Gravity and the bootstrap program

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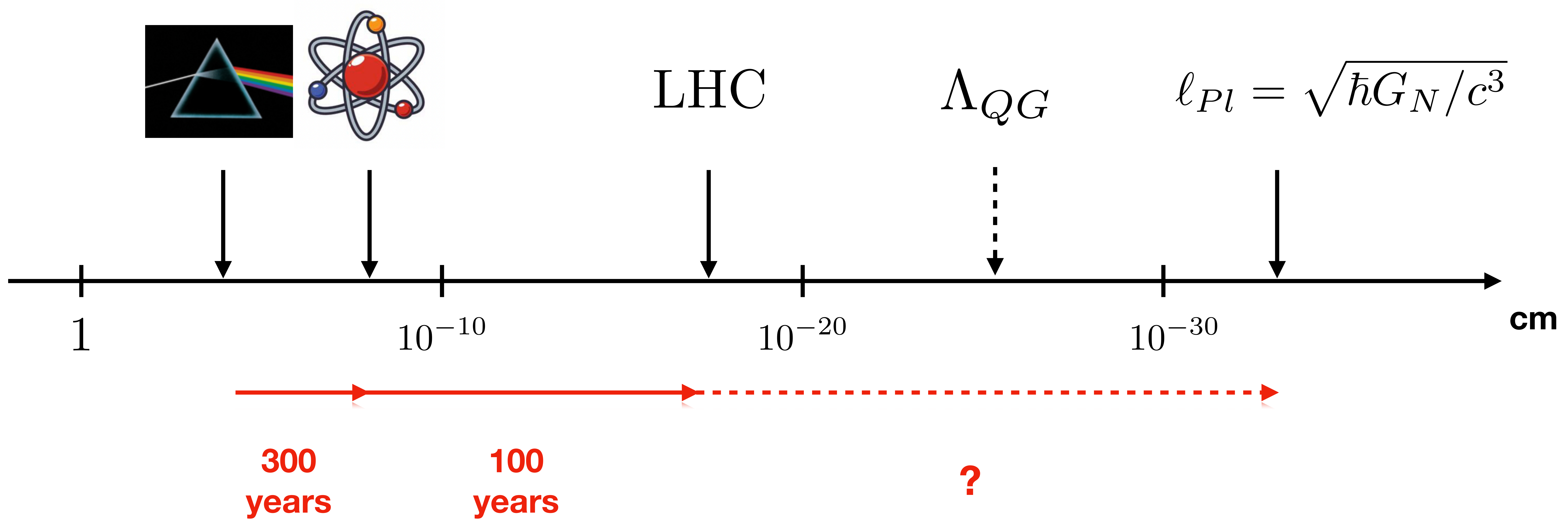
Celestial Holography Satellite Meeting 2025, NY



Given no direct experimental input, we learn from string theory:

- Holography (dual microscopic formulation) [top-down]

- **Bootstrap** (causality, unitarity/QM) [bottom-up]

The bottom-up approach proceeds as follows:

1. Identify an observable

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- *AdS* (CFT bootstrap)
- $\mathbb{M}_{d>4}$ (S-matrix bootstrap)

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- *AdS* (CFT bootstrap)
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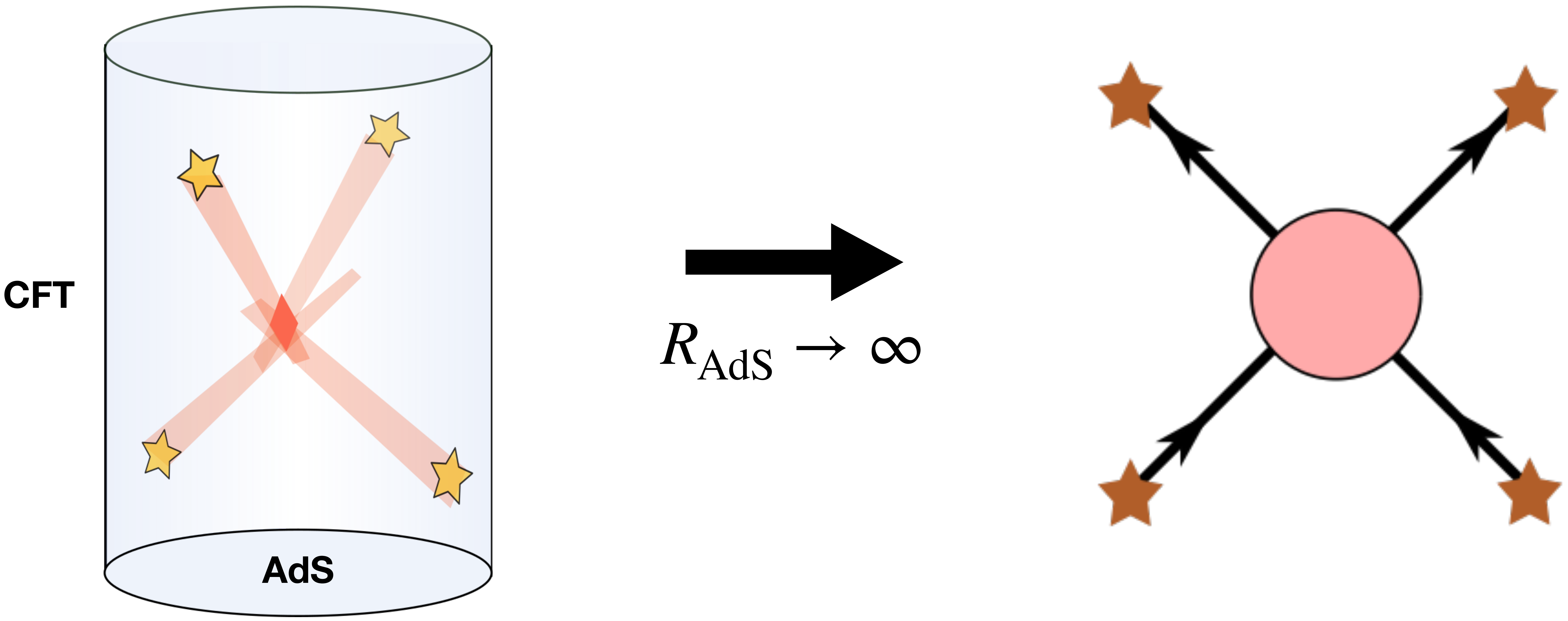
In other cases, it could be that we are **unlucky**, and no fruitful bootom-up approach is possible ($\mathbb{M}_4$, *dS*, cosmology). Hard to be sure.

A convenient observable to study is a **2-2 scattering amplitude**



[Rastelli]

There is indeed an interesting tension between (**analyticity/causality, crossing and unitarity**).

Mathematically, it is expressed, for example, in the existence of the **2SDR** (twice-subtracted dispersion relations).

$s + t + u = 4m^2$

discontinuity

$$T(s,t) = g(t) + \frac{1}{\pi}\int_{m_{gap}^2}^{\infty} ds' \frac{T_s(s',t)}{s'^2}\left(\frac{s^2}{s'-s} + \frac{u^2}{s'-u}\right)$$

