Amplitudes to shockwaves in QCD and gravity in Regge asymptotics

Raju Venugopalan

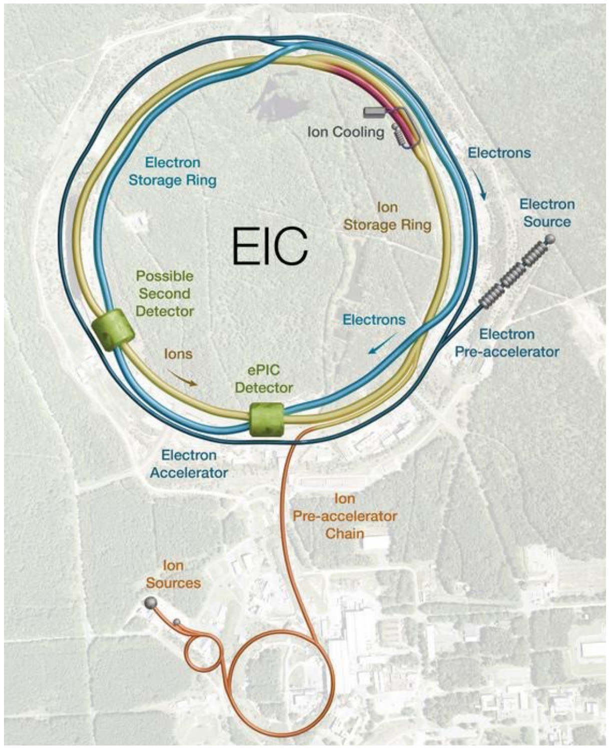
BNL and CFNS, Stony Brook

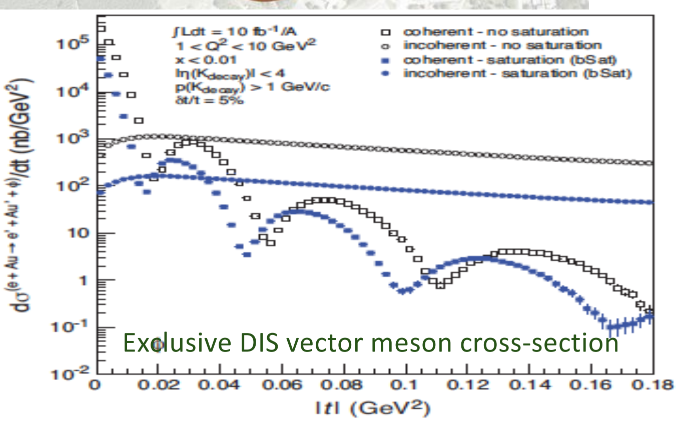
Celestial Holography satellite meeting, April 15-16, 2025

Based in part on recent published work

(arXiv:2311.03463, 2312.03507, 2312. 11652, 2406.10483,+ **2505.XXXXX in preparation**)

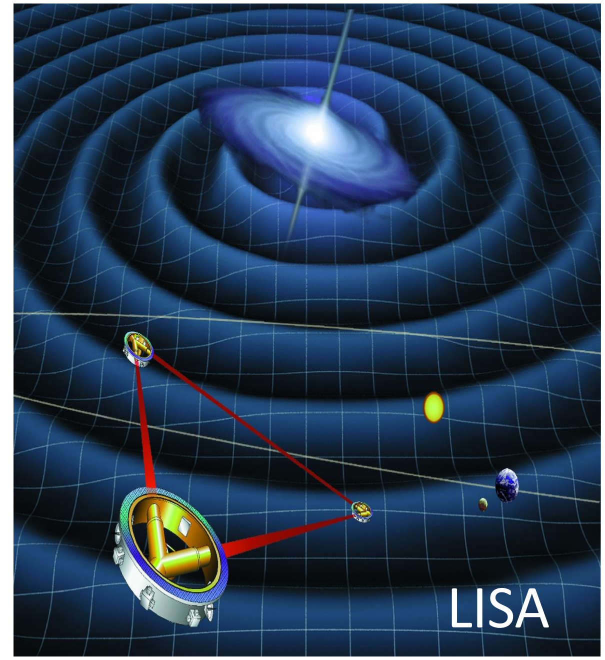
with Himanshu Raj (Simons Confinement+ QCD Strings Collaboration Fellow at Stony Brook)

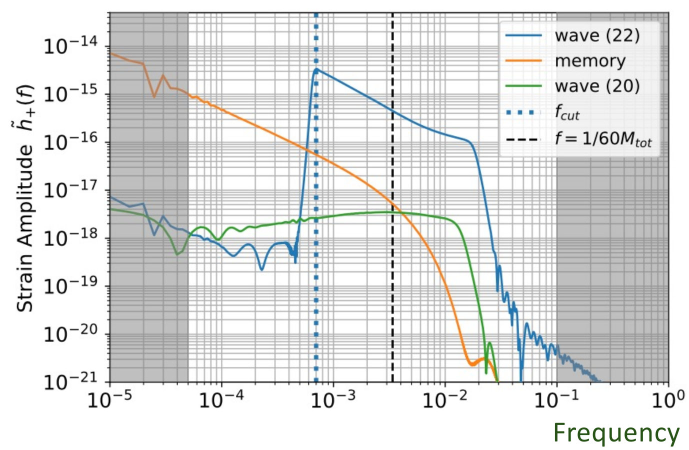




Color memory

GR memory





Color Memory

Monica Pate, Ana-Maria Raclariu and Andrew Strominger

Abstract

A transient color flux across null infinity in classical Yang-Mills theory is considered. It is shown that a pair of test 'quarks' initially in a color singlet generically acquire net color as a result of the flux. A nonlinear formula is derived for the relative color rotation of the quarks. For weak color flux the formula linearizes to the Fourier transform of the soft gluon theorem. This color memory effect is the Yang-Mills analog of the gravitational memory effect.

arXiv:1707.08016

Measuring Color Memory in a Color Glass Condensate at Electron-Ion Colliders

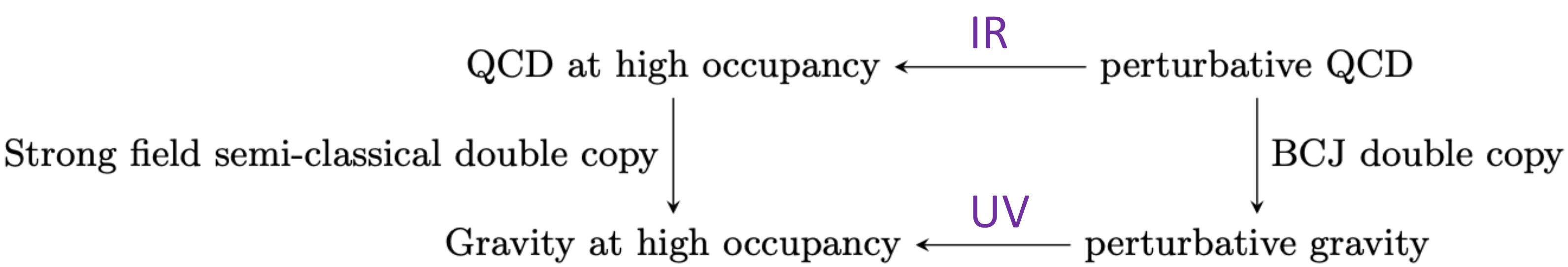
Adam Ball[1], Monica Pate[1], Ana-Maria Raclariu[1], Andrew Strominger[1] and Raju Venugopalan[2]

Abstract

The color memory effect is the non-abelian gauge theory analog of the gravitational memory effect, in which the passage of color radiation induces a net relative SU(3) color rotation of a pair of nearby quarks. It is proposed that this effect can be measured in the Regge limit of deeply inelastic scattering at electron-ion colliders.

arXiv:1805.12224

Double Copy: gluon → gravitational radiation in shockwave collisions



Monteiro,O'Connell,White, arXIv:1410.0239
Goldberger, Ridgeway, arXiv:1611.03493

Bern, Carrasco, Johannson,
arXiv: 1004.0476

Road map of my talk

I) Why can we measure color memory?

A brief DIS primer. The BFKL equation in multi-Regge asymptotics.

Breakdown of the OPE, classical lumps, their quantum descendants.

Color memory in the Color Glass Condensate.

Precision measurements of the CGC at the Electron-Ion Collider.

II) QCD-Gravity double copy in Regge asymptotics

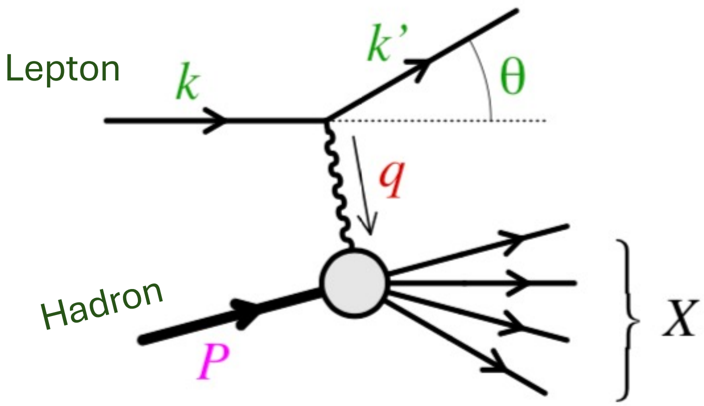
2→ N scattering, the Lipatov double copy and reggeization.

From amplitudes to shockwaves.

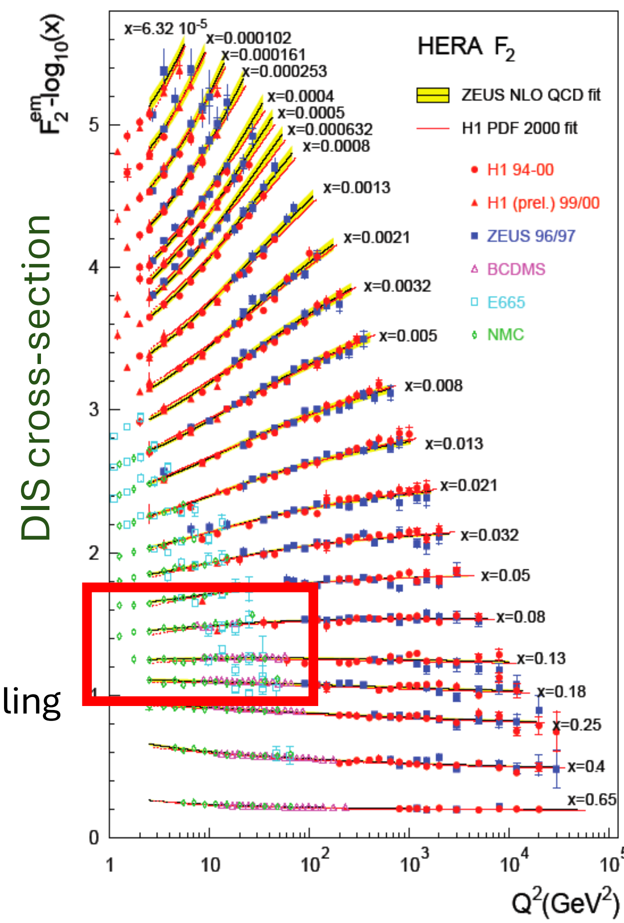
The Lipatov vertex and shockwave propagators.

