A two-cluster approach for halo nuclei

Presentation by

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Exotic nuclei

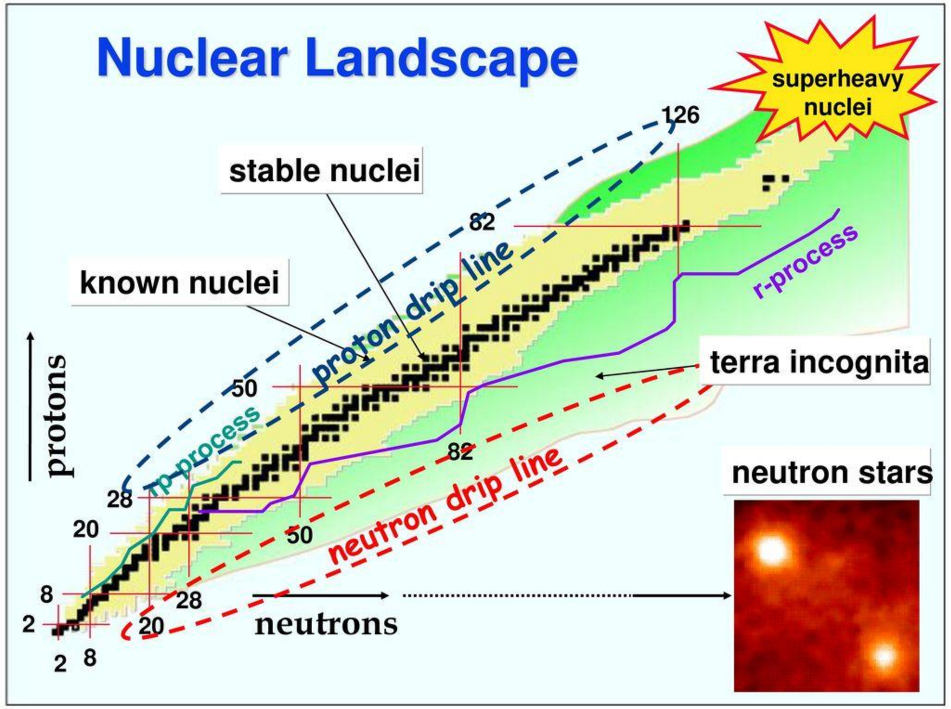
- About 7000 bound nuclei exist
- over 3000 nuclides of 118 elements are known
- another 1500 nuclides are waiting to be discovered
- 285 stable isotopes of the elements that build up nature

Exotic nuclei

- contain many more or many fewer neutrons than a stable isotope.
- lie far away from stability line in the chart of nuclei.
- can be found in the crust of neutron's stars.
- are so short lived and rapidly decayed.

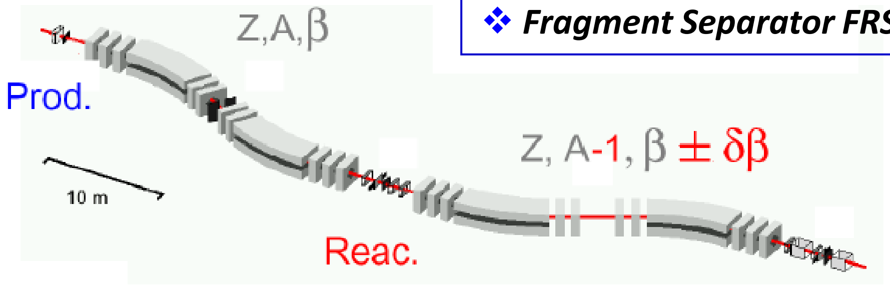
Tanihata experiment

- A series of experiments with RNB led to the discovery of the neutron halo structure in light nuclei near the neutron dripline.
- In 1985, Tanihata and his collaborators at Lawrence Berkeley Laboratory measured the interaction radius of exotic nuclei as He,Li,Be isotopes.
- They discovered abnormally spatially extended nuclei (^{6}He,^{11}Li,^{11}Be)
- The rms radius deduced about 1.5 times larger than a stable nucleus as proportional ~$1.18\ A^{1/3}$



$$\sigma_I(p,t)=\pi\left[R_I(p)+R_I(t)\right]^2$$

- ❖ *800 MeV/u ^{11}B primary beam*
- ❖ *Fragmentation*
- ❖ *Fragment Separator FRS*

