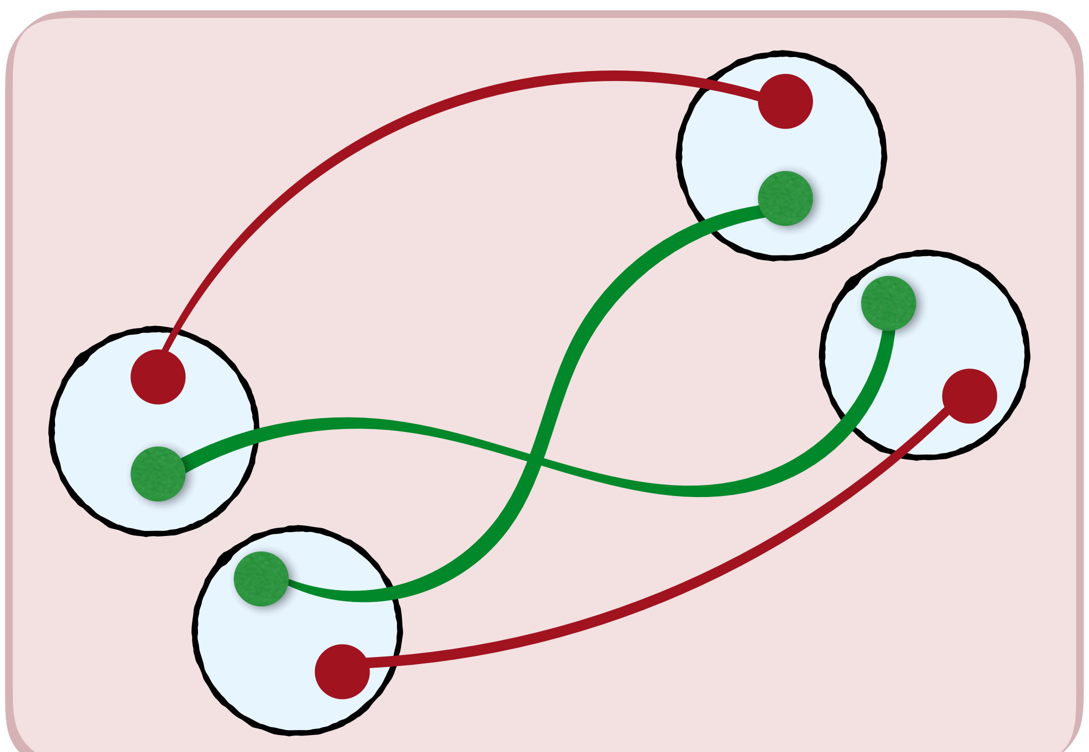

A (Bigger Picture) Lattice Spectrum Calculation: The Two-Point Correlator $C_{ab}^{\text{2pt.}}(t) = \langle 0|\mathcal{O}_b(y_4)\mathcal{O}_a^\dagger(x_4)|0\rangle$

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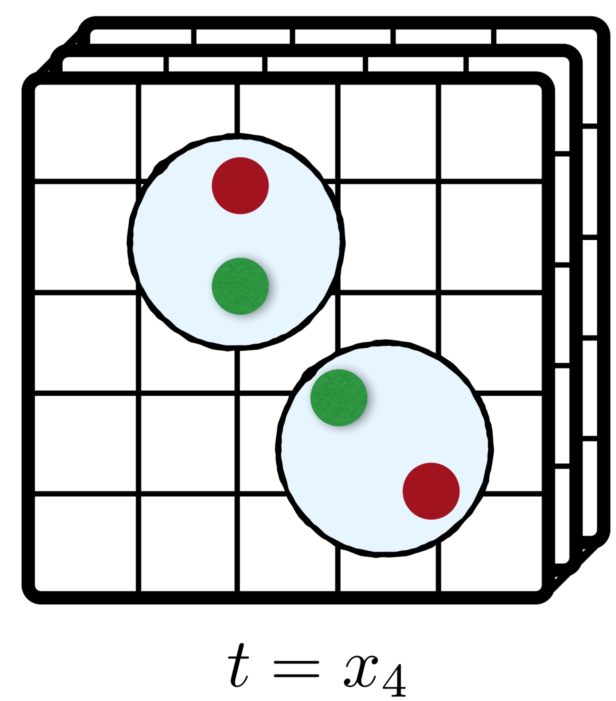

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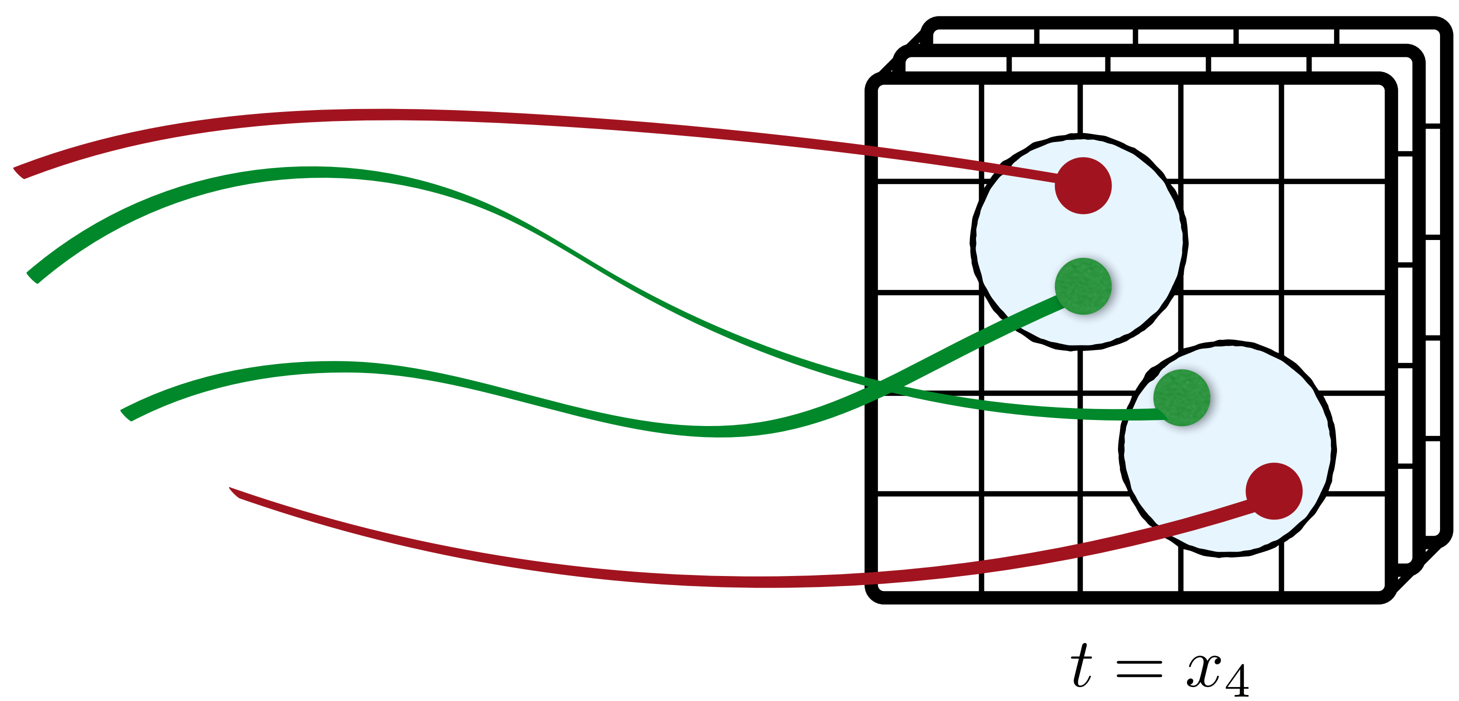

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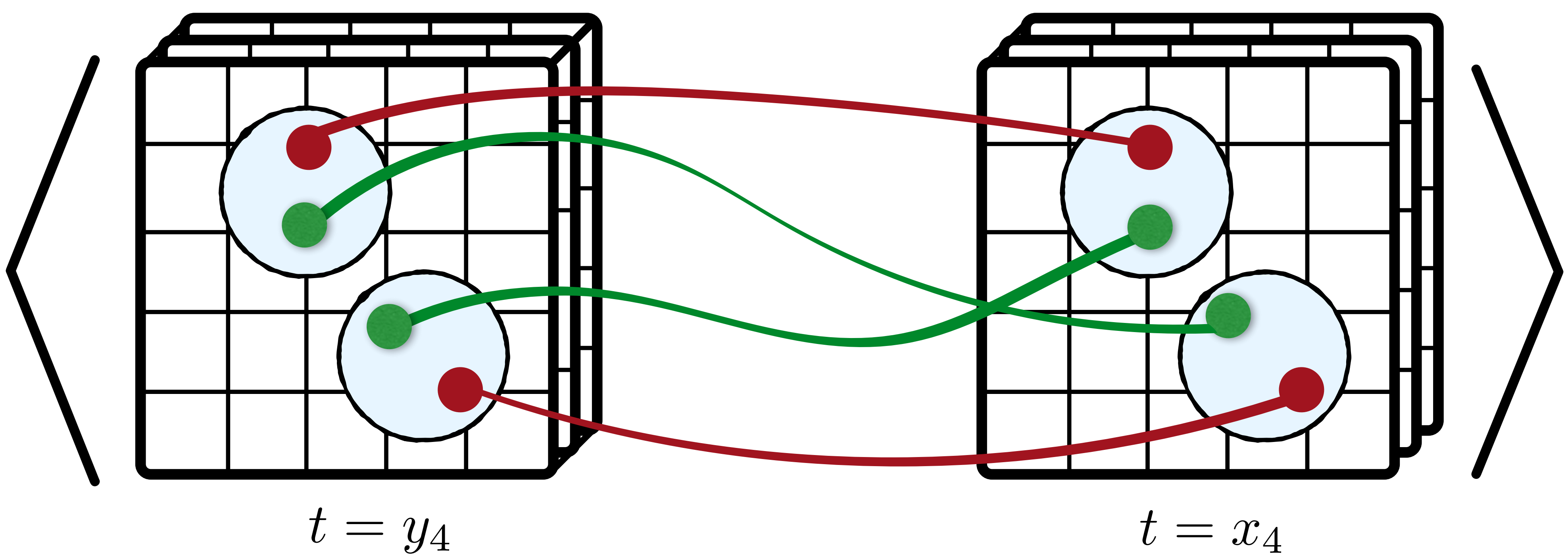

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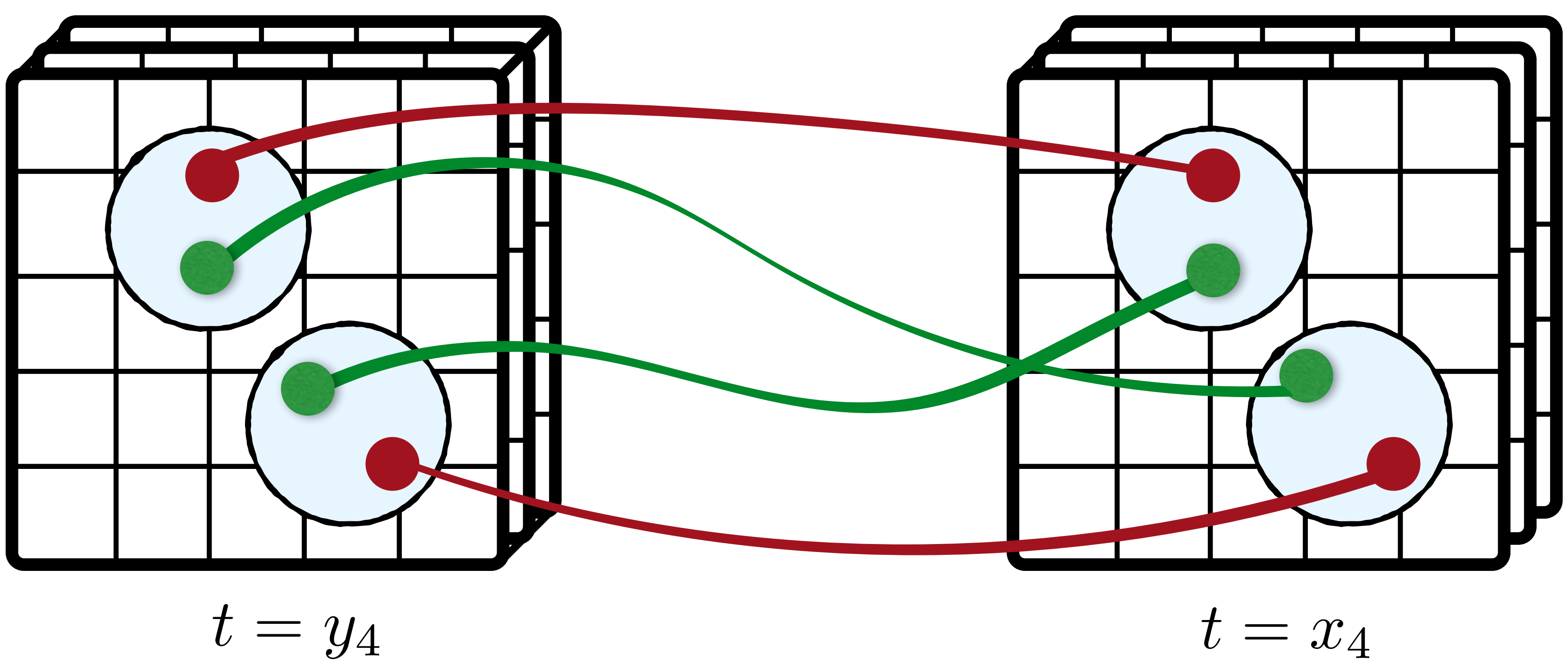
$$C_{ab}^{\text{2pt.}}(x_4 - y_4) = \Big\langle$$  $$\Big\rangle$$

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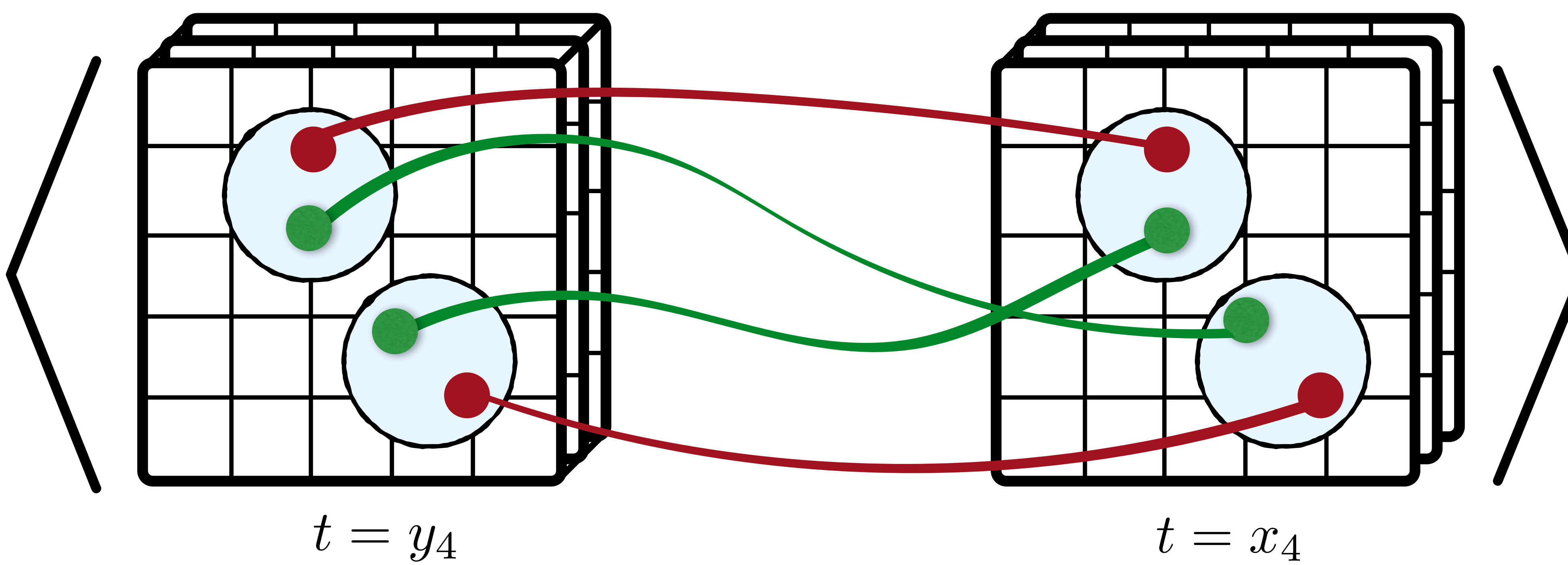
$$C^{2\text{pt.}}_{ab}(x_4 - y_4) =$$



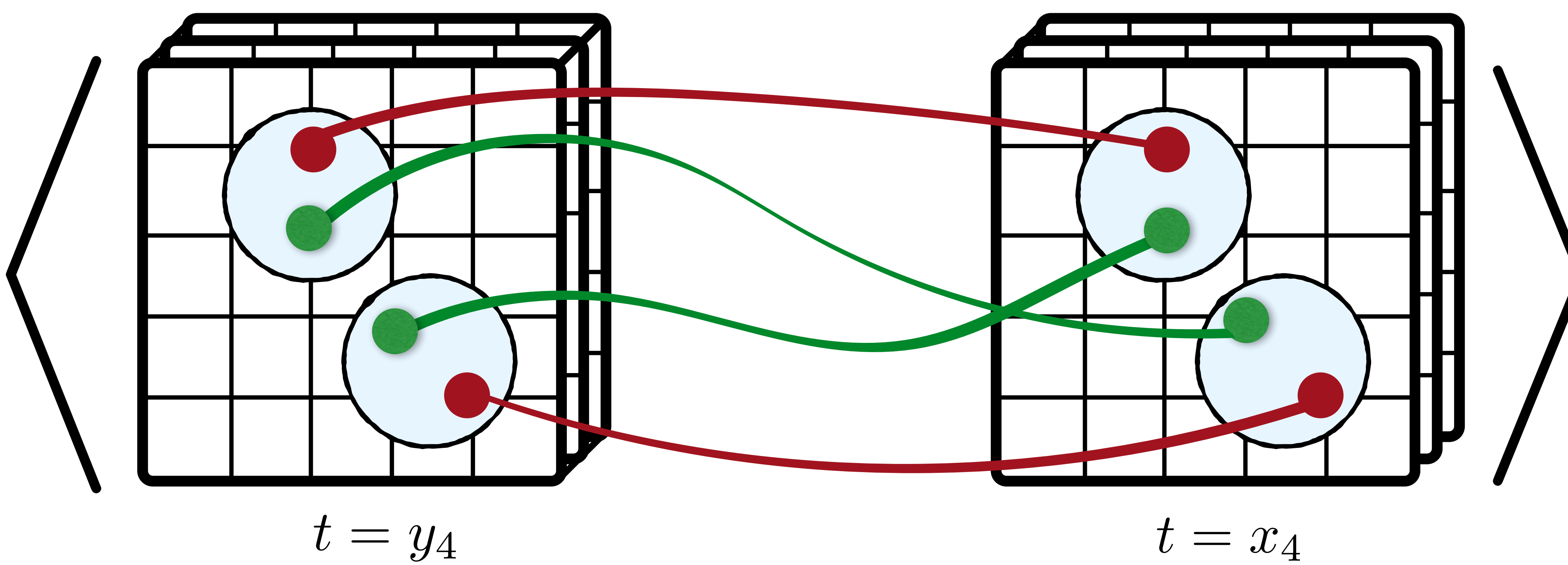
A (Bigger Picture) Lattice Spectrum Calculation: The Two-Point Correlator $C_{ab}^{2\text{pt.}}(t) = \langle 0|\mathcal{O}_b(y_4)\mathcal{O}_a^\dagger(x_4)|0\rangle$

- **Hilbert Space Formalism:**
  - Insert a complete set of QCD eigenstates

$$C_{ab}^{2pt.}(t, \mathbf{P}) \equiv \langle 0|\mathcal{O}_b(t, \mathbf{P})\mathcal{O}_a^\dagger(0, \mathbf{P})|0\rangle = \sum_n Z_{b,n} Z_{a,n}^\dagger e^{-E_n t}$$

Extract QCD Energy Eigenstates

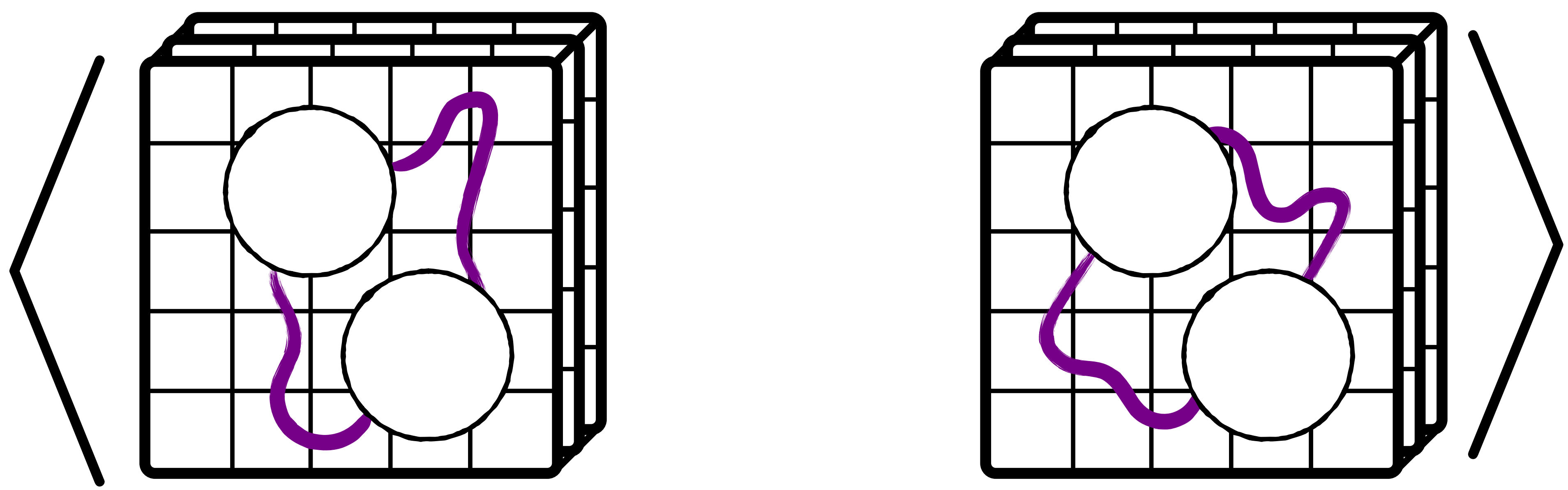
$$C_{ab}^{2\text{pt.}}(x_4 - y_4) =$$



Operators Used for 0^{++} $\mathbf{2b2\bar{b}}$ State

0^{++}	
source	sink
$\mathcal{O}^{A_1}_{(\eta_b,\eta_b)}$	

$$C^{2\text{pt.}}_{ab}(x_4 - y_4) =$$



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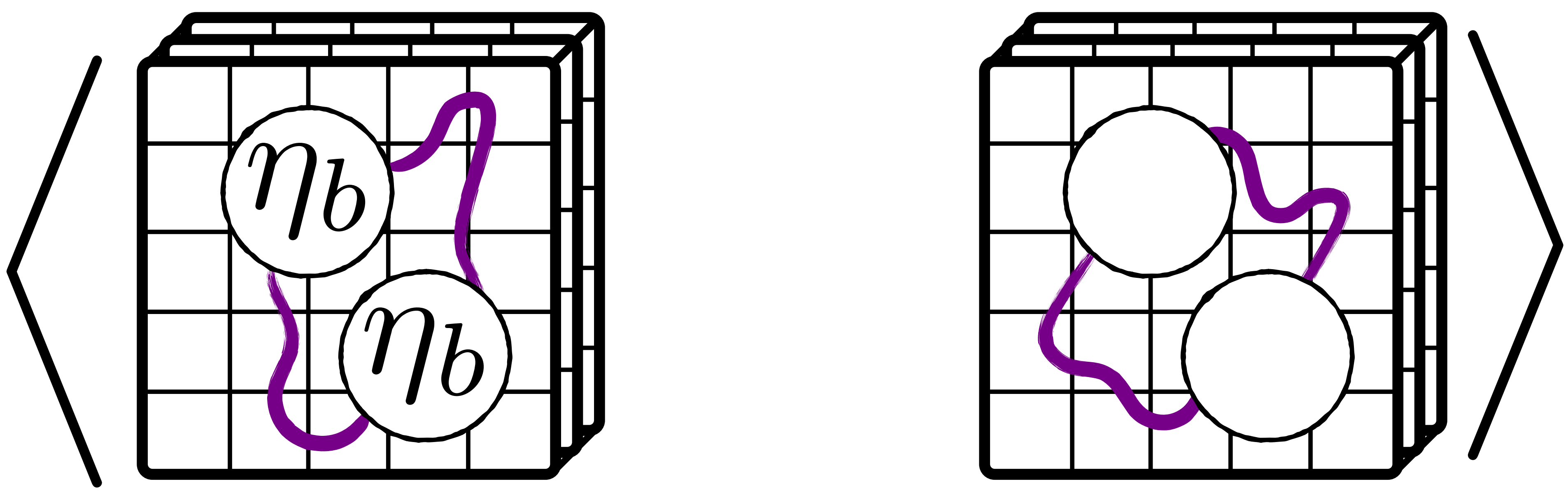

Operators Used for 0^{++} $\mathbf{2b2\bar{b}}$ State

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source	sink
$\mathcal{O}^{A_1}_{(\eta_b,\eta_b)}$	$\mathcal{O}^{A_1}_{(\eta_b,\eta_b)}$

