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Memory of Robinson-Trautman waves

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Plan of the talk

1. Robinson-Trautman waves
2. Memory in asymptotically flat spacetimes
3. Making RT waves asymptotically flat
4. Memory of RT waves
5. Application & discussion

in collaboration with $\boxed{\text{Ali Seraj}}$

1 RT (1960) spherical waves

$$ds^2 = -2U\,du^2 + 2\,du\,dr + r^2\,ds_2^2\,,$$

conf. flat 2d metric $\quad ds_2^2 = -\frac{2}{P^2}\,d\zeta\,d\bar{\zeta}$

$P(u,\zeta,\bar{\zeta}) = e^{-\varphi}$ $\quad \boxed{\partial_u \varphi = -\frac{1}{12M}\,\Delta_2^2\,\varphi}$ $\quad U = -r\,\partial_u\varphi - \frac{1}{4}R_2 + \frac{M}{r}$

Tod (1989) 2d Calabi flow $\quad \partial_u g_2 = \frac{1}{12M}\,\Delta_2 R_2\, g_2$

Calabi (1983)

(1a) NP tetrad $\ell = \partial_r$ $\quad n = \partial_u + U\partial_r$ $\quad m = r^{-1} P \bar{\partial}$ not round enough

m: covariant derivative $\quad \eth \eta^s = P^{1-s} \bar{\partial}(P^s \eta^s)$ $\quad \Delta_2 \eta^0 = 2 \eth \bar{\eth} \eta^0$

not flat enough $\quad \gamma = \frac{1}{2} \partial_u \varphi + \mathcal{O}(r^{-1})$, $\quad \nu = \bar{\eth} \partial_u \varphi + \mathcal{O}(r^{-1})$

(1b) algebraically special $\psi_0 = \psi_1 = 0$

ℓ: repeated null direction, generator of null geodesic congruence

twist-free $\rho = \bar{\rho}$ $\quad$ shear-free $\sigma = 0$ $\quad$ expansion $-\mathrm{Re}\,\rho = \frac{1}{r}$

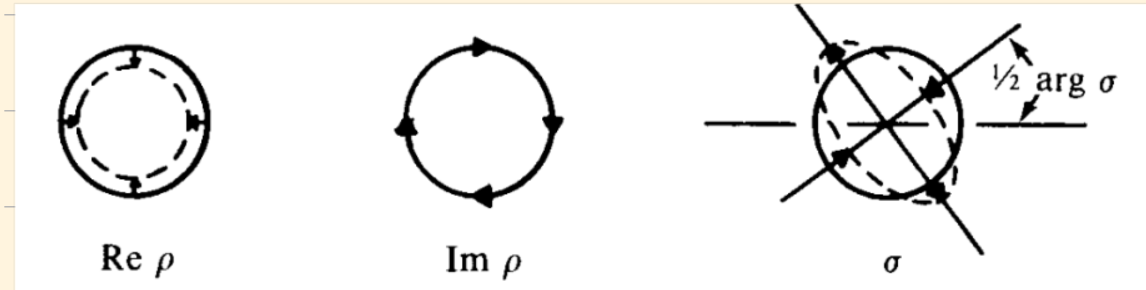


FIG. 1. The geometrical interpretation of the optical scalars in terms of the effect of propagation of a small circle perpendicular to the beam.

Sachs (1961)

Chandrasekhar (1983)

1c trivial constant solution S^2 $\quad P_0 = \frac{1}{\sqrt{2}}(1+\zeta\bar{\zeta})$ $\quad \varphi_0 = -\ln P_0$

space-time solution Schwarzschild black hole

deviation $\Phi = \varphi - \varphi_0$

$$-3M e^{4\Phi} J_u \Phi = \left[(f_0 \bar{f}_0)^2 + (f_0 \bar{f}_0) \right] \Phi$$

$$-2 f_0 \Phi \bar{f}_0 \Phi - 2 \bar{f}_0 \Phi f_0 \bar{f}_0 f_0 \Phi - 2 f_0 \Phi \bar{f}_0 f_0 \bar{f}_0 \Phi - 2 (f_0 \bar{f}_0 \Phi)^2 + 4 f_0 \Phi \bar{f}_0 \Phi f_0 \bar{f}_0 \Phi$$

