Machine learning for Lattice gauge theories

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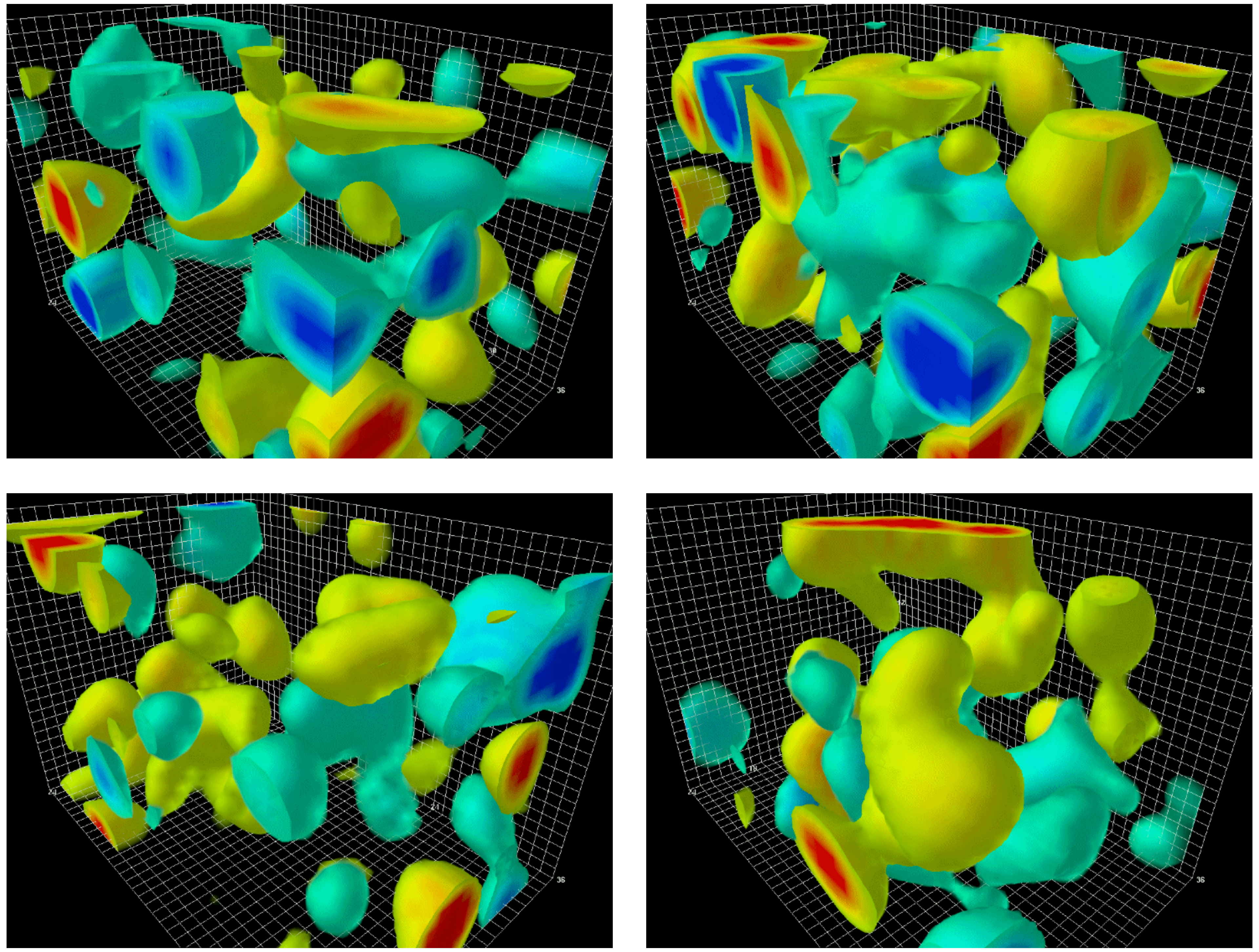
Lattice simulations

High-dimensional path integral over degrees of freedom assigned to points and edges of a lattice

- Boltzmann weight $e^{-S(\phi)}$ encodes distribution over "typical" configurations