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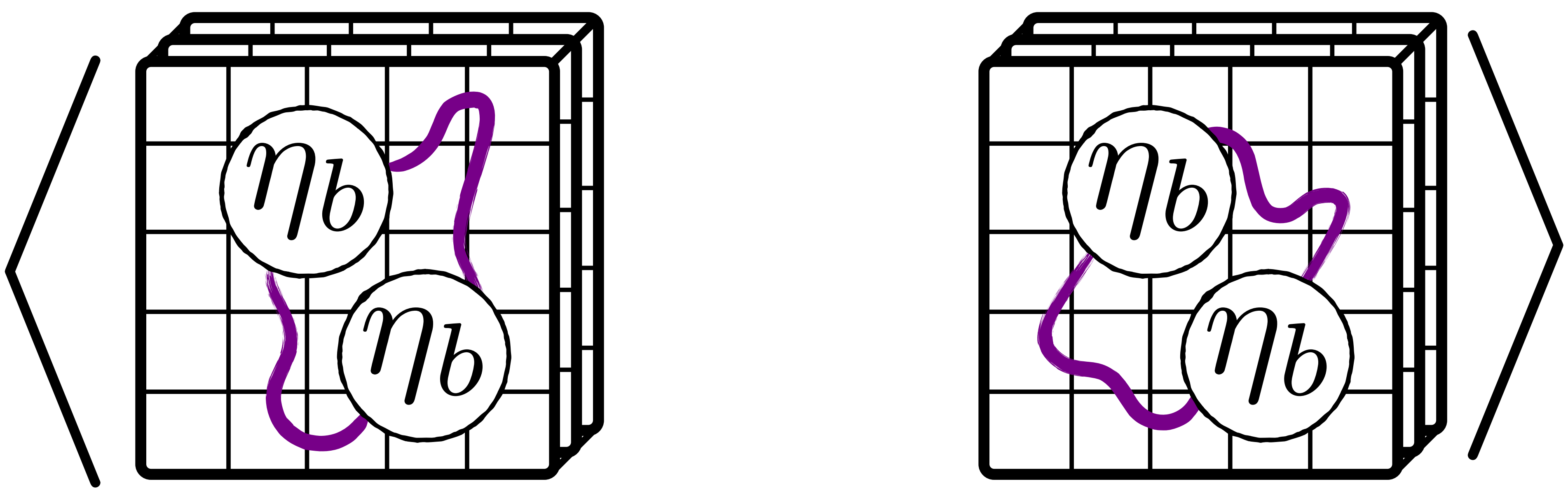
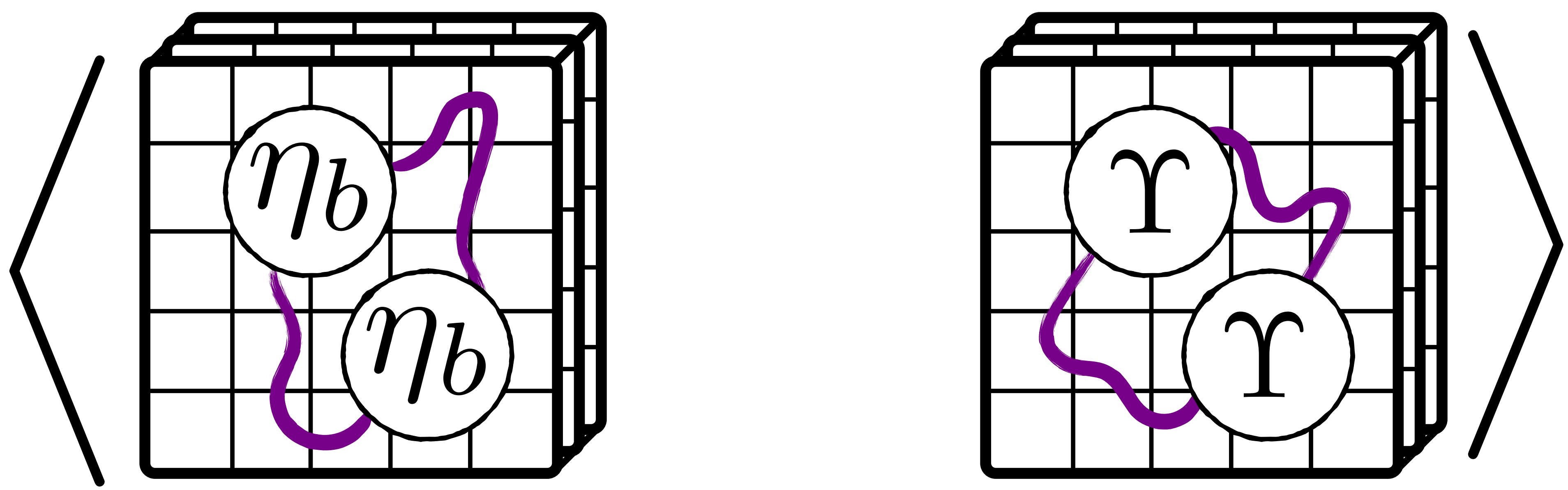


Operators Used for 0^{++} $\mathbf{2b2\bar{b}}$ State

0^{++}	
source	sink
$\mathcal{O}^{A_1}_{(\eta_b,\eta_b)}$	$\mathcal{O}^{A_1}_{(\eta_b,\eta_b)}$
$\mathcal{O}^{A_1}_{(\eta_b,\eta_b)}$	$\mathcal{O}^{A_1}_{(\Upsilon,\Upsilon)}$

$$C^{2\text{pt.}}_{ab}(x_4 - y_4) =$$



Operators Used for 0^{++} $\mathbf{2b2\bar{b}}$ State

0^{++}	
source	sink
$\mathcal{O}^{A_1}_{(\eta_b,\eta_b)}$	$\mathcal{O}^{A_1}_{(\eta_b,\eta_b)}$
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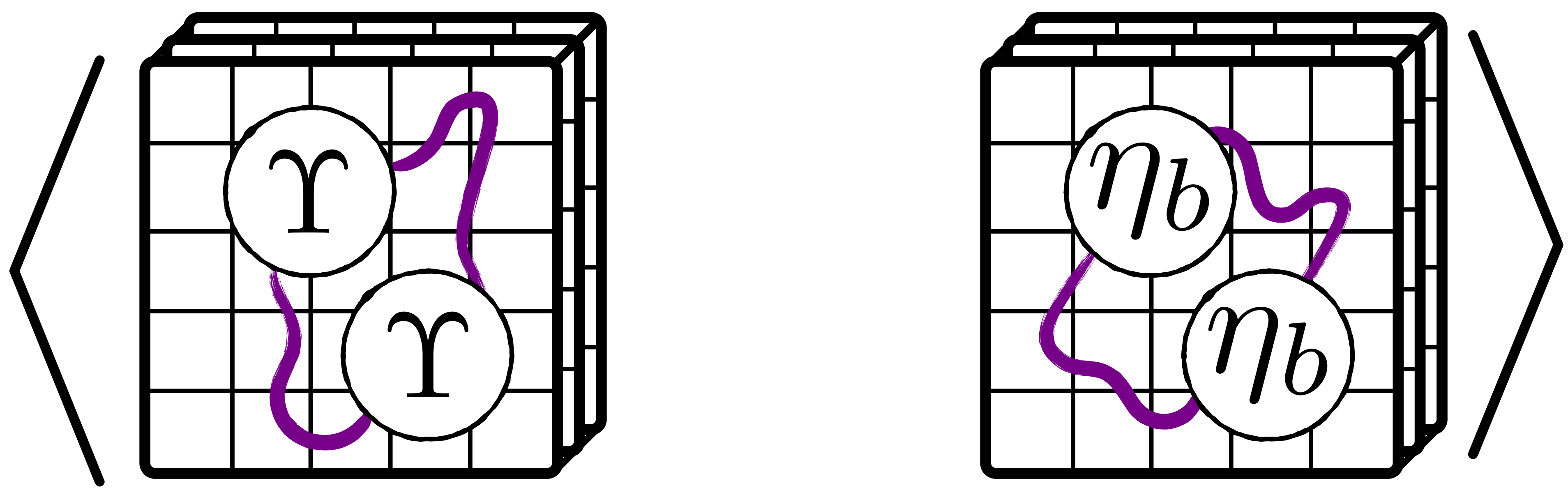

Operators Used for 0^{++} $\mathbf{2b2\bar{b}}$ State

0^{++}	
source	sink
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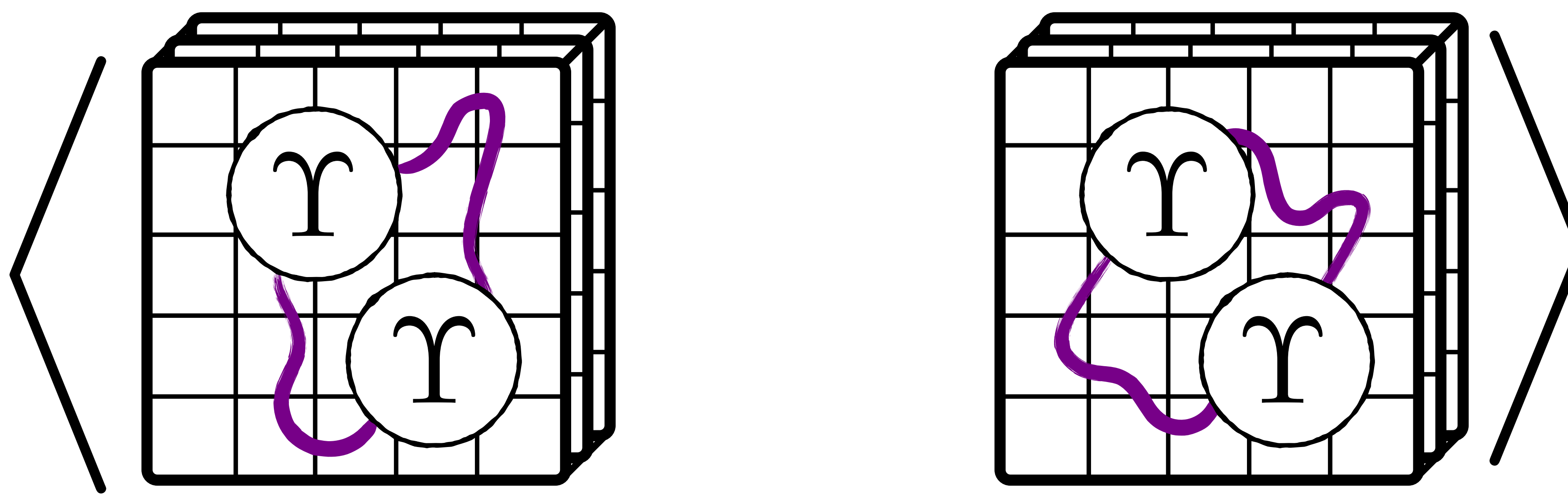

Operators Used for 0^{++} $\mathbf{2b2\bar{b}}$ State

0^{++}	
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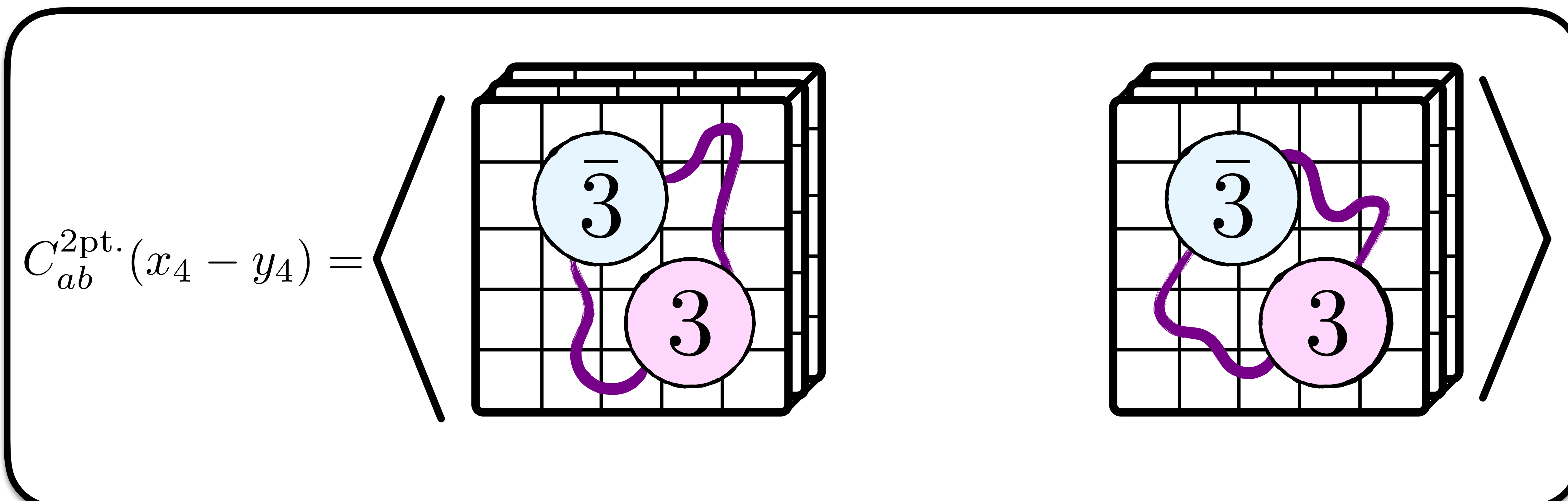
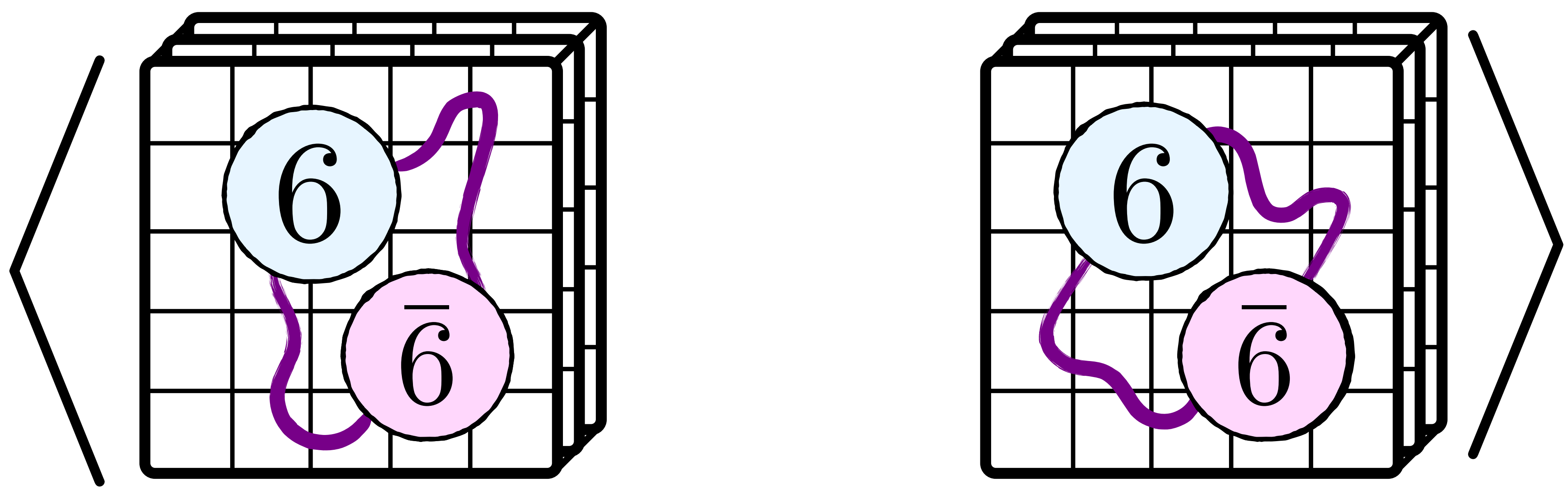
$\mathcal{O}^{A_1}_{(\eta_b,\eta_b)}$	$\mathcal{O}^{A_1}_{(\eta_b,\eta_b)}$
$\mathcal{O}^{A_1}_{(\eta_b,\eta_b)}$	$\mathcal{O}^{A_1}_{(\Upsilon,\Upsilon)}$
$\mathcal{O}^{A_1}_{(\Upsilon,\Upsilon)}$	$\mathcal{O}^{A_1}_{(\eta_b,\eta_b)}$
$\mathcal{O}^{A_1}_{(\Upsilon,\Upsilon)}$	$\mathcal{O}^{A_1}_{(\Upsilon,\Upsilon)}$
$\mathcal{O}^{A_1}_{(D_{\bar{3}_c},A_{3_c})}$	$\mathcal{O}^{A_1}_{(D_{\bar{3}_c},A_{3_c})}$



Operators Used for 0^{++} $\mathbf{2b2\bar{b}}$ State

0^{++}	
source	sink
$\mathcal{O}^{A_1}_{(\eta_b,\eta_b)}$	$\mathcal{O}^{A_1}_{(\eta_b,\eta_b)}$
$\mathcal{O}^{A_1}_{(\eta_b,\eta_b)}$	$\mathcal{O}^{A_1}_{(\Upsilon,\Upsilon)}$
$\mathcal{O}^{A_1}_{(\Upsilon,\Upsilon)}$	$\mathcal{O}^{A_1}_{(\eta_b,\eta_b)}$
$\mathcal{O}^{A_1}_{(\Upsilon,\Upsilon)}$	$\mathcal{O}^{A_1}_{(\Upsilon,\Upsilon)}$
$\mathcal{O}^{A_1}_{(D_{\bar{3}_c},A_{3_c})}$	$\mathcal{O}^{A_1}_{(D_{\bar{3}_c},A_{3_c})}$
$\mathcal{O}^{A_1}_{(D_{6_c},A_{\bar{6}_c})}$	$\mathcal{O}^{A_1}_{(D_{6_c},A_{\bar{6}_c})}$

$C^{2\mathrm{pt.}}_{ab}(x_4 - y_4) =$



We perform a Bayesian fit to all the data within a certain channel

- We perform a Bayesian fit to all the data within a certain channel
- But you want to see the actual data! What can we easily show?

The Lattice Effective Mass

$$aE^{\text{eff}} = \log\left(\frac{C(t)}{C(t+1)}\right)$$

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$$aE^{\text{eff}} = \log\left(\frac{C(t)}{C(t+1)}\right)$$

$$= aE_0 + \frac{Z_1^2}{Z_0^2} e^{-(E_1 - E_0)t}(1 - e^{-(E_1 - E_0)}) + \ldots$$

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Excited State Contributions Exponentially decay away with time

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$$= aE_0 + \frac{Z_1^2}{Z_0^2} e^{-(E_1-E_0)t}(1 - e^{-(E_1-E_0)}) + \dots \xrightarrow{t\to\infty} aE_0$$

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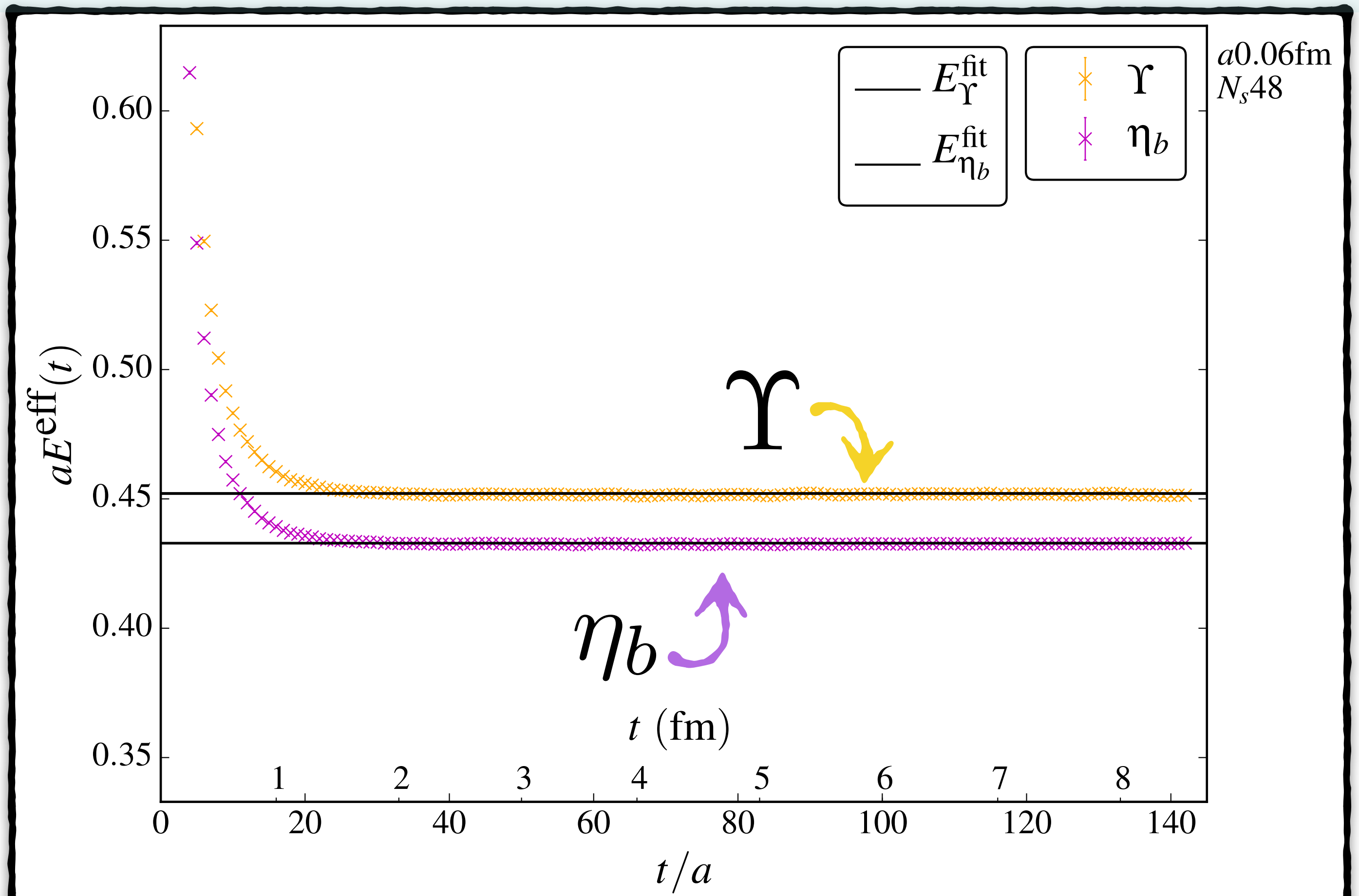
Excited State Contributions Exponentially decay away with time



Fake 0^{++} data on the $a \approx 0.06$ fm ensemble

"What might you expect to see?"

Fake 0^{++} data on the $a \approx 0.06$ fm ensemble

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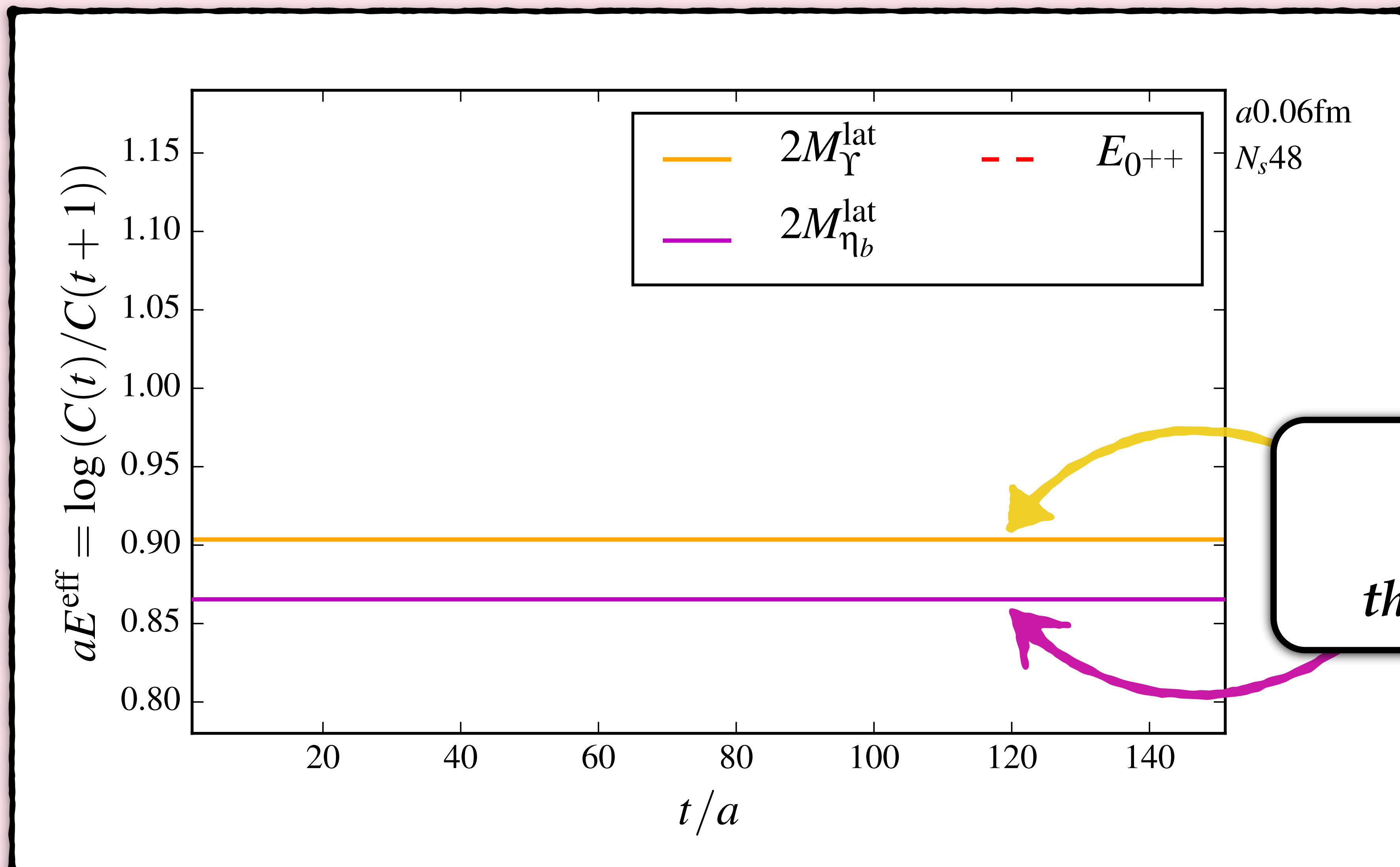
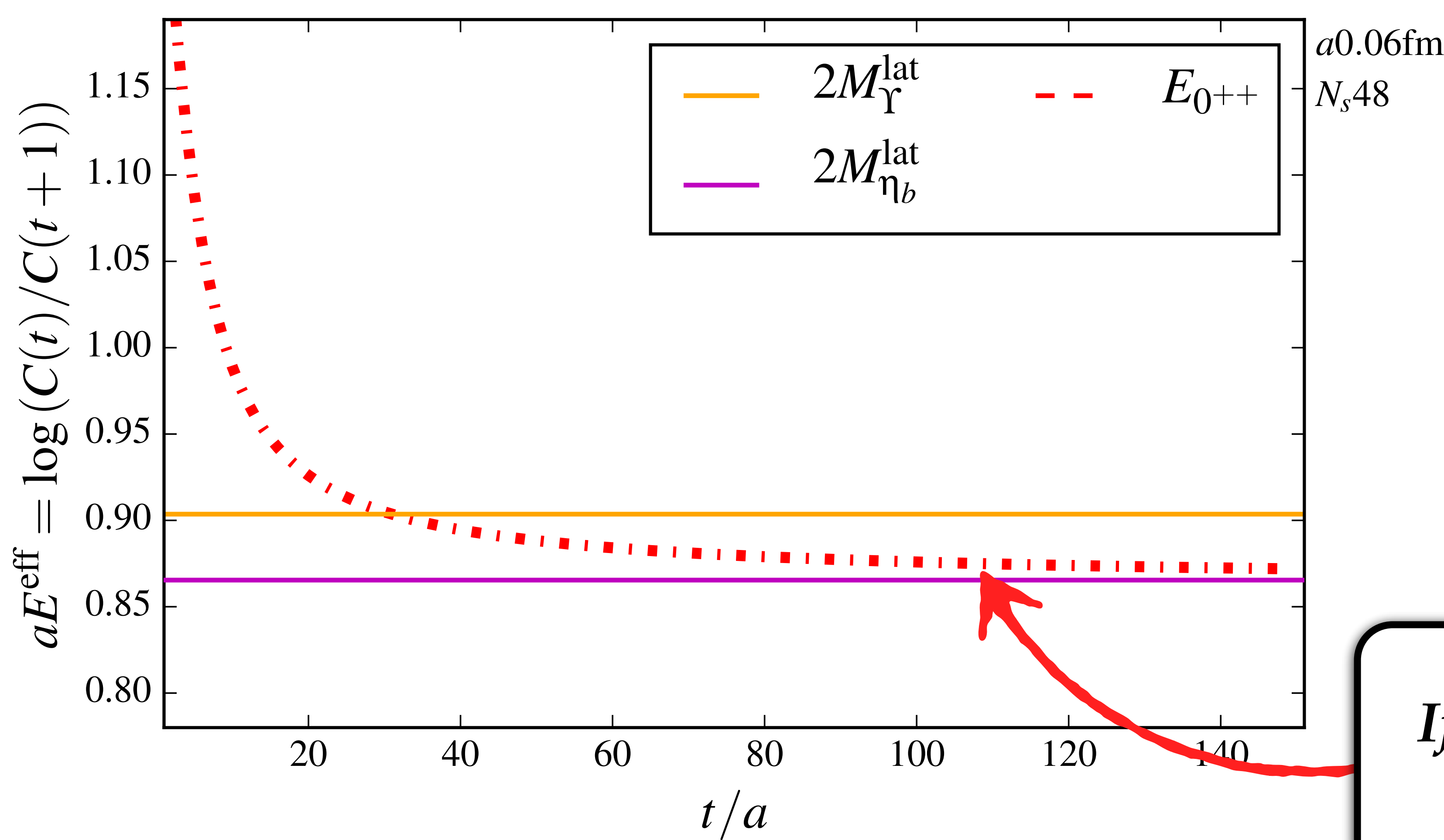