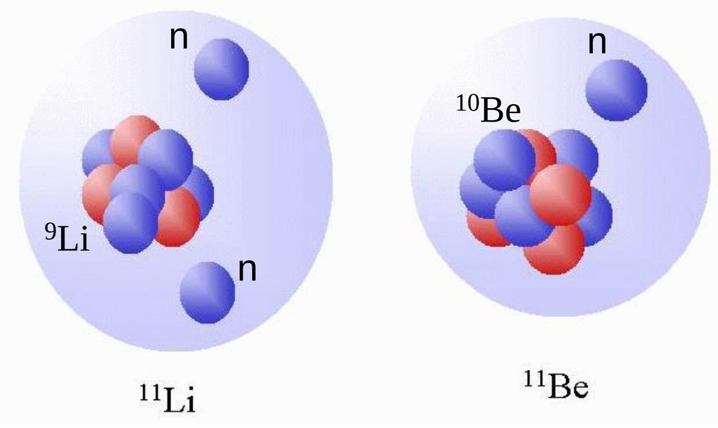


Halo nuclei I

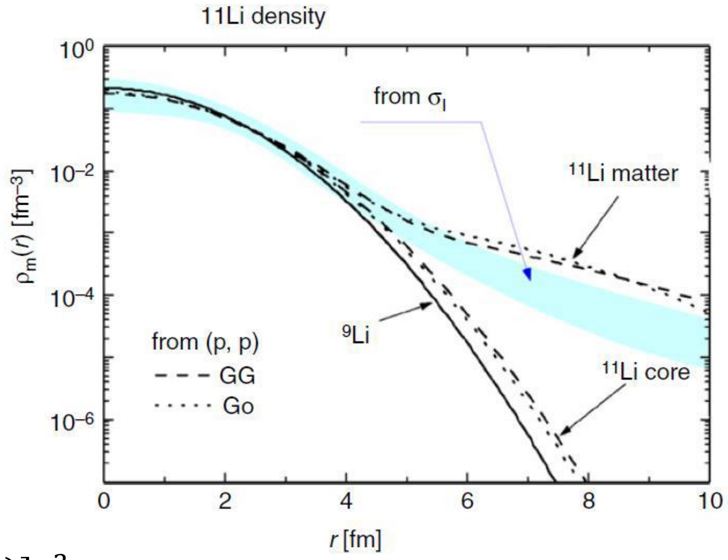
Halo nuclei are exotic nuclei with the following properties:

- Strong cluster structure they are described as a core plus halo neutrons.

2*n* halo nucleus
S_{2n}=369 keV



1*n* halo nucleus
S_n=504 keV



$$S_n = [m_n + m(Z, A-1) - m(Z, A)]c^2$$

is energy required to separate neutron from a nucleus

- Weakly bound Their separation energy less than 1 MeV whereas in stable nuclei is about 6 ~ 8 MeV. They typically decay by β emission.

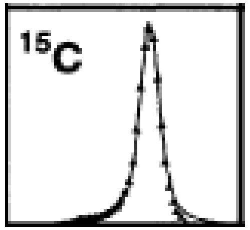
$$\Psi(r) \propto \frac{e^{-\kappa r}}{r}$$

$$\kappa = \sqrt{\frac{2\mu S_{n/2n}}{\hbar^2}}$$

- Extend density Their neutron density distribution shows an extremely long tail.
- Very narrow momentum distributions in comparison with stable nuclei.

$$\Delta p \cdot \Delta x \geq \hbar / 2$$

small → large

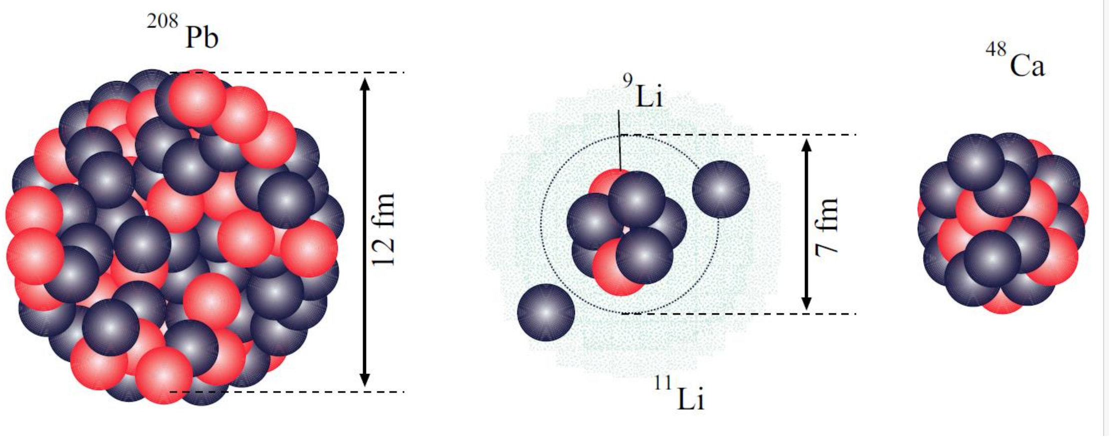


- Lower Complete Fusion at energies above the Coulomb barrier.

