IMPERIAL

MoEDAL

Magnetic monopole pair production in strong magnetic fields

Arttu Rajantie

First Quantisation for Physics in Strong Fields,
Edinburgh, 25 February 2025

Maxwell Equations

$$\vec{\nabla} \cdot \vec{E} = \rho_{\mathrm{E}}$$

$$\vec{\nabla} \cdot \vec{B} = 0$$

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

$$\vec{\nabla} \times \vec{B} = -\frac{\partial \vec{E}}{\partial t} + \vec{j}_{\mathrm{E}}$$

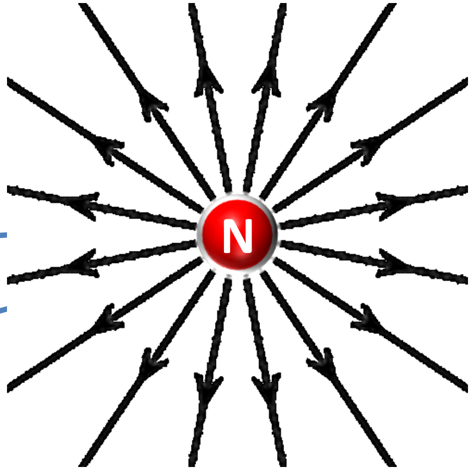
No magnetic charges

377.] The quantity of magnetism at one pole of a magnet is always equal and opposite to that at the other, or more generally thus:—

In every Magnet the total quantity of Magnetism (reckoned algebraically) *is zero.* (Maxwell, Treatise on Electricity and Magnetism, 1873)

Magnetic Charges

$$\vec{\nabla} \cdot \vec{E} = \rho_{\mathrm{E}}$$
$$\vec{\nabla} \cdot \vec{B} = \rho_{\mathrm{M}}$$
$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} - \vec{J}_{\mathrm{M}}$$
$$\vec{\nabla} \times \vec{B} = \frac{\partial \vec{E}}{\partial t} + \vec{J}_{\mathrm{E}}$$

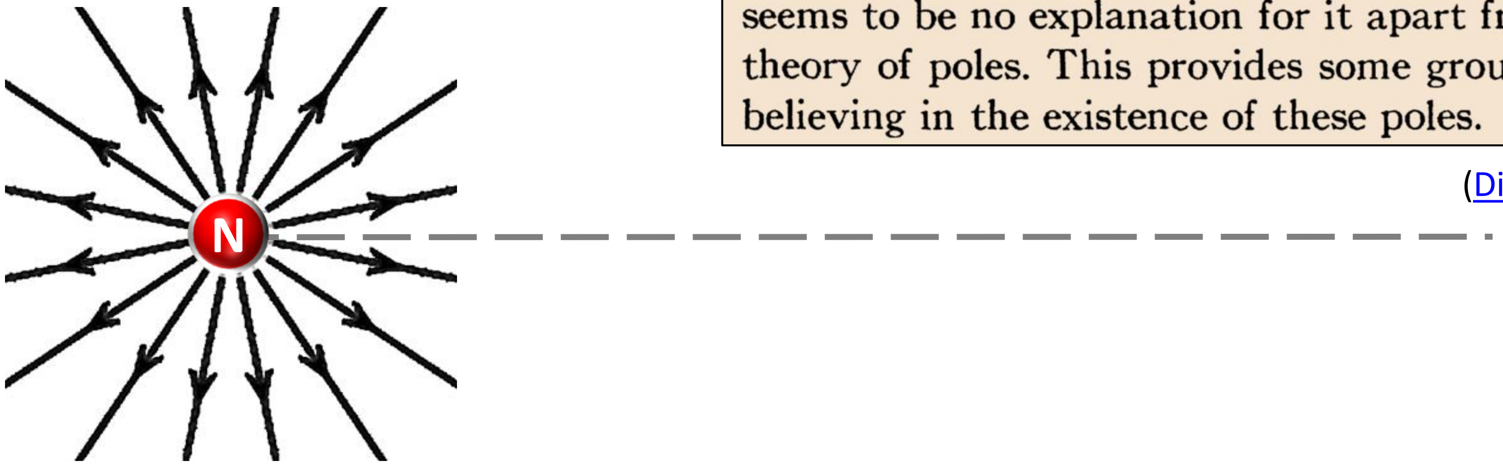


- Duality $\vec{E} + i\vec{B} \to e^{i\theta}(\vec{E} + i\vec{B})$
- Magnetic monopole = Particle with non-zero magnetic charge g

Dirac Monopole (1931)

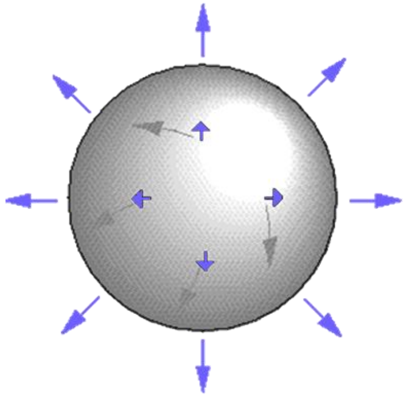
> require all poles to be quantized. The quantization of electricity is one of the most fundamental and striking features of atomic physics, and there seems to be no explanation for it apart from the theory of poles. This provides some grounds for believing in the existence of these poles.

(Dirac 1948)



- Quantum mechanics: Dirac monopole configuration (Dirac 1931)
- Dirac string unobservable if all electric charges e and magnetic charges g satisfy the Dirac quantisation condition $eg \in 2\pi\mathbb{Z}$
- The magnetic charge g must be an integer multiple of the Dirac charge $g_D = \frac{2\pi}{e_0} \approx 20.7$, where e_0 is the elementary electric charge

't Hooft-Polyakov Monopole



- Smooth "hedgehog" solution in SU(2) gauge theory with an adjoint scalar field ('t Hooft 1974, Polyakov 1974):

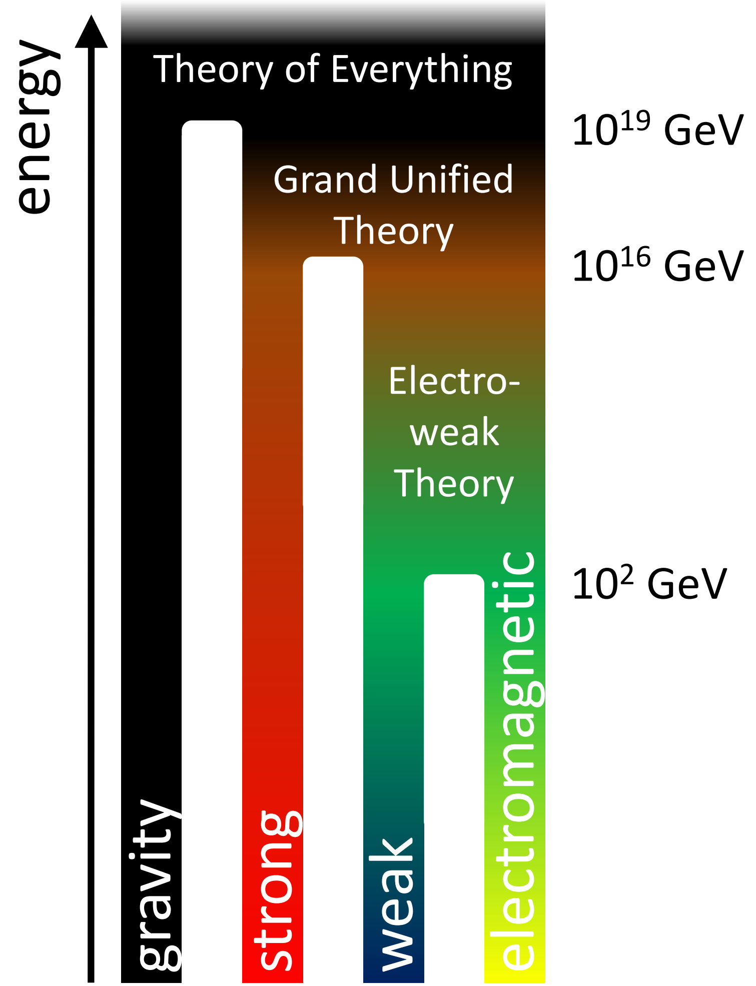
$$\Phi^a \propto x_a, \quad A_i^a \propto \epsilon_{iaj} x_j$$