Some consequences.

DIS and the QCD revolution



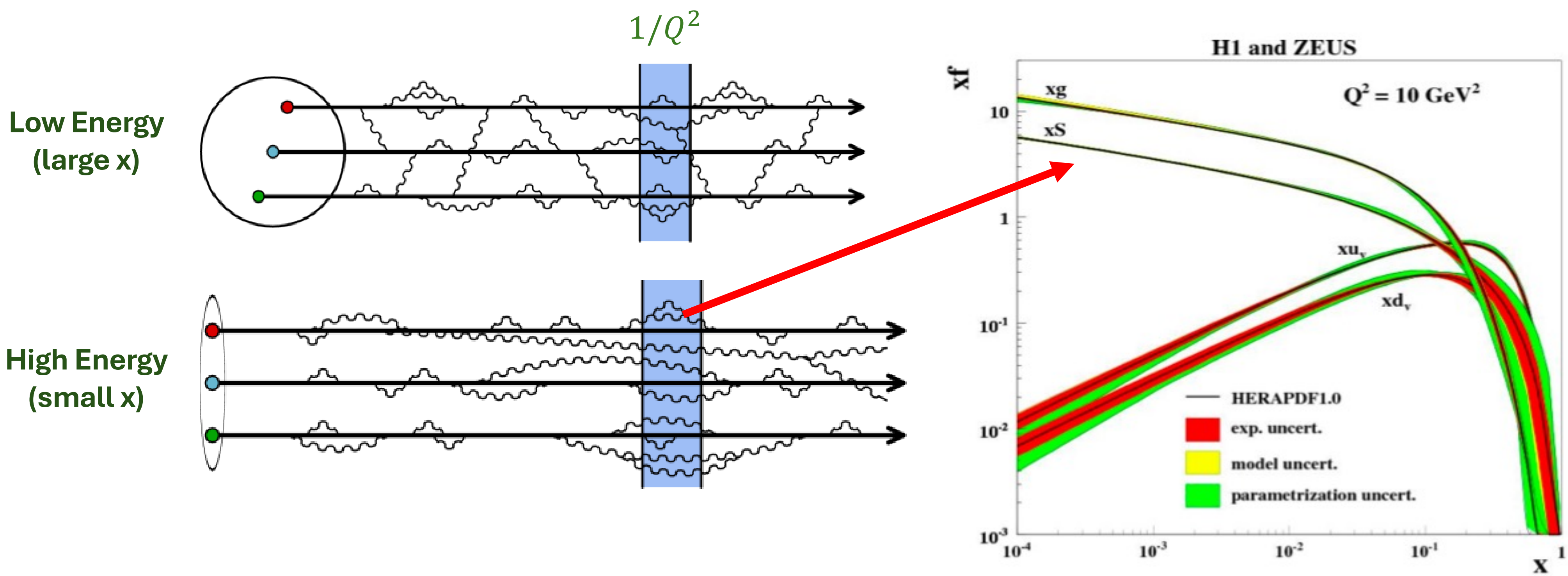
The discovery of Bjorken scaling in DIS led to the parton model, asymptotic freedom, and QCD, shattering existing paradigms…



Bjorken scaling

Bj in memorium:
https://indico.slac.stanford.edu/event/9148/timetable/

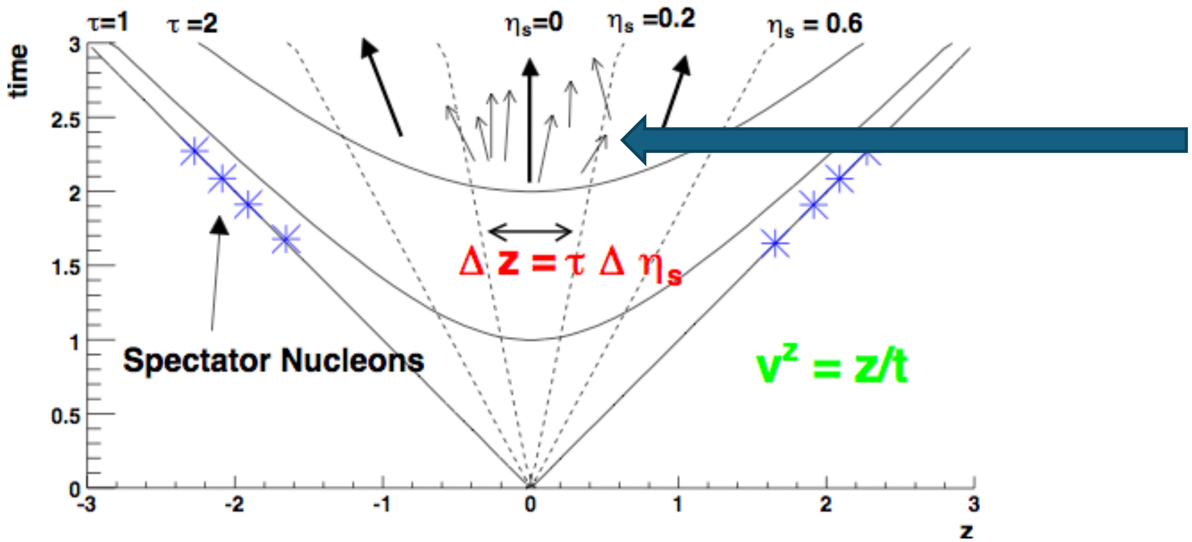
Spacetime picture of wee partons in a hadron

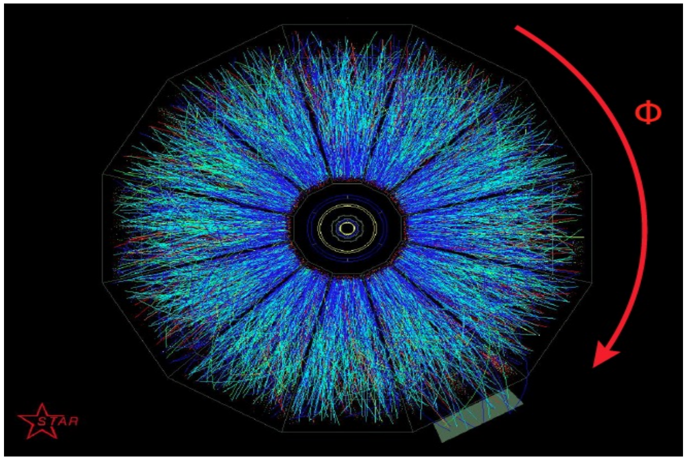


As the proton is boosted, “parton” fluctuations live longer -- released as Bremsstrahlung

Suppression in coupling compensated by large phase space for soft glue: $\alpha_S\, Ln\left(\frac{1}{x}\right) \sim 1$

Spacetime picture of a high energy hadron-hadron collision





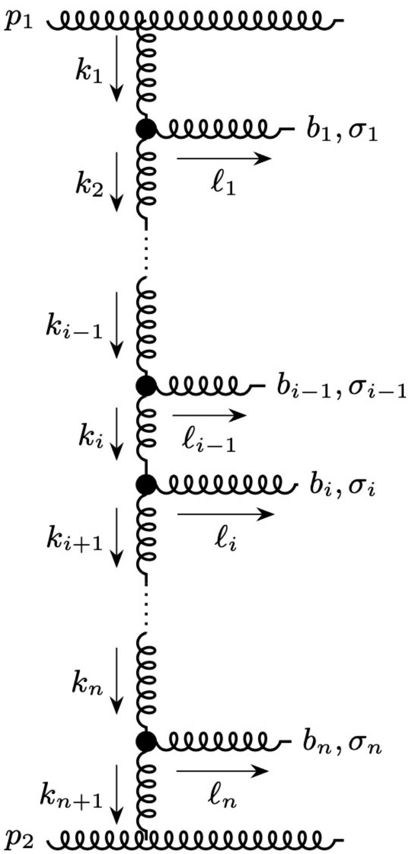
$\eta_s = \frac{1}{2}\,\mathrm{Ln}\,(\frac{t+z}{t-z}) \approx Y$

Fast "valence" partons populate fragmentation regions at large rapidities – "leading particle" effect

Slow "wee" partons populate central rapidities (mostly gluons and sea-quark pairs)
– they create a Quark-Gluon-Plasma

QCD thermalization: Ab initio approaches and interdisciplinary connections
Jürgen Berges, Michal P. Heller, Aleksas Mazeliauskas, RV
Rev. Mod. Phys. **93**, 035003 (2021)

BFKL: 2→ N QCD amplitudes in Regge asymptotics



Compute multiparticle in multi-Regge kinematics of QCD:

$$y_0^+ \gg y_1^+ \gg y_2^+ \gg \cdots \gg y_N^+ \gg y_{N+1}^+ \qquad \text{with} \qquad \boldsymbol{k}_i \simeq \boldsymbol{k}$$

BFKL ladder is ordered in rapidity . Produced partons are wee in longitudinal momentum(` `slow”) but hard in transverse momentum – weak coupling Regge regime of QCD

