Unified Worldline treatment of a Dirac particle with (pseudo)scalar and (axial)vector couplings

Based on work done with F. Bastianelli, J. P. Edwards, D. G. C. McKeon, C. Schubert

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Outline

1. Introduction

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2. Path integral for the Schrodinger-Pauli equation
   - Partition trace
   - Coherent state representations

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2. Path integral for the Schrodinger-Pauli equation
   - Partition trace
   - Coherent state representations
3. Worldline Formalism for a Dirac particle with various couplings
   - The one-loop effective action
   - Spinning particle model

Outline

Worldline methods: Quantum Field Theory results from qzn of QM models

- Main tools in use: particle actions

$$S[x,\psi;G]=\int_0^T d\tau\left(\dot{x}^2+\psi\dot{\psi}+V(x,\dot{x},\psi;G)\right)$$

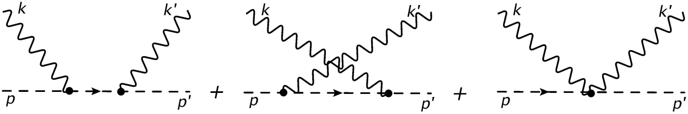
x bosonic ψ fermionic G external

- canonical qzn
- particle path integrals:
    - topology of a circle: PBC $\longrightarrow$ one loop effective action, one loop amplitudes

      review by Schubert '01
    - topology of a line: DBC $\longrightarrow$ dressed propagator, tree level amplitudes

Some Advantages of Worldline Formalism

- directly obtain off-shell Feynman amplitudes, rather than single Feynman diagrams. Ex: Compton scattering in scalar QED



- gauge-invariance efficiently guaranteed
- Dirac algebra and momentum integrals suitably obtained
- worldline formalism works well *also* with massive particle and at one loop
- provides a unique setup for QFT in external fields

Today's talk

What I ain't gonna talk about today:

The worldline approach to:

- Gravity
- QFT on spaces with boundary
- Non commutative QFT
- Higher Spin
- $\cdots$

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I'd like to discuss:

- couplings which present an even and/or **odd** number of gamma matrices
- keep it simple

Outline

An elementary model

Simplified SP hamiltonian:

- only spin, no orbital Dof's
- constant field B
- $D = 2$

$$H = -\sigma \cdot \mathbf{B}$$

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Partition trace

$$Z = \mathrm{tr}\; e^{\sigma \cdot \mathbf{B}} = 2 \cosh |\mathbf{B}|$$

Task: reproduce the latter with a coherent state Grassmann path integral

Outline

Fermionic harmonic oscillator:

$$a = \frac{1}{2}(\sigma_1 + i\sigma_2)\,, \quad a^\dagger = \frac{1}{2}(\sigma_1 - i\sigma_2)$$

$$\{a, a\} = \{a^\dagger, a^\dagger\} = 0\,, \quad \{a, a^\dagger\} = 1$$

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Hamiltonian

$$\sigma \cdot \mathbf{B} = a^\dagger B + a B^*\,, \quad B := B_1 + iB_2$$

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Coherent states:

$$a|z\rangle = z|z\rangle\,, \quad \langle\bar{z}|a^\dagger = \bar{z}\langle\bar{z}|\,, \quad \langle\bar{z}|z\rangle = e^{\bar{z}z}$$
$$\mathbb{1} = \int d\bar{z}dz\, e^{-\bar{z}z}\, |z\rangle\langle\bar{z}|$$
$$\mathrm{tr}\, A = \int d\bar{z}dz\, e^{-\bar{z}z}\, \langle -\bar{z}|A|z\rangle = \int dz d\bar{z}\, e^{\bar{z}z}\, \langle\bar{z}|A|z\rangle$$

