A New Twist on Spin

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<u>Goal</u>: Kinematic variables that make <u>all</u> constraints from conformal symmetry manifest:

* SO(1,4) + conservation *

Main Result:

These variables exist: TWISTORS!

$$P_\mu^2 = 0 \qquad\qquad \cancel{P}_{AB} = \lambda^a{}_A \lambda_{aB}$$

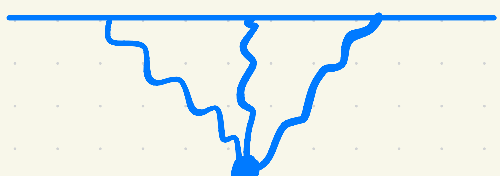
NULL CONE → SPINOR HELICITY VARS. →

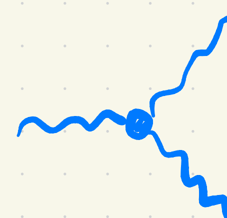
→ TWISTORS $Z_A = \pi_a \lambda^a{}_A$

$$J^{ab}(P) = \int_{\mathrm{Ker}\,\cancel{P}} DZ\, f(Z)\, \pi^a \pi^b$$

Beautiful connection to Flat Space Amplitudes!

$$\langle T\,T\,T \rangle = \int [DZ] (\ldots) \times \mathcal{M}_{flat}$$





Properly interpreted...

All ingredients present in the literature

Chiodaroli, Gunaydin, Johansson, Roiban '22

Ward '89

Neiman '14

Adamo, Skinner, Williams '17

Osborn, Petkou '93

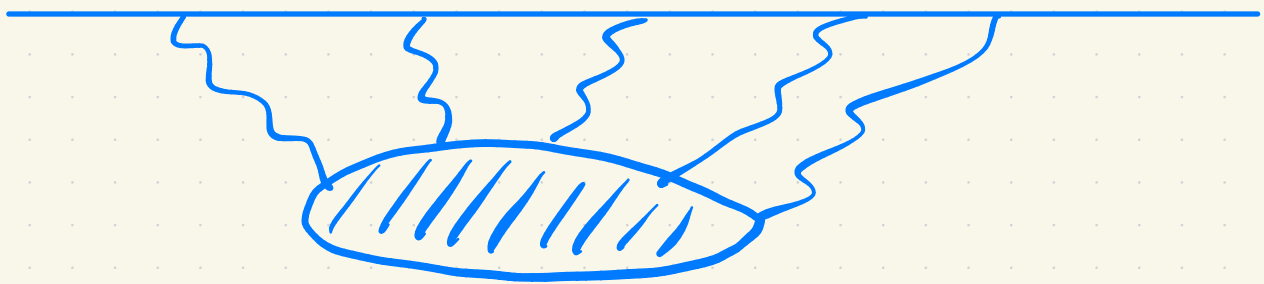
Costa, Penedones, Poland, Rychkov '11

We combined them in a new way.

Motivation:

* Cosmology
* Success story in flat space

Cosmology

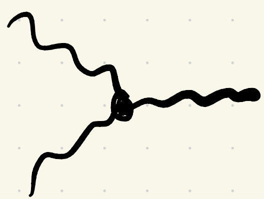


What is the cosmological correlator of $\begin{cases} \text{Y.M.} \\ \text{G.R.} \end{cases}$ at n- points, tree-level?

Flat Space

Right variables: Spinor-Helicity

$$p_\mu^2 = 0 \implies (p\cdot\vec{\sigma})_{\alpha\dot{\alpha}} = \lambda_\alpha \tilde{\lambda}_{\dot{\alpha}}$$



$$1^+ 2^+ 3^+ : \left([12][23][31]\right)^S$$

$$1^+ 2^+ 3^- : \left(\frac{[12]^3}{[23][31]}\right)^S$$

"INEVITABLE"

Current Approaches

* Embedding Space — $SO(1,4)$ manifest; Conservation imposed

* Momentum Space — $SO(1,4)$ not manifest; Conservation easy

* Mellin Space — Conservation not obvious; SCTs not manifest

First Idea

Embedding Space Rays are NULL MOMENTA

$$P_\mu^2 = 0 \longrightarrow P = \lambda\lambda$$

2D CFT: $$P_{\alpha\dot\alpha} = \lambda_\alpha \tilde{\lambda}_{\dot\alpha}$$

Works for any Δ. What's special about J/T?

