

Lattice QCD calculations for Muon $g-2$

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School of Physics and Astronomy
The University of Edinburgh

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THE UNIVERSITY
of EDINBURGH

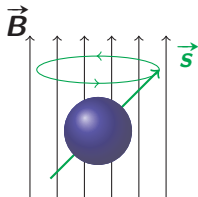
Magnetic Moment of the Muon

- ▶ magnetic moment $\vec{\mu}$ of the muon due to its spin $\vec{s}$ and electric charge e

$$\vec{\mu} = g \frac{e}{2m} \vec{s}$$

torque $\vec{\tau} = \vec{\mu} \times \vec{B}$

- ▶ gyromagnetic-factor (g -factor) of the muon



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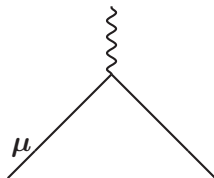
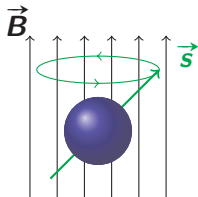
$$\vec{\mu} = g \frac{e}{2m} \vec{s}$$

torque $\vec{\tau} = \vec{\mu} \times \vec{B}$

- ▶ gyromagnetic-factor (g -factor) of the muon

without quantum effects:

$$g = 2$$

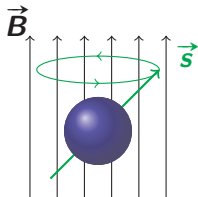


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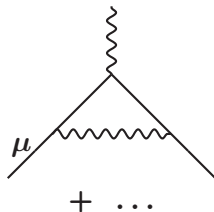
- ▶ gyromagnetic-factor (g -factor) of the muon

with quantum effects:

$$g = 2.00233 \dots$$

anomalous magnetic moment of the muon
“Muon $g-2$ ”

$$a_{\mu} = \frac{g - 2}{2}$$



Muon g-2: Experimental measurement

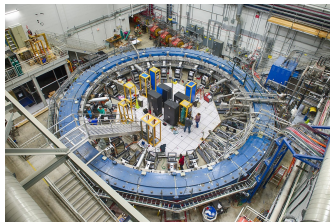
Previous: Muon g-2 @ BNL (2006)

[Phys.Rev. D73, 072003 (2006)]

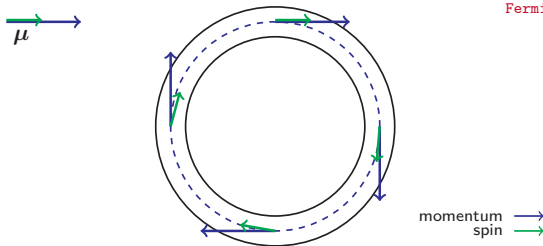
New: Muon g-2 @ FNAL (2021, 2023)

[PhysRevLett.126.141801 (2021)]

measure precession frequency of
muons in magnetic field:



[[https://commons.wikimedia.org/wiki/File:Fermilab_g-2_\(E989\)_ring.jpg](https://commons.wikimedia.org/wiki/File:Fermilab_g-2_(E989)_ring.jpg)]



$$\omega_a = a_\mu \frac{eB}{m_\mu}$$

$$a_\mu(\text{exp}) = 11659205.9(2.2) \times 10^{-10}$$

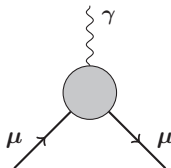
[Phys. Rev. Lett. 131, 161802 (2023)]

Muon g-2: Standard Model Prediction

White Paper (2020) of the Muon g-2 Theory initiative

[Phys.Rept. 887 (2020) 1-166]

[<https://muon-gm2-theory.illinois.edu/>]

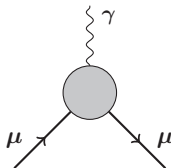


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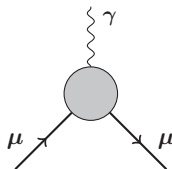