The non-interacting $2\eta_b$ and 2Υ thresholds for reference

Fake 0^{++} data on the $a \approx 0.06$ fm ensemble

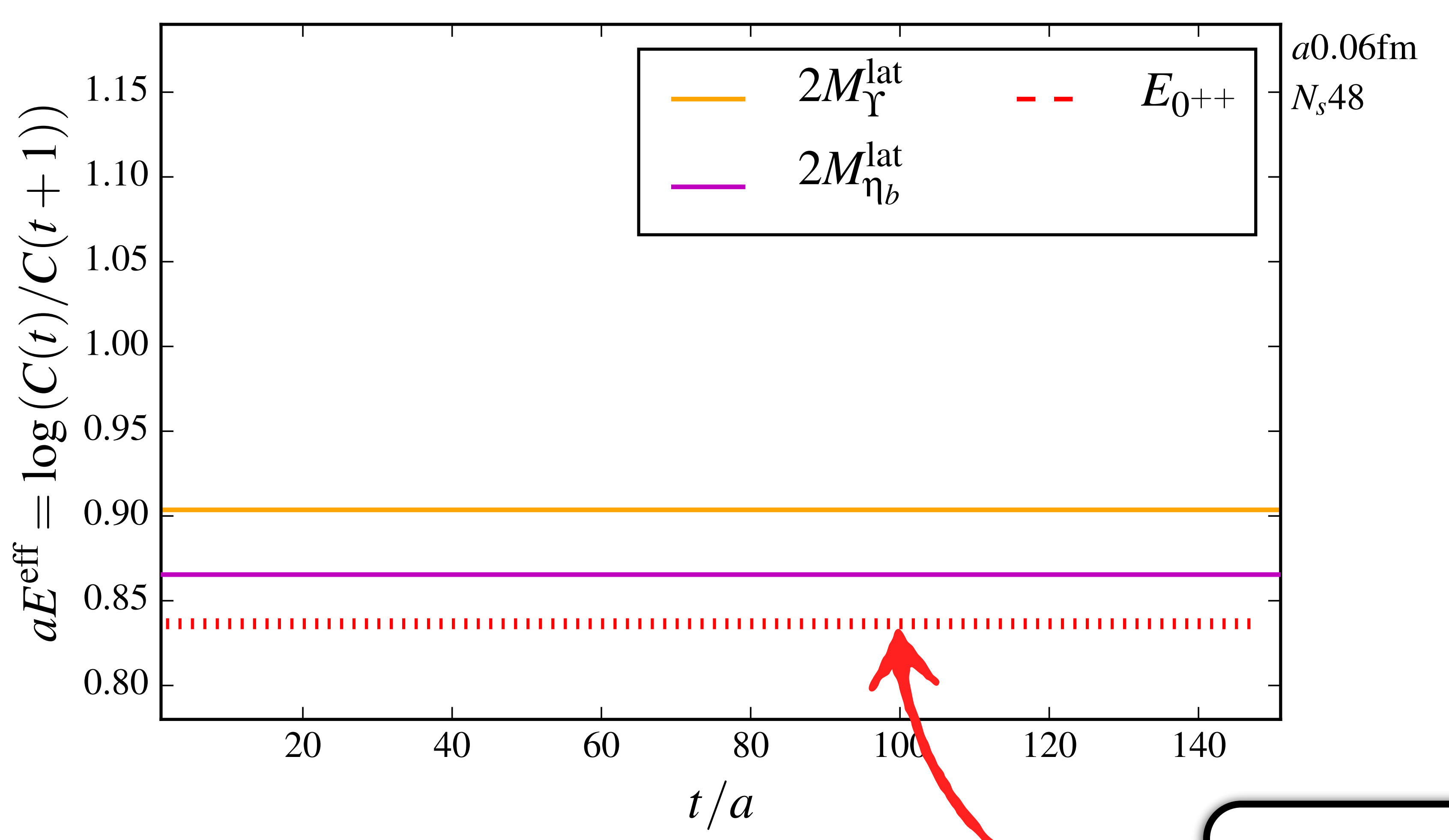
"What might you expect to see?"



If there was no new stable tetraquark

Fake 0^{++} data on the $a \approx 0.06$ fm ensemble

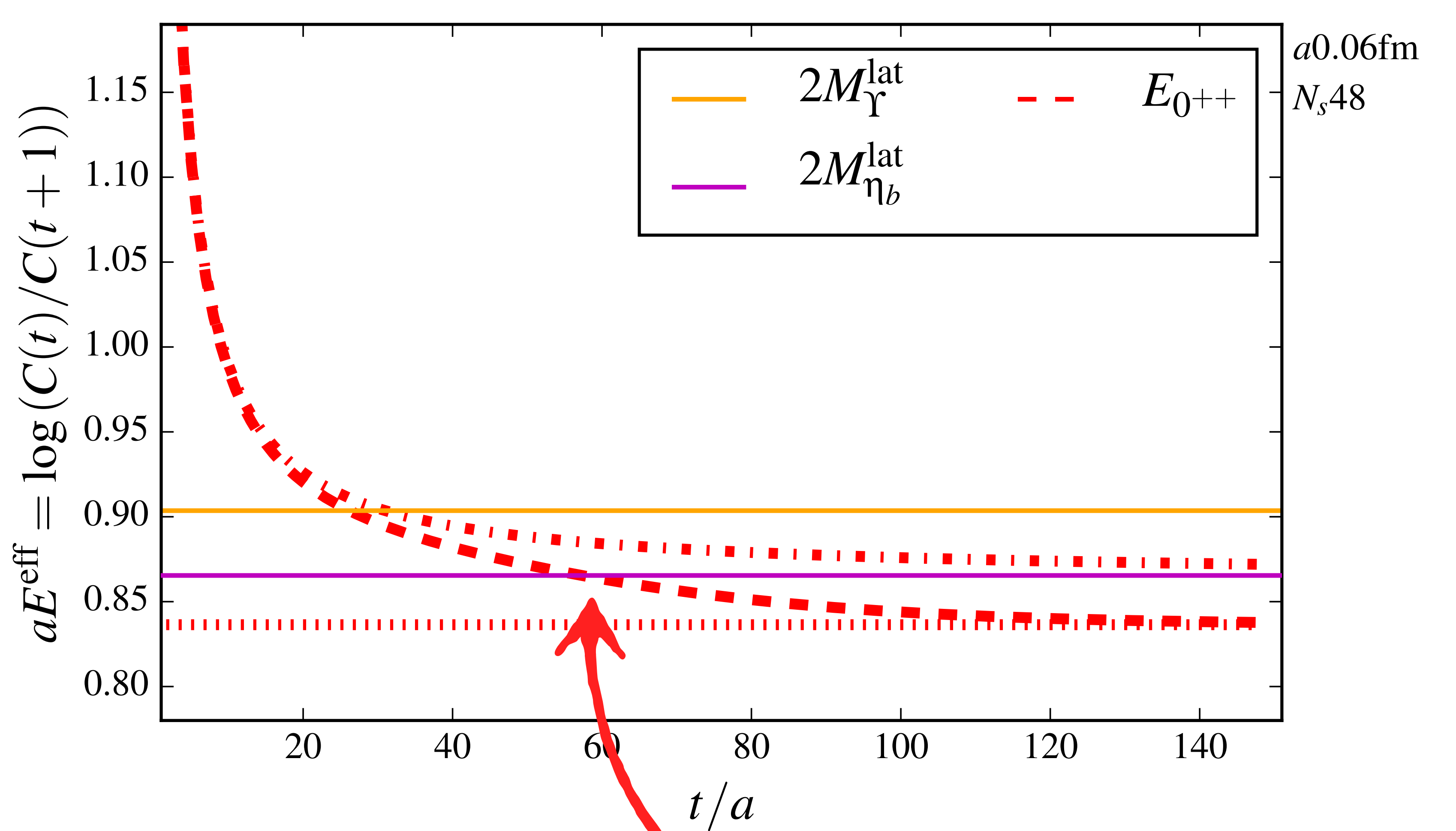
"What might you expect to see?"



If there was <u>ONLY</u> a new state 100 MeV below threshold

Fake 0^{++} data on the $a \approx 0.06$ fm ensemble

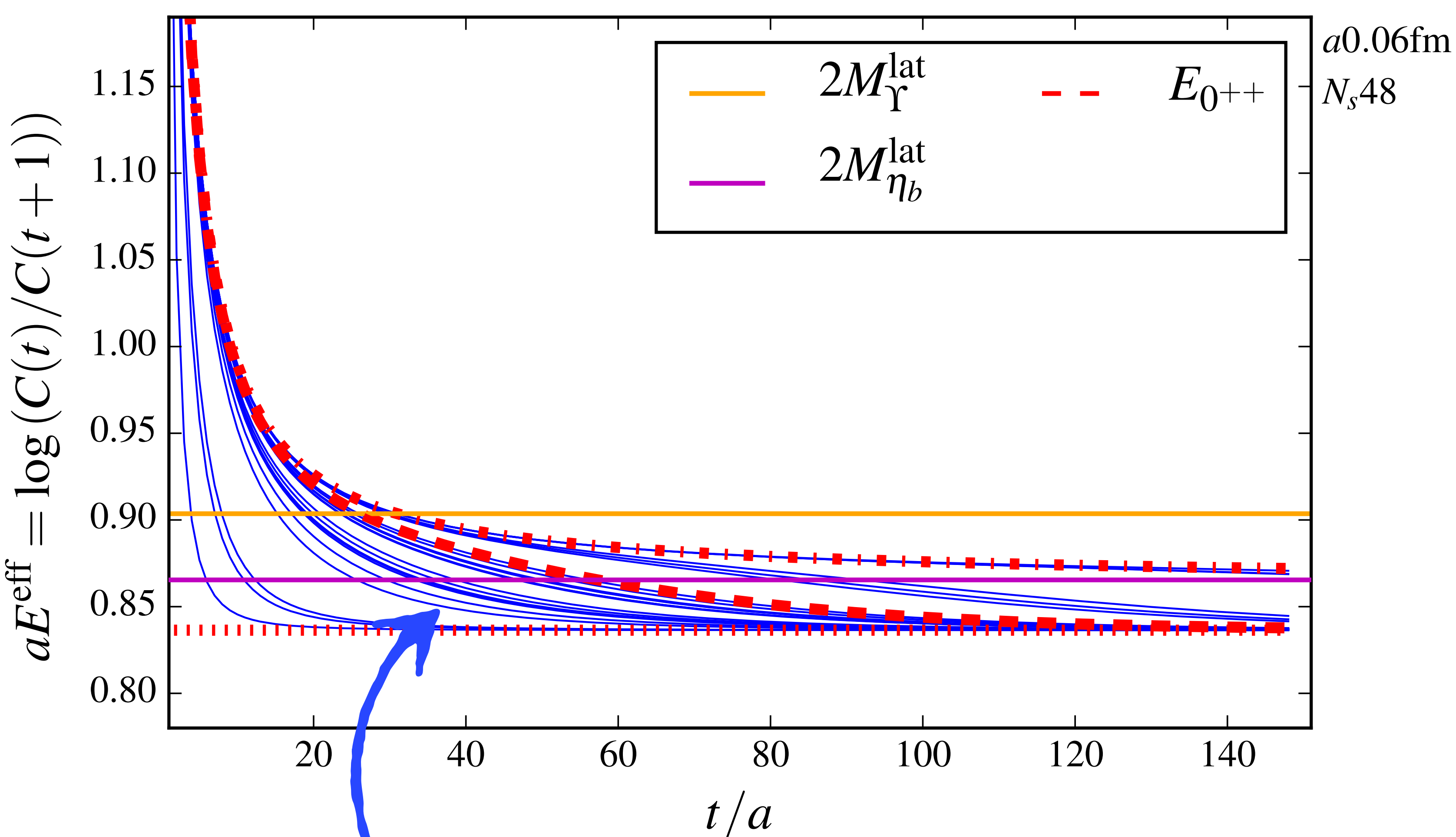
"What might you expect to see?"



If the new state was 100 MeV below threshold and had $Z_0 = Z_1$

Fake 0^{++} data on the $a \approx 0.06$ fm ensemble

"What might you expect to see?"



If the new state was 100 MeV below threshold and had different Z_0, Z_1

Fake 0^{++} data on the $a \approx 0.06$ fm ensemble

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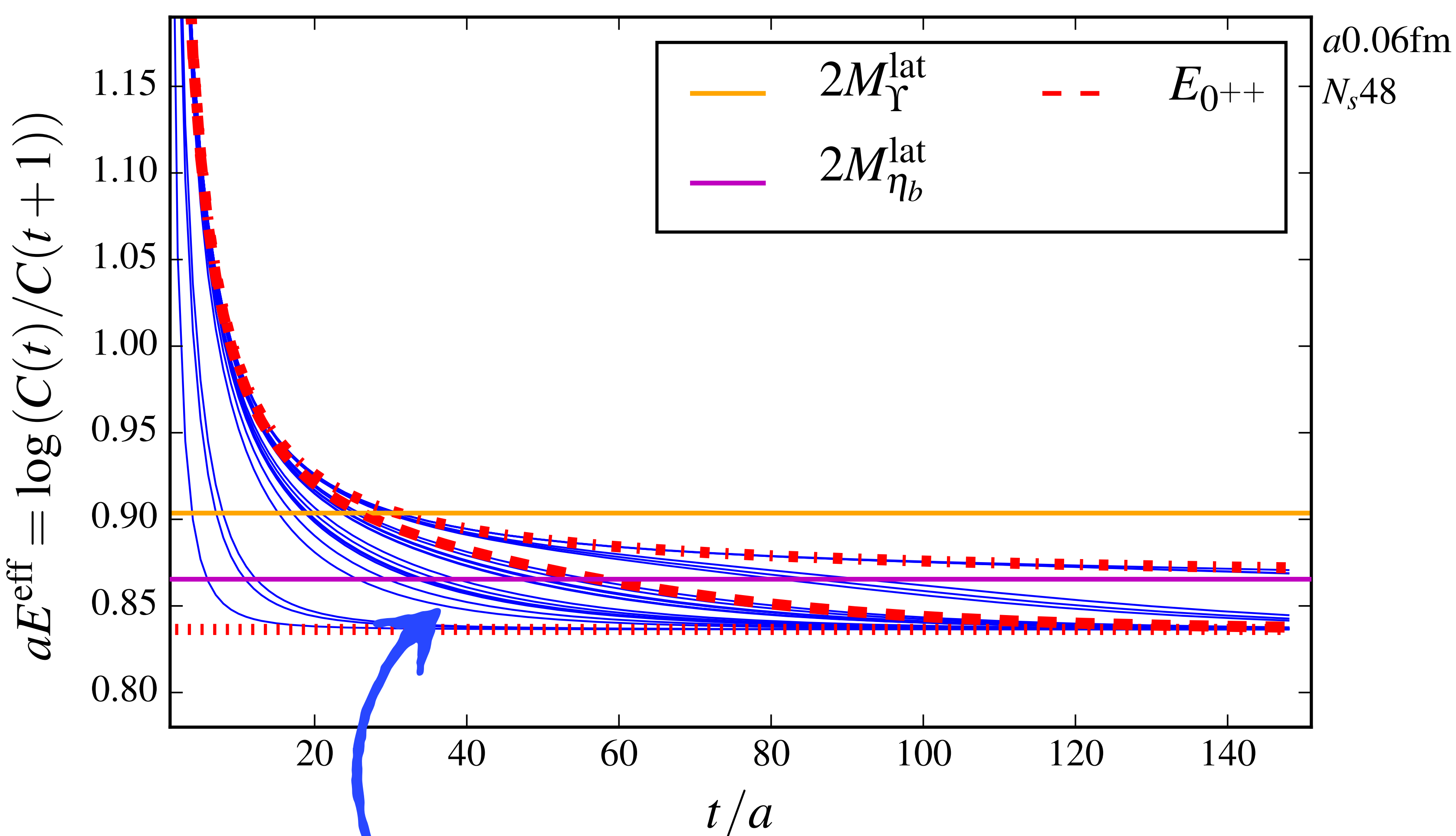
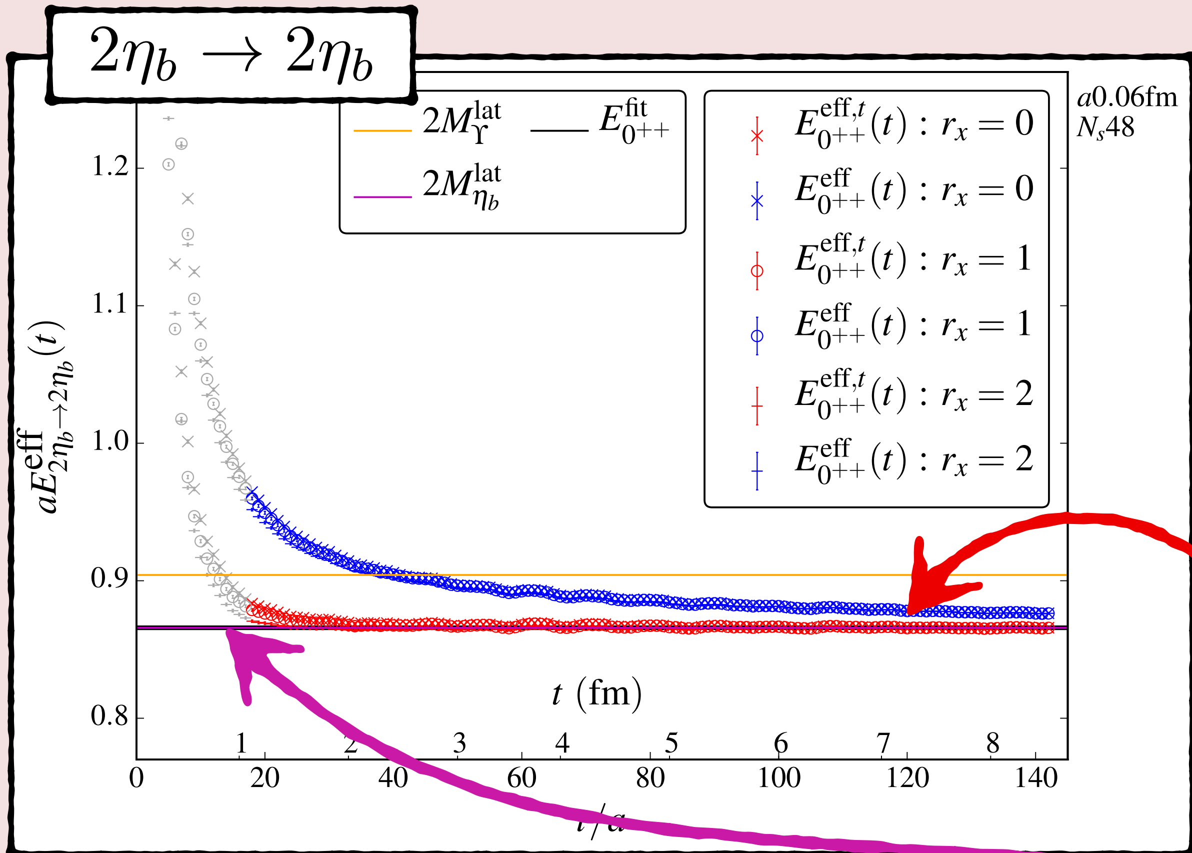


If the new state was 100 MeV below threshold and had different Z_0, Z_1

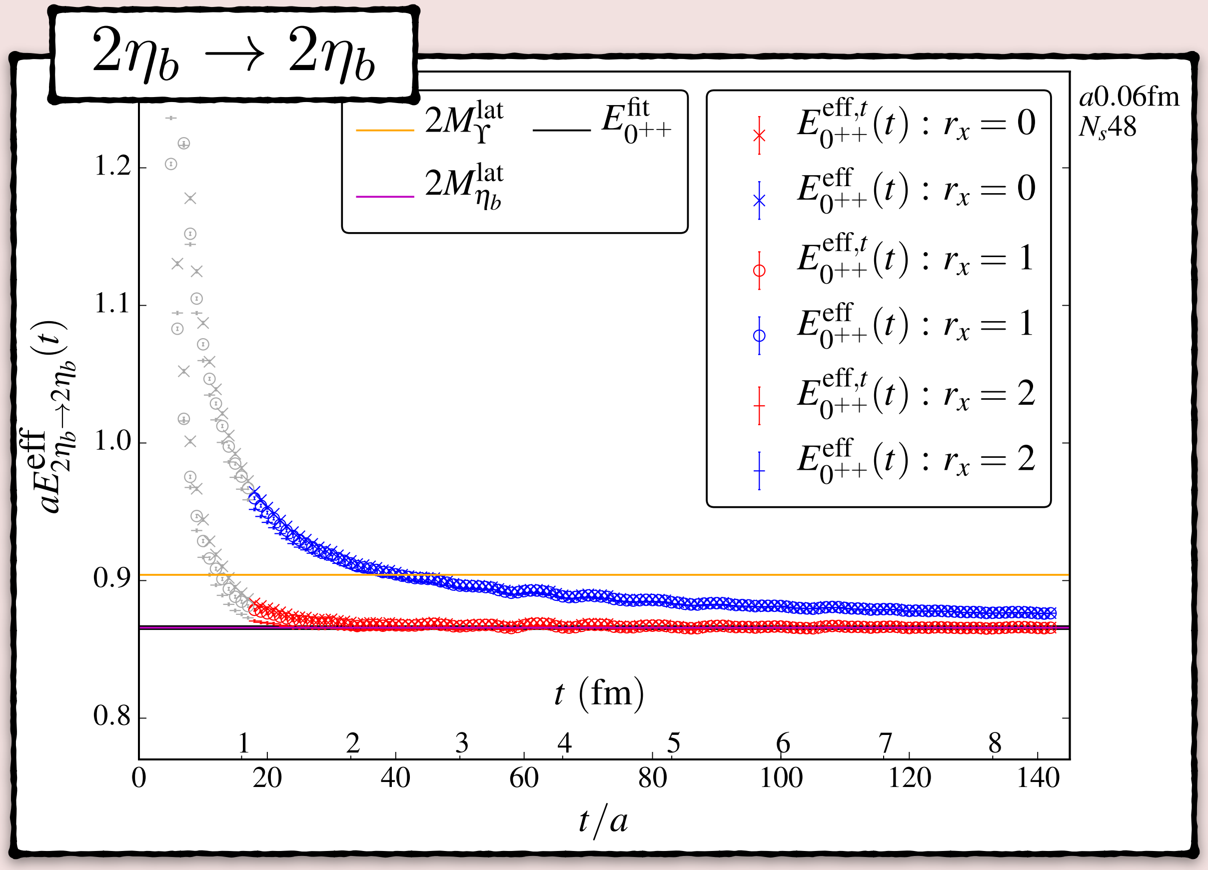
If tetraquark exists we should see a fall below threshold and a clean signal as in blue curve!