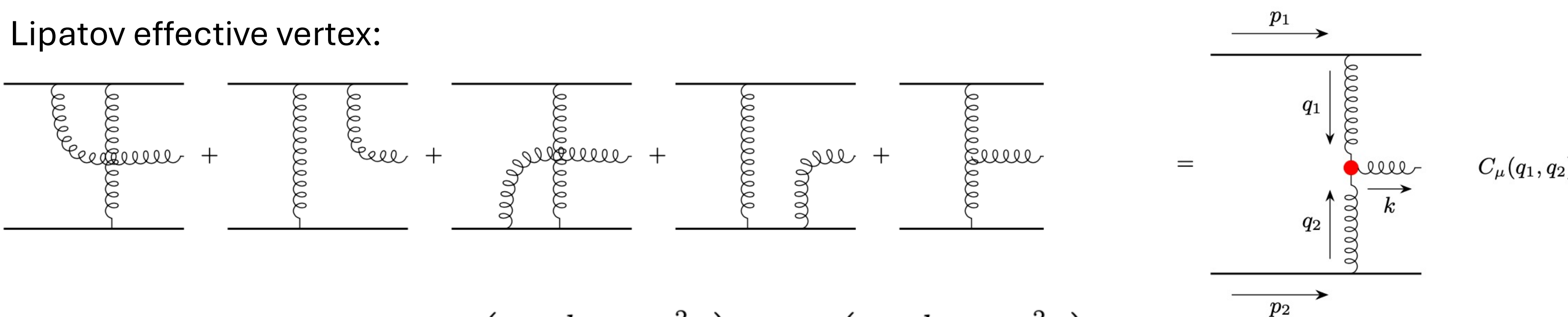
RG description rapidity of evolution given by the BFKL Hamiltonian
Very rapid growth of the amplitude with energy

A(s,t) = $s^{\alpha(t)}$ with $\alpha(t) = \alpha_0 + \alpha' \, |t|$ BFKL pomeron

BFKL: Balitsky-Fadin-Kuraev-Lipatov (1976-1978)

BFKL: Building blocks

Lipatov effective vertex:

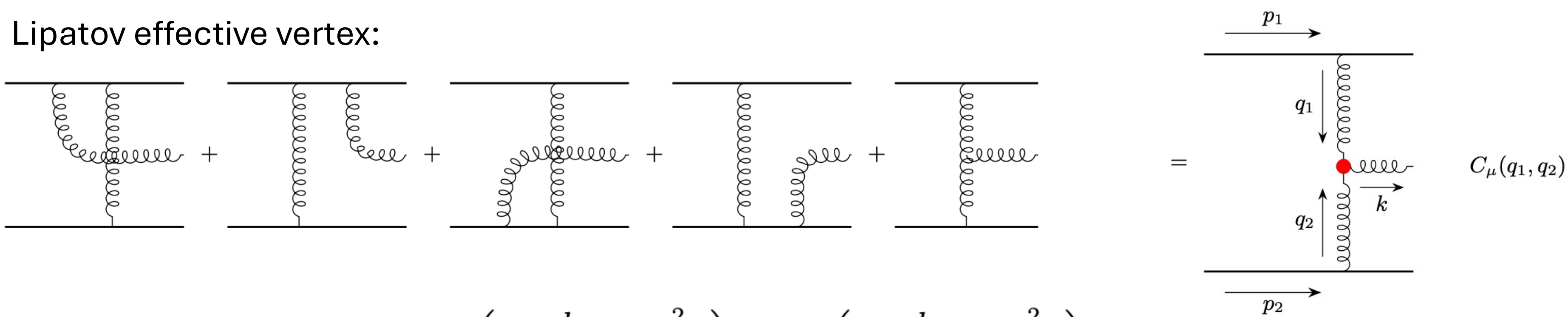


$$C_\mu(\boldsymbol{q}_1, \boldsymbol{q}_2) \simeq -\boldsymbol{q}_{1\mu} + \boldsymbol{q}_{2\mu} + p_{1\mu}\left(\frac{p_2 \cdot k}{p_1 \cdot p_2} - \frac{\boldsymbol{q}_1^2}{p_1 \cdot k}\right) - p_{2\mu}\left(\frac{p_1 \cdot k}{p_1 \cdot p_2} - \frac{\boldsymbol{q}_2^2}{p_2 \cdot k}\right)$$

Gauge covariant, satisfies $k_\mu\, C^\mu$ =0

BFKL: Building blocks

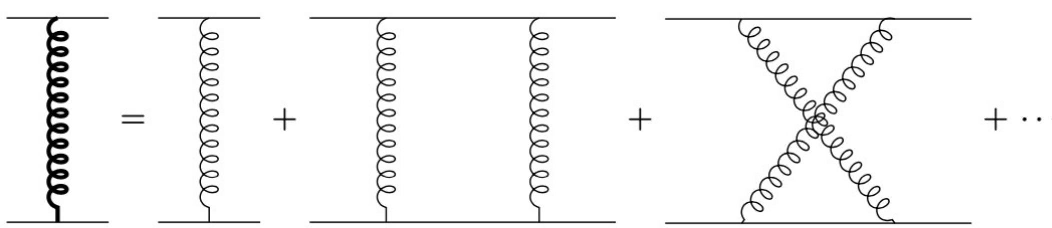
Lipatov effective vertex:



$$C_\mu(\boldsymbol{q}_1, \boldsymbol{q}_2) \simeq -\boldsymbol{q}_{1\mu} + \boldsymbol{q}_{2\mu} + p_{1\mu}\left(\frac{p_2 \cdot k}{p_1 \cdot p_2} - \frac{\boldsymbol{q}_1^2}{p_1 \cdot k}\right) - p_{2\mu}\left(\frac{p_1 \cdot k}{p_1 \cdot p_2} - \frac{\boldsymbol{q}_2^2}{p_2 \cdot k}\right)$$

Gauge covariant, satisfies $k_\mu \, C^\mu = 0$

Reggeized gluon:



$$\frac{1}{t_i} \to \frac{1}{t_i} e^{\alpha(t_i)(y_{i-1} - y_i)}$$

$$\alpha(t) = \alpha_s N_c \, t \int \frac{d\boldsymbol{k}}{(2\pi)^2} \frac{1}{\boldsymbol{k}^2 (\boldsymbol{q} - \boldsymbol{k})^2} \,, \qquad t = -\boldsymbol{q}^2$$

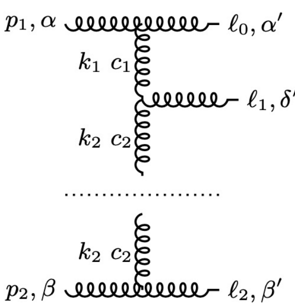
Some features of the BFKL equation and its solution

Key to the construction of the BFKL ladder are multi-Regge asymptotics and dispersive techniques

Example: 2→ 3 ampltude

3g vertex

Building block is the 2 → 2 Born amplitude and three gluon vertex $\mathcal{A}^{\alpha\alpha'\beta\beta'}_{2\to 2, p_1+p_2\to \ell_0+\ell_1} = \Gamma^{\alpha\alpha' c}_{p_1\ell_0}\frac{s}{t}\Gamma^{\beta\beta' c}_{p_2\ell_1}$



For 2→ 3, pole contribution in k_2 can be expressed in terms of the 2 → 2 amplitude

$$P_{k_2^2}\mathcal{A}^{\alpha\alpha'\beta\beta'\delta'}_{2\to 2+1} = \mathcal{A}^{\alpha\alpha' c_2\delta'}_{p_1+(-k_2)\to \ell_0+\ell_1}\frac{s}{k_2^2}\Gamma^{\beta\beta' c_2}_{p_2\ell_2} \Rightarrow -ig\frac{s}{k_1^2 k_2^2}\Gamma^{\alpha\alpha' c_1}_{p_1\ell_0}\Gamma^{\beta\beta' c_2}_{p_2\ell_2} f^{c_2\delta' c_1} \times kinematic\ factors$$

The k_1 pole can be expressed similarly allowing one to reconstruct the full amplitude

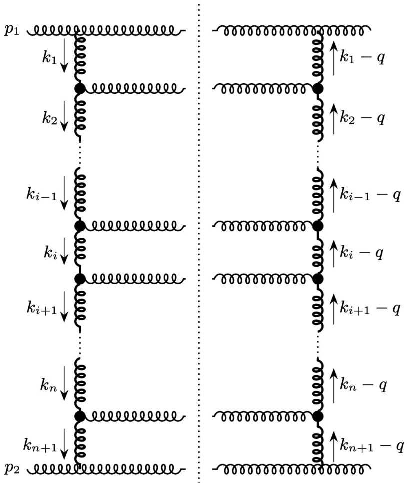
$$\mathcal{A}^{\alpha\alpha'\beta\beta'\delta'}_{2\to 2+1} = ig\frac{s}{k_1^2 k_2^2}\Gamma^{\alpha\alpha' c_1}_{p_1\ell_0}\Gamma^{\beta\beta' c_2}_{p_2\ell_2} f^{c_1\delta' c_2} C_\nu(k_1, k_2)\epsilon^\nu(\ell_1)$$

Lipatov vertex

This process is iterated to all orders to construct the BFKL ladder – the same procedure will apply in gravity…

$2 \to \mathrm{N} + 2$ amplitude in the Regge limit: the BFKL equation

BFKL Pomeron: compound color singlet state of two reggeized gluons



The imaginary part of this $2 \to \mathrm{N} + 2$ amplitude simplifies greatly in Mellin space