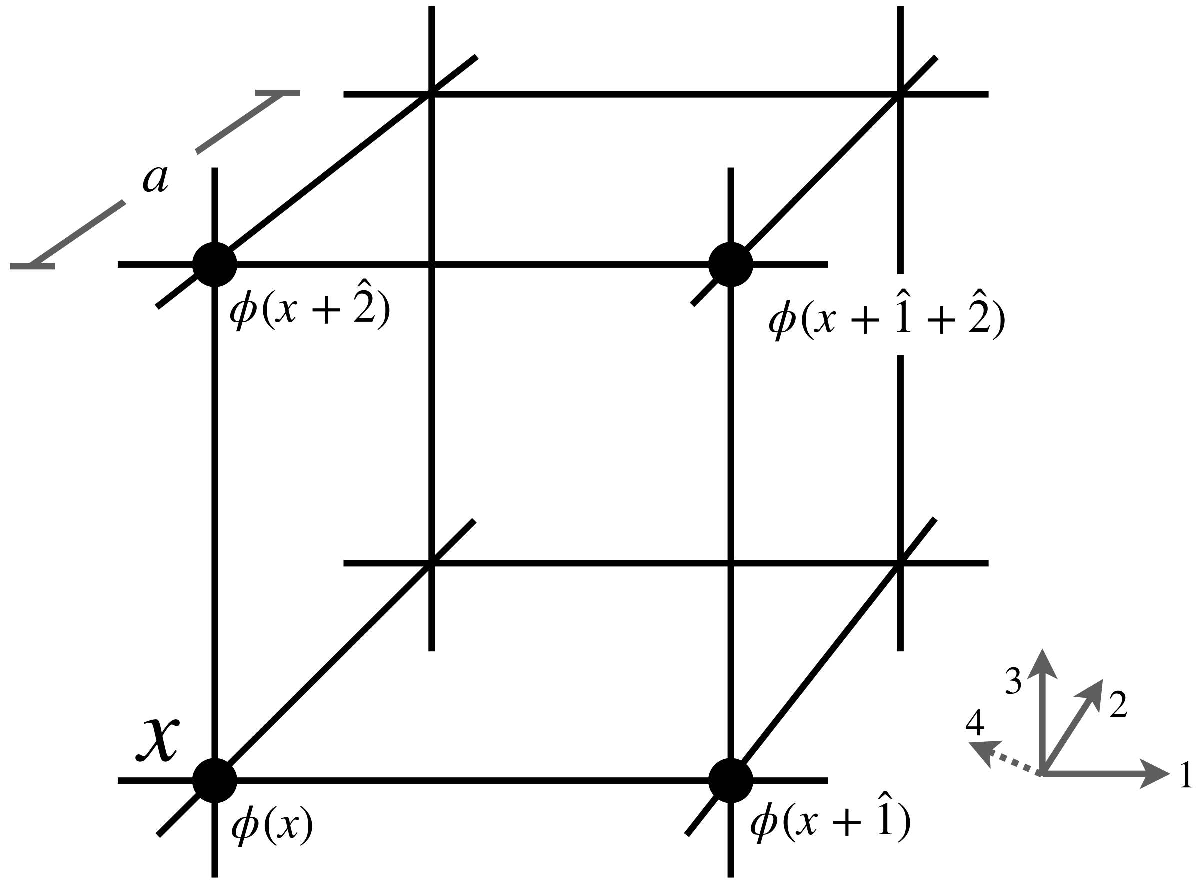


Partition function

$$Z \equiv \left[\prod_x \int_{-\infty}^{\infty} d\phi(x)\right] e^{-S(\phi)}$$

Thermal expt. value of operator $\mathcal{O}$

$$\langle \mathcal{O} \rangle = \left[\prod_x \int_{-\infty}^{\infty} d\phi(x)\right] \mathcal{O}(\phi)\, e^{-S(\phi)}/Z$$



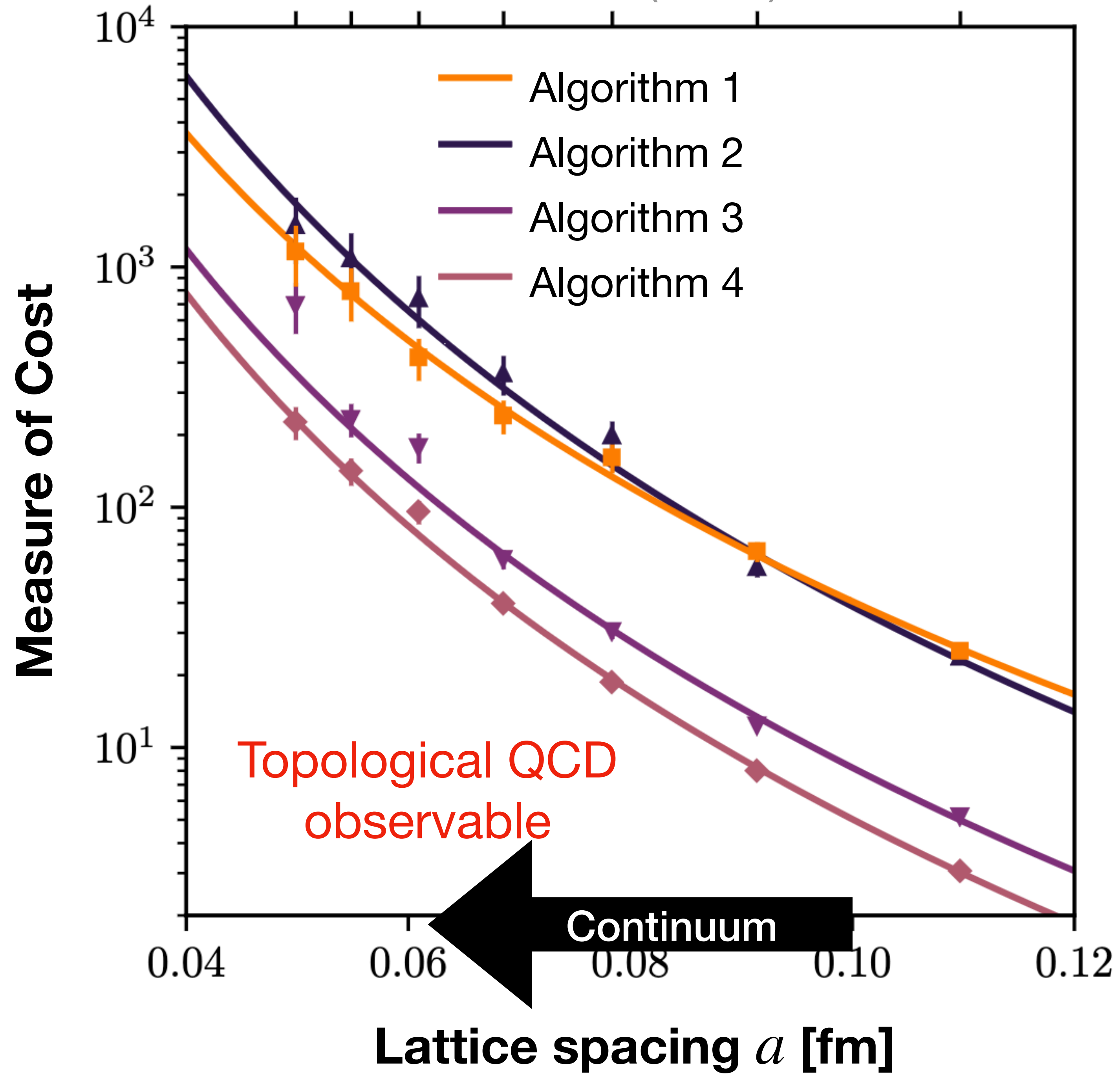
Why machine learning?

