# Updates on Sp(4) gauge theories for BSM

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# Outline

- Composite Higgs Models
- Why Sp(4)?
- Two recent investigations
- Conclusions

# Composite Higgs Models

$$\mathcal{L} = -\frac{1}{2} V_{\mu\nu} V^{\mu\nu} + \frac{1}{2} \left( i q_a^{j\dagger} \bar{\sigma}^\mu D_\mu q^{ja} - i \left( D_\mu q_a^j \right)^\dagger \bar{\sigma}^\mu q^{ja} \right)$$

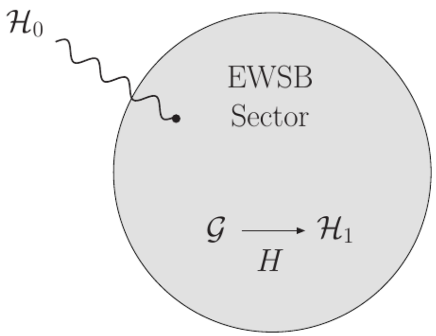
Consider a theory with gauge group $G_{HC}$ and LH Weyl fermions transforming as

$$\mathcal{G} = n_1 R_1 \oplus n_2 R_2 \oplus \ldots$$

At energy scale $f \ll \Lambda_{UV}$ , the strong coupling drives the spontaneous breaking

$$\mathcal{G} \longrightarrow \mathcal{H}_1$$

which delivers $n = \dim \mathcal{G} - \dim \mathcal{H}_1$ massless NG bosons.




**Vacuum Misalignement**: The Gauging of $\mathcal{H}_0$ by external vector bosons breaks the symmetry explicitly and gives mass to some of the NG boson.

Kaplan & Georgi, PLB136(1984)

Taken from R. Contino, "The Higgs as a composite Nambu-Goldstone boson".

**Top Partial Compositeness**: The (heavy) physical top is a mixture of SM top and a composite chimera baryons,

$$|phys.\rangle = \alpha |top\rangle + \beta |\chi\rangle$$

Kaplan, NPB365(1991)

# Why Sp(4)?

What does the UV completion of a CHM look like?

$\mathcal{G}$, $\mathcal{H}_1$ and $\mathcal{H}_0$ should be chosen so that:

- The gauge theory should be Asymptotically Free.
- The Breaking $\mathcal{G} \longmapsto \mathcal{H}_1 \supset \mathcal{H}_{\mathcal{EW}}$ should be possible
- $\mathcal{G}/\mathcal{H}_1$ can accomodate at least one Higgs multiplet.
- Composite states can be used as partners to SM fermions.
- $G_{HC}$ is free of gauge (global) anomalies,
- $\mathcal{H}_0$ free of 't Hooft anomalies.

These requests restrict $N_c$ and $n_i$.

Symplectic Unitary group defined as follows

$$Sp(2N_c) = \{U \in SU(2N_c) \quad | \quad \Omega U \Omega^T = U^\star\}$$

where

$$\Omega = \begin{pmatrix} 0 & \mathbb{1} \\ -\mathbb{1} & 0 \end{pmatrix}$$

| $G_{HC}$ | $R_1$ | $R_2$ | constraint |
|---|---|---|---|
| $Sp(2N_c)$ | 5xAd | 6xF | $2N_c \geq 12$ |
| $Sp(2N_c)$ | 5x$A_2$ | 6xF | $2N_c \geq 4$ |
| **$Sp(2N_c)$** | **4xF** | **6x$A_2$** | **$2N_c \leq 36$** |
| $SO(N_c)$ | 5x$S_2$ | 6xF | $2N_c \geq 55$ |
| $SO(N_c)$ | 5xAd | 6xF | $2N_c \geq 15$ |
| $SO(N_c)$ | 5xF | 6xSpin | $N_c$ =7, 9, 10, 11, 13, 14 |
| $SO(N_c)$ | 5xF | 6xF | $N_c$ = 7, 9 |
| $SO(N_c)$ | 4XF | 6xF | $N_c$ = 11, 13 |

Ferretti, Karateev JHEP03(14)077

Barnard, Gherghetta, Ray JHEP02(2014)002

**Note**: $Sp(2N_c)$ also interesting to study confinement and large-N limit in gauge theories (not in this talk)

# The Sp(4) gauge theory

$$\mathcal{L} = -\frac{1}{2} V_{\mu\nu} V^{\mu\nu} + \frac{1}{2} \left( i q_a^{j\dagger} \bar{\sigma}^\mu D_\mu q^{ja} - i \left( D_\mu q_a^j \right)^\dagger \bar{\sigma}^\mu q^{ja} \right) - \frac{1}{2} M \Omega_{jk} \Omega_{ab} \left( q^{jaT} \tilde{C} q^{kb} - q_a^{j\dagger} \tilde{C} q_b^{k\star} \right)$$

Where $V_\mu = V_\mu^a T_a$ and $D_\mu = \partial_\mu - i g_0 V_\mu^a T^a$ with $a = 1, ..., 4, j = 1, ..., 4$

