Novel static and dynamic properties of gauge theories through the lens of Quantum Link Models

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UKLFT Annual Meeting

University of Edinburgh

Outline

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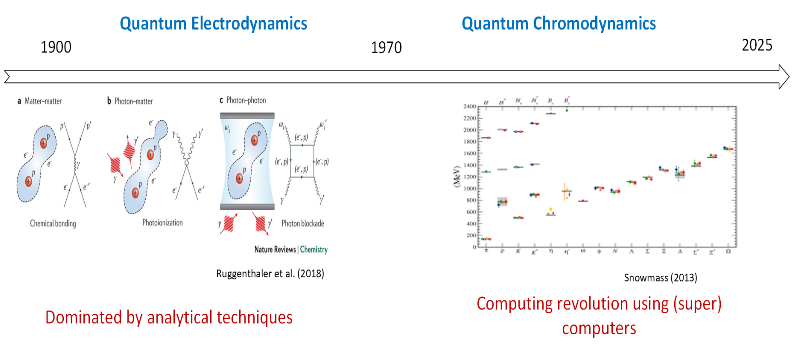
Computing in QFT

Familiar physics with Quantum Links

Surprises in static and dynamic phenomena

Applications in Quantum Computing

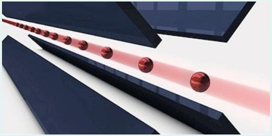
Classical techniques for QFTs

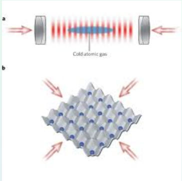


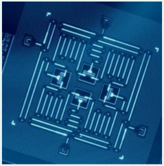
- Diagrammatics in QED explain light-matter interactions.
- QED created new domains: electronics, solid state, lasers, $\cdots$.
- Development of QCD largely through HPC classical computing.

Quantum simulators and computers: experiments

- Precise control of quantum degrees of freedom can potentially revolutionize computing paradigms: ion-traps, optical lattices, superconducting qubits.

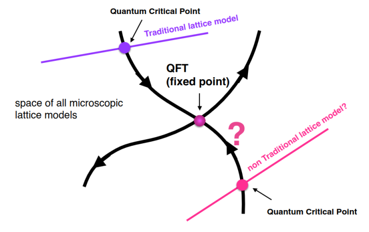




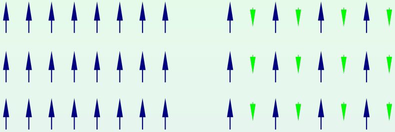


- Experiments with ion-traps and Rydberg atoms have already demonstrated significant progress, especially for realizing constrained and gauge theories.
- Precise experimental control on finite number of quantum states.
- Novel theory formulations to take advantage of the hardware?
- Outstanding problems: finite density phases, real-time dynamics.

Explorations guided by RG

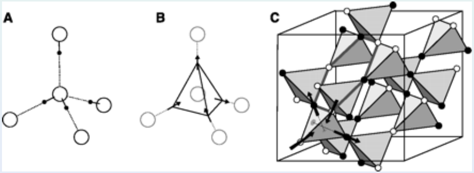


Wilson (1975): Calculable models constructed using symmetry to extract universal answers applicable to a large class of phenomena.



Ferro- and Ferri-magnetism.

Fig: Michael Schmid, Wikipedia.



Frustrated Magnets (pyrochlores).

Fig: Bramwell, Gringras.

CMP phenomena: emergent gauge invariance

What are Quantum Links?

- Generalized lattice gauge theories:
  - Horn (1981); Orland, Rohrlich (1990);
  - Chandrasekharan, Wiese (1997); + Brower (1999)
  - Rokhsar, Kivelson (1988); Moessner, Sondhi, Fradkin (2002)
    $\rightarrow$ Tangible connection to CMP (e.g. dimer model, magnets).
- Continuous gauge symmetries with discrete link operators
  $\rightarrow$ finite dimensional Hilbert space
  $\rightarrow$ extension of Wilson LGTs.
  $\rightarrow$ possibility of new physics scenarios.
- Excellent candidate models for quantum simulators.
- Testbed for analytical (e.g. EFTs) and numerical (e.g. Monte Carlo, Tensor Network) methods.
- Very close to Qubit Regularizations; several common aspects with Loop-String-Hadrons.

