Heavy mesons semileptonic decays with the heavy-HISQ method

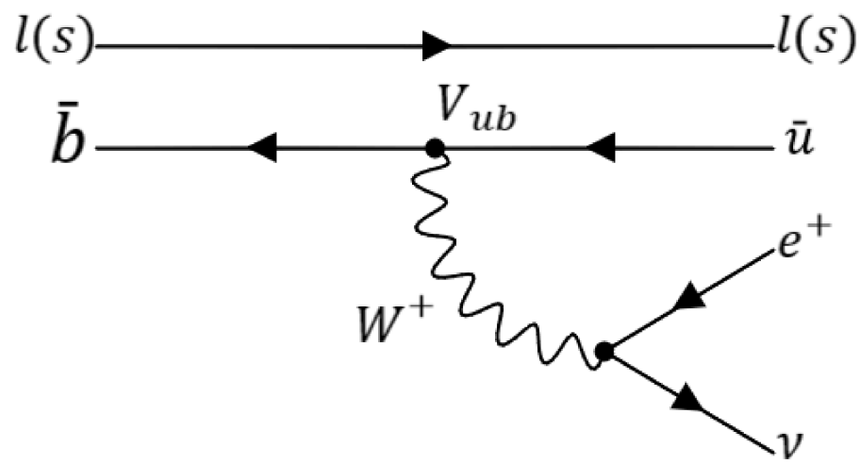
Olmo Francesconi

UKLFT Annual Meeting, 28th April 2025

- Form factors updates $B \to \pi \ell \nu$ $B_s \to K \ell \nu$

- Physical continuum extrapolation strategy

Why B semileptonic decays?



$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

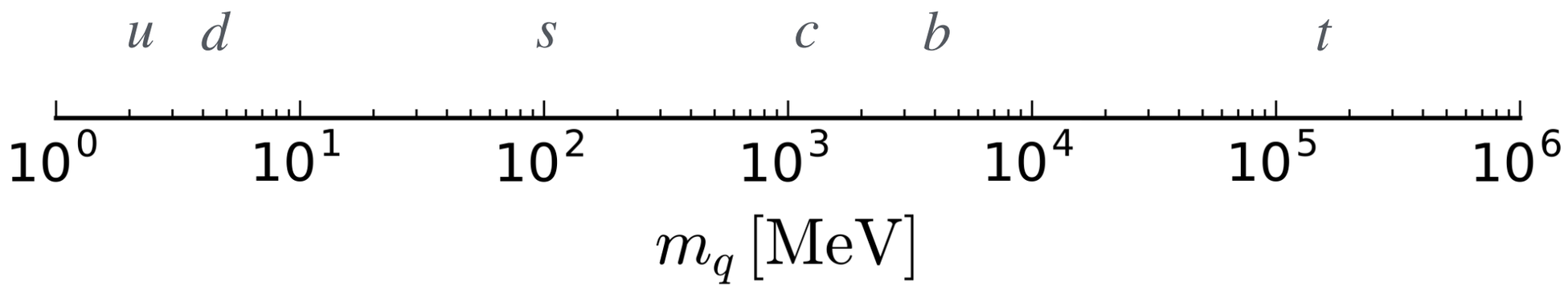
Test SM by determining CKM matrix elements

Experiment | Theory

$$\frac{d^2\Gamma(B \to \pi \ell \nu)}{dq^2} \approx |V_{ub}|^2 \times \left[|f_+(q^2)|^2 \mathcal{K}_1 + |f_0(q^2)|^2 \mathcal{K}_2 \right]$$

Form factors accessible non-perturbatively by computing low energy matrix elements via lattice calculation