The 0^{++} data on the $a \approx 0.06$ fm ensemble



- For this talk, focus on red data points
- Then compare to $2\eta_b$ non-interacting threshold to determine binding

The 0^{++} data on the $a \approx 0.06$ fm ensemble



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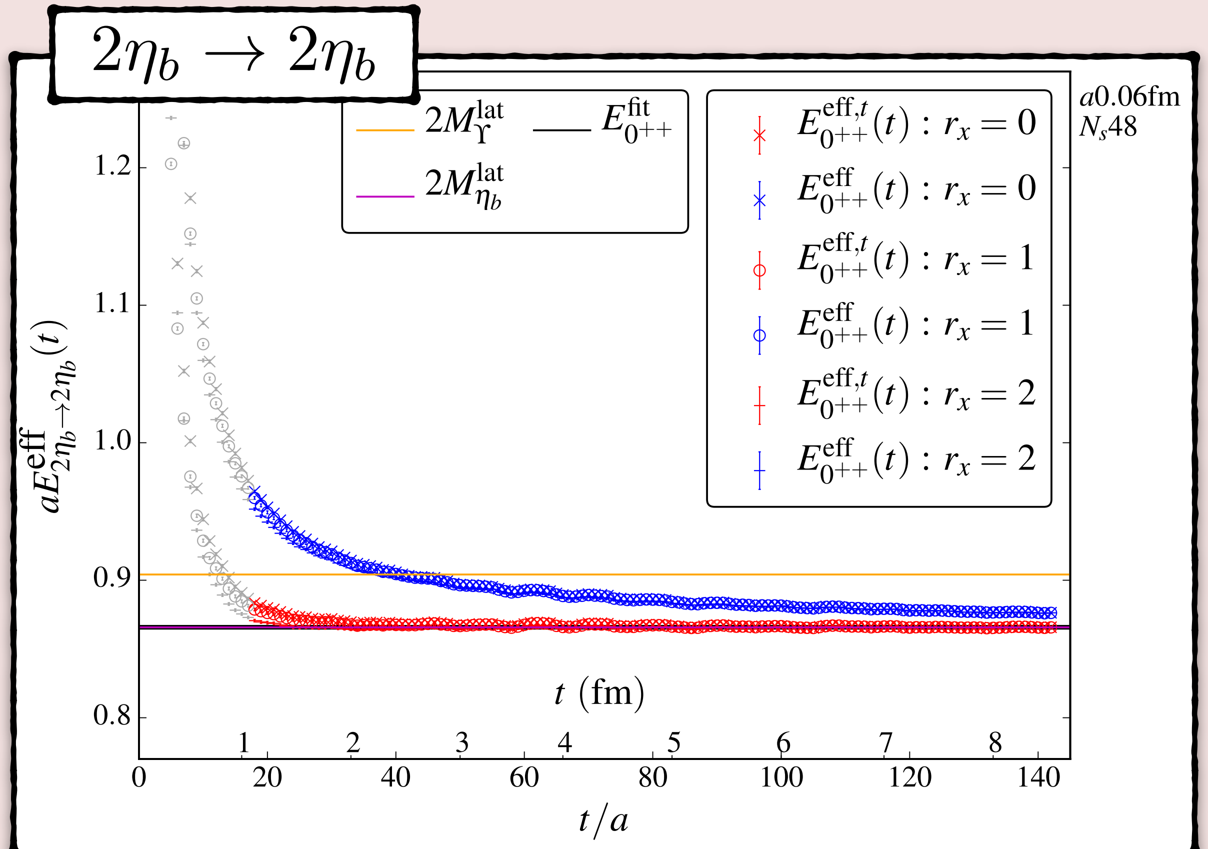

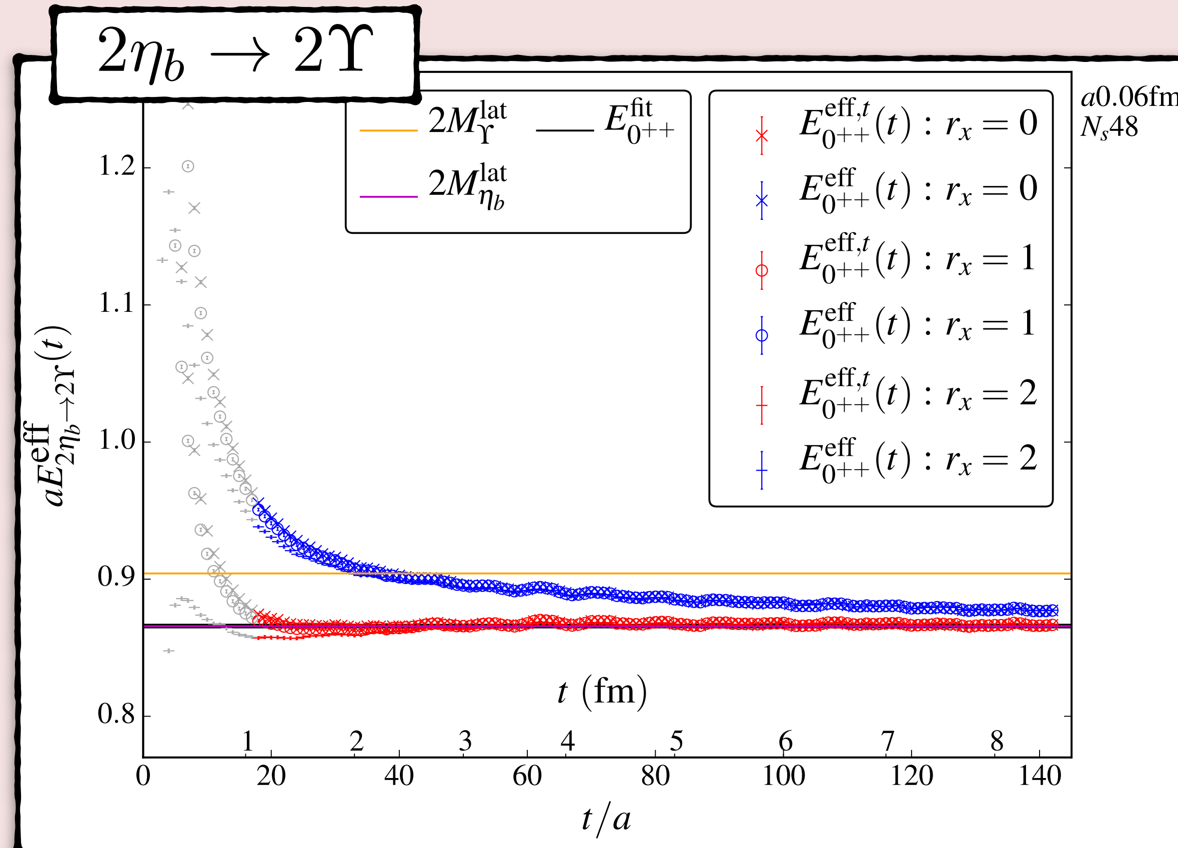

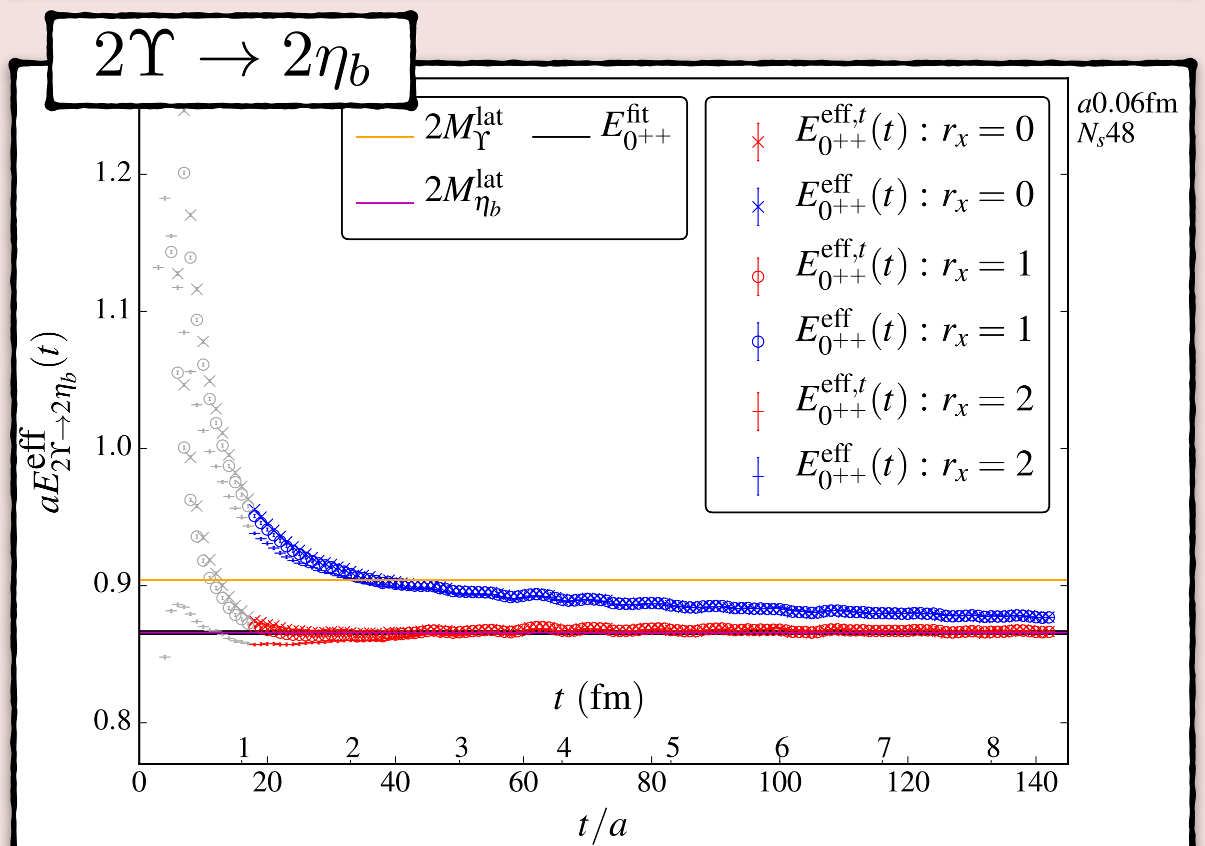

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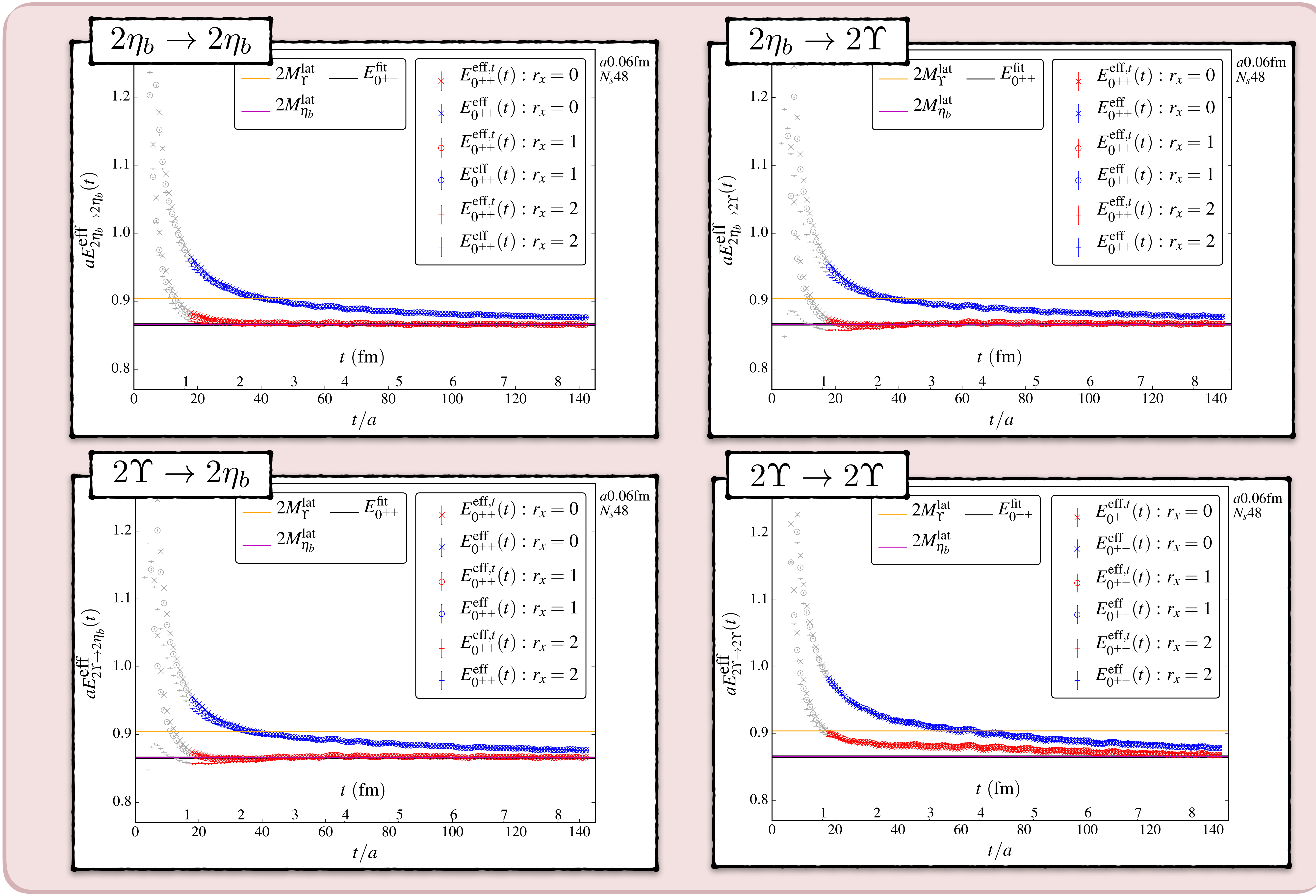

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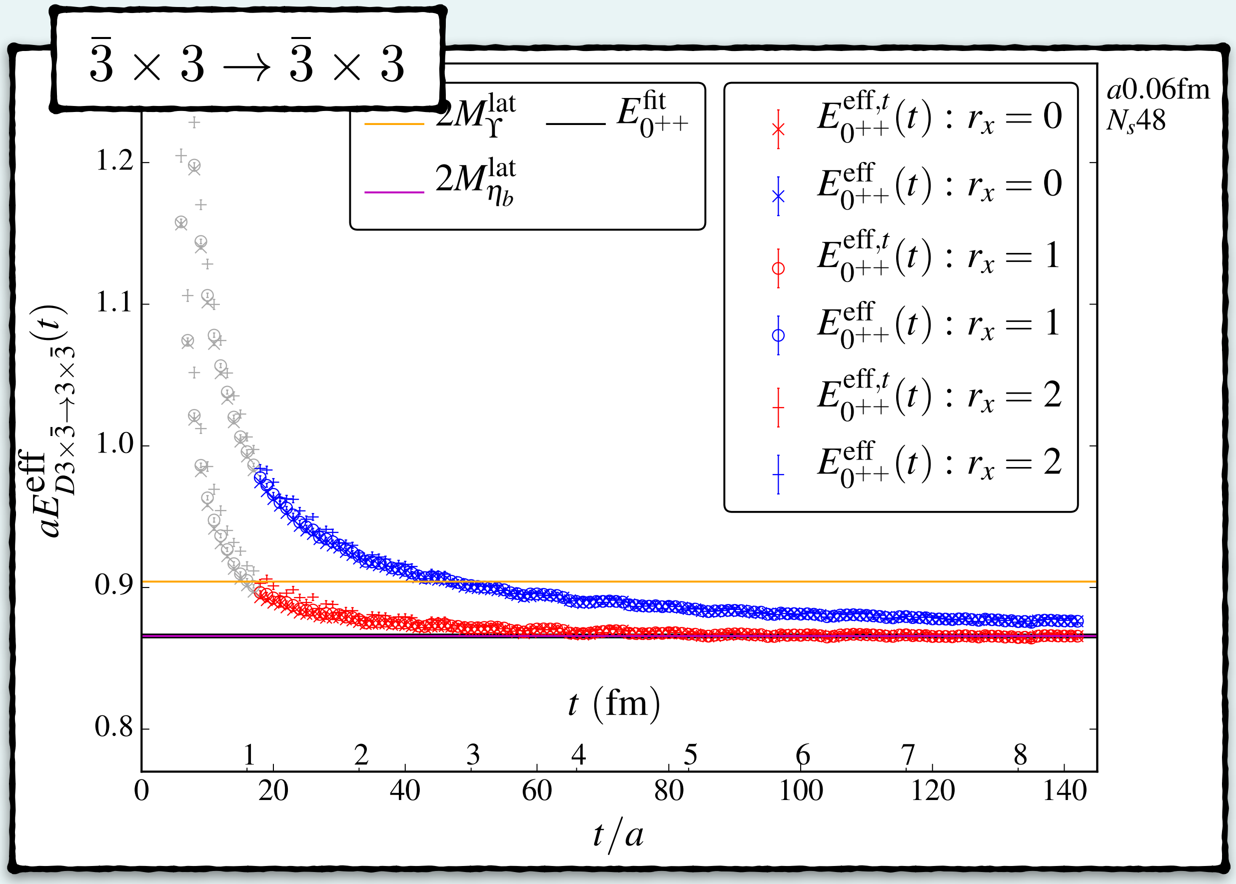
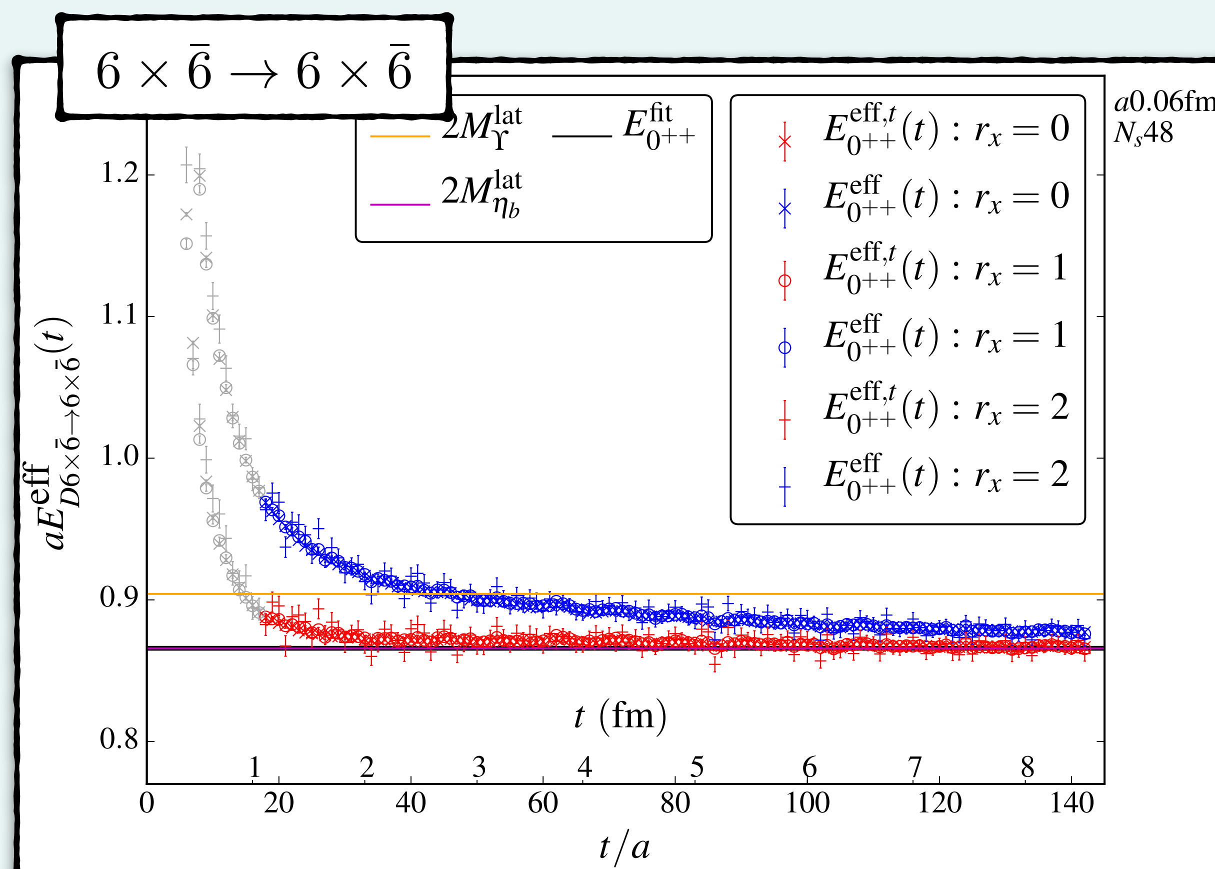
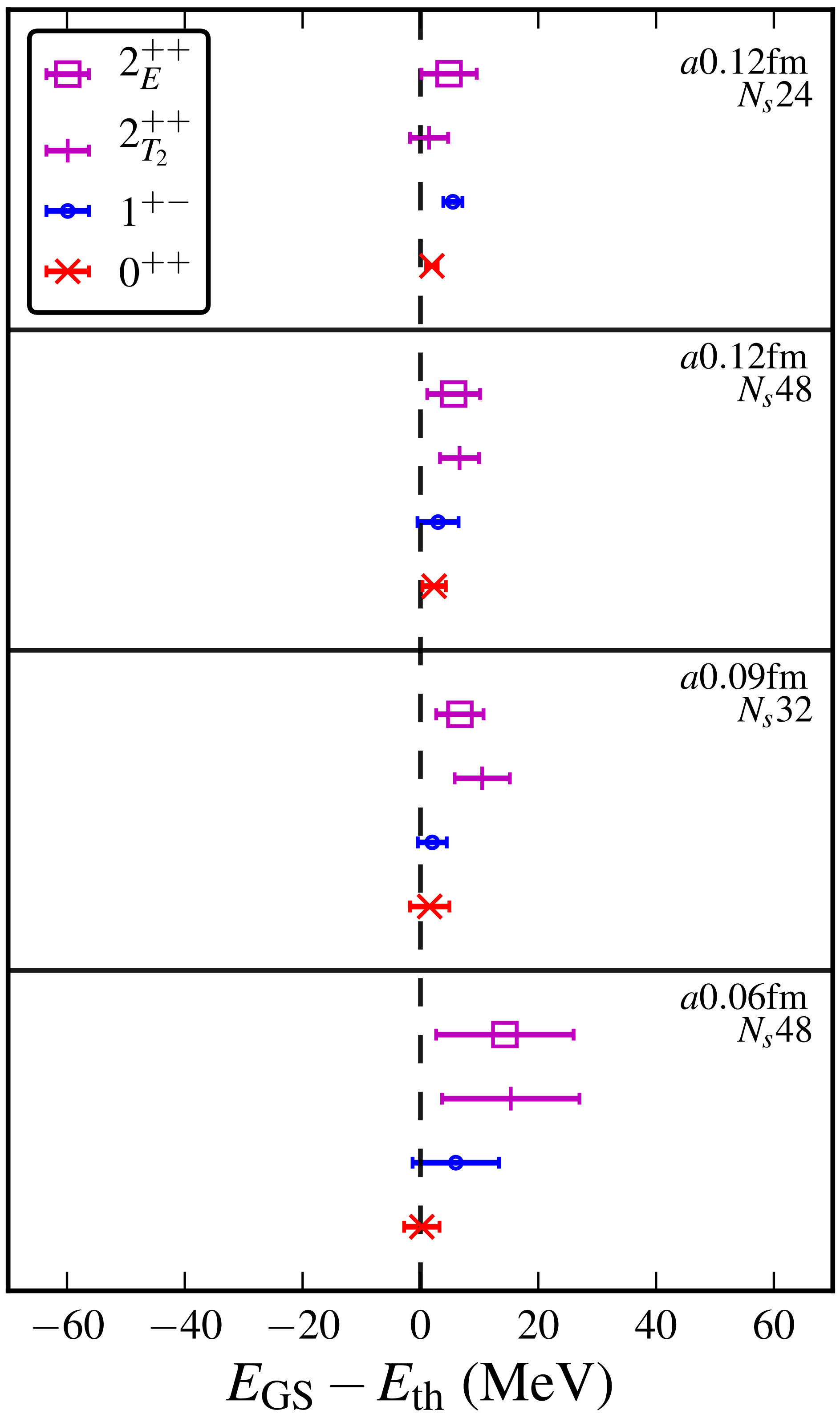




Summary of Energies from Lattice



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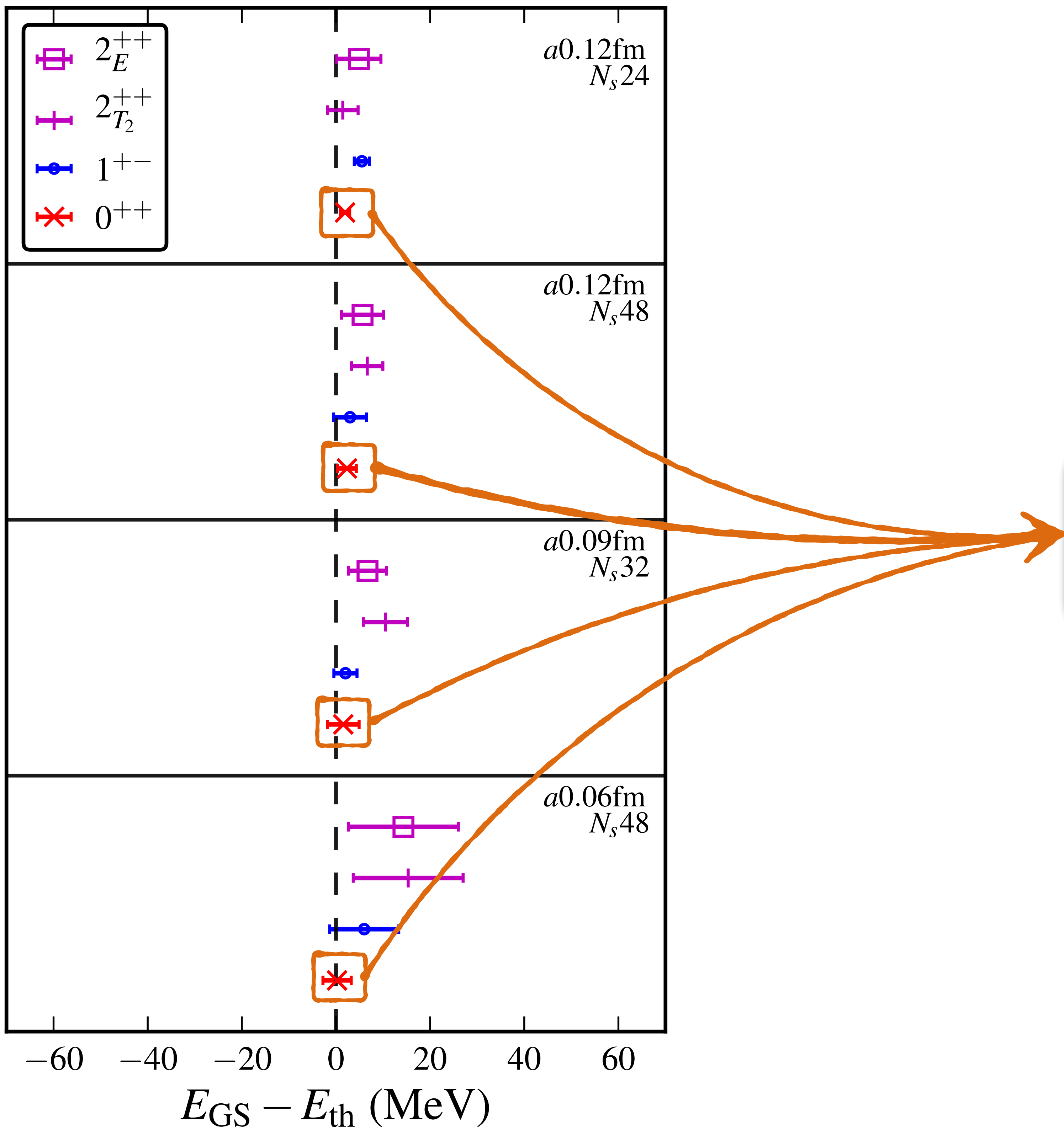
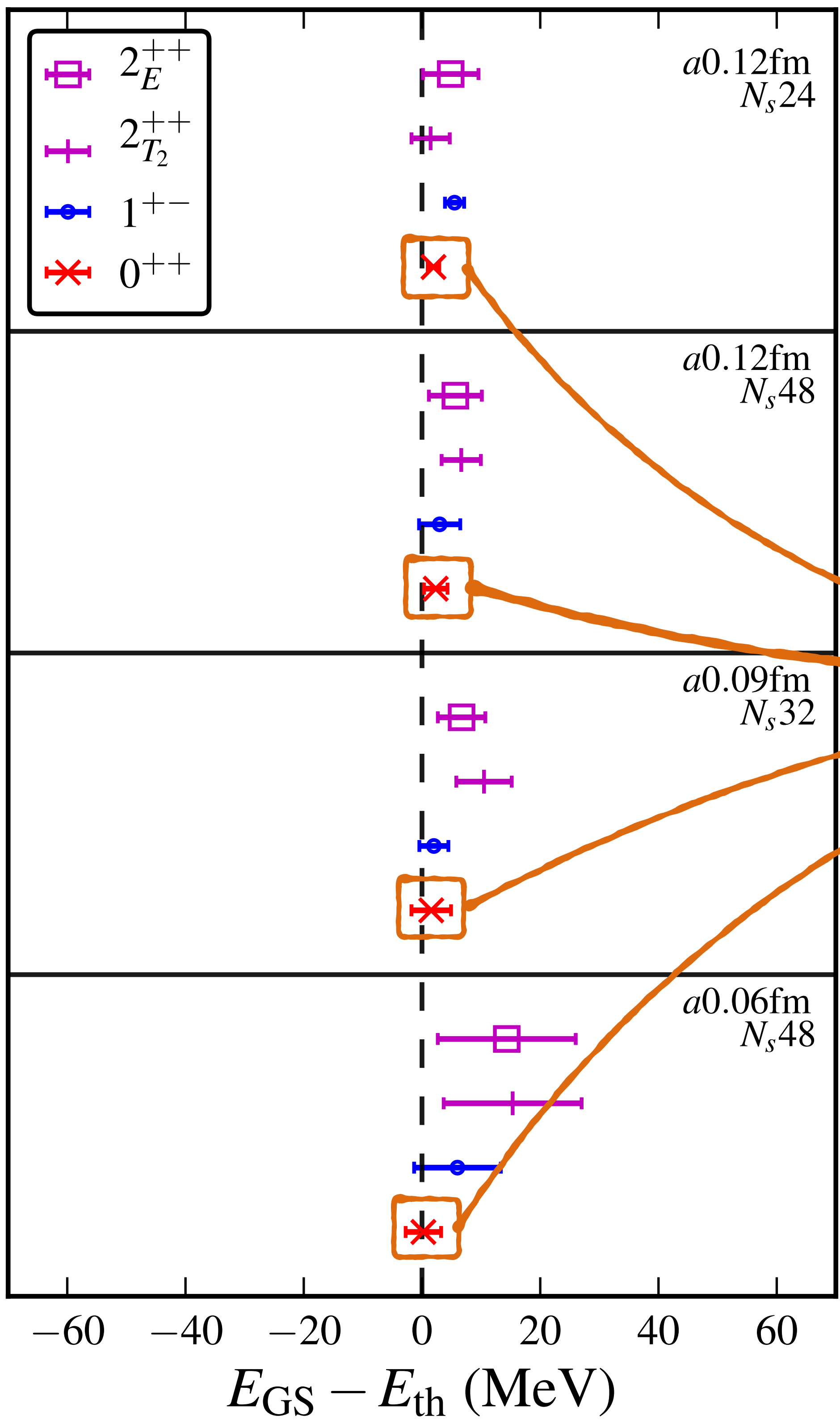


No evidence of 0^{++} below $2\eta_b$ threshold

Summary of Energies from Lattice



No evidence of 0^{++} below $2\eta_b$ threshold

If you don't observe a process, need to determine a bound, e.g, proton decay.