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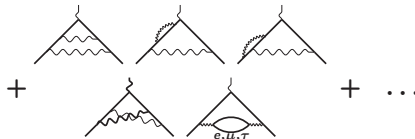
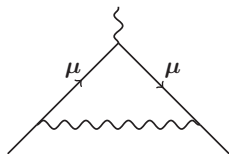
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electro-magnetism

$$11658471.8931(104) \times 10^{-10}$$



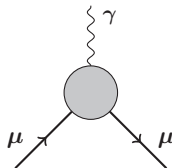
$O(10^4)$ diagrams
at $O(\alpha^5)$

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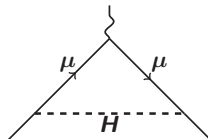
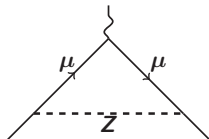
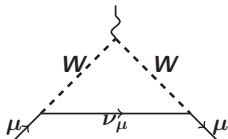


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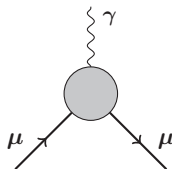


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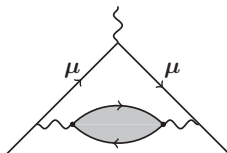
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Hadronic Vacuum Polarisation (HVP)

$$693.1(4.0) \times 10^{-10}$$

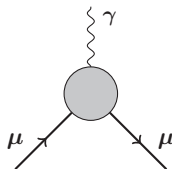


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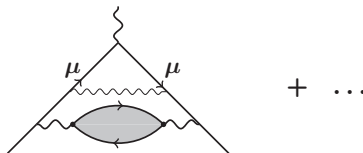
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HVP(α^3, α^4)

$$-8.59(7) \times 10^{-10}$$

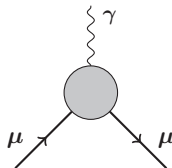


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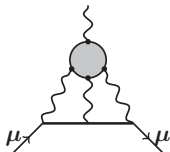
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HVP(α^3, α^4)

$$-8.59(7) \times 10^{-10}$$

Hadronic light-by-light scattering

$$9.2(1.8) \times 10^{-10}$$



Experiment vs Standard Model prediction

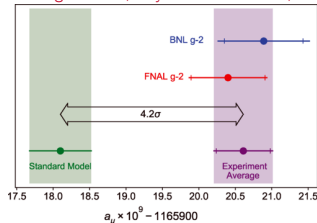
Exp ('23): $a_\mu = 0.00116592059(22)$

Exp ('21): $a_\mu = 0.00116592061(41)$

SM: $a_\mu = 0.00116591810(43)$

► new physics?

Muon $g-2$ Coll., Phys. Rev. Lett. 126, 141801



Experiment vs Standard Model prediction

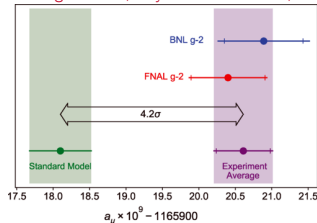
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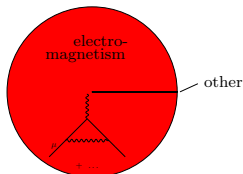
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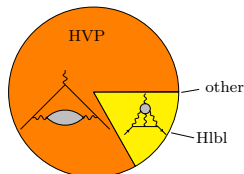
► FNAL to publish final result in 2025 (0.14 ppm), new upcoming experiment @JPARC

► Breakdown of Standard Model Prediction

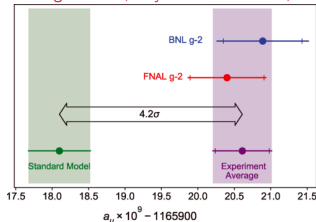
contribution to a_μ



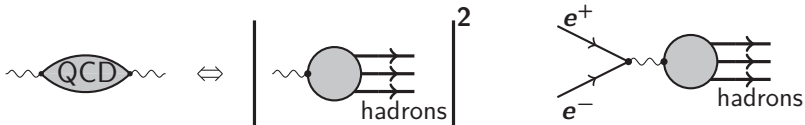
contribution to variance $\Delta^2 a_\mu$



