GRAVITATIONAL WAVES FROM WORLDLINE QUANTUM FIELD THEORY

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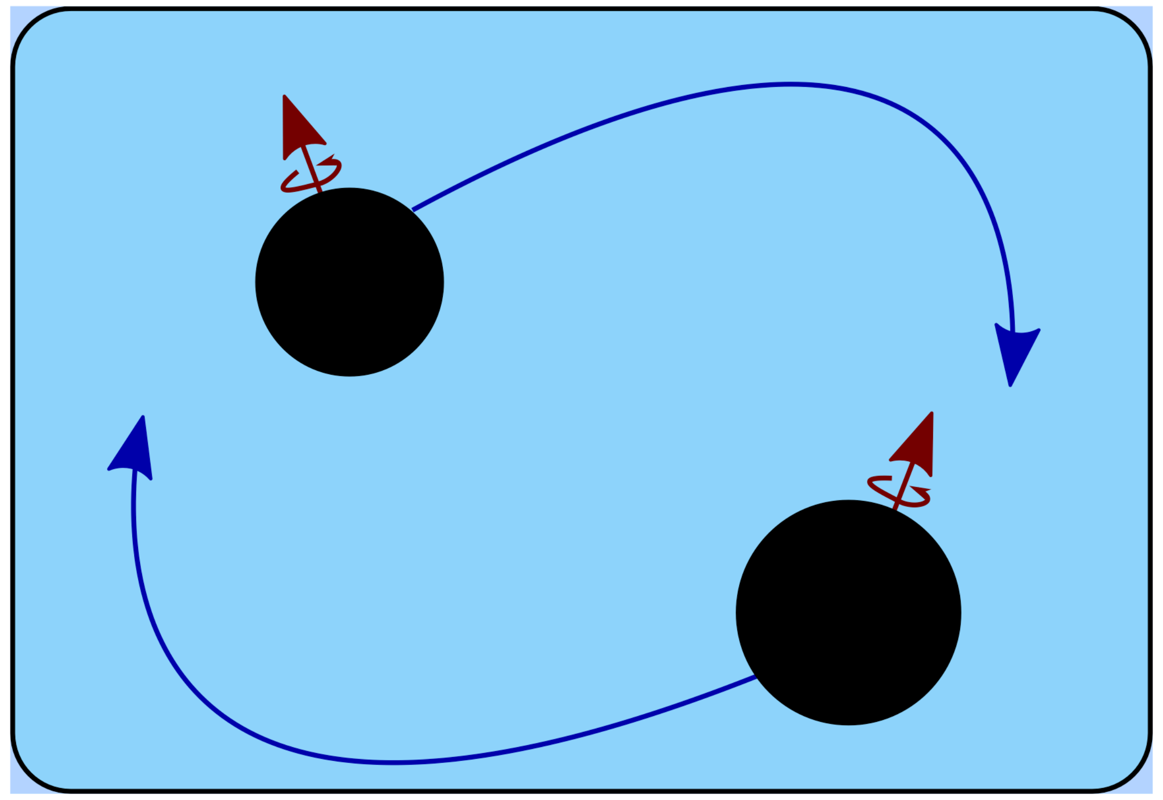
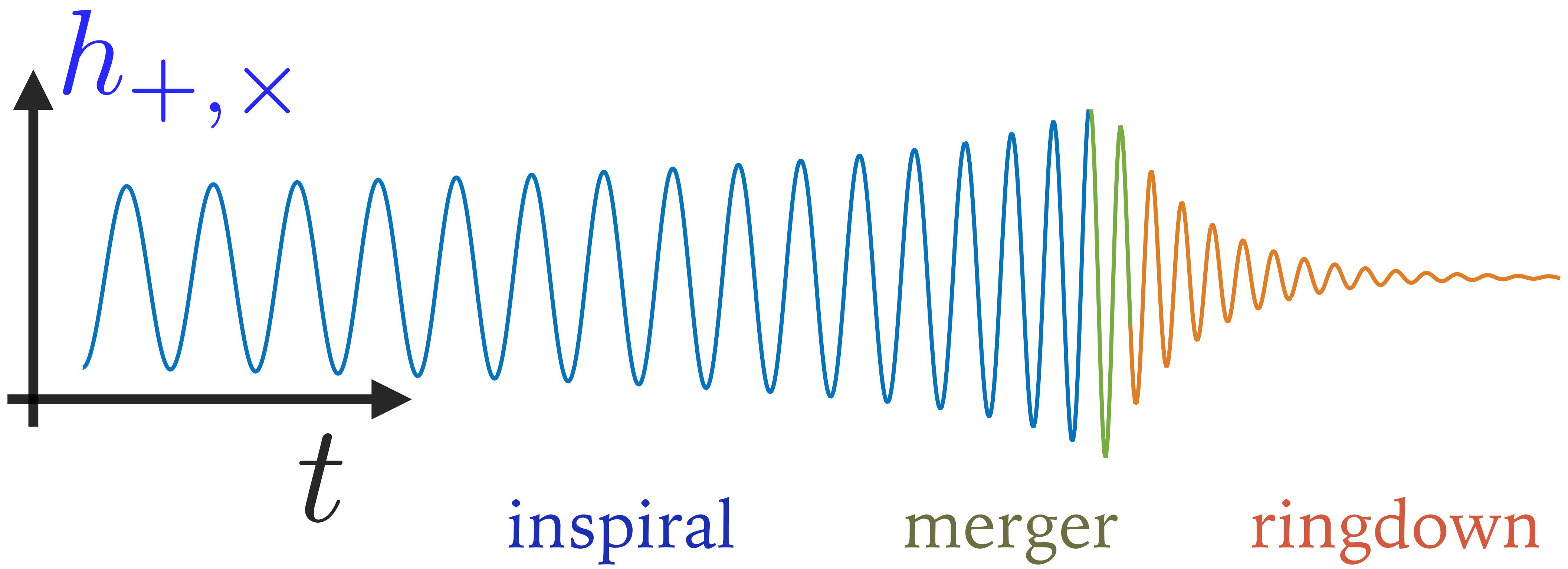
THE GRAVITATIONAL TWO-BODY PROBLEM

The **next generation** of gravitational wave detectors — **LISA, Einstein Telescope, Cosmic Explorer** — will have **100x signal-to-noise ratio!** We require **high-precision gravitational wave templates:**





$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

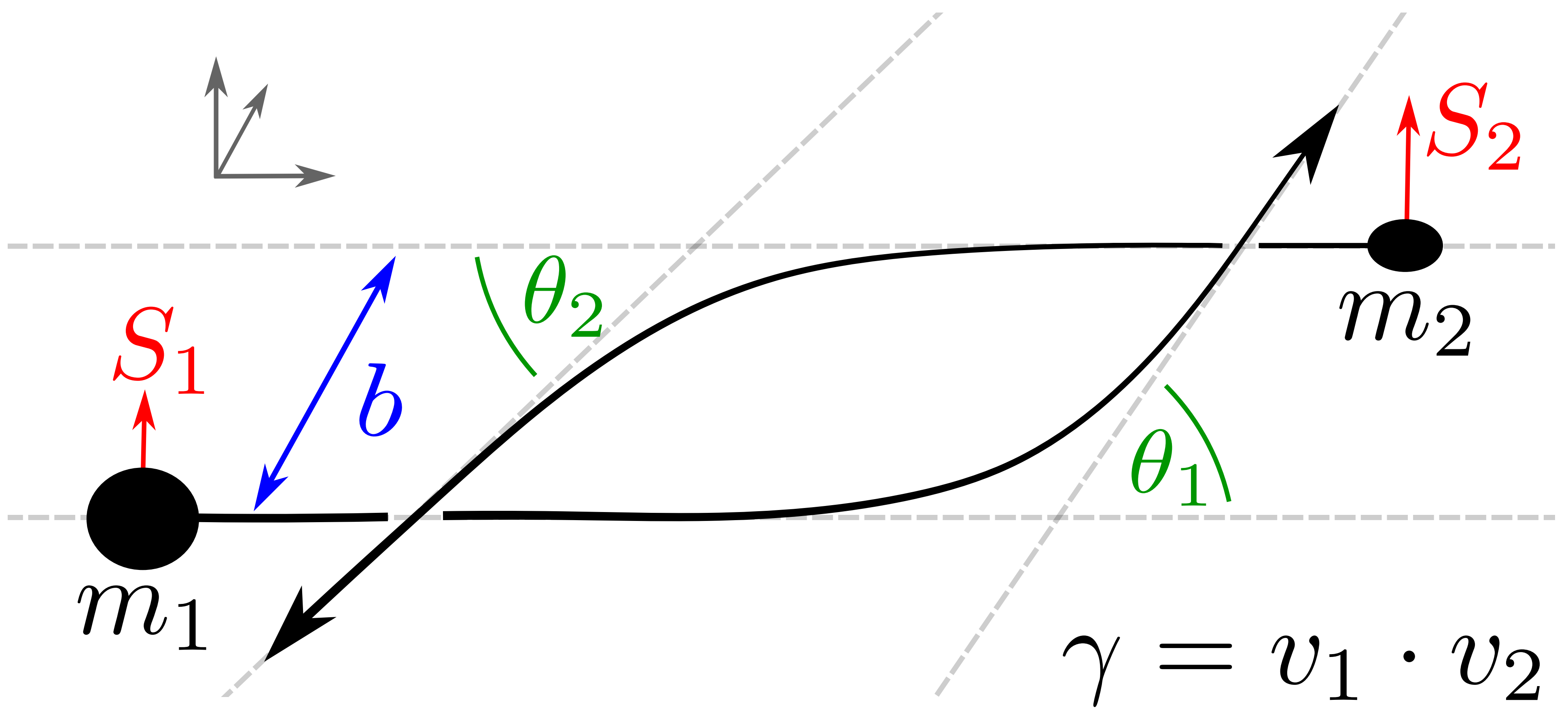
$$\ddot{x}^{\mu} + \Gamma^{\mu}{}_{\nu\rho} \dot{x}^{\nu} \dot{x}^{\rho} = 0$$

Einstein's equations **cannot be solved exactly**. Two ways to approximate:

- **Numerical relativity**: good for the **merger** (**strong fields, short duration**)
- **Perturbation theory**: good for the **inspiral** (**weak fields, many cycles**)

Our approach: use Quantum Field Theory (QFT) to perturbatively solve the classical 2-body problem

