

Edinburgh, 21–23 February 2018

Rare kaon decay phenomenology

Giancarlo D'Ambrosio
INFN Sezione di Napoli

Collaboration with Teppei Kitahara arXiv:
1707.06999 PRL

Collaboration with Teppei Kitahara, Isabel Fernández Suárez, Miriam Lucio Martínez,
Diego Martínez Santos, Veronika Georgieva Chobanova arXiv:1711.11030

Rare n Strange 2017: strange physics at LHCb

GD, Lewis Tunstall, Diego Martinez Santos, Veronika Chobanova,
Xabier Cid Vidal, Francesco Dettori, Marc-Olivier Bettler

Collaboration with Crivellin, A., Kitahara, T and Nierste, U.
e-Print: arXiv:1703.05786

Closing in on the radiative weak chiral couplings
Luigi Cappiello, Oscar Cata, Giancarlo D'Ambrosio.
arXiv:1712.10270

Collaboration with M. Knecht, L. E. Greynat, D.

Collaboration with Abhishek Iyer

Flavour issues in warped custodial models: B
anomalies and rare K decays
Giancarlo D'Ambrosio, Abhishek M. Iyer. Dec 21, 2017. 22 p
arXiv:1712.08122

Outline

- the weak chiral lagrangian: attempt to determine the coefficients
- $K_{S,L} \rightarrow \mu\mu$

QCD and EFT

Chiral Perturbation theory

χ PT effective field theory approach based on **two** assumptions

- π 's Goldstone bosons of $SU(3)_L \times SU(3)_R \rightarrow SU(3)_V$ $SU(3)_L \times SU(3)_R$ symm. \mathcal{L}_{QCD} $m_q = 0$
- **(chiral) power counting** There is a small expansion parameter $p^2/\Lambda^2_{\chi\text{SB}}$

$$\Lambda_{\chi\text{SB}} \approx 4 \pi F_\pi \sim 1.2 \text{ GeV}$$

$$U = e^{\frac{i\sqrt{2}}{F_\pi} \pi} \quad U \xrightarrow{SU(3)_L \times SU(3)_R} g_L U g_R^\dagger$$

π non-lin. real U lin. transf.

Chiral sym. breaking through dim. parameter

F_π, χ related to $\langle 0 | J_{5\mu} | \pi \rangle, \langle 0 | \bar{q}_L q_R | 0 \rangle$

$F_\pi \approx 93 \text{ MeV}$

$m_\pi^2 \sim \langle 0 | \bar{q}_L q_R | 0 \rangle m_q$
GOR rel.

$$\mathcal{L}_{\Delta S=0} = \mathcal{L}_{\Delta S=0}^2 + \mathcal{L}_{\Delta S=0}^4 + \dots = \frac{F_\pi^2}{4} \overbrace{\langle D_\mu U D^\mu U^\dagger + \chi U^\dagger + U \chi^\dagger \rangle}^{\pi \rightarrow l\nu, \pi\pi \rightarrow \pi\pi, K \rightarrow \pi..} + \sum_i \overbrace{L_i O_i}^{K \rightarrow \pi..} + \dots$$

Fantastic chiral prediction

$$A_{\pi\pi} \sim (s - m_\pi^2) / F_\pi^2$$

L_i Gasser Leutwyler coeff determined from expts.
 O_i p^4 operator

Vector Meson Dominance in the strong sector

Ecker, Gasser, de Rafael, Pich

L_i	L_i expts	V	A	Total (Scalar incl.)	Total QCD rel. incl.
L_1	0.4 ± 0.3	0,6	0	0,6	0,9
L_2	1.4 ± 0.3	1,2	0	1,2	1,8
L_3	-3.5 ± 1.1	-3,6	0	-3,0	-4,9
L_4	-0.3 ± 0.5	0	0	0	0
L_5	1.4 ± 0.5	0	0	1,4	1,4
L_6	-0.2 ± 0.3	0	0	0	0
L_7	-0.4 ± 0.2	0	0	-0,3	-0,3
L_8	0.9 ± 0.3	0	0	0,9	0,9
L_9	6.9 ± 0.7	6,9	0	6,9	7,3
L_{10}	-5.5 ± 0.7	-10	4	-6,0	-5,5