If M=0, the breaking SU(4) → Sp(4) is driven by the condensation of

$$\langle \Sigma^{nm} \rangle = \Omega_{ab} \langle q^{naT} C q^{mb} \rangle \neq 0$$

| Fields | Sp(4) | SU(4) | SU(6) |
|---|---|---|---|
| $V_\mu$ | 10 | 1 | 1 |
| q | 4 | 4 | 1 |
| ψ | 5 | 1 | 6 |

The gauge group is pseudo-real: $-T_a^\star = \Omega T_a \Omega^T$ and with

$$Q^a = \begin{pmatrix} q^a \\ -\Omega^{ab} C q_b^\star \end{pmatrix}$$

one can write $\mathcal{L}$ in terms of Dirac spinors,

$$\mathcal{L} = -\frac{1}{2} V_{\mu\nu} V^{\mu\nu} + i \bar{Q}_a^i \gamma^\mu \left( D_\mu Q^i \right)^a - m \bar{Q}_a^i Q^{ia}$$

where $M = m\Omega$

- In this last form, the theory can be discretized and investigated on the lattice.
- Other matter contents can be accomodated in the same way

# Lattice General Setup

$$\mathrm{S} = \beta \sum_{x,\mu<\nu} \left(1 - \tfrac{1}{4N_c} \Re \,\mathrm{Tr}\, \mathcal{P}_{\mu\nu}(x)\right) + a^4 \sum_x \bar{q}^f(x) D_m^f q^f(x) + a^4 \sum_x \bar{\Psi}^{as}(x) D_m^{as} \Psi^{as}(x)$$

where

$$D_m^R \psi(x) = \left(\frac{4}{a} + m_0^R\right)\psi(x) - \frac{1}{2a}\sum_\mu \{(1-\gamma_\mu)U_\mu^R(x)\psi(x+\mu)$$
$$+(1+\gamma_\mu)U_\mu^R(x-\mu)\psi(x-\mu)\}$$

for R representation of the gauge group.

| Field | Sp(4) | SU(4) | SU(6) |
|---|---|---|---|
| V | 10 | 1 | 1 |
| q | 4 | 4 | 1 |
| ψ | 5 | 1 | 6 |

Observables:

- Spectrum and decay constants from

$$C_{\alpha\beta}(t) = \langle 0|\mathcal{O}_\alpha(t)\mathcal{O}_\beta^\dagger(0)|0\rangle$$

- $\chi$-Symmetry breaking pattern from spectrum of $D_m^R$
- Topological susceptibility

Algorithms:

- HB+OR for pure gauge
- HMC for dynamical fermions
- Wilson Flow or sigma for scale setting
- Most of the technology for SU(N) can be reused.

TELOS, JHEP03(2018)185

Code Bases:

- HiREp (Del Debbio, Patella, Pica, PRD81(19)),
- Grid (Boyle et al. 1512.03487)

# Summary of lattice results

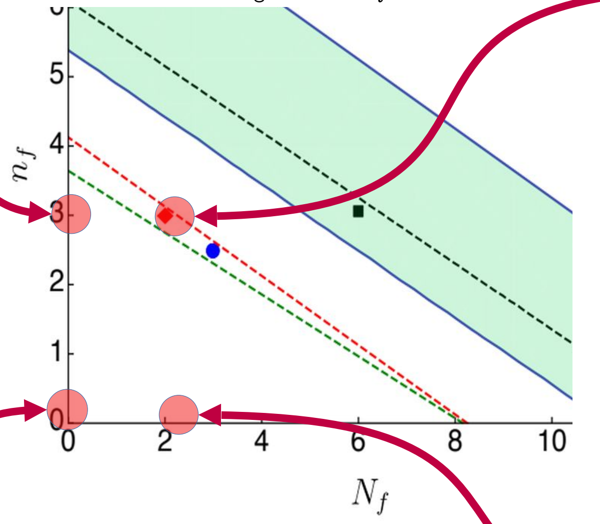
nf=3

- Bulk phase diagram
- Eigenvalues of Dirac Operator (TELOS, RD111(25))

Quenched theory

- Glueball, Meson, Chimera Baryon spectrum (TELOS, PRD103(),PRD109(24)).
- Topological Susceptibility for $Sp(2N_c)$ (TELOS, PRD106(22)).
- Deconfinement transition for Sp(4) (TELOS, PRD108(23)).

Image: courtesy of J-W Lee




Nf=2, nf=3

- $N_{fund}=2$, $N_{as}=3$ (TELOS, PRD106(22))
- Spectral densities (TELOS, PRD110(24))
- Mixing of flavor singlets, $N_{fund}=2$, $N_{as}=3$ (TELOS, PRD110(24)

Nf=2

- Spectrum & decay constants (TELOS, JHEP12(19))

# Chimera Baryons - Quenched analysis

- Couple Sp(4) theory to multirep hyperquarks, e.g. Sp(4)xSO(6).
- Gauge the SU(3) subgroup of SO(6) and idenfity it with QCD.
- Chimera Baryons are the states formed by one ψ and two q.
- Chimera Baryons are the top partners: they mix with SM top quarks and generate their large mass.