- Magnetic charge $g = \int d\vec{S} \cdot \vec{B} = 2g_D = 4\pi/e$
- Finite, semiclassically calculable mass $M \approx \frac{4\pi v}{e} \sim \frac{m}{e^2}$

GUT Monopoles

- Grand Unified Theory (GUT): Electroweak & strong forces unified above ~ $10^{16}\mathrm{GeV}$
  → 't Hooft-Polyakov monopoles of mass ~ $10^{17}\mathrm{GeV}$
- String theory monopoles typically ~ $10^{20}\mathrm{GeV}$ ([Gross&Perry 1983])
- Lower mass in some models, e.g,. Pati-Salam – like $SU(3)_c \times SU(3)_L \times SU(3)_R$ trinification ([Raut, Shafi & Thapa 2022]):

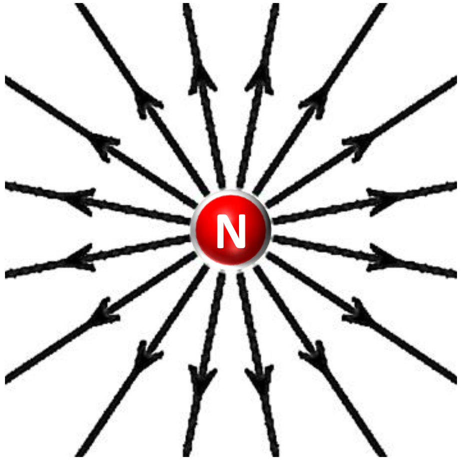
$$M \sim 160\ \mathrm{TeV}$$



Electroweak Monopoles

- Standard Model: No finite-energy magnetic monopole solution
- UV modification ⇒ Dirac-like singular monopole with finite energy (Cho&Maison 1996)
  - Mass bound $M \gtrsim 5.5\ \mathrm{TeV}$ (Cho et al 2012, Ellis et al 2016)
  - Another variant $M \gtrsim 2.4\ \mathrm{TeV}$ (Beneš&Blaschke 2020)
- With additional scalar triplets ⇒ Non-singular monopole solution (Hung 2020; Ellis, Hung & Mavromatos 2020)
  - Mass $900\ \mathrm{GeV} \lesssim M \lesssim 3\ \mathrm{TeV}$
  - Also explains $\sin^2\theta_W \approx 0.231$ and neutrino masses

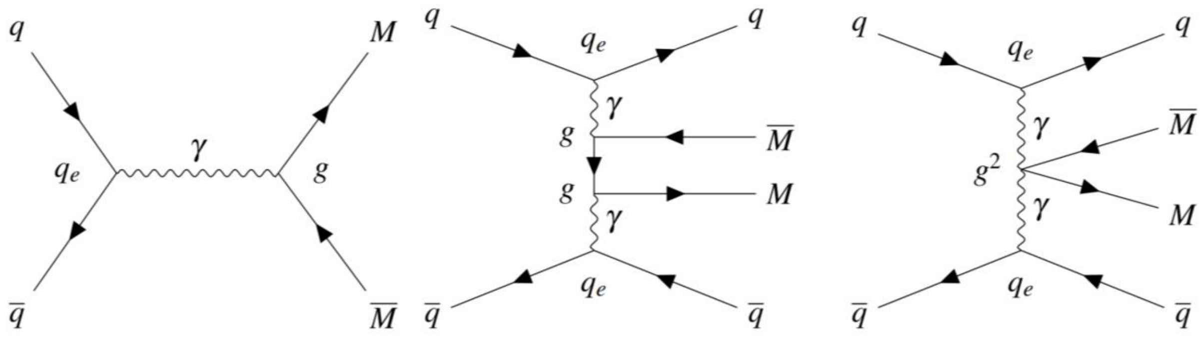
Elementary Monopoles



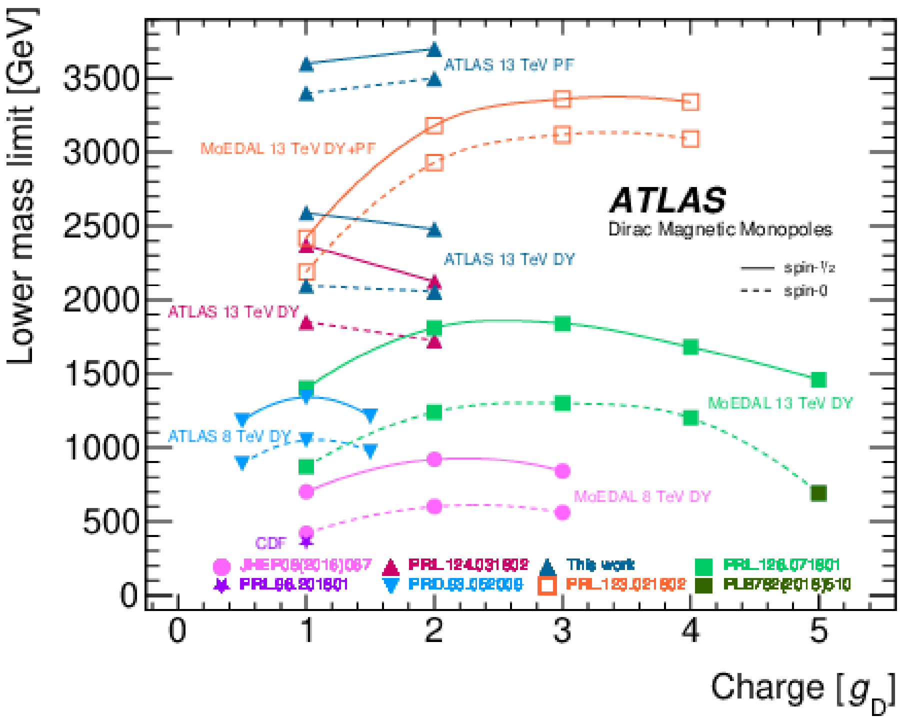
- Elementary particle with a magnetic charge, **arbitrary mass**
- Quantum field theory formulations: Schwinger 1966, Zwanziger 1971
  - Also lattice formulation (Farakos et al 2024)
- Dirac quantisation condition $g = 2\pi/e \approx 20.7 \gg 1$ ⇒ Non-perturbative
- Quantum effects ⇒ Effective size $R \sim g^2/4\pi M$? (Goebel 1970)

LHC pp Results

- Nominal mass bound $M \gtrsim 3\ \mathrm{TeV}$
- Based on tree-level Drell-Yan/photon fusion processes
- But perturbation theory is **not valid** because $g_\mathrm{D} = 2\pi/e \approx 20.7 \gg 1$



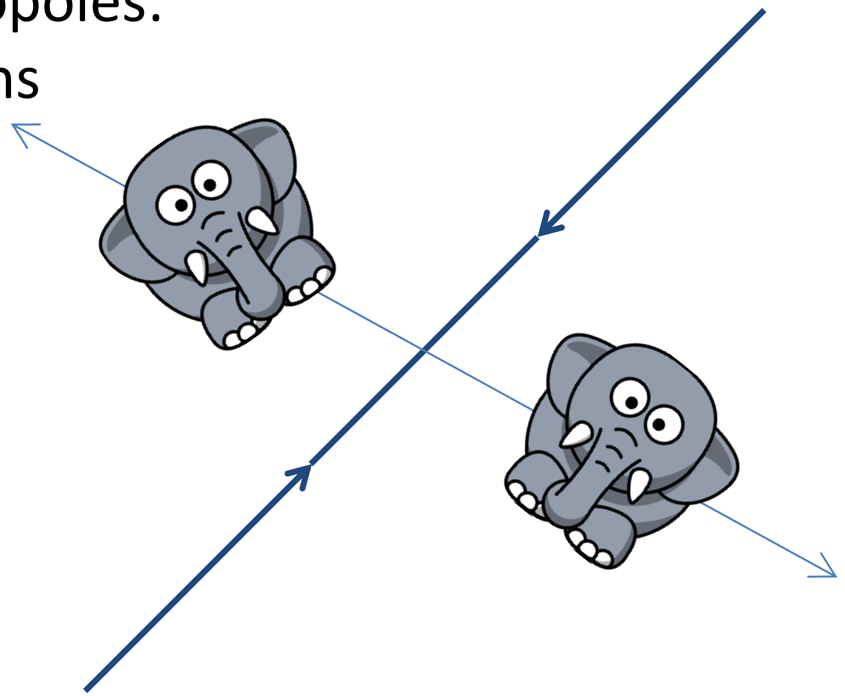
(a) DY monopole production. (b) Three-vertex PF process. (c) Four-vertex PF process.