FROM BLACK HOLE SCATTERING TO GRAVITATIONAL WAVEFORMS



$$\Delta p_1^\mu = p_1(\tau = +\infty) - p_1(\tau = -\infty)$$

$$\Delta S_1^\mu = S_1(\tau = +\infty) - S_1(\tau = -\infty)$$

$$\Delta p_1^\mu = G\Delta p_1^{(1)\mu} + G^2\Delta p_1^{(2)\mu} + G^3\Delta p_1^{(3)\mu} + \cdots$$

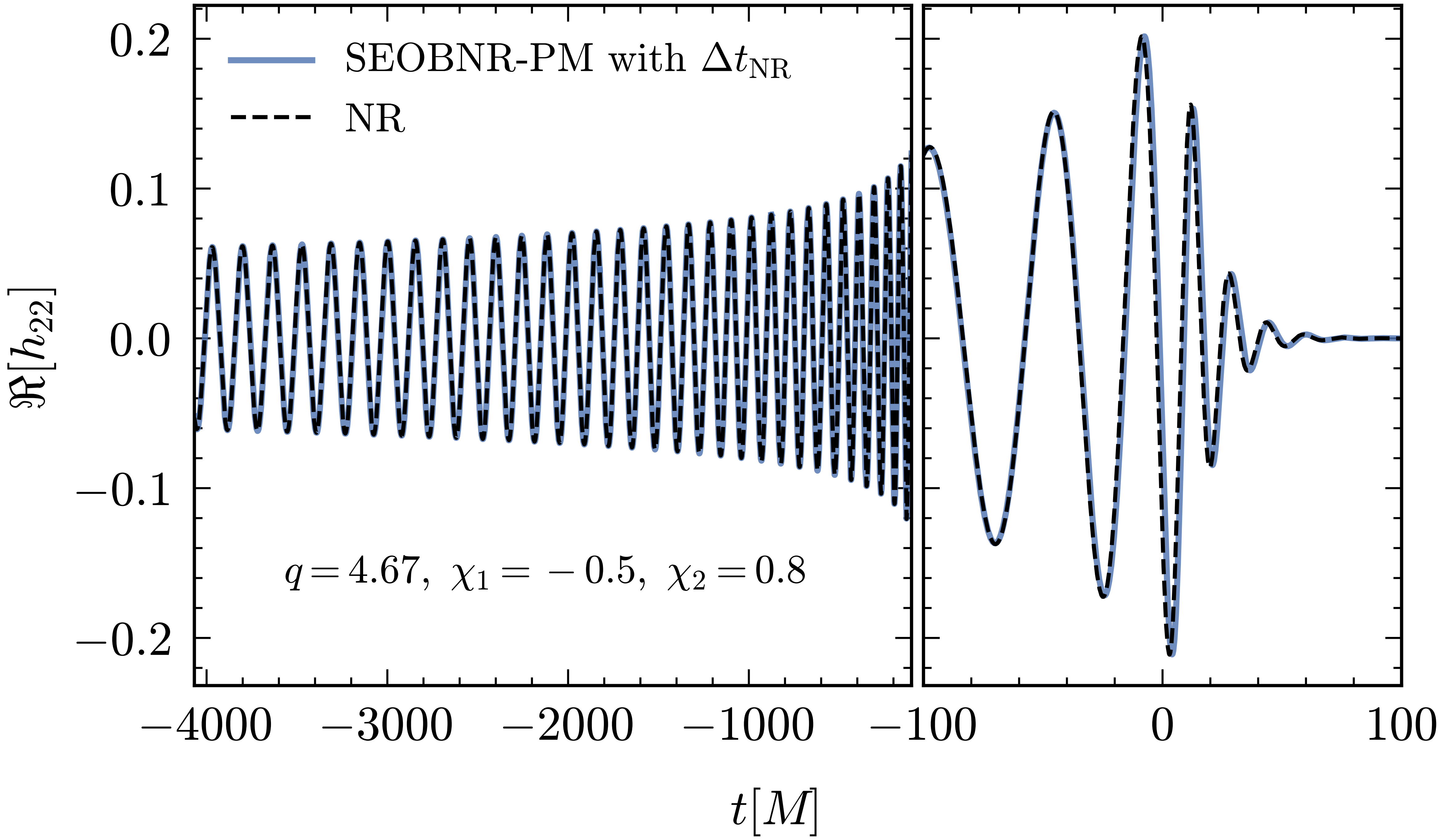
1PM **2PM** **3PM**

Good reasons to focus on **black hole scattering**:

- Natural post-Minkowskian (PM) expansion in G, allows for **arbitrary fast velocities**
- **Gauge-invariant scattering observables**: impulse, spin kick, scattering angle, fluxes, …
- Mature **QFT-based scattering technology**

Scattering observables are **input data for Effective-One-Body (EOB) gravitational waveform models, e.g. SEOBNR-PM**

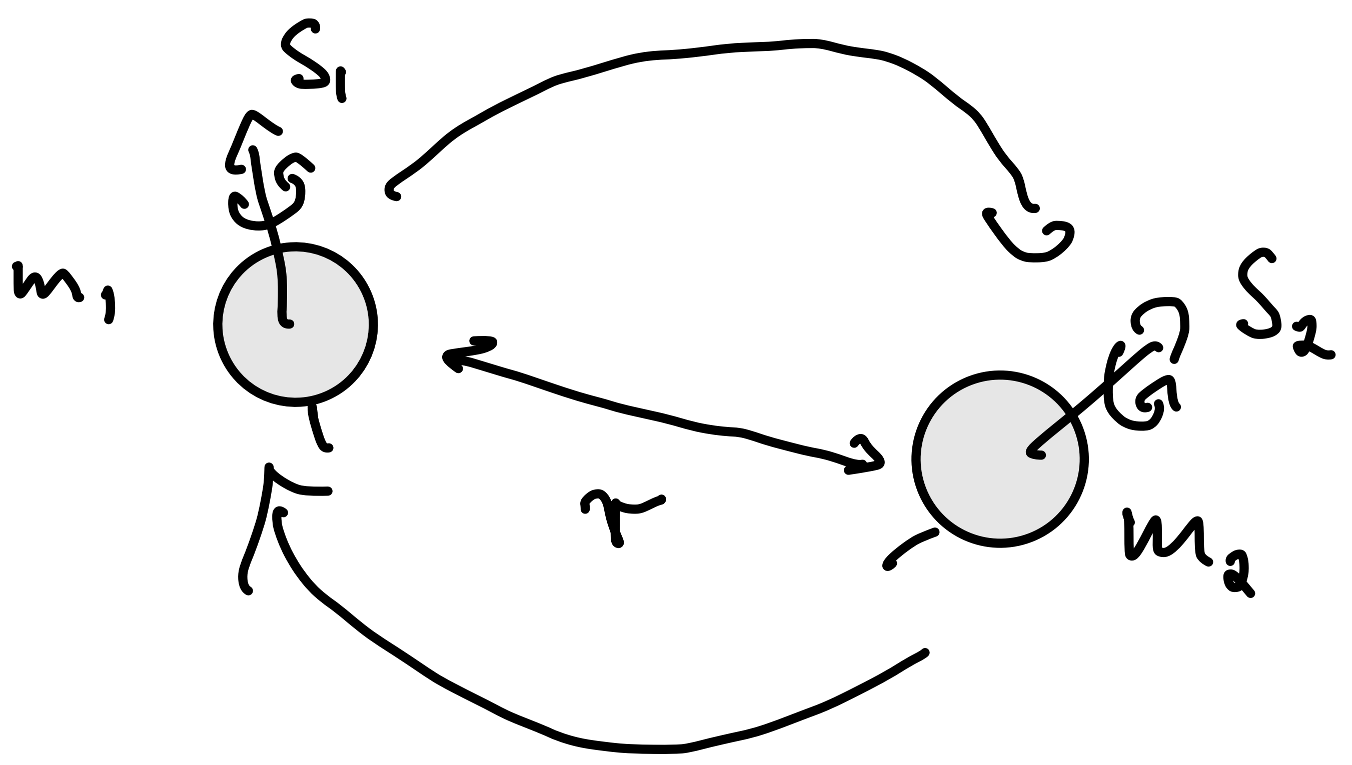
*Buonanno, **GM**, Patil, Pompili* ***Phys. Rev. Lett. 133 (2024)***



WORLDLINE EFFECTIVE FIELD THEORY (EFT)

Goldberger, Rothstein, Steinhoff, Porto, Levi, Foffa, Sturani, …

During the **inspiral phase**, the bound two-body problem enjoys a **separation of scales**:



$$\frac{r_S}{r} \ll 1 \implies \frac{Gm}{r} \ll 1$$

$$r_s << r << R$$

Internal Zone **Near Zone** **Far Zone**

Far away, black holes & neutron stars appear like point particles!

Tail effect ⟹ breakdown near/far

Motivates a **point-particle, worldline-based description** of astrophysical compact objects:

$$S = -m\int \mathrm{d}\tau \left(\underbrace{\frac{1}{2e} g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu + \frac{e}{2}} + \underbrace{(Gm)^4 \left(\frac{c_{E^2}}{e^3} E_{\mu\nu}E^{\mu\nu} + \frac{c_{B^2}}{e^3} B_{\mu\nu}B^{\mu\nu} \right)} + \cdots \right)$$

Point-particle action *Finite-size corrections, tidal effects*

Can also consider Einstein-Hilbert extensions, beyond GR, **particular interest in spin!**

$$E_{\mu\nu} = R_{\mu\alpha\nu\beta}\pi^\alpha\pi^\beta$$
$$B_{\mu\nu} = R^*_{\mu\alpha\nu\beta}\pi^\alpha\pi^\beta$$

SPINNING PHASE SPACE

Ambrosi, Kunar, Van Holten '15

Spinning degrees of freedom captured by the position x^μ, momentum p_μ, spin tensor S^{ab}

$$\boxed{\begin{aligned} \{x^\mu, p_\nu\} &= \delta^\mu_\nu\,, \\ \{S^{ab}, S^{cd}\} &= \eta^{bd}\,S^{ac} + \eta^{ac}\,S^{bd} - \eta^{bc}\,S^{ad} - \eta^{ad}\,S^{bc}\,, \end{aligned}}$$

$$\begin{aligned} \pi_\mu &:= p_\mu + \frac{1}{2}\omega_{\mu,ab}S^{ab}\,, \\ S^{\mu\nu} &:= e^\mu_a e^\nu_b S^{ab}\,, \end{aligned}$$

