Università di Genova

INFN
Istituto Nazionale di Fisica Nucleare

Resummations and pair creation in the first quantised approach

S. Franchino-Viñas

Edinburgh, February 2025

Main
Organizers
Invited Speakers
Program
Application
Venue

zurück zu allen Veranstaltungen

New Trends in First Quantisation: Field Theory, Gravity and Quantum Computing

831. WE-Heraeus-Seminar

13 Apr - 17 Apr 2025

Where: Physikzentrum Bad Honnef

Scientific organizers: Dr. Sebastián Franchino-Viñas, HZDR Dresden-Rossendorf • Dr. James Edwards, U Plymouth/UK • Prof. Dr. Holger Gies, U Jena

Our current best understanding of nature at its fundamental level is based on field theories: Einstein's theory of gravity on cosmological scales, or quantum field theory describing interactions between elementary particles on sub-microscopic scales. Complementary to long-established conventional methods, several theoretical branches of fundamental physics based on the framework of first quantisation have blossomed during recent years, driven either by technological, observational or experimental advances.

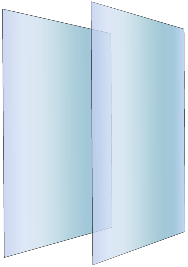
The aim of this workshop is to cultivate a common set of strategies among these research

Partly based on 2312.16303 [PLB], 2412.03340, 2501.17094: A. Boasso, F. Fecit, SFV, C. Garcia, D. Mazzitelli, S. Pla, V. Vitagliano, U. Wainstein.

Outline

Quantum field theory in backgrounds

Classical vacuum



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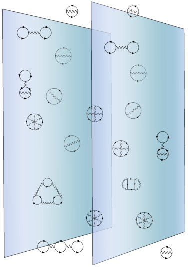
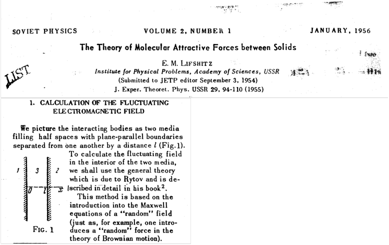
Quantum Field Theory vacuum





SOVIET PHYSICS VOLUME 2, NUMBER 1 JANUARY, 1956

The Theory of Molecular Attractive Forces between Solids

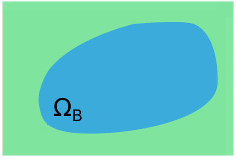
E. M. LIFSHITZ
Institute for Physical Problems, Academy of Sciences, USSR
(Submitted to JETP editor September 3, 1954)
J. Exper. Theoret. Phys. USSR 29, 94-110 (1955)

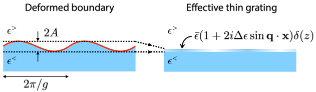
1. CALCULATION OF THE FLUCTUATING ELECTROMAGNETIC FIELD

We picture the interacting bodies as two media filling half spaces with plane-parallel boundaries separated from one another by a distance l (Fig.1).

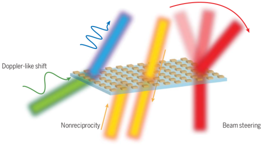
To calculate the fluctuating field in the interior of the two media, we shall use the general theory which is due to Rytov and is described in detail in his book[2]. This method is based on the introduction into the Maxwell equations of a "random" field (just as, for example, one introduces a "random" force in the theory of Brownian motion).

FIG. 1





Oue et al., 2021



Optical phenomena that can be realized with spatiotemporal metasurfaces. Wavelength

Shaltout et al., Science 364 (2019)

Examples

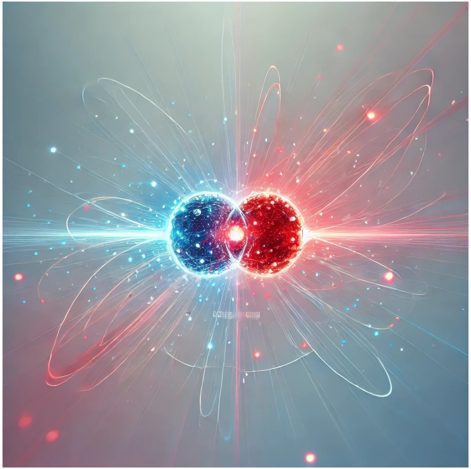
$$S := \frac{1}{2}\int \mathrm{d}^D x \sqrt{-g}\left[|(\nabla - \mathcal{A}(x))\phi(x)|^2 + H^2(x)|\phi(x)|^2\right].$$