Muon g-2 Coll., Phys. Rev. Lett. 126, 141801

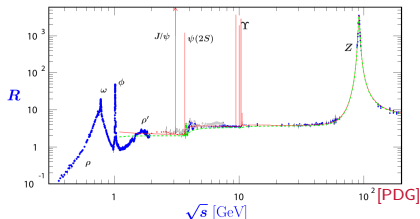


The HVP from R-ratio



$$R(s) = \frac{\sigma(e^+ e^- \rightarrow \text{hadrons}, s)}{\sigma(e^+ e^- \rightarrow \mu^+ \mu^-, s)}$$

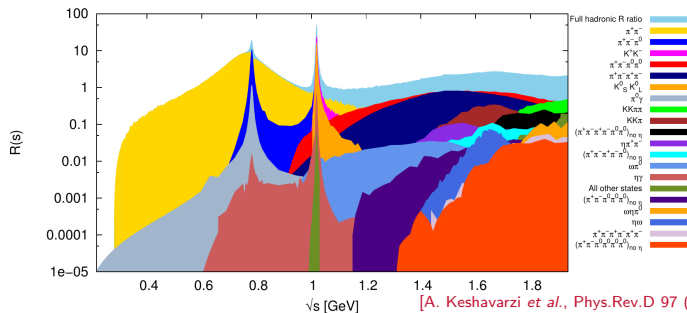
$$a_\mu^{\text{HVP}} = \left(\frac{\alpha m_\mu}{3\pi} \right)^2 \int_{m_\pi^2}^{\infty} ds \frac{R(s)K(s)}{s^2}$$



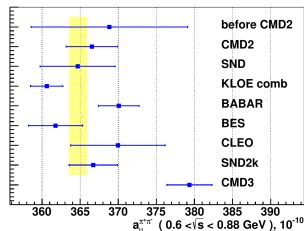
$a_\mu^{\text{hvp}} = 689.46(3.25)$	[Jegerlehner 18]
$a_\mu^{\text{hvp}} = 693.9(4.0)$	[DHMZ 19]
$a_\mu^{\text{hvp}} = 693.37(2.46)$	[KNT 18]
$a_\mu^{\text{hvp}} = 693.1(4.0)$	[white paper]

Tensions in R-ratio

- various channels contributing to R -ratio

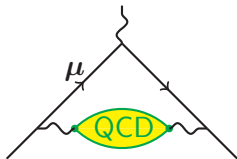


- 2023 results for $e^+e^- \rightarrow \pi^+\pi^-(\gamma)$ from CMD-3 disagree with KLOE and BABAR [CMD-3, arXiv:2302.08834]



Hadronic Vacuum Polarisation (HVP) from the lattice

- ▶ calculate **hadronic** part on the lattice



- ▶ vector two-point function

$$\mathbf{C}_{\mu\nu}(t) = \sum_{\vec{x}} \langle J_{\mu}(t, \vec{x}) J_{\nu}(0) \rangle$$

- ▶ electromagnetic current

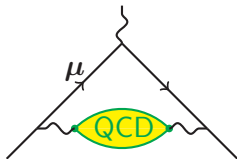
$$J_{\mu} = \frac{2}{3} \bar{u} \gamma_{\mu} u - \frac{1}{3} \bar{d} \gamma_{\mu} d - \frac{1}{3} \bar{s} \gamma_{\mu} s + \dots$$

- ▶ a_{μ} from $\mathbf{C}(t)$ [T. Blum, Phys.Rev.Lett.**91**, 052001 (2003); Bernecker and Meyer, Eur.Phys.J.**A47**, 148 (2011)]

$$a_{\mu}^{\text{HVP}} = \sum_t w_t C_{ii}(t) \quad \text{with kernel function } w_t$$

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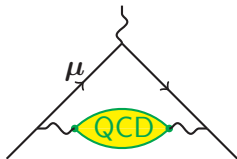
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- ▶ flavour decomposition (isospin symmetric QCD $u = d = \ell$)

$$\mathbf{C}(t) = \frac{5}{9} \mathbf{C}^{\ell}(t) + \frac{1}{9} \mathbf{C}^s(t) + \frac{4}{9} \mathbf{C}^c(t)$$