Bound on 0^{++} $\mathbf{2b2\bar{b}}$ state to be stable

"How would it have missed?"

- If stable tetraquark exists, at a particular time t^*,

$$C(t^*) = |\langle 0|\mathcal{O}|4b\rangle|^2 e^{-aE_{4b}t^*} + |\langle 0|\mathcal{O}|2\eta_b\rangle|^2 e^{-aE_{2\eta_b}t^*}$$

$$= \quad Z_{4b}^2 \; e^{-aE_{4b}t^*} + \quad \tilde{Z}_{2\eta_b}^2 \, e^{-aE_{2\eta_b}t^*}$$

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Input Data

$$= Z_{4b}^2 \; e^{-aE_{4b}t^*} + \tilde{Z}_{2\eta_b}^2 \, e^{-aE_{2\eta_b}t^*}$$

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Input Data

Output Constraint

$$= Z_{4b}^2 \; e^{-aE_{4b}t^*} + \tilde{Z}_{2\eta_b}^2 \, e^{-aE_{2\eta_b}t^*}$$

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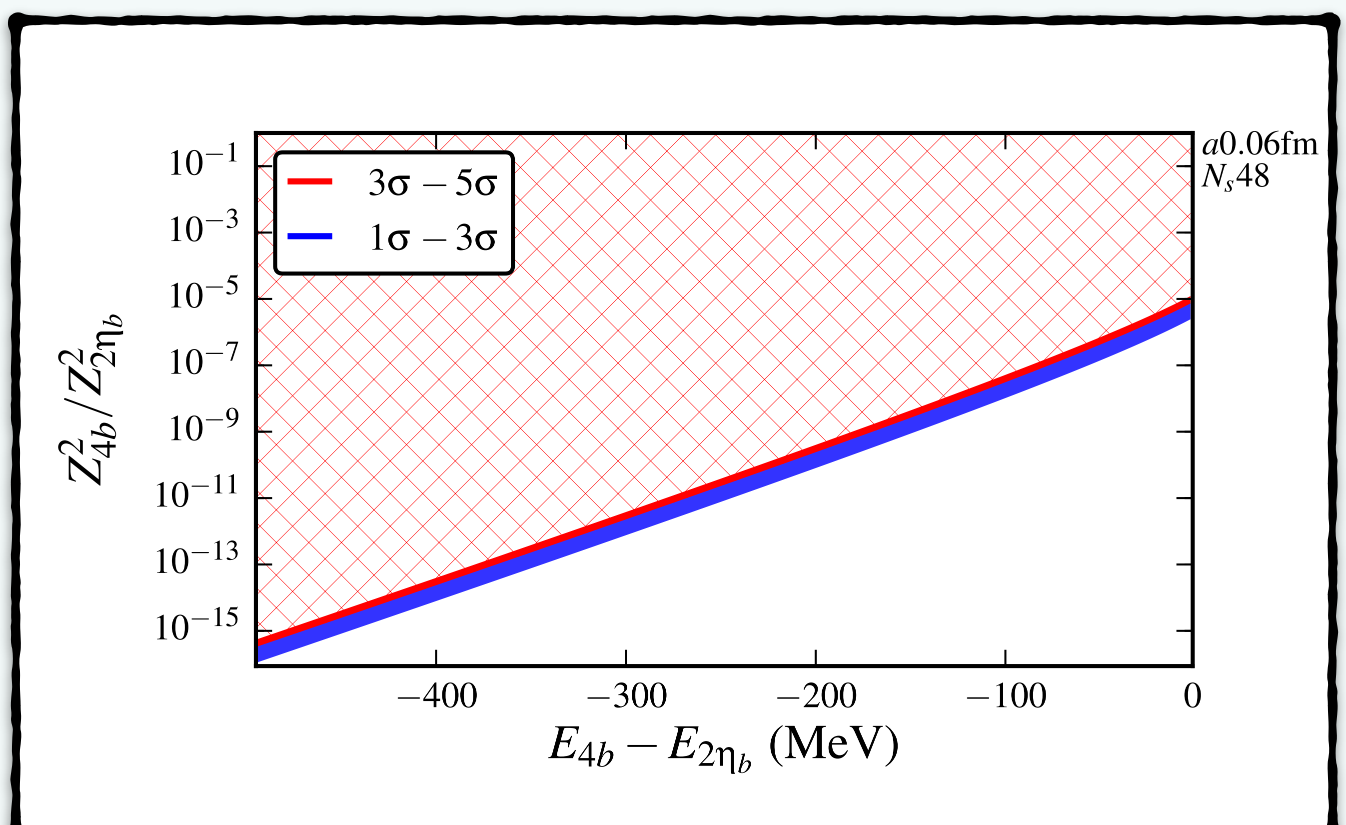
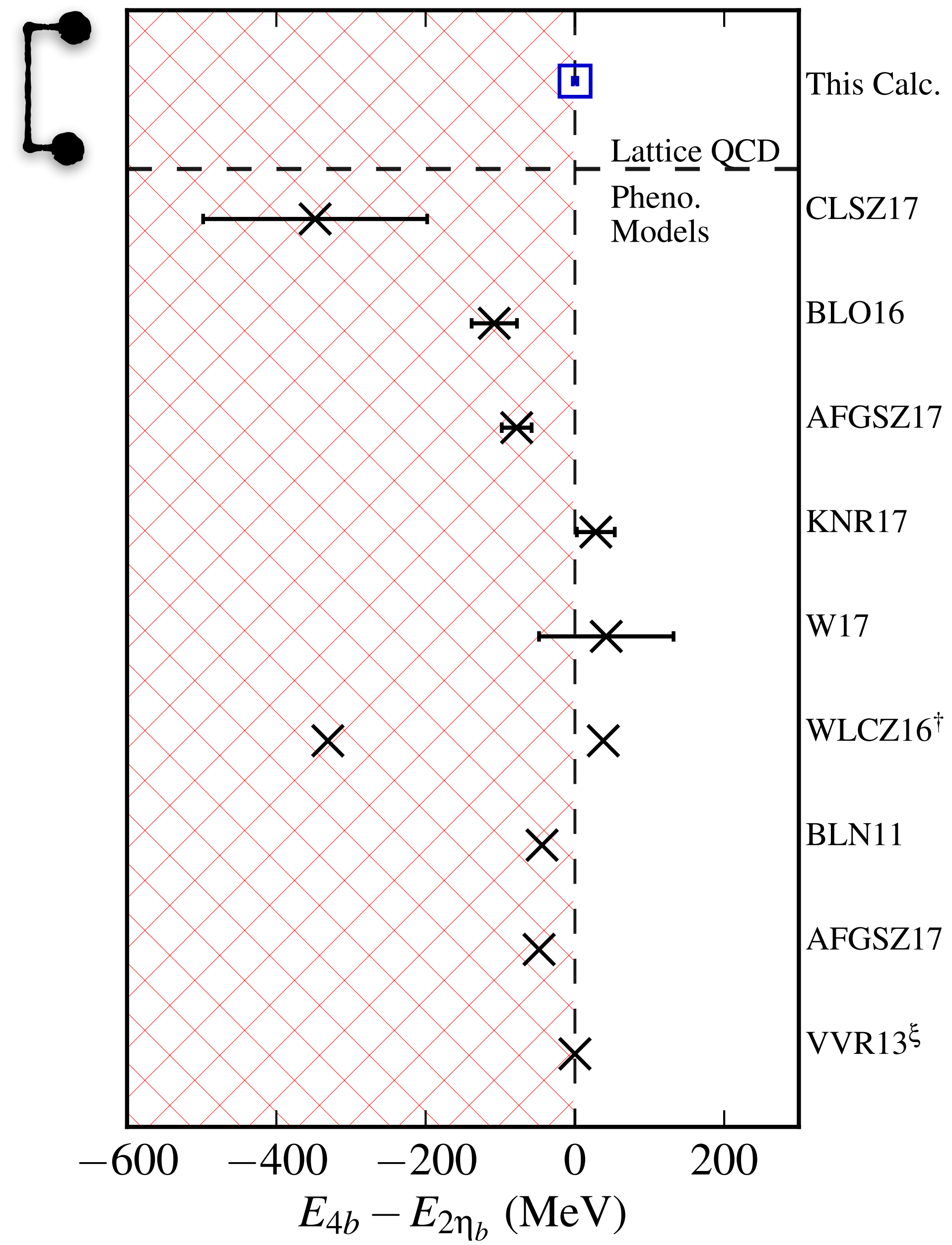


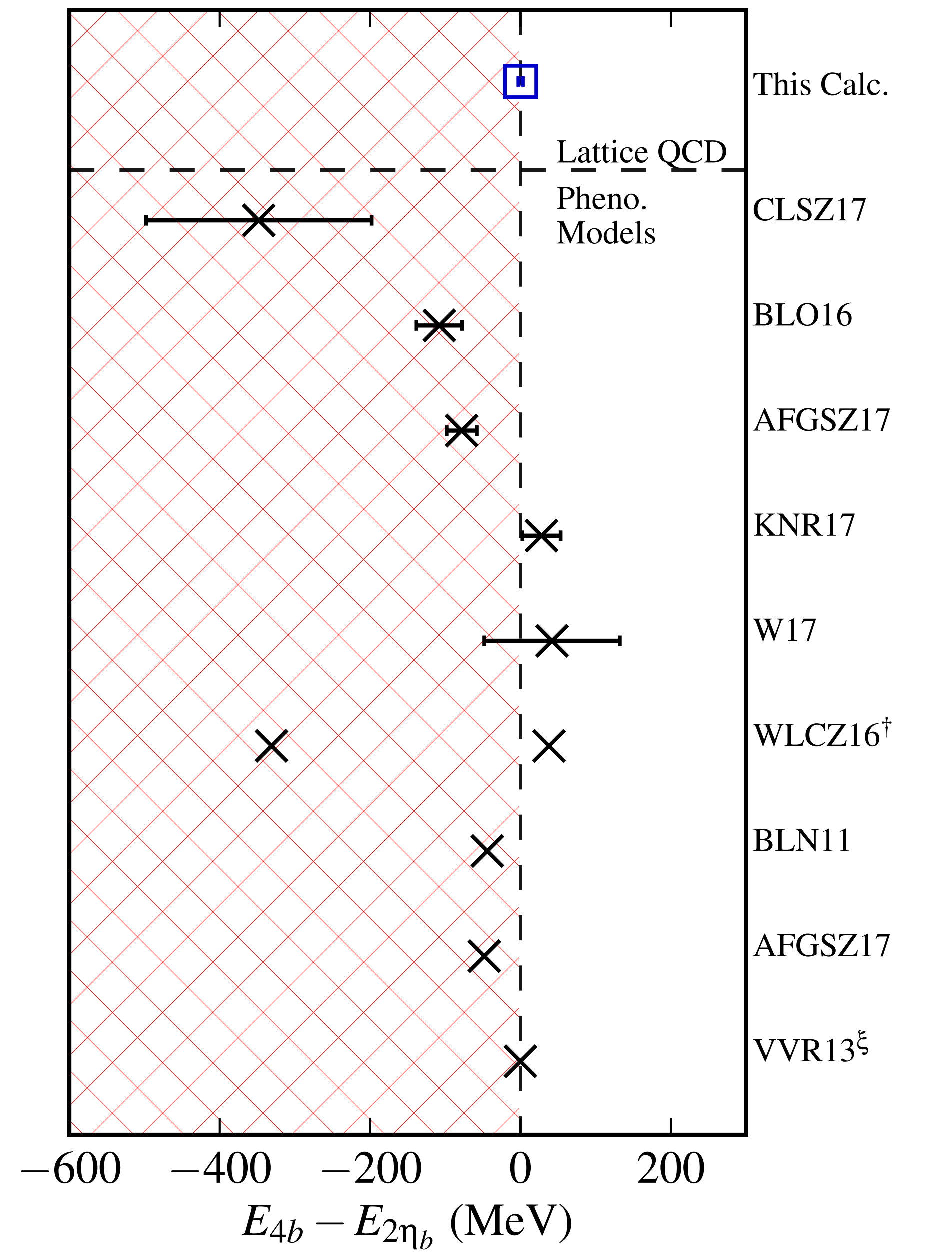
Summary

- In Summary, lattice QCD finds no evidence of a stable $2b2\bar{b}$ tetraquark



What The Models Need!

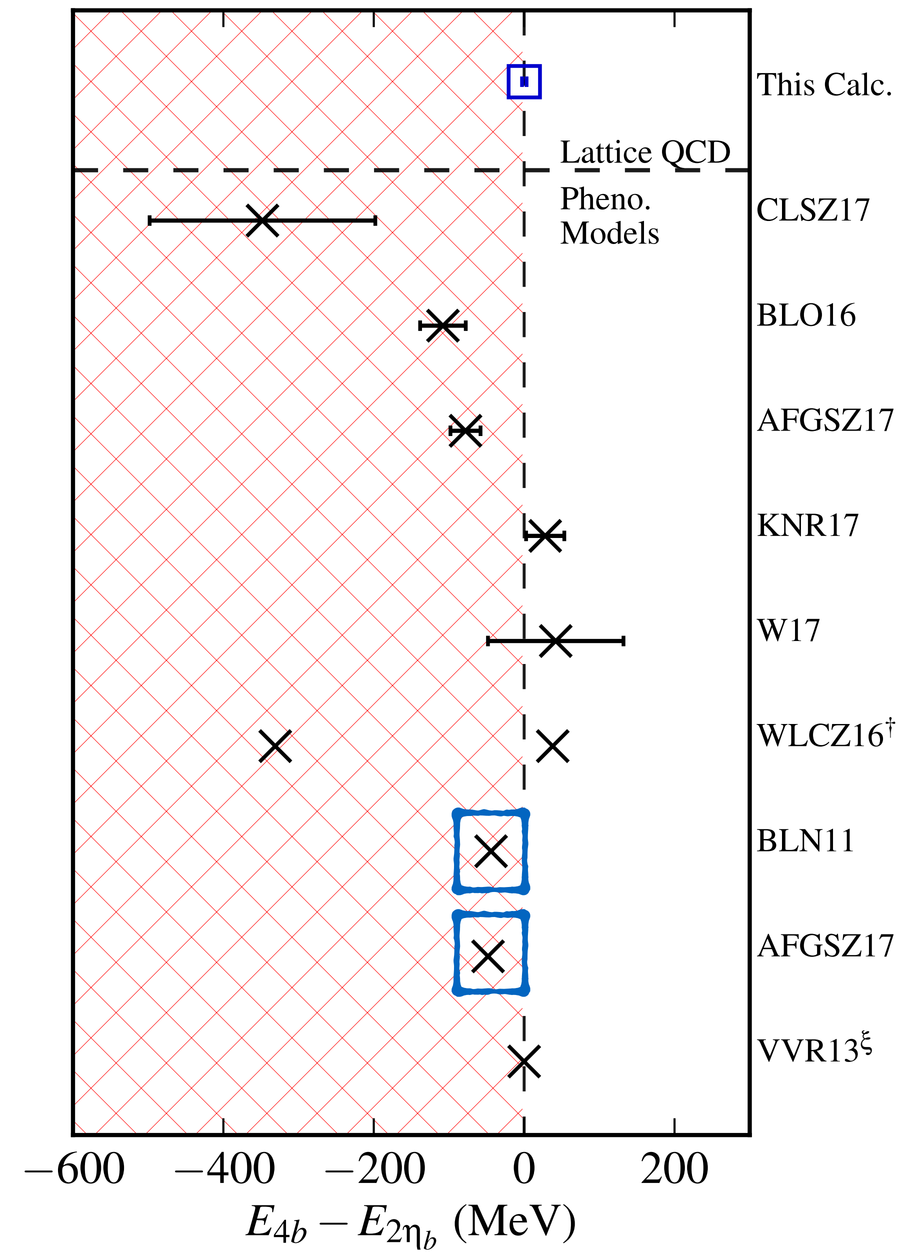
	Have quarks as elementary particles?	Full 2×2 potential matrix (including mixing between different color components)?	Both Short and Long distance effects in Gluon exchange?



What The Models Need!

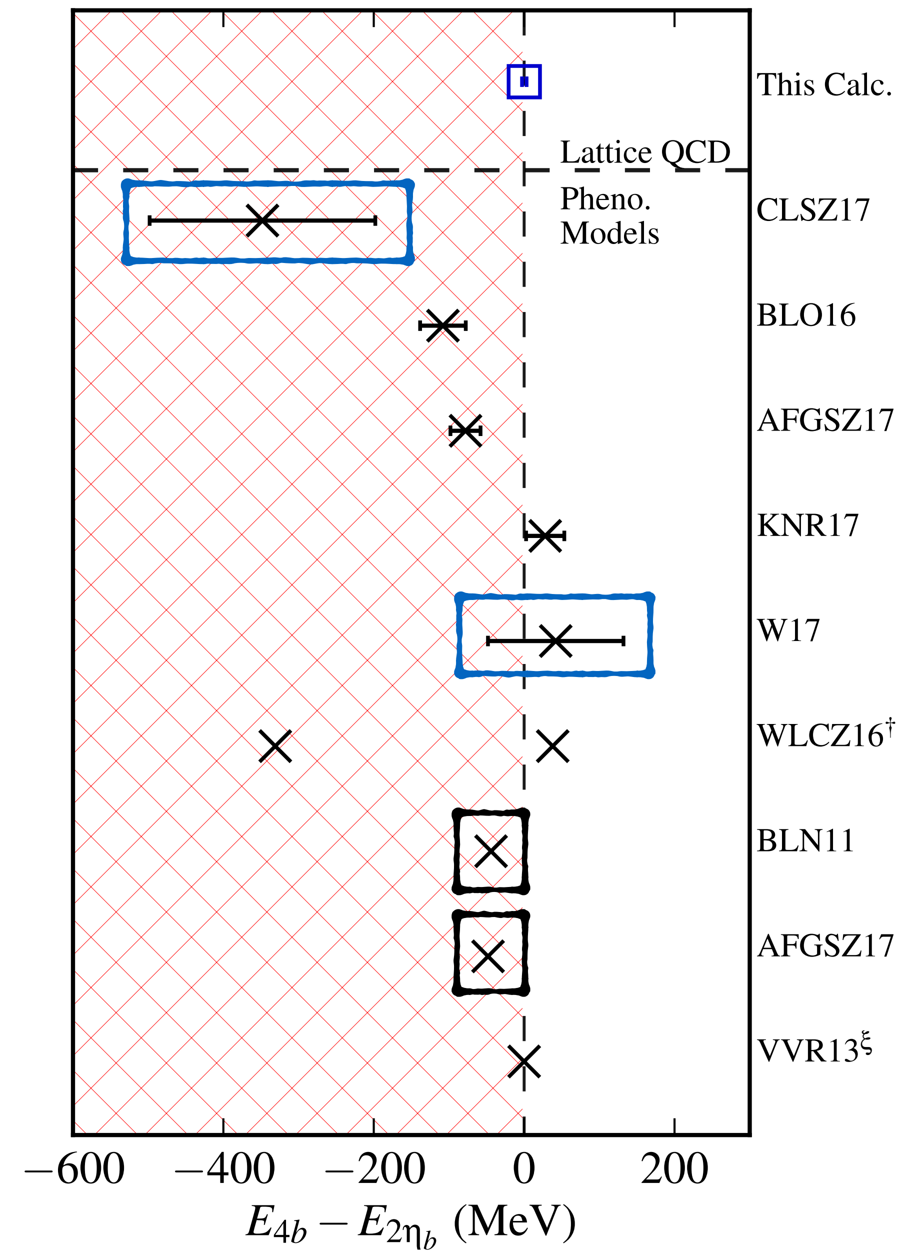
Diquarks

	Have quarks as elementary particles?	Full 2×2 potential matrix (including mixing between different color components)?	Both Short and Long distance effects in Gluon exchange?
1110.1867	✗	✓	✗
1710.0254	✗	✓	✓



What The Models Need!

		Have quarks as elementary particles?	Full 2×2 potential matrix (including mixing between different color components)?	Both Short and Long distance effects in Gluon exchange?
Diquarks	1110.1867	✗	✓	✗
	1710.0254	✗	✓	✓
Sum-Rules	1605.01647	✓	✓	✗
	1701.04285	✓	✓	✗



What The Models Need!

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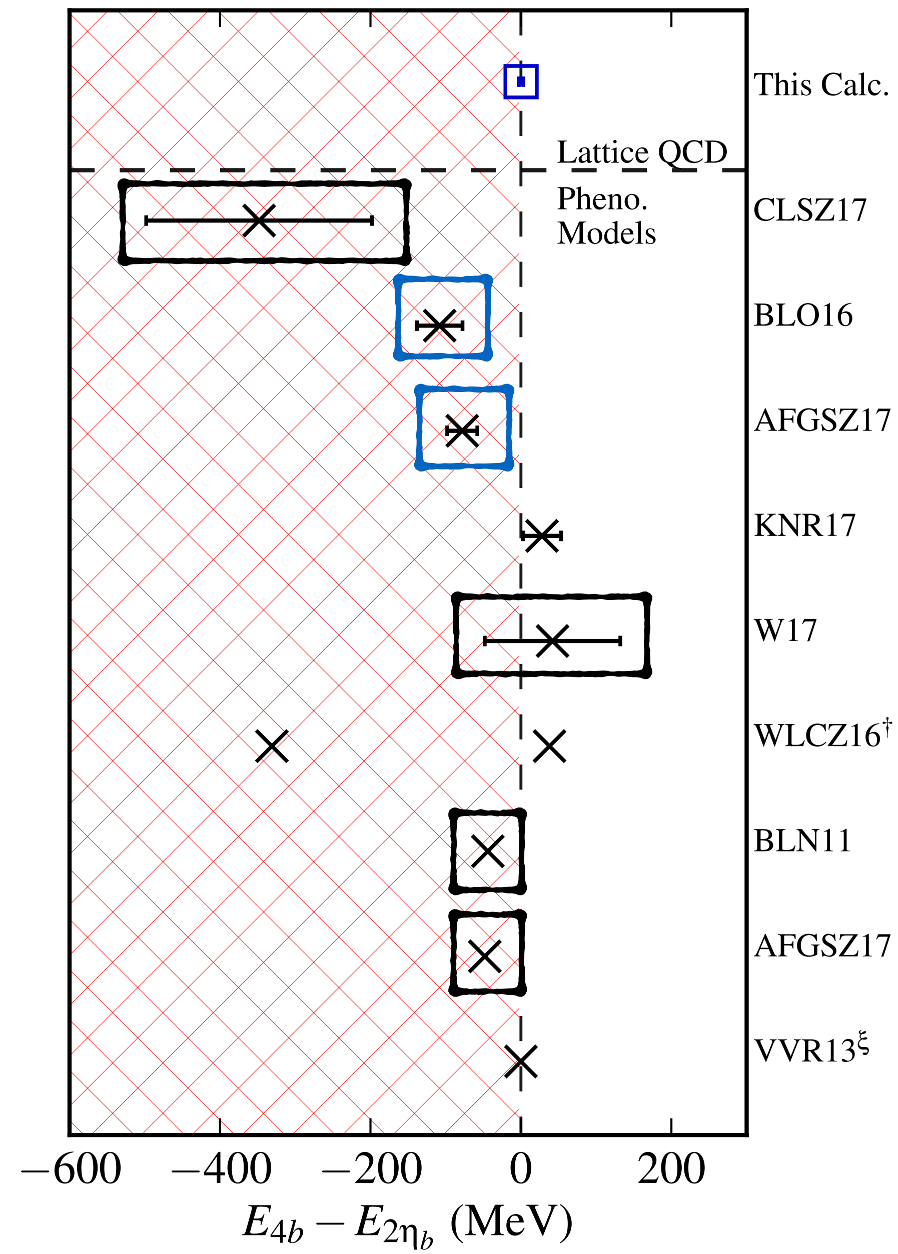
Diquarks	1110.1867	✗	✓	✗
	1710.0254	✗	✓	✓
Sum-Rules	1605.01647	✓	✓	✗
	1701.04285	✓	✓	✗
Schrodinger Equation	1710.0254	✓	✗	✗
	1612.00012	✓	✗	✓



What The Models Need!