QCD inspired relations

$$F_V = 2G_V = \sqrt{2}f_\pi$$

$$F_A = f_\pi$$

$$M_A = \sqrt{2}M_V$$

KSFR: $G_V = \sqrt{2} F_\pi$
determined by dominance
of pion, V,A to recover
QCD short distance
constraints

$$L_1^V = \frac{L_2^V}{2} = -\frac{L_3^V}{6} = \frac{G_V^2}{8M_V^2}, \quad L_9^V = \frac{F_V G_V}{2M_V^2}, \quad L_{10}^{V+A} = -\frac{F_V^2}{4M_V^2} + \frac{F_A^2}{4M_A^2}$$

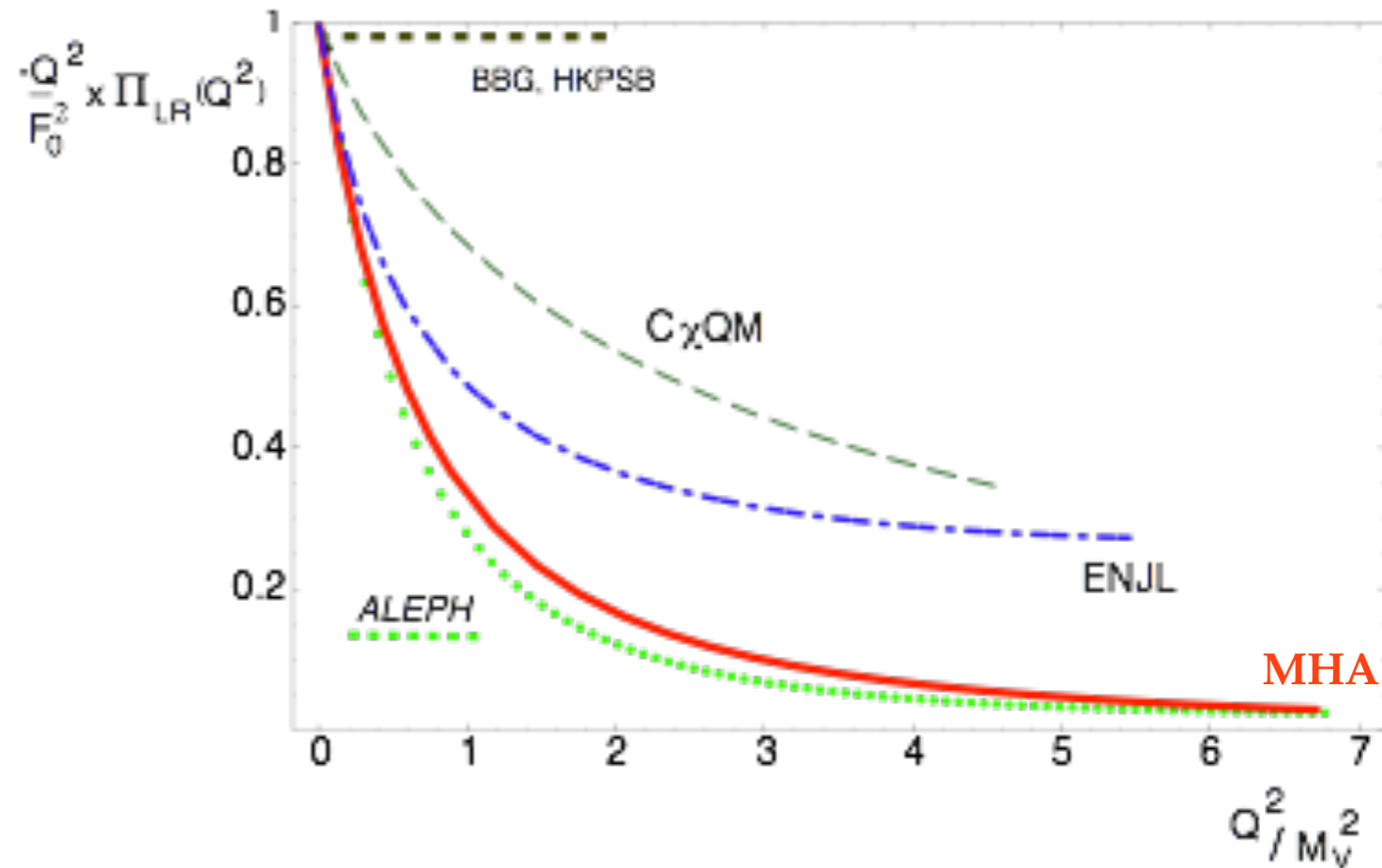
QCD inspired relations

$$L_1^V = L_2^V / 2 = -L_3^V / 6 = L_9^V / 8 = -L_{10}^{V+A} / 6 = f_\pi^2 / (16M_V^2)$$

Minimal Hadronic Ansatz

(MHA)

- Traditional wisdom: low energy VERY WELL approximated by π 's, V, A
- Short distance: QCD
- A good interpolation among the two regimes is sufficient for a good description of the correlators



De Rafael

Weak interaction

The symmetry of the short distance hamiltonian $-\frac{G_F}{\sqrt{2}}V_{ud}V_{us}^*C_-(\bar{s}_L\gamma^\mu u_L)(\bar{u}_L\gamma_\mu d_L)$

described in CHPT

$$\mathcal{L}_{\Delta S=1} = \mathcal{L}_{\Delta S=1}^2 + \mathcal{L}_{\Delta S=1}^4 + \dots = G_8 F^4 \underbrace{\langle \lambda_6 D_\mu U^\dagger D^\mu U \rangle}_{K \rightarrow 2\pi/3\pi} + \underbrace{G_8 F^2 \sum_i N_i W_i}_{K^+ \rightarrow \pi^+ \gamma \gamma, K \rightarrow \pi l^+ l^-} + \dots$$