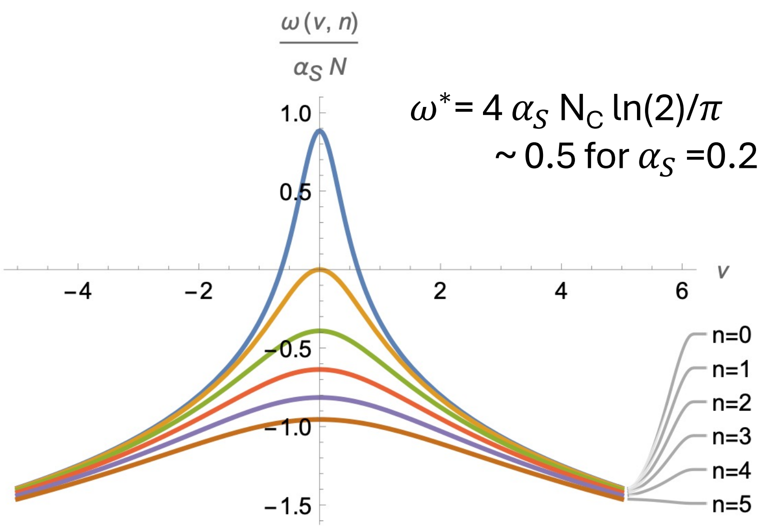
$$\mathcal{M}_\ell(\boldsymbol{q}^2) \equiv \int_1^\infty d\left(\frac{s}{\boldsymbol{k}^2}\right) \frac{\mathrm{Im}\, \mathcal{A}_{2\to 2}^{\mu\mu'\nu\nu'}(s,t)}{\mathcal{A}_0^{\mu\mu'\nu\nu'}(s,t)} \left(\frac{s}{\boldsymbol{k}^2}\right)^{-\ell-1}$$

with $$\mathcal{M}_\ell(\boldsymbol{q}^2) = 2\pi \boldsymbol{q}^2 \alpha_s N_c^2 (N_c^2 - 1) \int \frac{d^2\boldsymbol{k}}{(2\pi)^2} \frac{1}{\boldsymbol{k}^2(\boldsymbol{q}-\boldsymbol{k})^2} f_\ell(\boldsymbol{k},\boldsymbol{q})$$

where $f_l(k,q)$ satisfies the BFKL integral equation

$$(\ell - \alpha(\boldsymbol{k}^2) - \alpha((\boldsymbol{q}-\boldsymbol{k})^2)) f_\ell(\boldsymbol{k},\boldsymbol{q}) = 1 - 2\alpha_s N_c \int \frac{d^2\boldsymbol{k}'}{(2\pi)^2} \frac{f_\ell(\boldsymbol{k}',\boldsymbol{q})}{\boldsymbol{k}'^2(\boldsymbol{q}-\boldsymbol{k}')^2} \left(\boldsymbol{q}^2 - \frac{\boldsymbol{k}^2(\boldsymbol{q}-\boldsymbol{k}')^2 + \boldsymbol{k}'^2(\boldsymbol{q}-\boldsymbol{k})^2}{(\boldsymbol{k}-\boldsymbol{k}')^2}\right)$$

$2 \to N + 2$ amplitude in the Regge limit: the BFKL equation



BFKL eigenvalues

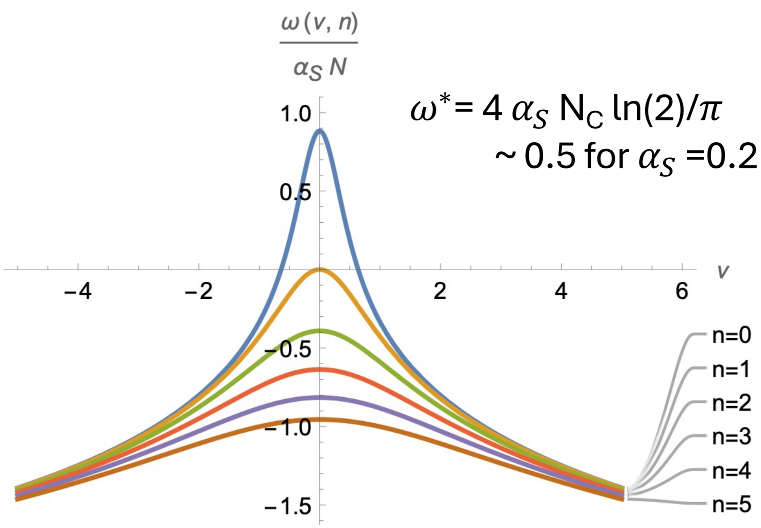
Solved as a simple eigenvalue equation as a function of Fourier conjugate variables -conformal spin ν and azimuthal variable n

Performing the inverse Mellin transform, one obtains

$\sigma(s) \sim s^{\omega^*} = s^{0.5}$

a much faster growth than the $Ln^2(s)$ predicted by Froissart…

$2 \rightarrow \mathrm{N}+2$ amplitude in the Regge limit: the BFKL equation



$\omega^{*} = 4\,\alpha_S\,N_C \ln(2)/\pi$
~ 0.5 for $\alpha_S = 0.2$

BFKL eigenvalues

This so-called LLx (leading log in x) result has been extended to NLLx accuracy.

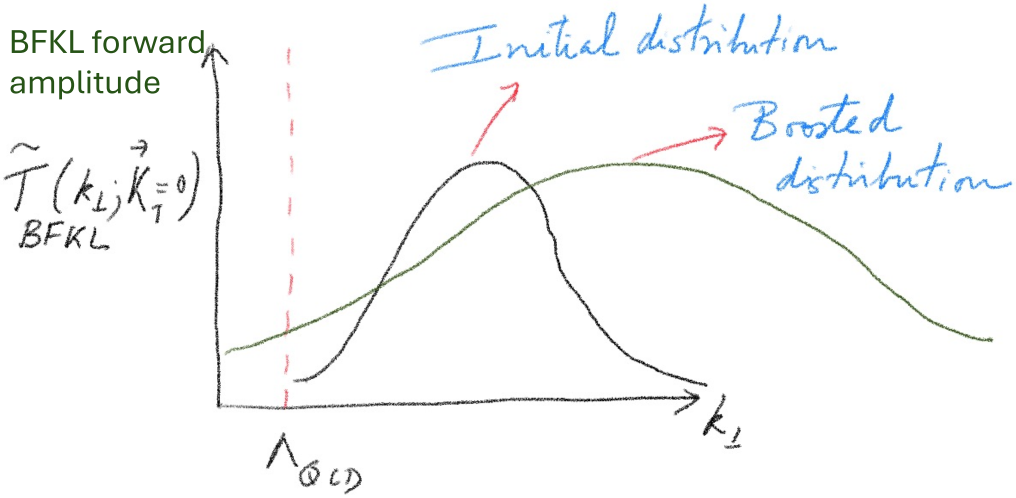
Excellent review of state-of-the-art:
Del Duca, Dixon, arXiv:2203.13026

After much sophisticated analysis, this gives

$\sigma(s) \sim s^{0.3}$ - in reasonable agreement with HERA data

Not the full story…
and only a preview to a richer, many-body picture

BFKL: infrared diffusion and gluon saturation



For a fixed large Q^2 there is an $x_0(Q^2)$ such that below x_0 the OPE breaks down…

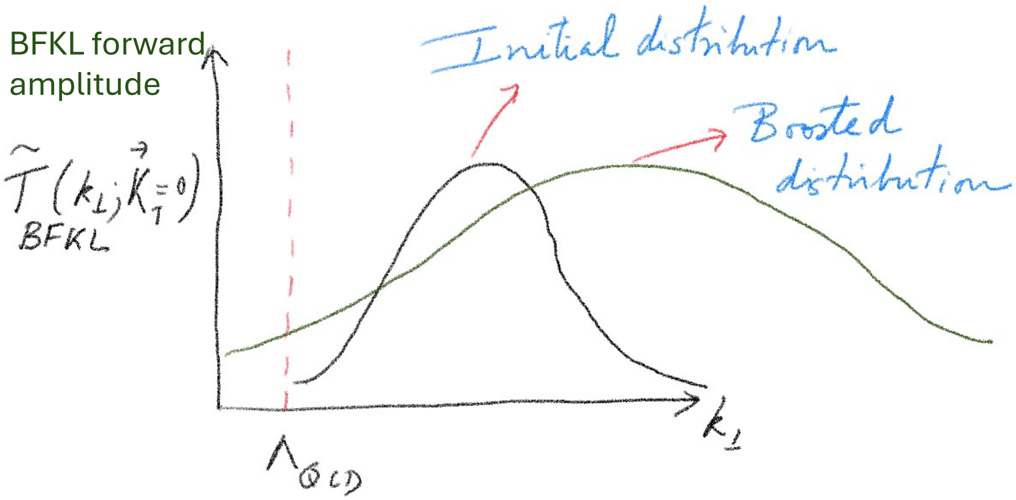
significant nonperturbative corrections in the leading twist coefficient and anomalous dimension functions due to diffusion of gluons to small values of transverse momentum.

A. H. Mueller, PLB 396 (1997) 251

NLLx BFKL does not cure infrared diffusion

Gluon saturation cures infrared diffusion

BFKL: infrared diffusion and gluon saturation



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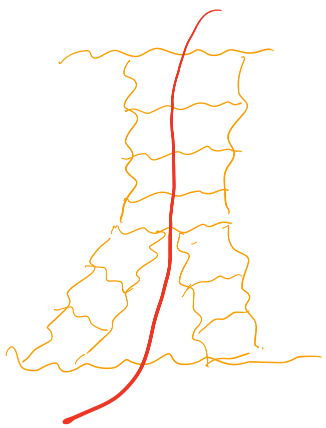
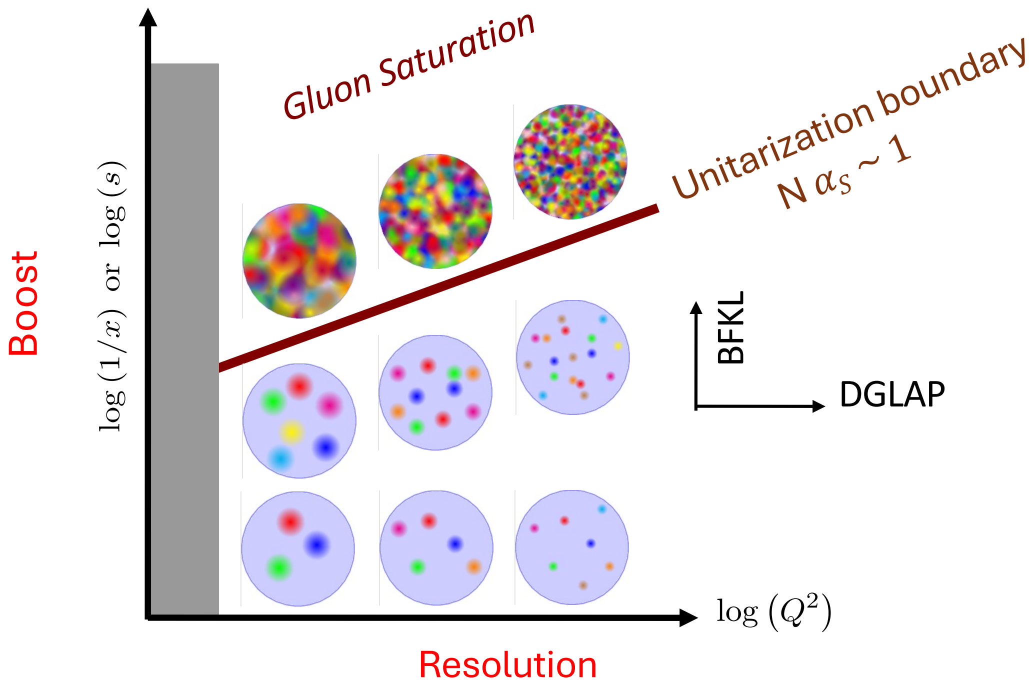
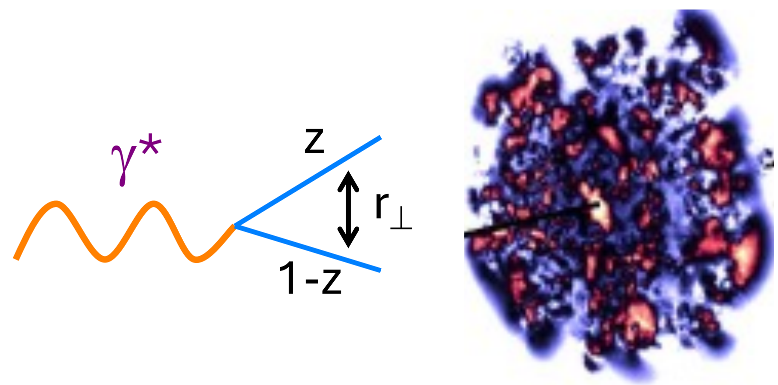