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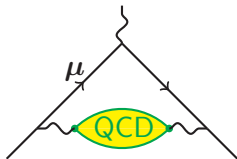
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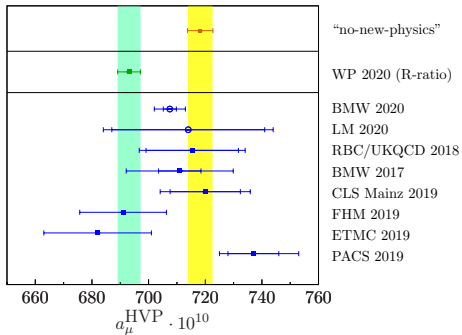
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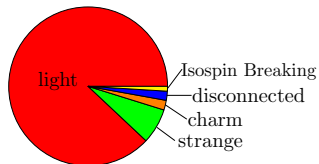
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Lattice Calculations of HVP (2020 WP status)

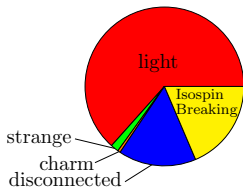
- White Paper lattice average $a_\mu^{\text{HVP}}(\text{lat}) = 711.6(18.4) \times 10^{-10}$



contributions to a^{HVP}



contributions to $\Delta a_\mu^{\text{HVP}}$

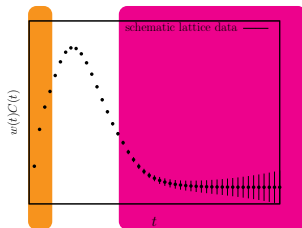


- BMW 2020

$$a_\mu^{\text{HVP}}(\text{BMW}) = 707.5(5.5) \times 10^{-10}$$

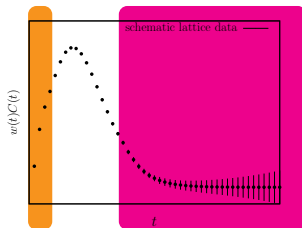
The window method

- ▶ main challenges:
 - ▶ statistical noise at large t
 - ▶ finite volume effects
(largest at large t)
 - ▶ discretisation effects at small t



The window method

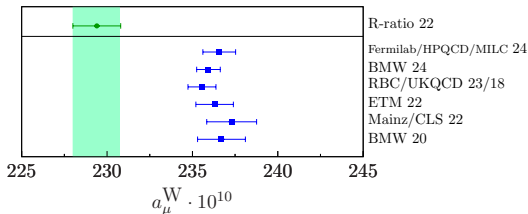
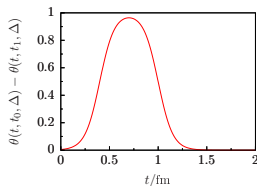
- ▶ main challenges:
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 - ▶ discretisation effects at small t



- ▶ window method

$$a_\mu = a_\mu^{\text{SD}} + a_\mu^{\text{W}} + a_\mu^{\text{LD}}$$

$$a_\mu^{\text{W}} = \sum_t w_t C(t) [\theta(t, t_0, \Delta) - \theta(t, t_1, \Delta)]$$



The long distance contribution to the HVP

- ▶ tame **statistical noise** by using knowledge of the spectral representation of the vector correlator $\mathbf{C}(t)$

$$\mathbf{C}(t) = \frac{1}{3} \sum_j \sum_x \langle J_j(t, \mathbf{x}) J_j(0) \rangle = \sum_i \mathbf{A}_i^2 e^{-E_i t} \quad \text{with} \quad \mathbf{A}_i^2 > 0$$

- ▶ lowest states: two pions with back-to-back momentum
- ▶ bounding method [S. Borsanyi *et al.*, (2017)], [T. Blum, VG, *et al.*, (2018)]

$$0 \leq \mathbf{C}(t_c) e^{-E_{t_c}(t-t_c)} \leq \mathbf{C}(t) \leq \mathbf{C}(t_c) e^{-E_0(t-t_c)}$$