(ATLAS Collaboration, 2023)

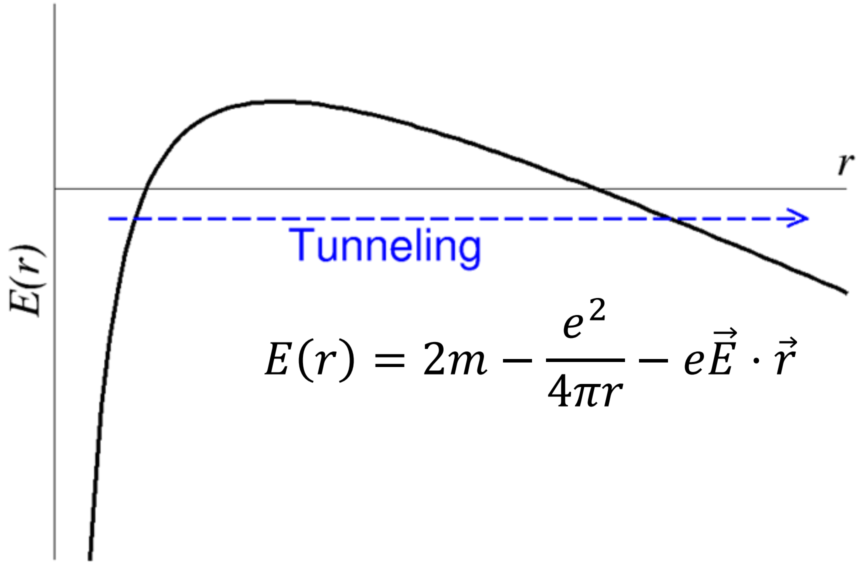
Exponential Suppression

- Semiclassical argument for solitonic monopoles: pair production from two-particle collisions suppressed by $\sim e^{-4/\alpha} \sim 10^{-238}$ (Witten 1979, Drukier&Nussinov 1982)
- Confirmed numerically for kinks in 1+1D (Demidov&Levkov 2011)
- Production of solitonic (and elementary?) monopoles may be practically impossible in two-particle collisions



Schwinger Pair Production

- Consider instead pair production from a strong magnetic field
- Electromagnetic dual to usual Schwinger pair production:
  (Sauter 1931, Heisenberg&Euler 1936, [Schwinger 1951](#))
  Tunneling through Coulomb potential barrier
- No direct experimental confirmation yet:
  - e^+e^- pairs need
    $|\vec{E}| \gg \frac{m^2}{e} \sim 10^{18}\ \mathrm{V/m}$
  - 1000 x stronger than the most powerful lasers



Worldline Calculation

- Dunne&Schubert 2005:
  Decay rate $\Gamma = 2\mathcal{V}^{-1}\,\mathrm{Im}\left(-\log\langle\Omega|\hat{S}_E|\Omega\rangle\right)$, where $\hat{S}_E$ is the Euclidean S-matrix

$$\langle\Omega|\hat{S}_E|\Omega\rangle = \int \mathcal{D}A_\mu \mathcal{D}\phi\, e^{-S}$$

$$= \int \mathcal{D}A_\mu \exp\left[-\mathrm{Tr}\log\left(-D^2+m^2\right) - \int d^4x \frac{1}{4} F_{\mu\nu}F^{\mu\nu}\right]$$

- Express the tracelog as a worldline

$$e^{-\mathrm{Tr}\log(-D^2+m^2)} = \sum_k \frac{1}{k!}\left[\int_0^\infty \frac{ds}{s} \int \mathcal{D}x_\mu\, e^{-S_0[x_\mu;s]}\right]^k,$$

where $S_0[x_\mu;s] = \frac{1}{4s}\int_0^1 d\tau\, \dot{x}^\mu\, \dot{x}_\mu + m^2 s + e\oint dx^\mu\, A_\mu$

Worldline Calculation

- Integrate out the photon

$$\Gamma = -\frac{2}{\mathcal{V}} \operatorname{Im} \log \sum_k \frac{1}{k!} \int \left(\prod_{j=1}^{k} \frac{ds_j}{s_j} \mathcal{D}x_j^\mu \right)$$

$$\times \exp\left[-\sum_{j=1}^{k} S_0\left[x_j^\mu; s_j\right] - \frac{e^2}{2} \sum_{ij} \oint dx_i^\mu \oint dx_j^\nu \, G_{\mu\nu}(x_i, x_j) \right]$$

where

$$G_{\mu\nu}(x_i, x_j) = -\frac{\delta_{\mu\nu}}{4\pi^2 |x_i - x_j|^2}$$

Worldline Calculation

- Dilute instanton gas approximation: $k = 1$

$$\Gamma = -\frac{2}{\mathcal{V}}\,\mathrm{Im}\log\int\frac{ds}{s}\mathcal{D}x^{\mu}$$

$$\times\exp\left[-S_0[x^{\mu};s] - \frac{e^2}{2}\int_0^1 d\tau\,d\tau'\,\dot{x}^{\mu}(\tau)\dot{x}^{\nu}(\tau')G_{\mu\nu}(x(\tau),x(\tau'))\right]$$

- Weak field $e\left|\vec{E}\right|/m^2 \ll 1 \;\Rightarrow$ Saddle point approximation $\Gamma \propto e^{-S[x]}$, where

$$S[x] = m\left(\int d\tau\dot{x}_{\mu}\dot{x}_{\mu}\right)^{1/2} + g\int d\tau\dot{x}_{\mu}A_{\mu}^{\mathrm{ext}}(x)$$
$$-\frac{e^2}{2}\int_0^1 d\tau\,d\tau'\,\dot{x}^{\mu}(\tau)\dot{x}^{\nu}(\tau')G_{\mu\nu}(x(\tau),x(\tau'))$$

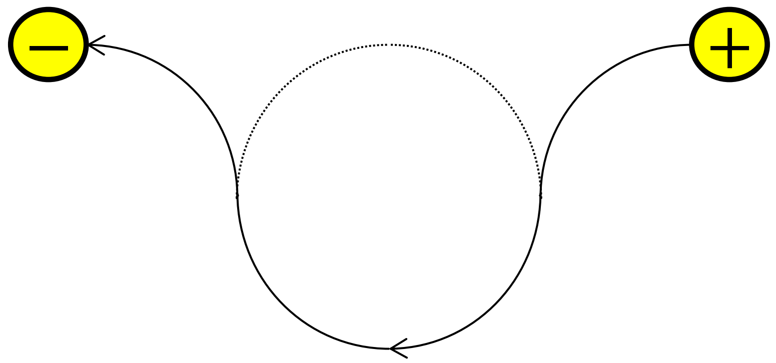
- Prefactor = functional determinant

Schwinger Rate in Constant Field

- Pair production rate per spacetime volume $\Gamma \sim \exp(-S_{\rm inst})$
  - $S_{\rm inst}$ is the instanton action
  - Prefactor from functional determinant
- Worldline instanton: (Affleck, Alvarez & Manton 1982)
  - Circle of radius $r = m/e|\vec{E}|$
  - Action $S_{\rm inst} = \frac{\pi m^2}{e|\vec{E}|} - \frac{e^2}{4}$

$$\Rightarrow \Gamma = \frac{e^2|\vec{E}|^2}{8\pi^3} e^{-\frac{\pi m^2}{e|\vec{E}|}+\frac{e^2}{4}}$$