State-of-the-art LGT calculations require **enormous computational cost.**

- $\gtrsim 10^9$ degrees of freedom
- "Critical slowing down" as $a \to 0$
- Costly matrix inversion for propagators $\langle \psi \bar{\psi} \rangle$ (especially as $m_q \to 0$)

This **limits the precision of physics results** (challenging uncertainties from $a \to 0$, $m_\pi \to \sim 140\text{MeV}$, and $V \to \infty$ limits!)

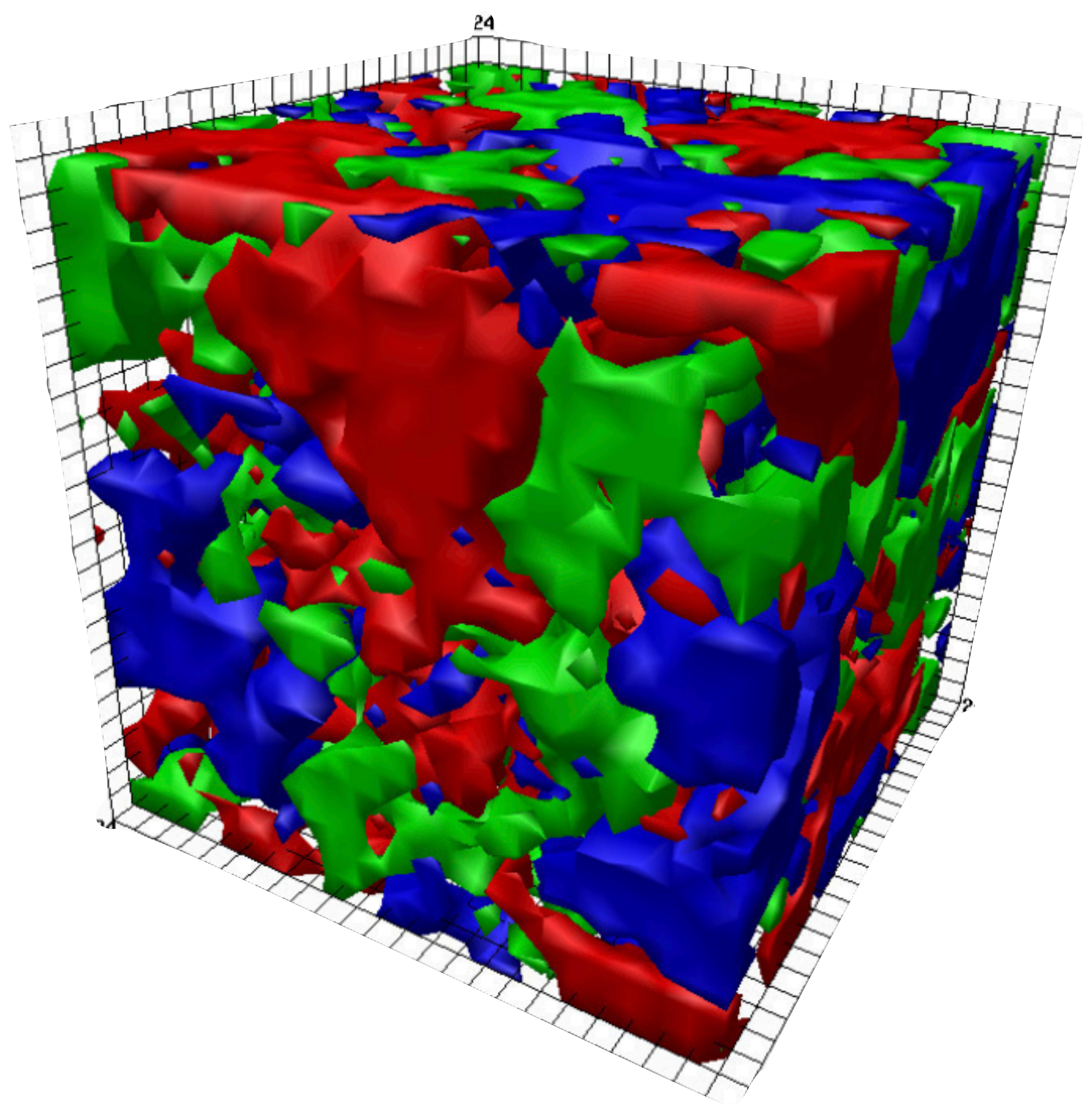
Eichhorn, et al. (2023) 2307.04742



Why machine learning?

Lattice field theories may be well-suited for application of ML

- Problem involving **lots** of well-structured data
- Analytically-known target
- Generative models with exactness now exist