subtraction term

2SDR

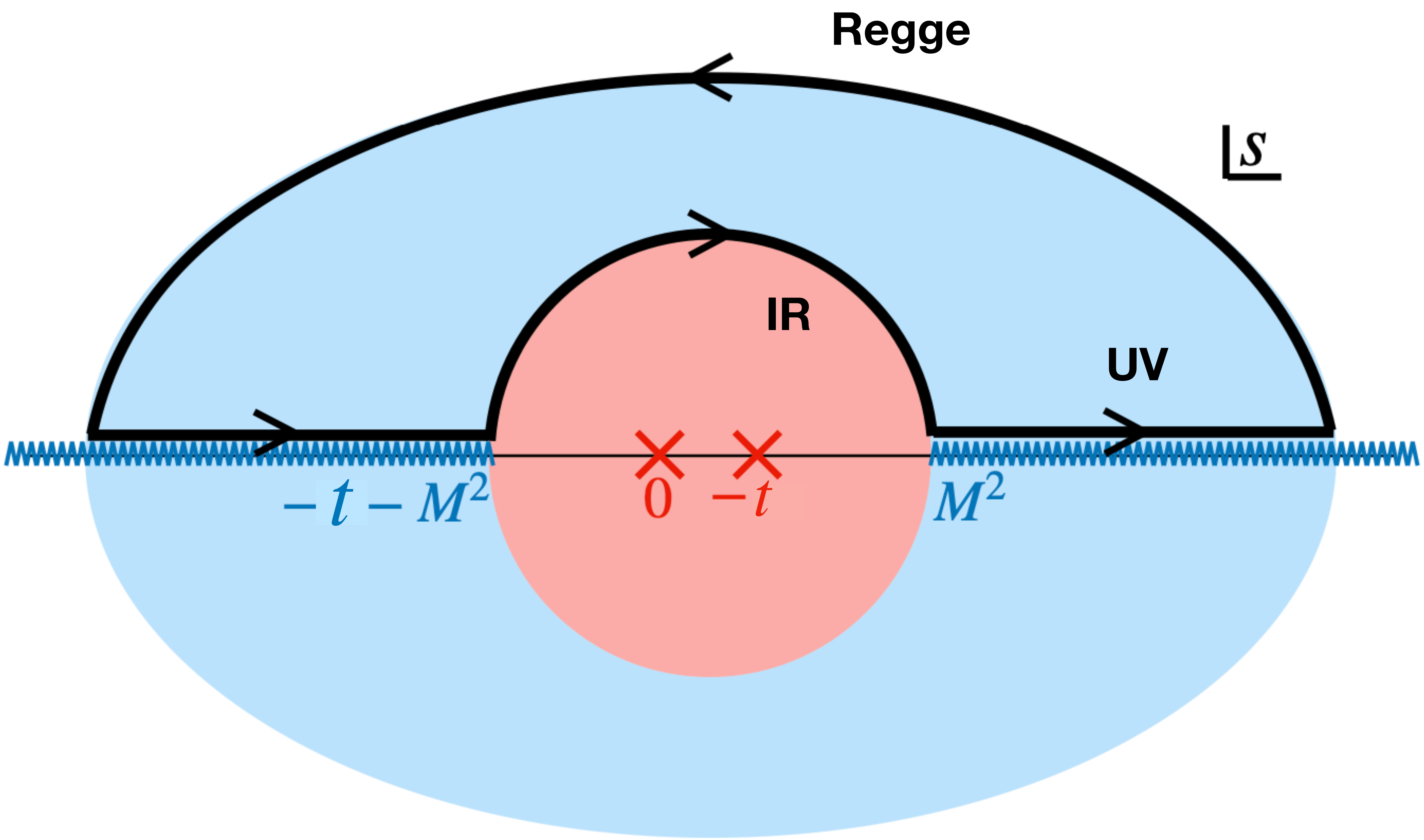
Unitarity: $T_s(m^2,t) = \sum_{J=0}^{\infty} \text{Im}\, a_J(m^2) P_J\left(1 + \frac{2t}{m^2}\right), \quad 2 \geq \text{Im}\, a_J(m^2) \geq 0 \; .$

Crossing:

$T(s,t) = T(u,t)$ ✔

$T(s,t) = T(t,s)$ ✘ **extra sum rules**

[Essentially the same in AdS]

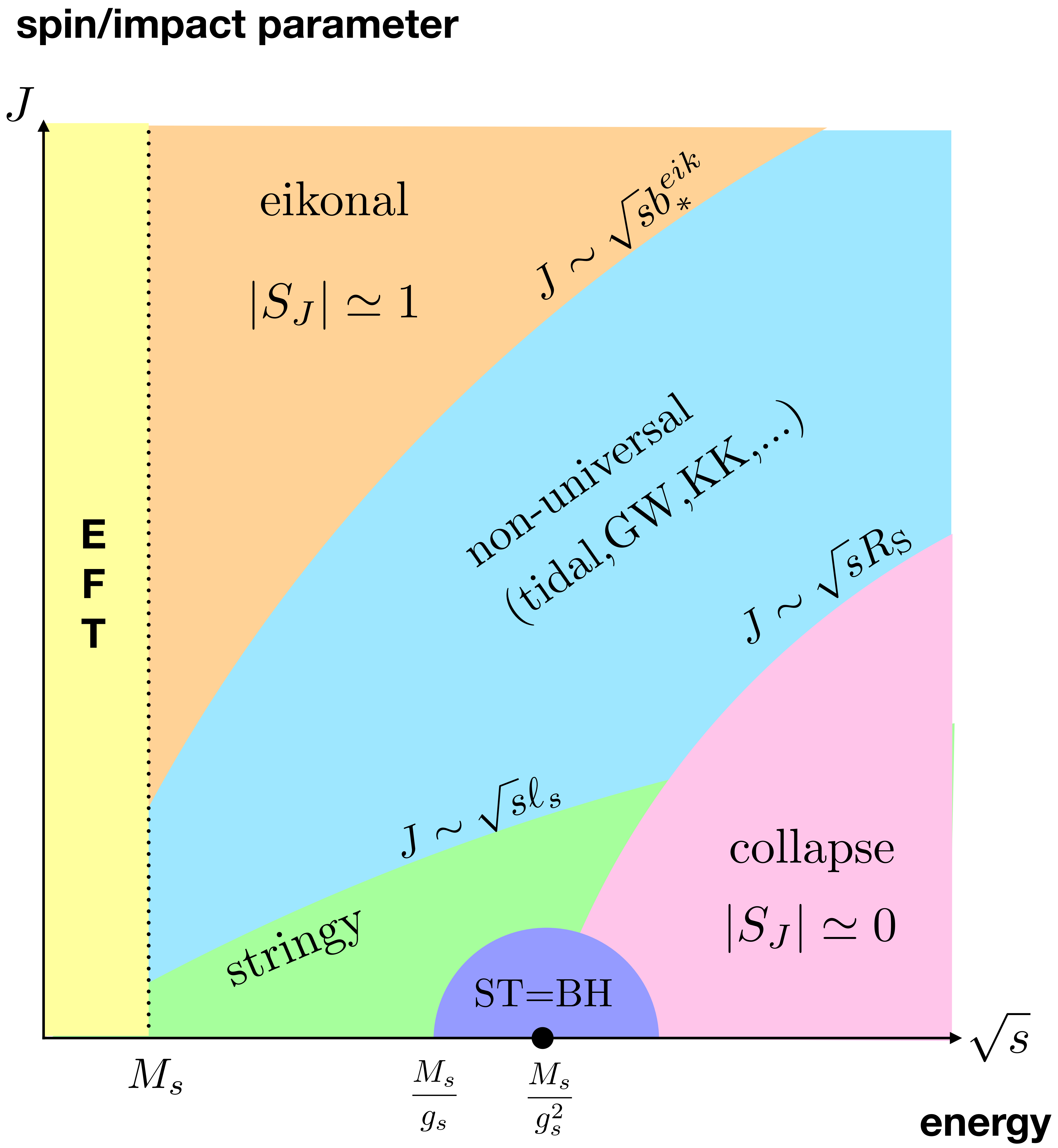


[Rastelli]

$$\oint_{\infty} \frac{ds}{2\pi i} f(s,t) T(s,t) = 0$$

IR+UV=0

[Amati, Ciafaloni, Veneziano '87]
[di Vecchia, Heissenberg, Russo, Veneziano '23]