$$| Phys\rangle = \alpha\ |SM\rangle\ +\ \beta\ |qq\psi\rangle$$

$$O_{\rho}^{ijk,\mu} = q_{\alpha}^{i\,a}(C\gamma^{\mu})_{\alpha\beta}q_{\beta}^{j\,b}\Omega^{ad}\Omega^{bc}\Psi_{\rho}^{k\,cd}$$

$$O_{\rho}^{ijk,5} = q_{\alpha}^{i\,a}(C\gamma^{5})_{\alpha\beta}q_{\beta}^{j\,b}\Omega^{ad}\Omega^{bc}\Psi_{\rho}^{k\,cd}$$

$$S = \beta \sum_{x,\mu<\nu} \left(1 - \frac{1}{4}\,\Re\,\mathrm{Tr}\,\mathcal{P}_{\mu\nu}(x)\right)$$

On the Lattice:

- Quenched theory, ~600 configurations per ensemble.
- Observables: 2-points correlators, with APE and Wuppertal smearing,
- Scale Setting: Wilson flow.
- No Flavor singlets!

| Ens. | β | $TxL^3$ | <P> | $\omega_0/a$ |
|---|---|---|---|---|
| QB1 | 7.62 | $48\times24^3$ | 0.6018898(94) | 1.448(3) |
| QB2 | 7.7 | $60\times48^3$ | 0.6088000(35) | 1.6070(19) |
| QB3 | 7.85 | $60\times48^3$ | 0.6203809(28) | 1.944(3) |
| QB4 | 8.0 | $60\times48^3$ | 0.6307425(27) | 2.3149(12) |
| QB5 | 8.2 | $60\times48^3$ | 0.6432302(25) | 2.8812(21) |

# Chimera Baryons - Quenched analysis

$$C_M(t) = \langle 0|\mathcal{O}_M(t)\mathcal{O}_M^\dagger(0)|0\rangle \longmapsto_{t\to\infty} \frac{|\langle 0|\mathcal{O}_M|M\rangle|^2}{2m_M} e^{-m_M t}$$

For Chimera baryons:

$\Sigma_{CB}/\Sigma_{CB}^*$ $$O_\rho^{ijk,\mu} = q_\alpha^{i\,a}(C\gamma^\mu)_{\alpha\beta} q_\beta^{j\,b}\Omega^{ad}\Omega^{bc}\Psi_\rho^{k\,cd}$$

$\Lambda_{CB}$ $$O_\rho^{ijk,5} = q_\alpha^{i\,a}(C\gamma^5)_{\alpha\beta} q_\beta^{j\,b}\Omega^{ad}\Omega^{bc}\Psi_\rho^{k\,cd}$$

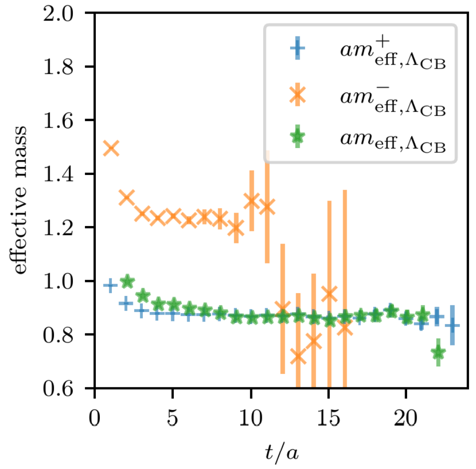
Note: these operators source states of different parity and spin, projection has to be performed

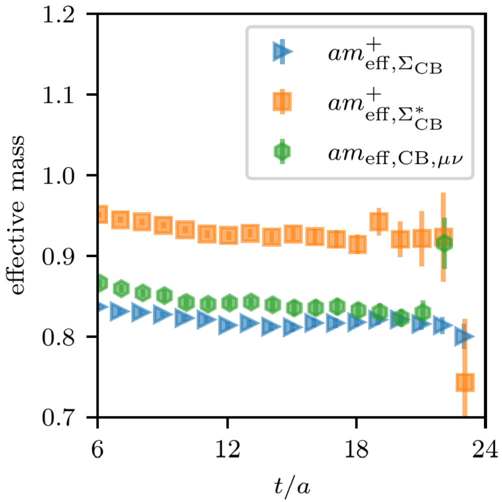
For (PseudoScalar) Mesons: $O_{PS}^{(f)} = \bar{q}\gamma^5 q$ $O_{ps}^{(as)} = \bar{\Psi}\gamma^5\Psi$

We expect $m_{PS,ps}^2 \propto m_{f,as}$ at small $m_{f,as}$

Questions:

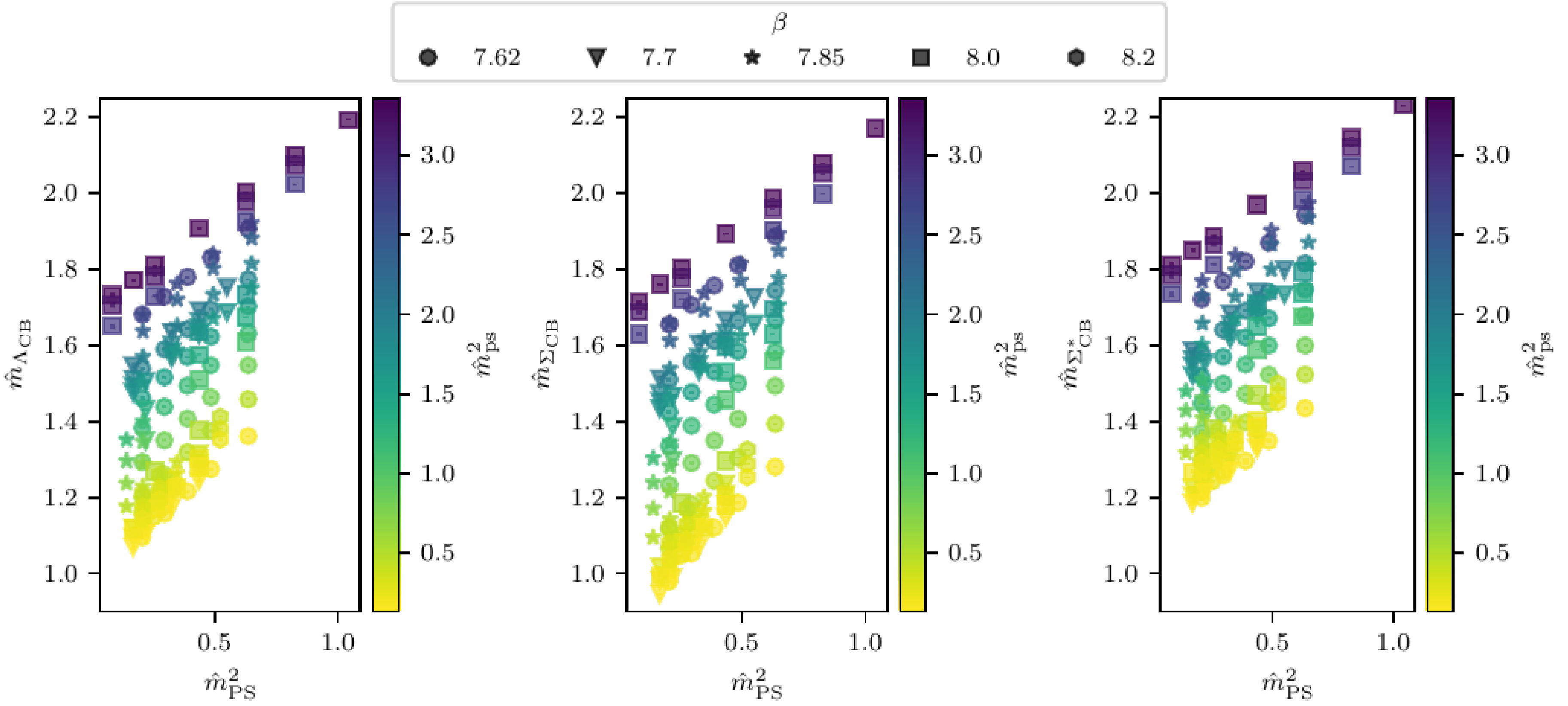
- Is there a hierarchy between m(Λ), m(Σ) and m(Σ*) and how does it depend on $m_{f,as}$ and on $m_{PS,ps}^2$ ?
- How to take the Continuum and massless limits?