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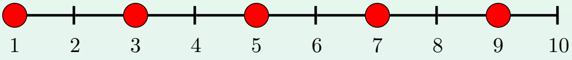
Quantum Link Schwinger Model

$$H = \frac{g^2}{2} \sum_{xy} E_{xy}{}^2 - \kappa \sum_{xy} \left[\psi_x^\dagger U_{xy} \psi_y + \mathrm{h.c.} \right] + m \sum_{x} (-1)^x \psi_x^\dagger \psi_x$$



- Quantum links: Finite dimensional gauge invariant representations possible.
- $U = S^+;\; U^\dagger = S^-;\;\; E = S^z$

$$[E_{xy}, U_{xy}] = U_{xy}$$
$$[E_{xy}, U_{xy}^\dagger] = -U_{xy}^\dagger$$
$$[U_{xy}, U_{xy}^\dagger] = 2E_{xy}$$

- Gauge symmetry: $[G_x, H] = 0$; where $G_x = (\nabla \cdot E_x - \rho_x)$ selects the physical states: $G_x|\Psi\rangle = 0$

$$V = \prod_x \exp\left(iq\theta_x G_x\right)$$
$$\tilde{H} = VHV^\dagger = H$$

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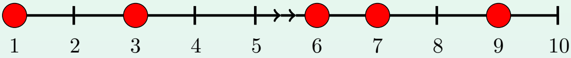

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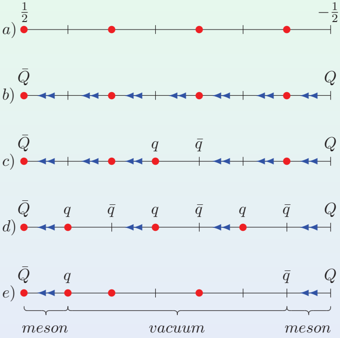
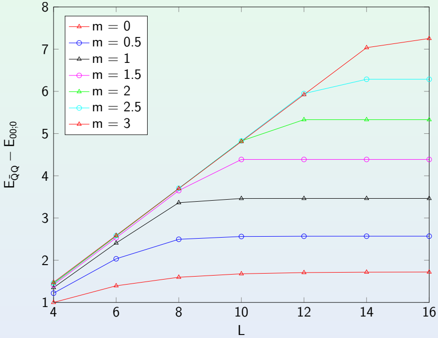
String breaking

Using $S = 1$, a confining string can be realized.





String breaking in a quantum simulator.
DB, Dalmonte, Müller, Rico Ortega, Stebler, Wiese, Zoller (PRL, 2012).
Experiment:

Yang, Sun, Ott, Wang, Zache, Halimeh, Yuan, Hauke, Yang (Nature, 2020)

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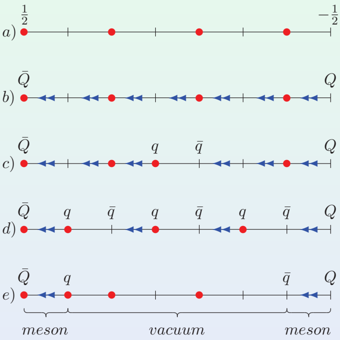