We only seek to describe **rotations** $S^i = \frac{1}{2}\epsilon^{ijk}S^{ij}$, not **boosts** S^{0i}. So we need a **spin-supplementary condition (SSC)**:

$$S_\mu = \frac{1}{2}\varepsilon_{\mu\nu\rho\kappa}\hat{\pi}^\nu S^{\rho\kappa} \qquad Z^\mu = S^{\mu\nu}\hat{\pi}_\nu$$

Decompose in the inertial frame π^μ

Objective: pick a **total Hamiltonian** H_T that **preserves the covariant SSC** $Z^\mu = 0$

$$\boxed{\dot{Z}^\mu = \{Z^\mu, H_T\} \approx 0}$$

Weak equality $\approx$ means vanishing when $H = Z^\mu = 0$

N=2 SUSY (SPIN-1 PARTICLE)

*Jakobsen, **GM**, Plefka, Steinhoff* ***Phys. Rev. Lett. 128 (2022)***

One possible realisation using an $\mathcal{N}=2$ SUSY algebra:

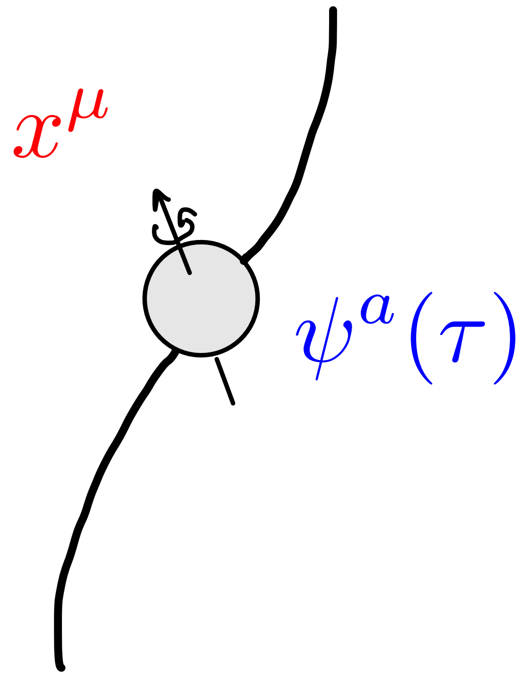
$$\{\psi^a,\bar\psi^b\} = -i\eta^{ab}$$

$$\{Q,\bar Q\} = -2iH\,,\quad \{J,Q\} = -iQ\,,\quad \{J,\bar Q\} = i\bar Q$$

$$H = \frac{1}{2}(g^{\mu\nu}\pi_\mu\pi_\nu - m^2 - R_{abcd}\bar\psi^a\psi^b\bar\psi^c\psi^d)\,,$$

$$J = \eta_{ab}\bar\psi^a\psi^b\,,\quad Q = \psi^a e^\mu_a\pi_\mu\,,\quad \bar Q = \bar\psi^a e^\mu_a\pi_\mu$$

These are a set of **first-class constraints**, and the SSC is **automatically enforced:**



$$S^{ab} = -2i\bar\psi^{[a}\psi^{b]} \qquad\qquad S^{\mu\nu}\pi_\nu = -i(\bar\psi^\mu Q + \psi^\mu\bar Q) \approx 0$$

Corresponding Lagrangian description, up to **quadratic spins**:

$$S_{\mathrm{BH/NS}} = -m\int d\tau\left[\tfrac{1}{2}g_{\mu\nu}\dot x^\mu\,\dot x^\nu + i\bar\psi D_\tau\psi + \tfrac{1}{2}R_{abcd}\bar\psi^a\psi^b\bar\psi^c\psi^d + C_E R_{a\mu b\nu}\dot x^\mu\dot x^\nu\bar\psi^a\psi^b\,\bar\psi\cdot\psi\right]$$

spin degrees of freedom

neutron star term

But… **no realisation beyond $\mathcal{N}=2$!**

HAMILTONIAN DESCRIPTION OF SPINNING BODIES

The SSC is a **second-class constraint** (Dirac), so we can solve for a **Lagrange multiplier** ζ_μ:

$$H_T = eH[e^a_\mu(x), p_\mu, S^\mu] + \zeta_\mu Z^\mu \qquad S_\mu = \frac{1}{2}\varepsilon_{\mu\nu\rho\kappa}\hat{\pi}^\nu S^{\rho\kappa} \qquad Z^\mu = S^{\mu\nu}\hat{\pi}_\nu$$

We are free to make an ansatz for *H*, in terms of only the **spin vector** S^μ:

$$H_{RS^n} = \begin{cases} \frac{C_{ES^n}}{m^{n-2}}\left(\nabla_S\right)^{n-2} E_{SS}\,, & n \text{ even}\,, \\ \frac{C_{BS^n}}{m^{n-2}}\left(\nabla_S\right)^{n-2} B_{SS}\,, & n \text{ odd}\,. \end{cases}$$

$$\begin{aligned} H_{R^2} &= (Gm)^4 m^2 \left(C_{E^2} E_{\mu\nu}E^{\mu\nu} + C_{B^2} B_{\mu\nu}B^{\mu\nu}\right), \\ H_{R^2S^2} &= (Gm)^4 m^2 \left(C_{E^2S^2} E_{\hat{S}\mu}E_{\hat{S}}{}^{\mu} + C_{B^2S^2} B_{\hat{S}\mu}B_{\hat{S}}{}^{\mu}\right), \\ H_{R^2S^4} &= (Gm)^4 m^2 \left(C_{E^2S^4} E_{\hat{S}\hat{S}}E_{\hat{S}\hat{S}} + C_{B^2S^4} B_{\hat{S}\hat{S}}B_{\hat{S}\hat{S}}\right), \end{aligned}$$

Z-corrections are then determined order-by-order in spins, giving the **total Hamiltonian**:

$$H_T = \frac{e}{2}\left(g^{\mu\nu}\pi_\mu\pi_\nu - m^2 \underbrace{- 2B_{SZ}}_{\text{Linear-curvature correction}} \underbrace{- \frac{2(C_{ES^2}-1)}{|\pi|^2}\left(E_{SS}B_{ZS} - Z\cdot S\, E_{S\nu}B_S{}^{\nu}\right)}_{\text{Quadratic-curvature correction}} - \sum_{n>1}\left(\tilde{H}_{RS^n} + \tilde{H}_{R^2S^n}\right)\right)$$

$$\tilde{H}_{R^mS^n} = \left(1 + |\pi|^{-1}\nabla_Z\right)H_{R^mS^n}$$

BOSONIC OSCILLATORS

*Haddad, Jakobsen, **GM**, Plefka '24*

We seek a **worldline action**, i.e. a **Lagrangian description**. So introduce new variables:

$$\{\alpha^a, \bar{\alpha}^b\} = -i\,\eta^{ab} \qquad\qquad S^{\mu\nu} = -2i\,\bar{\alpha}^{[\mu}\alpha^{\nu]}$$

Desired Lagrangian is given by a **Legendre transform**, solve for the **canonical momentum:**

$$\tilde{S} = -\int \mathrm{d}\tau \left[p_\mu \dot{x}^\mu - i\bar{\alpha}^a \dot{\alpha}^b \eta_{ab} - H_T\right]. \qquad \dot{x}^\mu = \{x^\mu, H_T\} = m^{-1}\pi^\mu + \cdots \text{corrections}$$

Resulting worldline action, up to linear curvature:

$$\begin{aligned} S := -\int \mathrm{d}\tau \Bigg[& \frac{m}{2} g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu - i\bar{\alpha}_a \frac{D\alpha^a}{d\tau} - B_{SZ} \\ & - m \sum_{n>1} \frac{1}{n!} \begin{Bmatrix} C_{ES^n}(1+|\dot{x}|^{-1}\nabla_Z)(\nabla_S)^{n-2} E_{SS}\,, & n \text{ even}, \\ -iC_{BS^n}(1+|\dot{x}|^{-1}\nabla_Z)(\nabla_S)^{n-2} B_{SS}\,, & n \text{ odd}, \end{Bmatrix} \Bigg], \end{aligned}$$

We have now performed spinning black hole calculations using WQFT up to S^4!

WORLDLINE QUANTUM FIELD THEORY (WQFT)

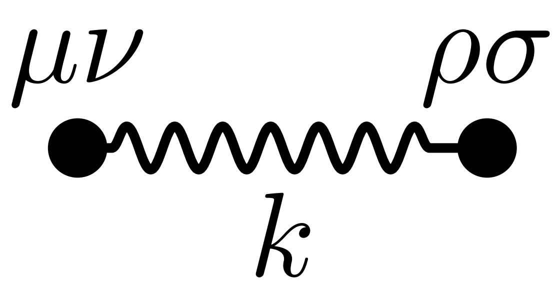
GM**, Plefka, Steinhoff* ***JHEP 02 (2021)

*Jakobsen, **GM**, Plefka, Sauer* ***JHEP 10 (2022)***

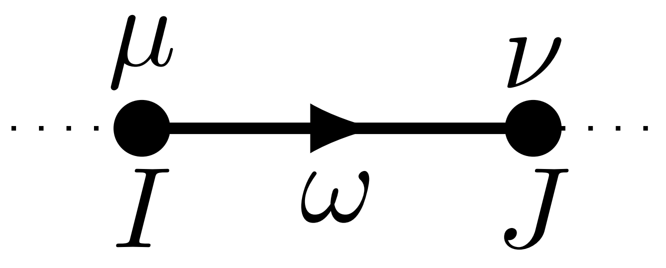
$$S=-\frac{2}{\kappa^2}\int \mathrm{d}^4x\sqrt{-g}R-\sum_{i=1}^{2}\int \mathrm{d}\tau_i\left[\frac{m_i}{2}g_{\mu\nu}\dot{x}_i^\mu\dot{x}_i^\nu-i\bar{\alpha}_{i,a}\frac{\mathrm{D}\alpha_i^a}{\mathrm{d}\tau_i}+\cdots\right]$$

$$x_i^\mu(\tau_i)=b_i^\mu+v_i^\mu\tau_i+z_i^\mu(\tau_i),$$
$$\alpha_i^\mu(\tau_i)=\alpha_{-\infty,i}^\mu+\alpha_i'^{\,\mu}(\tau_i)\,,$$
$$g_{\mu\nu}(x)=\eta_{\mu\nu}+\kappa\,h_{\mu\nu}(x)\,.$$

