DEFINING THE ENTIRE SET OF CELESTIAL MARGINAL OPERATORS

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BROAD MOTIVATION

The duality between gauge and gravity theories in 4D asymptotically flat space and a 2D celestial conformal field theory has allowed us to learn a lot about aspects of the boundary theory.

One of the primary goals was to use 2D CFT techniques to better our understanding of the bulk theories.

To do this we need some kind of intrinsic definition of the boundary theory.

Can understanding the space of marginal deformations help us do this?

WHAT ARE MARGINAL OPERATORS

Generally, there are three types of operators in a CFT: (for d=2)

Relevant: $\Delta < 2$

Irrelevant: $\Delta > 2$

Marginal: $\Delta = 2$

Preserve conformality

Deformations by these control the renormalization group flow

If there are operators that we are allowed to add to the boundary theory that do not change it from a CFT, then they must be of relevance in the bulk!

OPE of marginal operators

$$\mathcal{M}_I(x)\mathcal{M}_J(y) \sim \underbrace{\frac{g_{IJ}}{|x-y|^4}}_{\text{Metric}} + \overbrace{\Gamma^K_{IJ}\mathcal{M}_K(y)\delta^{(2)}(x-y)}^{\text{Connections}} + \cdots$$

Curvature, etc..

(D. Kutasov, 1989)

MARGINALITY AND CELESTIAL SYMMETRIES

Initially we studied this problem in the context of the non-linear sigma model where the marginal particles were identified with soft scalars (D. Kapec, Y.A.Law, SN, 2022)

Scalars are already spin-0, so one only needed to find a way to make them dimension $\Delta = 2$

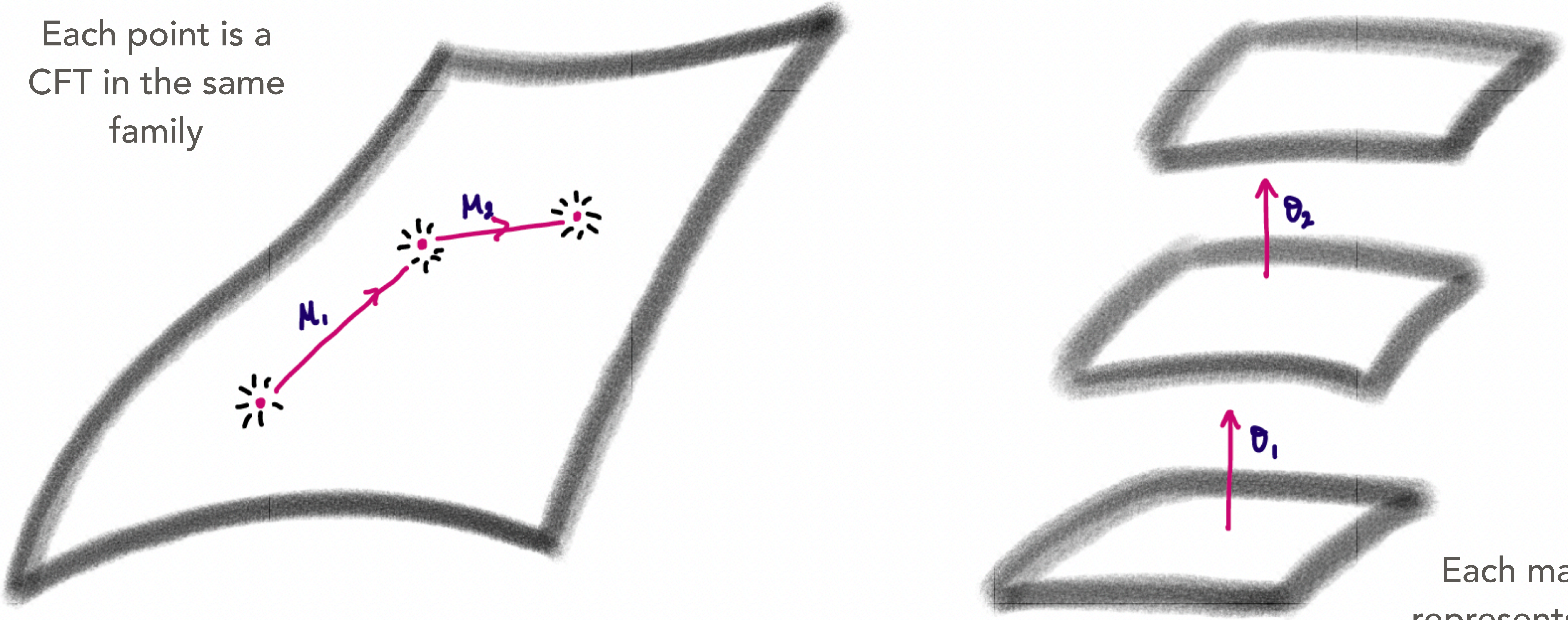
We were able to study the four point functions of these operators to understand aspects of the curvature of the manifold

Then Kapec studied what happens in the case of gluons and gravitons where one identifies the marginal operators as the shadow of soft operators (D. Kapec, 2022)

These are **quasi-marginal** because they are not spin-0!