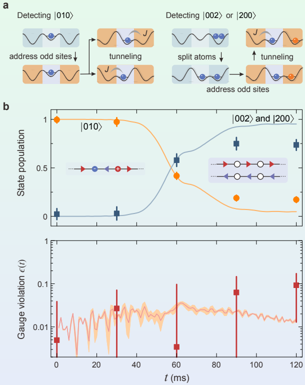

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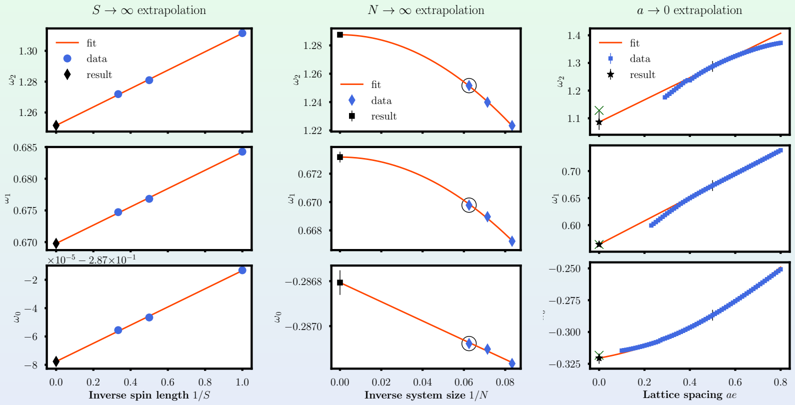
Extrapolating to the continuum limit



Possible to reliably reproduce the low-lying meson spectra for moderate values of spin-representation S. Zache, van Damme, Halimeh, Hauke, DB (PRD, 2021).

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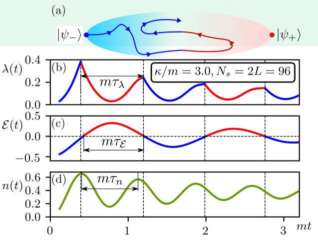
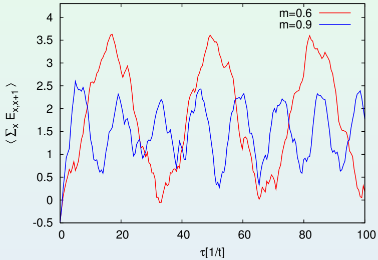
Fractional Electric Fluxes

$S = \frac{1}{2}$ can realize $S_{\text{top}} = \frac{i\theta}{4\pi} \int d^2x\, \epsilon_{\mu\nu} F_{\mu\nu}$ with $\theta = \pi$.
Coleman's prediction of a phase transition at finite m/g for $\theta = \pi$.

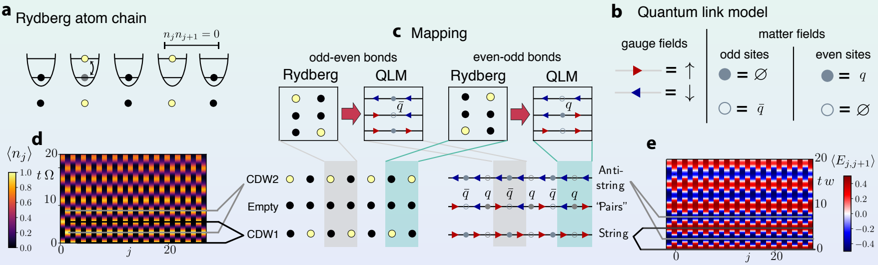




- $Z(2)$ CP symmetry breaking/restoration transition.
- Coherent long-lived oscillation of electric flux across the phase transitions, signatures of dynamical quantum phase transitions.
  Huang, DB, Heyl (PRL, 2019),
  Deasules, DB, Hudomal, Papic, Sen, Halimeh (2022).

Connection with experiments

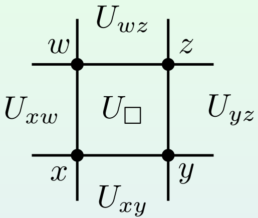
- Rydberg chains: observed many-body dynamics with a 51-atom quantum simulator Bernien et. al. (Nature, 2017).
- Directly related to the spin $S = \frac{1}{2}$ QLM Surace, Mazza, Giudici, Lerose, Gambassi, Dalmonte (PRX, 2020)



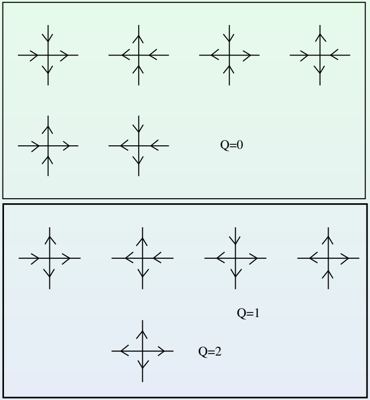
- Anomalous thermalization $\longrightarrow$ quantum scars.

  Turner, Michailidis, Abanin, Serbyn, Papic (Nature Phys, 2018).