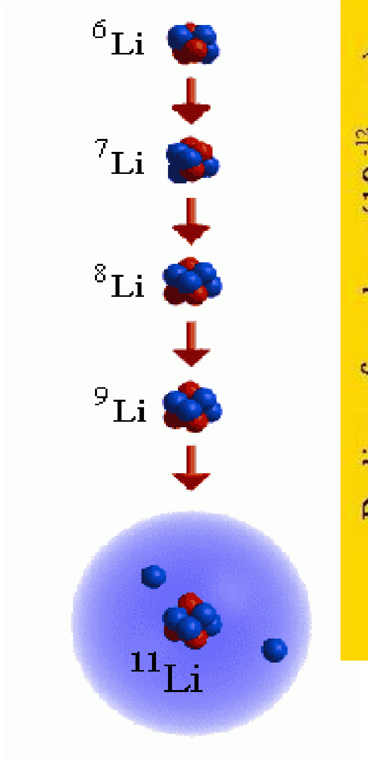
It may be reduced by breakup and incomplete fusion.

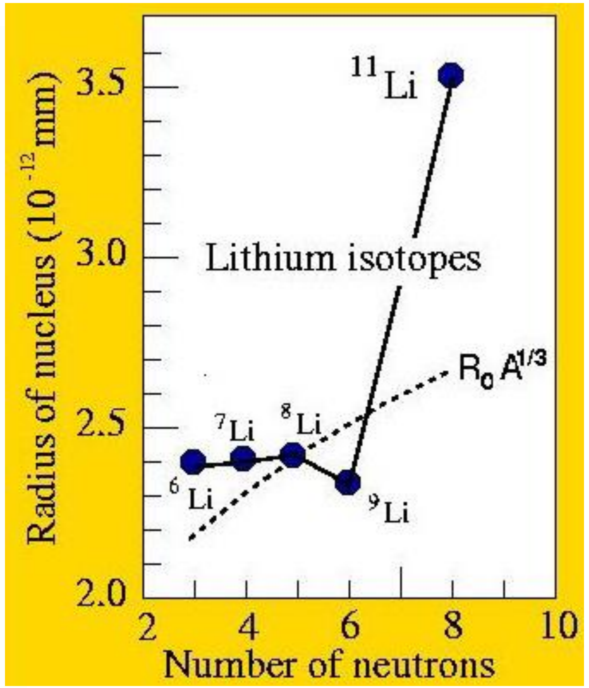
Halo nuclei II

- **Large root-mean-square radius** Their rms is quite large and the valence neutrons are mostly located far from the core.

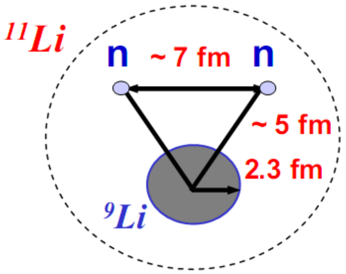


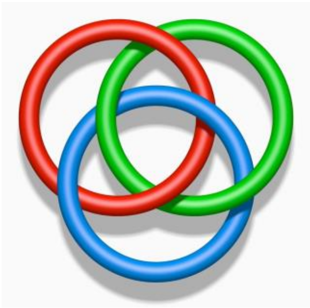
- **Short lived** They have decay lifetime in order of ms~s and cannot be used as targets. Instead, direct reactions done in inverse kinematics.
- **Large electromagnetic dissociation cross section** close to one barn. This was interpreted as evidence of an enhanced electric dipole (E1) response at low excitation energies, called **Soft E1 excitation**.
- **Borromean nuclei** Three-body systems with no bound binary sub-systems like ^{6}He and ^{11}Li.