\+ other higher twist cuts of O(1) when gluon occupancy $N \equiv \frac{xG_A(x, Q_S^2)}{2(N_c^2 - 1)\pi R_A^2 Q_S^2} = \frac{1}{\alpha_S(Q_S)}$

Classicalization when $\alpha_S(Q_S) \ll 1$ for saturation scale $Q_S \gg \Lambda_{QCD}$

Maximal packing of gluons: Gluon saturation





Color screened wee partons
live on surface of sphere of radius
$1/k^+ \sim 1/k_\perp \sim 1/Q_S$ (x, b)

Emergent dynamics of semi-classical lumps that unitarize the cross-section described by the saturation scale Q_S(x,b).

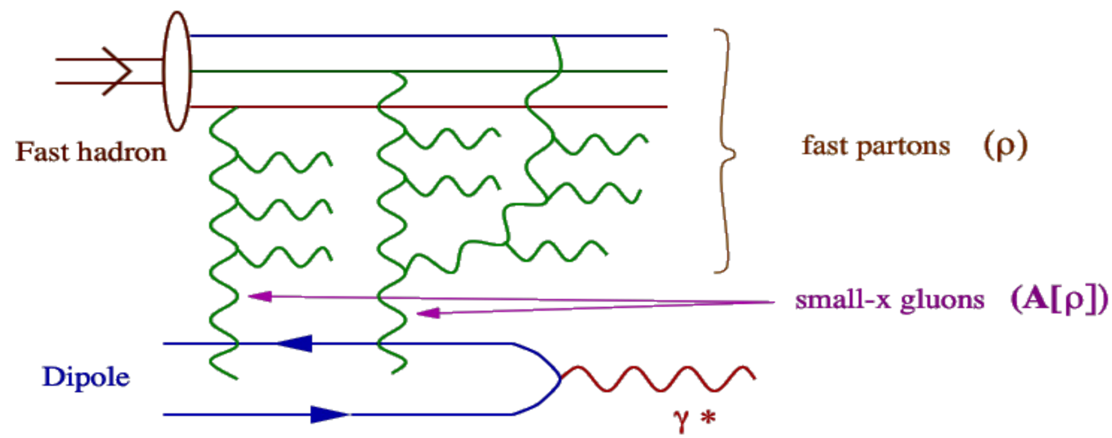
Due to asymptotic freedom, the many-body strong field dynamics of these lumps can be computed in weak coupling

Color Glass Condensate: classical EFT for Regge asymptotics

Born-Oppenheimer separation between fast and slow light-front modes (Galilean EFT)

Large x (P^+) modes: static, strong (~1/g) color sources ρ^a

Small x ($k^+ << P^+$) modes: fully dynamical gauge fields A_μ^a



$$\mathcal{Z}[j] = \int [d\rho] W_{\Lambda^+}[\rho] \left\{ \frac{\int^{\Lambda^+} [dA]\delta(A^+) e^{iS_{\Lambda^+}[A,\rho] - \int j\cdot A}}{\int^{\Lambda^+} [dA]\delta(A^+) e^{iS_{\Lambda^+}[A,\rho]}} \right\}$$

$W_{\Lambda^+}[\rho]$: nonpert. gauge inv. weight functional defined at initial $x_0 = \Lambda^+ / P^+$
$S_{\Lambda^+}[A, \rho]$: Yang-Mills action + gauge-inv. coupling of sources to fields (Wilson line)

CGC review: Gelis,Iancu,Jalilian-Marian,RV:arXiv 1002.0333

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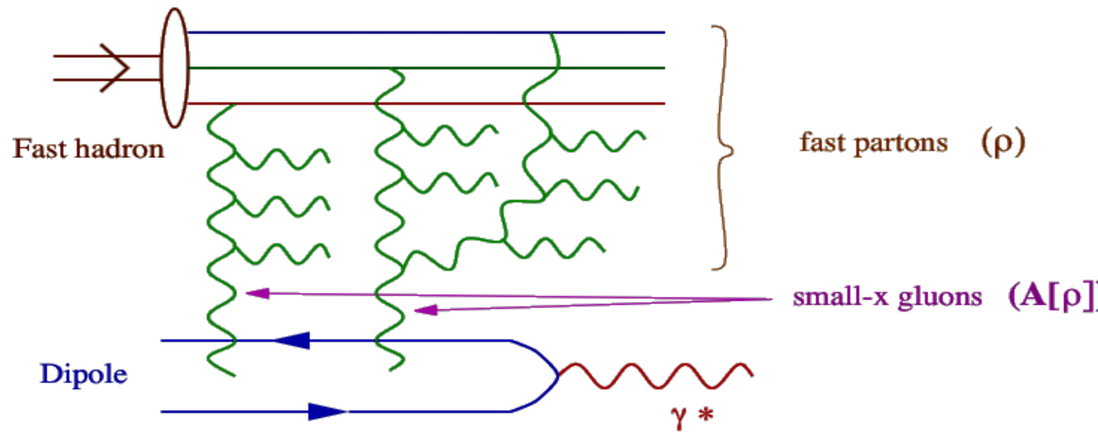


Explicit construction for large nuclei (= large number of coherent sources of color charge at small x)

$$W_{\Lambda^+}[\rho] \to \int [d\rho] \exp\left(-\int d^2x_\perp \left[\frac{\rho^a\rho^a}{2\mu_A^2} - \frac{d_{abc}\rho^a\rho^b\rho^c}{\kappa_A}\right]\right)$$

Pomeron configurations

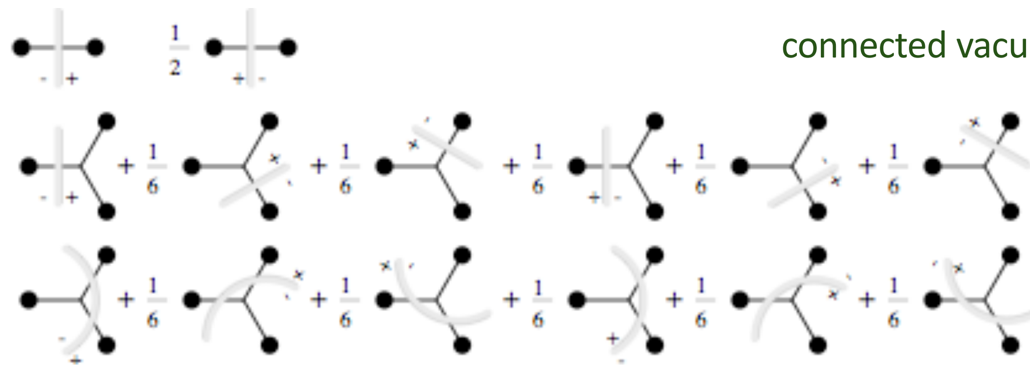
Odderon configurations

For A>>1, $\mu_A^2 \sim Q_S^2 \propto A^{1/3} >> \Lambda_{QCD}^2$
weak coupling EFT for large parton densities!

McLerran,RV (1993)

General all-order formalism: Cutkosky's rules in strong fields

$$2\,\mathrm{Im}\sum_{\mathrm{conn.}} V =$$



connected vacuum graphs in $\lambda\phi^3$

Propagators on Schwinger-Keldysh contour

Well-known example: Schwinger pair production in strong field QED

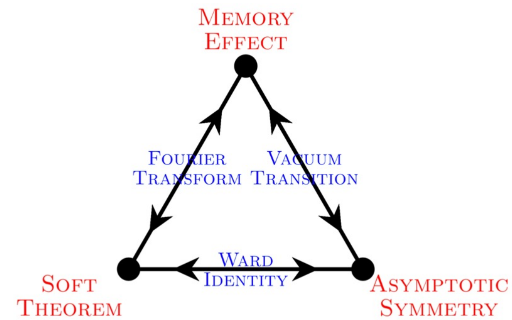
Simple understanding of "AGK cutting rules" of Reggeon Field Theory: combinatorics of cut and uncut sub-graphs contributing to a given multiplicity

AGK: Abramovsy,Gribov,Kancheli

- Very general consequence of unitarity in strong fields
- Independent of language of Pomerons and Reggeons

Gelis,RV: hep-ph/0601209, hep-ph/0608117

Color memory in the CGC



Strominger,arXiv:1703.05448

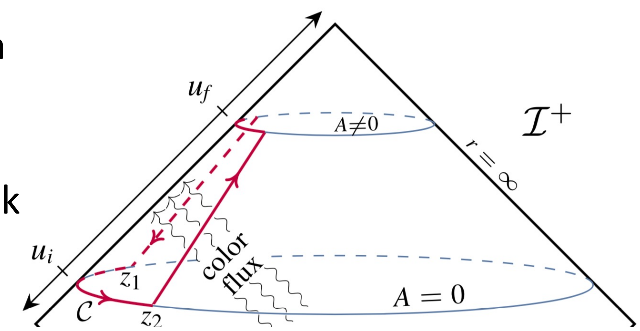
Static Yang-Mills shockwave wave solution in LC gauge

$A_i = 0$ | $A_i = -\frac{-1}{ig} U \partial_i U^\dagger$

$x^- = 0$

Transverse dynamics can be mapped on to celestial sphere at null infinity:

Kick Q_S suffered by dipole crossing shock is the color memory effect



$$(r,u,z,\bar{z}) \to (\lambda r, \lambda^{-1} u, \lambda^{-1} z, \lambda^{-1} \bar{z})$$