Promote gravitons, deflections to **propagating d.o.f's:**



$$=i\frac{\eta_{\mu(\rho}\eta_{\sigma)\nu}-\frac{1}{D-2}\eta_{\mu\nu}\eta_{\rho\sigma}}{(k^0+i0^+)^2-\mathbf{k}^2}$$

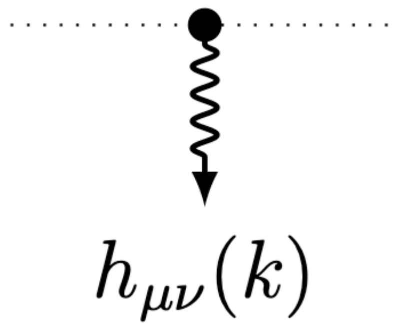


$$=-i\frac{\eta^{\mu\nu}}{m}\begin{pmatrix}\frac{1}{(\omega+i0)^2} & & \\ & & -\frac{1}{\omega+i0} \\ & \frac{1}{\omega+i0} & \end{pmatrix}_{IJ}$$

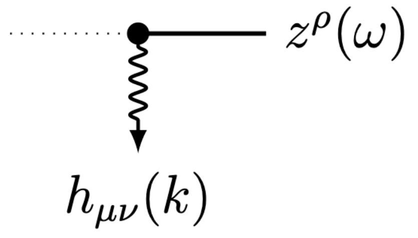
$$\mathcal{Z}_I^\mu(\omega)=\{z^\mu(\omega),\alpha'^\mu(\omega),\bar{\alpha}'^\mu(\omega)\}$$

Causality demands use of **retarded propagators** (from **Schwinger-Keldysh in-in formalism)**

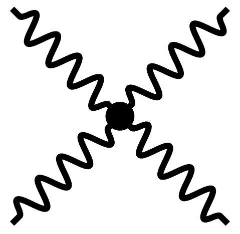
Gravitons live in the bulk, carry **momentum; deflections** live on the worldline, carry **energy**.



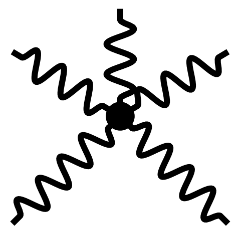
$$=-i\frac{m}{2m_{\mathrm{Pl}}}e^{ik\cdot b}\delta\!\!\!{}^{-}(k\cdot v)v^\mu v^\nu\,,$$



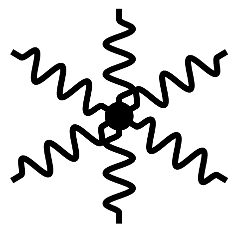
$$=\frac{m}{2m_{\mathrm{Pl}}}e^{ik\cdot b}\delta\!\!\!{}^{-}(k\cdot v+\omega)\left(2\omega v^{(\mu}\delta_\rho^{\nu)}+v^\mu v^\nu k_\rho\right)$$



$$\sim\sqrt{G}^2k^2,$$



$$\sim\sqrt{G}^3k^2$$

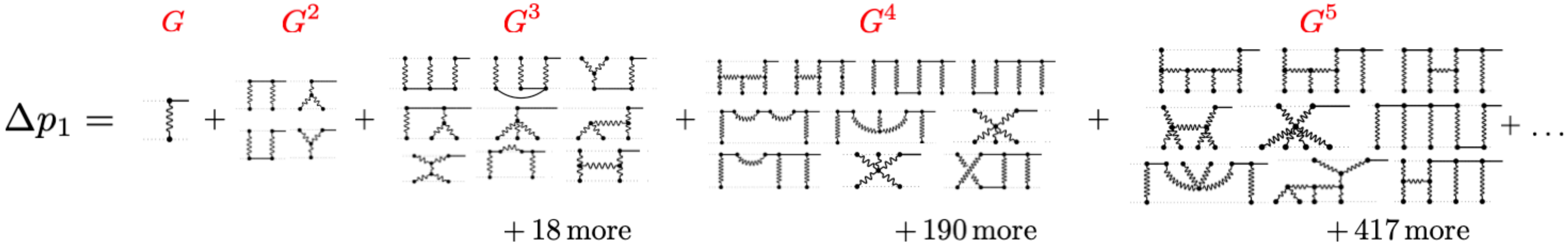


$$\sim\sqrt{G}^4k^2,\ldots$$

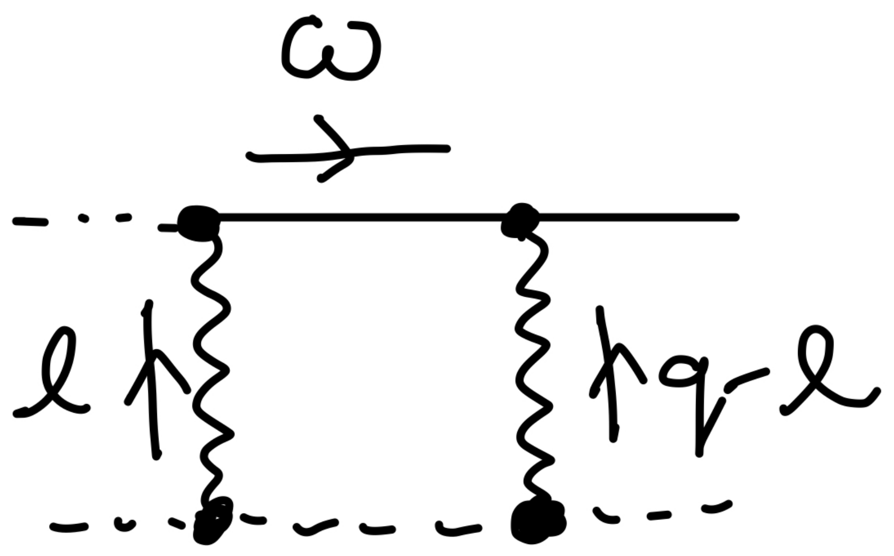
Tree-level one-point functions = Solutions to classical equations of motion

SCATTERING OBSERVABLES

For **momentum impulse** draw tree diagrams with 1 outgoing line:



All graphs are trees. Integrate on internal energies/momenta:



$$= \int_{q,\ell,\omega} \frac{\delta(\omega - \ell\cdot v_1)\delta(\omega + (q-\ell)\cdot v_1)\delta(\ell\cdot v_2)\delta((q-\ell)\cdot v_2)}{(\omega + i0)^2 \ell^2 (\ell - q)^2} e^{iq\cdot b}$$

$$= \int_q \delta(q\cdot v_1)\delta(q\cdot v_2) e^{iq\cdot b} \int_\ell \frac{\delta(\ell\cdot v_2)}{(\ell\cdot v_1 + i0)^2 \ell^2 (\ell - q)^2}$$

OSF scale-free loop integration

Loop integrals arise from **lack of momentum conservation:**

Loop integrals from tree-level diagrams

IN-IN FORMALISM

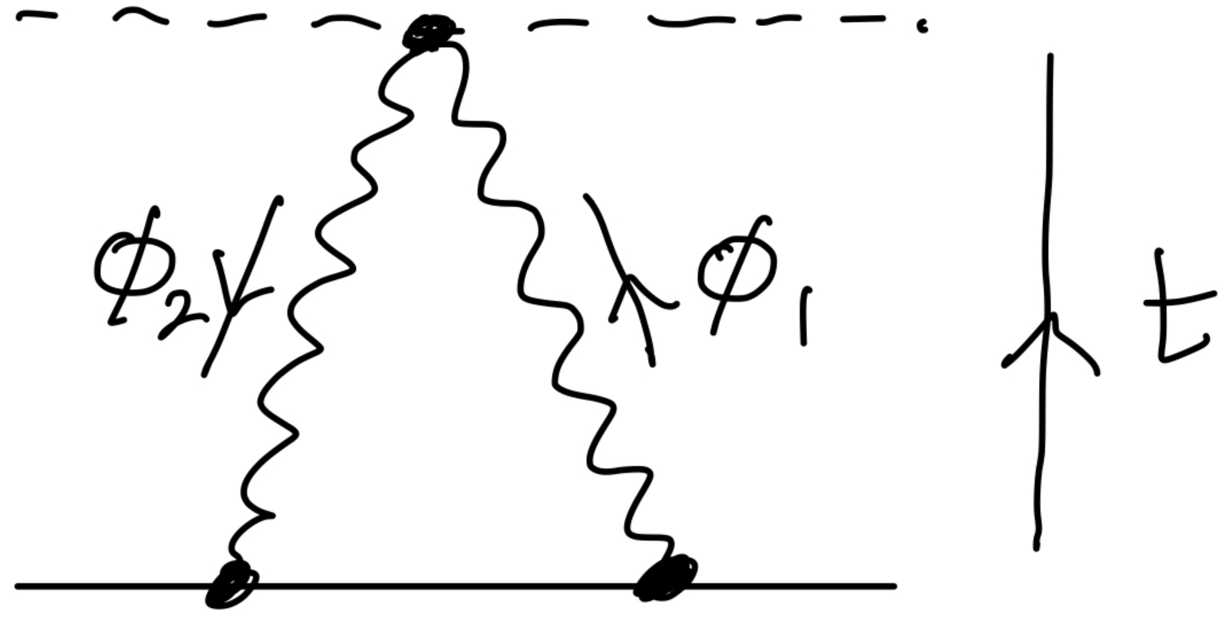
*Jakobsen, **GM**, Plefka, Sauer JHEP 10 (2022)*

- Also known as **Schwinger-Keldysh, or Closed-Time Path (CTP):**

$$\langle \mathcal{O}(t,\mathbf{x})\rangle_{\text{in}-\text{in}} := {}_{\text{in}}\langle 0|U(-\infty,t)\mathcal{O}(t,\mathbf{x})U(t,-\infty)|0\rangle_{\text{in}}$$