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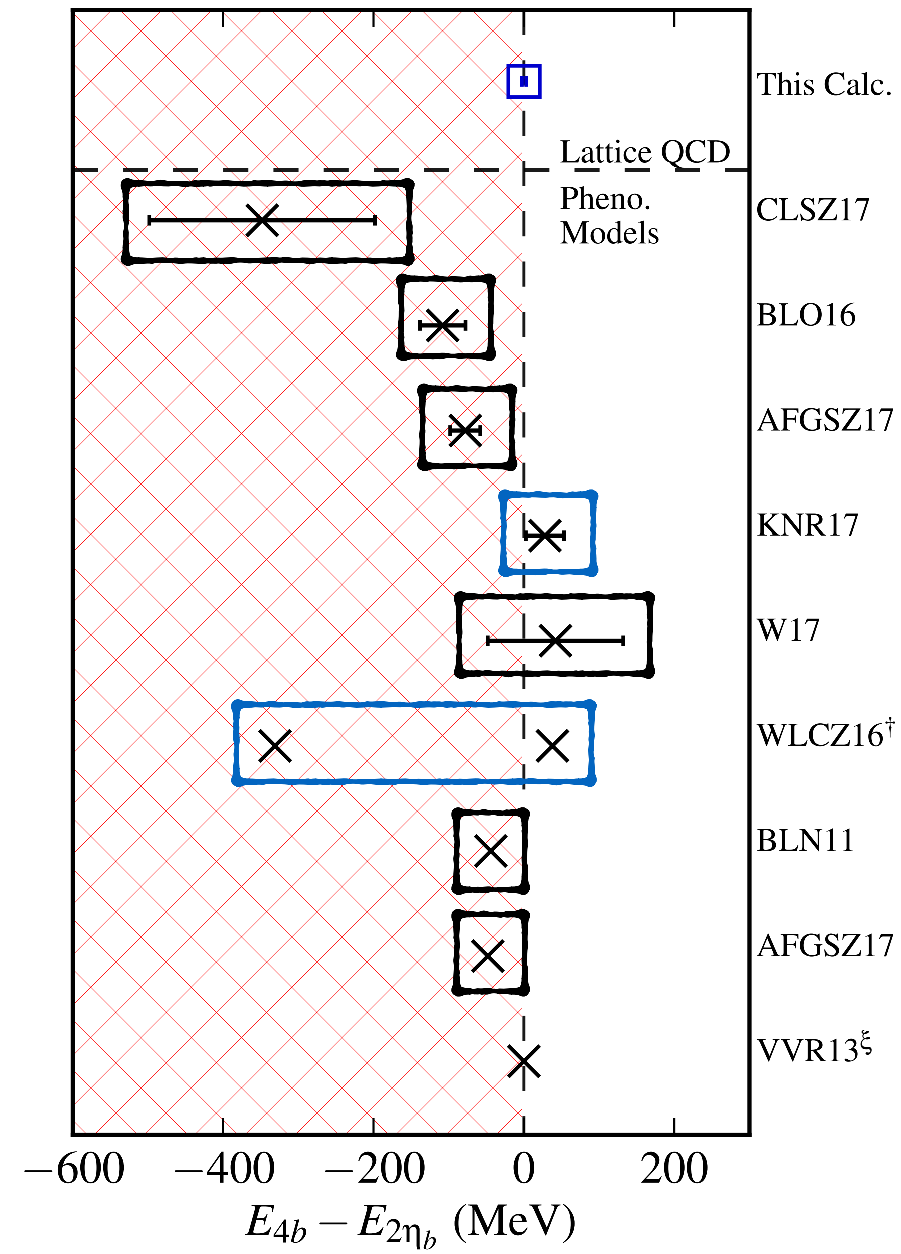
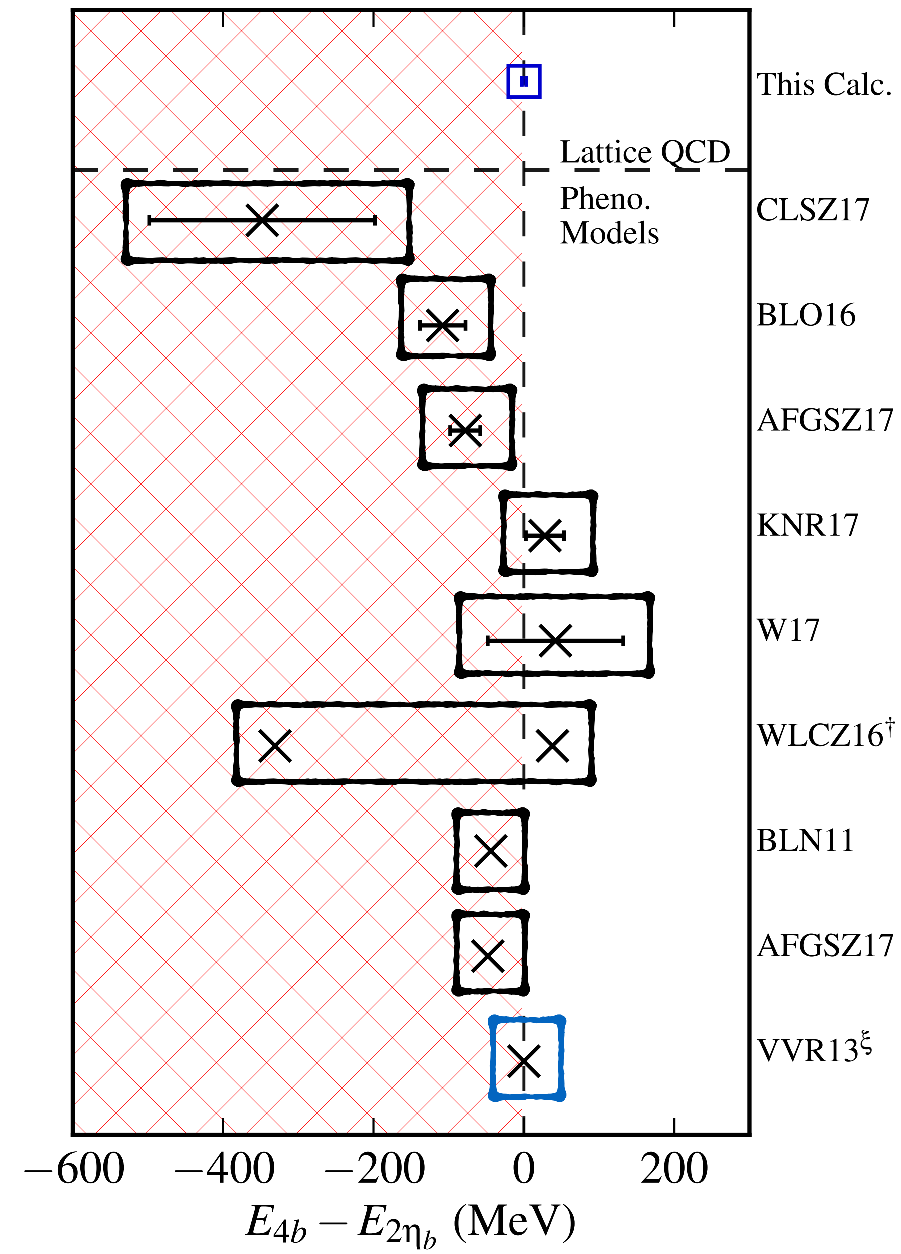
Diquarks	1710.0254	✗	✓	✓
Sum-Rules	1605.01647	✓	✓	✗
Sum-Rules	1701.04285	✓	✓	✗
Schrodinger Equation	1710.0254	✓	✗	✗
Schrodinger Equation	1612.00012	✓	✗	✓
Pheno.	1605.01134	✓	✗	✗
Pheno.	1611.00348	✓	✗	✓



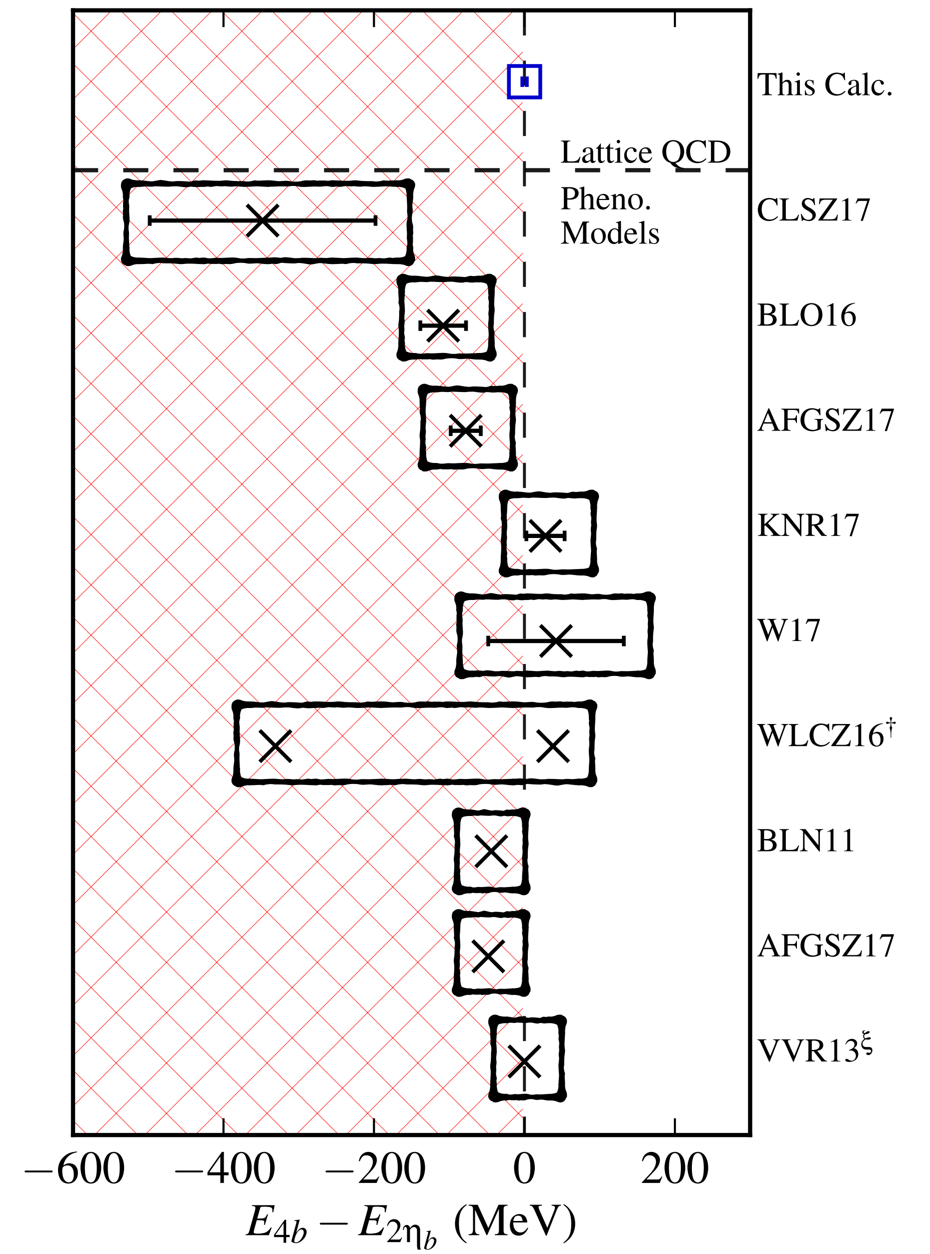
What The Models Need!

		Have quarks as elementary particles?	Full 2×2 potential matrix (including mixing between different color components)?	Both Short and Long distance effects in Gluon exchange?
Diquarks	1110.1867	✗	✓	✗
	1710.0254	✗	✓	✓
Sum-Rules	1605.01647	✓	✓	✗
	1701.04285	✓	✓	✗
Schrodinger Equation	1710.0254	✓	✗	✗
	1612.00012	✓	✗	✓
Pheno.	1605.01134	✓	✗	✗
	1611.00348	✓	✗	✓
String	1703.00783	✓	✓	✗

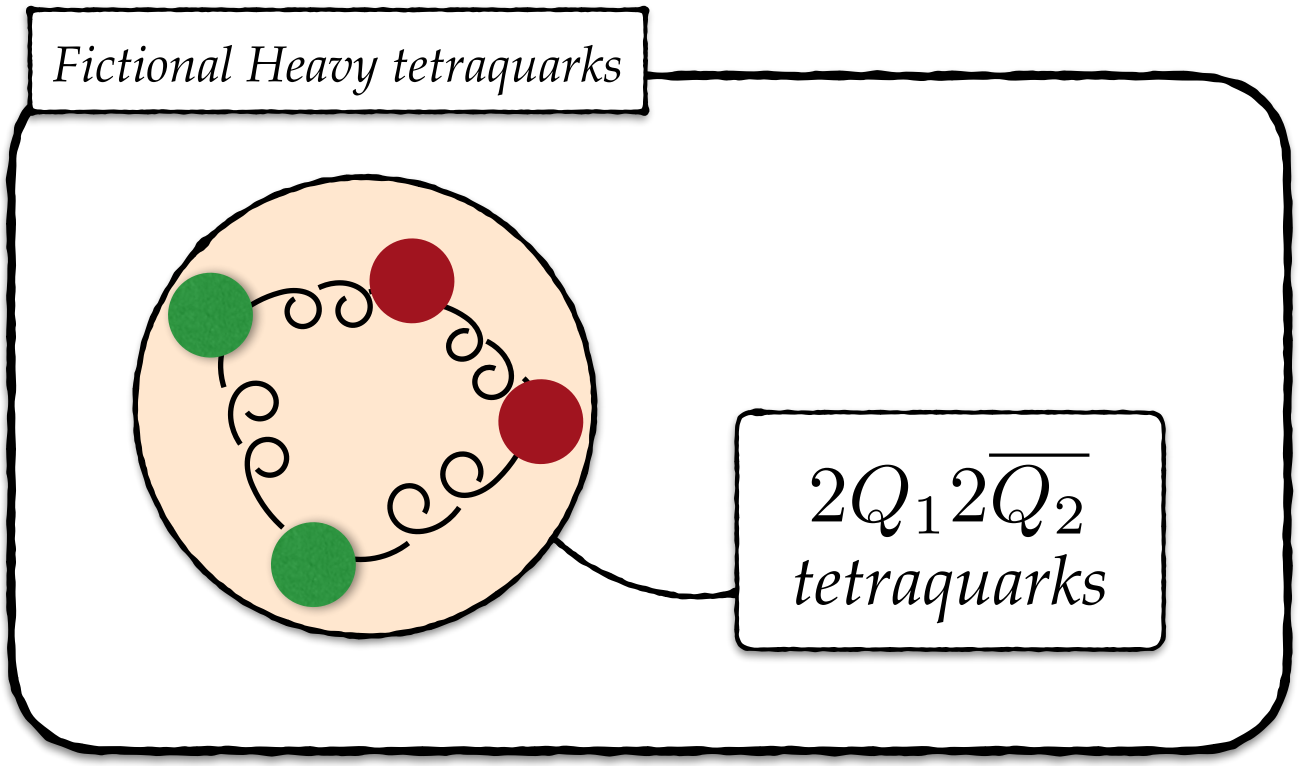


What The Models Need!

	N.B., The lattice is **NOT** a model	Have quarks as elementary particles?	Full 2×2 potential matrix (including mixing between different color components)?	Both Short and Long distance effects in Gluon exchange?
Diquarks	1110.1867	✗	✓	✗
	1710.0254	✗	✓	✓
Sum-Rules	1605.01647	✓	✓	✗
	1701.04285	✓	✓	✗
Schrodinger Equation	1710.0254	✓	✗	✗
	1612.00012	✓	✗	✓
Pheno.	1605.01134	✓	✗	✗
	1611.00348	✓	✗	✓
String	1703.00783	✓	✓	✗

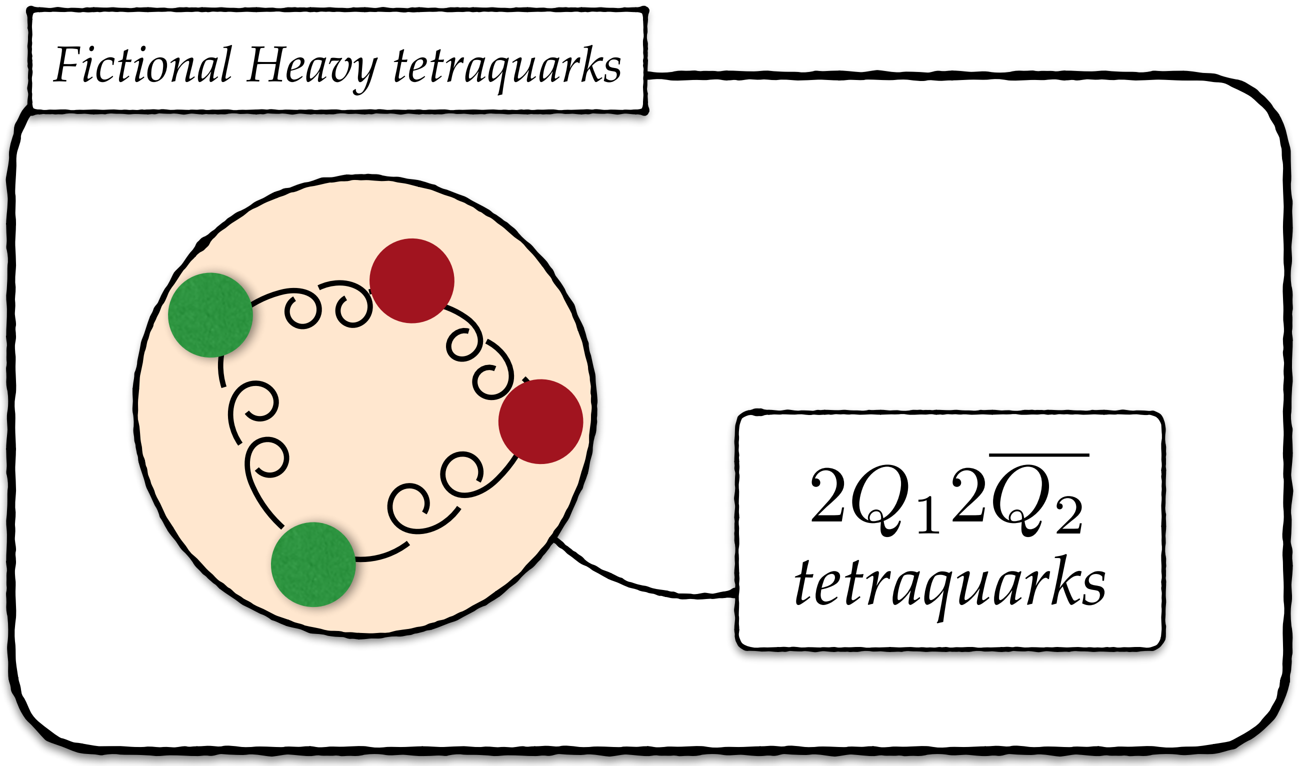


Future Work



Fictional Heavy tetraquarks
$2Q_1 2\overline{Q_2}$
tetraquarks

Future Work



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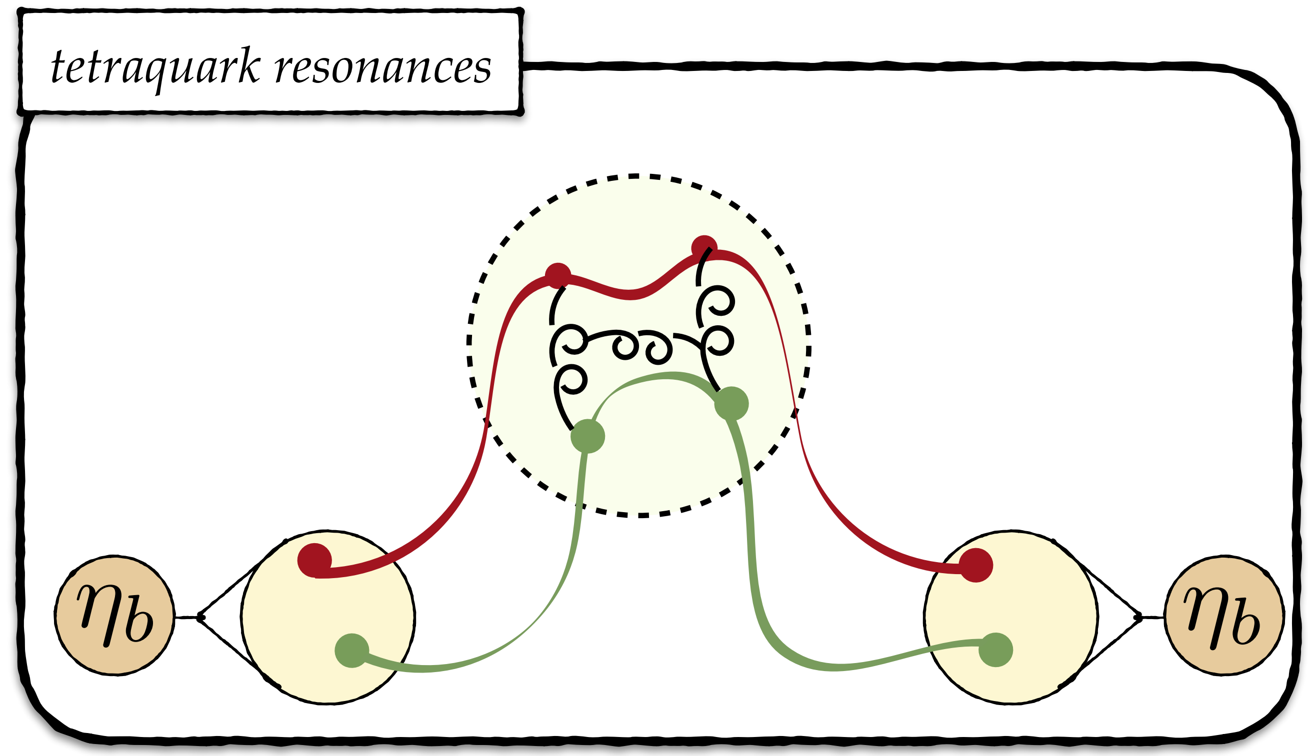
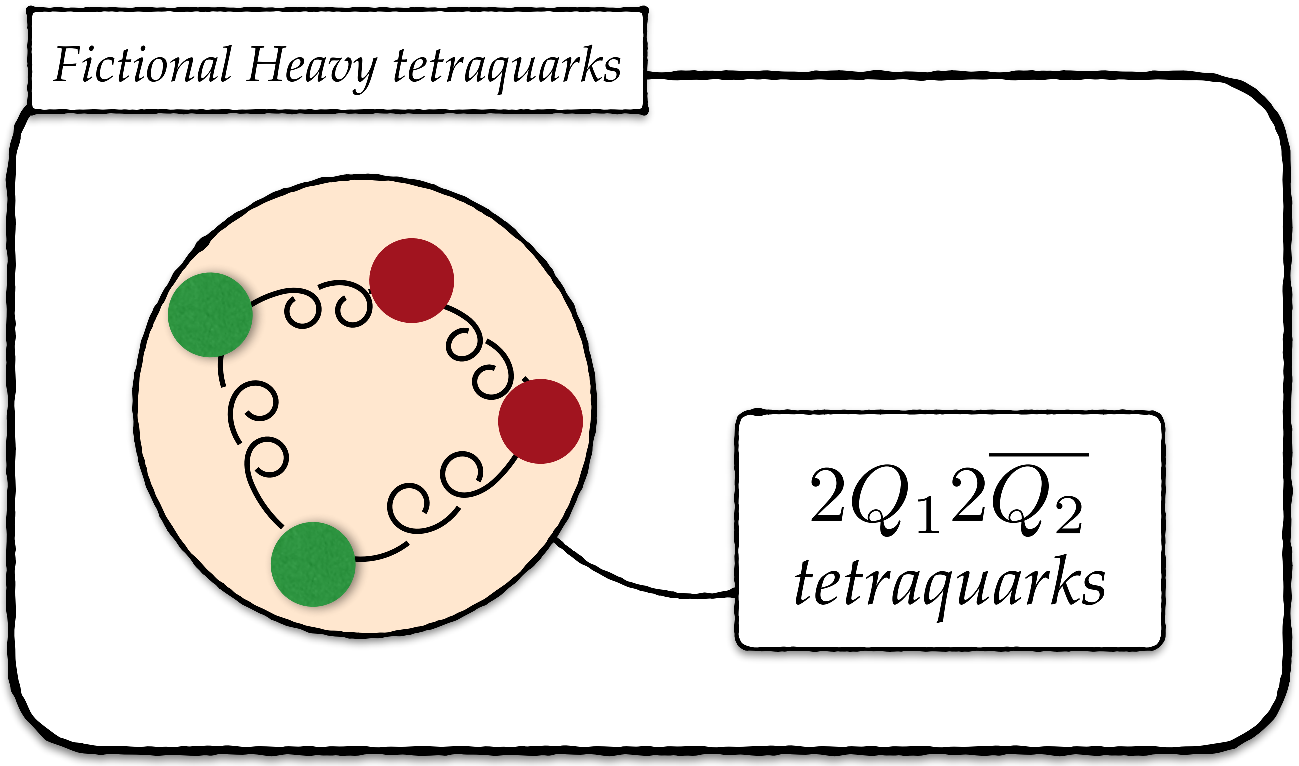


tetraquark resonances
η_b
η_b

Future Work



Fictional Heavy tetraquarks
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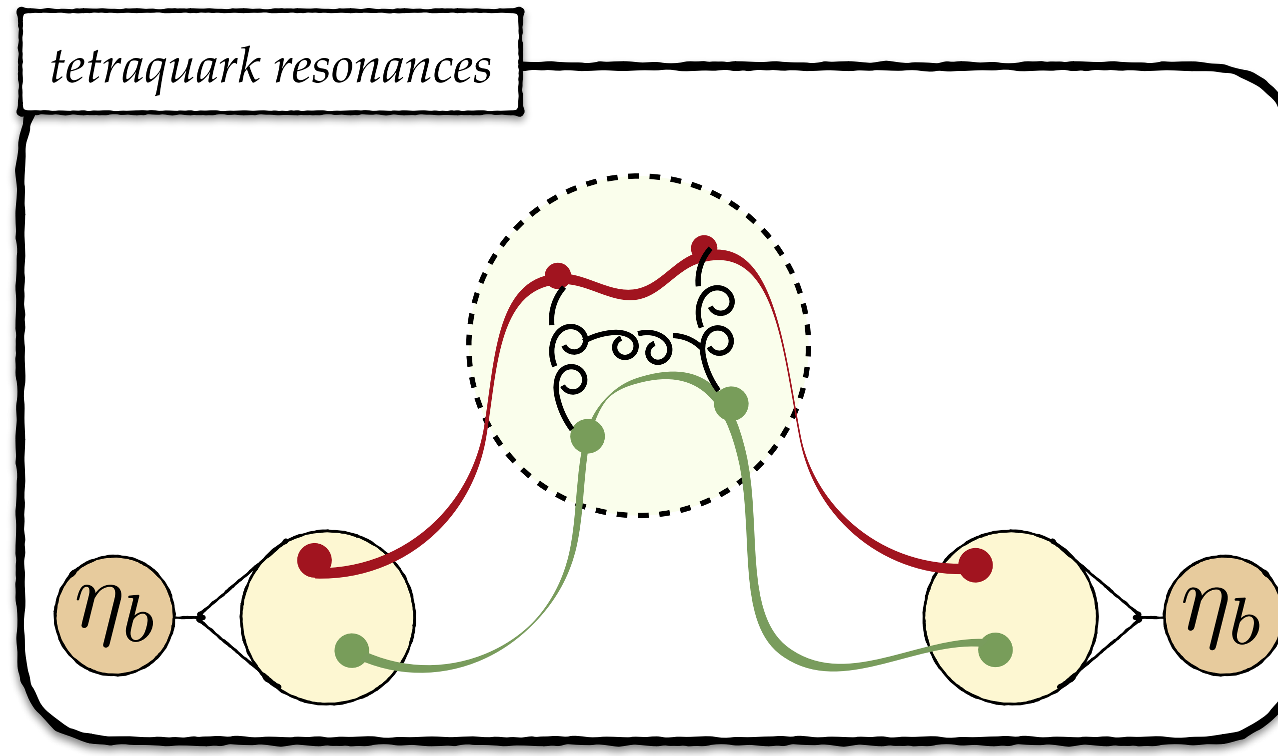
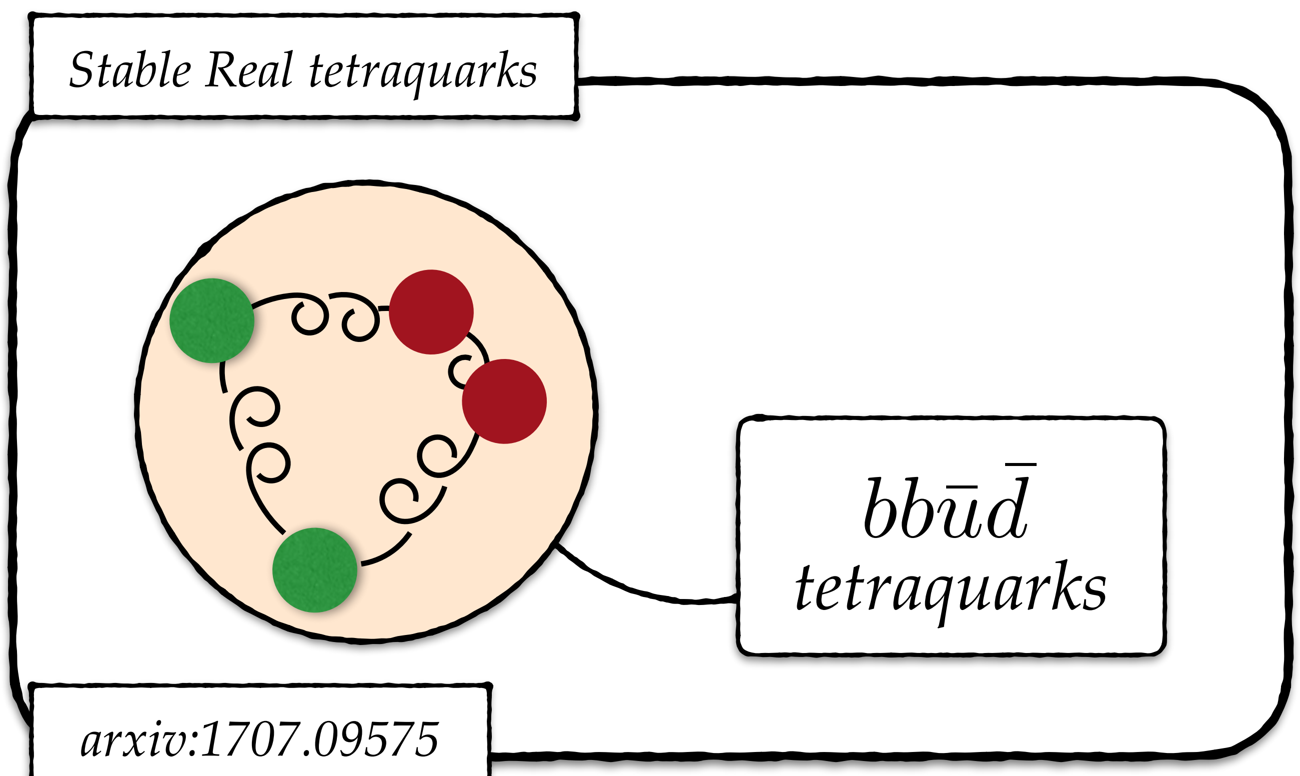