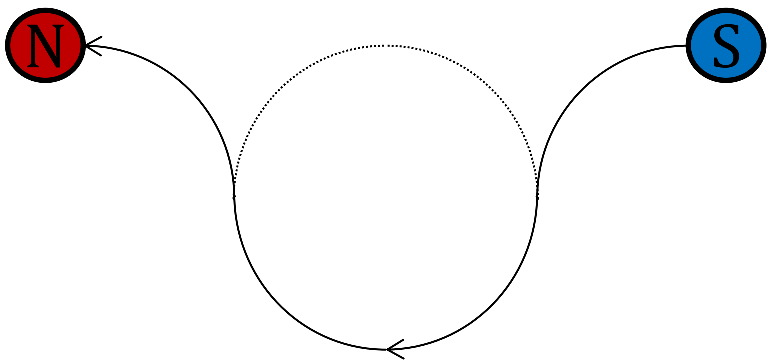
- Arbitrary coupling but weak field $e|\vec{E}| \ll m^2$



Monopoles from Schwinger Process

- The calculation does not require weak coupling: $g \gg 1$ is not a problem
- Largely independent of microscopic structure of monopoles (elementary or solitonic)
- Pair production rate (constant field, $T = 0$):
  (Affleck&Manton 1982; Gould&AR 2017)

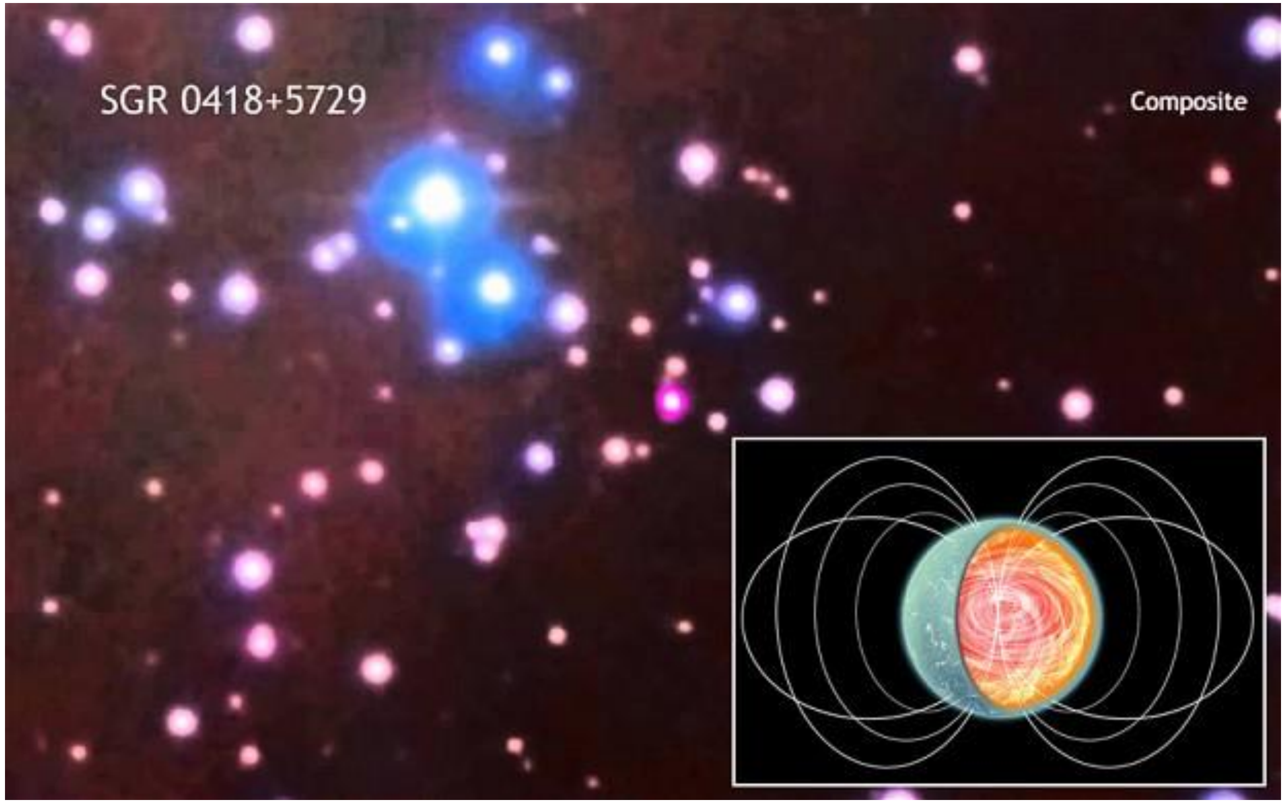
$$\Gamma = \frac{g^2 |\vec{B}|^2}{8\pi^3} e^{-\frac{\pi M^2}{g|\vec{B}|} + \frac{g^2}{4}}$$



- Pair production if $M \lesssim \left(\frac{g^3 |\vec{B}|}{4\pi}\right)^{1/2}$
- Strongest magnetic fields in lab $|\vec{B}| \sim 100\ \mathrm{T} \sim 10^{-13}\ \mathrm{GeV}^2$
  $\Rightarrow$ Mass bound $M \gtrsim 10\ \mathrm{keV}$

Magnetars

- Neutron stars with very strong magnetic fields
- Exterior: $B \sim 10^{11}\ \mathrm{T} \approx 10^{-4}\ \mathrm{GeV}^2$
  - Monopole pair production would make the field decay
  - Bound
    $M \gtrsim 0.3\ \mathrm{GeV}, g = g_D$
    $M \gtrsim 0.7\ \mathrm{GeV}, g = 2g_D$
    (Gould&AR 2017)
- Interior: $B \sim 10^{13}\ \mathrm{T}$?
  - Monopoles production would lead to field fluctuations (Klyuev 2024)



LHC Heavy Ion Collisions

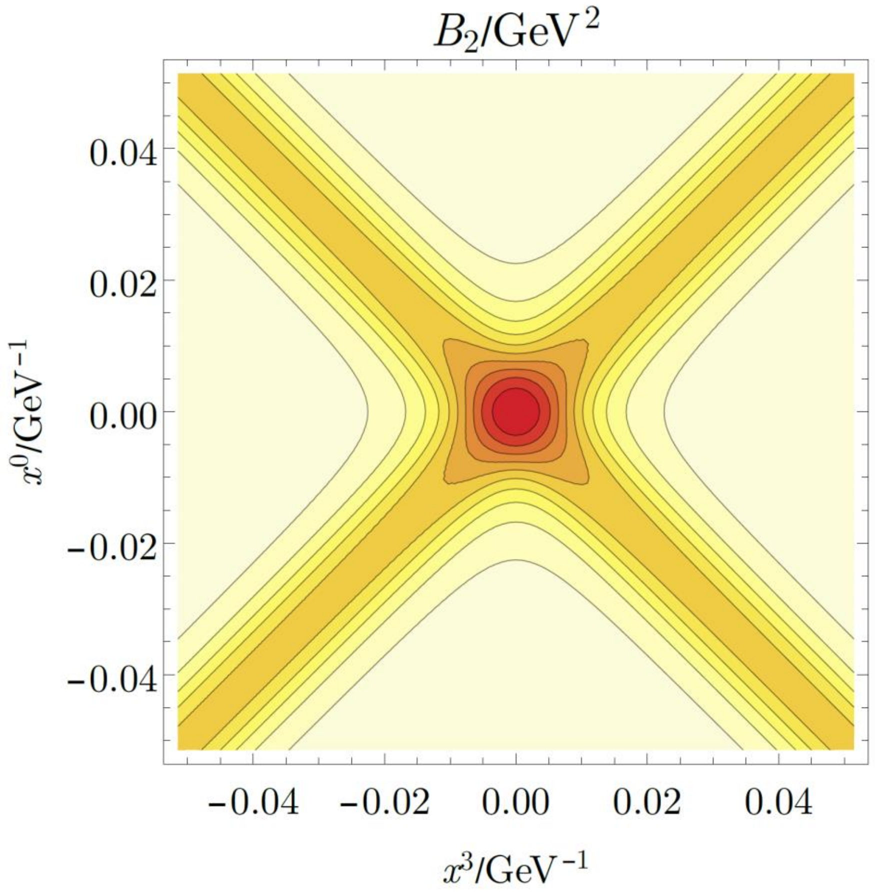
- Strongest magnetic fields in the Universe: Ideal for monopole search (Gould&AR 2017)
- Run 2: Pb-Pb collisions with $\sqrt{s_{NN}} = 5.02\ \mathrm{TeV}$
- Ultraperipheral collision: Time-dependent field