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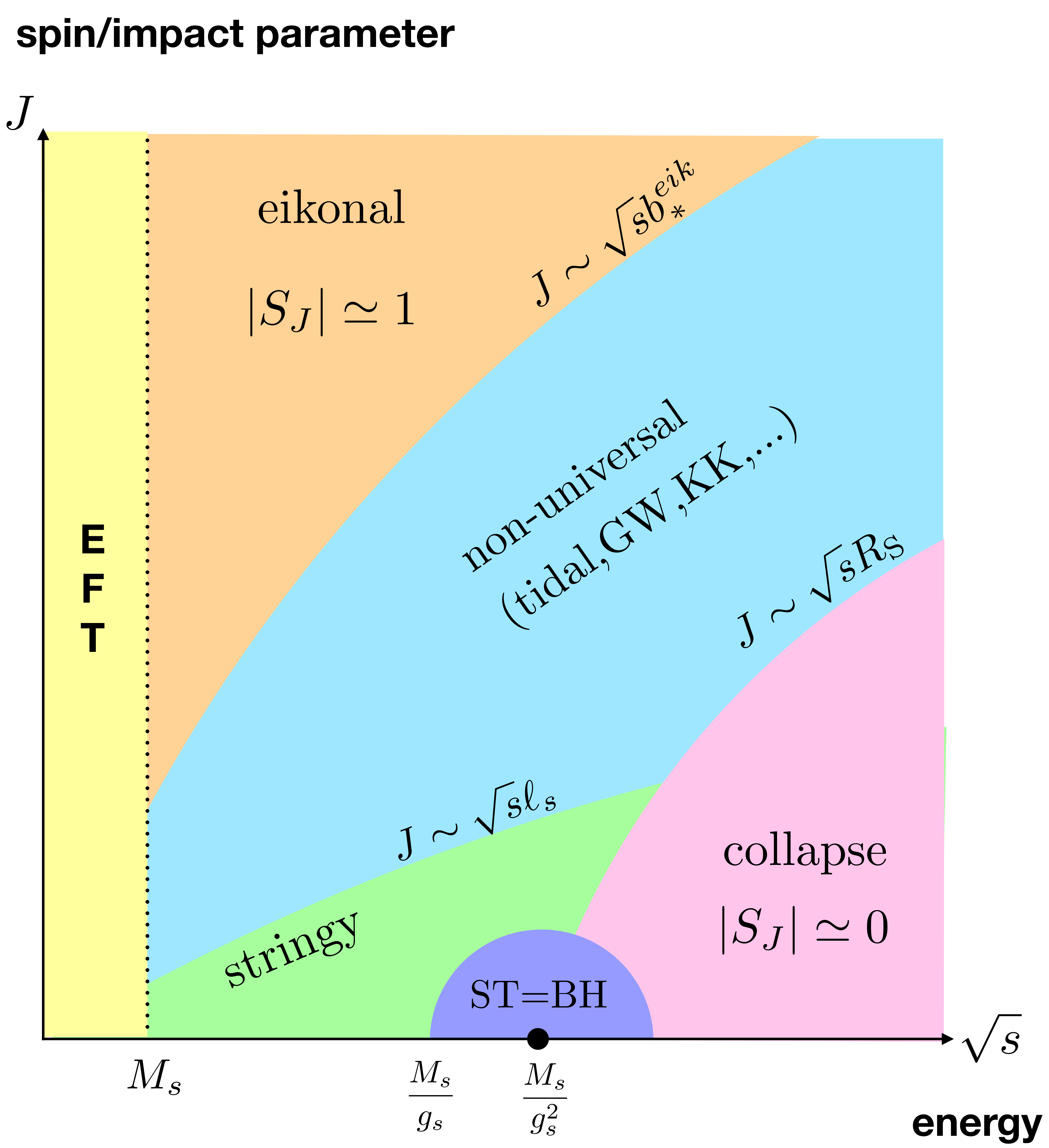
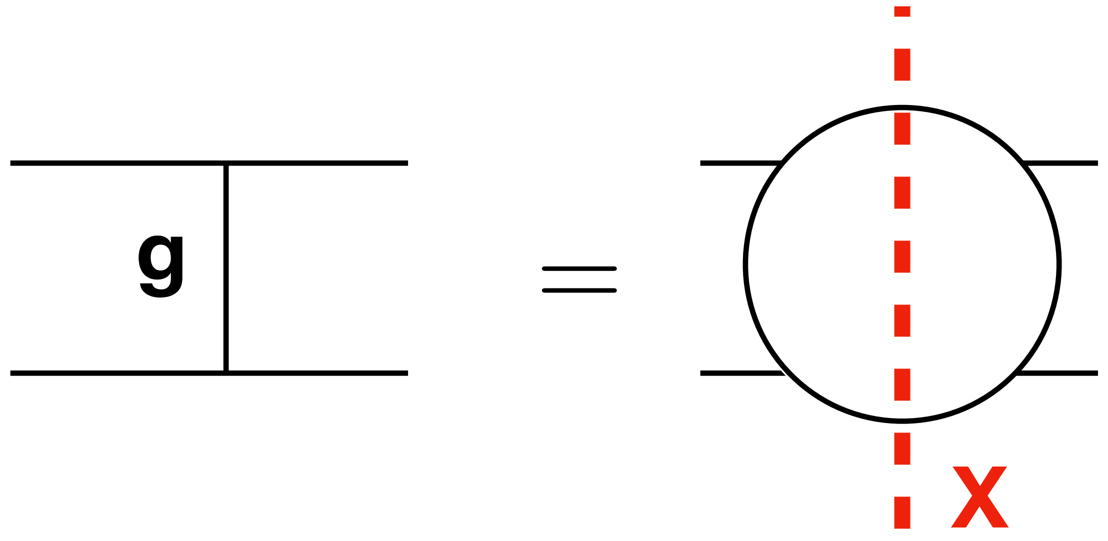


$$S = \frac{1}{16\pi G_N} \int d^D x \sqrt{-g} \left(R - 2\Lambda + \dots \right)$$



G_N is the overall UV budget.

$$\frac{8\pi G_N s^2}{-t} + \dots = s^2 \int_0^\infty \frac{ds'}{\pi} \frac{2T_s(s',t)}{(s')^3}, \quad t < 0$$

It is convenient also to study it in the impact parameter space ($d = 4$, $\delta = -2G_N s \log b/L_{\text{IR}}$)

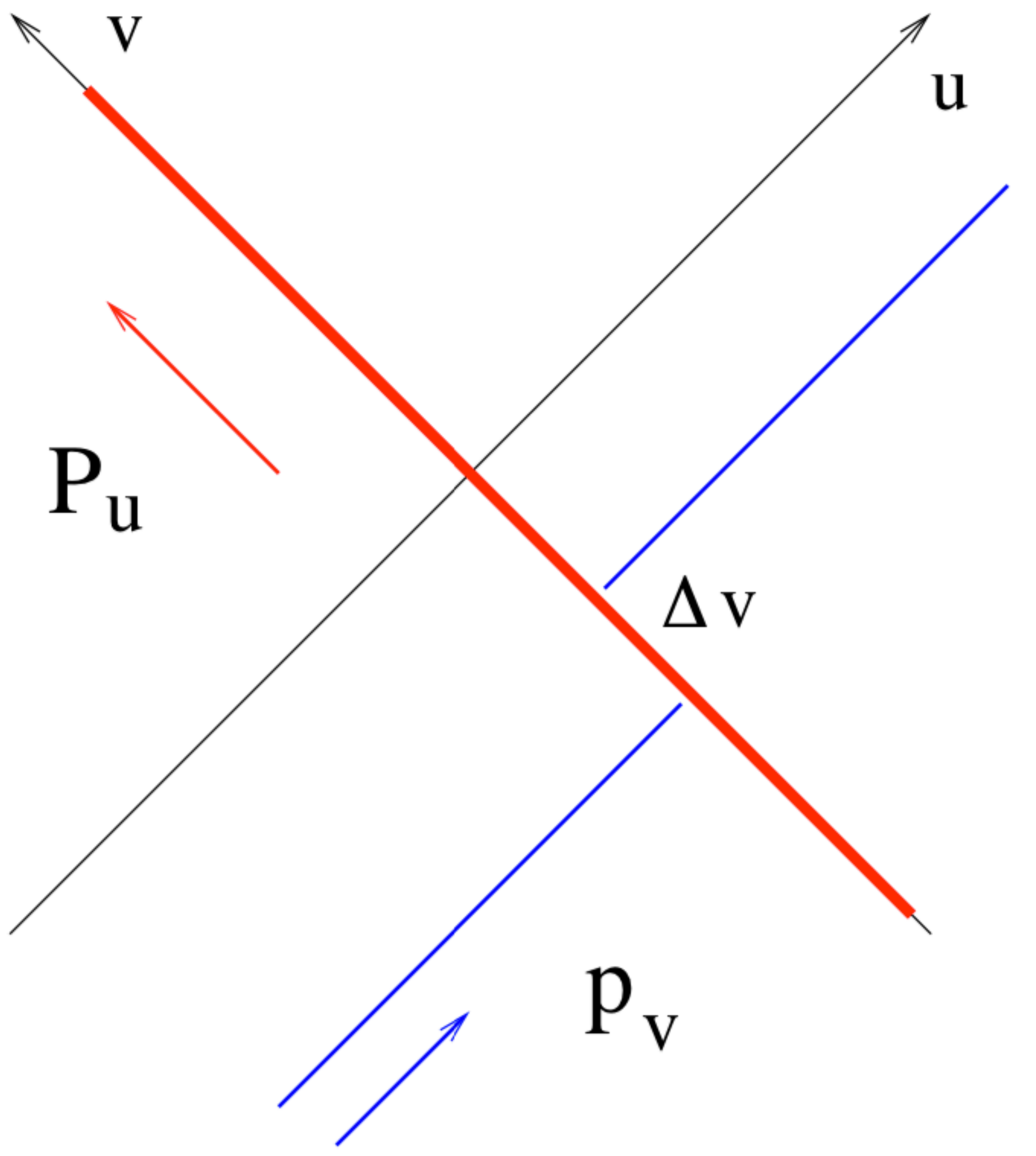
$$i(1 - e^{2i\delta(s,b)}) = \int_0^\infty \frac{ds'}{\pi} \frac{2\sin^2 \delta(s',b)}{s'} \left(\frac{s}{s'-s} + \frac{s}{s'+s} \right)$$

[Caron-Huot,Li '24] [Haring,AZ '24] [Cheng,Stanford]

Higher derivatives = massive higher spins

What controls corrections to general relativity?