Map: $x^+ = \sqrt{2r}$, $x^- = \frac{1}{\sqrt{2}}(u + rz\bar{z})$,

$x^1 + i x^2 = 2\, r\, z$

For $\lambda \to \infty$:

$r \to \infty \;\to\; x^+ \to \infty, x^- \to 0$

Pate, Raclariu, Strominger, PRL (2017)

Ball,Pate,Raclariu,Strominger,RV, Ann. Phys. 407 (2019) 15

The Wilson lines $U = P \exp\left(i\int_y^\infty dy' \frac{\rho(x_t, y')}{\nabla_t^2} \right)$ are vertex operators on the celestial sphere

Satisfy a 2-D conformal Kac-Moody algebra

V.P. Nair (1988)

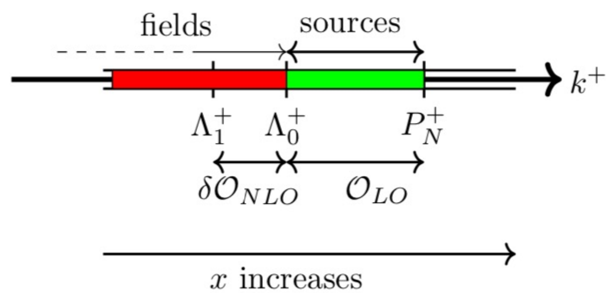
He,Mitra,Strominger (2015)

Boosting memory in CGC EFT: RG hierarchy of many-body correlators

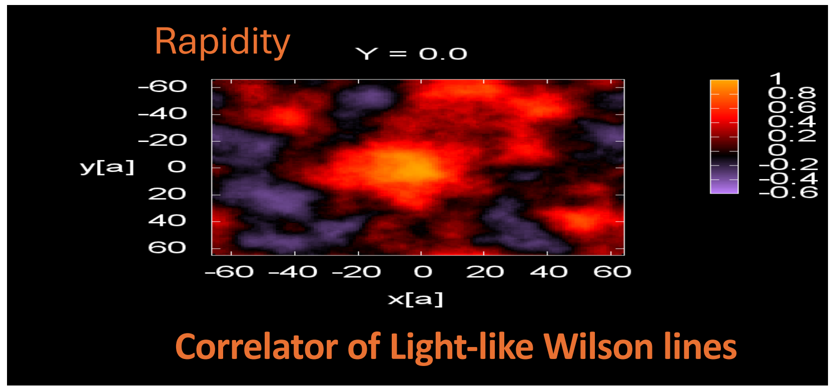
$$\frac{\partial}{\partial Y}\langle \mathcal{O}[\rho]\rangle_Y = \frac{1}{2}\langle \int_{x,y} \frac{\partial}{\partial \rho^a(x)} \chi^{ab}_{x,y} \frac{\partial}{\partial \rho^b(y)} \mathcal{O}[\rho]\rangle_Y$$

Rapidity → ``time”

“diffusion coefficient”: retarded Green function in strong field background



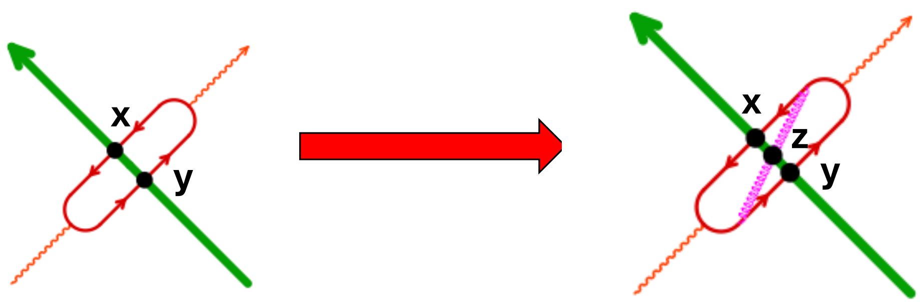
Langevin diffusion of “wee” partons
in functional space of color fields
B-JIMWLK hierarchy of n-point Wilson line correlators



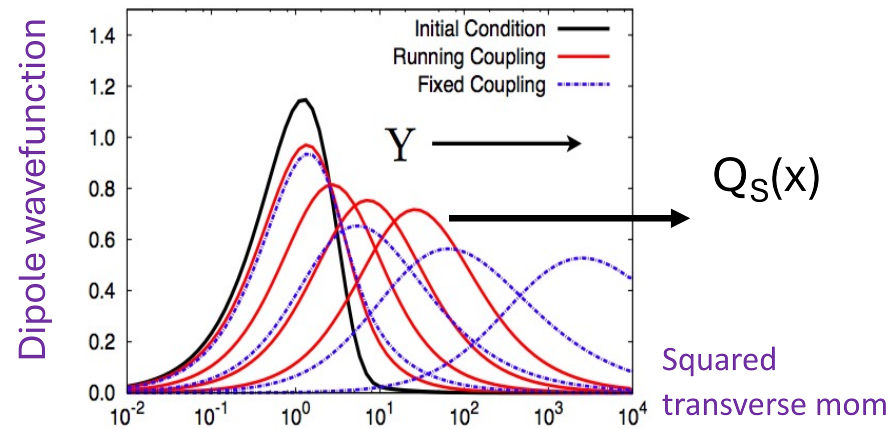
Dumitru,Jalilian-Marian,Lappi,Schenke,RV
PLB706 (2011)219

Balitsky, hep-ph/9509348
Jalilian-Marian,Kovner, Leonidov, Weigert, hep-ph/9706377
Iancu, Leonidov, McLerran, hep-ph/0011241

How memory evolves in inclusive DIS



Evolution of dipole correlators of Wilson lines in shockwave background

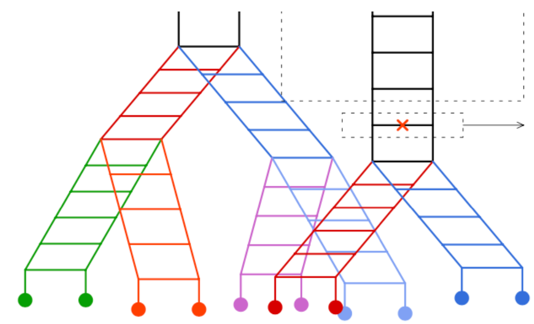


$$\frac{\partial}{\partial Y}\langle \mathrm{Tr}(V_x V_y^\dagger)\rangle_Y = -\frac{\alpha_S N_c}{2\pi^2}\int_{z_\perp} \frac{(x_\perp - y_\perp)^2}{(x_\perp - z_\perp)^2 (z_\perp - y_\perp)^2}\,\langle \mathrm{Tr}(V_x V_y^\dagger) - \frac{1}{N_c}\mathrm{Tr}(V_x V_z^\dagger)\,\mathrm{Tr}(V_z V_y^\dagger)\rangle_Y$$

Y = Ln(1/x)

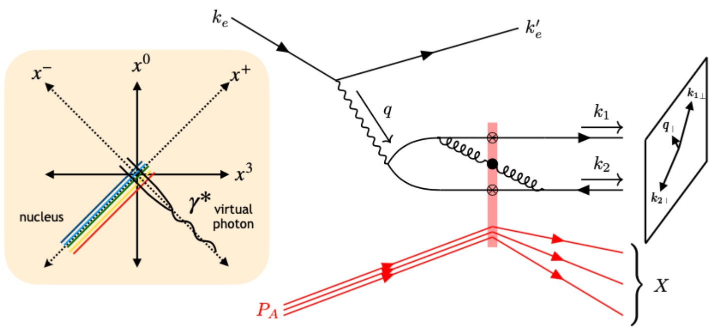
Closed form expression for A >>1, Nc→ ∞: non-linear Balitsky-Kovchegov (BK) eqn.

BFKL equation for dipoles is the "low density" linearized form of this equation!



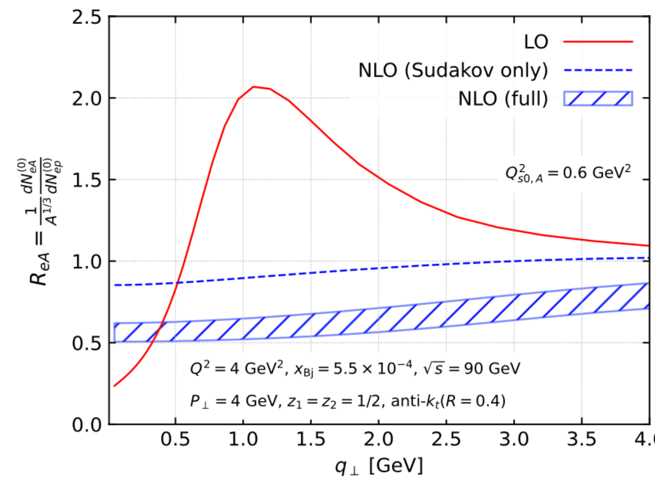
"Fan multi-Pomeron" diagrams

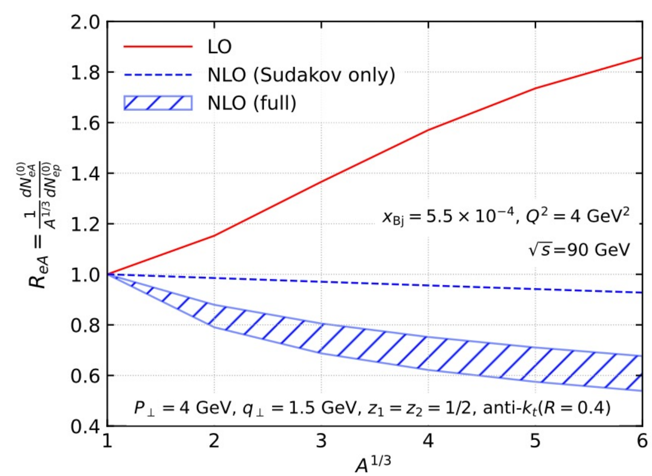
Precision measurements of memory at the EIC



Measure imbalance of back-to-back di-jets in DIS

Caucal,Salazar,Schenke,Stebel,RV, PRL 132 (2024) 8, 081902



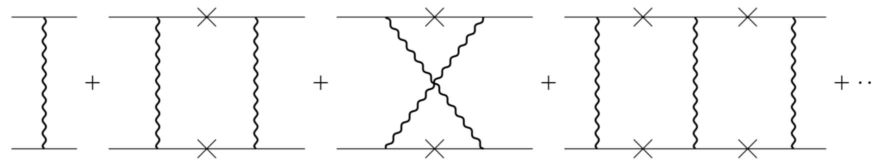


Global analyses to extract "universal" features
from p+A collisions at the LHC and e+A collisions from the EIC