Quantum Links in $(2+1)$-d



$$U_\square = U_{xy} U_{yz} U^\dagger_{wz} U^\dagger_{xw}$$



$$G_x = \sum_i (\mathrm{E}_{x,x+i} - \mathrm{E}_{x-i,x})$$

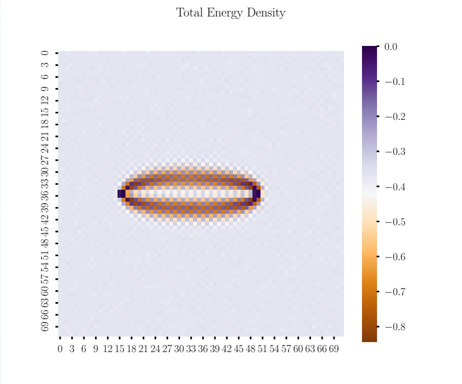
$$\begin{aligned} \mathrm{H} &= -J \sum_\square \left(U_\square + U^\dagger_\square\right) \\ &+ \lambda \sum_\square \left(U_\square + U^\dagger_\square\right)^2 \end{aligned}$$

E^2_{xy} is a constant: drops in H, but enters via G_x.

$Z = \mathrm{Tr}\ \left[e^{-\beta \mathrm{H}} \mathbb{P}_{\mathbb{G}}\right]$; $\mathbb{G} = \prod_x \delta(G_x)$

Flux fractionalization

Pure $(2+1)$-d U(1) gauge link models reveal phases with global symmetry breaking. Energy density of static $Q = \pm 1$ charges:



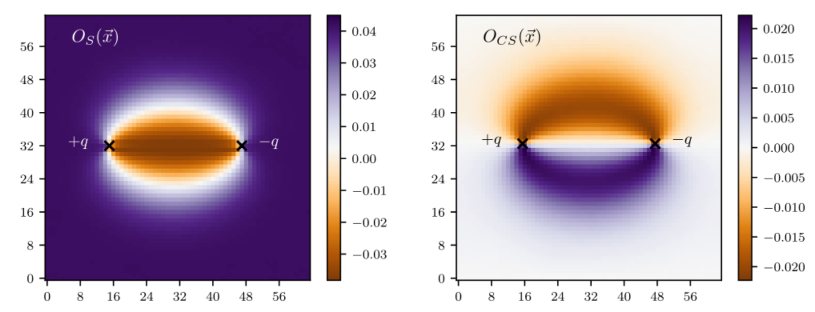
Strands carry fractional $E = \frac{1}{2}$, which can be identified with domain walls using effective field theory methods.

DB, Jiang, Widmer, Wiese (J Stat Mech, 2013) $\longrightarrow$ crystalline confinement.

DB, Caspar, Jiang, Peng, Wiese (Phys. Rev. Res., 2022) $\longrightarrow$ nematic confinement.

Flux fractionalization

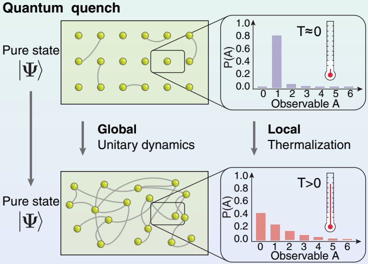
Such correlated symmetry breaking and flux fractionalization are also seen in self-adjoint extension of Wilson LGT with $\theta = \pi$.



Banerjee, DB, Kanwar, Mariani, Rindlisbacher, Wiese (2024)

Dynamics: Thermalization and ETH

- Interacting quantum systems (mostly) thermalize. Fast? Slow? Evade?
- However, QM is unitary: $|\psi(t)\rangle = \exp(-iHt)\,|\psi(0)\rangle$.
- Approach to equilibrium generally guided via Eigenstate Thermalization Hypothesis (ETH). Deutsch (PRA 1991), Srednicki (PRE, 1994).



Information about the initial state converted into (non-local) correlations through spreading of quantum entanglement.

Nandkishore, Huse (Ann. Rev. of CMP, 2015).