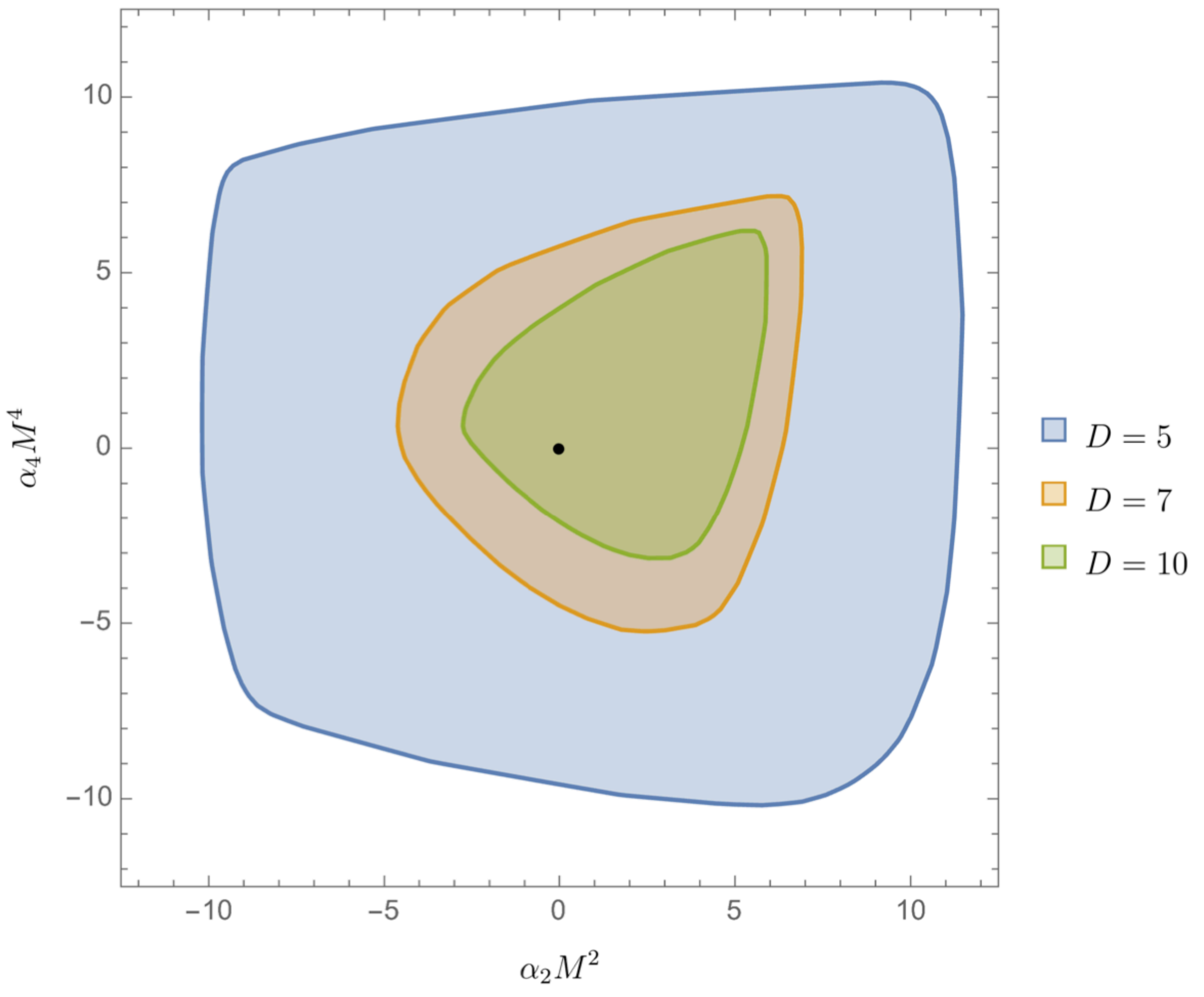
$$S = \frac{1}{16\pi G_N} \int d^D x \sqrt{-g} \left(R - 2\Lambda + \alpha_2 R_{\mu\nu\rho\sigma} R^{\mu\nu\rho\sigma} + \dots \right)$$



$b^2 \sim |\alpha_2|$

$$|\alpha_2| \lesssim \frac{1}{M_{\text{HS}}^2}$$

$$|\beta_{R^3}| \lesssim \frac{\log m_{\text{HS}} L_{\text{IR}}}{m_{\text{HS}}^4}$$



[Camanho, Edelstein, Maldacena, AZ '14]

[Caron-Huot, Li, Parra-Martinez, Simmons-Duffin '22]

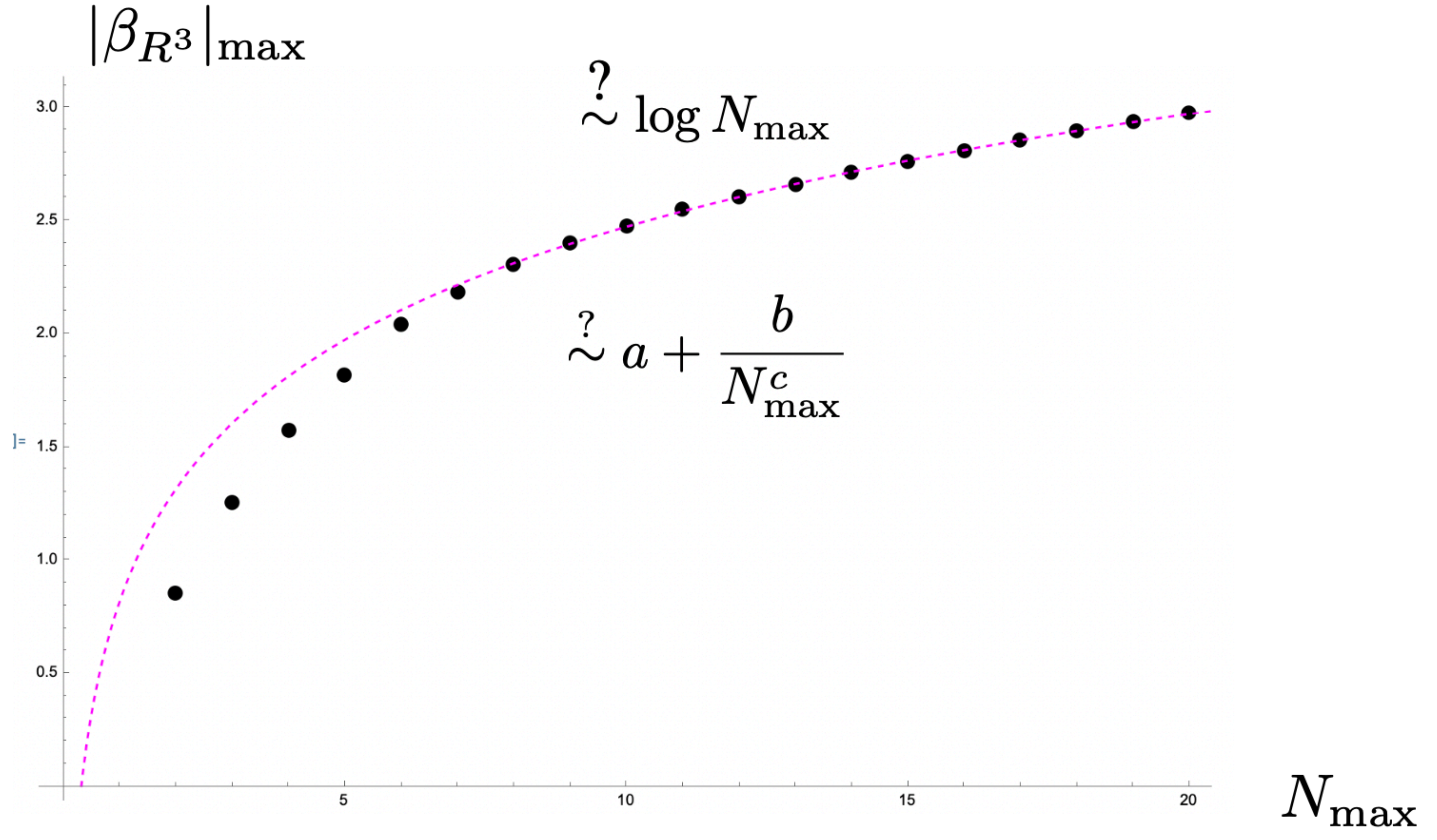
In $d = 4$, the bound depends on the IR cutoff. Consider an explicit ansatz for the 2-2 amplitude and maximize the three-point coupling

$$T_{--++}(s,t,u) = (\langle 12\rangle[34])^4 f(s|t,u)$$

$$f_{N_{\max}}(s|t,u) = -\frac{\Gamma(-s)\Gamma(-t)\Gamma(u)}{\Gamma(1+s)\Gamma(1+t)\Gamma(1+u)} + \sum_{c_s,c_t=0}^{N_{\max}} \sum_{d_s,d_t=1}^{2N_{\max}} \alpha_{c_s,c_t,d_s,d_t} \frac{\Gamma(c_s-s)\Gamma(c_t-t)\Gamma(c_t-u)}{\Gamma(d_s+s)\Gamma(d_t+t)\Gamma(d_t+u)}$$

$$f(s|t,u) = \frac{1}{stu} + |\beta_{R^3}|^2 \frac{tu}{s}$$

$N_{\max} = 20: \quad 1218 \quad \text{terms}$



$$|\beta_{R^3}| \lesssim \log N_{\max} \qquad |\beta_{R^3}| \lesssim \frac{\log m_{\text{HS}} L_{\text{IR}}}{m_{\text{HS}}^4}$$

Our explicit amplitude is consistent with loosing the bound.