- ▶ explicitly reconstruct the lowest N states
- ▶ improved bounding method [A. Meyer, *et al.*, (2019)]

$$\rightarrow \text{bounding method for } \mathbf{C}(t) - \sum_{i=1}^N \mathbf{A}_i^2 e^{-E_i t}$$

RBC/UKQCD collaboration

Boston University

Nobuyuki Matsumoto

BNL and BNL/RBRC

Peter Boyle

Taku Izubuchi

Christopher Kelly

Shigemi Ohta (KEK)

Amarjit Soni

Masaaki Tomii

Xin-Yu Tuo Shuhei Yamamoto

University of Cambridge

Nelson Lachini

CERN

Matteo Di Carlo

Felix Erben

Andreas Jüttner (Southampton)

Tobias Tsang

Columbia University

Norman Christ

Sarah Fields

Ceran Hu

Yikai Huo

Yong-Chull Jang

Joseph Karpie (JLab)

Erik Lundstrum

Bob Mawhinney

Bigeng Wang (Kentucky)

University of Connecticut

Tom Blum

Jonas Hildebrand

Luchang Jin

Vaishakhi Moningi

Anton Shcherbakov

Douglas Stewart

Joshua Swaim

DESY Zeuthen

Raoul Hodgson

Edinburgh University

Luigi Del Debbio

Vera Gülpers

Maxwell T. Hansen

Nils Hermansson-Truedsson

Ryan Hill

Antonin Portelli

Azusa Yamaguchi

University of Liverpool

Nicolas Garron

LLNL

Aaron Meyer

Autonomous University of

Madrid

Nikolai Husung

Milano Bicocca

Mattia Bruno

Nara Women's University

Hiroshi Ohki

Peking University

Xu Feng

Tian Lin

University of Regensburg

Andreas Hackl

Danile Knüttel

Christoph Lehner (BNL)

Sebastian Spiegel

RIKEN CCS

Yasumichi Aoki

University of Siegen

Matthew Black

Anastasia Boushmelev

Oliver Witzel

University of Southampton

Bipasha Chakraborty

Ahmed Elgaziari

Jonathan Flynn

Joe McKeon

Rajnandini Mukherjee

Callum Radley-Scott

Chris Sachrajda

Stony Brook University

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The long-distance window of the hadronic vacuum polarization for the muon $g - 2$

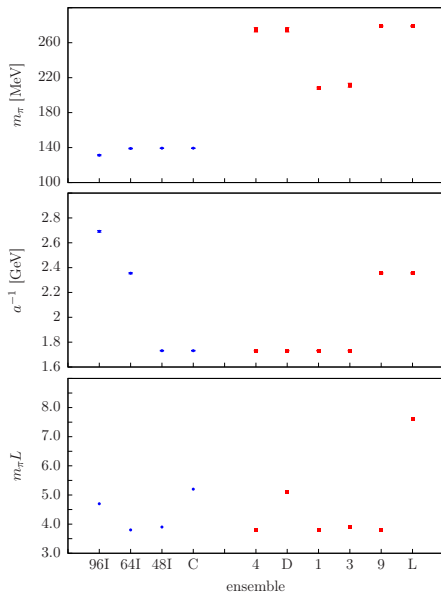
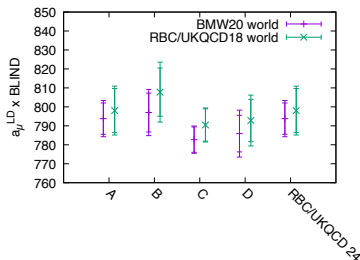
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(RBC and UKQCD Collaborations)

arXiv:2410.20590

RBC/UKQCD long distance result – computational details

- ▶ $N_f = 2 + 1$ Domain Wall Fermions
- ▶ four physical point ensembles
- ▶ three lattice spacings
- ▶ several volumes
- ▶ four groups
- ▶ blinding (overall factor)