Coherent state path integral for the SP model

Slice the exponential and take the trace

$$\mathrm{tr}\, e^{\sigma\cdot\mathbf{B}} = \int dz d\bar{z}\, e^{\bar{z}z}\, \langle\bar{z}|\, \underbrace{e^{\sigma\cdot\mathbf{B}/N}\cdots e^{\sigma\cdot\mathbf{B}/N}}_{N\ \text{times}} |z\rangle$$

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Trotter's

$$e^{\sigma\cdot\mathbf{B}/N} \approx e^{aB^*/N}\, e^{a^\dagger B/N} + O(N^2)$$

- Insert N unities $\int d\bar{z}_i dz_i\, e^{-\bar{z}_i z_i}\, |z_i\rangle\langle\bar{z}_i|$
- integrate over $\bar{z}$ and z: sets $z =: z_0 = -z_N$

Antiperiodic Boundary Conditions

Coherent state path integral for the SP model

Inserting σ_3 (chirality matrix in $D = 2$)

$$\mathrm{tr}\, \sigma_3 e^{\sigma\cdot\mathbf{B}} = \int dz d\bar{z}\, e^{\bar{z}z} \, \langle \bar{z}|\sigma_3 \underbrace{e^{\sigma\cdot\mathbf{B}/N} \cdots e^{\sigma\cdot\mathbf{B}/N}}_{N \text{ times}} |z\rangle$$

$$= \int dz d\bar{z}\, e^{\bar{z}z} \, \langle -\bar{z}| e^{\sigma\cdot\mathbf{B}/N} \cdots e^{\sigma\cdot\mathbf{B}/N} |z\rangle$$

Since, $\langle \bar{z}|\sigma_3 = \langle -\bar{z}|$

- Insert N unities $\int d\bar{z}_i dz_i \, e^{-\bar{z}_i z_i} \, |z_i\rangle\langle\bar{z}_i|$
- integrate over $\bar{z}$ and z: sets $z =: z_0 = +z_N$

Periodic Boundary Conditions

Coherent state path integral for the SP model

Left with

$$\mathrm{tr}\, e^{\sigma\cdot\mathbf{B}} = \int dz_1 \ldots d\bar{z}_N \, e^{-\sum_i \bar{z}_i(z_i - z_{i-1})} \prod_{i=N}^{1} e^{(\bar{z}_i B + z_{i-1} B^*)/N}$$

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Two ways to proceed:

1. Naive one:

$$\prod_{i=N}^{1} e^{(\bar{z}_i B + z_{i-1} B^*)/N} = e^{\sum_i (\bar{z}_i B + z_{i-1} B^*)/N}$$

$$\longrightarrow \quad e^{\int_0^1 d\tau (\bar{z}(\tau) B + z(\tau) B^*)}$$

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2. Correct one: must keep exponentials (time)ordered

$$\prod_{i=N}^{1} e^{(\bar{z}_i B + z_{i-1} B^*)/N} \longrightarrow \mathbb{P}\, e^{\int_0^1 d\tau (\bar{z}(\tau) B + z(\tau) B^*)}$$

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Thus (for the naive way, drop $\mathbb{P}$),

$$\mathrm{tr}\, e^{\sigma\cdot\mathbf{B}} = \oint_{ABC} D\bar{z}(\tau) Dz(\tau)\, e^{-\int_0^1 d\tau \bar{z}\dot{z}(\tau)}\ \mathbb{P}\, e^{\int_0^1 d\tau (\bar{z}(\tau) B + z(\tau) B^*)}$$

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$$\mathrm{tr}\, e^{\sigma\cdot\mathbf{B}} = \oint_{ABC} D\psi(\tau)\, e^{-\frac{1}{4}\int_0^1 d\tau\, \psi_a \dot{\psi}_a(\tau)}\ \mathbb{P}\, e^{\int_0^1 d\tau\, \psi_a(\tau) B_a}$$

$$\mathrm{tr}\, \sigma_3\, e^{\sigma\cdot\mathbf{B}} = \oint_{PBC} D\psi(\tau)\, e^{-\frac{1}{4}\int_0^1 d\tau\, \psi_a \dot{\psi}_a(\tau)}\ \mathbb{P}\, e^{\int_0^1 d\tau\, \psi_a(\tau) B_a}$$