- Simple example of a **scalar field** — path integral involves **2 copies of the theory:**

$$Z[J_1,J_2] = \int \mathrm{D}[\phi_1,\phi_2]\exp\left\{\frac{i}{\hbar}\left[(S[\phi_1]-S[\phi_2]) + \int \mathrm{d}^4x\big(J_1(x)\phi_1(x) - J_2(x)\phi_2(x)\big)\right]\right\}$$



$$\begin{aligned}\phi_1(t=+\infty,\mathbf{x}) =& \phi_2(t=+\infty,\mathbf{x})\\ \phi_1(t=-\infty,\mathbf{x}) =& \phi_2(t=-\infty,\mathbf{x}) = 0\end{aligned}$$

$$\langle \phi_A(x)\,\phi_B(y)\rangle = \begin{pmatrix}\langle 0|\mathcal{T}\phi(x)\phi(y)|0\rangle & \langle 0|\phi(y)\phi(x)|0\rangle\\ \langle 0|\phi(x)\phi(y)|0\rangle & \langle 0|\mathcal{T}^*\phi(x)\phi(y)|0\rangle\end{pmatrix} = \begin{pmatrix}D_F(x,y) & D_-(x,y)\\ D_+(x,y) & D_D(x,y)\end{pmatrix}$$

KELDYSH BASIS

- **Huge simplification** in the **Keldysh basis**, with **advanced & retarded propagators:**

$$\langle \phi_a(x)\,\phi_b(y)\rangle = \begin{pmatrix} \frac{1}{2}D_H(x,y) & D_{\rm ret}(x,y) \\ -D_{\rm adv}(x,y) & 0 \end{pmatrix}$$

$$\phi_+ = \frac{1}{2}(\phi_1+\phi_2)$$

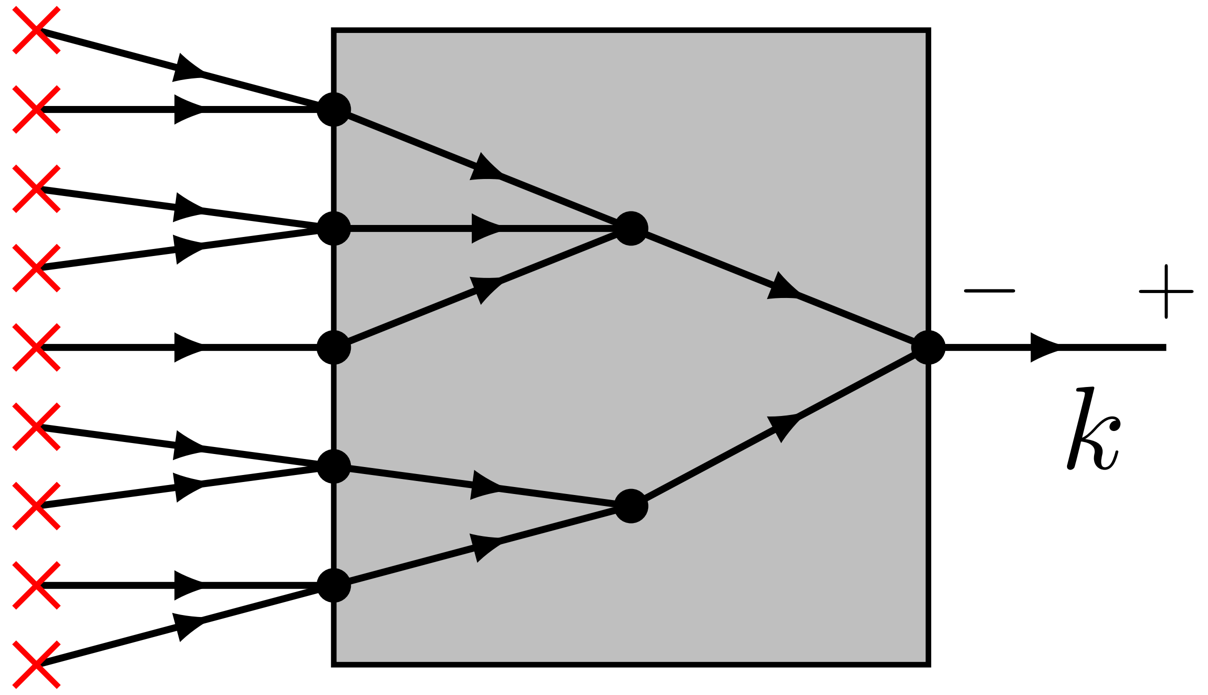
$$\phi_- = \phi_1 - \phi_2$$

$$\widetilde{D}_{\rm ret}(k) = \underset{-}{\bullet}\!\!\longrightarrow\!\!\underset{+}{\bullet} = \frac{-i}{(k^0+i0)^2-\mathbf{k}^2}\,,$$

$$\widetilde{D}_{\rm adv}(k) = \underset{+}{\bullet}\!\!\longleftarrow\!\!\underset{-}{\bullet} = \frac{-i}{(k^0-i0)^2-\mathbf{k}^2}\,,$$

- Diagrams conspire to ensure **forward-in-time flow of causality:**

$$\langle \phi(k)\rangle_{\text{in-in}} =$$



Absorbs the cuts present in e.g. KMOC formalism

$$\frac{1}{k^2+\text{sgn}(k^0)i0} = \frac{1}{k^2+i0} + 2i\pi\theta(-k^0)\delta(k^2)$$

- Upshot: calculate tree-level 1-point functions using **in-out Feynman rules + retarded propagators**.

- We use this in WQFT, with **retarded bulk and worldline propagators**

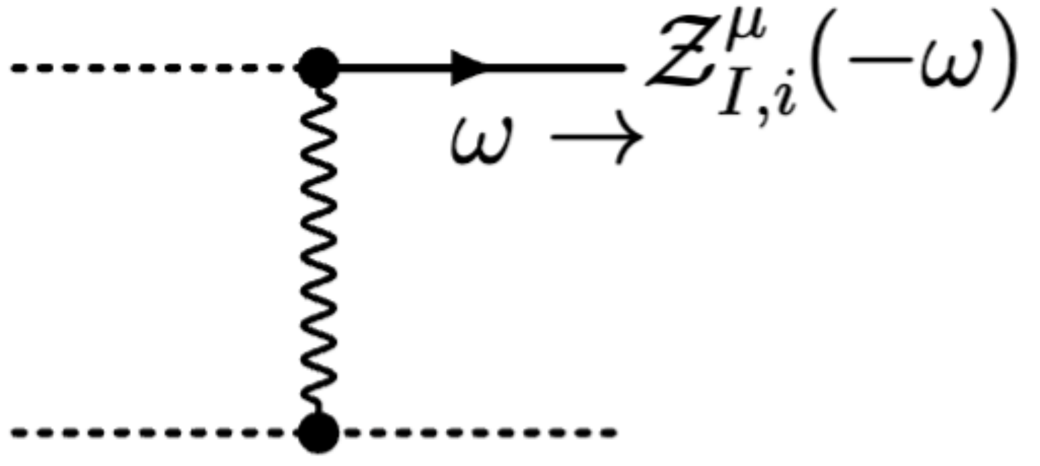
SPINNING SCATTERING OBSERVABLES

Haddad, Jakobsen, ***GM****, Plefka '24*

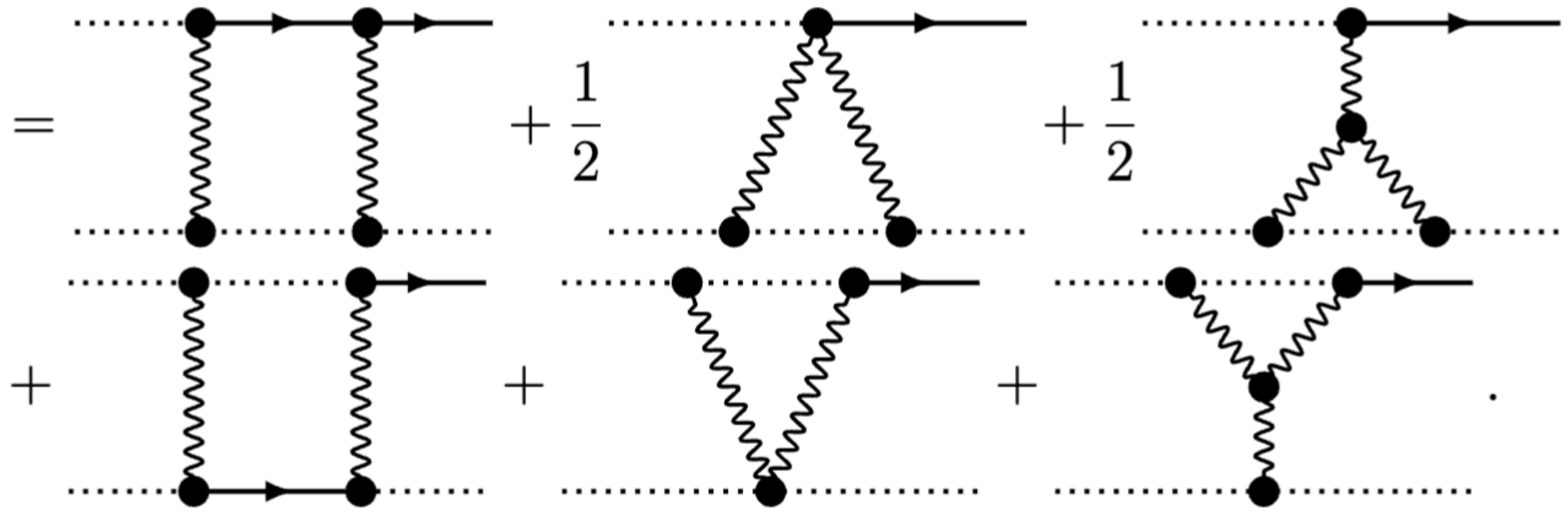
In order to test our **spinning Lagrangian**, we compute **scattering observables** up to S^4:

$$\boxed{\mathcal{Z}_I^{\mu}(\omega) = \{z^{\mu}(\omega), \alpha'^{\mu}(\omega), \bar{\alpha}'^{\mu}(\omega)\}}$$

$$\langle \mathcal{Z}^{\mu}_{I,i}(-\omega)\rangle_{\mathrm{LO}} =$$



$$\langle \mathcal{Z}^{\mu}_{I,i}(-\omega)\rangle_{\mathrm{NLO}}$$



Observables appear as the components of $\mathcal{Z}_I^{\mu}$ with a cut external line:

$$\Delta p_i^{\mu} = -m_i\omega^2 \left.\langle z_i^{\mu}(-\omega)\rangle\right|_{\omega\to 0}\,,$$
$$\Delta\alpha_i^{\mu} = \mathrm{i}\,\omega \left.\langle \alpha_i'^{\mu}(-\omega)\rangle\right|_{\omega\to 0}\,.$$

$$\Delta S_i^{\mu\nu} = -2\mathrm{i} m_i(\bar{\alpha}^{[\mu}_{-\infty,i}\Delta\alpha_i^{\nu]} + \Delta\bar{\alpha}_i^{[\mu}\alpha^{\nu]}_{-\infty,i} + \Delta\bar{\alpha}_i^{[\mu}\Delta\alpha_i^{\nu]})$$

We reproduce all relevant results from the literature up to S^4!