Heavy quarks on the lattice



$m_\pi L \gtrsim 4$

$a^{-1} \gg$ Mass scale

$m_\pi \approx 140$ Mev

$\bar{m}_b \approx 4.2$ Gev

$\Big\} \rightarrow$

$a \approx 0.05$ fm

$N = L/a \approx 120$

Very expensive even in 2025

Heavy-HISQ →

Simulate multiple heavy quark masses

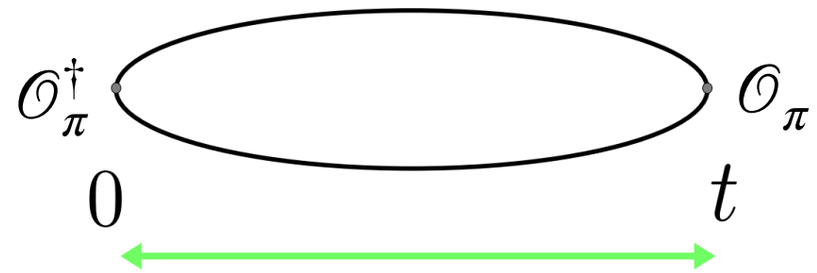
Extrapolate to the physical q mass

Lattice setup

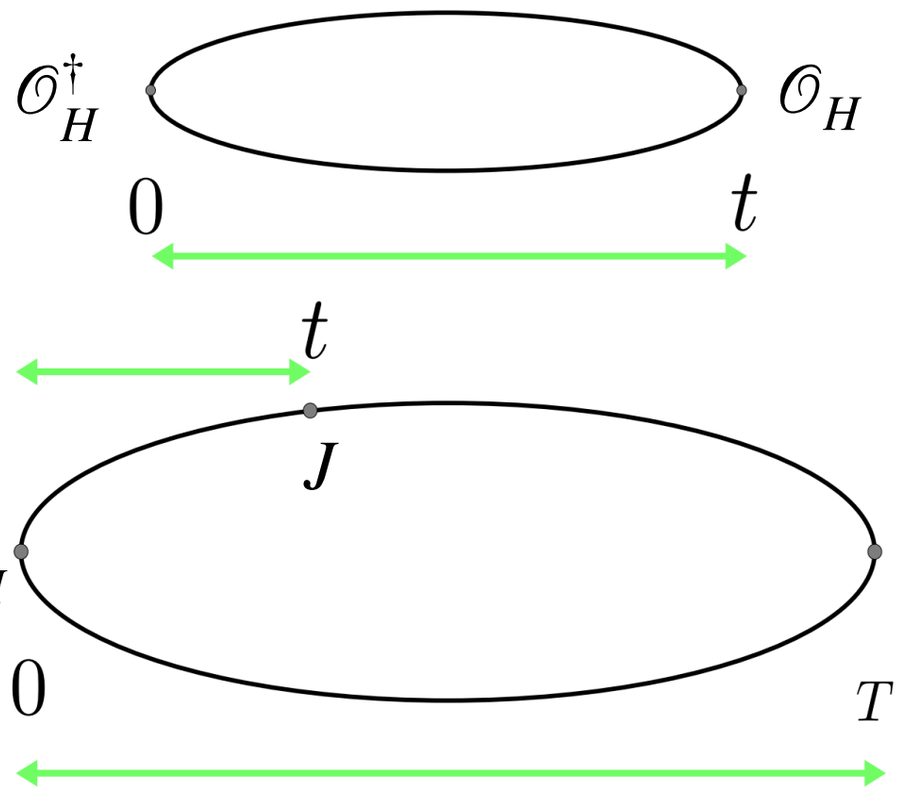
- 2nd Generation MILC gluon fields $N_f = 2 + 1 + 1$
- Highly Improved Staggered Quark (HISQ) action for all quarks
- Smallest heavy mass tuned to charm
- Fully relativistic calculations
- Covers almost all the kinematic range

Set	$\approx a$(fm)	m_s/m_l	$N_x^3 \times N_t$	am_h range	$\lvert\overrightarrow{p}^{\pi,K}_{\max}\rvert$ [MeV/c^2]	T range
f-5	0.09	5	$32^3 \times 96$	0.450-0.8	311	15-24
f-phys	0.09	27	$64^3 \times 96$	0.433-0.8	330	15-24
sf-5	0.06	5	$48^3 \times 144$	0.274-0.8	622	22-31
sf-phys	0.06	27	$96^3 \times 192$	0.2585-0.8	648	22-31
uf-5	0.04	5	$64^3 \times 192$	0.194-0.8	583	29-44

Correlators



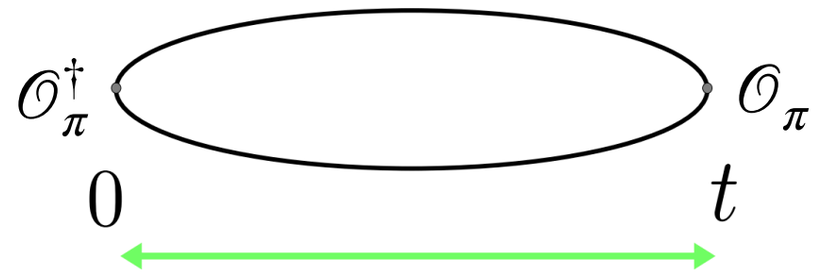
$$C_2^\pi(t) = \sum_{i=0}^{N_{exp}-1} \left[|A_i^{\pi,n}|^2 \left(e^{-E_i^{\pi,n} t} + e^{-E_i^{\pi,n}(N_t - t)} \right) + (oscillating) \right]$$



$$C_2^H(t) = \sum_{i=0}^{N_{exp}-1} \left[|A_i^{H,n}|^2 \left(e^{-E_i^{H,n} t} + e^{-E_i^{H,n}(N_t - t)} \right) + (oscillating) \right]$$

$$C_3^{H,\pi}(t,T) = \sum_{i,j=0}^{N_{exp}-1} \left[A_i^{\pi,n} V_{ij}^{nn} A_j^{H,n} e^{-E_i^{\pi,n} t} e^{-E_i^{H,n}(T-t)} + \ldots \right]$$