with Majorana variables, $\psi_1 = z + \bar{z}\,,\ \psi_2 = -i(z - \bar{z})$

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with Majorana variables, $\psi_1 = z + \bar z\,,\ \ \psi_2 = -i(z - \bar z)$

$$\oint_{ABC} D\psi(\tau)\, e^{-\frac{1}{4}\int_0^1 d\tau\, \psi_a\dot\psi_a(\tau)} = \mathrm{tr}\,\mathbb{1} = 2$$

$$\underbrace{\oint_{PBC} D\psi(\tau)\, e^{-\frac{1}{4}\int_0^1 d\tau\, \psi_a\dot\psi_a(\tau)}}_{\text{zero modes present}} = \mathrm{tr}\,\sigma_3 = 0$$

Coherent state path integral for the SP model

above,

$$\mathbb{P}\, e^{\int_0^1 d\tau\, \psi_a(\tau) B_a} = 1 + \int_0^1 d\tau\, \psi_a(\tau) B_a$$

$$+ \int_0^1 d\tau_1 \int_0^{\tau_1} d\tau_2\, \psi_a(\tau_1)\psi_b(\tau_2)\, B_a B_b + \cdots$$

Correlation functions:

$$\big\langle \psi_a(\tau)\psi_b(\tau') \big\rangle = \delta_{ab}\, \epsilon(\tau - \tau')$$

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Computation of the partition trace

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Coherent state path integral for the SP model

Computation of the partition trace

- In both ways, only even terms (in B, hence ψ) are nonvanishing
- In the naive case one only has terms proportional to

$$\int_0^1 d\tau \int_0^1 d\tau' \, \epsilon(\tau - \tau') = 0 \ .$$

Hence,

$$\mathrm{tr}\, e^{\sigma \cdot \mathbf{B}} = 2 \, ,$$

which is obviously WRONG!

Coherent state path integral for the SP model

- In the correct case, at order $2n$, the correlator yields

$$\int_0^1 d\tau_1 \int_0^{\tau_1} d\tau_2 \cdots \int_0^{\tau_{2n-1}} d\tau_{2n}\ \epsilon(\tau_1 - \tau_2) \cdots \epsilon(\tau_{2n-1} - \tau_{2n})$$
$$= \frac{1}{(2n)!}$$

which is the volume of the hyper-triangle; all other contractions cancel pairwise.

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It allows us to reproduce

$$\mathrm{tr}\, e^{\sigma\cdot \mathbf{B}} = 2\cosh(|\mathbf{B}|)\,,$$

the CORRECT result!

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Main message:

- if your action involves Grassmann odd parts
  $\Rightarrow$ you ought to path-order....
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Main message:

- if your action involves Grassmann odd parts
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  terms which involve those parts
- Grassmann even parts: no need of path-ordering
- Let's run another test with an even crazyer toy-model, a **Grassmann-odd magnetic field, in 2d**:

$$\underbrace{H = -\sigma \cdot \boldsymbol{\Omega}}_{\text{SP}' \text{ model}}$$

$\Rightarrow$ the lagrangian will be Grassmann-even

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- In $D = 2d$ it would be 2^d $(\sigma_a \ \to \ \gamma_a)$
- From the path integral:

$$
\begin{aligned}
\mathrm{tr}\, e^{\gamma\cdot\boldsymbol{\Omega}} &= \oint_{ABC} D\psi(\tau)\, e^{-\frac{1}{4}\int_0^1 d\tau\, \psi_a\dot{\psi}_a(\tau)}\, e^{\int_0^1 d\tau\, \psi_a(\tau)\Omega_a} \\
&= \oint_{ABC} D\psi(\tau)\, e^{-\frac{1}{4}\int_0^1 d\tau\, \psi_a\dot{\psi}_a(\tau)} = \mathrm{tr}\, \mathbb{1} = 2^d \qquad \checkmark
\end{aligned}
$$