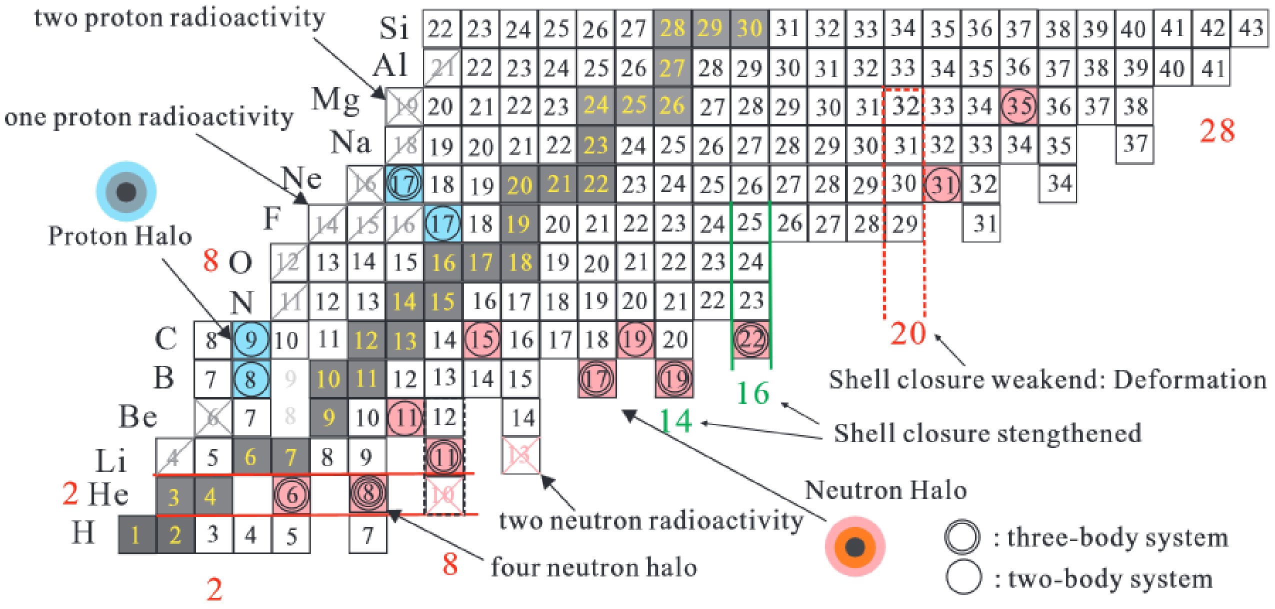
Coat of arms of the Italian aristocratic family Borromeo from Milan.





Examples of Halo Nuclei

Candidates	Valence nucleons	Separation energy (MeV)	**Half-time** ms
^{6}He	2n	0.975	801(10)
^{11}Li	2n	0.369	8.75(14)
^{11}Be	1n	0.502	13810(80)
^{19}C	1n	0.580	2.92(13)
^{8}B	1p	0.140	770(3)
^{17}Ne	2p	0.933	109.2(6)



Study of the halo nuclei:

- Exploring many nuclear structure and reaction models.
- Extend our understanding of the nuclear force.
- Check the limits of validity of structure models.
- Medium mass halo nuclei: ^{22}C, ^{29}F, ^{37}Mg
- The future of halos looks promising where theoretical studies predict halo-candidate nuclei such as ^{29}Ne, ^{31}F, ^{34,39}Na, ^{40}Mg, ^{42}Al, and even in heavier isotopes like ^{62,72}Ca.

New two-cluster approach

- We derived the expression for the distance between two clusters in weakly-bound and halo nuclei.

$$r_{cv}^{rms} = \sqrt{\frac{A}{A_c A_v}\left(A\langle r_m^2\rangle - A_c\langle r_{mc}^2\rangle - A_v\langle r_{mv}^2\rangle\right)}$$