Correlators



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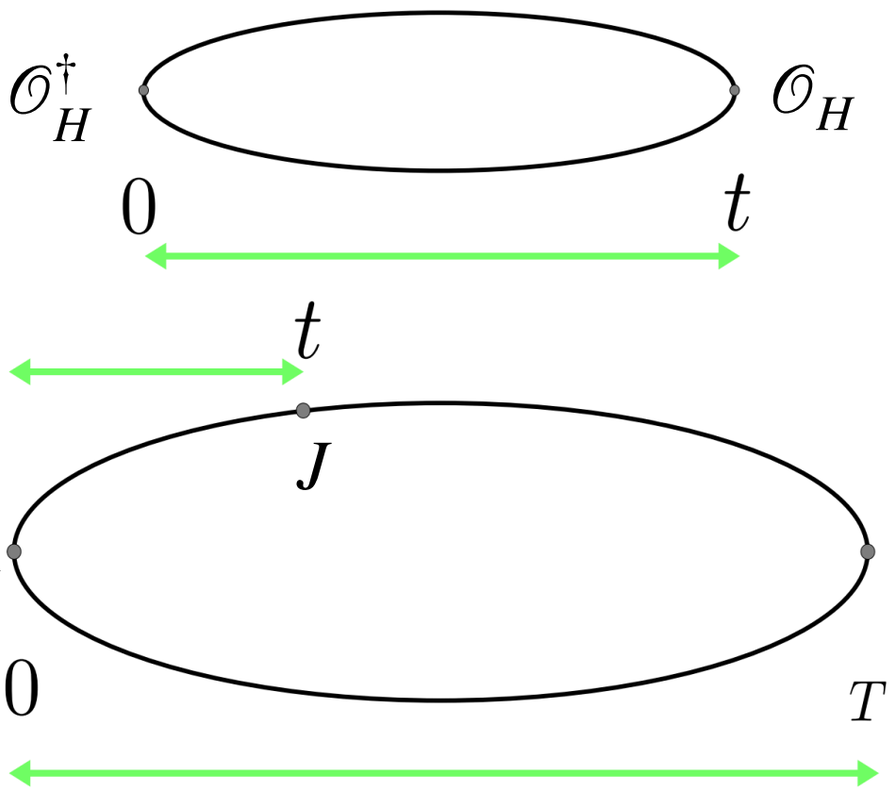

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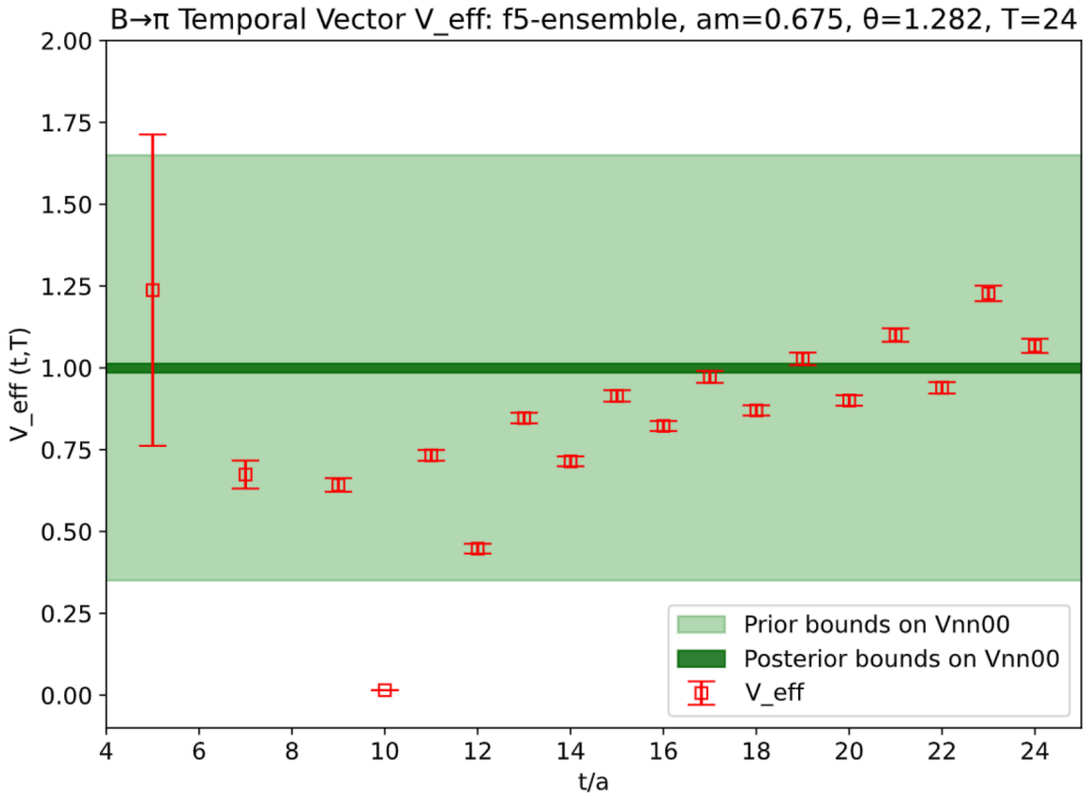
$$V_{00}^{nn} \quad \sim \quad \langle \pi | \, J \, | H \rangle \quad \rightarrow \quad f(q^2)$$