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$$= \oint_{ABC} D\psi(\tau)\, e^{-\frac{1}{4}\int_0^1 d\tau\, \psi_a\dot{\psi}_a(\tau)} = \mathrm{tr}\, \mathbb{1} = 2^d \quad \checkmark$$

- Let's move on to more realistic models

Outline

The QFT model

Dirac particle coupled to external scalar, pseudoscalar, vector and axialvector fields Mondragón et al '95, D'Hoker, Gagné '95

$$S[\psi;\phi,\phi_5,A,A_5] = \int d^4x\ \bar{\psi}\Big[i\partial\!\!\!/ - m - g\phi - ig_5\gamma_5\phi_5 - eA\!\!\!/ - e_5\gamma_5 A\!\!\!/_5\Big]\psi(x)$$

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One-loop effective action (coupling constants absorbed into fields):

$$\Gamma[\phi,\phi_5,A,A_5] = \ln \mathrm{Det}\Big[\underbrace{m + p\!\!\!/ + \phi + i\gamma_5\phi_5 + A\!\!\!/ + \gamma_5 A\!\!\!/_5}_{m+\mathcal{O}}\Big]$$

The QFT model

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- the operator above is in the canonical first-order formalism:

$$\mathcal{O} = \not{p} + \cdots$$

- want to use a second-order fornalism:

$$H = p^2 + \cdots$$

to interpret H as a particle hamiltonian

The QFT model

Using that $\gamma_5^2 = \mathbb{1}$

$$\mathrm{Det}\big(m + \mathcal{O}\big) = \mathrm{Det}\big(m + \tilde{\mathcal{O}}\big) = \mathrm{Det}^{1/2}\Big[\big(m + \mathcal{O}\big)\big(m + \tilde{\mathcal{O}}\big)\Big]$$

where

$$\tilde{\mathcal{O}} := \gamma_5 \mathcal{O} \gamma_5$$

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where

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Thus,

$$\Gamma[\phi, \phi_5, A, A_5] = \frac{1}{2} \ln \mathrm{Det}\Big[m^2 - (\partial_\mu + i\mathcal{A}_\mu)^2 + \mathcal{V}\Big]$$

The QFT model

where we have an effective matrix-valued connection, which is non-Abelian,

$$\mathcal{A}_\mu = \underbrace{A_\mu + \gamma_{\mu\nu}\gamma_5 A_5^\nu}_{\textbf{even}} + \underbrace{i\gamma_\mu\gamma_5\phi_5}_{\textbf{odd}}\,,$$

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$$\mathcal{A}_\mu = \underbrace{A_\mu + \gamma_{\mu\nu}\gamma_5 A_5^\nu}_{\textbf{even}} + \underbrace{i\gamma_\mu\gamma_5\phi_5}_{\textbf{odd}},$$

and an effective matrix-valued potential,

$$\mathcal{V} = \underbrace{\frac{i}{2}\gamma^{\mu\nu}F_{\mu\nu} + i\gamma_5\partial\cdot A_5 + (D-2)A_5^2 + (D-1)\phi_5^2 + \phi^2}_{\textbf{even}}$$

$$+ \underbrace{2i\phi\phi_5\gamma_5 + 2m(\phi + i\gamma_5\phi_5)}_{\textbf{even}} + \underbrace{2i(D-2)\phi_5 \not{A}_5 - i\gamma^\mu\partial_\mu\phi}_{\textbf{odd}}$$

The QFT model

The 'ln' can exponentiated *à la Schwinger*,

$$\Gamma[\phi, \phi_5, A, A_5] = -\frac{1}{2}\int_0^\infty \frac{dT}{T} \times \mathrm{Tr}\exp\Big\{ -T\Big[-(\partial_\mu + i\mathcal{A}_\mu)^2 + \mathcal{V} + m^2\Big]\Big\}$$

and,

$$\mathrm{Tr}\exp\{\} \sim \frac{1}{(4\pi T)^{D/2}} \sum_{n=0}^\infty T^n \int d^D x \ \underbrace{a_n(x,x)}_{\text{SDW coefficients}}$$