[Haring,AZ '23]
[Chang, Parra-Martinez '25]

Minimal correction to GR

There are situations when $\alpha_2 = \alpha_3 = 0$ (maximal SUSY).

$$S = \frac{1}{16\pi G_N}\int d^Dx\sqrt{-g}\left(R + \alpha \ell_{Pl}^6\; t_8 t_8 R^4 + \dots\right) \qquad \alpha \geq 0$$

[Gross, Witten, Green, Vanhove, Gutperle, Russo, ...]

$$\frac{T(s,t,u)}{8\pi G_N} = s^4\left(\frac{1}{stu} + \alpha\,\ell_P^6 + \dots\right)$$

$$\alpha \ell_P^6 = \frac{2}{\pi}\int_0^\infty \frac{ds}{s^5} T_s(s,0)$$

$$\frac{T(s,t,u)}{8\pi G_N} = s^4\Bigg(\underbrace{\frac{1}{stu}}_{\texttt{SUGRA}} + \underbrace{\prod_{A=s,t,u}(\rho_A+1)^2 \sum_{a+b+c\leq N}{}' \alpha_{(abc)}\rho_s^a \rho_t^b \rho_u^c}_{\texttt{UV completion}}\Bigg)$$

[Paulos, Penedones, van Rees, Vieira '17]

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[Paulos, Penedones, van Rees, Vieira '17]

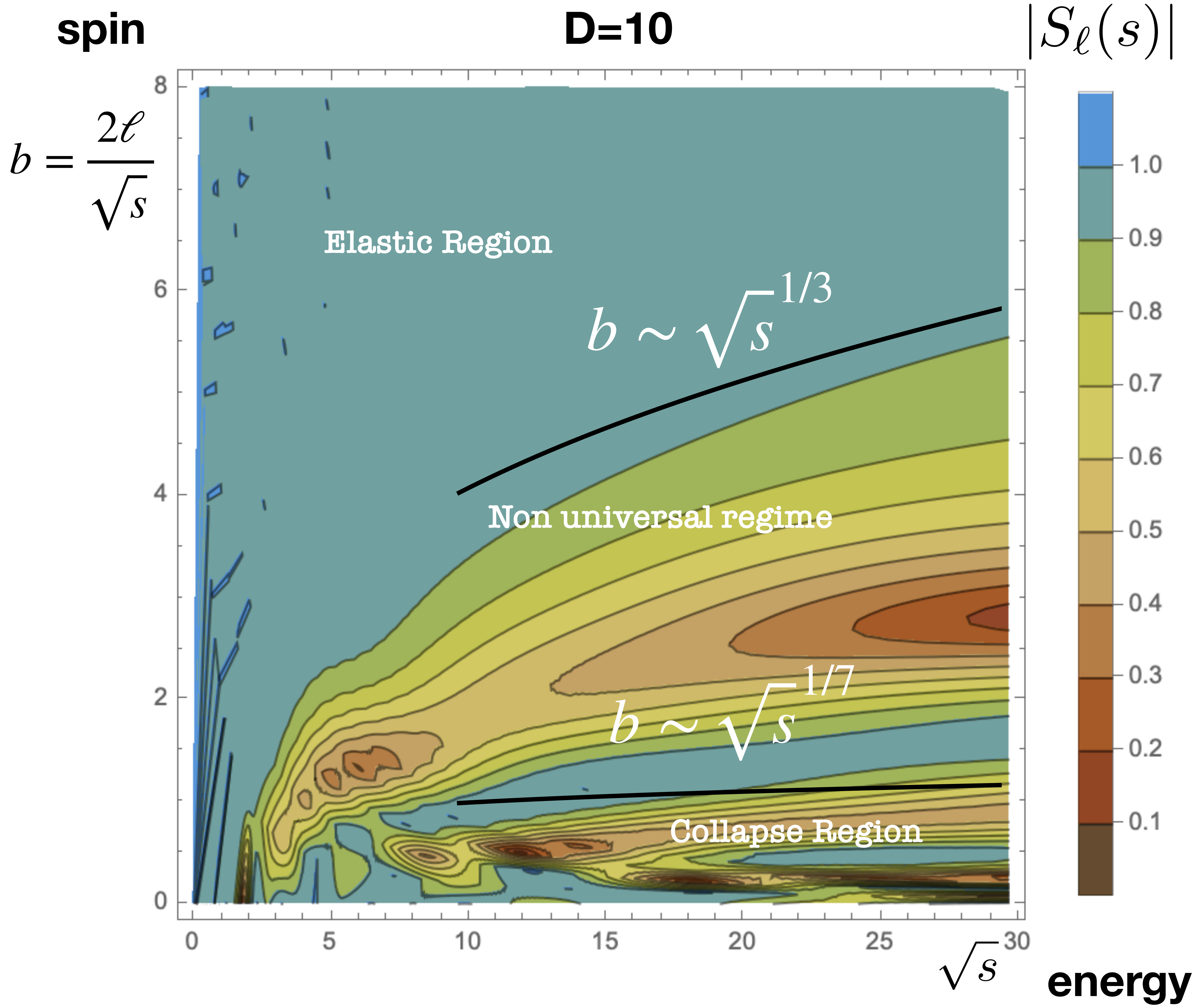
$$\alpha = \frac{(2\pi)^2}{3\ 2^7} \simeq 0.1028\,.$$

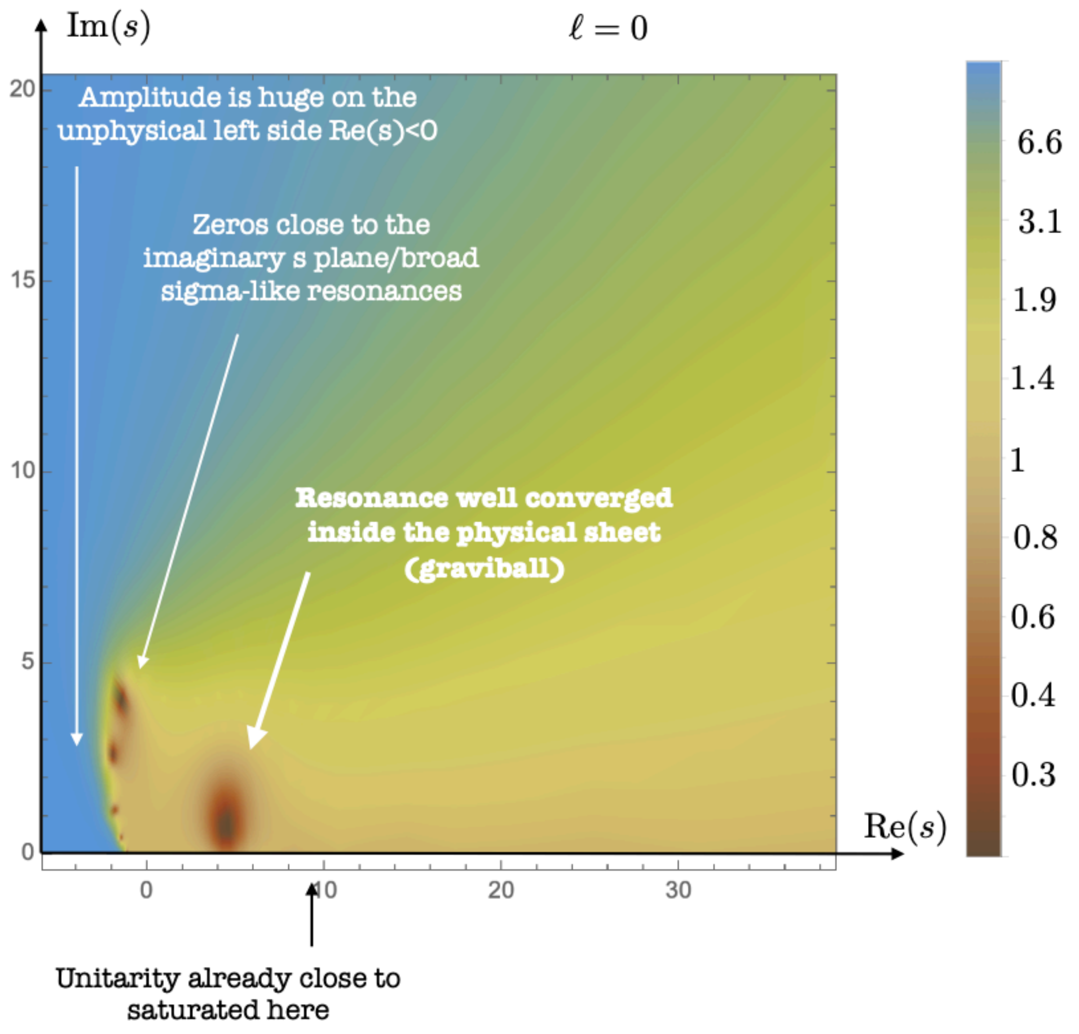
$$\alpha^{\text{IIB}} = \frac{1}{2^6} E_{\frac{3}{2}}(\tau, \bar\tau) \geq \frac{1}{2^6} E_{\frac{3}{2}}(e^{i\pi/3}, e^{-i\pi/3}) \approx 0.1389$$