tetraquark resonances
η_b
η_b



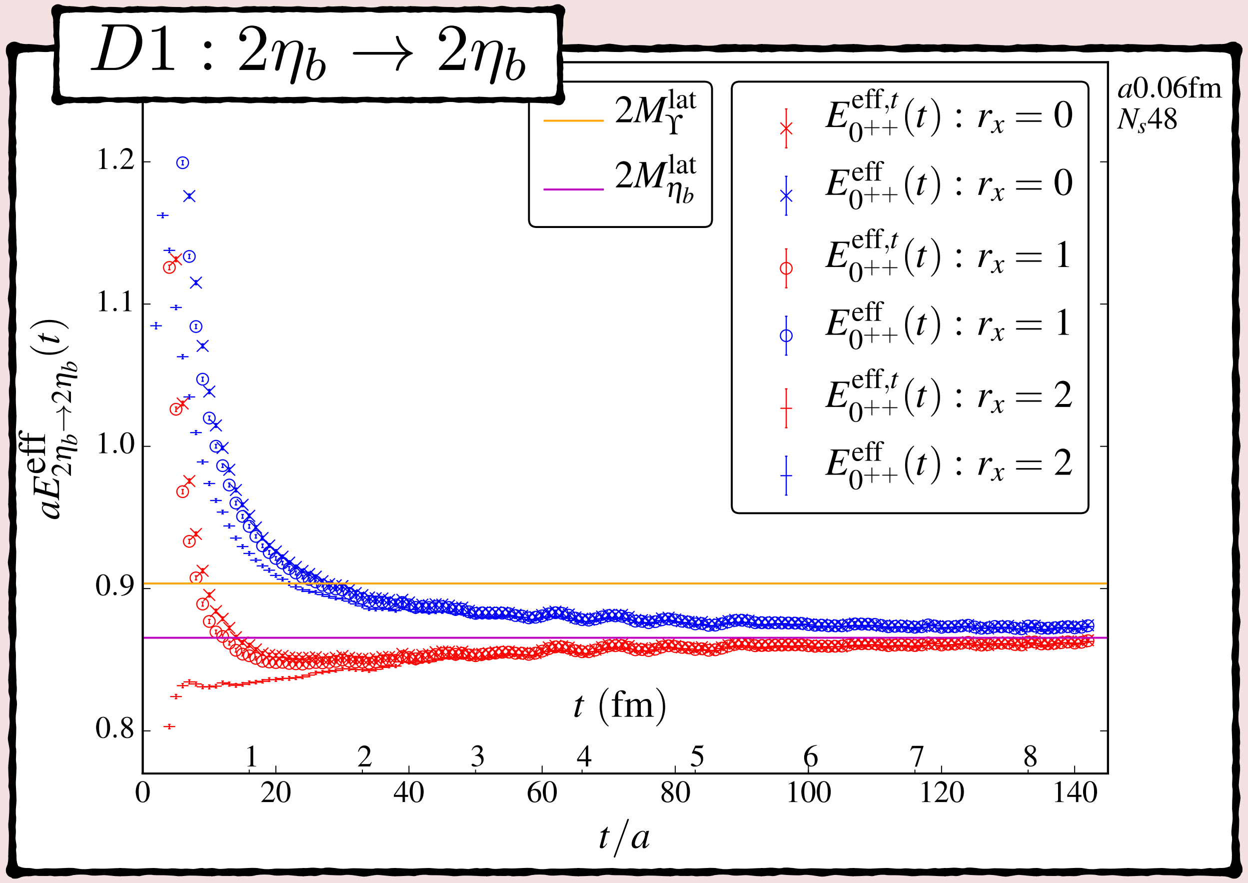
Stable Real tetraquarks
$bb\bar{u}\bar{d}$
tetraquarks
arxiv:1707.09575

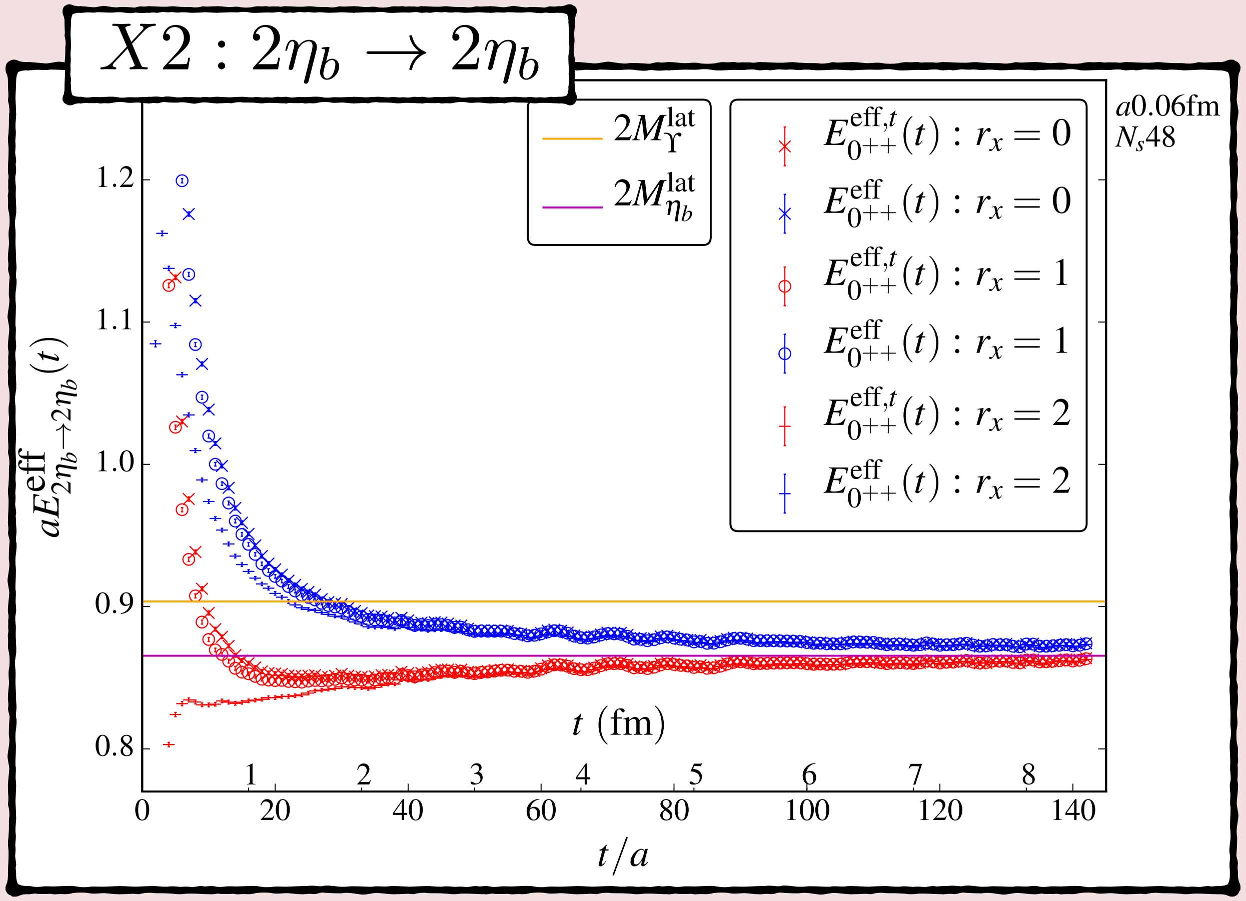
Thank you!

Thank You to Raul Briceno for slide template and pretty graphics!

Back-Up Slides

Individual Wick Contraction Correlator Data





Lattice QCD Methodology

1. Take Euclidean QCD and discretise it on a **Finite-Volume Lattice** of length $\mathbf{L}$ and spacing $\mathbf{a}$

Complication: b***-quarks do not fit on current lattices!!***

Solution: Use a Non-Relativistic Effective Field Theory to simulate the b***-quarks***

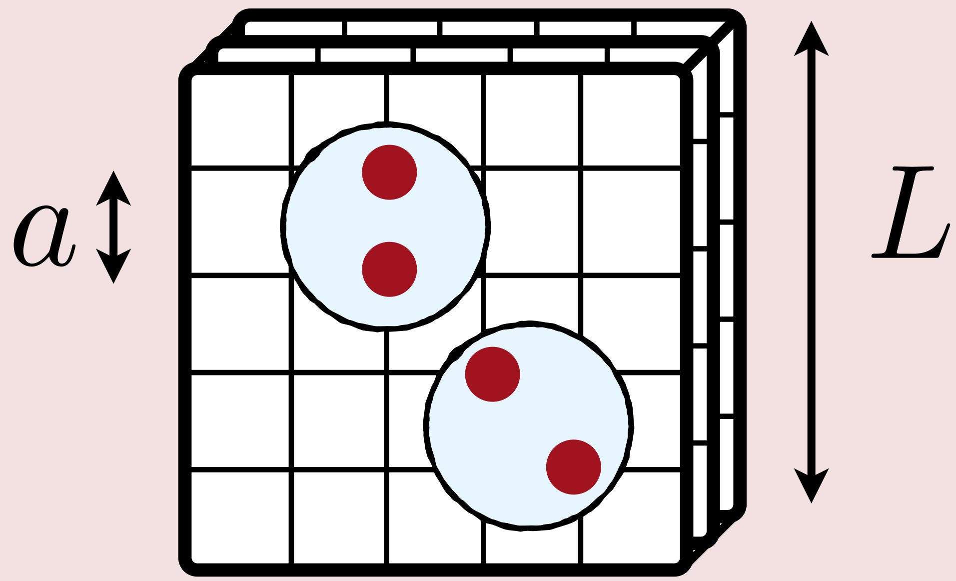
- Has Expansion Parameter $\mathbf{v^2 \sim 0.1}$
- N.B.: Matching Coefficients Need to be Calculated in Lattice Perturbation Theory

Lattice QCD Methodology

Down the Rabbit Hole

Lattice QCD Methodology

1. Take Euclidean QCD and discretise it on a **Finite-Volume Lattice** of length **L** and spacing $\mathbf{a}$



Lattice QCD Methodology

1. Take Euclidean QCD and discretise it on a **Finite-Volume Lattice** of length **L** and spacing **a**
2. Get one of these:

3. Buy one of these:

4. Numerically evaluate the Feynman Path Integral (the first-principles approach to QFT)
5. Do all the computations/analysis
6. Pay the Electricity Bill….

Lattice QCD Methodology

1. Take Euclidean QCD and discretise it on a **Finite-Volume Lattice** of length **L** and spacing $\mathbf{a}$

$$
\begin{aligned}
a\delta H &= aH_0 + a\delta H_{v^4} + a\delta H_{v^6};\\
aH_0 &= -\frac{\Delta^{(2)}}{2am_b}\\
a\delta H_{v^4} &= -c_1\frac{(\Delta^{(2)})^2}{8(am_b)^3} + c_2\frac{i}{8(am_b)^2}\left(\nabla\cdot\tilde{\mathbf{E}} - \tilde{\mathbf{E}}\cdot\nabla\right)\\
&\quad - c_3\frac{1}{8(am_b)^2}\sigma\cdot\left(\tilde{\nabla}\times\tilde{\mathbf{E}} - \tilde{\mathbf{E}}\times\tilde{\nabla}\right)\\
&\quad - c_4\frac{1}{2am_b}\sigma\cdot\tilde{\mathbf{B}} + c_5\frac{\Delta^{(4)}}{24am_b} - c_6\frac{(\Delta^{(2)})^2}{16n(am_b)^2}\\
a\delta H_{v^6} &= -c_7\frac{1}{8(am_b)^3}\left\{\boldsymbol{\Delta}^{(\mathbf{2})}, \sigma\cdot\tilde{\mathbf{B}}\right\}\\
&\quad - c_8\frac{3}{64(am_b)^4}\left\{\boldsymbol{\Delta}^{(\mathbf{2})}, \sigma\cdot\left(\tilde{\nabla}\times\tilde{\mathbf{E}} - \tilde{\mathbf{E}}\times\tilde{\nabla}\right)\right\}\\
&\quad - c_9\frac{i}{8(am_b)^3}\sigma\cdot\tilde{\mathbf{E}}\times\tilde{\mathbf{E}}
\end{aligned}
$$

Lattice QCD Methodology

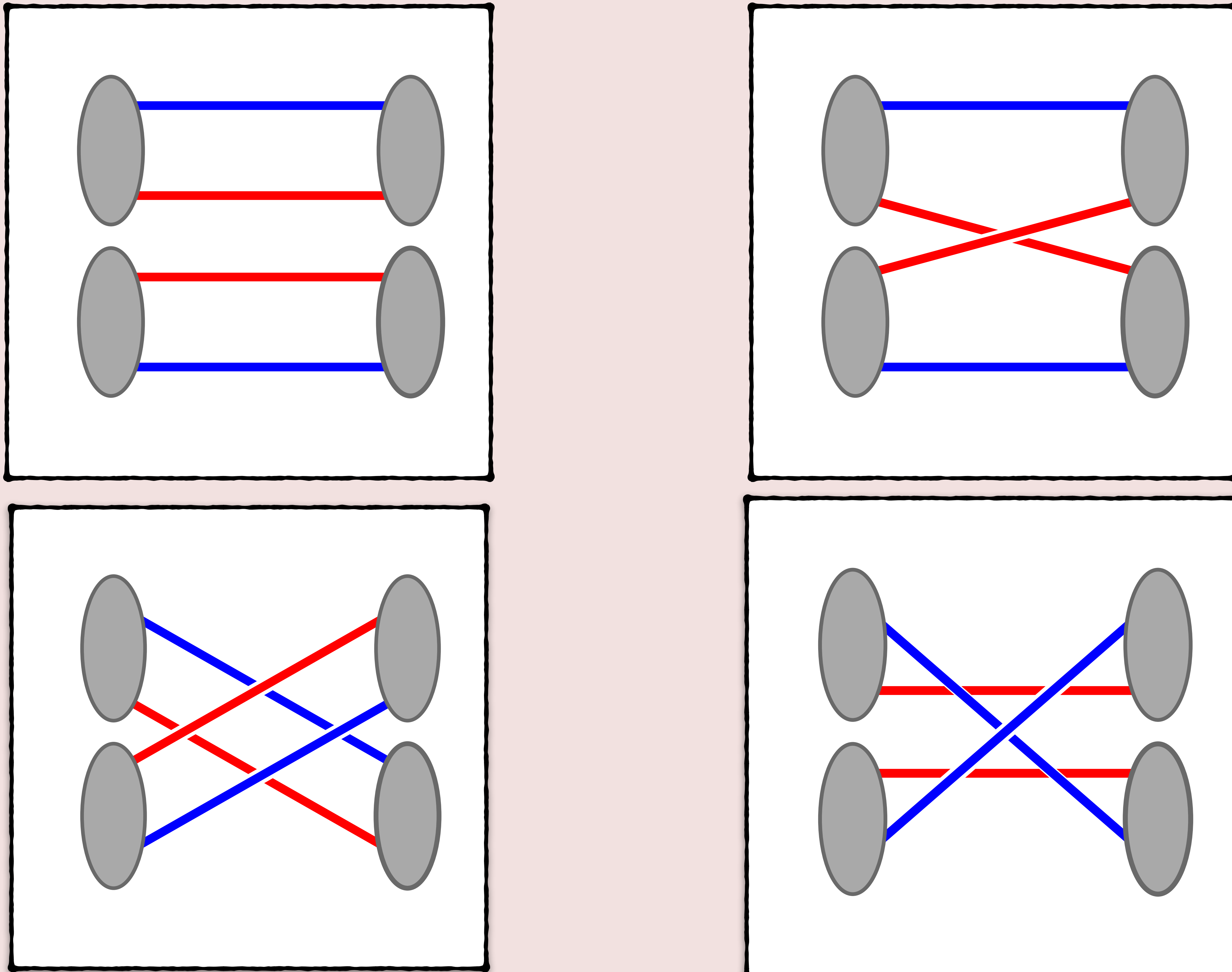
4. Numerically evaluate the Feynman Path Integral (the first-principles approach to QFT)

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int \mathcal{D}U \mathcal{D}\psi \mathcal{D}\bar{\psi} e^{-S} \mathcal{O}[U, \psi, \bar{\psi}]$$

$$\approx \frac{1}{N} \sum_{i=1}^{N} \mathcal{O}[G^{(i)}]$$

- where the integral is approximated as a sum over configurations $\{\mathbf{G^{(i)}}\}$ distributed according to the probability density: $\exp(-S_{YM}) \prod \det(D + m_q)$

Two-Meson Wick Contractions

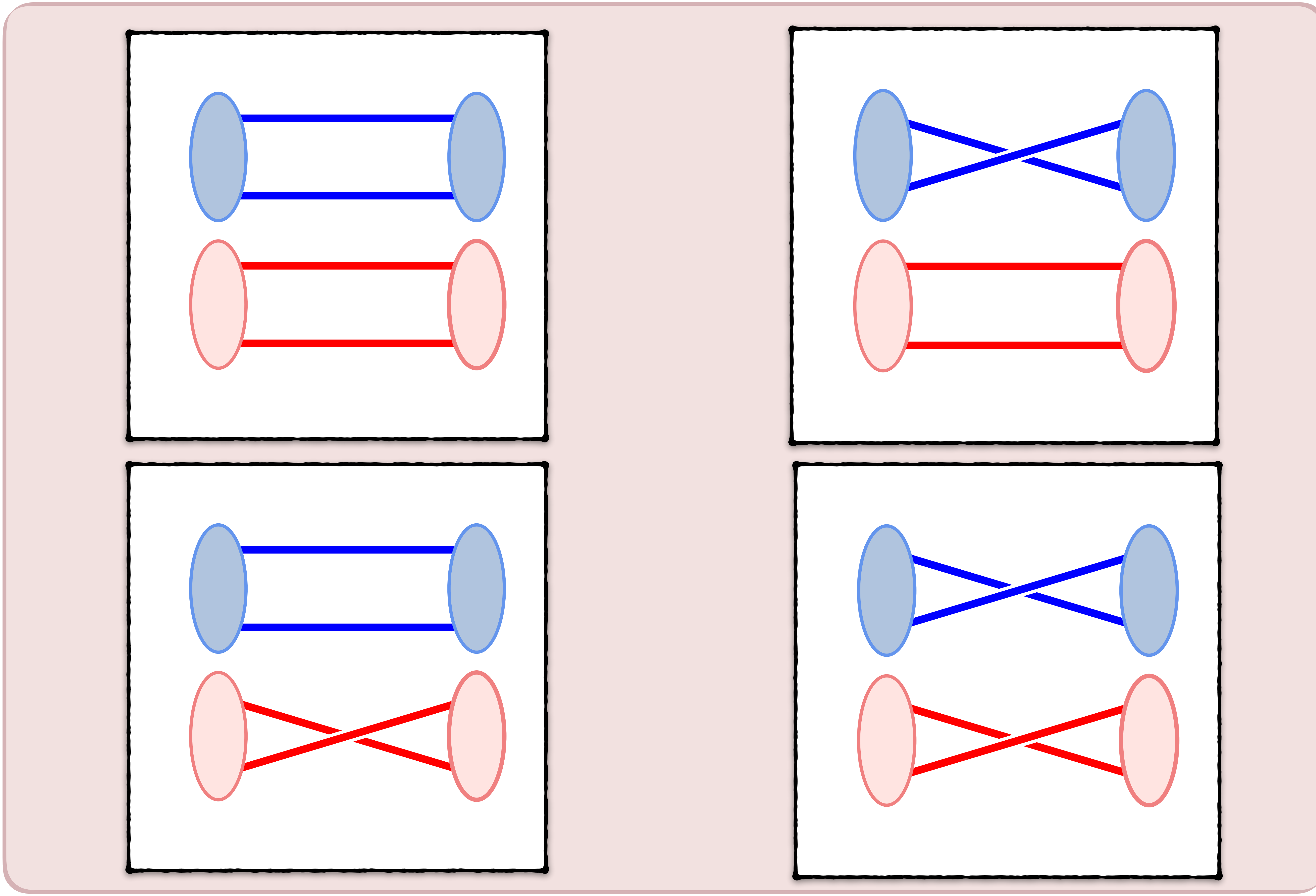


Fierz Relations

Table 1: Fierz relations in the $\bar{b}\bar{b}bb$ system relating the two-meson and the diquark-antidiquark bilinears.

J^{PC}	Diquark-AntiDiquark	Two-Meson
0^{++}	$\bar{3}_c \times 3_c$	$-\frac{1}{2}\lvert 0; \Upsilon\Upsilon\rangle + \frac{\sqrt{3}}{2}\lvert 0; \eta_b\eta_b\rangle$
0^{++}	$6_c \times \bar{6}_c$	$\frac{\sqrt{3}}{2}\lvert 0; \Upsilon\Upsilon\rangle + \frac{1}{2}\lvert 0; \eta_b\eta_b\rangle$
1^{+-}	$\bar{3}_c \times 3_c$	$\frac{1}{\sqrt{2}}\left(\lvert 1; \Upsilon\eta_b\rangle + \lvert 1; \eta_b\Upsilon\rangle\right)$
2^{++}	$\bar{3}_c \times 3_c$	$\lvert 2; \Upsilon\Upsilon\rangle$

Diquark-Antidiquark Wick Contractions



Correlator Data With Harmonic Oscillator

- Add to the NRQCD Hamiltonian the harmonic oscillator scalar potential

$$\delta H_{HO} = \frac{m_b \omega^2}{2} |\mathbf{x} - \mathbf{x_0}|^2$$

This would bind a hypothetical compact tetraquark more, relative to the lowest threshold, and hence this hypothetical tetraquark would show up more easily in our calculation

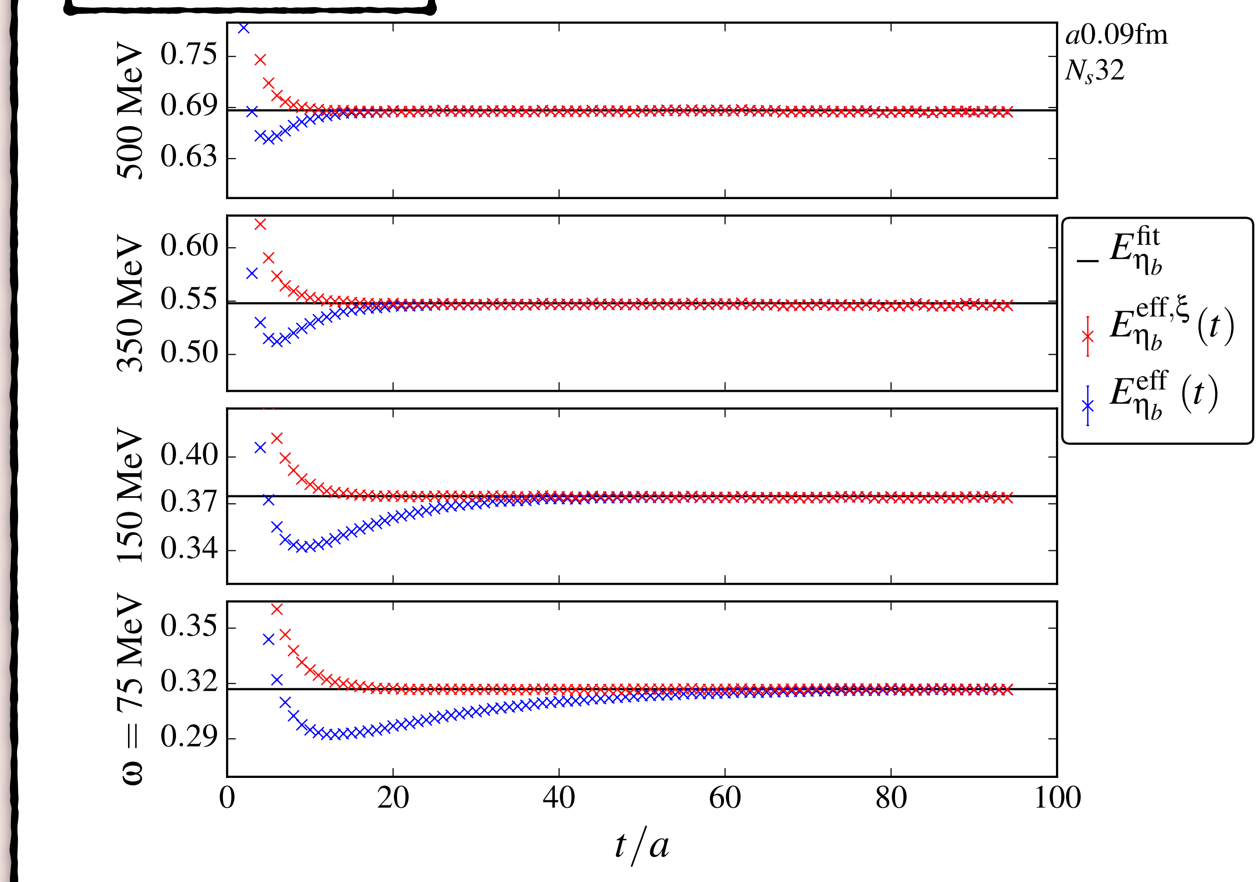
Correlator Data With Harmonic Oscillator

