How the dual CFT emerges from the worldsheet

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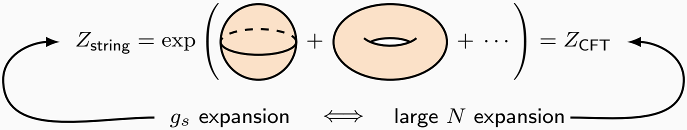
Based on work with **Andrea Dei, Kiarash Naderi, Sean Seet, Vit Sriprachyakul**

Motivation

The idea that large-N gauge theories are secretly string theories goes back to 't Hooft ['t Hooft '74]



This idea is made **very concrete** by the AdS/CFT correspondence

Except for in **very special** examples, it is not clear exactly how the worldsheet path integral reproduces dual CFT observables

Can we find a family of string backgrounds for which the worldsheet/dual CFT dictionary can be very well understood?

Motivation

AdS$_3$ string theory with pure NS-NS flux is among the most well-understood holographic backgrounds

- Worldsheet theory described by a WZW model on SL$(2,\mathbb{R})$ [Giveon, Kutasov, Seiberg; Maldacena, Ooguri]
- The full worldsheet spectrum is known, and the worldsheet CFT is in principle solvable

Despite computational power in the bulk, the dual CFT is much more mysterious

- It has a continuous spectrum (contrast to AdS$_4$, AdS$_5$, AdS$_7$...)
- It's correlation functions are generically divergent [Seiberg, Witten '99]

Motivation

Recently, a CFT dual was proposed which matches the perturbative spectrum of pure NS-NS (bosonic) strings on $\mathrm{AdS}_3 \times C$ [Balthazar, Giveon, Kutasov, Martinec '21] [Eberhardt '21]

Based on a deformation of a **symmetric product CFT**

$$\mathrm{Sym}(\mathbb{R}_\varphi \times C) + \mu \int e^{-\alpha\varphi} \sigma_2$$

Continuous spectrum exactly reproduces continuous string spectrum in the bulk [Eberhardt, Gaberdiel '19]

The CFT is a perturbation* of a free CFT

$$S = S_{\mathrm{free}} + \mu \int \mathcal{O}_{\mathrm{pert}}$$

Motivation

Observables can be computed computed in conformal perturbation theory

$$\langle \Phi_1 \cdots \Phi_n \rangle = \sum_{m=0}^{\infty} \frac{(-\mu)^m}{m!} \left\langle \left(\int \mathcal{O}_{\text{pert}} \right)^m \Phi_1 \cdots \Phi_n \right\rangle_{\text{free}}$$

A perturbative matching between worldsheet and boundary CFT correlators has been achieved [Dei, Eberhardt '21; Hikida, Schomerus '23; BK, Seet, Sriprachyakul '23-'24]

The matching of [Dei, Eberhardt '21] was based on a series of miraculous identities relating worldsheet correlation functions to the dual CFT answers

The reason **why** the worldsheet and CFT answers matched was unclear

Motivation

Our recent work proposes an extremely **simple** explanation for the perturbative series in terms of worldsheet string theory:

$$\langle \Phi_1 \cdots \Phi_n \rangle = \underbrace{\sum_{m=0}^{\infty} \frac{(-\mu)^m}{m!}}_{\text{`winding sectors' on the worldsheet}} \underbrace{\int \mathrm{d}^2\xi_1 \cdots \mathrm{d}^2\xi_m}_{\text{`transverse' string motion}} \underbrace{\langle \mathcal{O}_{\mathsf{pert}}(\xi_1) \cdots \mathcal{O}_{\mathsf{pert}}(\xi_m) \Phi_1 \cdots \Phi_n \rangle_{\mathsf{free}}}_{\text{`radial' string motion}}$$

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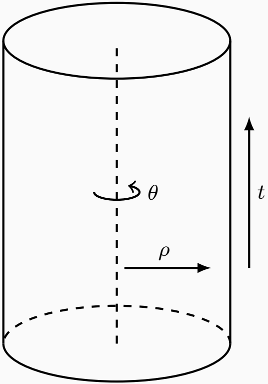
$$\mathrm{d}s^2 = -\cosh^2(\rho)\,\mathrm{d}t^2 + \mathrm{d}\rho^2 + \sinh^2(\rho)\,\mathrm{d}\theta^2$$



To study string theory on this background, we need to introduce fluxes

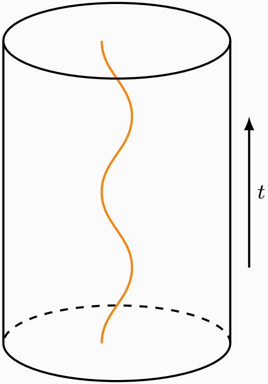
For pure NS-NS backgrounds, one introduces a nonzero 3-form flux

$$H = 4\sinh(\rho)\cosh(\rho)\,\mathrm{d}\rho \wedge \mathrm{d}t \wedge \mathrm{d}\theta$$

Solves the low-energy equations of motion:

$$R_{\mu\nu} \propto H_{\mu\rho\sigma} H_{\nu}{}^{\rho\sigma}$$

String theory in AdS$_3$



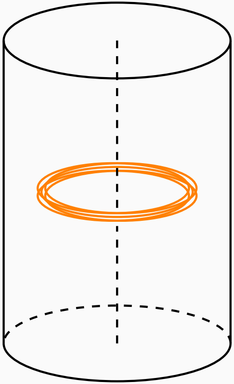
We usually think of AdS as a **bounding box**

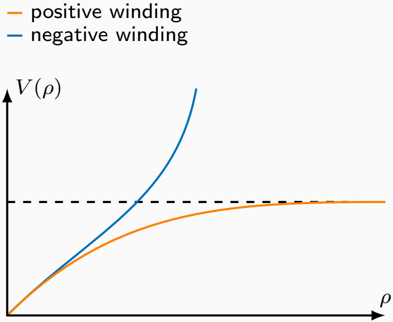
Massive particles living in AdS$_3$ require infinite energy to reach the boundary

For generic values of NS-NS and RR fluxes, this holds in string theory too

For **pure NS-NS flux**, this breaks down!