Halo nucleus rms radius

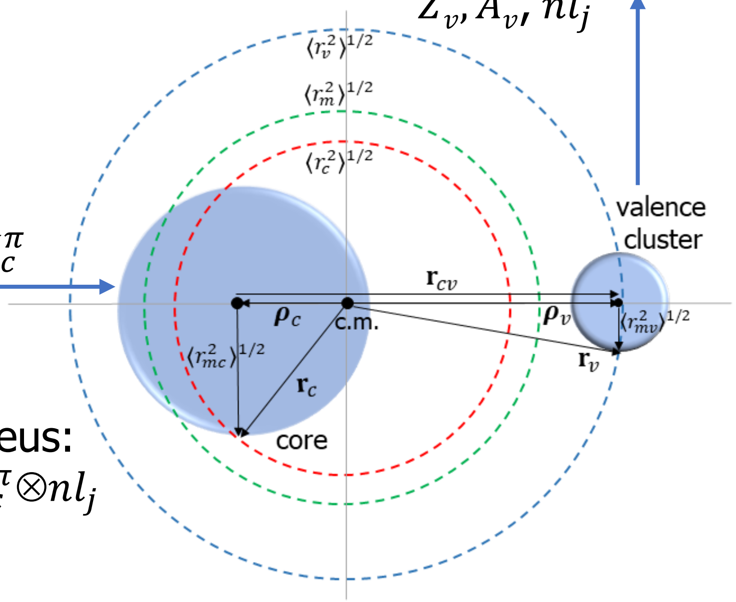
core rms radius

Valence cluster rms radius

preliminary

neutron or
Cluster of neutrons:
Z_v, A_v, nl_j

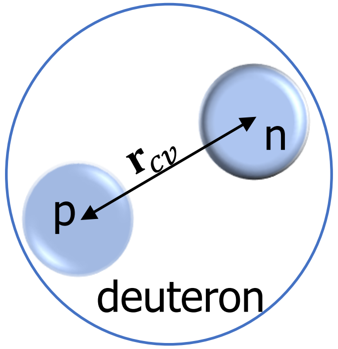
Core: Z_c, A_c, I_c^π



Halo nucleus:
$Z, A, J^\pi = I_c^\pi \otimes nl_j$

H.M. Maridi, J. Singh, N.R. Walet, D.K. Sharp, arXiv: 2407.03044

Nucleus	Core	Sep. en. (MeV)	r_{cm} (fm)	r_m (fm)	r_{cv} (fm)	exp. r_{cv} (fm)
^{2}H	p	2.225	0.84	1.98	3.60	3.94(1)
^{6}He	^{4}He	0.975	1.57	2.48	3.79	3.36(39),3.9(2)
^{11}Be	^{10}Be	0.502	2.39	2.91	6.15	5.77(16),6.1(5)
^{11}Li	^{9}Li	0.369	3.12	2.53	4.94	5.01(32)
^{15}C	^{14}C	1.218	2.43	2.60	4.36	4.15(50),4.5(2)
^{19}C	^{18}C	0.580	2.75	3.0	6.06	5.5(3), 6.6(5)



Is the deuteron a halo nucleus?

Weakly-bound but stable

r.m.s. radius of		
	d	2fm
	^{4}He	1.46-1.67fm

New bound-state wave functions

- We presented ground-state wave functions, $u_{lj}(r)$, as a combination of *s, p, d* harmonic oscillator (HO) states, $R_{nl}(r)$, with
- The HO size related to the separation energy as $R_0 = \sqrt{3\hbar^2/2\mu\varepsilon_0}$.
- the bound wave function may be given as a linear combination of different core spins with valence nucleon(s) spin

$$|J_0M_0\rangle = \sum_{n_0\ell_0 j_0} \alpha(I_c^\pi, n_0\ell_0 j_0)|I_c^\pi \otimes n\ell_0 j_0\rangle \qquad \sum \alpha^2 = 1$$

- the ground state wave functions are used as inputs of dipole response function *dB(E1)/dε* calculations

$$\frac{dB(E1)}{dE_{\rm rel}} = |\langle \mathbf{q}|\frac{Ze}{A} r Y_m^1 |\Phi(\mathbf{r})\rangle|^2$$

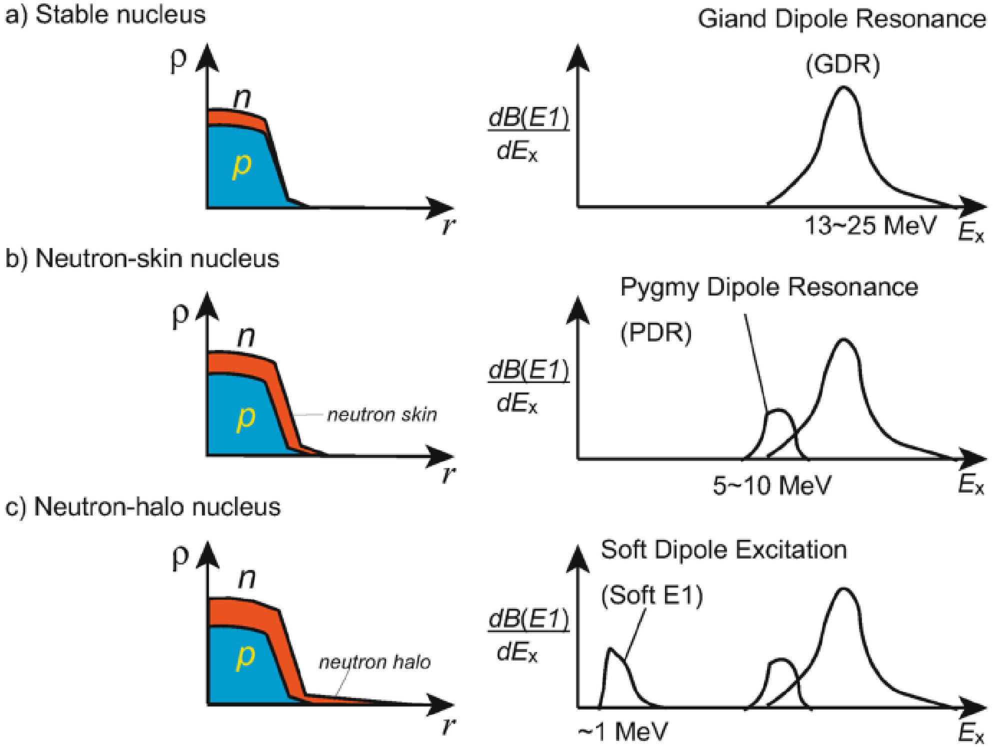
continuum w.f.