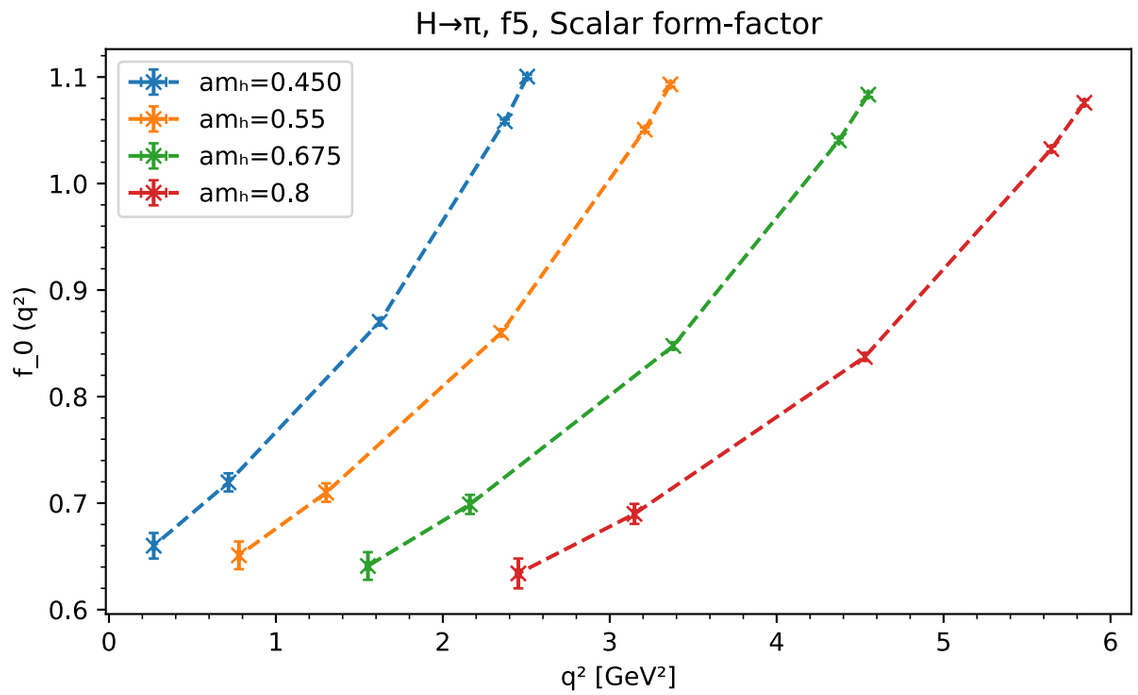
Fitting strategy

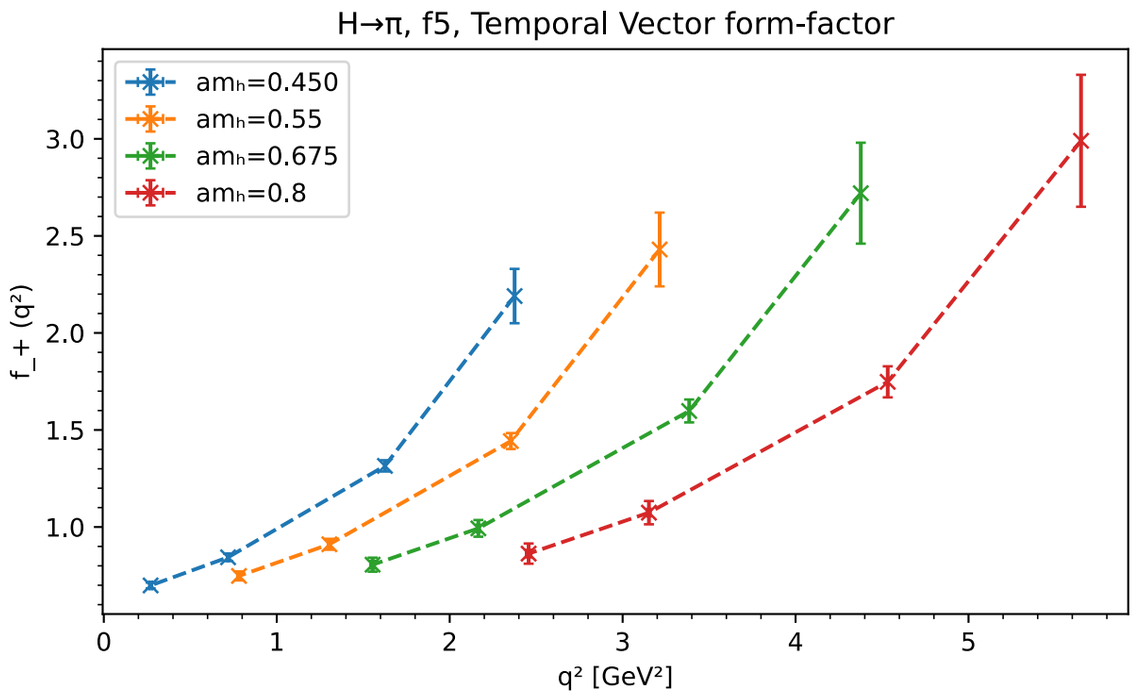
[2501.18586] Roberts, Bouchard, Francesconi, Parrot