$$B(t) \approx \frac{B_{\max}}{(1+\omega^2 t^2)^{\frac{3}{2}}}$$

  with peak strength

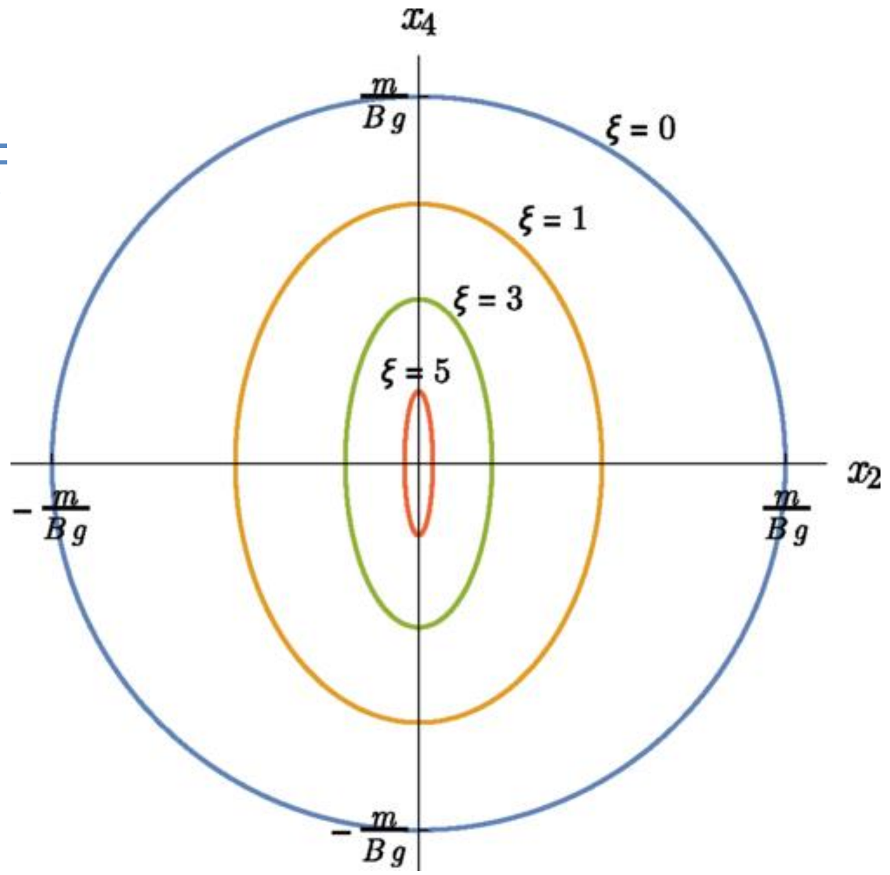
$$B_{\max} \approx \frac{Ze\gamma}{2\pi R^2} \approx 7.3\ \mathrm{GeV}^2 \sim 10^{16}\ \mathrm{T}$$

  and $\omega \approx \gamma/R \approx 73\ \mathrm{GeV}$
- Highly time-dependent



Worldline Instanton

$$S[x] = m\left(\int d\tau \dot{x}_\mu \dot{x}_\mu\right)^{1/2} + gB_{\max}\int d\tau \frac{\dot{x}_4 x_2}{\sqrt{1-\omega^2 x_4^2}} - \frac{g^2}{8\pi^2}\int_0^1 d\tau\, d\tau' \frac{\dot{x}_\mu(\tau)\dot{x}_\mu(\tau')}{|x(\tau)-x(\tau')|^2}$$



- Gould, Ho & AR 2019 :
  - Leading order in g^3B/m^2: Ellipse
  - Action to NLO:

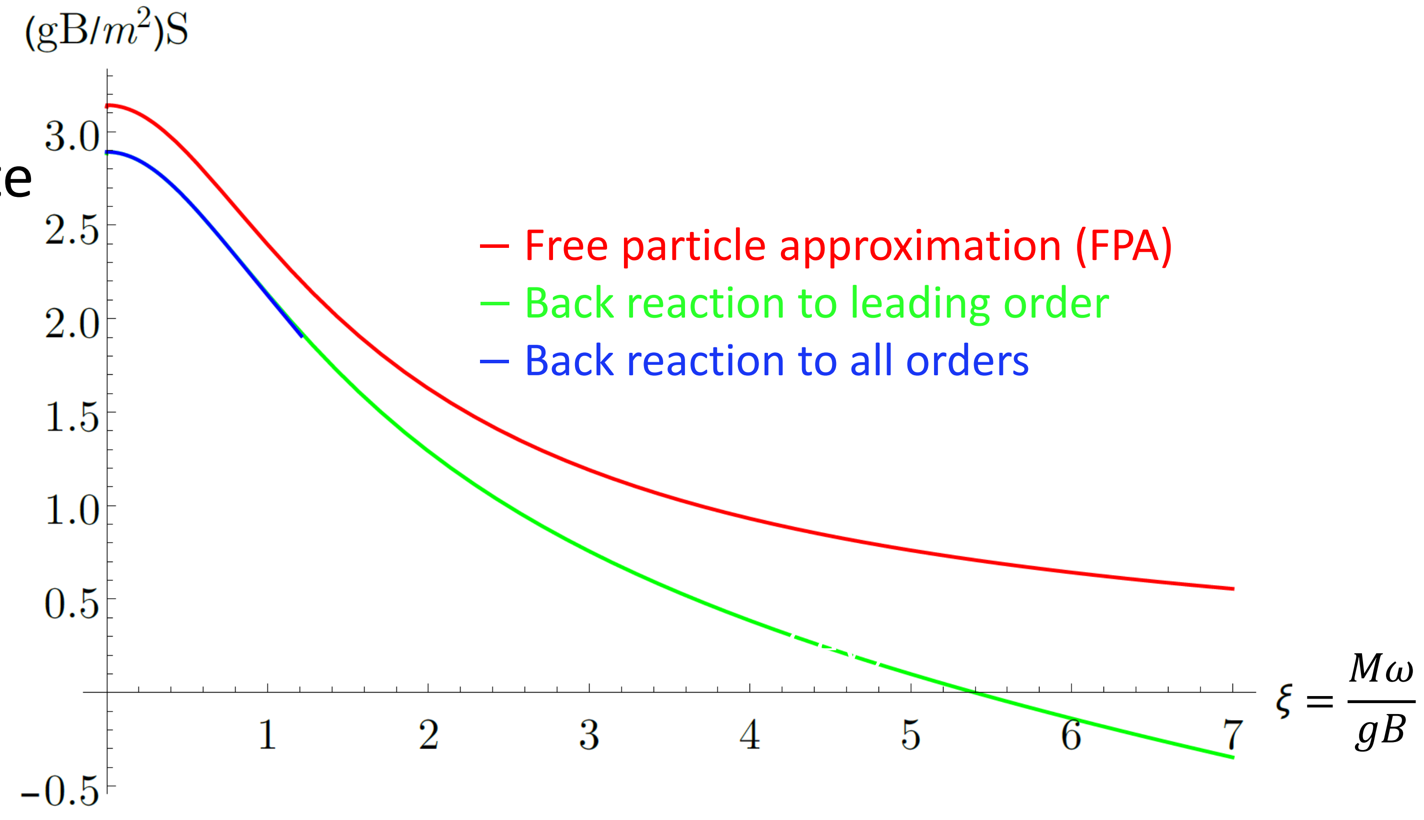
$$S[x] = \frac{4m^2}{gB\xi^2}\left(\mathbf{E}(-\xi^2) - \mathbf{K}(-\xi^2)\right) - \frac{g^2}{8}\frac{2+\xi^2}{\sqrt{1+\xi^2}}$$