linearization $\quad \Phi = \epsilon \Phi_1$

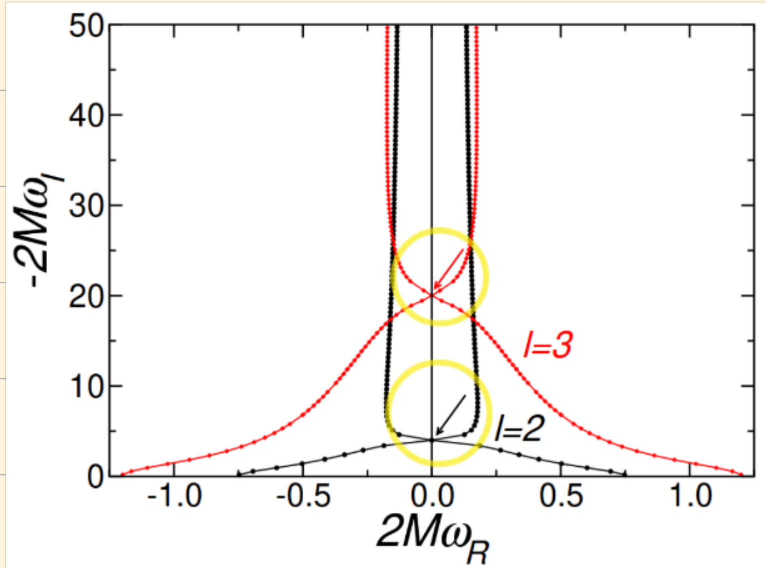
$$-3M J_u \Phi_1 = \left[(f_0 \bar{f}_0)^2 + (f_0 \bar{f}_0) \right] \Phi_1$$

$$\Phi_1 = a^{jm}(u)\ {}_0Y_{jm} + c.c.$$

$$a^{jm}(u) = c^{jm}, \quad j \leq 1 \qquad a^{jm}(u) = c^{jm} e^{-\frac{\omega_j}{M} u}, \quad j > 1$$

algebraically special quasi-normal modes

$$\boxed{\omega_j = \frac{(j-1)j(j+1)(j+2)}{12}}$$



Couch & Newman (1973)

Chandrasekhar (1984)

Qi & Schutz (1993)

Berti et al. (2009)

2 asymptotically flat : $\rho' = \bar{\rho}'$ $\epsilon' = \pi' = \kappa' = 0$ $\tau' = \bar{\alpha}' + \beta'$

$\exists$ Bondi frame $l' = \partial_{r'}$ $n' = \partial_{u'} - \frac{1}{2}\partial_{r'} + \mathcal{O}(r'^{-1})$

coordinate conditions $m' = r'^{-1} P_0 \bar{\eth}' + \mathcal{O}(r'^{-2})$

r' dependence $D'\rho' = \rho'^2 + \sigma'\bar{\sigma}'$ $D'\sigma' = 2\rho'\sigma' + \Psi_0'$

$\Psi_0' = \mathcal{O}(r'^{-5})$ $-\rho' = +\frac{1}{r'} + \mathcal{O}(r'^{-3})$ $\sigma' = \sigma'^0 r'^{-2} + \mathcal{O}(r'^{-2})$

incoming radiation expansion shear

news $\lambda'^0 = \partial_{u'} \bar{\sigma}'^0$ $\psi_3'^0 = -\eth_0' \lambda'^0$ $\psi_4'^0 = -\partial_{u'} \bar{\lambda}'^0$

evolution $\partial_{u'} \psi_2'^0 = \eth_0' \psi_3'^0 + \sigma'^0 \psi_4'^0$

(non-linear) memory   Christodoulou (1991) [Frauendiener (1992)]

$$\partial_{u'} t_0^{\prime 2} \bar\sigma'^{0} = -\partial_{u'}\left[\psi_2'^{0} + \sigma'^{0} \bar\lambda'^{0}\right] + \boxed{\lambda'^{0} \bar\lambda'^{0}} \qquad (x)$$

Bondi mass aspect $m'_B = -\frac{1}{2}\left(\psi_2'^{0} + \sigma'^{0}\lambda'^{0} + c.c.\right)$

$\int_{S^2} (x) \cdot {}_0\bar Y_{jm}$ → $j \leq 1$ conservation laws

$\int_{S^2} (x) \cdot {}_0\bar Y_{jm}$ → $j \geq 2$ $\frac{1}{2}\left(\eth' \bar\sigma'^{0}_{jm} + c.c.\right) = \Delta\left(t_0^{-2} m'_B\right)_{jm} + \int_{u'_i}^{u'_f} du \left(t_0^{-2} |\lambda'^{0}|^2\right)_{jm}$

generalized mass aspect $m'_G = -\left(\psi_2'^{0} + \sigma'^{0}\lambda'^{0} + t_0^{2}\,\bar\sigma'^{0}\right) = \overline{m'_G}$