VMD not as successful, in particular for $K \rightarrow 3\pi$, where in principle large VMD important

π	2π	3π	N_i
$\pi^+\gamma^*$ $\pi^0\gamma^* (S)$ $\pi^+\gamma\gamma$	$\pi^+\pi^0\gamma^*$ $\pi^0\pi^0\gamma^* (L)$ $\pi^+\pi^0\gamma\gamma$ $\pi^+\pi^-\gamma\gamma (S)$ $\pi^+\pi^0\gamma$ $\pi^+\pi^-\gamma (S)$ $\pi^+\pi^-\gamma^* (L)$ $\pi^+\pi^-\gamma^* (S)$ $\pi^+\pi^0\gamma^*$	$\pi^+\pi^+\pi^-\gamma$ $\pi^+\pi^0\pi^0\gamma$ $\pi^+\pi^-\pi^0\gamma (L)$ $\pi^+\pi^-\pi^0\gamma (S)$	$N_{14}^r - N_{15}^r$ $2N_{14}^r + N_{15}^r$ $N_{14} - N_{15} - 2N_{18}$ " $N_{14} - N_{15} - N_{16} - N_{17}$ " " $7(N_{14}^r - N_{16}^r) + 5(N_{15}^r + N_{17}^r)$ $N_{14}^r - N_{15}^r - 3(N_{16}^r - N_{17}^r)$ $N_{14}^r - N_{15}^r - 3(N_{16}^r + N_{17}^r)$ $N_{14}^r + 2N_{15}^r - 3(N_{16}^r - N_{17}^r)$
	$\pi^+\pi^-\gamma (L)$ $\pi^+\pi^0\gamma$	$\pi^+\pi^-\pi^0\gamma (S)$ $\pi^+\pi^+\pi^-\gamma$ $\pi^+\pi^0\pi^0\gamma$ $\pi^+\pi^-\pi^0\gamma (S)$ $\pi^+\pi^-\pi^0\gamma (L)$	$N_{29} + N_{31}$ " $3N_{29} - N_{30}$ $5N_{29} - N_{30} + 2N_{31}$ $6N_{28} + 3N_{29} - 5N_{30}$

$$K(p_K) \rightarrow \pi(p_1)\pi(p_2)\gamma(q)$$

- Lorentz + gauge invariance \Rightarrow Electric (E) and Magnetic (M) amplitude

$$A(K \rightarrow \pi\pi\gamma) = F^{\mu\nu} [E \partial_\mu K \partial_\nu \pi + M \varepsilon_{\mu\nu\rho\sigma} \partial^\rho K \partial^\sigma \pi]$$