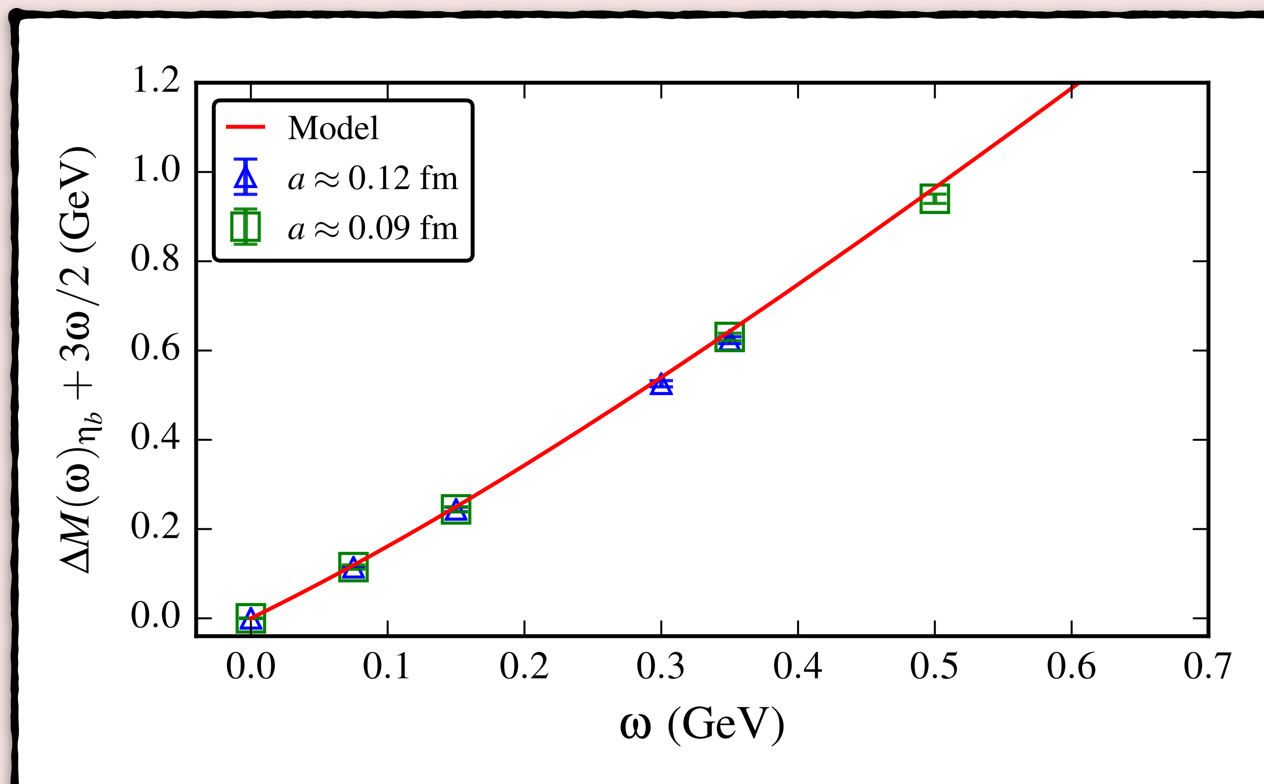
$$C^{JPC}_{i,j}(t,\omega) = \sum_n \frac{Z^i_n Z^{j,*}_n}{(1+e^{-2\omega t})^{\frac{3}{2}}} e^{-(M(\omega)_n + \frac{3}{2}\omega)t}$$

$$C^{JPC}_{i,j}(t,\omega) = \sum_{X_2} Z^i_{X_2} Z^{j,*}_{X_2} \left(\frac{2\omega\mu_r\pi^{-1}}{1-e^{-4\omega t}}\right)^{\frac{3}{2}} \times e^{-(M^S_1(\omega)+M^S_2(\omega)+3\omega)t} + \cdots$$

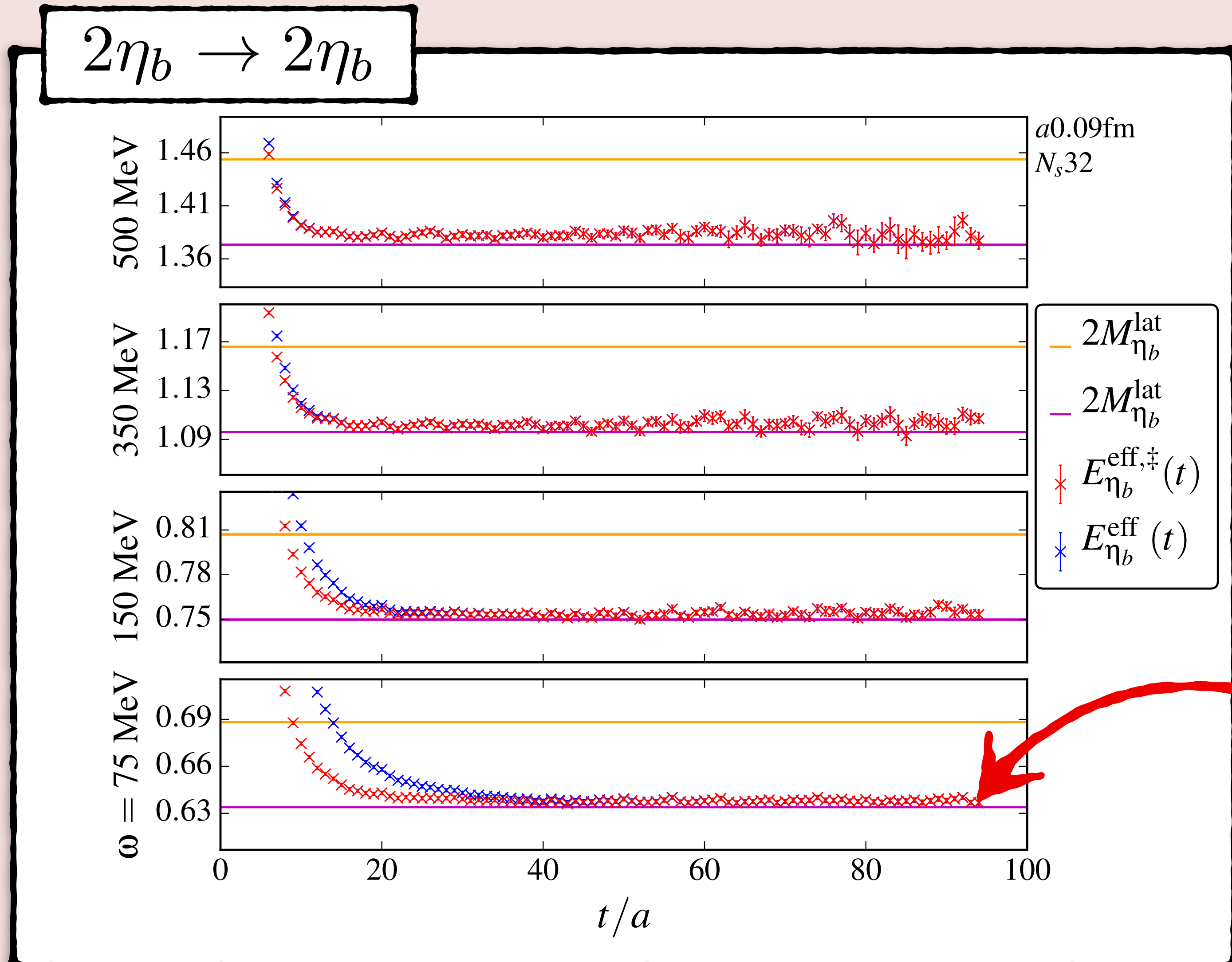
The single and two-particle correlators get modified in the presence of the HO

$\eta_b \to \eta_b$





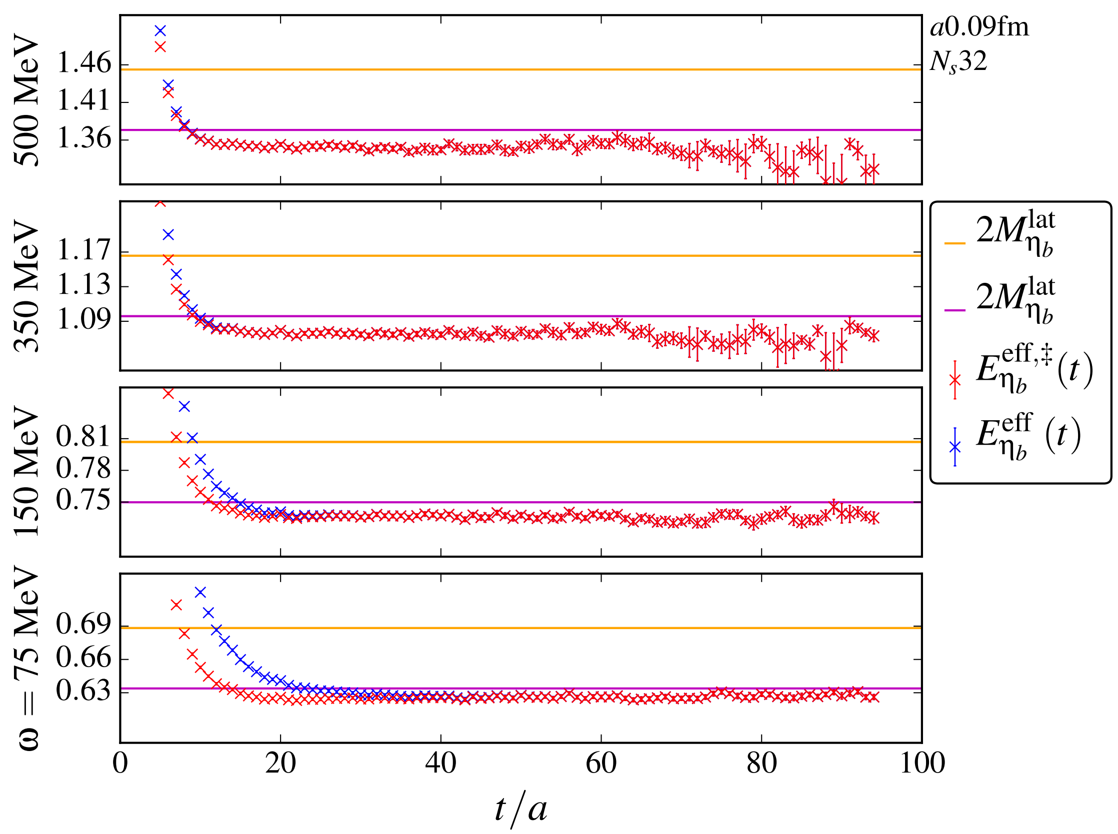
Correlator Data With Harmonic Oscillator



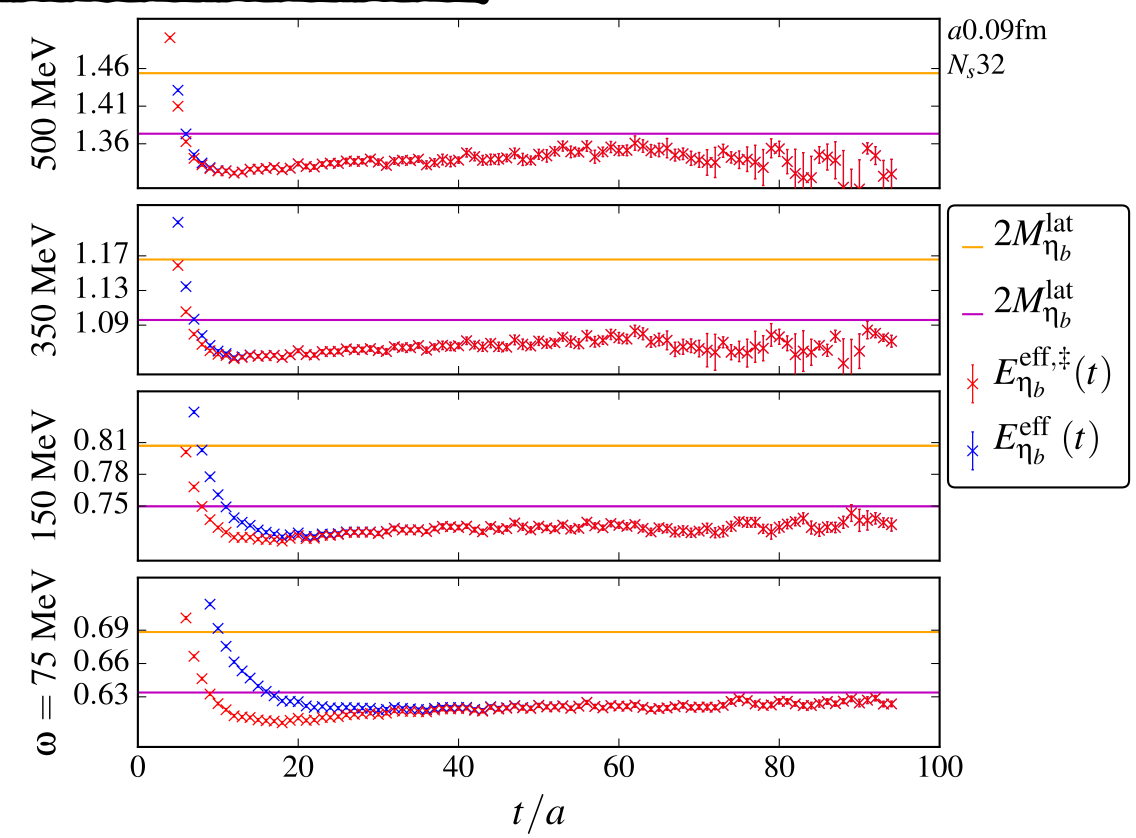
- No indication of a new bound state despite the addition of the scalar potential!!!

Individual Wick Contraction Correlator Data HO

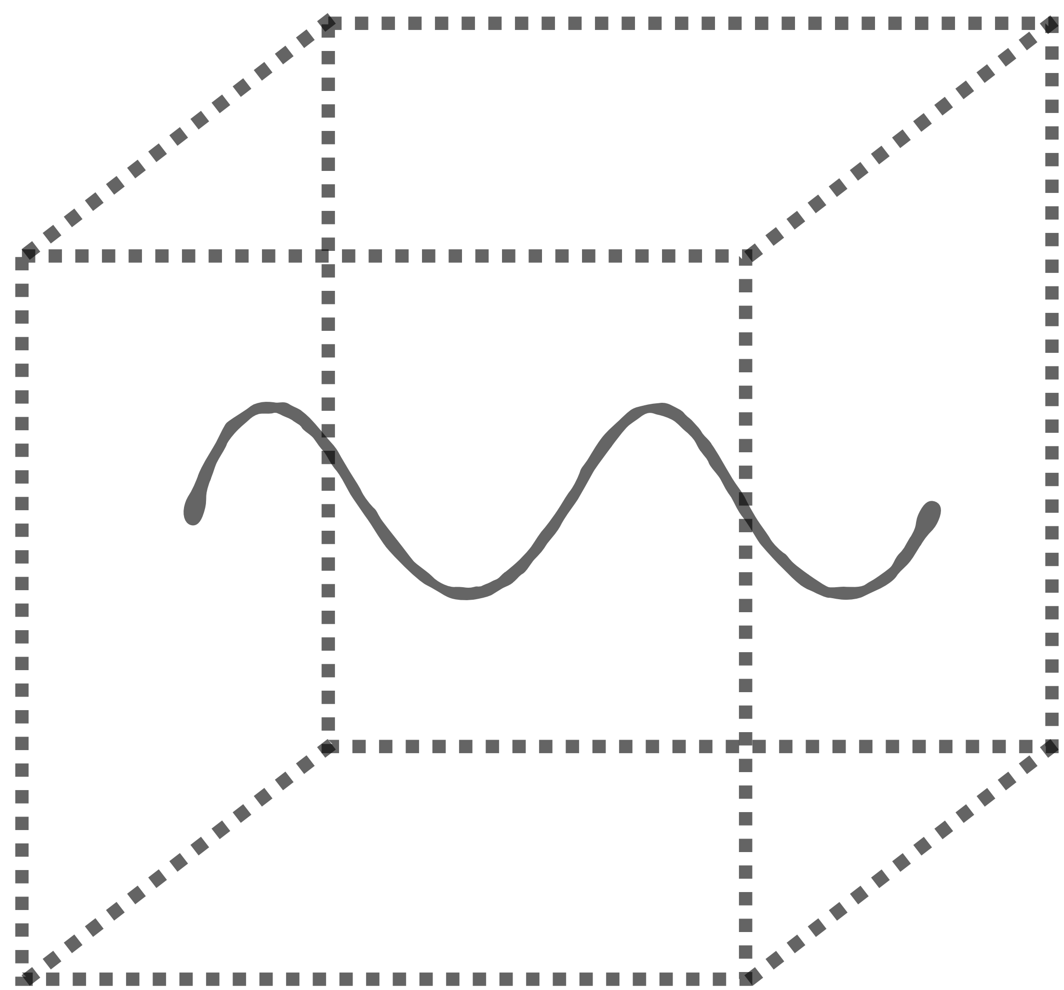
$D1 : 2\eta_b \to 2\eta_b$



$X2 : 2\eta_b \to 2\eta_b$



Bottomonium Elastic Scattering States in FV



"only a discrete number of modes can exist in a finite volume"

Spatially Periodic Box:

$$p_i \in \frac{2\pi}{L} \times \mathbb{Z}$$

Bottomonium Elastic Scattering States in FV

$$E(X^2) = \sqrt{M_1^2 + |\mathbf{k}|^2} + \sqrt{M_2^2 + |\mathbf{k}|^2}$$
$$\approx M_1^S + M_2^S + \frac{|\mathbf{k}|^2}{2\mu_r}$$

where we have defined the static, kinetic and reduced masses by M^S, M^K and $\mu_r = M_1^K M_2^K/(M_1^K + M_2^K)$

back-to-back states on our ensembles. As an example, examining the $a = 0.09$ fm ensemble, and taking $M_{\eta_b} = 9.399(2)$ GeV from the PDG [4], the smallest allowed $|\mathbf{k}|^2/2\mu_r \approx 20$ MeV or 0.0092 in lattice units with all other back-to-back states separated by multiples of

$$C(t) = \sum_{X^2} \int \frac{d^3k}{(2\pi)^3} Z_{X^2}(\mathbf{k})^2 e^{-E(X^2)t}$$

$$C(t) = \sum_{X^2} e^{-(M_1^S + M_2^S)t} \sum_k \left\{ \sum_{i=0}^{\infty} Z_{X^2}^{2l} \frac{|\mathbf{k}|^{2l}}{\mu_r^{2l}} \right\} e^{-\frac{|\mathbf{k}|^2}{2\mu_r}t} \tag{A5}$$

Bottomonium Elastic Scattering States in FV

When does the two-body scattering states look like a continuum within stat. precision?

$$I^{(l)}(t) = \frac{1}{\mu_r^{2l}} \int_{-\infty}^{\infty} d|\mathbf{k}||\mathbf{k}|^{2l+2} e^{-\frac{|\mathbf{k}|^2}{2\mu_r}t}$$

$$D^{(l)}(t) = \frac{1}{\mu_r^{2l}} \sum_{|\mathbf{k}|} |\mathbf{k}|^{2l+2} e^{-\frac{|\mathbf{k}|^2}{2\mu_r}t}.$$

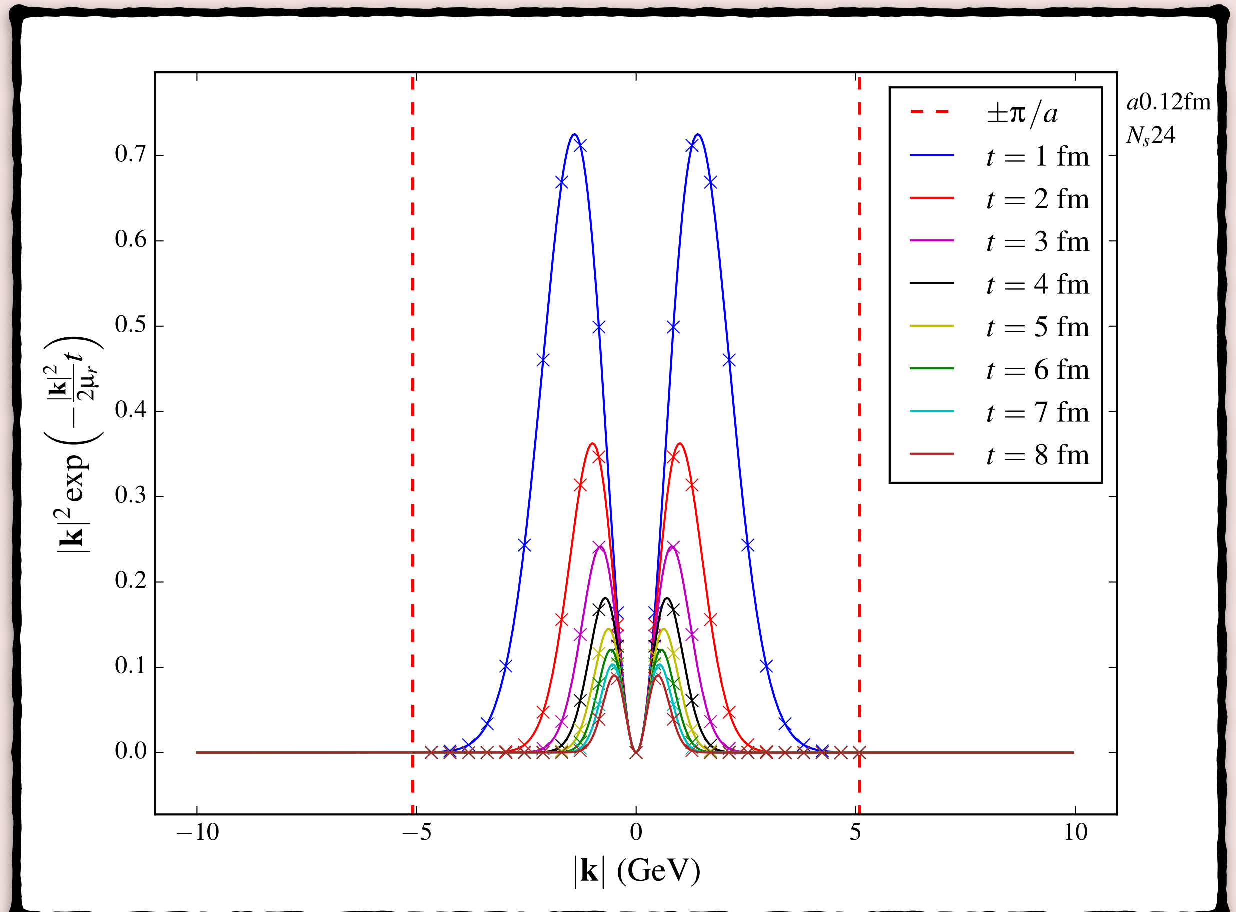
$$\left| \frac{\sum_{l=0}^{l_{max}} Z^{2,(l)} I^{(l)}(t) - \sum_{l=0}^{\infty} Z^{2,(l)} D^{(l)}(t)}{\sum_{l=0}^{l_{max}} Z^{2,(l)} I^{(l)}(t)} \right|$$

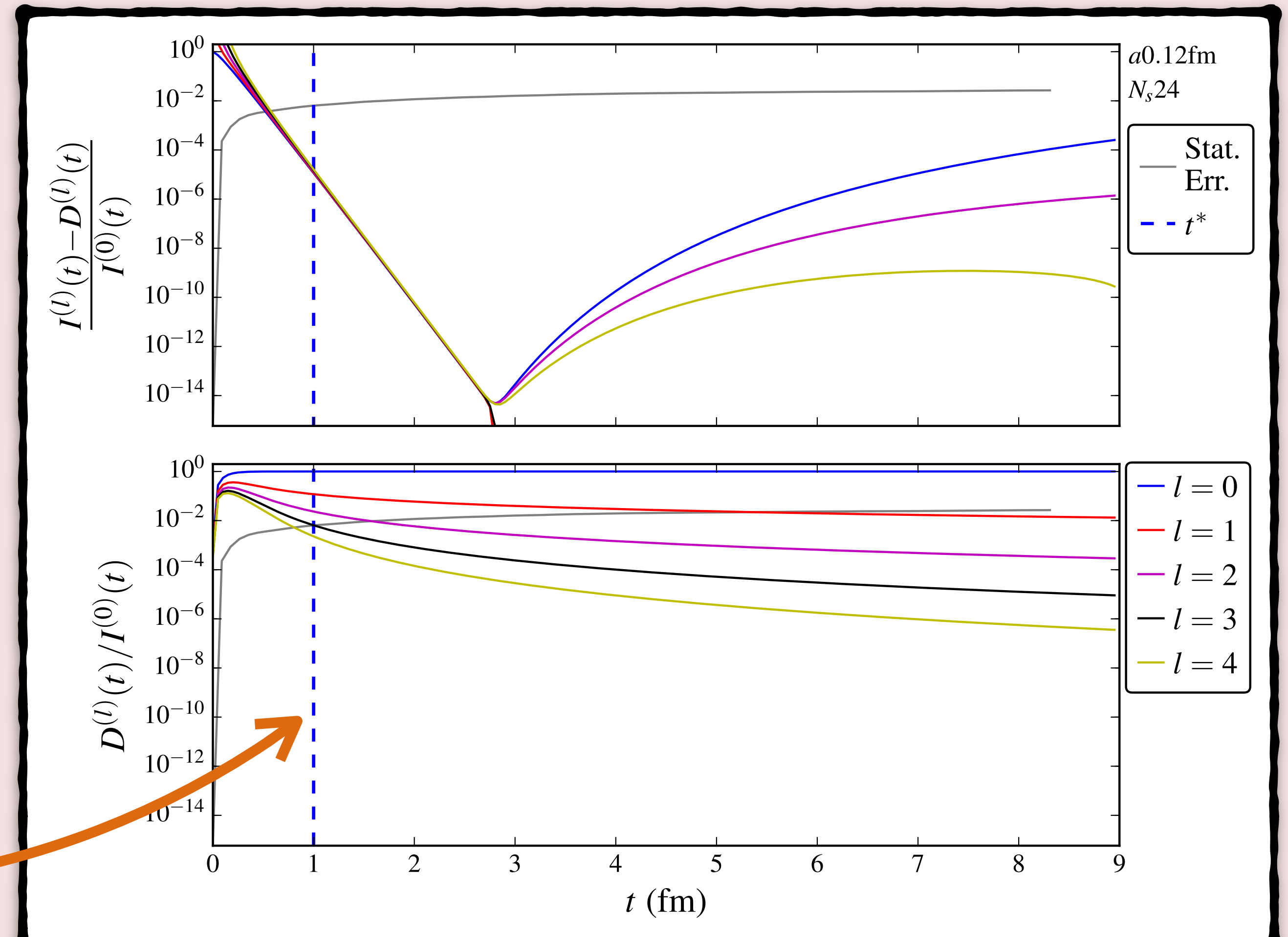
$$\leq \sum_{l=0}^{l_{max}} \frac{\left| I^{(l)}(t) - D^{(l)}(t) \right|}{I^{(0)}} + \sum_{l=l_{max}+1}^{\infty} \frac{D^{(l)}(t)}{I^{(0)}}$$

$$\leq \frac{\delta C(t)}{C(t)}$$

Bottomonium Elastic Scattering States in FV

When does the two-body scattering states look like a continuum within stat. precision?





After 1 fm!!