(one-loop) Effective action $\rightarrow \Gamma = \mathrm{Log\,Det}\,[\mathcal{Q}]\,, \quad \mathcal{Q} := -(\nabla - A)^2 + V(x).$

- Casimir effect (Bordag's book)
- Semiclassical gravity:
  - Hawking's radiation
  - black hole's quantum hair (Calmet et al., 2023)
  - gravitational chiral gap (Flachi et al., 2014)
  - ...

- Strong-field QED (Fedotov et al., 2022):
  - birrefringence (Mignani et al., 2017, star polarimetry);
  - Creation of light pseudoscalars (Masso et al., 2004);
  - Schwinger effect (Domcke et al., 2020, axion inflation);
  - ...

Pair creation



- How many?
- Where?
- When?
- How?

⋮

Outline

Folgerungen aus der Diracschen Theorie des Positrons.

Von **W. Heisenberg** und **H. Euler** in Leipzig.

Mit 2 Abbildungen. (Eingegangen am 22. Dezember 1935.)

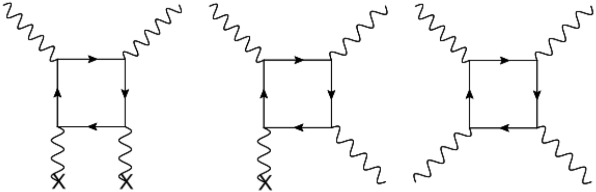
Aus der Diracschen Theorie des Positrons folgt, da jedes elektromagnetische Feld zur Paarerzeugung neigt, eine Abänderung der Maxwellschen Gleichungen des Vakuums. Diese Abänderungen werden für den speziellen Fall berechnet, in dem keine wirklichen Elektronen und Positronen vorhanden sind, und in dem sich das Feld auf Strecken der Compton-Wellenlänge nur wenig ändert. Es ergibt sich für das Feld eine Lagrange-Funktion:

$$\mathfrak{L} = \frac{1}{2}(\mathfrak{E}^2 - \mathfrak{B}^2) + \frac{e^2}{hc}\int_0^\infty e^{-\eta}\frac{d\eta}{\eta^3}\left\{ i\eta^2(\mathfrak{E}\mathfrak{B})\cdot\frac{\cos\left(\frac{\eta}{|\mathfrak{E}_k|}\sqrt{\mathfrak{E}^2-\mathfrak{B}^2+2i(\mathfrak{E}\mathfrak{B})}\right)+\text{konj}}{\cos\left(\frac{\eta}{|\mathfrak{E}_k|}\sqrt{\mathfrak{E}^2-\mathfrak{B}^2+2i(\mathfrak{E}\mathfrak{B})}\right)-\text{konj}} + |\mathfrak{E}_k|^2 + \frac{\eta^2}{3}(\mathfrak{B}^2-\mathfrak{E}^2)\right\}.$$

For small fields (*a* and *b* are used to parametrize the higher-order contributions to the Lagrangian.)

$$\mathcal{L} = \tfrac{1}{2}(\mathbf{E}^2 - c^2\mathbf{B}^2) + a(\mathbf{E}^2 - c^2\mathbf{B}^2)^2 + b(\mathbf{E}\cdot\mathbf{B})^2 + \cdots,$$

LbL scattering



- (g-2) experiments
- Lamb-shift
- Akhmadaliev et al. (2002, photon splitting $Bi_4Ge_3O_{12}$)
- ATLAS Collaboration (2017, Pb-Pb collisions)
- PVLAS experiment (1995, . . .)
- Optical polarimetry of a neutron star (Mignani et al., 2017)
- DeLLight experiment (IJCLab, running)

Scales

$$\omega^2/m_e^2 = O(10^{-6}),$$
$$E^2/E_c^2 = (10^{-8}),$$
$$E_c := \frac{m_e^2}{e} \approx 1.3 \times 10^{18}\ \text{V/m}.$$