MARGINAL VS. QUASI-MARGINAL

Each point is a CFT in the same family



Each manifold represents a copy of the conformal group

POSSIBLE SINGLE-PARTICLE MARGINAL OPERATORS

So, other than scalars, can we construct other marginal operators?

We need to shift their dimension and spin. We need transformations that preserve the Casimir

$$C_2(\Delta, J) = \Delta(\Delta - 2) + J^2 = 2\left(h\left(h-1\right) + \bar{h}\left(\bar{h}-1\right)\right)$$

Transform	(Δ', J')	$(h', \bar{h}')$	Order	Comparison to [24]
$\mathbb{I}$	(Δ, J)	$(h, \bar{h})$	1	1 (Identity)
$\mathcal{L}^+$	$(1-J, 1-\Delta)$	$(1-h, \bar{h})$	2	L (Light)
$\mathcal{L}^-$	$(J+1, \Delta-1)$	$(h, 1-\bar{h})$	2	F (Floodlight)
$\mathcal{S} \equiv \mathcal{L}^+\mathcal{L}^-$	$(2-\Delta, -J)$	$(1-h, 1-\bar{h})$	2	S (Full Shadow)
$\mathcal{P}$	$(\Delta, -J)$	$(\bar{h}, h)$	2	S_J (Spin-Shadow)
$\mathcal{P}\mathcal{L}^+\mathcal{L}^- \equiv \mathcal{P}\mathcal{S}$	$(2-\Delta, J)$	$(1-\bar{h}, 1-h)$	2	S_Δ (Euclidean Shadow)
$\mathcal{P}\mathcal{L}^+$	$(1-J, \Delta-1)$	$(\bar{h}, 1-h)$	4	R
$\mathcal{P}\mathcal{L}^-$	$(J+1, 1-\Delta)$	$(1-\bar{h}, h)$	4	$\overline{\mathrm{R}}$

(P. Kravchuk, D. Simmons-Duffin 2018)

(SN 2024)

MARGINALITY FROM GLUONS

For gluons, they have spin $J = \pm 1$ and dimension Δ

Transform	(Δ', J')	$(h', \bar{h}')$	Order	Comparison to [24]
$\mathbb{I}$	(Δ, J)	$(h, \bar{h})$	1	1 (Identity)
$\mathcal{L}^+$	$(1-J, 1-\Delta)$	$(1-h, \bar{h})$	2	L (Light)
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$\mathcal{P}\mathcal{L}^-$	$(J+1, 1-\Delta)$	$(1-\bar{h}, h)$	4	$\overline{\mathrm{R}}$

So, we need to light transform them!

$$\mathcal{M}_{\mathcal{L}^-}(z,\bar{z}) \equiv -\frac{\alpha_+}{\sqrt{2}} T^a \mathcal{L}^- \left[R^{+,a}\right] = \frac{\alpha_+ T^a}{\sqrt{2\pi}} \int \frac{d\bar{w}}{(\bar{z}-\bar{w})^2} R^{+,a}(z,\bar{w}).$$

$$\mathcal{M}_{\mathcal{L}^+}(z,\bar{z}) \equiv \frac{\alpha_-}{\sqrt{2}} T^a \mathcal{L}^+ \left[R^{-,a}\right] = \frac{\alpha_- T^a}{\sqrt{2\pi}} \int \frac{dw}{(z-w)^2} R^{-,a}(w,\bar{z}).$$

We can compute the OPEs from three-point functions.

(SN 2024)

MARGINAL GLUON OPE

Using the three point function of the light transformed operators we obtain the following OPE

$$\mathcal{M}_{\mathcal{L}^-}(z,\bar{z})\mathcal{M}_J(w,\bar{w}) \sim \frac{(N^2-1)\delta_{J\mathcal{L}^+}}{2|z-w|^4} - \frac{N(N^2-1)}{4\sqrt{2\pi}}\delta_{J\mathcal{L}^-}\mathcal{M}_{\mathcal{L}^-}(w,\bar{w})\delta^{(2)}(z-w)$$

$$\mathcal{M}_{\mathcal{L}^+}(z,\bar{z})\mathcal{M}_J(w,\bar{w}) \sim \frac{(N^2-1)\delta_{J\mathcal{L}^-}}{2|z-w|^4} - \frac{N(N^2-1)}{4\sqrt{2\pi}}\delta_{J\mathcal{L}^+}\mathcal{M}_{\mathcal{L}^+}(w,\bar{w})\delta^{(2)}(z-w)$$

$$\mathcal{M}_I(x)\mathcal{M}_J(y) \sim \frac{g_{IJ}}{|x-y|^4} + \Gamma^K_{IJ}\mathcal{M}_K(y)\delta^{(2)}(x-y) + \cdots$$

We can read off the associated metric components and the connections which are non-zero

(SN 2024)

THE PROBLEM WITH GRAVITONS

Gravitons have spin $J = \pm 2$ and dimension Δ

Transform	(Δ', J')	$(h', \bar{h}')$	Order	Comparison to [24]
$\mathbb{I}$	(Δ, J)	$(h, \bar{h})$	1	1 (Identity)
$\mathcal{L}^+$	$(1-J, 1-\Delta)$	$(1-h, \bar{h})$	2	L (Light)
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$\mathcal{P}\mathcal{L}^+$	$(1-J, \Delta-1)$	$(\bar{h}, 1-h)$	4	R
$\mathcal{P}\mathcal{L}^-$	$(J+1, 1-\Delta)$	$(1-\bar{h}, h)$	4	$\overline{\mathrm{R}}$

None of the allowed transformations will let us get a marginal operator from a single graviton!