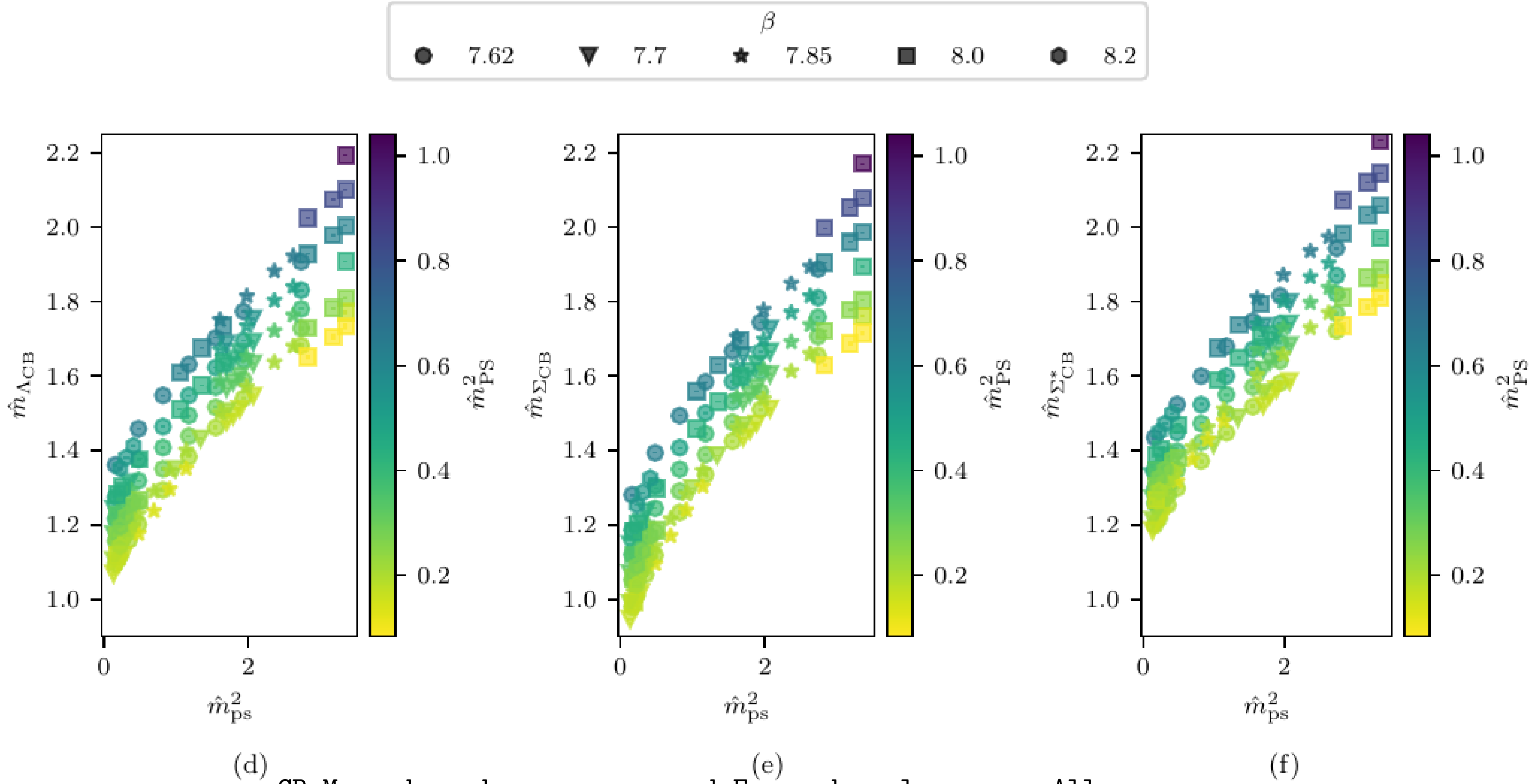

Mass plateaux obtained for QB1 at $am_0^f$ =-0.77 and $am_0^{as}$ =-1.1. Taken from PRD109(24). Unprojected plateau should be compatible with the lightest projected plateau

# Chimera Baryons - mass dependence




CB Mass dependence on squared AS pseudoscalar mass. All quantities are calculated in Wilson Flow units. Taken from TELOS PRD109(24).

# Chimera Baryons - mass dependence




CB Mass dependence on squared F pseudoscalar mass. All quantities are calculated in Wilson Flow units. Taken from TELOS, PRD109(24).

# Chimera Baryons - Massless and Continuum extrapolations

- From a QCD-inspired ansatz from Heavy-Baryon ChiPT,

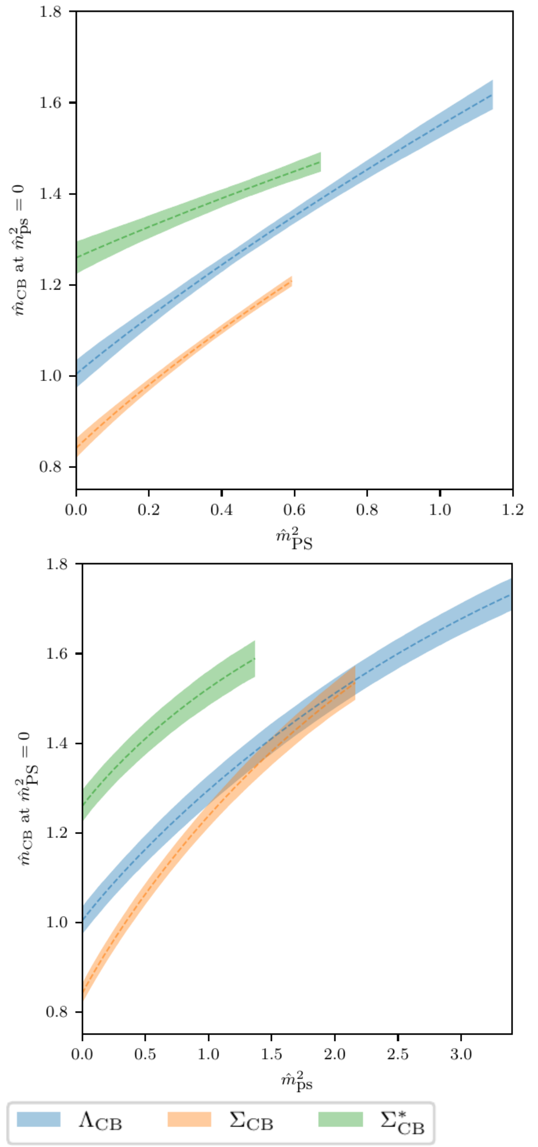
$$\begin{aligned}\hat{m}_{CB} =& \hat{m}_{CB}^{\chi} \\ &+F_2\hat{m}_{PS}^2 + A_2\hat{m}_{ps}^2 + L_1\hat{a} \\ &+L_{2F}\hat{a}\hat{m}_{PS}^2 + L_{2A}\hat{m}_{ps}^2\hat{a} \\ &+F_4\hat{m}_{PS}^4 + A_4 m_{ps}^4 + C_4\hat{m}_{ps}^2\hat{m}_{PS}^2\end{aligned}$$