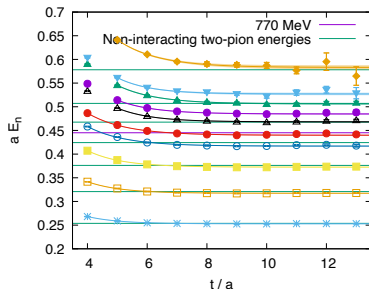
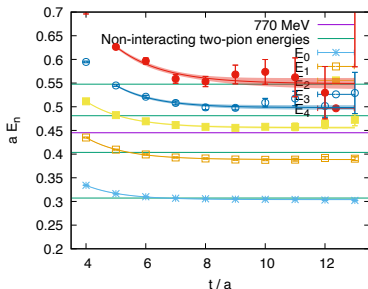
RBC/UKQCD long distance result – spectrum

- spectrum/overlaps of vector correlator using GEVP

[M. Luscher and U. Wolff (1990); B. Blossier *et al* (2009)]

$$C^{ij}(t) = \langle O^i(t) O^j(t)^\dagger \rangle$$

- vector and two-pion operators with momenta up to $(2, 0, 0)$, or $(2, 2, 0)$ (96l, C), using distillation [M. Peardon *et al*, Phys. Rev. D 80, 054506 (2009)]
- spectrum ensembles 48l (left) and C (right)

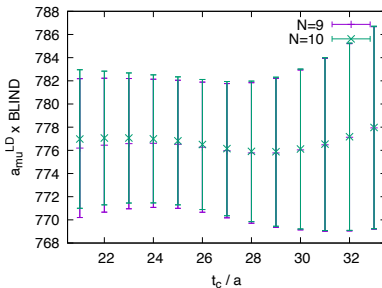
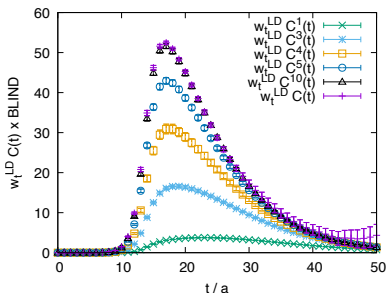


RBC/UKQCD long distance result – reconstruction

- ▶ long-distance $\mathbf{a}_\mu$ from vector correlator $\mathbf{a}_\mu^{\text{LD}} = \sum_t \mathbf{w}_t \mathbf{C}(t) \theta(t, t_1, \Delta)$
- ▶ for large times $t > t_c$: use reconstruction

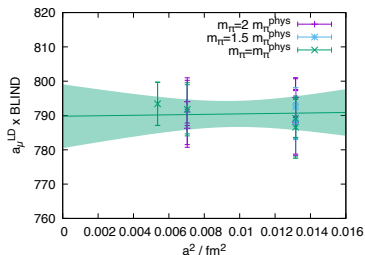
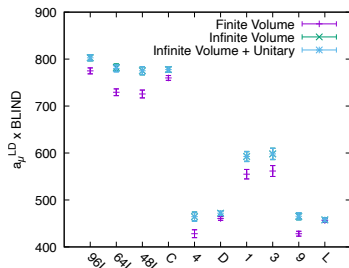
$$\mathbf{C}^N(t) = \sum_{i=1}^N \mathbf{A}_i^2 e^{-E_i t}$$

- ▶ Left: reconstruction for 96l
- ▶ Right: dependence of $\mathbf{a}_\mu$ on t_c and number of states N for 96l



RBC/UKQCD long distance result – FV and continuum

- ▶ FV corrections using Hansen-Patella formalism [M. T. Hansen and A. Patella (2019), (2020)]
- ▶ ensembles that only differ by volume (48l&C, 4&D, 9&L) agree after FV corrections
- ▶ combined extrapolation: continuum and “physical” isospin-symmetric point
- ▶ continuum extrapolation: a^2 -term
- ▶ two choices of $\mathbf{Z}_V$