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$$\mathfrak{L} = \frac{1}{2}(\mathfrak{E}^2-\mathfrak{B}^2) + \frac{e^2}{\hbar c}\int_0^\infty e^{-\eta}\frac{d\eta}{\eta^3}\left\{ i\eta^2(\mathfrak{E}\mathfrak{B})\cdot\frac{\cos\left(\frac{\eta}{|\mathfrak{E}_k|}\sqrt{\mathfrak{E}^2-\mathfrak{B}^2+2i(\mathfrak{E}\mathfrak{B})}\right)+\text{konj}}{\cos\left(\frac{\eta}{|\mathfrak{E}_k|}\sqrt{\mathfrak{E}^2-\mathfrak{B}^2+2i(\mathfrak{E}\mathfrak{B})}\right)-\text{konj}} + |\mathfrak{E}_k|^2 + \frac{\eta^2}{3}(\mathfrak{B}^2-\mathfrak{E}^2)\right\}.$$

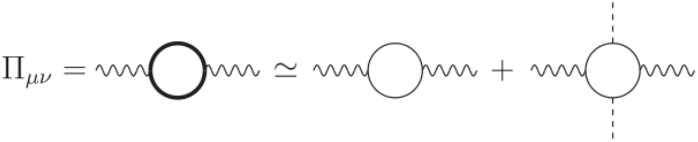
$$\mathcal{L} = \tfrac{1}{2}(\mathbf{E}^2 - c^2\mathbf{B}^2) + a(\mathbf{E}^2 - c^2\mathbf{B}^2)^2 + b(\mathbf{E}\cdot\mathbf{B})^2 + \cdots,$$

where a and b are used to parametrize the higher order contributions to the Lagrangian. The latter will be kept arbitrary in our analysis; however, keep in mind that the EH case corresponds to

$$\frac{b_{\mathrm{EH}}}{7} = a_{\mathrm{EH}} := \frac{2\alpha_{\mathrm{QED}}^2}{45 m^4}\,\frac{\epsilon_0^2\hbar^3}{c^5}.$$

Born-Infeld, axions, etc.

Scattering



$$\mathcal{L}_{\rm eff} := \tfrac{1}{2}\left[\mathbf{E}_x \cdot (\epsilon_0 \mathbb{I} + \delta\epsilon)\cdot \mathbf{E}_x - \mathbf{B}_x \cdot \left(\mu_0^{-1}\mathbb{I} - \frac{\delta\mu}{\mu_0^2}\right)\cdot \mathbf{B}_x\right] + \mathbf{E}_x \cdot \delta\boldsymbol{\Psi}\cdot \mathbf{B}_x,$$

where we have defined the vacuum polarization tensors

$$\begin{aligned}
\delta\epsilon^{ij} &:= 2\left[2a(\mathbf{E}_L^2 - \mathbf{B}_L^2 c^2)\delta^{ij} + bB_L{}^i B_L{}^j + 4aE_L{}^i E_L{}^j\right],\\
\delta\boldsymbol{\Psi}^{ij} &:= 2(b\mathbf{E}_L \cdot \mathbf{B}_L \delta^{ij} + bB_L{}^i E_L{}^j - 4ac^2 B_L{}^j E_L{}^i),\\
\frac{\delta\mu^{ij}}{\mu_0^2} &:= 2\left[2ac^2(\mathbf{B}_L^2 c^2 - \mathbf{E}_L^2)\delta^{ij} + 4ac^4 B_L{}^i B_L{}^j + bE_L{}^i E_L{}^j\right]\\
&= \delta\epsilon^{ij}\{E_i \leftrightarrow c\,B_i\}.
\end{aligned}$$

Maxwell eqs. $\Longrightarrow$

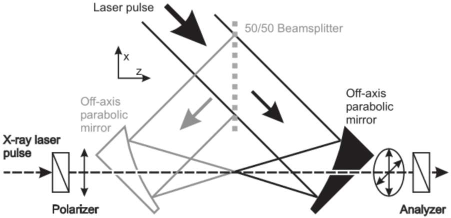
$$\Box\mathbf{D} = \nabla \times [\nabla \times (\mathbf{D} - \mathbf{E})] + \partial_t [\nabla \times (\mathbf{H} - \mathbf{B})] = \mathbf{J}^{\rm eff} \Longrightarrow$$ intense fields, energies as large as possible.