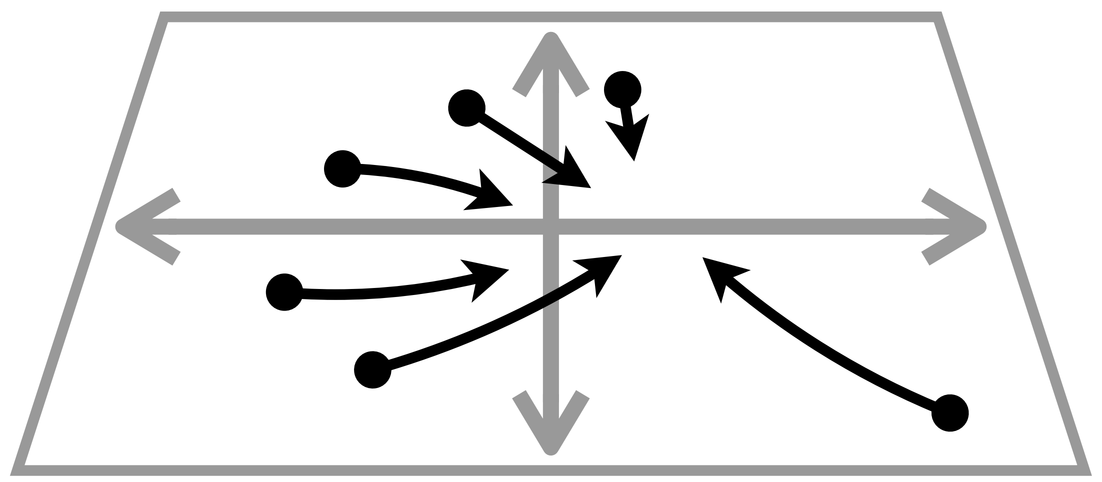
Stokes, Kamleh, Leinweber 1312.0991

Some applications of ML

Two major components to a lattice calculation.
Ongoing efforts to apply ML to both of these.

See e.g. Boyda, et al.
Snowmass 2022, 2202.05838

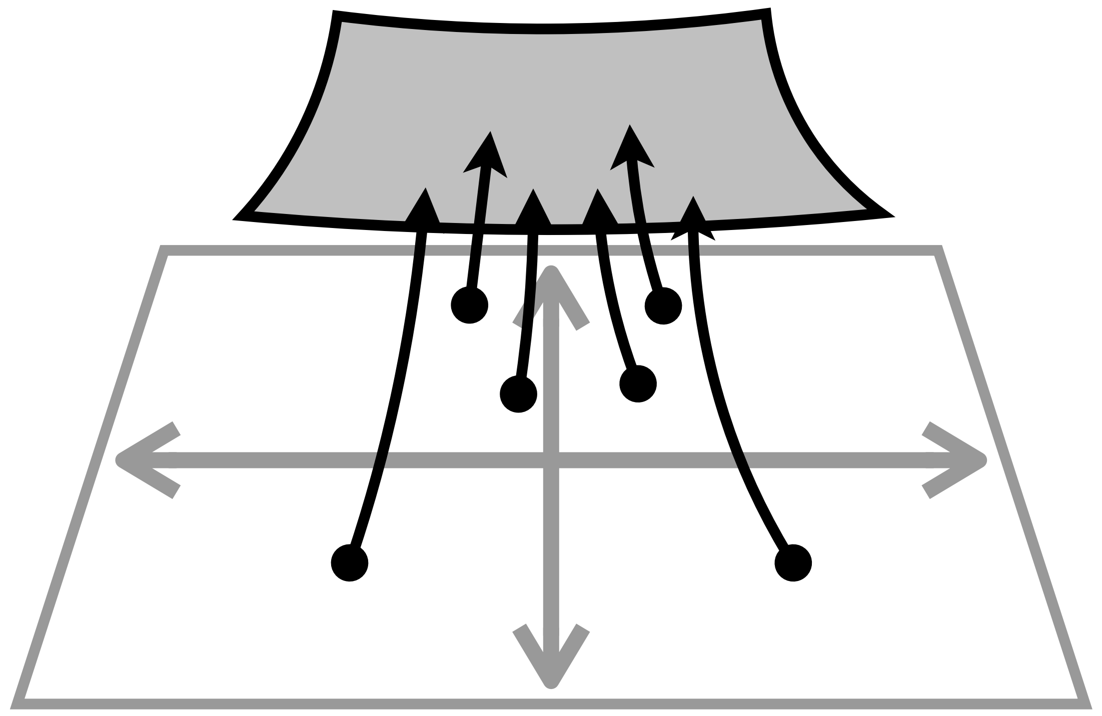
1. Ensemble generation



Normalizing flow models

- PRD100 (2019) 034515, 2101.08176, 2107.00734
- PRL125 (2020) 121601, ICML (2020) 2002.02428, PRD103 (2021) 074504, 2305.02402
- PRD104 (2021) 114507, PRD106 (2022) 014514, PRD106 (2022) 074506, PoSLATTICE (2022) 036
- 2211.07541, 2401.10874, 2404.10819, 2404.11674