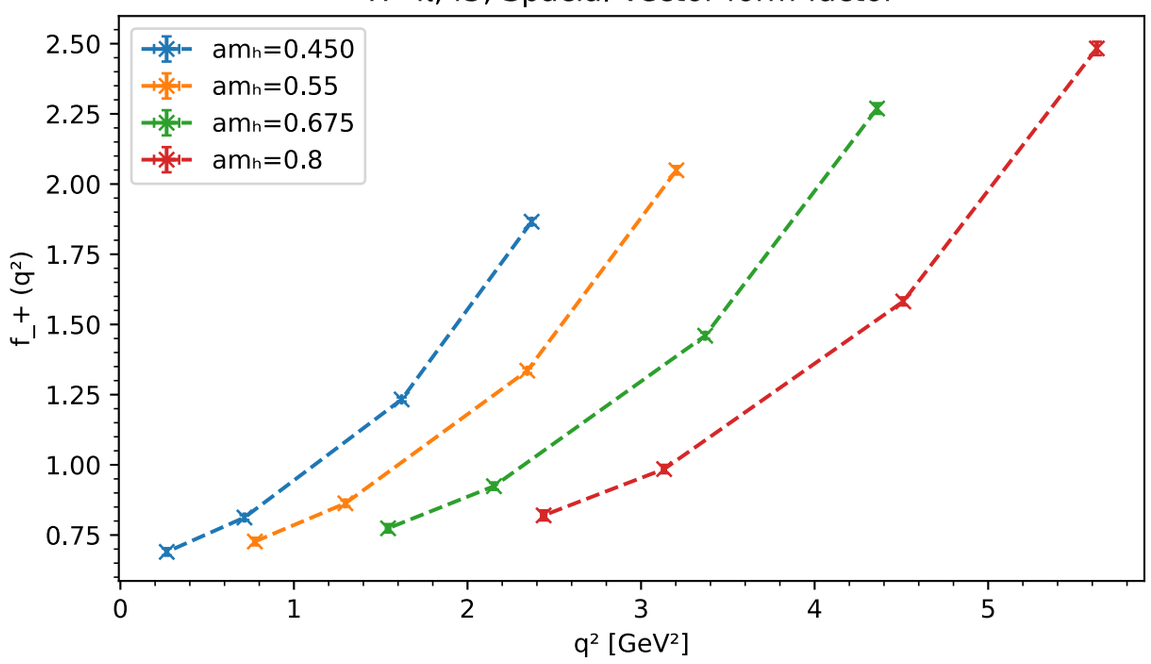
- Extensive correlation analysis
- Fully correlated multi exponential bayesian fitting procedure [corrfitter, lsqfit]
- Concurrent fit for:
  - Heavy masses
  - Twists
  - 3-pt time separations
  - 3-pt current components
  - $\mathcal{O}(100)$ parameters