g.s w.f.

Electric dipole strength distributions

- Halo nuclei have the so-called soft dipole excitations
- *dB(E1)/dε can be extracted from the measured Coulomb dissociation cross sections*

$$\frac{d\sigma_{\rm CD}}{dE_{\rm rel}} = \frac{16\pi^3}{9\hbar c} N_{E1}(E_x) \frac{dB(E1)}{dE_{\rm rel}}$$



Dipole response functions

- This combination is applied successfully to the reproduce *dB(E1)/dε* data by fitting the *spd* mixing.

$$\frac{dB(E\lambda)}{d\varepsilon} = C_0^{2\lambda}\frac{(2\lambda+1)}{4\pi}(Z_{\rm eff}^{(\lambda)}e)^2 \sum_{n_0\ell_0 j_0}[\alpha(I_c^\pi, n_0\ell_0 j_0)]^2$$

$$\times \sum_{\ell}\langle \ell_0 0 \lambda 0|\ell 0\rangle^2 \left|\int_0^\infty dr\, r^\lambda\, u^J_{\varepsilon\ell j}(r)\; u^{J_0}_{\ell_0 j_0}(r)\right|^2$$

wights

$\sum \alpha^2 = 1$

$C_0^2 = r_{cv}^2/R_0^2$

continuum w.f.

ground-state w.f.

$$\boxed{B(E1) = (3/4\pi)(Ze/A)^2\langle r^2\rangle}$$

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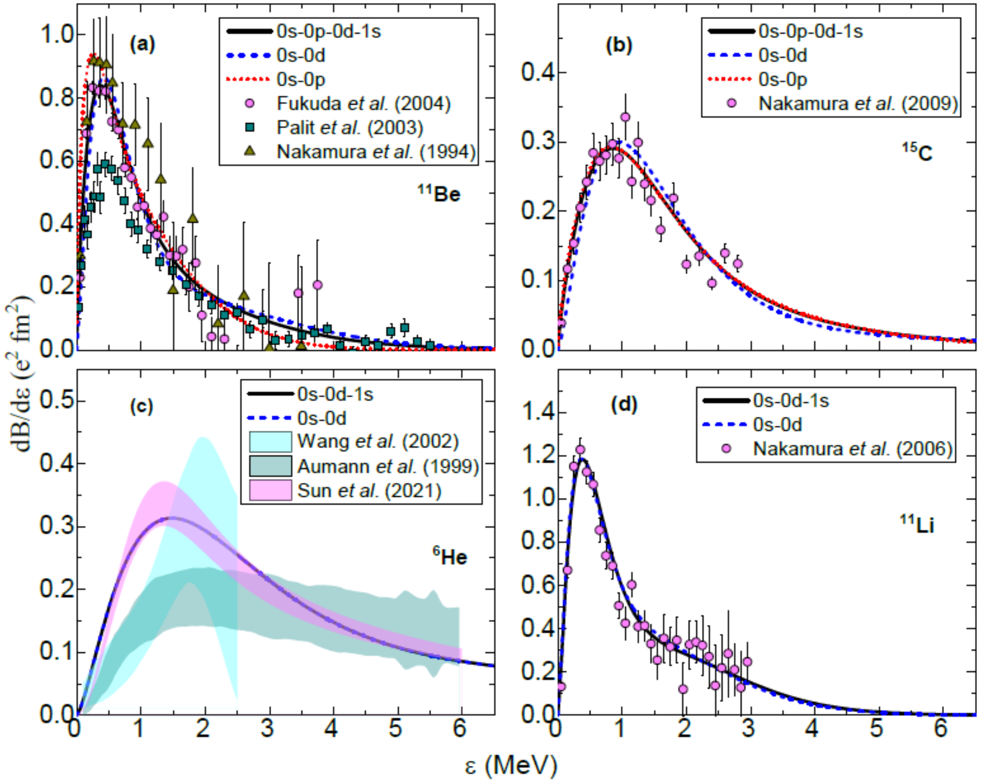
System	weight α^2					$B(E1)$ (e^2fm^2)		r_{cv}^{rms} (fm)	
	$0s/0S$	$0p$	$0d/0D$	$1s/1S$	χ^2	Calc.	exp.	Calc.	exp.
^{10}Be+n	0.35 0.45 0.30	0.29 0.70	0.36 0.55	0.0	2.91 9.52 27.4	1.19	0.90(6) [11], 1.3(3) [10], 1.05(6) [12]	6.15	6.4(7) [10], 5.7(4) [11], 5.77(16), 6.1(5) [12]
^{14}C+n	0.64 0.75 0.62	0.31 0.38	0.05 0.25	0.0	3.13 6.02 4.15	0.73	0.53(5), 0.77(7) [14]	4.36	4.5(2) [14]
^{4}He+2n	0.38		0.62	0.0	10.4	1.58	1.2(2) [4], 1.6(2) [6]	3.86	3.36(39) [4], 3.9(2) [6]
^{9}Li+2n	0.21 0.28		0.67 0.72	0.11	2.19 2.52	1.60	1.42(18), 1.78(22) [9]	4.75	5.01(32) [9], 6.2(5) [77]