2. Observable measurements & analysis



Learned contour deformations

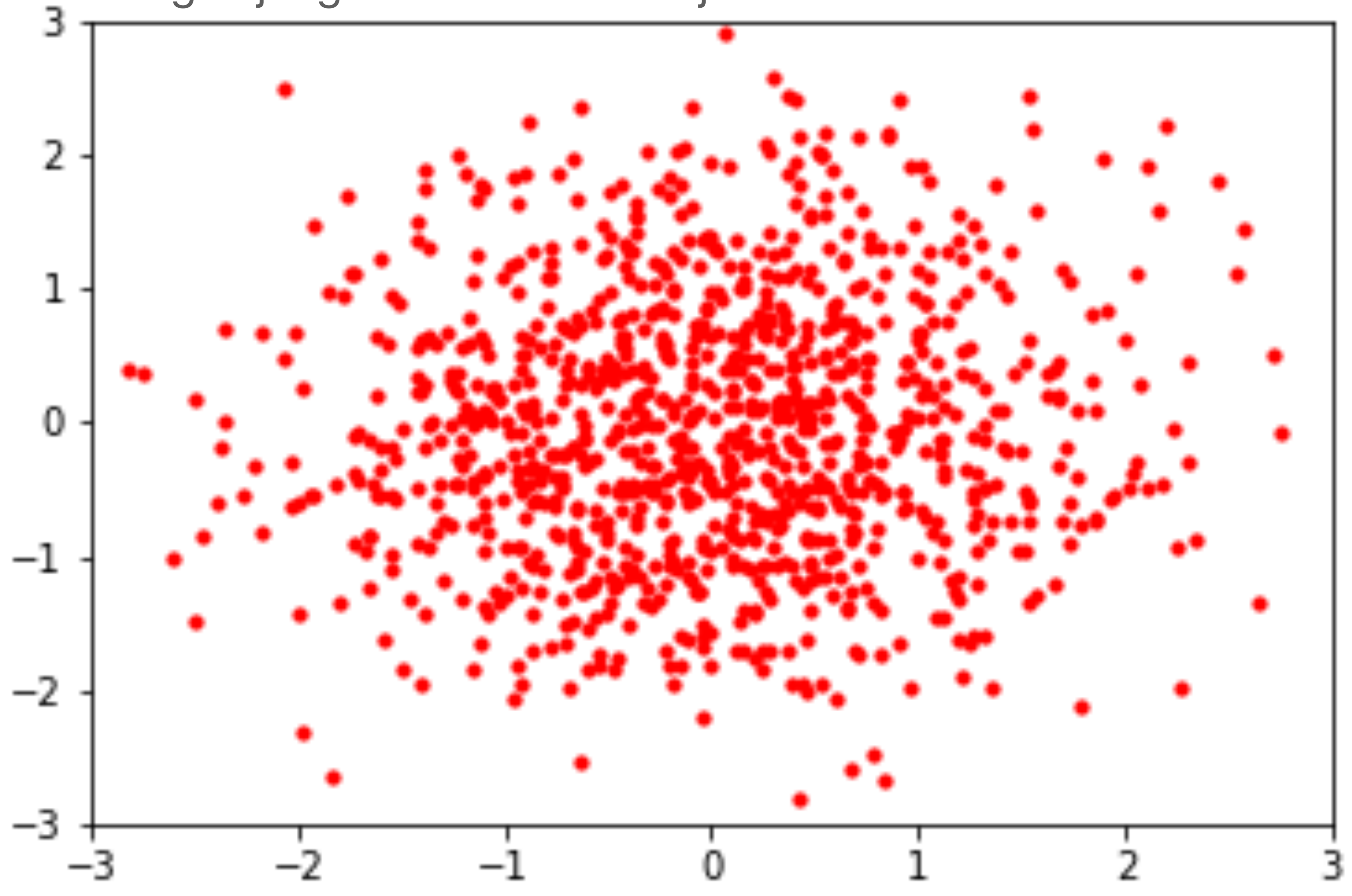
- PRD98 (2018) 074511, PoS LATTICE2018 176
- PRD102 (2020) 014514
- PRD103 (2021) 094517
- 2309.00600, NeurIPS ML4PS (2023)

Normalizing flow models

Tabak & Vanden-Eijnden CMS8 (2010) 217
Tabak & Turner CPA66 (2013) 145

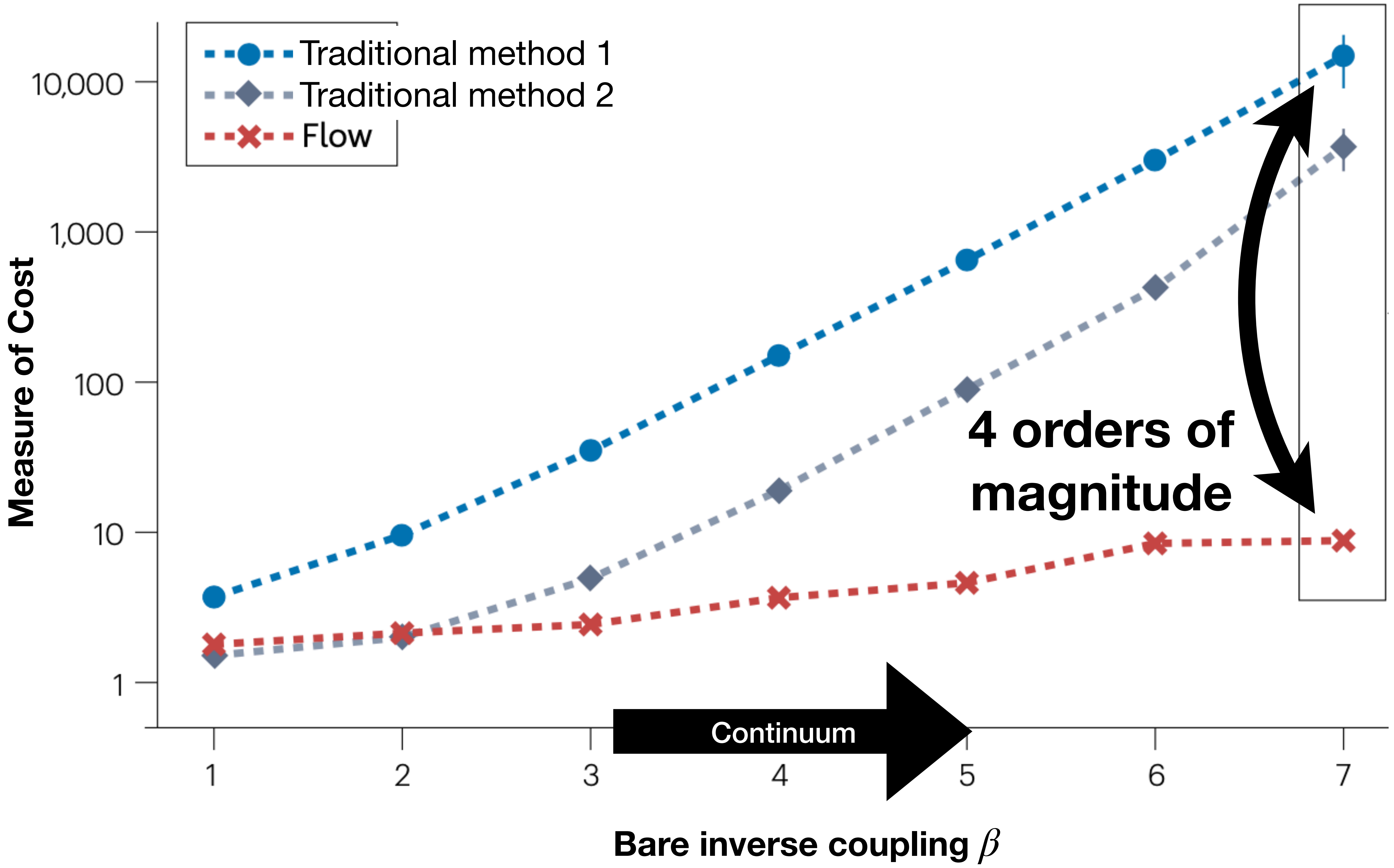
- Sample from "easy" prior density $r(\xi)$
- Apply parametrized diffeomorphism f (the "flow")
- Output samples follow new "model density" $q(\phi) = r(\xi)\,\det|\partial f(\xi)/\partial\xi|^{-1}$
- Flow f can be **learned** to match target density!