SPINNING LAGRANGIAN ANSATZ

To verify our Hamiltonian-based analysis, start from a **Lagrangian Ansatz:**

$$S = -m \int \mathrm{d}\tau \left(\frac{1}{2} g_{\mu\nu} \dot{x}^{\mu} \dot{x}^{\nu} - i \eta_{ab} \bar{\alpha}^{a} \frac{\mathrm{D}\alpha^{b}}{\mathrm{d}\tau} - \mathcal{L}_{\mathrm{nm}} \right)$$

$$\mathcal{L}_{\mathrm{nm}}^{(a)} = \sum_{n=1}^{\infty} (\nabla_{\mathfrak{a}})^{2n-2} \left(\frac{C_{ES^{2n}}}{(2n)!} \mathcal{E}_{\mathfrak{aa}} - \frac{\mathrm{i} C_{BS^{2n+1}}}{(2n+1)!} \nabla_{\mathfrak{a}} \mathcal{B}_{\mathfrak{aa}} \right),$$

$$\mathcal{L}_{\mathrm{nm}}^{(Z)} = \frac{C_2^{(Z)}}{2} B_{\mathfrak{az}} - \frac{\nabla_{\mathfrak{z}}}{|\dot{x}|} \sum_{n=2}^{\infty} (\nabla_{\mathfrak{a}})^{2n-4} \left(\frac{C_{2n}^{(Z)}}{(2n)!} \nabla_{\mathfrak{a}} \mathcal{B}_{\mathfrak{aa}} + \frac{\mathrm{i} C_{2n-1}^{(Z)}}{(2n-1)!} \mathcal{E}_{\mathfrak{aa}} \right)$$

We seek to fix the coefficients $C_n^{(Z)}$. Demand **no change in the SSC vector**:

$$\boxed{\Delta Z_i^{\mu} = \Delta p_{i,\nu} S_{-\infty,i}^{\mu\nu} + p_{i,\nu} \Delta S_i^{\mu\nu} + \Delta p_{i,\nu} \Delta S_i^{\mu\nu} = 0}$$

Constrains coefficient values to those agreeing with the Hamiltonian:

$$C_n^{(Z)} = n \times \begin{cases} C_{ES^{n-1}}, & n \text{ odd}, \\ C_{BS^{n-1}}, & n \text{ even}, \end{cases}$$

$$C_{R^2,1}^{(Z)} = 0, \quad C_{R^2,2}^{(Z)} = C_{ES^2}(C_{ES^2} - 1),$$
$$C_{R^2,3}^{(Z)} = -C_{R^2,4}^{(Z)} = C_{ES^2} - 1.$$

Linear Curvature

Quadratic Curvature

SCATTERING SPINNING BLACK HOLES: STATE-OF-THE-ART

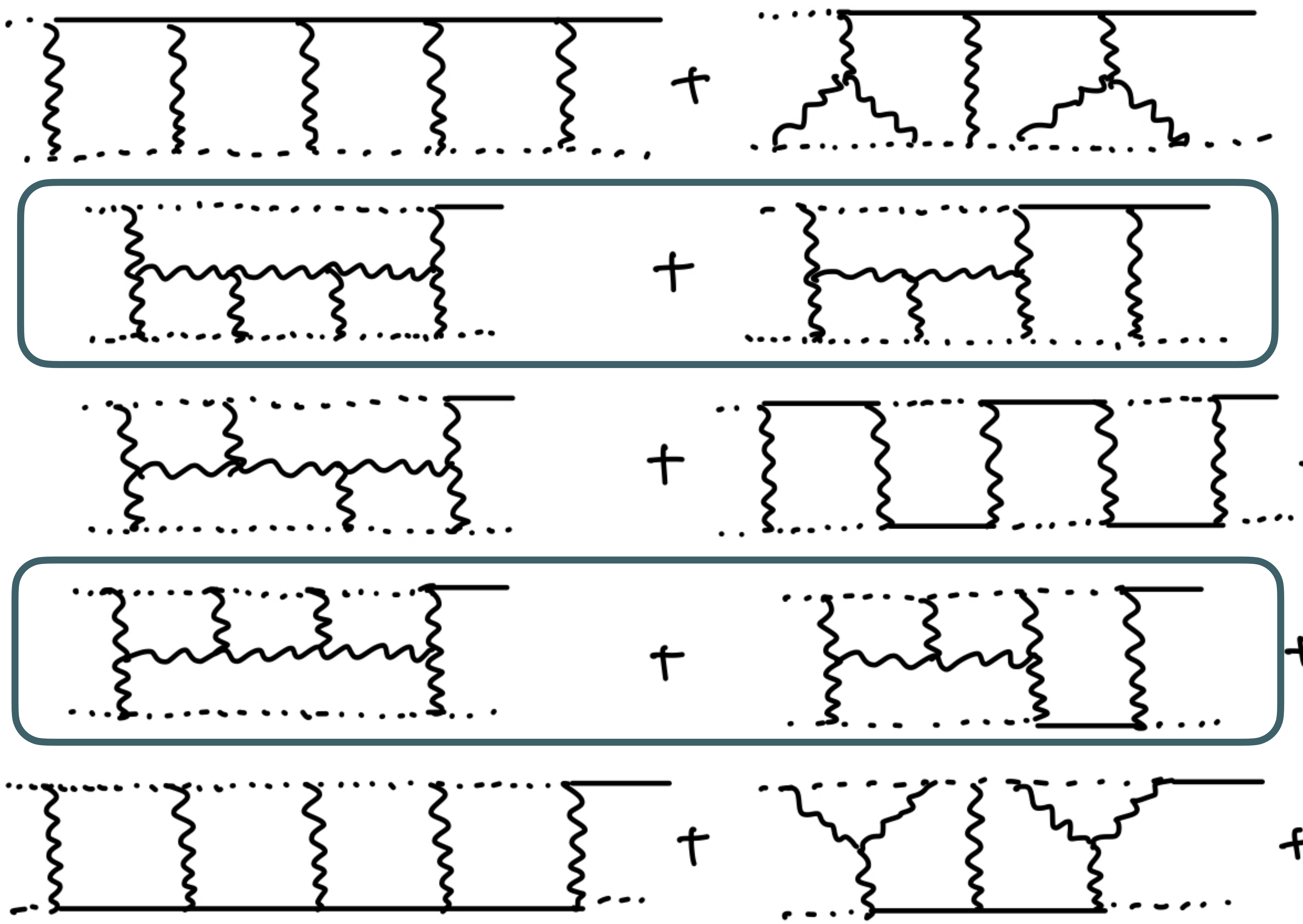
Higher orders in spin are **suppressed by physical PM counting**: $a_i^\mu = G m_i \chi_i^\mu$, $|\chi_i| < 1$

	S^0 (Spin-0)	S^1 (Spin-1/2)	S^2 (Spin-1)	S^3 (Spin-3/2)	S^4 (Spin-2)	S^5 (Spin-5/2)
1PM (tree level)	G	G^2	G^3	G^4	G^5	G^6
2PM (1 loop)	G^2	G^3	G^4	G^5	Guevara, Ochirov, Vines ’18 Chen, Chung,Huang, Kim ’21 Haddad, Jakobsen, **GM**, Plefka ‘24	Bern, Kosmopoulos, Luna, Roiban, Teng ’22; Aoude, Haddad, Helset ’23; Bautista ’23; Chen, Wang ’24; Bohnenblust, Cangemi, Johansson, Pichini ‘24
3PM (2 loops)	Bern, Cheung, Parra-Martinez, Ruf, Herrmann, Roiban, Shen, Solon, Zeng, Kälin, Liu, Porto, Di Vecchia, Heissenberg, Russo, Veneziano, Travaglini, Brandhuber, Damgaard, Planté, Vanhove, Bjerrum-Bohr	**Jakobsen, GM ’22** Akpinar, Cordero, Kraus, Ruf, Zeng ‘24	**Jakobsen, GM ’22** Akpinar, Cordero, Kraus, Ruf, Zeng ‘24	Akpinar, Cordero, Kraus, Smirnov, Zeng ‘25	Akpinar, Cordero, Kraus, Smirnov, Zeng ‘25	G^8
4PM (3 loops)	Dlapa, Kälin, Liu, Neef, Porto, Damgaard, Hansen, Planté, Vanhove, Bern, Parra-Martinez, Roiban, Ruf, Shen Solon, Zeng	**Jakobsen, GM, Plefka, Sauer, Xu ’23**	G^6	G^7	G^8	***Tail effect*** log *v*’s ↓
5PM (4 loops)	**Driesse, Jakobsen, Klemm, GM, Nega, Plefka, Sauer, Usovitsch ‘24**	G^6	G^7	G^8	G^9	G^{10}

5PM MOMENTUM IMPULSE

0SF: 63 diagrams   1SF: 426 diagrams

$$\Delta p_1^{(5)\mu} =$$



Our first 5PM calculation

Very difficult!

When conservative, $\Delta p_1^\mu = -\Delta p_2^\mu$

Very easy!

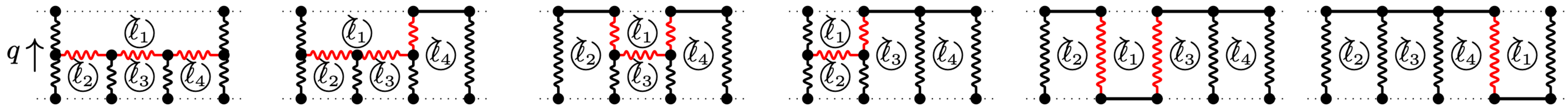
$m_1 m_2^5$	0SF
$m_1^2 m_2^4$	1SF
$m_1^3 m_2^3$	2SF
$m_1^4 m_2^2$	$\overline{\text{1SF}}$
$m_1^5 m_2$	$\overline{\text{0SF}}$

New results!