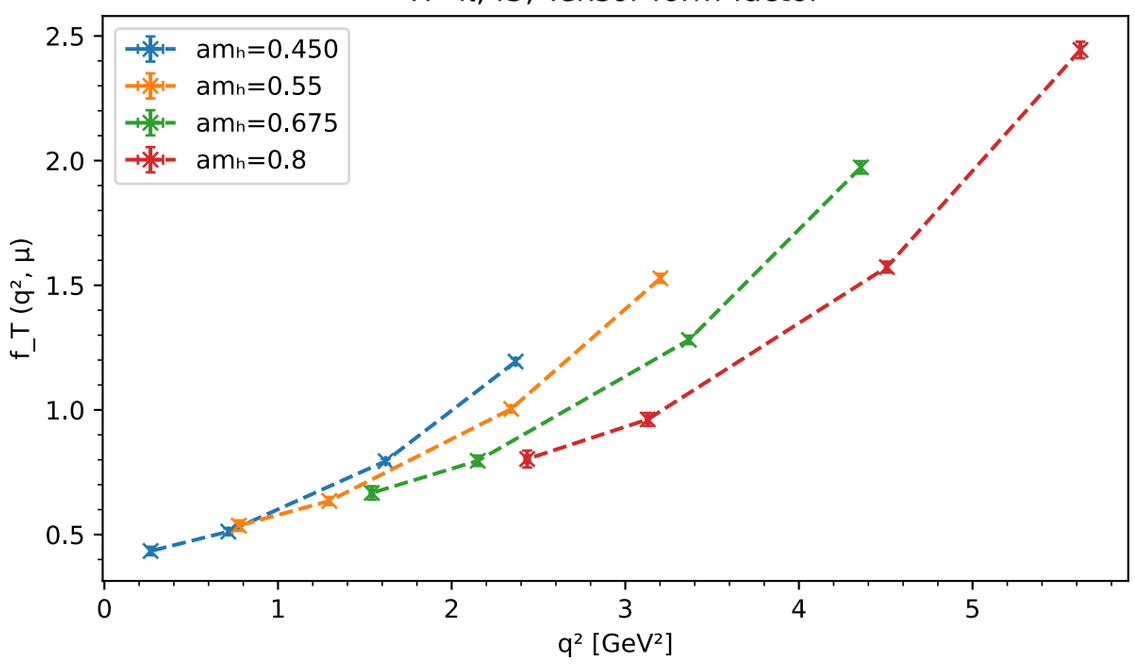


Fit results - Form Factors









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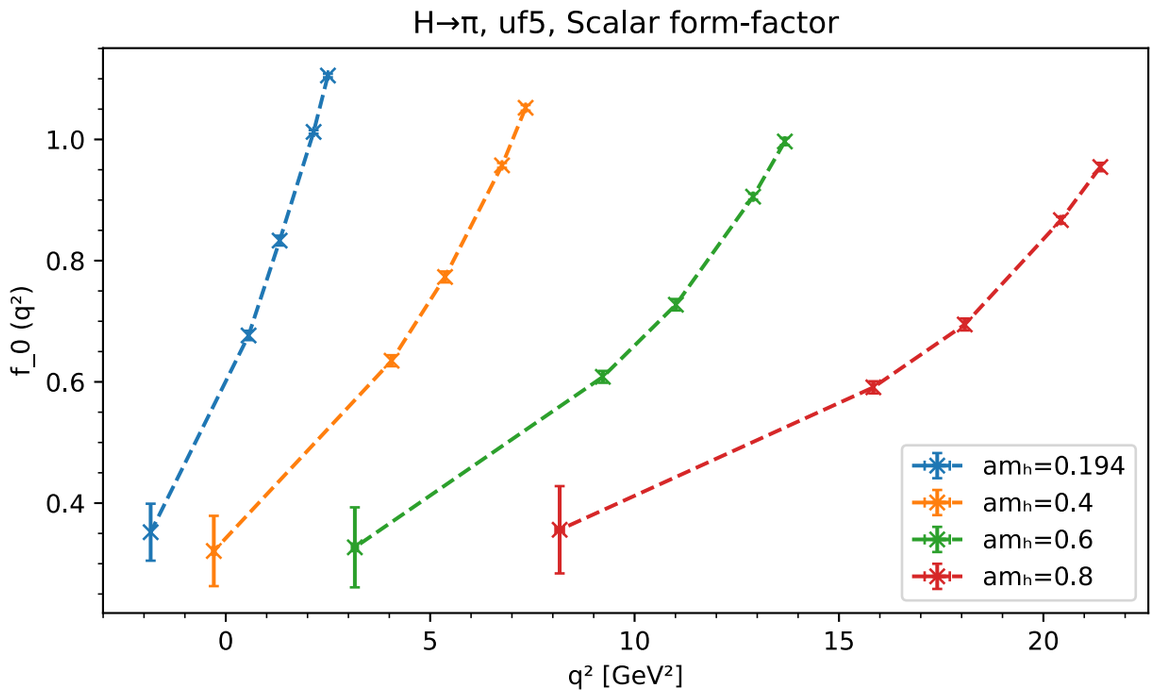