$\mathbf{E}_x = \text{XFEL} \sim \text{keV}$, $\mathbf{E}_L = \text{high power optical lasers} \sim \text{eV}$; for plane waves

Effects

$$\mathfrak{A} \approx 8\pi^2 \mathfrak{F} E_1 E_2 \omega_{\text{out}}^2 \delta^3(\mathbf{k}_{\text{out}} - \mathbf{k}_{\text{in}} - \mathbf{k}_1 + \mathbf{k}_2),$$

where $\mathfrak{F}(\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_{\text{in}}, \mathbf{e}_{\text{out}}, \mathbf{n}_1, \mathbf{n}_2, \mathbf{n}_{\text{in}}, \mathbf{n}_{\text{out}})$ is a purely algebraic expression of the four polarization vectors $\mathbf{e}_l$ and propagation direction unit vectors $\mathbf{n}_l = \mathbf{k}_l/\omega_l$.



$$\mathfrak{F} = 16a(\mathbf{e}_{\text{in}} \cdot \mathbf{e}_1)(\mathbf{e}_{\text{out}} \cdot \mathbf{e}_1) + 4b\mathbf{e}_{\text{in}} \cdot (\mathbf{n}_{\text{in}} \times \mathbf{e}_1)\, \mathbf{e}_{\text{out}} \cdot (\mathbf{n}_{\text{in}} \times \mathbf{e}_1)\,.$$

$\Downarrow$

Birefringence

(Heinzl et al. 2006)

Folgerungen aus der Diracschen Theorie des Positrons.

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- For small fields (a and b are used to parametrize the higher-order contributions to the Lagrangian.)

$$\mathcal{L} = \tfrac{1}{2}(\mathbf{E}^2 - c^2\mathbf{B}^2) + a(\mathbf{E}^2 - c^2\mathbf{B}^2)^2 + b(\mathbf{E}\cdot\mathbf{B})^2 + \cdots .$$

- For large fields, trigonometric function in the denominator!

On the heat kernel

Consider the Euclidean effective action

$$\Gamma = -\int_0^\infty \frac{\mathrm{d}\tau}{\tau} \int \mathrm{d}^d x\, K(x,x;\tau),$$

where we have introduced the heat kernel

$$\begin{aligned} K(x,x';\tau) &= e^{-\tau\mathcal{Q}}(x,x';\tau), \quad \mathcal{Q} := -(\nabla - A)^2 + V(x), \\ &= \sum_{n=0}^{\infty} \phi_n^* e^{-\tau\lambda_n}\phi_n, \quad \mathcal{Q}\phi_n = \lambda_n\phi_n \end{aligned} \tag{1}$$

which satisfies the PDE (+ appropriate initial condition)

$$[\partial_\tau + \mathcal{Q}]K(x,x';\tau) = 0.$$

Can one hear the shape of a drum?

The Schwinger–DeWitt expansion seems natural; if M is the mass, $\sigma = \text{dist}^2/2$,

$$K(x, x'; \tau) = \frac{1}{(4\pi)^{D/2}} e^{-\tau M^2 - \frac{\sigma(x,x')}{2\tau}} \widetilde{\Omega}(x, x'; \tau),$$

where $\widetilde{\Omega}$ in general has an asymptotic expansion in terms of geometric invariants:

$$\widetilde{\Omega}(x, x'; \tau) =: \sum_{j=0}^{\infty} a_j(x, x') \tau^{j-D/2}.$$

Examples (scalar field):

$$a_0(x, x') = 1, \quad a_1(x, x) = V(x) + R(x).$$

Heat kernel resummations

First consider a Yukawa coupling. We can resum an infinite tower of invariants (SFV et al., 2023):

$$\mathcal{K}_Y = \{V,\ \mathrm{tr}\left[(\nabla_{\alpha\beta}V)^j\right],\ \nabla^\alpha V(\nabla_{\alpha\beta}V)^j\nabla^\beta V, \quad j \geq 0\}.$$

The proof is based on an expansion beyond the Schwinger-DeWitt Ansatz:

$$K_Y(x,x';\tau) =: \frac{1}{(4\pi)^{d/2}} \frac{e^{-\tau V(x') - \frac{1}{4}\tilde{\sigma}^\mu A^{-1}_{\mu\nu}(x';\tau)\tilde{\sigma}^\nu - C(x';\tau)}}{\det^{1/2}(\tau^{-1}A(x';\tau))} \Omega_Y(x,x';\tau),$$

where $A_{\mu\nu}$, C depend on V; $\tilde{\sigma}$ is a deformed square root of Synge's function (distance).