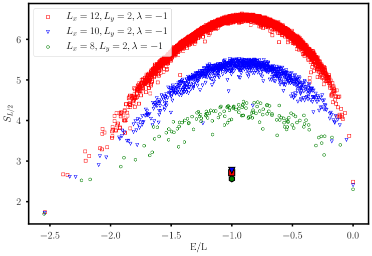
Kaufman et. al., (Science, 2016).

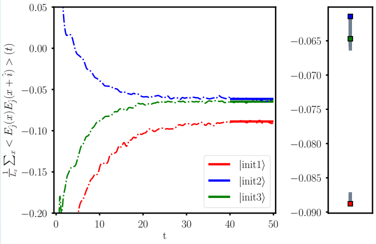
- Increasing examples of translational-invariant interacting systems showing (weak-) ergodicity breaking: quantum many-body scars.

Quantum Scars

Excited spectrum contains states with anomalous entanglement entropy and are localized in Hilbert (Fock) space.

DB, Sen (PRL, 2021).



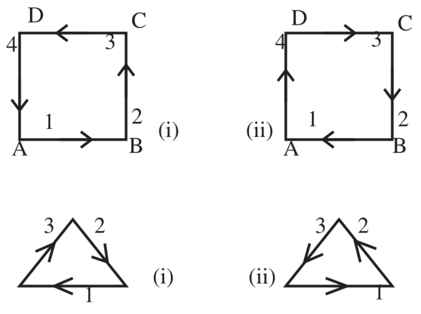


Larger variety of scars in QLMs and QDMs.

Even non-Abelian gauge theories/QCD show such states?

Outline

Real-time dynamics of plaquettes



$$H_\square = -g(S_1^+ S_2^+ S_3^- S_4^- + S_1^- S_2^- S_3^+ S_4^+),$$
$$H_\triangle = -g(S_1^+ S_2^+ S_3^+ + S_1^- S_2^- S_3^-),$$

$$H_{\mathrm{N}} = -g\sigma^3_{xy}\sigma^3_{yz}\sigma^3_{zw}\sigma^3_{wx},$$

$$U_{\mathrm{S,A}}(t) = \exp\left[i\frac{\pi}{4}\sigma^3_{\mathrm{A}}\sum_{j=1}^{N}\sigma^3_j\right]\exp\left[igt\sigma^1_{\mathrm{A}}\right] \times \exp\left[-i\frac{\pi}{4}\sigma^3_{\mathrm{A}}\sum_{j=1}^{N}\sigma^3_j\right]$$

For $N = 4$,

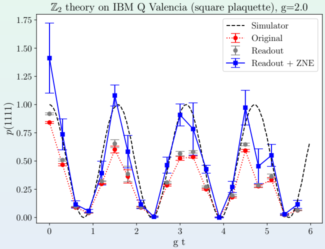
$$\begin{aligned} U_{\mathrm{S,A}}(t) &= \exp\left[(-i)^5 gt\sigma^1_{\mathrm{A}}(\sigma^3_{\mathrm{A}}\sigma^3_1)(\sigma^3_{\mathrm{A}}\sigma^3_2)(\sigma^3_{\mathrm{A}}\sigma^3_3)(\sigma^3_{\mathrm{A}}\sigma^3_4)\right] \\ &= \exp\left[-igt\sigma^1_{\mathrm{A}}\sigma^3_1\sigma^3_2\sigma^3_3\sigma^3_4\right] \end{aligned}$$

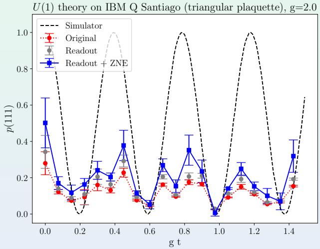
Huffman, Garcia, DB (Phys. Rev. D. 2021).

Tracking return probability

$$\mathcal{L}(t) = |\mathcal{G}(t)|^2; \quad \mathcal{G}(t) = \langle \psi_0 | e^{-iHt} | \psi_0 \rangle$$

Results from IBMQ Valencia (5 qubit) and IBMQ Santiago (5 qubit).