$$J(\lambda);\, T(\lambda)$$ HOLOMORPHICITY

in progress: BCFW ≡ BPZ

First Idea

3D CFT

Embedding Space Rays are NULL MOMENTA

$$P_\mu^2 = 0$$

Symplectic 4x4 matrix

$$(P \cdot \Gamma) = P_{AB} \quad A = 1-4 \qquad \eta_{\mu\nu} \longrightarrow \Omega_{AB}$$

$$\det P_{AB} = (P_\mu^2)^2 \xrightarrow{\text{rank } 2} P_{AB} = \lambda_{aA} \lambda^a{}_B$$

$a = 1,2$ little group

$A = 1-4$ $SO(1,4)$ index

$P_{AB} = \lambda_{aA} \lambda^a_B$ **Constraint**: $\Omega^{AB} \lambda^a_A \lambda^b_B \equiv$

$\equiv \langle \lambda^a \lambda^b \rangle = 0$

Very useful even for generic Δ.

Holomorphicity more confusing.

Idea needed...

Second Idea

(in hindsight...)

λ_{aA}'s span Kernel of P_{AB}, 2D space $\lambda^a \cdot P = 0$

Take $\quad Z_A \equiv \pi_a \lambda^a{}_A$, some linear combination of λ's

HOLOMORPHICITY: $f(Z)$ only

$$J^{ab}(\lambda) = \int \langle \pi d\pi \rangle \pi^a \pi^b f(Z)$$

$$T^{abcd}(\lambda) = \int \langle \pi d\pi \rangle \pi^a \ldots \pi^d f(Z)$$

CONSERVED!

Rules

* SPACETIME: $Z^i_A Z^j_B \Omega^{AB}$ only +
  + Scaling

* CONSERVATION: $f(Z)$'s only
  + $\int (\ )$ invariant under
  $\pi \longrightarrow c\pi$

PINS $\Delta = S+1$.

Helicity (?)

Instead of (π^a), more invariant way

$$J^+ \equiv \tilde{\jmath}_a \tilde{\jmath}_b J^{ab} = \int_{\mathrm{Ker}P} DZ\, (\sigma^* \cdot Z)^2 f^+(Z)$$

$(\mathrm{Ker}P)^*$

$$J^- = \int DZ\, \left(\sigma^* \cdot \frac{\partial}{\partial Z}\right) f^-(Z)$$

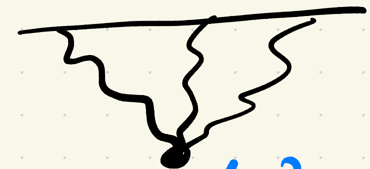
Ansatz

$$\langle J\,J\,J\rangle = \begin{cases} \displaystyle\int [DZ_m]\, dc_{mn}\, \left(\sigma_m^* \cdot Z_m\right)^2 \, \mathrm{Exp}\left[i\, c_{mn}\, Z_m \cdot Z_n\right] \times f_{+++}\left(c_{12}, c_{23}, c_{31}\right) \\ \displaystyle\int [DZ_m]\, dc_{mn} \left(\sigma_1^* \cdot \frac{\partial}{\partial Z_1}\right)^2 \left(\sigma_2^* \cdot \frac{\partial}{\partial Z_2}\right)^2 \left(\sigma_3^* \cdot \frac{\partial}{\partial Z_3}\right)^2 \times \mathrm{Exp}\left[i\, c_{mn}\, Z_m \cdot Z_n\right] \times f_{---}\left(c_{12}, c_{23}, c_{31}\right) \end{cases}$$

Counting Rules are (almost) the same as for flat space Amplitudes!!

$$[mn] \rightarrow c_{mn}$$

$$f(c_{12}, c_{23}, c_{31}) = (c_{12}c_{23}c_{31})^{\pm S}$$



Easy to unpack: δ-function + Wick contractions

Reproduces famous results.

Now + Future

P_μ

* 2D, 4D
* 4-particle correlators
* Factorization
* ½ Fourier Transform
  ((2,2) dS?)

λ^a_A

$\mathcal{L}_A$

$$\begin{tikzcd}[column sep=small] & \mathcal{L} \arrow[dl] \arrow[dr, leftrightarrow] & \\ P_\mu \arrow[rr, leftrightarrow] & & K_\mu \end{tikzcd}$$