$\Omega_Y(x,x;\tau)$ then does not depend on $\mathcal{K}_Y$.

The diagonal reads

$$K_Y(x,x;\tau) = \frac{e^{-\tau V + \nabla^\alpha V \left[\gamma^{-3}\left(\gamma\tau - 2\tanh(\frac{1}{2}\gamma\tau)\right)\right]_{\alpha\beta}\nabla^\beta V}}{(4\pi)^{d/2}\,\det^{1/2}\left((\gamma\tau)^{-1}\sinh(\gamma\tau)\right)}\Omega_Y(x,x;\tau),$$

$$\gamma^2_{\alpha\beta} := 2\nabla_{\alpha\beta}V,$$

$$\Gamma \propto \int_0^\infty \frac{\mathrm{d}\tau}{\tau} K(x,x;\tau).$$

Wick-rotating to Minkowski, depending on the features of V, the effective action might acquire an imaginary part (nontrivial vacuum persistence amplitude).

A corollary is a (generalization and) proof of the conjecture in Navarro-Salas and Pla, 2020:

"in (S)QED, $d = 4$ the invariants $F^{\mu\nu}F_{\mu\nu}$ and $F^{\mu\nu}\tilde{F}_{\mu\nu}$ appear only in the prefactor of the effective action".

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The proof goes as follows (in the Fock–Schwinger gauge)

$$[\partial_\tau - \nabla^2 + 2A_\mu(x)\nabla^\mu + V_{EM}(x)]K_{EM}(x, x'; \tau) = 0,$$

where the effective scalar potential is given by

$$V_{EM}(x) := \nabla_\mu A^\mu - A_\mu A^\mu,$$

and one then resums (at the local level in the HK; derivatives are enclosed in Ω)

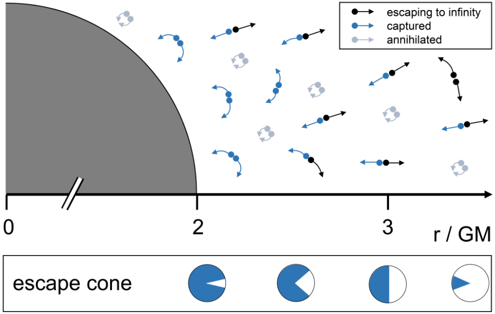
$$\mathcal{K}_{EM} = \{(F^n)^\mu{}_\mu, n \geq 0\}.$$

Outline

Gravitational pair creation

The gravitational case is not an exception; just more involved. Let us consider not strong fields, but weak, rapidly varying ones.

It has recently proposed that in this scenario a new type of Hawking radiation is generated [Wondrak et al., 2024]; comments by Navarro-Salas and Pla, Akhmedov, Schubert et al, . . .



[Wondrak et al. 2024]

Using a Avramidi–Barvinsky–Vilkovisky resummation for the heat kernel, the effective action one gets is [SFV et al. 2024]

$$\Gamma^{(2)}_{\mathrm{E}} := \frac{1}{2(4\pi)^{D/2}} \int \mathrm{d}^D\mathbf{x}\,\sqrt{g}\,\Big[\sigma^2\beta^{(1)}(\Box_E)\sigma^2 - 2\sigma^2\beta^{(2)}(\Box_E)R + \xi\left(\sigma^2\beta^{(1)}(\Box_E)R + R\beta^{(1)}(\Box_E)\sigma^2\right) + R\left(\xi^2\beta^{(1)}(\Box_E) - 2\xi\beta^{(2)}(\Box_E) + \beta^{(4)}(\Box_E)\right)R + R_{\mu\nu}\beta^{(3)}(\Box_E)R^{\mu\nu}\Big] + \cdots .$$