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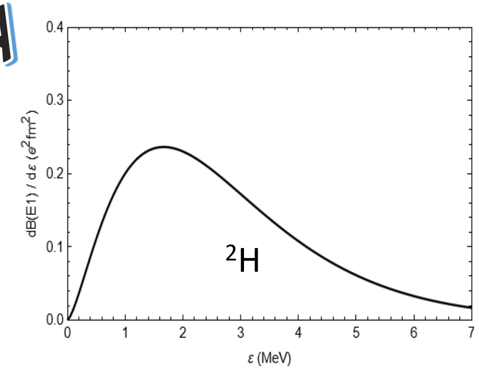
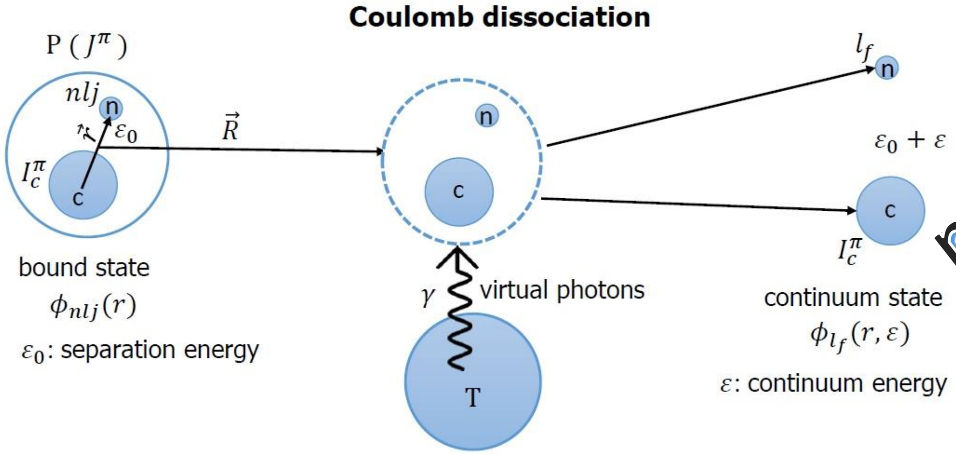