- Very many data points to fit, with very many parameters: total of 1315 analysis procedures. To assess them we use

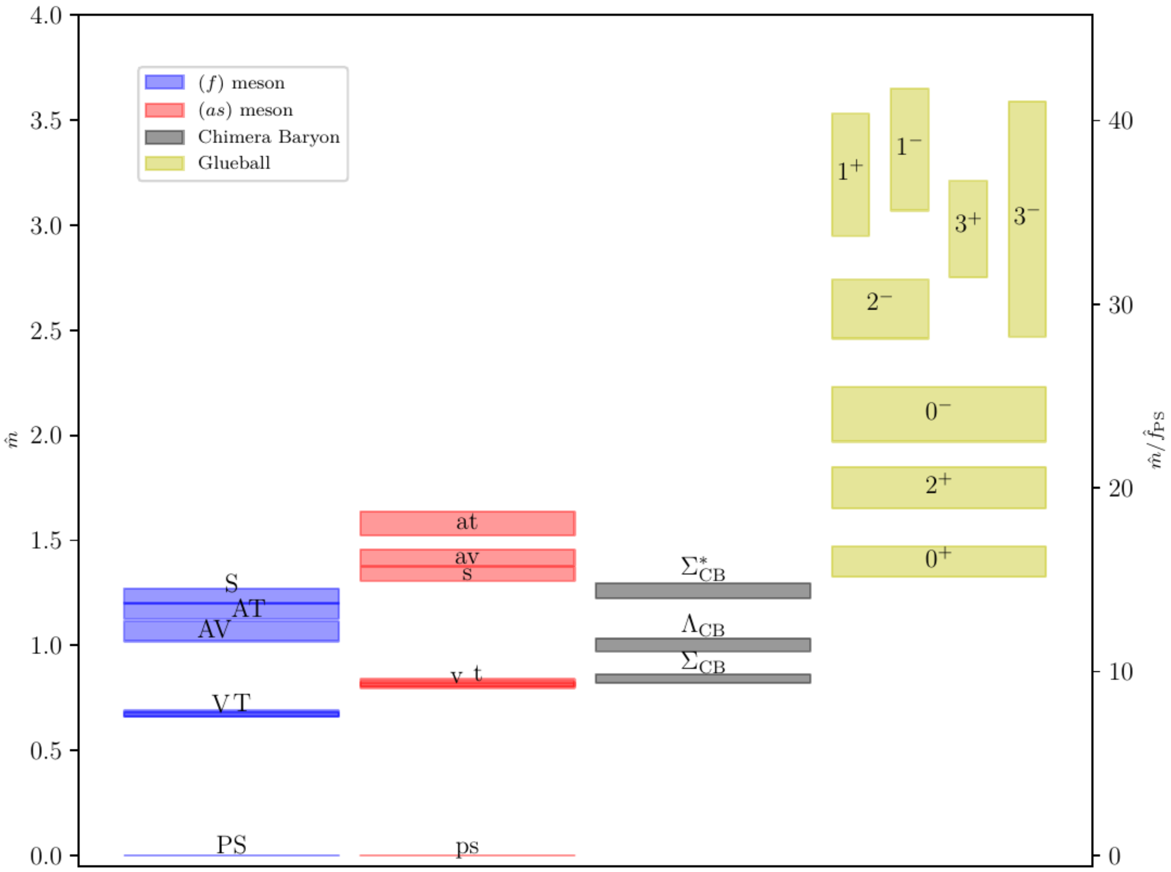
$$AIC = \chi^2 + 2k + N_{cut}$$

Where:

- k number of fitting parameters,
- $N_{cut}$ number of datapoint removed by restricting fitting range



# Full Quenched Spectrum




Low mass spectrum of the quenched 2f+3as theory, together with glueball states. Taken from TELOS, PRD109(24).

# Mixing of Flavor Singlets - Lattice Setup

- Flavor singlets: Spin 0, negative parity.
- EFT description related to Axion-like particles. (Bellazzini et al., PRL119)
- Symmetry of the Lagrangian: U(1) x U(1) x SU(6)x SU(4).
- We expect the lightest states to mix, analogously to a and η in QCD
- Anomaly can only break one linear combination of the two U(1)s.
- Disconnected diagrams present that reduce the signal-to-noise ratio.
- Mixing with glueballs? See poster by N. Brito.