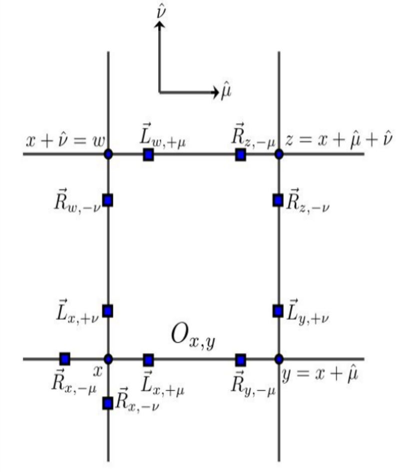




With error-mitigation techniques (zero-noise extrapolation + readout correct), we were able to handle (sometimes) complexities above the quantum volume of the computers : $V_Q = 2^{\min(d,m)}$

Huffman, Garcia, DB (Phys. Rev. D. 2021).

Non-Abelian QLMs



Hamiltonian: $\mathcal{H} = \mathcal{H}_E + \mathcal{H}_B$

$$\mathcal{H}_E = \frac{g^2}{2}\sum_{x,\mu}\left(L^a_{x,+\mu}L^a_{x,+\mu} + R^a_{x+\mu,-\mu}R^a_{x+\mu,-\mu}\right)$$

$$\mathcal{H}_B = -\frac{1}{4g^2}\sum_{\square}\mathrm{Tr}\,\mathcal{O}_\square$$

where $\mathcal{O}^{ab}_\square = O^{am}_{x,y}\;O^{mn}_{y,z}\;O^{np}_{z,w}\;O^{pb}_{w,x}$

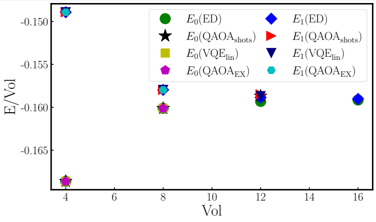
$$G^a_x = \sum_\mu (L^a_{x,+\mu} + R^a_{x,-\mu}), \quad [G^a, G^b] = 2i\varepsilon^{abc}G^c,$$

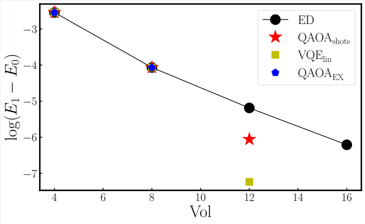
$$[\mathcal{H}, {G^a}_x] = 0$$

Physical states are gauge invariant: $G_x|\psi\rangle = 0$

Non-Abelian LGTs: low-energy states

- Variational algorithms can be used to find ground states.
- SSB of discrete symmetry $\rightarrow$ (almost) degenerate GS.
- How good are quantum algorithms to extract such states?
- Matter-free $SO(3)$ quantum link model in $d = 2 + 1$.
- Results from classical simulators for E_0 and E_1:

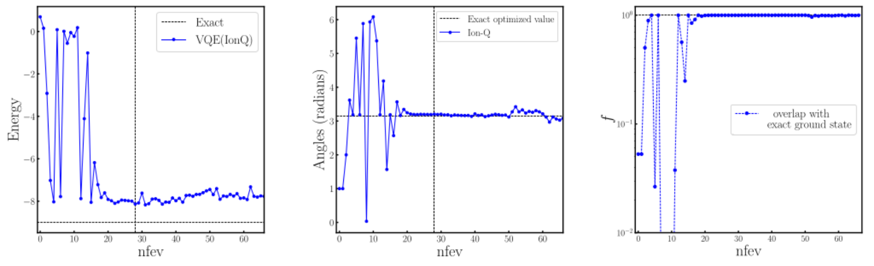




Maiti, DB, Chakraborty, Huffman, (Phys Rev Res, 2025).

Results from IonQ Hardware: SO(3) QLM

Energy extraction using gauge-invariant parameterization of the wave-function gives promising results in a IonQ hardware.



Tests on larger systems in progress.

Maiti, DB, Chakraborty, Huffman, (Phys Rev Res, 2025).