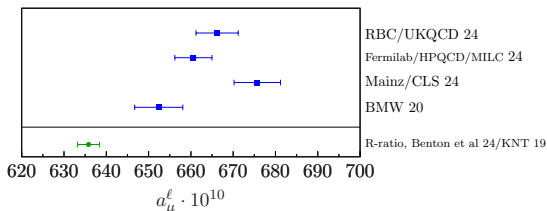
Comparison

- ▶ Final results (BMW isospin symmetric scheme):

$$a_{\mu}^{\text{LD},\ell} = 411.4(4.3)(2.4) \times 10^{-10}$$

$$a_{\mu}^{\ell} = 666.2(4.3)(2.5) \times 10^{-10}$$

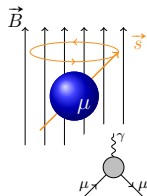
- ▶ systematic error dominated by choices of extrapolation (2.2×10^{-10})
- ▶ comparison with other (recent and precise) light-quark results



- ▶ good agreement between lattice calculations
- ▶ **Disclaimer:** not all results shifted to same IB symmetric point

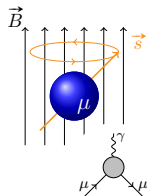
Summary

- ▶ Muon $g - 2$ promising quantity for finding new physics
→ currently: 4.2σ tension between experiment and theory
- ▶ Theory error dominated by Hadronic Vacuum Polarisation
→ R-ratio input: tensions between experiments
- ▶ precision of lattice calculations now competitive with R -ratio, further improvements needed to match precision of FNAL experiment
- ▶ recent lattice calculations closer to experiment, in tension with R -ratio
- ▶ Are we about to find new physics with a_μ ?



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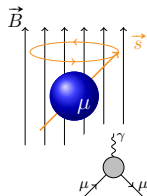


Probably not.



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Probably not.

Thank you!



Backup

Isospin symmetric schemes

- ▶ Decomposing into isospin symmetric and isospin breaking

$$\boxed{X^\phi} = \boxed{\bar{X}} + \boxed{X_{SU(2)}} + \boxed{X_\gamma} \quad \hat{X} \equiv \bar{X} + X_{SU(2)}$$

QED isospin breaking
 strong isospin breaking
 isospin symmetric
 observable at the physical point

- ▶ observable at the physical point unambiguously defined
- ▶ separation into $\bar{X}$, $X_{SU(2)}$ and X_γ requires separation scheme
 → define arbitrary values $\bar{\Pi}$ and $\hat{\Pi}$ for a set of quantities Π
- ▶ RBC/UKQCD long-distance calculation uses two schemes:
 - ▶ BMW20: $m_\pi = 0.13497\text{GeV}$, $m_{ss^*} = 0.6898\text{GeV}$, $w_0 = 0.17236\text{fm}$
 - ▶ RBC/UKQCD18: $m_\pi = 0.135\text{GeV}$, $m_K = 0.4957\text{GeV}$,
 $m_\Omega = 1.67225\text{GeV}$

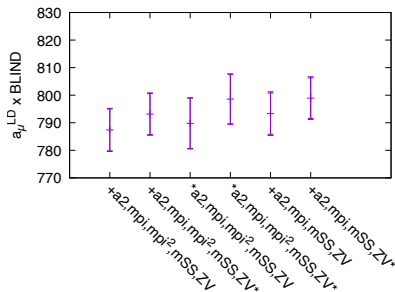
a_μ^{LD} combined extrapolation

- ▶ two fit-ansatze (e.g., for BMW scheme):

$$f_+ = f_0 + f_1 a^2 + f_2 \left(w_0 m_\pi - (w_0 m_\pi)_{\text{phys}} \right) + f_3 \left(w_0 m_\pi - (w_0 m_\pi)_{\text{phys}} \right)^2 + f_4 \left(w_0 m_{\text{ss}*} - (w_0 m_{\text{ss}*})_{\text{phys}} \right)$$

$$f_* = f_0 \left(1 + f_1 a^2 \right) \left(1 + f_2 \left(w_0 m_\pi - (w_0 m_\pi)_{\text{phys}} \right) + f_3 \left(w_0 m_\pi - (w_0 m_\pi)_{\text{phys}} \right)^2 + f_4 \left(w_0 m_{\text{ss}*} - (w_0 m_{\text{ss}*})_{\text{phys}} \right) \right)$$