where $\xi = m\omega/gB$

  - All orders: Solve numerically

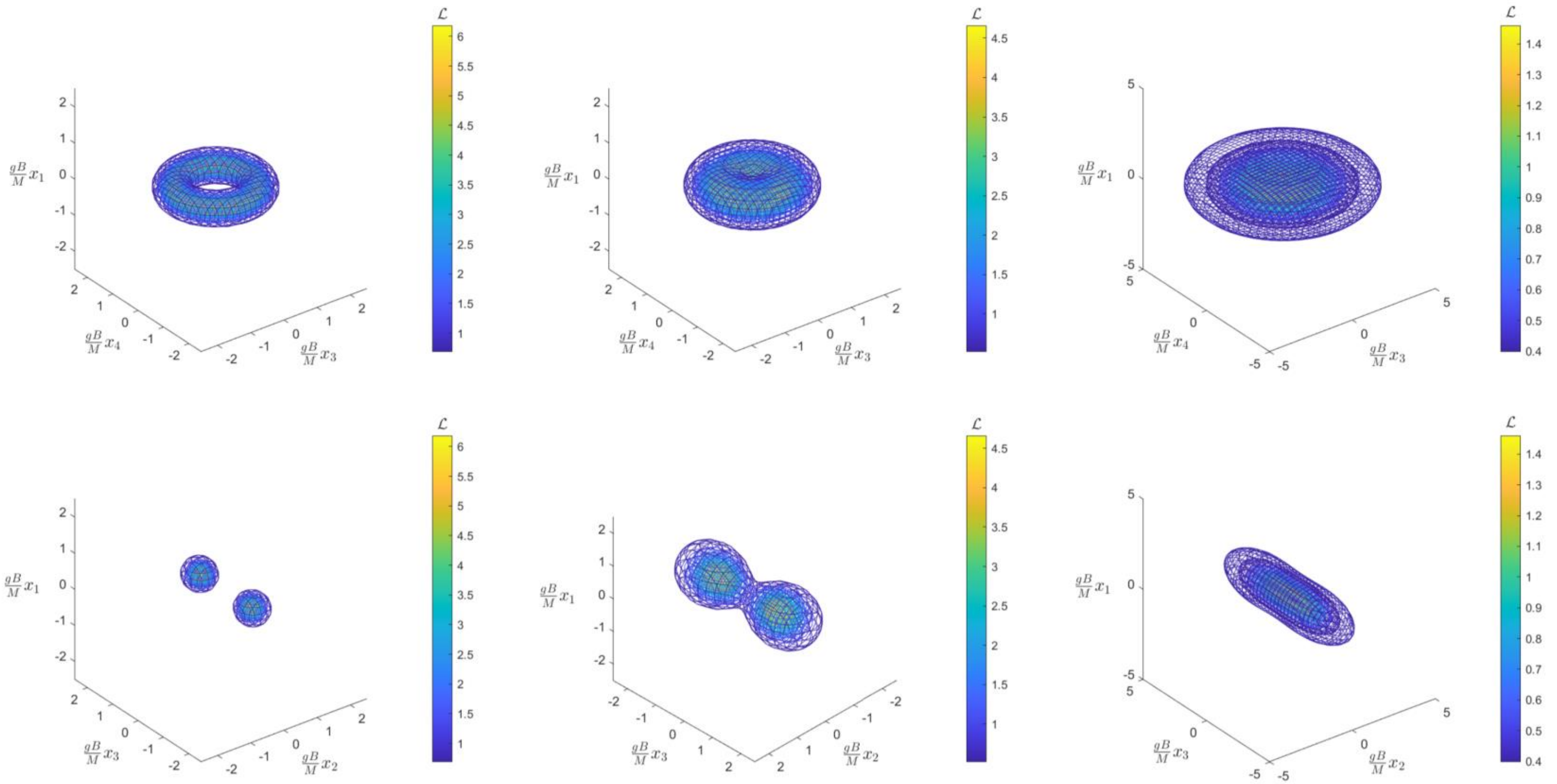
Time Dependence

- Time dependence and interactions both **enhance** the production rate (Gould, Ho & AR, 2019)
  $\Rightarrow$ Stronger mass bounds
- Numerical calculation cannot reach LHC parameters yet:
  Need $\xi \gtrsim 40$



Effect of Monopole Size

- 4D field theory instanton solution (Ho&AR 2021)
- Georgi-Glashow model: SU(2)+adjoint scalar
- Action determines the quantum Schwinger rate $\Gamma \propto \exp(-S_{\text{inst}})$



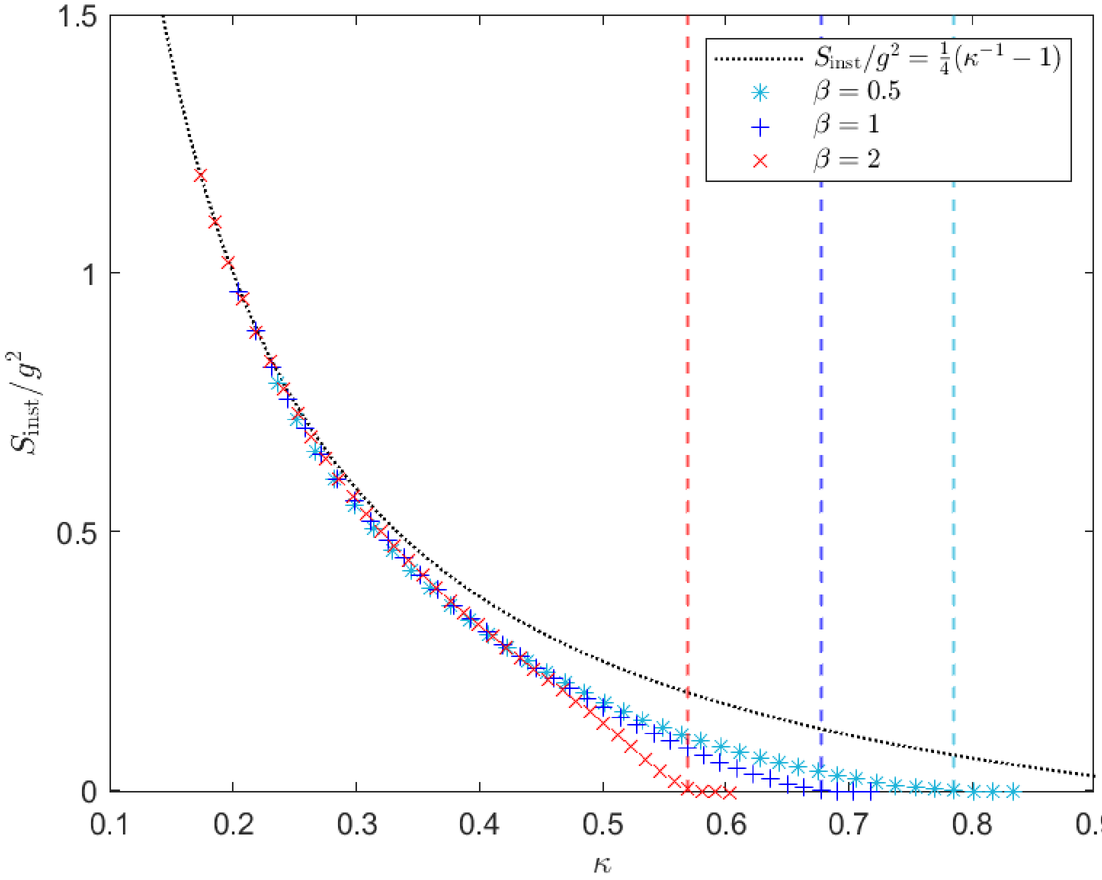
(a) $B/B_{\text{crit}} = 0.3$ (b) $B/B_{\text{crit}} = 0.6$ (c) $B/B_{\text{crit}} = 0.9$