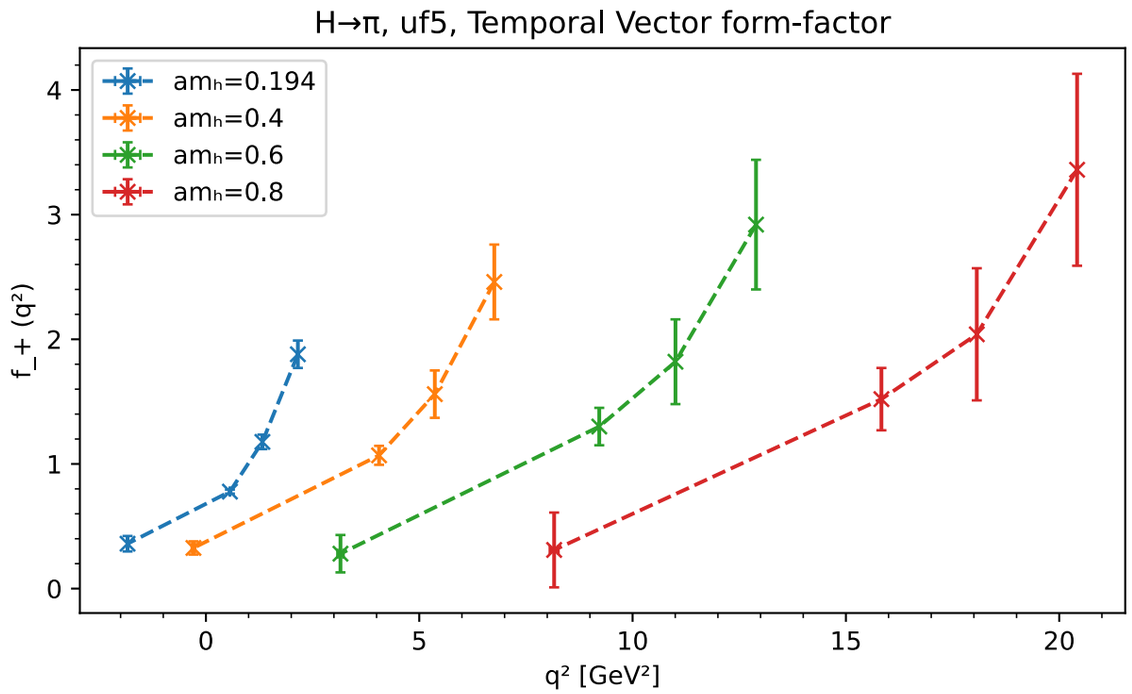
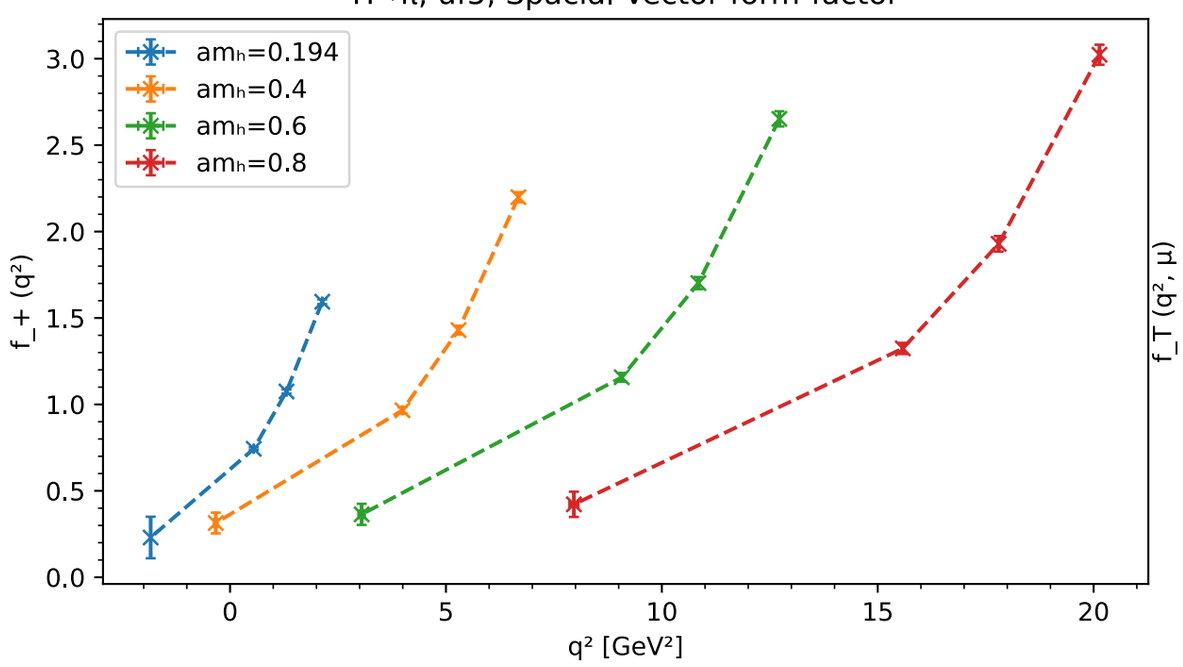
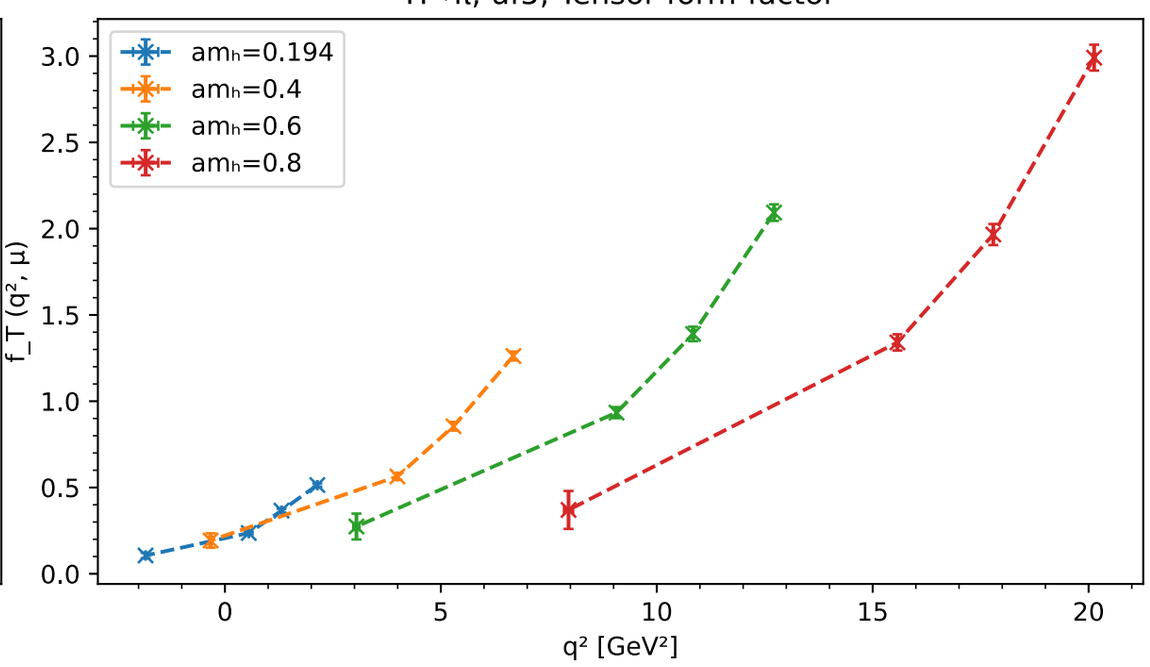