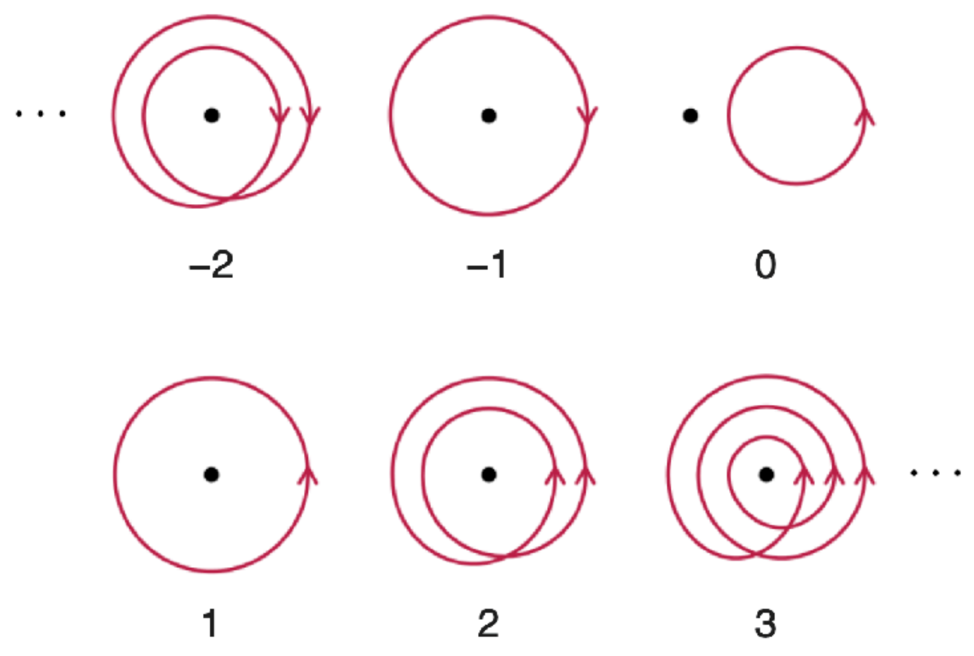
blog.evjang.com/2019/07/nf-jax.html



Simple prior distribution $r(\xi)$ → Complex model distribution $q(\phi)$

Example of lattice sampling success





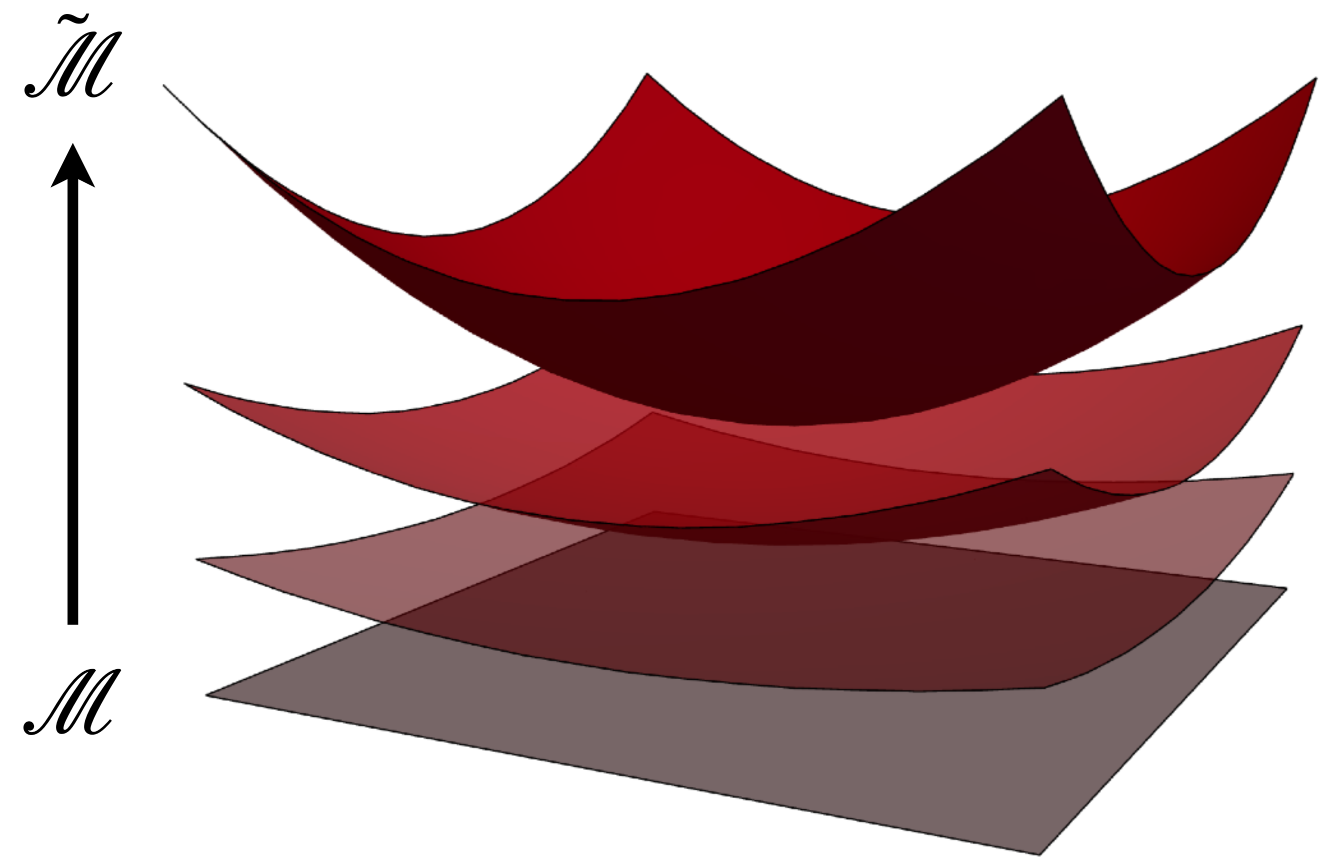
Cost of MCMC vastly reduced due to better topological mixing.