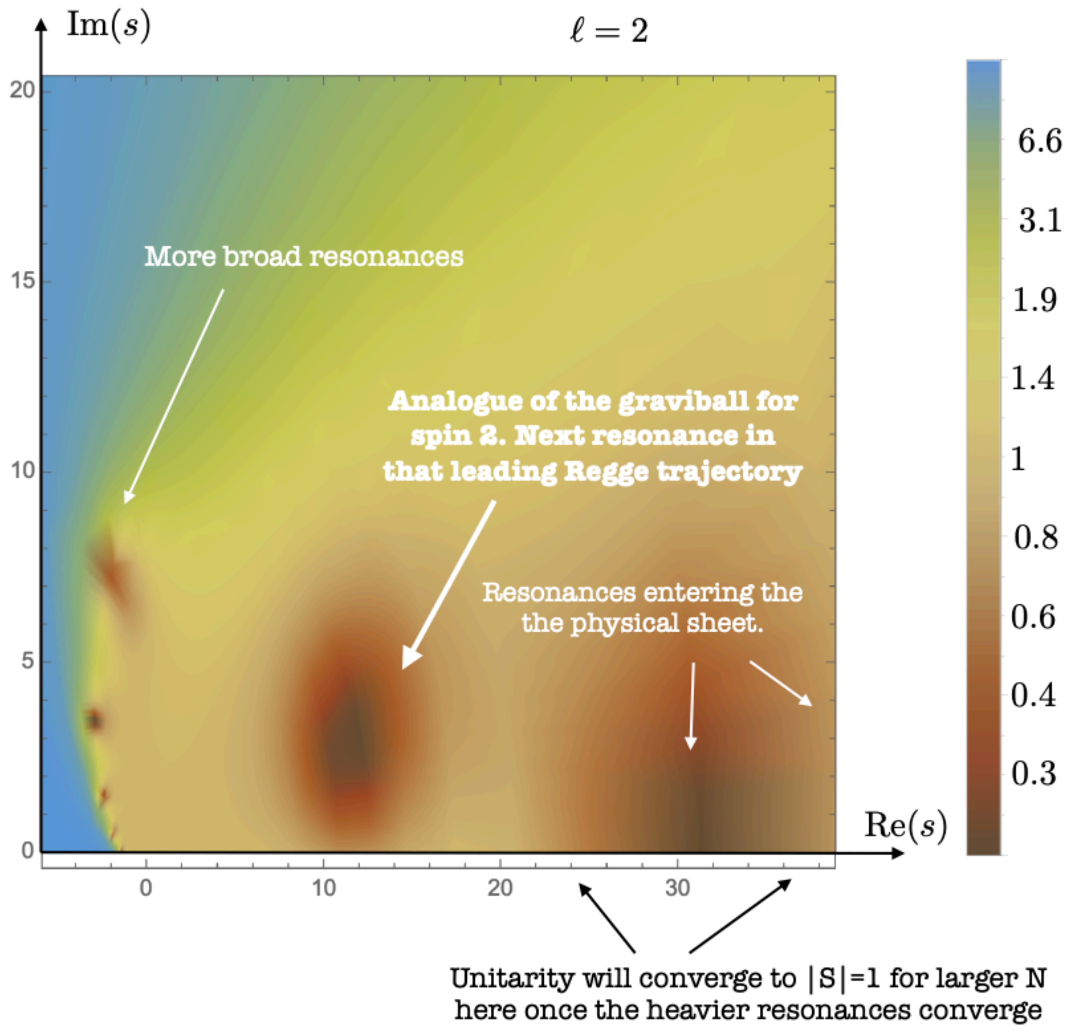
$$\alpha^{\text{IIA}} = \frac{\zeta(3)}{32 g_s^{3/2}} + g_s^{1/2} \frac{\pi^2}{96} \geq \frac{\pi^{3/2} (\zeta(3))^{1/4}}{24\sqrt{3}} \approx 0.1403$$

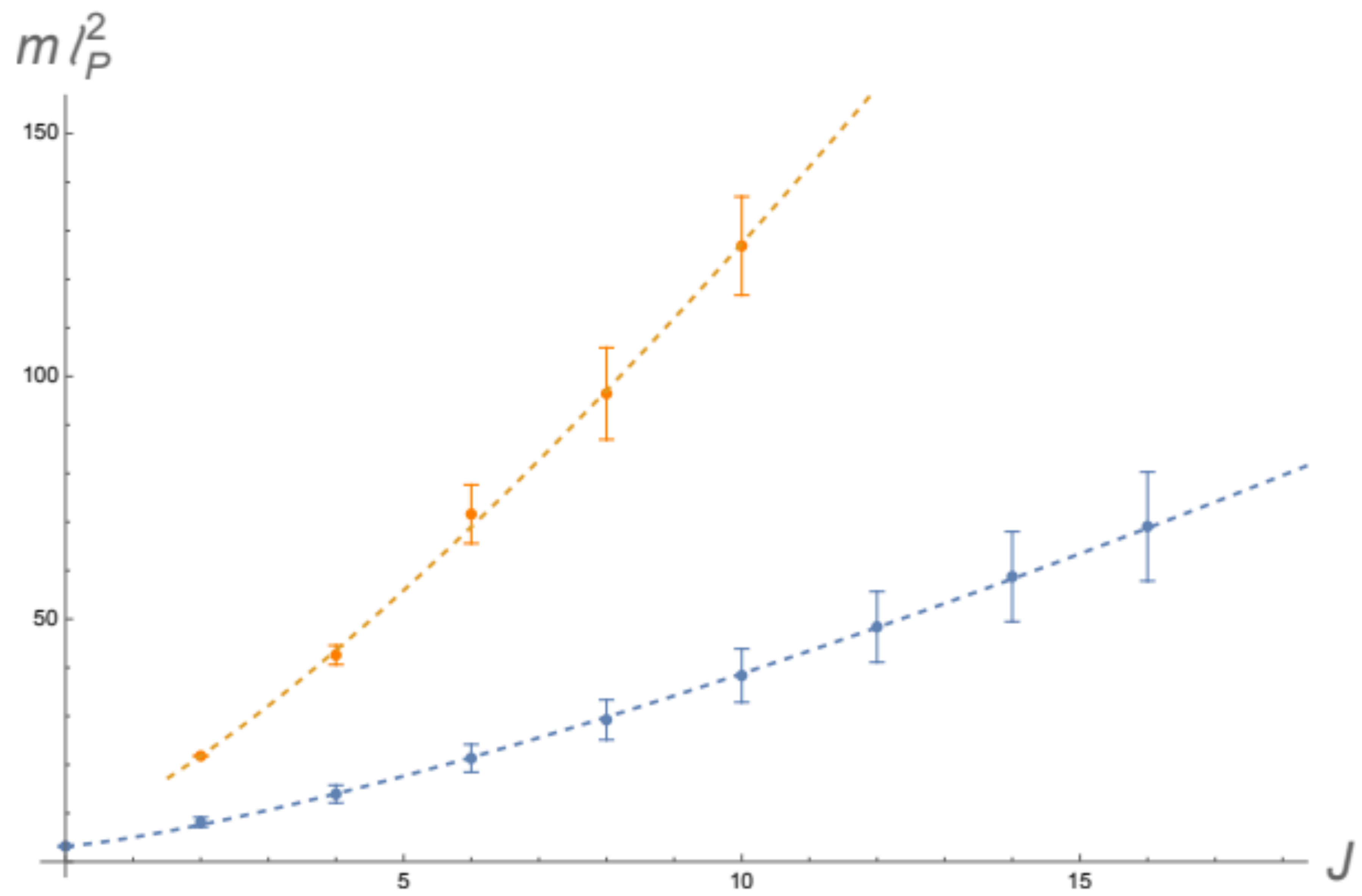
Dimension	Bootstrap	String/M–Theory
9	0.223 ± 0.002	0.241752
10	0.124 ± 0.003	0.138949
11	0.101 ± 0.005	0.102808

[Guerrieri, Murali, Penedones, Vieira '22]









[plots A. Guerrieri]

Can be pushed further, technically challenging.