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- Gamma matrices are kept explicit. e.g.

$$a_0 = \mathrm{tr}\,\mathbb{1} = 2^{D/2},$$
$$a_1 = -\mathrm{tr}\,[\mathcal{V}] = -2^{D/2}\left[\phi^2 + (D-1)\phi_5^2 + (D-2)A_5^2\right]$$

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- SDW coefficients provide benchmarks for our worldline construction
- In some cases (e.g. scalar coupled to vector and Yukawa) the expansion can be resummed

Franchino-Viñas et al '24, '25

Worldline construction

Issues to be dealt with:

- presence of terms with an odd number of γ's (issue not present in QED/QCD or pure vector/axial-vector)
  - odd number of Grassmann variables ψ^μ
  - need path ordering as we know from the toy model
  - worldline translation invariance broken: won't be able to fix the position of a vertex

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  - odd number of Grassmann variables ψ^μ
  - need path ordering as we know from the toy model
  - worldline translation invariance broken: won't be able to fix the position of a vertex
- axial couplings: presence of γ_5: not a big deal
  - when present in amplitudes, swap them around at the prices of minus signs and cancel them pairwise
  - if one remains (odd number) it changes the b.c.'s of the ψ's to PBC, as in the toy model

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  1. Time Slicing: first-principled
  2. Mode Regularization: like cutoff
  3. Dimensional Regularization: perturbative

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  Only TS appears to be suitable in the presence of $\mathbb{P}$
- γ_μ matrices are reproduced with Grassmann variables, as in the toy model $\gamma_\mu \;\rightarrow\; \psi_\mu$

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- it provides a 'counterterm' potential V_{TS}
- in our case

$$V_{TS} = -D\phi_5^2 - (D-1)\psi_\mu A_5^\mu \phi_5 - (D-1)A_5^2$$

Outline

The one-loop effective action

With the above prescriptions,

$$\Gamma[\phi, \phi_5, A, A_5] = -\int_0^\infty \frac{dT}{2T} e^{-m^2 T} \oint Dx \oint_{(A)PBC} D\psi \; e^{-\int_0^1 d\tau \, L(\tau)}$$

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where,

$$L(\tau) = \frac{1}{4}\dot{x}^2 + \frac{1}{2}\psi\cdot\dot{\psi} + i\dot{x}^\mu A_\mu - i\psi^\mu\psi^\nu F_{\mu\nu} - i\dot{x}\cdot\psi\psi\cdot A_5\hat{\gamma}_5$$
$$+ i\partial\cdot A_5\hat{\gamma}_5 - A_5^2 + \phi^2 - \phi_5^2 + i\phi\phi_5\hat{\gamma}_5 + m(\phi + i\phi_5\hat{\gamma}_5)$$
$$-\psi\cdot\partial\phi + i\dot{x}\cdot\psi\phi_5\hat{\gamma}_5 - \psi\cdot A_5\hat{\gamma}_5$$

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Summary

- Obtained a unified worldline description of a Dirac particle coupled to (psuedo) scalars and (axial) vectors
  - use vertex operators to compute one-loop amplitudes (various examples done)
  - inverse mass expansion $\equiv$ heat-kernel expansion

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- Tree-level version

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Outlook:

- Tree-level version
- higher loop contributions to Heisenberg-Euler lagrangians
- effective actions in (strong) fields

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  - use vertex operators to compute one-loop amplitudes (various examples done)
  - inverse mass expansion $\equiv$ heat-kernel expansion

Outlook:

- Tree-level version
- higher loop contributions to Heisenberg-Euler lagrangians
- effective actions in (strong) fields
- Coupling to gravity (n. l. σ models)