- ▶ either $f_3 = 0$ or $f_3 \neq 0$
- ▶ two different $\mathbf{Z}_V$
- ▶ total: $\rightarrow$ 8 fit choices
 $\rightarrow$ model average



RBC/UKQCD window result – ensembles

- Möbius Domain Wall Fermions
- three lattice spacings at the physical point with $N_f = 2 + 1$

ID	a^{-1}/GeV	$L^3 \times T \times L_s$	m_π/MeV	m_K/MeV
48l	1.7312(28)	$48^3 \times 96 \times 24$	139.32(30)	499.44(88)
64l	2.3549(49)	$64^3 \times 128 \times 12$	138.98(43)	507.5(1.5)
96l	2.6920(67)	$96^3 \times 192 \times 12$	131.29(66)	484.5(2.3)

- additional “helper” ensembles at heavier masses

ID	a^{-1}/GeV	N_f	$L^3 \times T \times L_s$	m_π/MeV	m_K/MeV	m_{D_s}/GeV
1	1.7310(35)	2+1	$32^3 \times 64 \times 24$	208.1(1.1)	514.0(1.8)	–
2	1.7257(74)	2+1	$24^3 \times 48 \times 32$	285.4(2.9)	537.8(4.6)	–
3	1.7306(46)	2+1	$32^3 \times 64 \times 24$	211.3(2.3)	603.8(6.1)	–
4	1.7400(73)	2+1	$24^3 \times 48 \times 24$	274.8(2.5)	530.1(3.1)	–
5	1.7498(73)	2+1+1	$24^3 \times 48 \times 24$	279.8(3.5)	539.1(5.3)	1.9902(69)
7	1.7566(81)	2+1+1	$24^3 \times 48 \times 24$	272.5(5.9)	523(10)	1.3882(57)
A	1.7556(83)	2+1	$24^3 \times 48 \times 8$	307.4(3.5)	557.3(5.7)	–

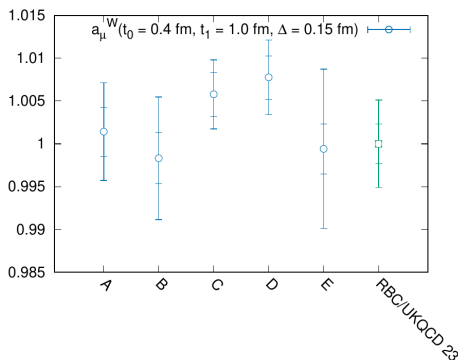
RBC/UKQCD window result – blinding

- ▶ blinded analysis, to avoid bias towards other results
- ▶ five different analysis groups
- ▶ vector two-point function blinded

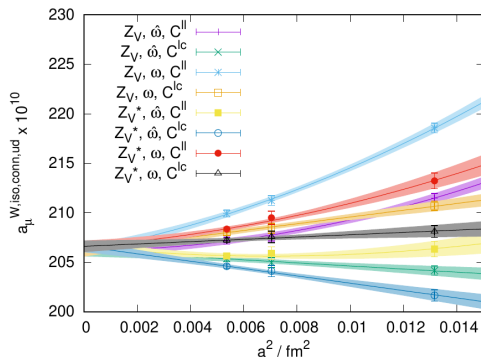
$$C_b(t) = (b_0 + b_1 a^2 + b_2 a^4) C_0(t)$$

with random coefficients b_0 , b_1 and b_2 different for each group

- ▶ relative unblinding



RBC/UKQCD window result – continuum extrapolation



- ▶ isospin symmetric light-quark connected window

$$a_{\mu}^{W, \text{ iso, conn, ud}} = 206.36(44)(43) \times 10^{-10}$$

- ▶ total window (using RBC/UKQCD 2018 results for other flavours)

$$a_{\mu}^W = 235.56(65)(50) \times 10^{-10}$$