The form factors involve nonanalytic functions,

$$\beta^{(i)}(\Box_E) := \frac{(-1)^{\frac{D}{2}}}{2\Gamma\left(\frac{D}{2}-1\right)} \int_0^1 \mathrm{d}z\, f^{(i)}(z)\left(m^2 - \gamma\Box_E\right)^{\frac{D}{2}-2} \begin{cases} \log\left(\frac{m^2-\gamma\Box_E}{\mu^2}\right), & \text{D even,} \\ (-1)^{-\frac{1}{2}}\pi, & \text{D odd,} \end{cases}$$

(μ is a scale of mass dimensions introduced by dimensional regularization, $f^{(i)}(z)$ polynomials in z, $\gamma := \frac{1-z^2}{4}$).

Wick-rotating, the probability of pair creation is

$$P = \frac{\pi^{1-D/2}}{2\Gamma\left(\frac{D}{2}-1\right)} \int \frac{\mathrm{d}^D p}{(4\pi)^D}\, \Theta\left(-p^2 - 4m^2\right)$$
$$\times \left(\alpha^{(1)}\sigma^2(-p)\,\sigma^2(p) + \alpha^{(2)} R(-p)\,\sigma^2(p) + \alpha^{(3)} R(-p)\,R(p) + \alpha^{(4)} R^{\mu\nu}(-p)\,R_{\mu\nu}(p)\right),$$

where the $\alpha^{(i)}$ functions are defined as

$$\alpha^{(1)} := \int_0^{z_{\text{max}}} \mathrm{d}z\, \left(-m^2 - \gamma p^2\right)^{\frac{D}{2}-2},$$
$$\vdots$$

The mass threshold arises from the analytic structure present in the resummation!

We can do better. In $D=4$ the electric and magnetic parts of the Weyl tensor can be defined as [Matte 1953]

$$\begin{cases} E_{ij} = \frac{1}{4}\epsilon_{abi}\epsilon_{cdj}C^{abcd}, \\ B_{ij} = -\frac{1}{2}\epsilon_{iab}C_{0j}{}^{ab}, \end{cases}$$

Using this decomposition, the $D=4$ massless pair production probability can be rewritten as

$$P = \frac{1}{240\pi}\int \frac{\mathrm{d}^4 p}{(2\pi)^4}\,\Theta\left(-p^2\right)\left[\frac{15}{2}\left(\xi-\frac{1}{6}\right)^2 R\left(-p\right)R\left(p\right)+E^{ij}\left(-p\right)E_{ij}\left(p\right)-B^{ij}\left(-p\right)B_{ij}\left(p\right)\right].$$

This is the analog of the usual QED textbook formula for the imaginary part of effective action, depending on the invariant $|F_{\mu\nu}|^2 = |\mathbf{E}\left(-p\right)|^2 - |\mathbf{B}\left(-p\right)|^2$.

Exercise for the participants: $\Theta(-p^2)(|E_{ij}|^2 - |B_{ij}|^2) \geq 0$.

Outline

Assisted pair creation

Actually we can catalyze the effect by using a mix of strong and rapidly varying potentials (mixed Toms–Parker and Barvinsky–Vilkovisky resummations).
For a Yukawa potential [Torgrimsson et al. 2017],

$$V(x) = V(x) + \epsilon\,\mathcal{V}(x)\ ,$$

The diagonal heat kernel can be written in terms of a generating functional (with higher derivatives of V) [SFV et al., to appear],

$$K(T) = \int \mathrm{d}^D\bar{x}\,\mathrm{e}^{-TV}\,\mathrm{e}^{-\int_t L_{\rm int}\left(-\frac{\delta}{\delta\eta(t)}\right)}\,Z[\eta](\bar{x};T)\Big|_{\eta=\partial V}\ .$$

while the expansion of Z is in powers of $\mathcal{V}$:

$$Z = Z_0 + \epsilon Z_1 + \epsilon^2 Z_2 + \dots\ .$$

For an arbitrary order the computation is tractable in the worldline formalism:

$$Z_n[\eta](\bar{x};T) = (-1)^n \int \left[\prod_{j=1}^{n} \hat{\mathrm{d}}q_j \tilde{\mathcal{V}}(q_1)\right] \int_{t_1\cdots} \oint \mathcal{D}s\,\mathrm{e}^{-i\sum_{l=1}^{n} q_l\cdot\bar{x} - \int_t \frac{\dot{s}^2}{4} + s^\mu s^\nu \Omega_{\mu\nu} + s^\mu \eta_\mu + is_\mu \sum_{i=1}^{n} q_i^\mu \delta(\tau - t_i)}\ .$$