To summarize, in AdS and $\mathbb{M}_{d>4}$ there is a powerful method to explore the space of theories. Currently, everything is done at the level of 2-2, going beyond:

- is possible in AdS and at tree-level in flat space (systematic)
- challenging in flat space finite G_N (M-theory)

[Strings 2025]

To summarize, in AdS and $\mathbb{M}_{d>4}$ there is a powerful method to explore the space of theories. Currently, everything is done at the level of 2-2, going beyond:

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[Strings 2025]

Going to $d = 4$, it is curious to ask the following question:

What is the simplest bootstrap-friendly IR safe quantity in 4d?

The usual 2-2 scattering amplitude implements momentum conservation, but in 4d this is extended to the **conservation of** $\mathcal{Q}_{\mathbf{BMS}}$.

[Strominger '13]

If we start with a familiar two-particle state, the final states are necessarily accompanied by nontrivial **memory**.

[Tolish,Wald '14]
[Strominger, AZ '14]

A formal construction of such (improper) states was recently given. How to perform systematic computations (bootstrap) of **the BMS-matrix elements** in a given gravitational EFT?

[Talk Freidel]
[Prabhu,Satishchandran,Wald '22,'24]
[Bekaert,Donnay,Herfray '24]

One way to avoid this is to consider **inclusive observables**. Consider an initial state

$$|\psi\rangle = \frac{1}{2!}\int \frac{d^3\vec{p}_1}{2|\vec{p}_1|(2\pi)^3}\frac{d^3\vec{p}_2}{2|\vec{p}_2|(2\pi)^3}\psi(\vec{p}_1,\vec{p}_2)|\vec{p}_1,\vec{p}_2\rangle$$

[Talk O'Connell]

on top of a **trivial memory vacuum**. The norm of the state

$$\langle\psi|\psi\rangle = \int d^{d-1}\vec{p}_1 d^{d-1}\vec{p}_2|\psi(\vec{p}_1,\vec{p}_2)|^2 = 1$$

Then energy correlators in such states have to be finite according to general arguments

positivity

$$\langle\psi|S^\dagger\mathcal{E}(\vec{n}_1)...\mathcal{E}(\vec{n}_k)S|\psi\rangle \leq \langle\psi|S^\dagger P_0^k S|\psi\rangle = \langle\psi|P_0^k|\psi\rangle < \infty$$

Wave packets complicate kinematics significantly. We can avoid talking about them by detecting several particles *x* off the beam axis

$$d\sigma_{\vec{p}_1,\vec{p}_2\to x+X}$$

- the initial state has an infinite norm
- soft radiation cancels against virtual corrections [Weinberg]
- collinear radiation is regular [Akhoury,Saotome,Sterman]

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We can then further get rid of of the energies of the final particles by measuring energy fluxes. The simplest thing to try is

$$\langle\vec{p},-\vec{p}|\mathcal{E}(\vec{n}_1)...\mathcal{E}(\vec{n}_k)|\vec{p},-\vec{p}\rangle \overset{?}{<} \infty$$

away from the beam. [LO manifestly IR finite] [Herrmann,Kologlu,Moult '24] [Kologlu,Parra-Martinez, wip]

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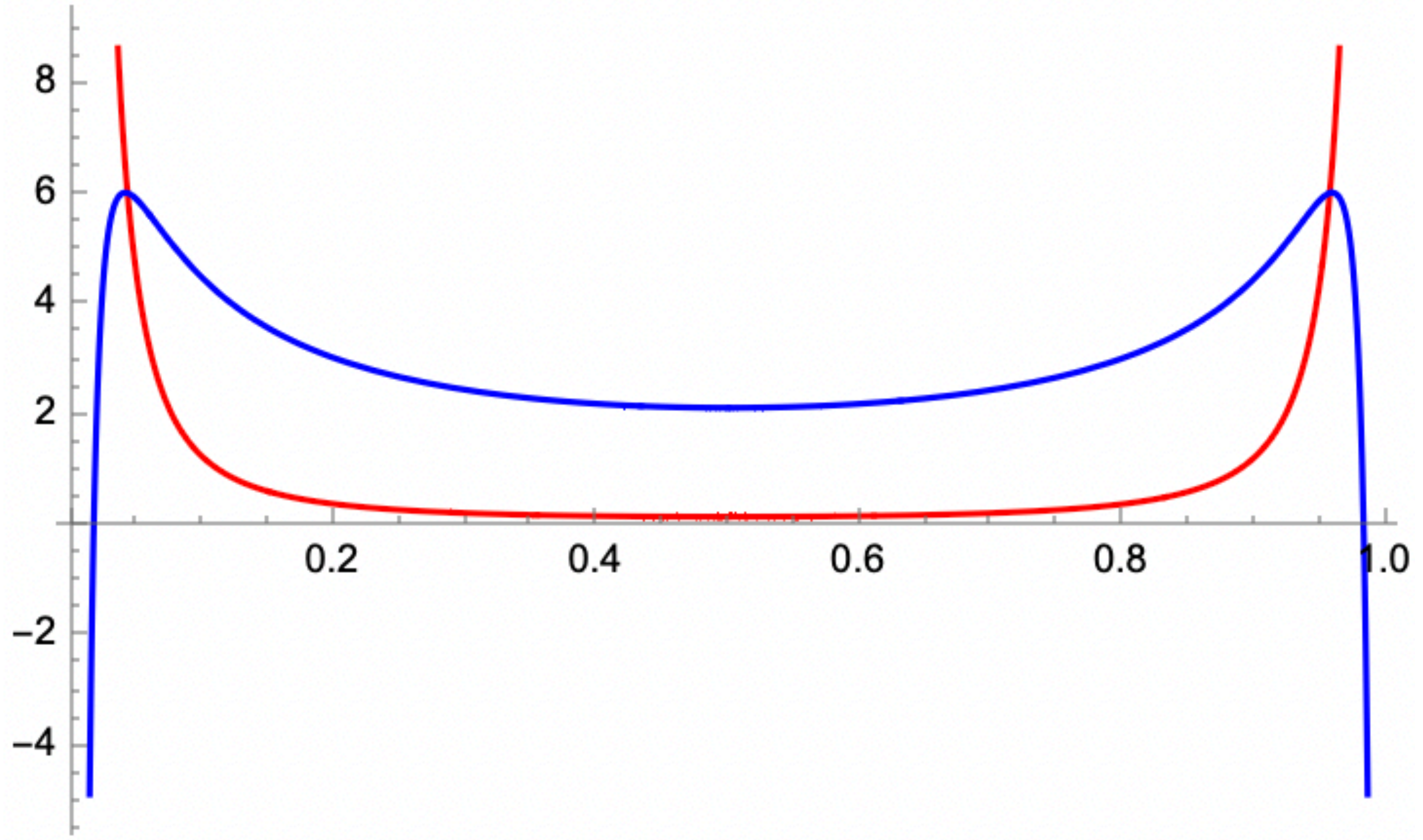
$$(G_N s)^3 \left[\frac{2(1-y)^3 \mathrm{Li}_2(y) - 2y^3 \mathrm{Li}_2(1-y)}{(1-2y)(1-y)^2 y^2} + \frac{2\log(1-y)\log(y)}{(1-y)^2 y^2} + \frac{2\pi^2}{3(1-y)y} + \frac{y \log^2(1-y) - (1-y)\log^2(y)}{(1-2y)(1-y)y} \right]$$