Towards the physical point

z-expansion $$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - q^2_{\text{max}}}}{\sqrt{t_+ - q^2} - \sqrt{t_+ - q^2_{\text{max}}}}$$

BGL-Parametrization $$f(z) = \frac{1}{\phi(z)\,P(z)} \sum_n a_n f(z,n)$$

Expansion in z (power series, orthonormal polynomial base)

Blashke factors: regularize sub-threshold poles

Outer functions:

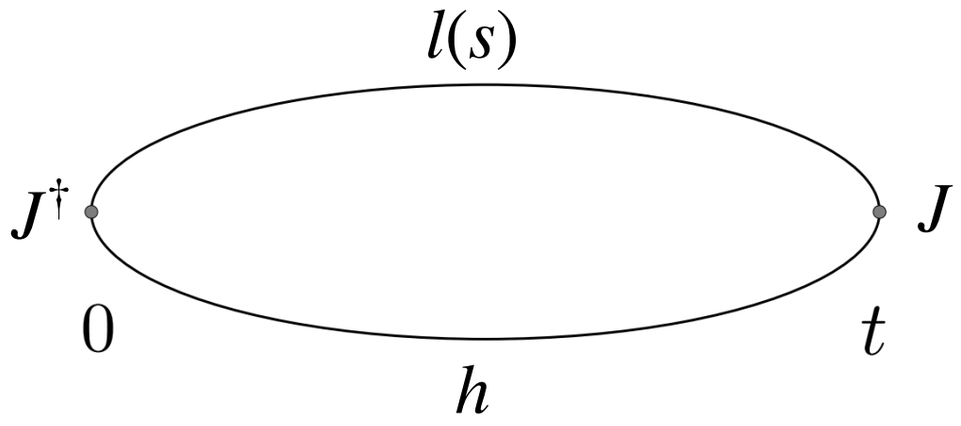
- Important to set fitting priors for a_n
- Depends on current susceptibilities
- For $b \to l(s)$ they are influenced by non perturbative condensate effects

Susceptibilities from the lattice

[2105.07851] Martinelli, Simula, Vittorio, [2405.01390] Harrison

$$\chi_{0^+} = \frac{1}{12}(m_b - m_l)^2 \int_0^\infty t^4 \; C_S(t) \; dt$$

$$C_X(t) = \ldots \int d^3x \; \langle 0 \,|\, T\bar{h}\Gamma_X l(x)(\bar{l})\Gamma_X h(0) \,|\, 0 \rangle$$



Computation of relevant correlators currently undergoing for $\bar{b}l$ and $\bar{b}s$ susceptibilities

Conclusions and Outlook

We have completed the extraction of raw form factors from correlator fits.

We are currently completing the lattice calculation of heavy-light and heavy-strange susceptibilities.

Finalize the extrapolation to the continuum and physical masses, using a modified z-expansion constrained by lattice-computed susceptibilities.