On the Lattice:

- ~500 configurations per ensemble.
- Ensemble generation and measurements: Grid & HiRep.
- 2-points (mixed) correlators, with APE smearing,
- Flavor singlets!
- Different $N_t$ to assess finite size effects (negligible!).

| Ens. | $am_0^f$ | $am_0^{as}$ | $\beta$ | $TxL^3$ | <P> | $\omega_0/a$ |
|---|---|---|---|---|---|---|
| M1 | -0.71 | -1.01 | 6.5 | $48x20^3$ | 0.585172(16) | 2.5200(50) |
| M2 | -0.71 | -1.01 | 6.5 | $64x20^3$ | 0.585172(12) | 2.5300(40) |
| M3 | -0.71 | -1.01 | 6.5 | $96x20^3$ | 0.585156(13) | 2.5170(40) |
| M4 | -0.70 | -1.01 | 6.5 | $64x20^3$ | 0.584228(12) | 2.3557(31) |
| M5 | -0.72 | -1.01 | 6.5 | $64x32^3$ | 0.5860810(93) | 2.6927(31) |

# Flavor singlets - Lattice Spectrum

$$\mathcal{O}_{\eta^f} = \frac{1}{N_f}\sum_{I=1}^{N_{as}} \bar{Q}^I\gamma^5 Q^I \qquad \mathcal{O}_{\eta^{as}} = \frac{1}{N_{as}}\sum_{k=1}^{N_{as}} \bar{\Psi}^k\gamma^5\Psi^k$$

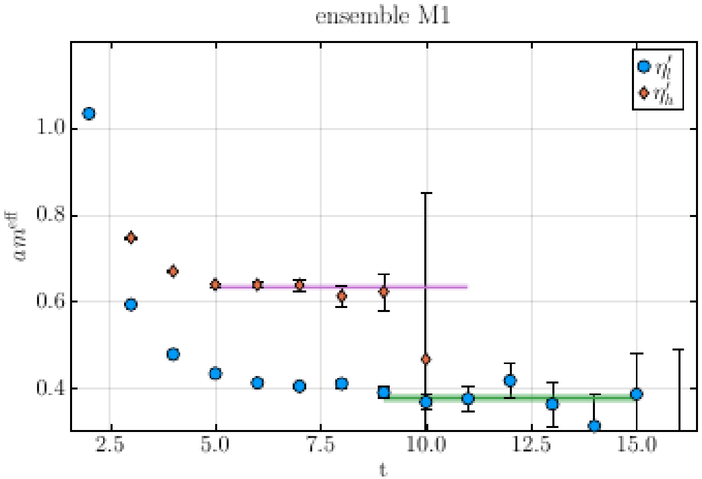
- Ground state + 1st excited state from GEVP analysis,

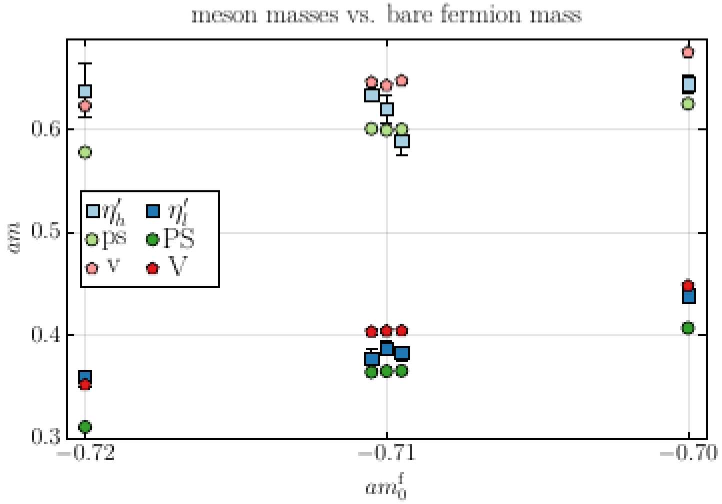
$$C(t,t_0)v_n(t,t_0) = \lambda_n(t,t_0)C(t_0)v_n(t,t_0), \quad \lambda_n(t,t_0) \sim Ae^{-E_n(t-t_0)}$$

- Masses similar to flavored mesons in PS&V channels,
- Errors up to one order of magnitude larger than for flavored mesons,
- No signal for axial-vectors,
- Heavy quark suppression effects of disc diagrams,
- State mixing taken as mixing of decay constants, angles given by

$$\begin{pmatrix} \langle 0|O_{\eta^f}|\eta_l'\rangle & \langle 0|O_{\eta^{as}}|\eta_l'\rangle \\ \langle 0|O_{\eta^f}|\eta_l'\rangle & \langle 0|O_{\eta^{as}}|\eta_h'\rangle \end{pmatrix} = \begin{pmatrix} A_f^{\eta_l'} & A_{as}^{\eta_l'} \\ A_f^{\eta_h'} & A_{as}^{\eta_h'} \end{pmatrix}$$

$$= \begin{pmatrix} A_{\eta_l'}\cos\phi_{\eta_l'} & A_{\eta_l'}\sin\phi_{\eta_l'} \\ -A_{\eta_h'}\sin\phi_{\eta_h'} & A_{\eta_h'}\cos\phi_{\eta_h'} \end{pmatrix}$$




Mass plateaux (top) and masses of pseudo-scalar flavor singlets, taken from TELOS, PRD110(24)