maximal transcedentality



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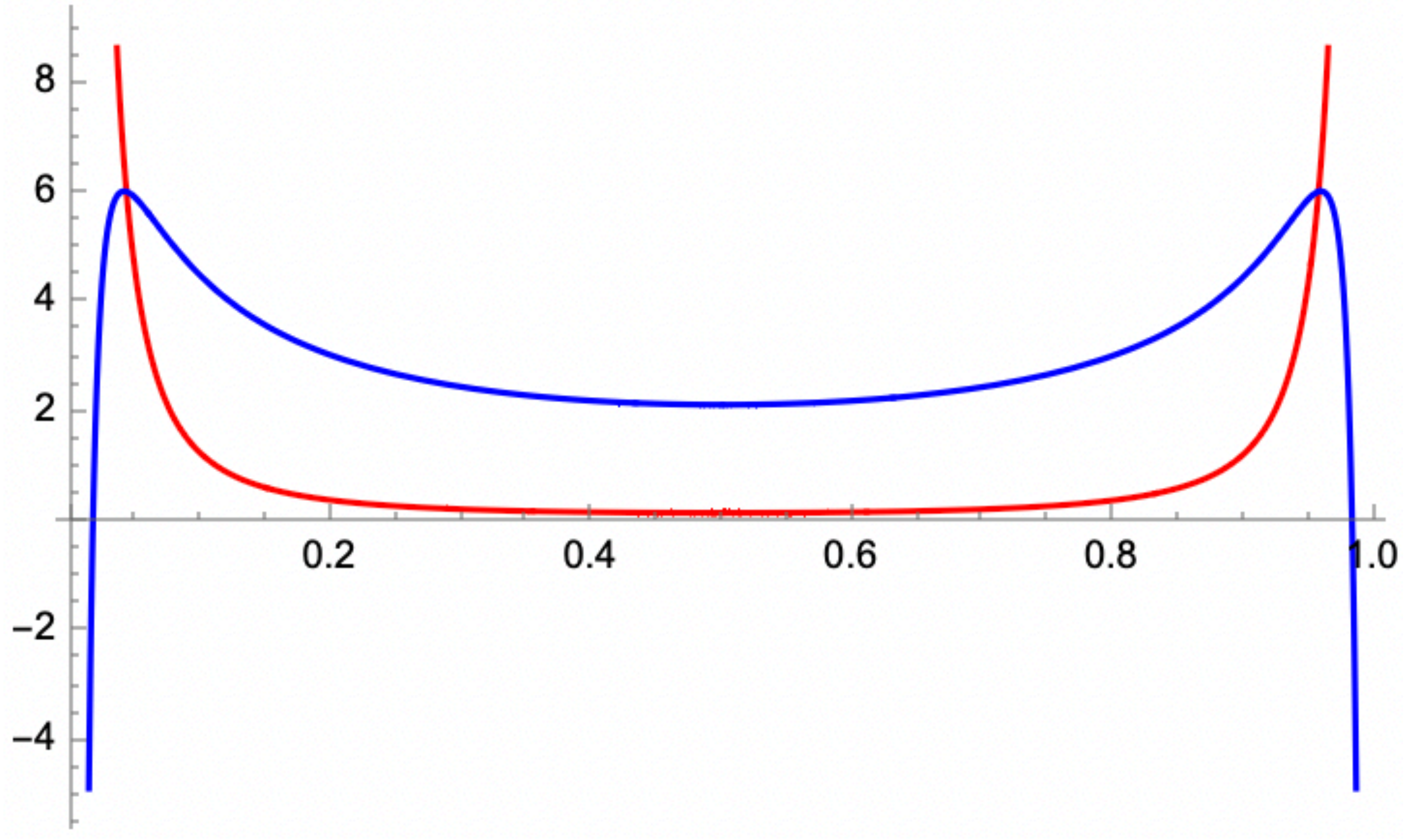

As $G_N s \to \infty$ we expect that

$$\langle \mathcal{E}(\vec{n}) \rangle \simeq \text{const}$$

due to particle/BH production.

[Chicherin,Korchemsky,Sokatchev,AZ]

In the two-point energy correlator $\langle \mathscr{E}(n_1)\mathscr{E}(n_2)\rangle$ IR divergencies cancel in the contact terms $\delta(z)$ and $\delta(1-z)$

$$z = \frac{1 - \cos \vec{n}_1 \vec{n}_2}{2}$$

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$\delta(1-z)$:

$$\frac{\log y\log(1-y)}{y^2(1-y)^2}$$

$0 < z < 1$ [averaged over the beam direction] :

$$\frac{(u^2+1)^2\left(-(1-iu)\mathrm{Li}_2(-iu)-(1+iu)\mathrm{Li}_2(iu)-2u\log(u)\tan^{-1}(u)+\frac{\pi^2}{3}\right)}{u^2} \qquad u=\sqrt{\frac{z}{1-z}}$$

soft theorem
+ t-channel pole

$$z\to 1: \quad \frac{\log^2(1-z)}{1-z}$$

in pure 4d gravity no maximal transcedentality

The **unnormalized off-beam energy correlators** are IR finite, computable, and kinematically simple in 4d. We checked explicitly at one loop, general arguments suggest that it is true all-loop, but it is not yet a theorem.

They obey positivity, analyticity, dispersion relations, permutation symmetry (crossing).

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In a collider context, **normalized energy correlators** have more positivity (in a spherically symmetric state, a BH decay?)

$$\langle\psi|\mathcal{E}(\theta)\mathcal{E}(0)|\psi\rangle = \sum_{J=0}^{\infty}(2J+1)H_J P_J(\cos\theta) \geq 0, \quad H_J \geq 0 \ .$$

[Fox,Wolfram '78]

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[Fox,Wolfram '78]

thank you!

Let us see how the usual total cross-section problem is solved

$$|\langle \vec{k}_1, \vec{k}_2 | \vec{p}_1, \vec{p}_2 \rangle|^2 \sim \frac{1}{t^2} \sim \frac{1}{(1-\cos\theta)^2} \sim \frac{1}{\theta^4}$$

This problem does not arise for the normalized states. Consider the ’t Hooft S-matrix

$$\langle \vec{k}_1, \vec{k}_2 | \vec{p}_1, \vec{p}_2 \rangle = (2\pi)^4 \delta^4(k_1 + k_2 - p_1 - p_2) \frac{\Gamma(1 - iG_N s)}{\Gamma(iG_N s)} \left(\frac{8\pi s}{-t}\right) \left(\frac{\mu_{IR}^2}{-t}\right)^{-iG_N s}$$

thanks to non-trivial **interference** this now leads to a finite result

$$\int d\mu(\tilde{p}_1, \tilde{p}_2) d\mu(p_1, p_2) d\mu(k_1, k_2) \psi^*(\tilde{p}_1, \tilde{p}_2) \psi(p_1, p_2) \langle \vec{\tilde{p}}_1, \vec{\tilde{p}}_2 | \vec{k}_1, \vec{k}_2 \rangle \langle \vec{k}_1, \vec{k}_2 | \vec{p}_1, \vec{p}_2 \rangle$$

which takes the form

$$\int d^{d-2}\Omega_{\vec{n}} \frac{(G_N s)^2}{(1 - \vec{n}\vec{n}_p)^{1-iG_N s}(1 - \vec{n}\vec{n}_{\tilde{p}})^{1+iG_N s}} = (2\pi)^2 \delta(\vec{n}_p - \vec{n}_{\tilde{p}}) + \mathcal{O}(G_N^2)$$

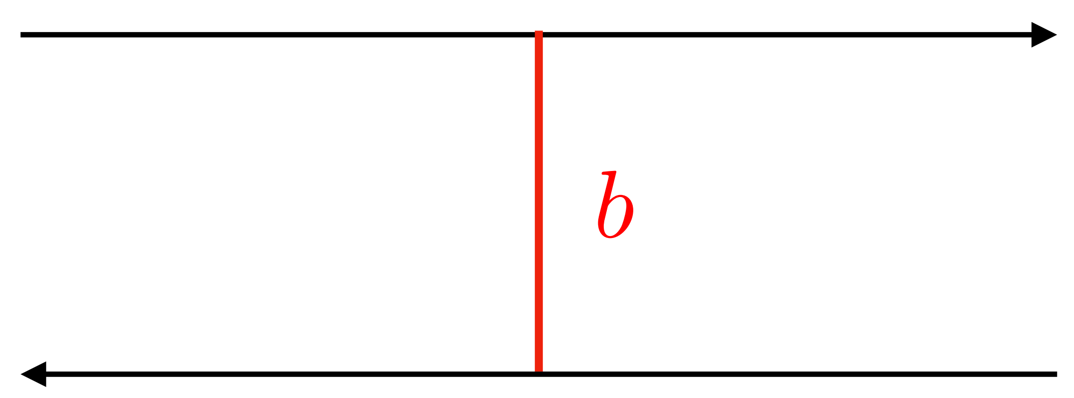
very similar to what happens for jets in QCD.

GN is the UV budget

The solution is to **smear** the amplitude

$$T_\psi(s) \equiv \int_0^{q_0} dq\,\psi(q)\,T(s, t = -q^2)$$

scattering experiment



$b \lesssim 1/q_0$

makes Legendres positive

$$\int_0^{q_0} dq\,\psi(q) P_J\left(1 - \frac{2q^2}{m^2}\right) \geq 0$$

[Caron-Huot,Mazac,Rastelli,Simmons-Duffin '21]

In this way, we can get the following (schematic) equation for graviton scattering

$$G_N = \int_0^\infty \frac{ds'}{\pi} \text{nonnegative}_\psi(s')$$

discontinuity of the amplitude

G_N is the overall UV budget.