- Integrand divides naturally into **mass sectors**, following **Self-Force (SF)**
- Quickly generate from **Feynman rules** using recursive **FORM** algorithm.
- All propagators are **retarded**, causality flow towards the **outgoing line** — in-in formalism

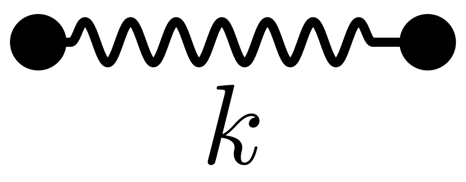
1SF LOOP INTEGRATION

At 1SF **all loop orders** we need only **1 planar integral basis** to handle all contributions:

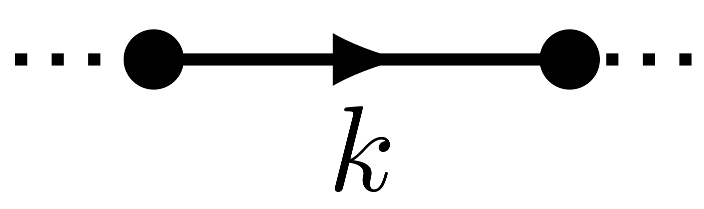


Red graviton propagators are active: **can go on-shell**, so the **i0 matters**.

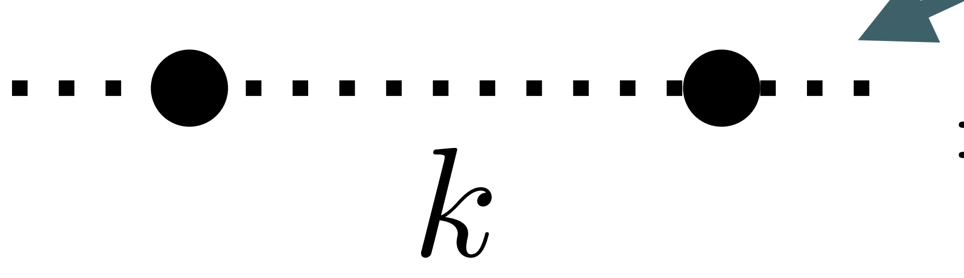
Interpret as cut propagator



$$= \frac{1}{(k^0 + i0^+)^2 - \mathbf{k}^2}$$



$$= \frac{1}{k \cdot v_i + i0^+}$$



$$= \delta(k \cdot v_i)$$

Integrals depend trivially on $|q|$, non-trivially on $\gamma = v_1 \cdot v_2$

$$\mathcal{I}^{\{\sigma\}}_{\{n\}} = \int_{\ell_1 \cdots \ell_L} \frac{\delta^{(\bar{n}_1 - 1)}(\ell_1 \cdot v_1) \prod_{i=2}^{L} \delta^{(\bar{n}_i - 1)}(\ell_i \cdot v_2)}{\prod_{i=1}^{L} D_i^{n_i}(\sigma_i) \prod_{I<J} D_{IJ}^{n_{IJ}}}$$

$$D_1 = \ell_1 \cdot v_2 + \sigma_1 i0^+ \,, \quad D_{i>1} = \ell_i \cdot v_1 + \sigma_i i0^+ \,,$$

$$D_{ij} = (\ell_i - \ell_j)^2 \,, \quad D_{qi} = (\ell_i + q)^2 \,, \quad D_{0i} = \ell_i^2$$

Loop integrals with retarded propagators:

1. Integration-by-Parts (IBPs) **unaffected**
2. Differential Equations (DEs) **unaffected**
3. Symmetries are **affected**
4. Boundary conditions ($v \to 0$) are **affected**

5PM-1SF CONSERVATIVE

Driesse, Jakobsen, ***GM****, Plefka, Sauer, Usovitsch* ***Phys. Rev. Lett 133 (2024)***

The **conservative** impulse has a **universal form:**

$$\Delta p^\mu_{1,\mathrm{cons}} = \underbrace{|p| \sin\theta_{\mathrm{cons}} \frac{b^\mu}{|b|}}_{b\text{-terms}} + \underbrace{(\cos\theta_{\mathrm{cons}} - 1) p^\mu}_{v\text{-terms}} \qquad p^\mu = (0, \mathbf{p}) = \frac{E_2 p_1^\mu - E_1 p_2^\mu}{M}$$

Result depends only on the **scattering angle**:

$$\boxed{\theta_{\mathrm{cons}} = \frac{E}{M} \sum_{n\geq 1} \left(\frac{GM}{|b|}\right)^n \left(\theta^{(n,0)}_{\mathrm{cons}}(\gamma) + \nu\theta^{(n,1)}_{\mathrm{cons}}(\gamma) + \cdots\right)} \qquad \nu = \frac{m_1 m_2}{M^2}$$

$\theta^{(5,1)}_{\mathrm{cons}}$ consists of **Multiple PolyLogarithms (MPLs)** up to **weight-3, alphabet** $a_i \in \{0, \pm 1, \pm i\}$

$$\theta^{(5,1)}_{\mathrm{cons}} = \sum_{k=1}^{31} c_k(\gamma) f_k(\gamma) \qquad G(a_1, \ldots, a_n; y) = \int_0^y \frac{\mathrm{d}t}{t - a_1} G(a_2, \ldots, a_n; t)$$

No other special functions in the conservative 5PM-1SF result!

NOW AVAILABLE: COMPLETE 5PM–1SF

*Driesse, Jakobsen, Klemm, **GM**, Nega, Plefka, Sauer, Usovitsch '24*

Accepted to Nature

We now have the **full 5PM-1SF impulse, including radiation-reaction!**

$$\Delta p^{(5)\mu}_{1\mathrm{SF}} = \frac{1}{b^5}\left(\hat{b}^\mu c_b(\gamma) + \check{v}_2^\mu c_v(\gamma) + \check{v}_1^\mu c'_v(\gamma)\right), \quad P^\mu_{\mathrm{rad}} = -\Delta p_1^\mu - \Delta p_2^\mu$$

$$\Delta p^{(5)\mu}_{\overline{1\mathrm{SF}}} = \frac{1}{b^5}\left(\hat{b}^\mu \bar{c}_b(\gamma) + \check{v}_2^\mu \bar{c}_v(\gamma) + \check{v}_1^\mu \bar{c}'_v(\gamma)\right), \quad E_{\mathrm{rad}} = \hat{P}_{\mathrm{com}} \cdot P_{\mathrm{rad}}$$

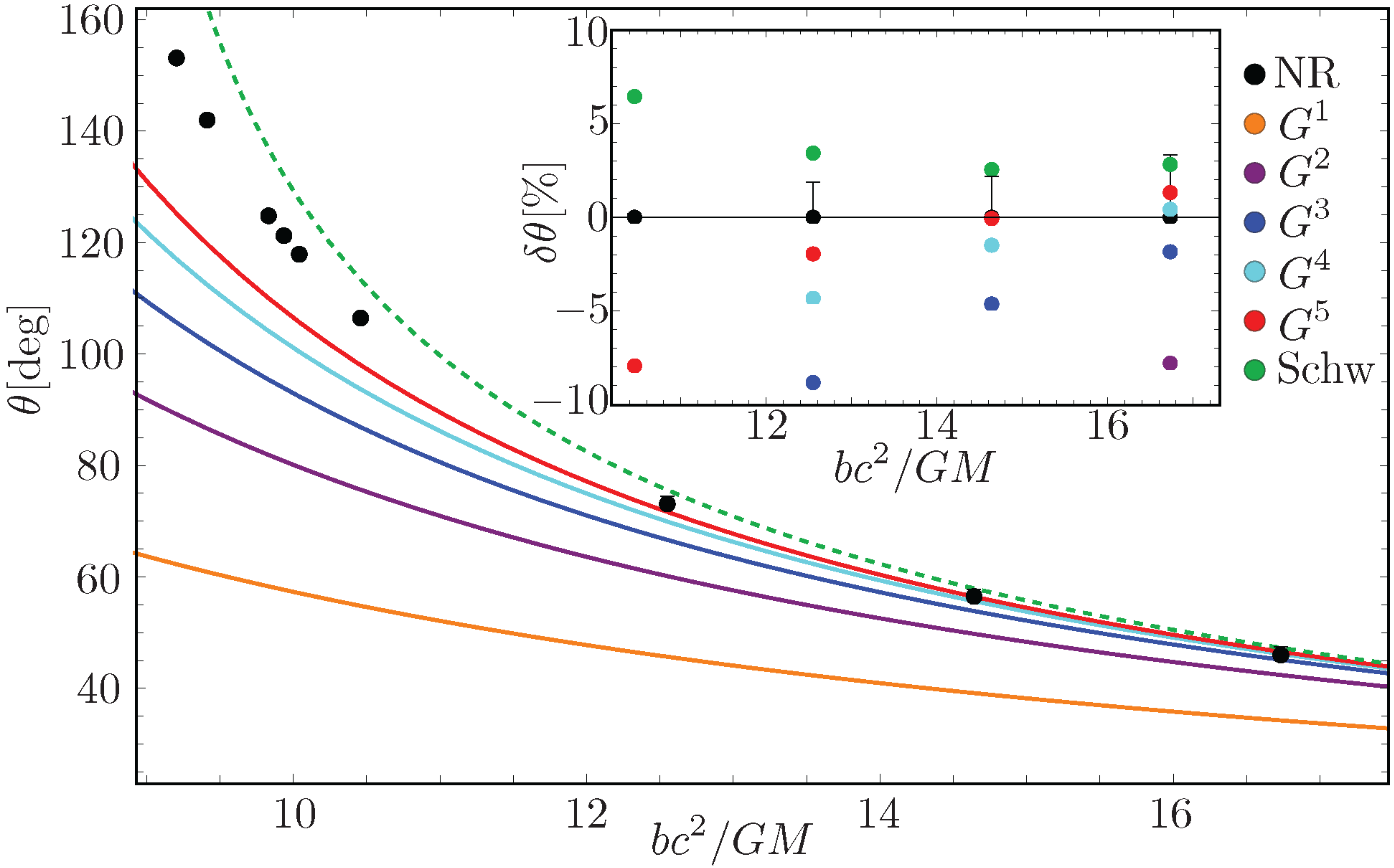
Encodes scattering angle, MPLs only
Even (pure real) integrals