Large # of inclusive, semi-inclusive, exclusive and diffractive final states

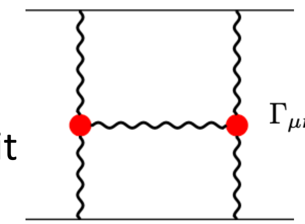
Multiparticle production in gravity: amplitudes to shockwave collisions

A) In gravity, the dominant contribution at large impact parameters is Eikonal scattering



B) Gravitational Lipatov vertex is dominant radiative correction in Regge kinematics – smoothly matches to Weinberg in the ultrarelativistic soft limit

Lipatov, PLB 116B (1982); JETP 82 (1982)



$$\Gamma_{\mu\nu}(\boldsymbol{q}_1,\boldsymbol{q}_2) \equiv \frac{1}{2}C_\mu(\boldsymbol{q}_1,\boldsymbol{q}_2)C_\nu(\boldsymbol{q}_1,\boldsymbol{q}_2) - \frac{1}{2}N_\mu(\boldsymbol{q}_1,\boldsymbol{q}_2)N_\nu(\boldsymbol{q}_1,\boldsymbol{q}_2)$$

Double copy of QCD Lipatov vertex

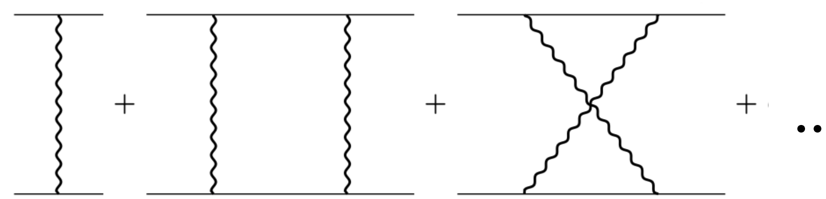
Double copy of QED bremsstrahlung vertex

C) Reggeization contributions suppressed except for b → R_s

$$\mathcal{M}^{(1)} \sim \frac{\kappa^2}{8\pi^2}\left(-i\pi s \log\left(\frac{-t}{\Lambda^2}\right) + t\log\left(\frac{s}{-t}\right)\log\left(\frac{-t}{\Lambda^2}\right)\right)$$

Eikonal phase

graviton Regge trajectory



Resummation a la BFKL – but additional ladder and non-ladder $\ln^2(s)$ contributions

Bartels,Lipatov,Sabio-Vera,arXiv:1208.3423
Melville,Naculich,Schnitzer,White, arXiv:1306.6019

Lipatov vertex from shockwave collisions

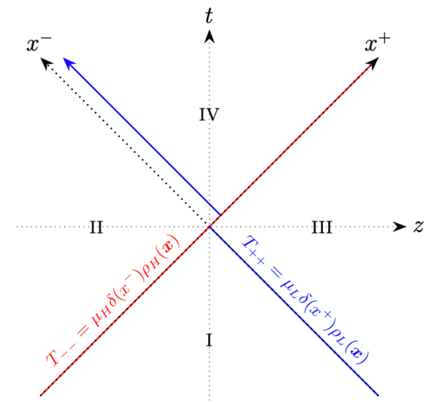
We will now sketch how the Lipatov vertex is recovered in shockwave collisions

Aichelburg-Sexl shockwave metric

$$ds^2 = 2dx^+dx^- - \delta_{ij}dx^i dx^j + f(x^-, \boldsymbol{x})\,(dx^-)^2$$

with $f(x^-, \boldsymbol{x}) = 2\kappa^2 \mu_H \delta(x^-) \dfrac{\rho_H(\boldsymbol{x})}{\Box_\perp} = \dfrac{\kappa^2}{\pi} \mu_H \delta(x^-) \displaystyle\int d^2\boldsymbol{y}\ \ln \Lambda |\boldsymbol{x} - \boldsymbol{y}| \rho_H(\boldsymbol{y})$

Soln of Einstein's eqns sourced by the EM tensor $T_{\mu\nu} = \delta_{\mu -}\delta_{\nu -}\mu_H \delta(x^-)\rho_H(\boldsymbol{x})$



$\mu_H = m_H\,\gamma$ = fixed for $\gamma \to \infty$

$\kappa^2 = 8\,\pi\, G$

Shockwave collisions: single shock background

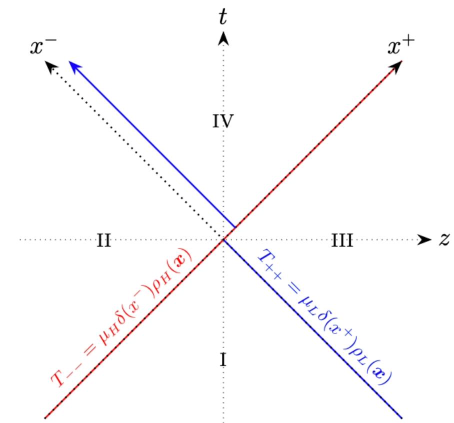
Linearizing around the metric $g_{\mu\nu} = \bar{g}_{\mu\nu} + \kappa\, h_{\mu\nu}$

fixing light cone gauge $h_{\mu +}$=0, find

$$h_{ij}(x^+, x^-, \boldsymbol{x}) = V(x^-, \boldsymbol{x}) h_{ij}(x^+, x^- = x_0^-, \boldsymbol{x})$$

with the gravitational Wilson line $V(x^-, \boldsymbol{x}) \equiv \exp\left(\frac{1}{2}\int_{x_0^-}^{x^-} dz^- \bar{g}_{--}(z^-, \boldsymbol{x})\, \partial_+\right)$

Exactly analogous to the QCD case
with $A_- \to\ g_{--}$ and $T^a \to \partial_+$



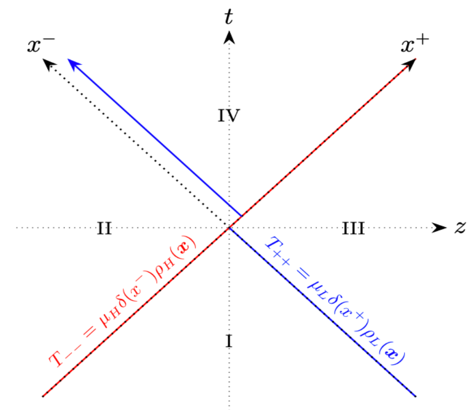
$\mu_H = m_H\, \gamma$ = fixed for $\gamma \to \infty$

κ^2=8 π G

Melville,Nachulich,Schnitzer,White,
arXiv:1306.6019

Shockwave collisions: "dilute-dense" approximation

Now consider the interaction of the "dilute" source ρ_L with the dense ρ_H shockwave:



$$T_{\mu\nu} = \delta_{\mu-}\delta_{\nu-}\mu_H \delta\left(x^-\right)\rho_H(\boldsymbol{x}) + \delta_{\mu+}\delta_{\nu+}\mu_L \delta\left(x^+\right)\rho_L(\boldsymbol{x})$$

Solve for metric in region IV – forward lightcone

$$g_{\mu\nu} = \bar{g}_{\mu\nu} + h_{\mu\nu} \qquad \bar{g}_{--} = 2\kappa\mu_H \delta\left(x^-\right)\frac{\rho_H(\boldsymbol{x})}{\Box_\perp}$$

$\mu_H = m_H\,\gamma$ = fixed for $\gamma \to \infty$

$\kappa^2 = 8\,\pi\,G$

We decompose the perturbation $h_{\mu\nu}$ into a term linear in ρ_L and one bi-linear in $\rho_L\rho_H$ (dilute-dilute limit)

Linearized Einstein's equations in light-cone gauge ($h_{+\mu}$=0) take the form

$$\bar{g}_{--}\partial_+^2\tilde{h}_{ij} - \Box\tilde{h}_{ij} = \kappa^2\left[\left(2\partial_i\partial_j - \Box_\perp\delta_{ij}\right)\frac{1}{\partial_+^2}T_{++} + 2T_{ij} - \delta_{ij}T - \frac{2}{\partial_+}\left(\partial_i T_{+j} + \partial_j T_{+i} - \delta_{ij}\partial_k T_{+k}\right)\right]$$

$$\tilde{h}_{ij} \equiv h_{ij} - \tfrac{1}{2}\delta_{ij}h \text{ where } h = \delta_{ij}h_{ij}$$

Shockwave collisions in general relativity: geodesics

Unlike QCD case, sub-eikonal contributions T_{+i}, T_{ij} are required for consistency of equations of motion

These are not uniquely fixed by energy-momentum conservation, the dynamics of the sources is needed to fix this. In the point particle approximation,

$$T^{\mu\nu}(x) = \frac{\mu_L}{\sqrt{-g}} \int_{-\infty}^{\infty} d\lambda \, \dot{X}^\mu \dot{X}^\nu \; \delta^{(4)}(x - X(\lambda))$$

Solution of the corresponding null geodesic equations $\quad \ddot{X}^\mu + \Gamma^\mu_{\nu\rho} \dot{X}^\nu \dot{X}^\rho = 0 \, , \qquad g_{\nu\rho} \dot{X}^\nu \dot{X}^\rho = 0$

in shockwave background given by $\quad X^- = \lambda \, , \quad X^i = b^i - \kappa^2 \mu_H X^- \Theta(X^-) \frac{\partial_i \rho_H(\boldsymbol{b})}{\Box_\perp}$

$$X^+ = -\kappa^2 \mu_H \Theta(X^-) \frac{\rho_H(\boldsymbol{b})}{\Box_\perp} + \frac{\kappa^4 \mu_H^2}{2} X^- \Theta(X^-) \left(\frac{\partial_i \rho_H(\boldsymbol{b})}{\Box_\perp} \right)^2$$

These geodesic solutions allow us to reconstruct the required components of the stress-energy tensor

Shockwave collisions in general relativity: Lipatov vertex

Solving eqns of motion, taking the Fourier transform, and putting the graviton momenta on-shell, one obtains