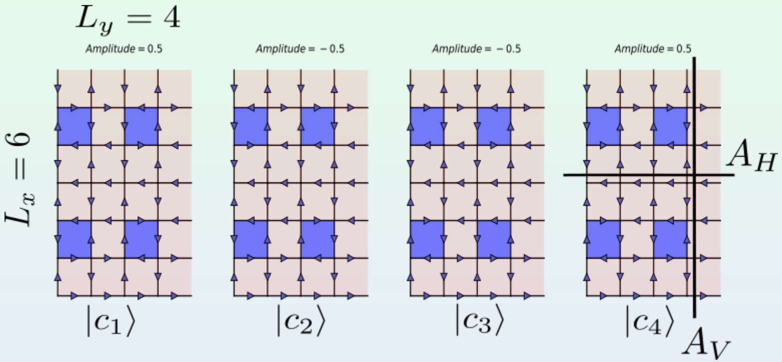
Overview

- QLMs exhibit rich physical phenomena in particle physics.
- Particle physics applications towards chiral symmetry breaking, dynamics of confinement, dense phases.
- Applications to frustrated magnetism and high Tc superconductors are relevant for condensed matter physics.
- Gauge-invariant states are useful for both algorithmic developments as well as quantum simulators.
- QLMs very useful for quantum computing applications.

THANK YOU FOR YOUR ATTENTION

Lego Scars in QDM

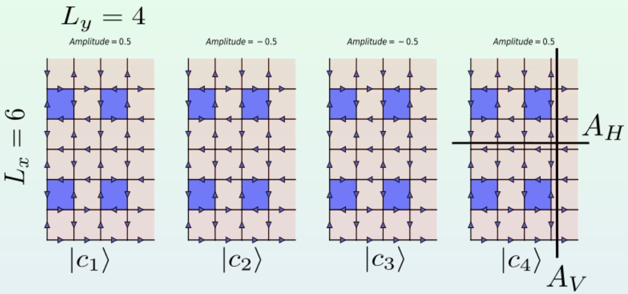
Certain scars in the QDM are exceptionally simple: Lego scars.



- $|\Psi_{\text{QMBS}}\rangle = \frac{1}{2}(|c_1\rangle - |c_2\rangle - |c_3\rangle + |c_4\rangle)$;
- $(\mathcal{O}_{\text{kin}}, \mathcal{O}_{\text{pot}}) = (0, 4)$; $\quad S_{L/2;A_H} = 0$; $\quad S_{L/2;A_V} = 2\ln(2)$.
- Can be written as a tensor product state: $|\Psi_{\text{QMBS}}\rangle = |\mathcal{L}_1\rangle \otimes |\mathcal{L}_2\rangle$

Quantum Caging

Certain scars in the QDM are exceptionally simple: Lego scars.



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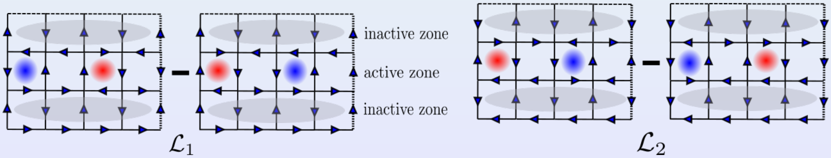
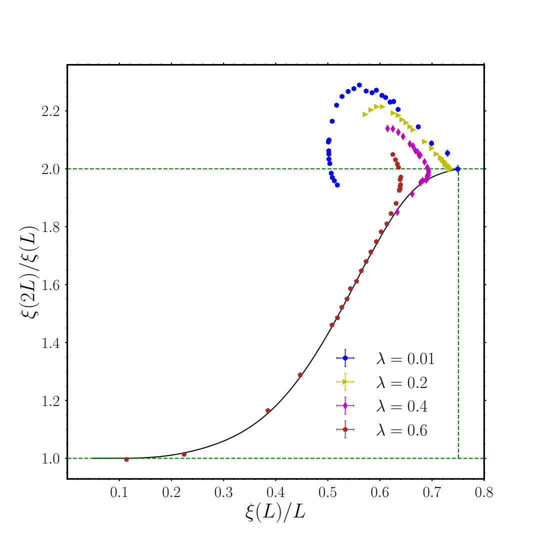


Biswas, DB, Sen (SciPost Physics, 2022).

Step Scaling Functions of XY model



Qubit theory reproduces the universal SSF.

Maiti, DB, Chandrasekharan, Marinkovic (PRL, 2024)