Encodes radiated momentum, contains K3 & CY3 kernels
Odd (pure imaginary) integrals

$$\theta^{(5,1)} = \frac{4}{5v^8} - \frac{137}{5v^6} + \frac{3008}{45v^5} + \frac{\frac{41\pi^2}{4} - \frac{3427}{6}}{v^4} + \frac{\frac{84\pi^2}{5} - \frac{4096}{1575}}{v^3} + \frac{-\frac{12544\log(2v)}{45} + \frac{3593\pi^2}{72} - \frac{445867}{432}}{v^2} + \frac{\frac{2144536}{11025} + \frac{453\pi^2}{35}}{v}$$
$$+ \left(-\frac{7552\log(2v)}{1575} + \frac{246527\pi^2}{1440} - \frac{1111790903}{756000}\right) + \left(\frac{19424344}{363825} + \frac{1787\pi^2}{672}\right)v$$
$$+ \left(-\frac{1762784\log(2v)}{11025} - \frac{184881\pi^2}{2240} + \frac{56424801733}{49392000}\right)v^2 + \left(\frac{16004496043}{104053950} - \frac{835619\pi^2}{59136}\right)v^3 + \cdots$$

PN Small-v Expansion

COMPARING THE SCATTERING ANGLE WITH NUMERICAL RELATIVITY (NR)



Increasing PM orders show **NR convergence**

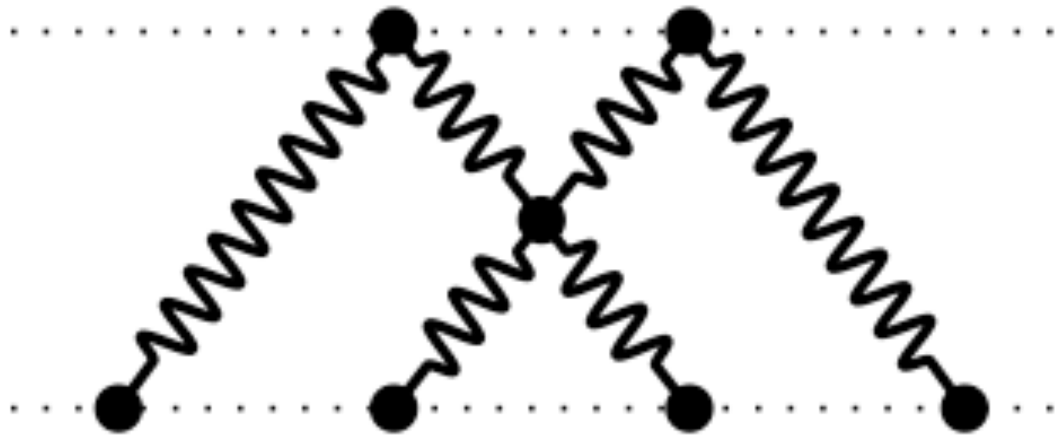
NR data: Damour, Rettegno, Pratten, Thomas, Schmidt ’23

Calls for EOB resummation!

CALABI-YAU THREE-FOLD (CY3)

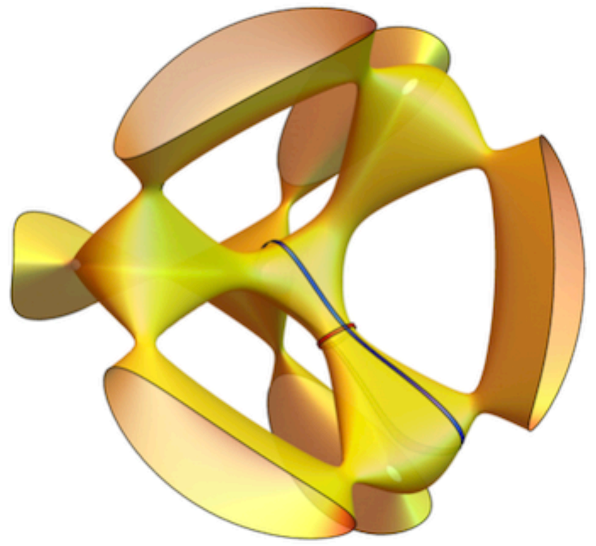
*Driesse, Jakobsen, Klemm, **GM**, Nega, Plefka, Sauer, Usovitsch ’24*
Accepted to Nature

The odd-in-*v* sector contains a **period of Calabi-Yau Three-fold**:

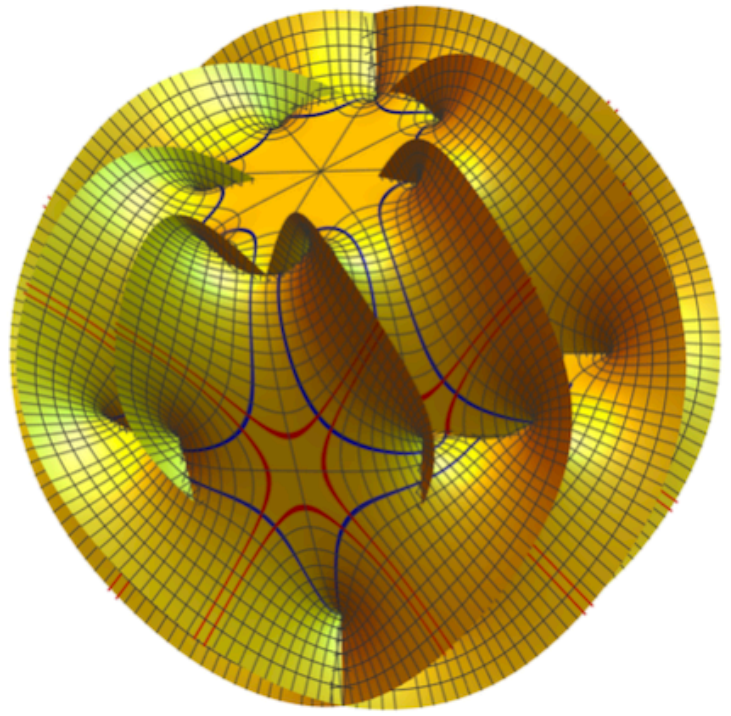


(CY3)

Torus $(n = 1)$



K3 $(n = 2)$



CY3 $(n = 3)$

$$\left[\left(z\frac{\mathrm{d}}{\mathrm{d}z}\right)^4 - z\left(z\frac{\mathrm{d}}{\mathrm{d}z} + \frac{1}{2}\right)^4\right]\varpi(z) = 0$$

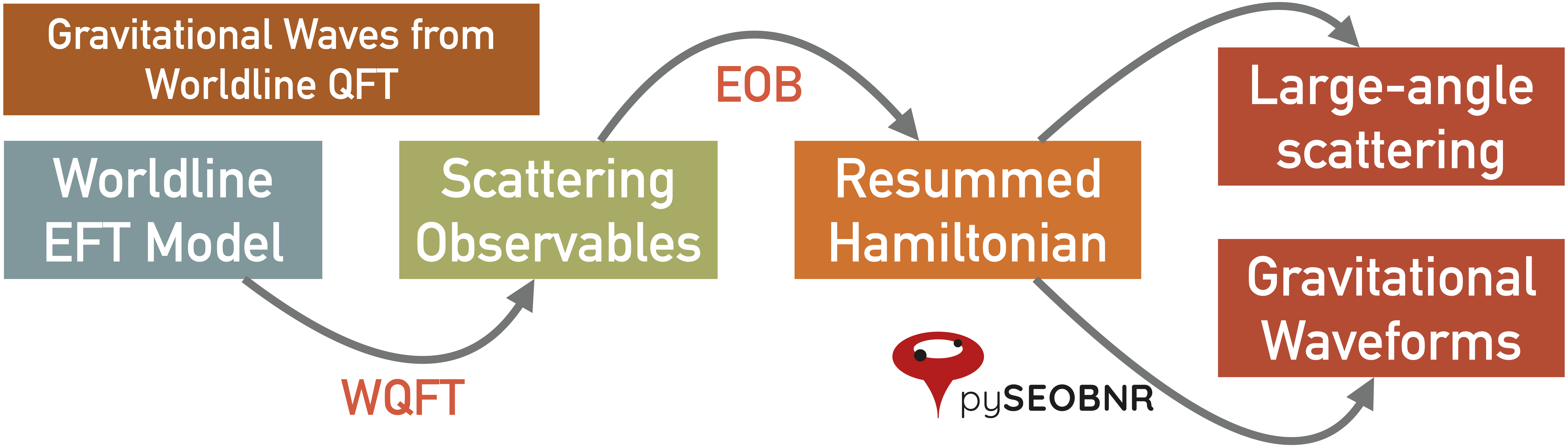
Picard-Fuchs Equation

We have **full analytic control**, can PN expand:

$$\alpha_1 = \frac{\varpi_0^2}{x(\varpi_0\varpi_1' - \varpi_0'\varpi_1)}$$

$$\begin{aligned} E_{\mathrm{rad}}^{(5)} = \frac{M^6\nu^2\pi}{5\Gamma b^5 v^3}\Big[&122 + \frac{3583}{56}v^2 + \frac{297\pi^2}{4}v^3 - \frac{71471}{504}v^4\left(\frac{9216}{7} - \frac{24993\pi^2}{224}\right)v^5 \\ &+ \left(\frac{2904562807}{6899200} + \frac{99\pi^2}{2} - \frac{10593}{70}\log\frac{v}{2}\right)v^6 + \left(\frac{7296}{7} - \frac{2927\pi^2}{28}\right)v^7 \\ &+ \left(\frac{4924457539}{29429400} + \frac{8301\pi^2}{112} - \frac{491013}{3920}\log\frac{v}{2}\right)v^8 + \left(\frac{99524416}{40425} - \frac{46290891\pi^2}{157696}\right)v^9 + \cdots\Big] \end{aligned}$$

CONCLUSIONS & OUTLOOK



Gravitational Waves from Worldline QFT

Worldline EFT Model

WQFT

Scattering Observables

EOB

Resummed Hamiltonian

pySEOBNR

Large-angle scattering

Gravitational Waveforms

Future directions

1. **Physical effects in worldline EFT description:** beyond GR, dynamical tides, higher spins, …
2. **Higher-PM scattering observables:** 5PM-2SF, 4PM-S^2, radial action, waveform, fluxes, …
3. **Improved EOB resummation:** inspiration from radial action, self-force expansion…
4. **Enhanced GWs:** more NR calibration, more PM data, fluxes, mode decomposition, …