(Ho and AR, 2021)

Effect of Monopole Size

- 4D field theory instanton solution (Ho&AR 2021)
- Georgi-Glashow model: SU(2)+adjoint scalar
- Action determines the quantum Schwinger rate $\Gamma \propto \exp(-S_{\text{inst}})$
  - **Enhanced** by nonzero size
  - Becomes unsuppressed at critical field $B_{\text{crit}} = m_{\text{v}}^2/e$

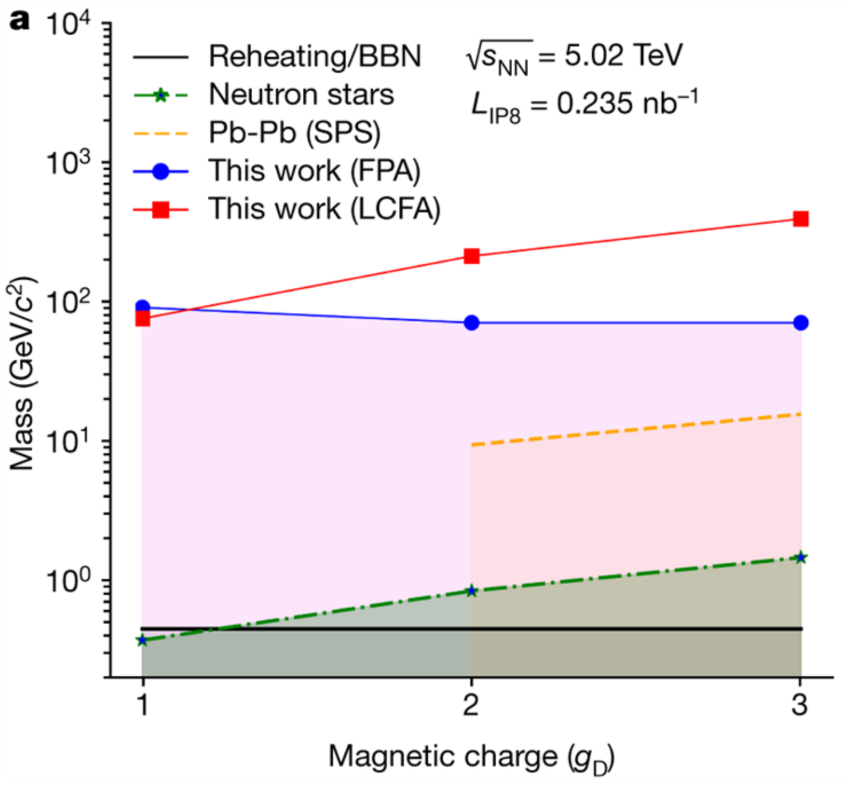
$\beta = m_{\text{s}}/m_{\text{v}}$



(Ho and AR, 2021)

MoEDAL Monopole Searches

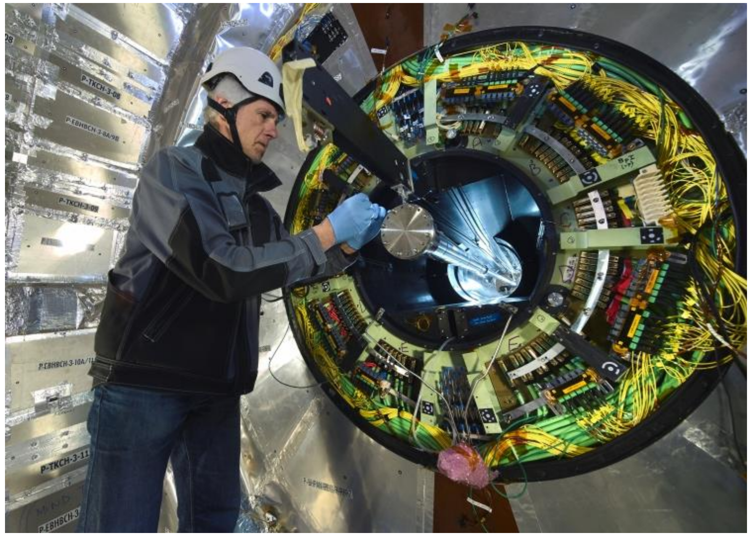
- Conservative bounds:
  Worldline instanton with
  the free particle approximation
- Momentum distribution: Gould, Ho & AR 2021
- MoEDAL, Nature 2022:
  - LHC Run 2:
    $235\ \mu\text{b}^{-1}$ at $\sqrt{s_{NN}} = 5.02\ \text{TeV}$
  - $800\ \text{kg}$ of aluminium trapping detectors
- Lower mass bound $M \gtrsim 80\ \text{GeV}$
  for all magnetic charges

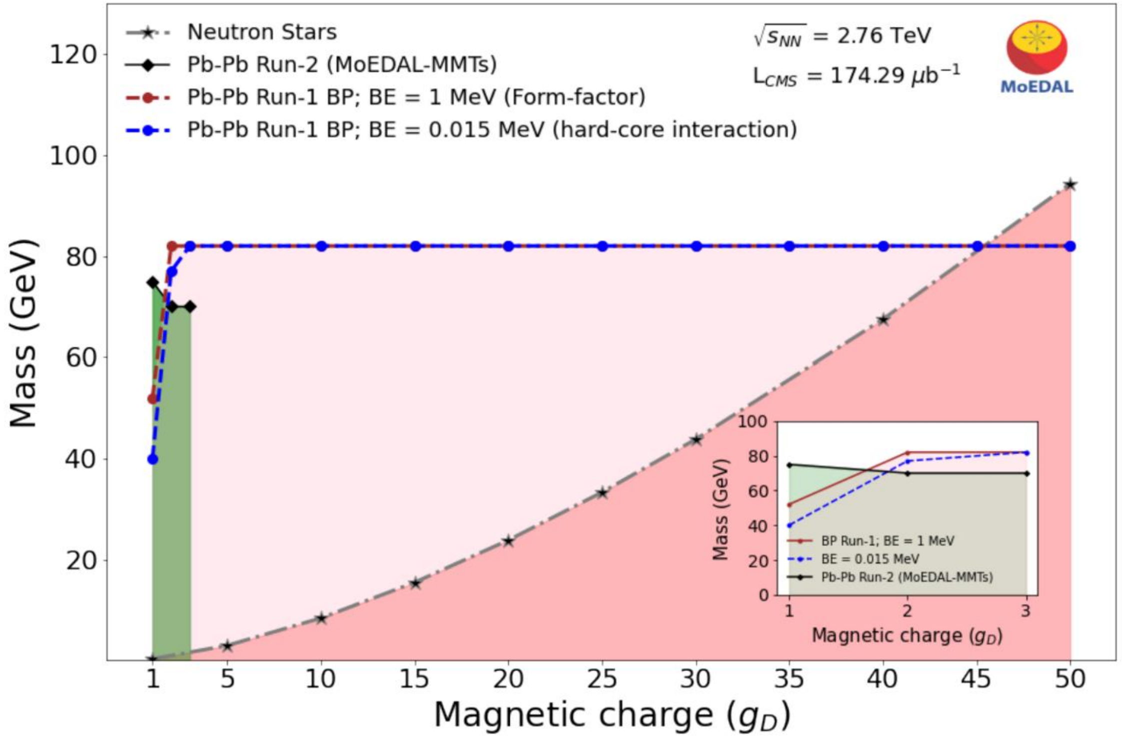


(MoEDAL Collaboration, Nature 2022)