Cranmer, GK, Racanière, Rezende, Shanahan Nature Reviews Physics 5 (2023) 526

Integral deformations for noisy observables

Lattice integrands are often holomorphic, allowing the integration contour to be deformed without bias.

$$\langle \mathcal{O} \rangle = \frac{1}{Z} \int_{\mathcal{M}} e^{-S(\phi)} \mathcal{O}(\phi) = \frac{1}{Z} \int_{\tilde{\mathcal{M}}} e^{-S(\tilde{\phi})} \mathcal{O}(\tilde{\phi})$$



- Defines a **modified observable**, which may have improved variance:

$$\mathcal{Q}(\phi) \equiv \det J(\phi) e^{-[S(\tilde{\phi}(\phi)) - S(\phi)]} \mathcal{O}(\tilde{\phi}(\phi))$$

$$\langle \mathcal{Q}(\phi) \rangle = \langle \mathcal{O}(\phi) \rangle$$

$$\mathrm{Var}[\mathcal{Q}(\phi)] \neq \mathrm{Var}[\mathcal{O}(\phi)]$$

Detmold, GK, Wagman, Warrington PRD102 (2020) 014514

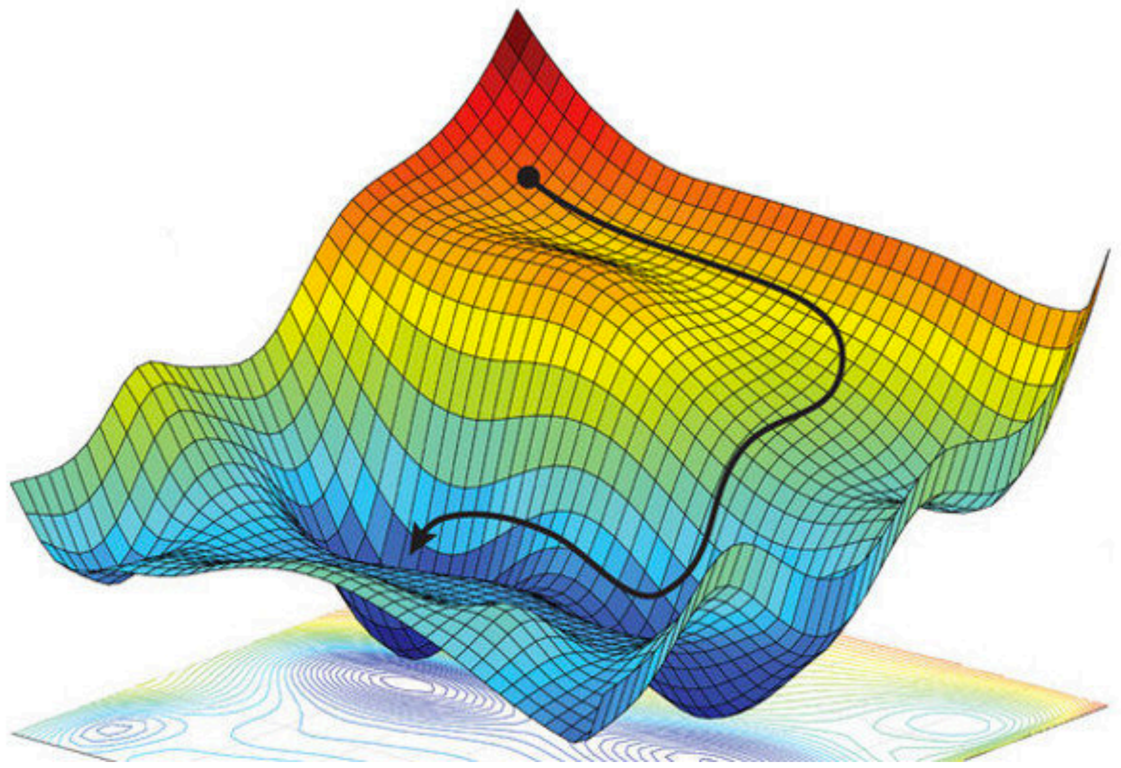
[Image credit: Neill Warrington]