# Flavor Singlets - Mixing Angle

$$\begin{pmatrix} \langle 0|O_{\eta^f}|\eta'_l\rangle & \langle 0|O_{\eta^{as}}|\eta'_l\rangle \\ \langle 0|O_{\eta^f}|\eta'_l\rangle & \langle 0|O_{\eta^{as}}|\eta'_h\rangle \end{pmatrix} = \begin{pmatrix} A_f^{\eta'_l} & A_{as}^{\eta'_l} \\ A_f^{\eta'_h} & A_{as}^{\eta'_h} \end{pmatrix}$$

$$= \begin{pmatrix} A_{\eta'_l}\cos\phi_{\eta'_l} & A_{\eta'_l}\sin\phi_{\eta'_l} \\ -A_{\eta'_h}\sin\phi_{\eta'_h} & A_{\eta'_h}\cos\phi_{\eta'_h} \end{pmatrix}$$

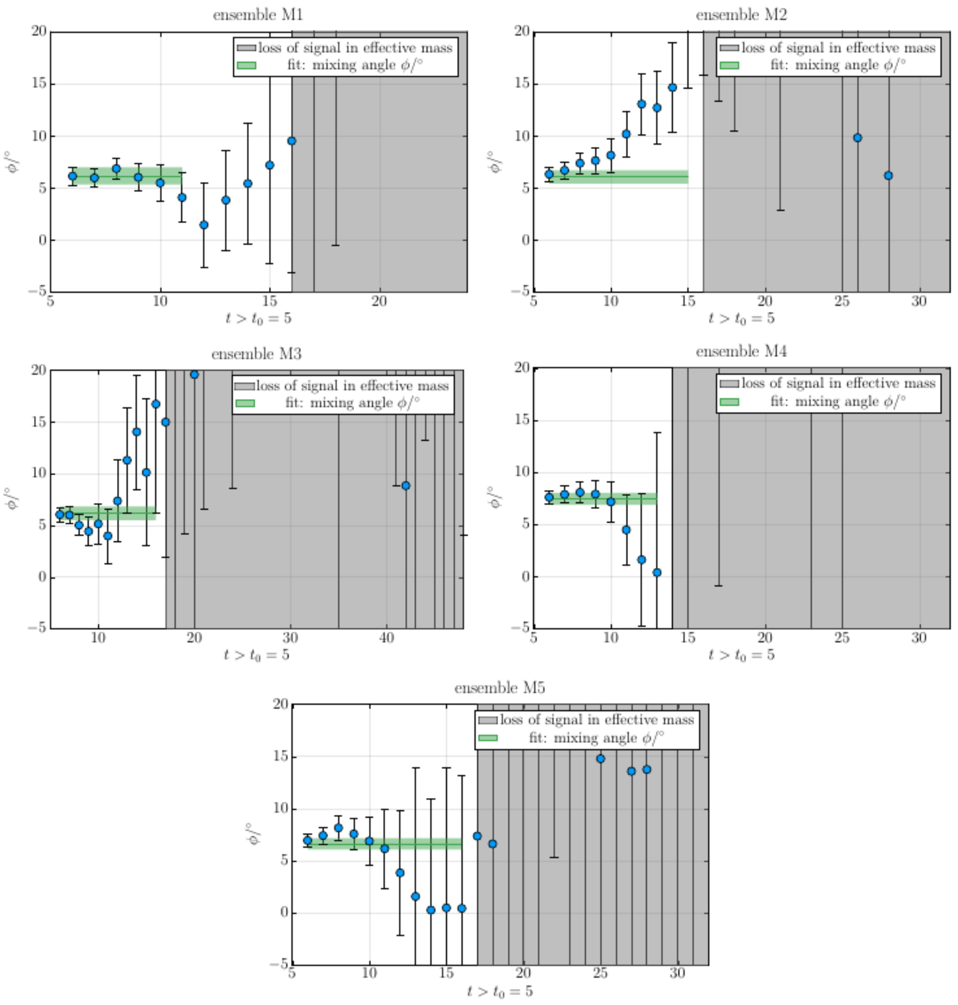
If $\phi_l \simeq \phi_h \simeq \phi$

$$-(\tan\phi)^2 = \frac{A_{as}^{\eta'_l} A_f^{\eta'_h}}{A_{as}^{\eta'_h} A_f^{\eta'_l}}$$

Measured for all the five ensembles:

- Angle extracted from a constant fit between $t_0$ and loss of signal ( remember $C(t,t_0)\,v(t,t_0) = \lambda_n(t,t_0)\,v(t,t_0)$ )
- Mixing angle generally small
- No Mass dependence detected.

$$\phi \sim 6^\circ$$




Value of the effective mixing angle as a function of t, taken from TELOS, PRD110(24)

# Conclusions

- Sp(4) gauge theories underlie attractive realizations of UV completions of CHMs.
- Their strongly coupled regime can be effetively studied on the lattice, two recent examples today:
  1) Chimera Baryons (Quenched) (TELOS, PRD109(24))
  2) Mixing of Flavor singlets (TELOS, PRD110(24)
- After years of exploratory studies, we are entering the precision era,
- Many datasets and codes produced: reproducibility strategy (Bennett, 2504.01876),
- Many results yet to come: scattering, decays, $2m_\pi < m_\rho$ regime.

Thank you