For backgrounds with pure NS-NS flux, strings with positive winding can reach the boundary at a finite cost of energy – **long strings**





In the Euclidean language, ‘long strings’ are finite-action worldsheets that live arbitrarily close to the conformal boundary of AdS$_3$

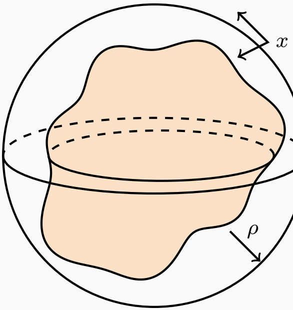
A long string is parametrized by:

- A map $x : \Sigma \to \partial\text{AdS}_3$
- A radial profile, given by a scalar ρ on the worldsheet

For the worldsheet action to be finite, the map x must be **orientation-preserving**

Different winding numbers give rise to different path integral sectors

$$Z^{\text{long}}_{\text{string}} = \sum_{N=0}^{\infty} \int_N \frac{\mathcal{D}x\,\mathcal{D}\rho}{\text{Diff}_\Sigma} e^{-S_{\text{string}}}$$



The long-string path integral

The long-string path integral does not capture all of the bulk physics, but will be enough to reproduce conformal perturbation theory in the dual CFT

It turns out that the long-string path integral is **exactly computable**

The strategy is to break up the path integral:

$$\int \frac{\mathcal{D}x}{\mathsf{Diff}_\Sigma} \int \mathcal{D}\rho \, e^{-S_{\mathsf{string}}}$$

Since $\partial\mathsf{AdS}_3$ is **two-dimensional**, and there are **two** worldsheet diffeomorphisms, the integral over $x : \Sigma \to \partial\mathsf{AdS}_3$ can **almost** be gauged away

The resulting integral over ρ can be computed using standard free-field techniques

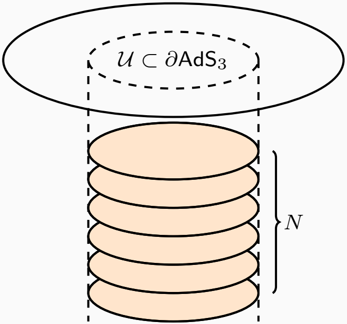
The long-string path integral

Question: Can we use worldsheet diffeomorphisms to completely fix x?

Take a worldsheet winding the boundary N times

The preimage $x^{-1}(\mathcal{U})$ of some disk generically looks like N disjoint disks on the worldsheet

We can locally use Diff_Σ to identify the worldsheet coordinates with the boundary coordinates on each disk



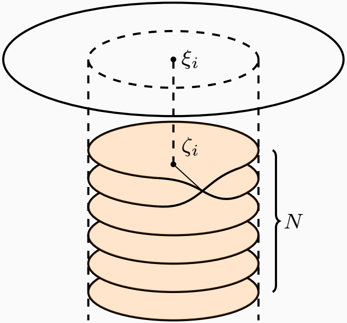
If $N > 1$, there will generically be points ξ_i on the boundary where two of the disks 'branch' into each other

The locations of the branch points ξ_i **cannot** be changed by worldsheet diffeomorphisms

The number of branch points is given at tree-level by

$$m = 2N - 2$$

The branch points represent **obstructions** to gauge-fixing the map $x : \Sigma \to \partial\mathrm{AdS}_3$



The long-string path integral

Upshot: can use worldsheet diffeomorphisms to trade integral over transverse motion x into integral over $2N-2$ branch points ξ_i on the boundary:

$$\sum_{N=0}^{\infty}\int_N \frac{\mathcal{D}x}{\mathsf{Diff}_\Sigma}\int \mathcal{D}\rho\, e^{-S_{\mathsf{string}}} = \sum_{m=0}^{\infty}\frac{1}{m!}\int \mathrm{d}^2\xi_1\cdots\mathrm{d}^2\xi_m \int \mathcal{D}\rho\, e^{-S_{\mathsf{string}}}$$

Thus, the integral over worldsheet configurations reproduces the perturbative integrals in the **dual CFT**

A little more work actually shows that the **integrands** are also perfectly reproduced

Summary

Sketched a natural mechanism for the perturbative CFT expansion to arise from string theory in AdS_3

Conformal perturbation theory in the boundary $\Longleftrightarrow$ integral over nontrivial **worldsheet configurations** of long strings in the bulk

With hindsight, the dual CFT could have been derived without doing **any real calculations**

One can use the logic of this talk to derived dual CFTs for both **type II superstrings** and **heterotic strings** on AdS_3 [BK, McStay, Sriprachyakul TBA]

Thank you.