Learning the integration contour

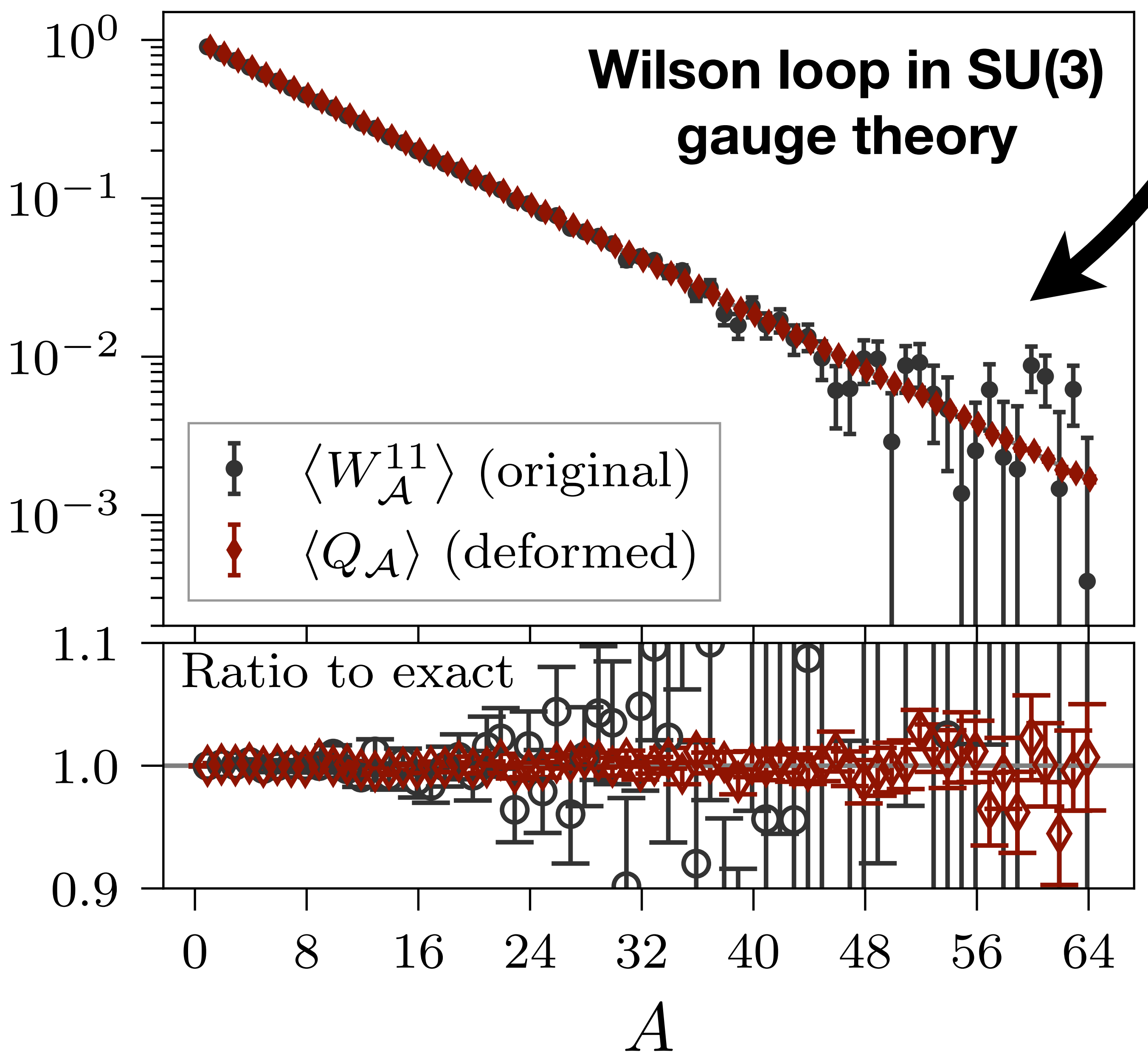
The choice of $f : \phi \mapsto \tilde{\phi}$ defines $\tilde{\mathcal{M}}$, $\mathcal{Q}(\phi)$, and the variance.

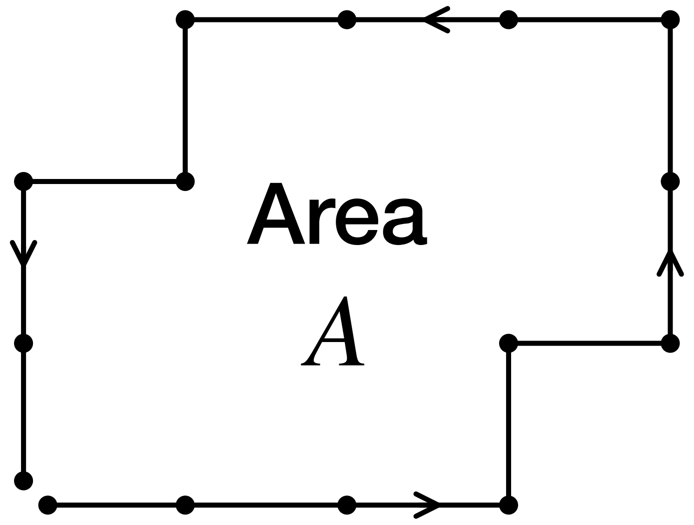
Parameterize $f(\phi; \omega)$ then **minimize variance**.

- Caveat: Complex analyticity
- Caveat: $SU(N)$ variables



[Image credit: 1805.04829]





Detmold, GK, Wagman, Warrington PRD102 (2020) 014514,
Detmold, GK, Lamm, Wagman, Warrington PRD103 (2021) 094517