Gravitational radiational field

$$\tilde{h}_{ij}^{(2)}(k) = \frac{2\kappa^3 \mu_H \mu_L}{k^2 + i\epsilon k^-} \int \frac{d^2 \boldsymbol{q}_2}{(2\pi)^2} \Gamma_{ij}(\boldsymbol{q}_1, \boldsymbol{q}_2) \frac{\rho_H}{\boldsymbol{q}_1^2} \frac{\rho_L}{\boldsymbol{q}_2^2}$$

Gravitational Lipatov vertex

likewise for other components, recovering $\Gamma_{\mu\nu}(\boldsymbol{q}_1, \boldsymbol{q}_2) \equiv \frac{1}{2} C_\mu(\boldsymbol{q}_1, \boldsymbol{q}_2) C_\nu(\boldsymbol{q}_1, \boldsymbol{q}_2) - \frac{1}{2} N_\mu(\boldsymbol{q}_1, \boldsymbol{q}_2) N_\nu(\boldsymbol{q}_1, \boldsymbol{q}_2)$

Compare to gauge theory radiation field

$$a_i(k) = \frac{g^3}{k^2 + i\epsilon k^-} \int \frac{d^2 \boldsymbol{q}_2}{(2\pi)^2} C_i(\boldsymbol{q}_1, \boldsymbol{q}_2) \frac{\rho_H \cdot T}{\boldsymbol{q}_1^2} \frac{\rho_L}{\boldsymbol{q}_2^2}$$

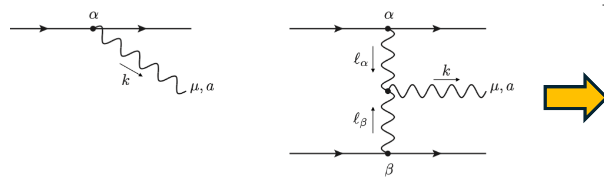
$-if^{abc} T_b T_c C_\mu(\boldsymbol{q}_1, \boldsymbol{q}_2)$ ⟷ $s\Gamma_{\mu\nu}(\boldsymbol{q}_1, \boldsymbol{q}_2)$

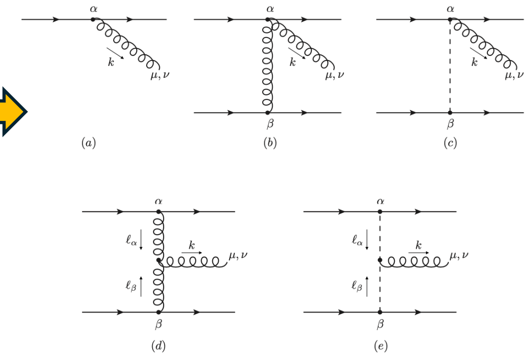
Is there a CK relation?

H.Johansson, A.Sabio Vera, E.Serna Campillo, and M.Vaszquez-Mozo, JHEP10,215(2013),arXiv:1307.3106 [hep-th]

Lipatov vertex from classical color-kinematic duality

Consider pert. solutions of Yang-Mills radiation field in collision of colored charges c^a_α (Wong equations)





Goldberger, Ridgway, arXiv:1611.03493

Taking ultrarelativistic limit
(keeping sub-eikonal terms, beyond leading QCD Lipatov vertex term)
and making replacements

$$c^a_\alpha \to p^\mu_\alpha$$

$$if^{a_1 a_2 a_3} \to \Gamma^{\nu_1\nu_2\nu_3}(q_1, q_2, q_3) = -\frac{1}{2}\left(\eta^{\nu_1\nu_3}(q_1 - q_3)^{\nu_2} + \eta^{\nu_1\nu_2}(q_2 - q_1)^{\nu_3} + \eta^{\nu_2\nu_3}(q_3 - q_2)^{\nu_1}\right)$$

$$g \to \kappa$$

recovers our previous result for the radiation field in terms of the gravitational Lipatov vertex

Raj,RV, arXiv:2312.03507

We can also show that the soft limit of the gravitational Lipatov vertex
gives the ultrarelativistic limit of the Weinberg soft graviton emission vertex

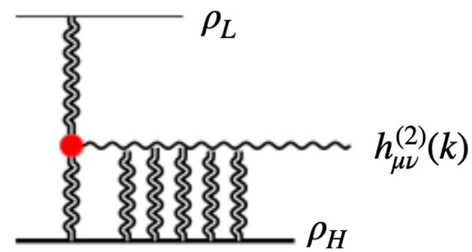
This is consistent with the observation that the soft limit of the classical double copy recovers the Weinberg emission vertex

P.V. Athira,A. Manu, arXiv:1907.10021

RG description in gravity a la CGC EFT in QCD?

In QCD, the CGC describes a semi-classical lump of size $1/Q_S$ that perturbatively unitarizes cross-sections. It corresponds to the non-trivial IR fixed point of an RG flow where the UV fixed point is BFKL

Can one obtain a similar RG picture in gravity in the shockwave framework? Reggeization must of single ladders must break down at at low impact parameter ($t \sim s$).



Instead, multiple scattering in the "dilute-dense" framework plays a central role
In a systematic treatment in this framework, we see that geodesic focusing (a la Raychaudhuri) of wee partons must play a role

Another key element in the RG description are shockwave propagators
- Again in GR, it may be important to keep non-eikonal contributions

Raj, RV, arXiv:2406.10483

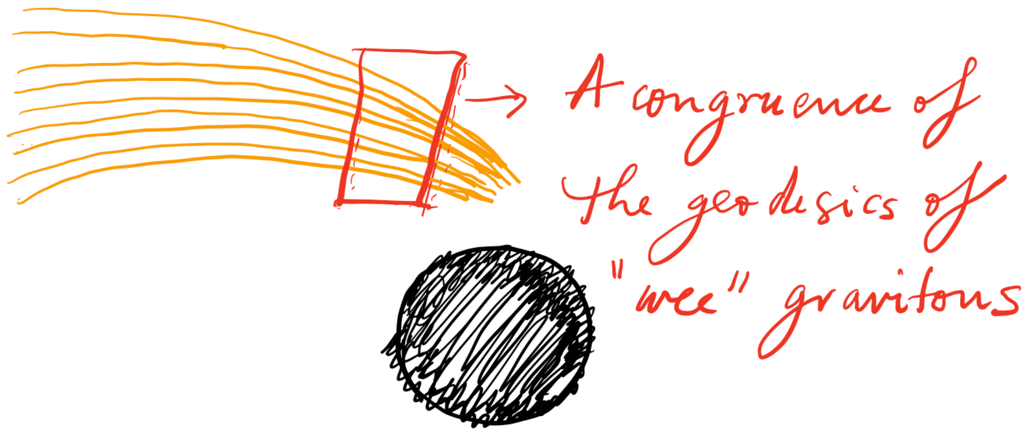
GR Wilson line

Qualitative ideas of a CGC- Black Hole correspondence as overoccupied gluon/graviton states have been discussed by myself and Gia Dvali

Dvali, RV, arXiv:11989

Geodesic congruence: the geometry of quantum information

The Raychaudhuri equation
- key in Hawking-Penrose singularity theorems :



Volume change of geodesic convergence

$$\dot{\theta} = -\Omega^i{}_j \Omega^j{}_i + K^i{}_i$$
$$= -\frac{1}{3}\theta^2 - \sigma_{ij}\sigma^{ij} + \omega_{ij}\omega^{ij} + K^i{}_i.$$

Bulk scalar Shear tensor Rotation tensor Includes Ricci curvature + stochastic graviton noise

H.-T. Cho and B.-L Hu, arxiv:2301.06325
M. Parikh, F. Wilczek, G. Zaharaide, PRL (2021)

Remarkably, the Raychaudhuri equation can be rephrased as the Bishop-Gromov upper bound on the "complexity volume in D-1 dimensions" of gate complexity
– realizing Nielsen's geometric picture in quantum information theory?

A. R. Brown, arXiv:2112.05724

Pure speculation: Can this complexity picture provide further insight into the RG fixed point of BH formation?

$2 \rightarrow N + 2$ amplitudes in trans-Planckian gravitation scattering: from wee partons to Black Holes

HIGH-ENERGY SCATTERING IN QCD AND IN QUANTUM GRAVITY AND TWO-DIMENSIONAL FIELD THEORIES

L.N. LIPATOV*

We construct effective actions describing high-energy processes in QCD and in quantum gravity with intermediate particles (gluons and gravitons) having the multi-Regge kinematics. The *S*-matrix for these effective scalar field models contains the results of the leading logarithmic approximation and is unitary. It can be expressed in terms of correlation functions for two field theories acting in longitudinal and transverse two-dimensional subspaces.

Effective action and all-order gravitational eikonal at planckian energies

AMATI,CIAFALONI,VENEZIANO **NPB403 (1993)707**

Building on previous work by us and by Lipatov, we present an effective action approach to the resummation of all semiclassical (i.e. $O(\hbar^{-1})$) contributions to the scattering phase arising in high-energy gravitational collisions. By using an infrared-safe expression for Lipatov's effective action, we derive an eikonal form of the scattering matrix and check that the superstring amplitude result is reproduced at first order in the expansion parameter R^2/b^2, where R, b are the gravitational radius and the impact parameter, respectively. If rescattering of produced gravitons is neglected, the longitudinal coordinate dependence can be explicitly factored out and exhibits the characteristics of a shock-wave metric while the transverse dynamics is described by a reduced two-dimensional effective action. Singular behaviours in the latter, signalling black hole formation, can be looked for.

The World as a Hologram

LEONARD SUSSKIND

Wee partons, by contrast, are not subject to Lorentz contraction. This implies that in the Feynman Bjorken model, the halo of wee partons eternally "floats" above the horizon at a distance of order $10^{-13} cm$ as it transversley spreads. The remaining valence partons carry the various currents which contract onto the horizon as in the Einstein Lorentz case.

By contrast, both the holographic theory and string theory require all partons to be wee. No Lorentz contraction takes place and the entire structure of the string floats on the stretched horizon. I have explained in previous articles how this behavior prevents the accumulation of arbitrarily large quantities of information near the horizon of a black hole. Thus we are led full circle back to Bekenstein's principle that black holes bound the entropy of a region of space to be proportional to its area.

***J.Math.Phys*. 36 (1995) 6377; > 4 K cites !**

In Acknowledgements:

Finally I benefitted from discussions with Kenneth Wilson and Robert Perry, about boosts and renormalization fixed points in light front quantum mechanics and Lev Lipatov about high energy scattering.

→ 30+ years of work by ACV et al. exploring gravitational shockwave collisions in this 2-D EFT