MoEDAL Monopole Searches

- MoEDAL, PRL 2024
  - CMS beam pipe from LHC Run 1: $174.29\ \mu b^{-1}$ at $\sqrt{s_{NN}} = 2.76$ TeV
- Lower mass bound $M \gtrsim 80$ GeV for all magnetic charges

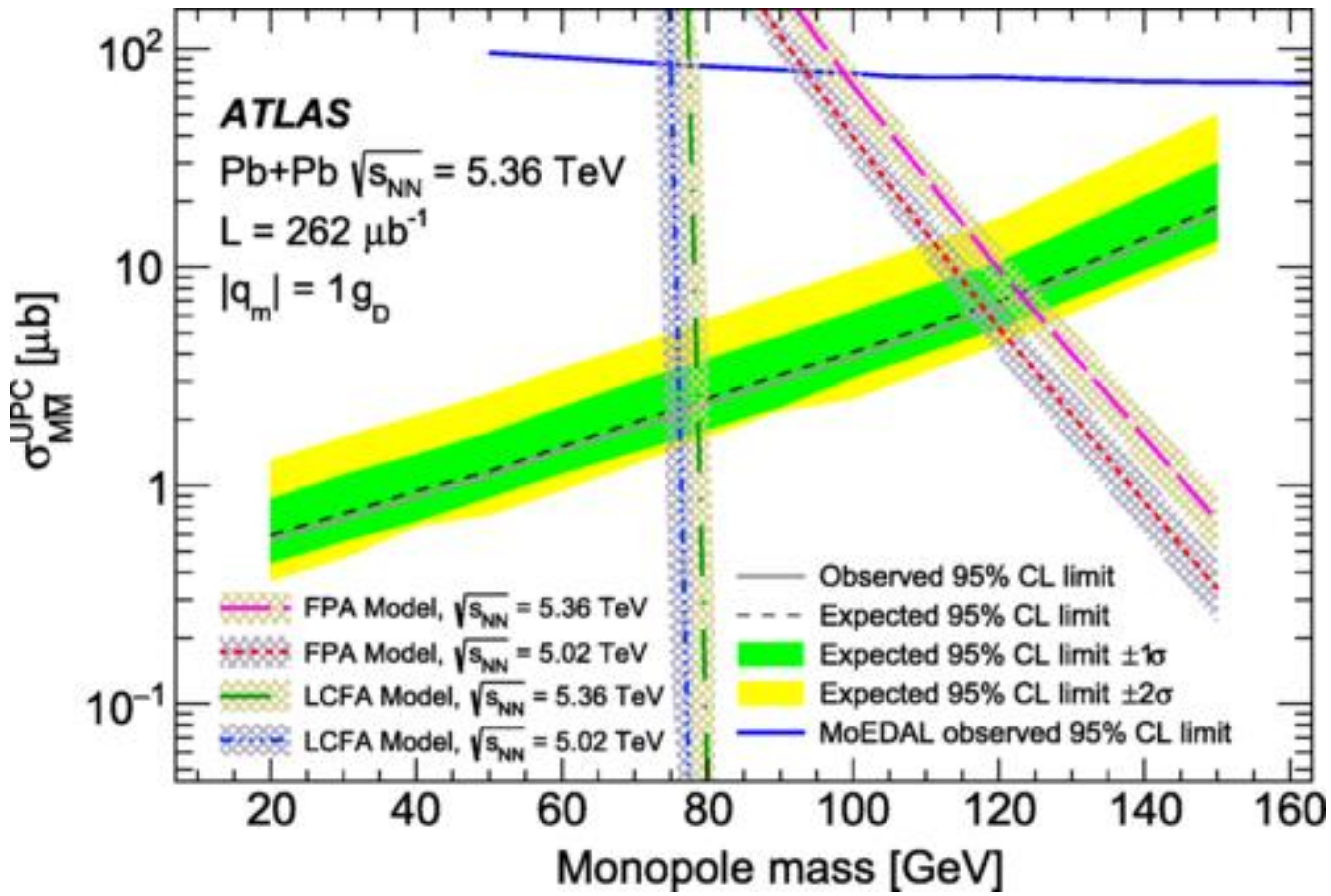




(MoEDAL Collaboration, PRL 2024)

ATLAS Monopole Search

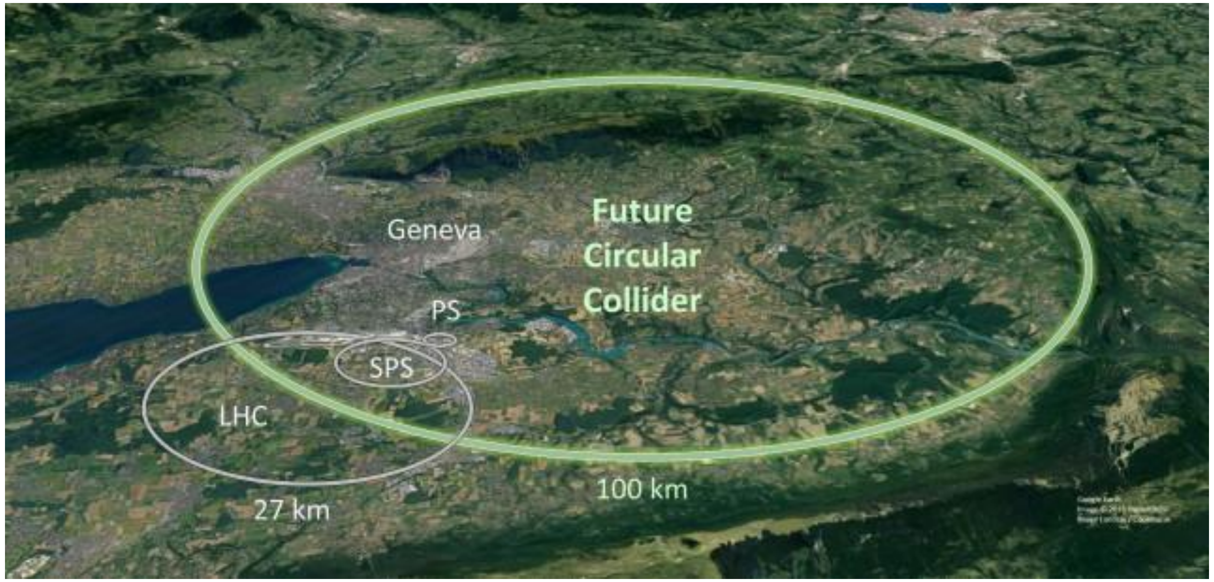
- ATLAS Collaboration, [PRL 2025]
- LHC Run 3: $262\mu\mathrm{b}^{-1}$ at $\sqrt{s_{NN}} = 5.36\ \mathrm{TeV}$
- High ionisation signal in the ATLAS pixel detector
- No events: Mass bound $M > 120\ \mathrm{GeV}$ for $g = 1g_D$ with free-particle approximation



Future Prospects

- More theoretical work is needed to fully account for finite size and spacetime dependence: Will improve current conservative bounds

- Free-particle approximation:
  Reach masses $M \sim \frac{\sqrt{s_{NN}}}{70}$
- FCC-hh : $\sqrt{s_{NN}} \sim 100\ \mathrm{TeV}$
  $\Rightarrow M \sim 1.4\ \mathrm{TeV}$ (EW monopoles!?)



Circular Collider on the Moon

- Beacham&Zimmermann 2021: Largest collider achievable with current technology
- Collision energy $\sqrt{s_{NN}} \sim 14\ \mathrm{PeV}$
- Enough to produce monopoles with mass $M \sim 200\ \mathrm{TeV}$ (e.g. Pati-Salam monopole!?)