Coulomb dissociation cross sections



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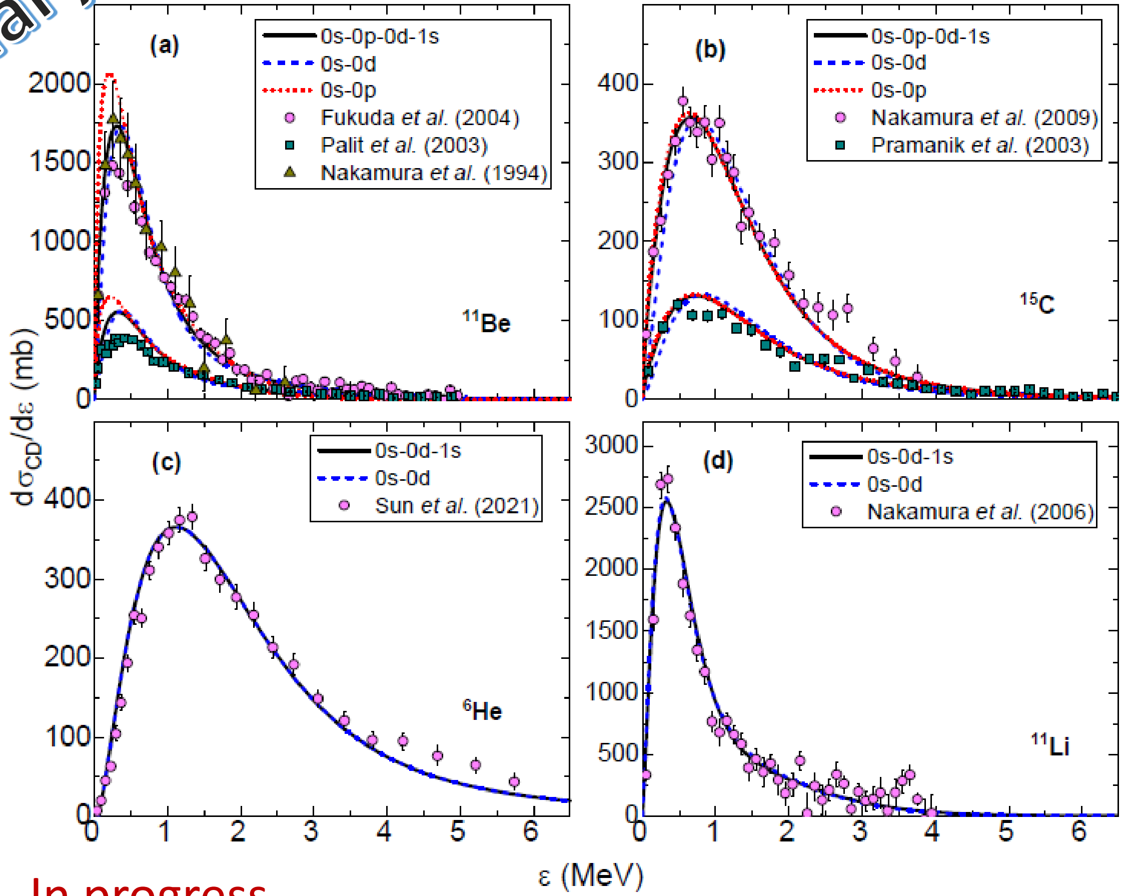

- The Coulomb dissociation can be taken place when a projectile with several hundred of MeV/nucleon passes a heavy ion target
- It may be excited by absorbing virtual photons from the target Coulomb field (mostly dipole excitations).
- It can be used to determine the astrophysical S(E) factor.
- The Coulomb dissociation cross sections *dσ(E1)/dε* are given as

Equivalent photon method

$$\frac{d\sigma_{\mathrm{CD}}}{dE_{\mathrm{rel}}} = \frac{16\pi^3}{9\hbar c} N_{E1}(E_x) \frac{dB(E1)}{dE_{\mathrm{rel}}}$$

H.M. Maridi, J. Singh, N.R. Walet, D.K. Sharp, arXiv: 2407.03044

In progress

✓ in transfer-reaction calculations?

Future

✓ estimating density distributions using Glauber theory.

Summary

This work

- The outlines about halo nuclei are presented.
- Presenting new cluster approach for halo nuclei.
- Presenting an expression for two-cluster distance in halo nuclei.
- Presenting new bound-state wave functions from *spd* mixing of harmonic oscillator functions
- This wave functions have succeeded to reproduce:
  - ✓ the dipole response functions $dB(E1)/d\varepsilon$ of one and two-neutron halo nuclei (^{11}Be, ^{15}C, ^{6}He, ^{11}Li)
  - ✓ the Coulomb dissociation $d\sigma(E1)/d\varepsilon$ of these nuclei.

Is deuteron a halo nucleus?

Future of this work

- Using the as inputs of the transfer-reaction calculations
- Estimating density distribution from these wave functions using Glauber theory.

Acknowledgments

Collaborators

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- J. Singh & N.R. Walet Theoretical Physics Group, University of Manchester, UK

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