Example: "Constant field strength" + Gaussian pulse;

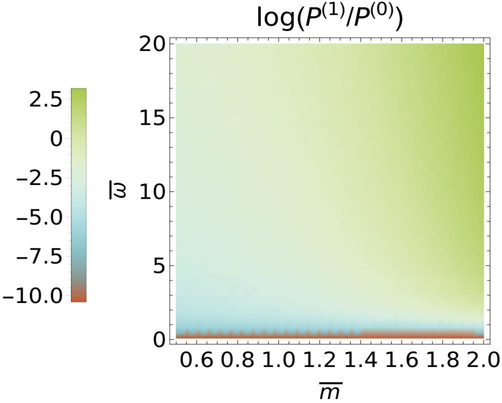
$$V_0(x) = v^2\, x_0^2 + m^2 \ , \quad v = \text{const} \ , \qquad \mathcal{V}(x) = \tfrac{\omega^2}{\pi}\, \mathrm{e}^{-\omega^2 x_0^2} \ .$$

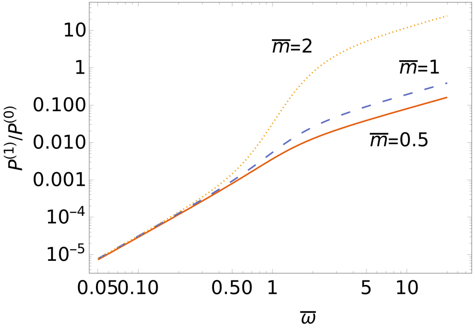
We get a modified analytic structure

$$\Gamma_{\mathrm{M}}^{(1)} = -\frac{\mathrm{Vol}_{(\mathrm{D}-1)}\epsilon\omega_{\mathrm{M}}}{2(4\pi)^{\frac{D}{2}}} \int_0^\infty \frac{\mathrm{d}\,T}{T^{\frac{(D-1)}{2}}} \frac{\mathrm{e}^{-m^2 T}}{\sin\left(v_{\mathrm{M}}\, T + i0\right)} \left(\frac{1}{\omega_{\mathrm{M}}^2} + \frac{\cot\left(v_{\mathrm{M}}\, T\right)}{v_{\mathrm{M}}} - i0 \right)^{-\frac{1}{2}} .$$

Exponential enhancement $e^{-\bar{m}^2 \bar{T}^*}$; Dimensionless variables $\bar{\omega} := \frac{\omega}{v_{\rm M}^{1/2}}$, $\bar{m} := \frac{m}{v_{\rm M}}$.

$$\mathrm{Im}\,\Gamma^{(1)}_{\rm M} = \frac{\pi \mathrm{Vol}_{(\mathrm{D}-1)}}{2(2\pi)^D} v_{\rm M}^{\frac{(D-1)}{2}} \epsilon \mathrm{e}^{-\bar{m}^2 \bar{T}^*} \bar{\omega} \int_0^{\pi - \bar{T}^*} \mathrm{d}T \frac{\mathrm{e}^{-\bar{m}^2 T}}{\sin\left(T + \bar{T}^*\right)}$$
$$\times \left|\bar{\omega}^{-2} + \cot(T + \bar{T}^*)\right|^{-\frac{1}{2}} \Phi\left(-\mathrm{e}^{-\pi \bar{m}^2}, \frac{D-1}{2}, \frac{T + \bar{T}^*}{\pi}\right),$$





Outline

Outlook

- Towards detection of the Schwinger effect: spectrum, real fields, ...
- What can we say about $G(x, x')$? (Locally-constant field approximation in SFQED and further resummations).
- Trace anomaly [Chernodub '23]? gravitational waves?
- Fermions and torsion? Paper coming.
- Self-interacting fields?
- Mixed backgrounds: gauge, gravitational ....

$$\vdots$$