(x) $\boxed{\partial_{u'} m'_G = -|\lambda'^{0}|^2}$ mass-loss for generalized aspect

③ making RT flat • generalized solution space $P(u,\zeta,\bar\zeta)$

• compute symmetry group BMS + (complex) Weyl trsf

• work out action on solution space G. B. & Troessaert (2016)

subtleties combined coord. trsf + frame rotation, 2nd order

$x^\mu = x^\mu(x'^\nu) \quad e_a'^{\,\nu} \dfrac{\partial x^\mu}{\partial x'^\nu} = \Lambda_a{}^b e_b{}^\mu$

finite Weyl trsf $P(u,\zeta,\bar\zeta) \to P_0(\zeta',\bar\zeta')$

Bondi time $u = u_0(u',\zeta',\bar\zeta') + O(r'^{-1})$ inverse $\boxed{u'_0(u,\zeta,\bar\zeta) = \int_0^u dv\, \frac{P}{P_0}}$

subleading class II rotation $b = r'^{-1} b_0 + O(r'^{-2})$ $\boxed{b_0 = \partial_0 u'_0 |}$

Results for RT $\boxed{\sigma'^{0} = \eth'^{2}_{0} u'_{0}|}$

$\psi'^{0}_{4} = \ldots \quad \psi'^{0}_{3} = \ldots \quad \psi'^{0}_{2} = -[e^{3\Phi}|M + 2b_0 \bar{\Psi}'^{0}_{3} + b_0^2 \bar{\Psi}'^{0}_{4}] \quad \bar{\Psi}'^{0}_{1} = \ldots$

$\Psi'^{0}_{0} = -[4b_0 \psi'^{1}_{0} + 6 b_0^2 \psi'^{0}_{2} + 4 b_0^3 \psi'^{0}_{3} + b_0^4 \bar{\Psi}'^{0}_{4}]$ Adamo et al. (2009)

$\boxed{\mathcal{M}'_{\mathcal{B}} = [e^{3\Phi} M - [(\eth_0 \bar{\eth}_0)^2 + (\eth_0 \bar{\eth}_0)] w'_0]|}$ Tafel (2000)

4 $\boxed{\lambda'^{0} = [-\bar{\eth}_0^2 \Phi + (\bar{\eth}_0 \Phi)^2]|}$

back to natural RT coordinates

$\boxed{\partial_u \mathcal{M}_{\mathcal{B}} = -e^{-\Phi} |\eth_0^2 \Phi - (\eth_0 \Phi)^2|^2}$ $\Leftarrow$ RT equation

decreasing $\mathcal{M}_{\mathcal{B}}(u_f) \leq \mathcal{M}_{\mathcal{B}}(u_i) \quad u_f > u_i$

useful for non-linear convergence to Schwarzschild ?

Rendall, Schmidt, Singleton, Chruściel (1988-92)

5 No news

$$\eth^2\varphi + (\eth\varphi)^2 = e^{-2\Phi}\left[\eth_0^2\Phi - (\eth_0\Phi)^2\right] = e^{-2\varphi}\left[\partial^2\varphi - (\partial\varphi)^2\right] = 0 \quad + \text{c.c.}$$

7 parameter family of constant "vacuum" solutions

$$\varphi_V = -\ln\sqrt{\frac{\mu}{4}} - \ln\left[|a\zeta+b|^2 + |c\zeta+d|^2\right], \quad a,b,c,d \in \mathbb{C} \quad ad-bc=1$$

$$\partial\bar\partial\varphi_V = -\frac{\mu}{4}e^{2\varphi_V} \qquad \varphi_0 = \varphi_V(\mu=2,\ a=1=d,\ b=0=c)$$

vacuum sector of Liouville $S = \int d\zeta d\bar\zeta\left[\frac{1}{2}\partial\phi_L\bar\partial\phi_L - \frac{\mu}{2}e^{\phi_L}\right] \quad \phi_L = \frac{1}{2}\varphi_V$

6 Summary

- explicit map of RT to asymptotically flat
- expressions for news and memory
- new (?) way of writing RT equation
- success of flat holography ?

RT solutions with $\Lambda \neq 0$ $\quad U \to U + \frac{\Lambda r^2}{6}$

same effective 2d dynamics Bakas & Skenderis (2014)