MULTI-PARTICLE OPERATORS

However, it is also possible to construct marginal operators as multi-particle operators

For example if O^+, O^- label positive and negative helicity spin-p particles, $\mathscr{M} = O^+O^-$ will be spin-0 and is a candidate for a marginal operator

This opens a whole realm of operators that we have not considered yet!

Computing the OPEs of multi-particle operators from amplitudes is a harder task than it is for single-particle operators.

(A. Guevara, Y. Hu, S. Pasterski, 2024)

OPE OF GLUON MULTI PARTICLE OPERATORS

We define the following operator

$$\mathcal{M}^{ab}_{\Delta_1+\Delta_2}(w,\bar{w}) =: \mathcal{O}^a_{\Delta_1,-}\mathcal{O}^b_{\Delta_2,+} : (w,\bar{w}) \equiv \oint_w \frac{dz}{2\pi i}\frac{1}{z-w}\oint_{\bar{w}}\frac{d\bar{z}}{2\pi i}\frac{1}{\bar{z}-\bar{w}}\mathcal{O}^a_{\Delta_1,-}(z,\bar{z})\mathcal{O}^b_{\Delta_2,+}(w,\bar{w}).$$

Given that for gluons

$$\begin{aligned}
\mathcal{O}^{a,-}_1(z_1,\bar{z}_1)\,\mathcal{O}^{b,-}_2(z_2,\bar{z}_2) &\sim \frac{\mathcal{F}^{a^-b^-}{}_{c^-}}{\bar{z}_{12}}\,\mathcal{O}^{c,-}_q(z_2,\bar{z}_2) + \ :\mathcal{O}^{a,-}_1\mathcal{O}^{b,-}_2:(z_2,\bar{z}_2) + \cdots \\
\mathcal{O}^{a,-}_1(z_1,\bar{z}_1)\,\mathcal{O}^{b,+}_2(z_2,\bar{z}_2) &\sim \frac{\mathcal{F}^{a^-b^+}{}_{c^-}}{z_{12}}\,\mathcal{O}^{c,-}_q(z_2,\bar{z}_2) + \frac{\mathcal{F}^{a^-b^+}{}_{c^+}}{\bar{z}_{12}}\,\mathcal{O}^{c,+}_q(z_2,\bar{z}_2) \\
&\quad + \ :\mathcal{O}^{a,-}_1\mathcal{O}^{b,+}_2:(z_2,\bar{z}_2) + \cdots \\
\mathcal{O}^{a,+}_1(z_1,\bar{z}_1)\,\mathcal{O}^{b,+}_2(z_2,\bar{z}_2) &\sim \frac{\mathcal{F}^{a^+b^+}{}_{c^+}}{z_{12}}\,\mathcal{O}^{c,+}_q(z_2,\bar{z}_2) + \ :\mathcal{O}^{a,+}_1\mathcal{O}^{b,+}_2:(z_2,\bar{z}_2) + \cdots
\end{aligned}$$

(A. Guevara, Y. Hu, S. Pasterski, 2024)

We find that the OPE of these operators is going to go like

$$\frac{1}{(z-w)^2}$$

with no delta function terms, which is not what we expect for usual marginal operators!

(M. Imseis, SN, A.W. Peet, WIP)

WHAT ABOUT FOR GRAVITY?

$$\mathcal{M}^{ab}_{\Delta_1+\Delta_2}(w,\bar{w}) =: \mathcal{O}^a_{\Delta_1,-}\mathcal{O}^b_{\Delta_2,+} : (w,\bar{w}) \equiv \oint_w \frac{dz}{2\pi i}\frac{1}{z-w}\oint_{\bar{w}} \frac{d\bar{z}}{2\pi i}\frac{1}{\bar{z}-\bar{w}}\mathcal{O}^a_{\Delta_1,-}(z,\bar{z})\mathcal{O}^b_{\Delta_2,+}(w,\bar{w}).$$

Ideally we would like to do this construction in the case of gravitons where we could not write down single particle marginal operators

The graviton-graviton OPE is non-singular (assuming $z, \bar{z}$ complex conjugates) and therefore Wick's theorem simply gives 0 for the OPE

In this case, we may need to construct more complex composite operators that potentially involve derivatives of the operators and then a linear combination to make them primaries…

(M. Imseis, SN, A.W. Peet, WIP)

THE BIG QUESTION: WHAT IS THE MANIFOLD?

While it seems that there is a much larger set of operators that one can deform by that are strictly marginal, it is not clear what physics lies in the structure of the manifold

Is there an explanation via symmetries?

Are there infinitely many directions?

Does it have anything to do with asymptotic structure?