- Unpolarized photons

$$\frac{d^2\Gamma}{dz_1 dz_2} \sim |E|^2 + |M|^2$$

$$|E^2| = |E_{IB}|^2 + 2\text{Re}(E_{IB}^* E_D) + |E_D|^2$$

↓

$$\text{Low Theorem} \Rightarrow E_{IB} \sim \frac{1}{E_\gamma^*} + c$$

E_D, M chiral tests

$$K^+ \rightarrow \pi^+ \pi^0 \gamma$$

$$A(K \rightarrow \pi \pi \gamma) = F^{\mu\nu} [E \partial_\mu K \partial_\nu \pi + M \varepsilon_{\mu\nu\rho\sigma} \partial^\rho K \partial^\sigma \pi]$$

$E1$ and $M1$ are measured with Dalitz plot

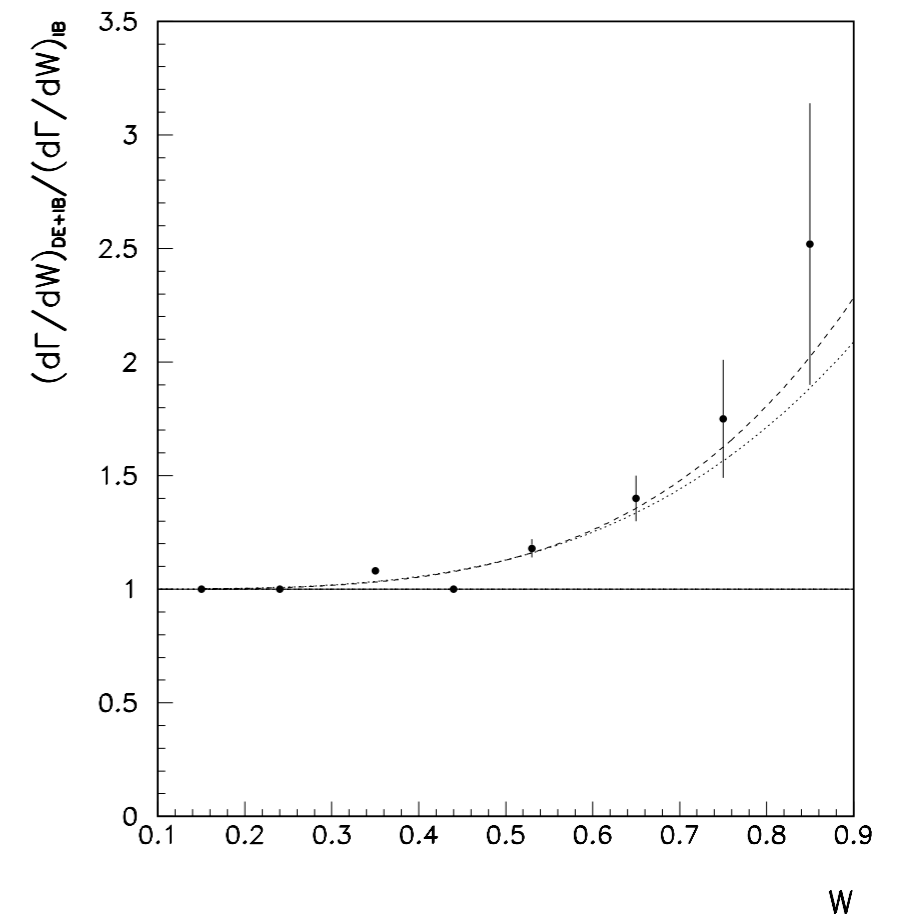
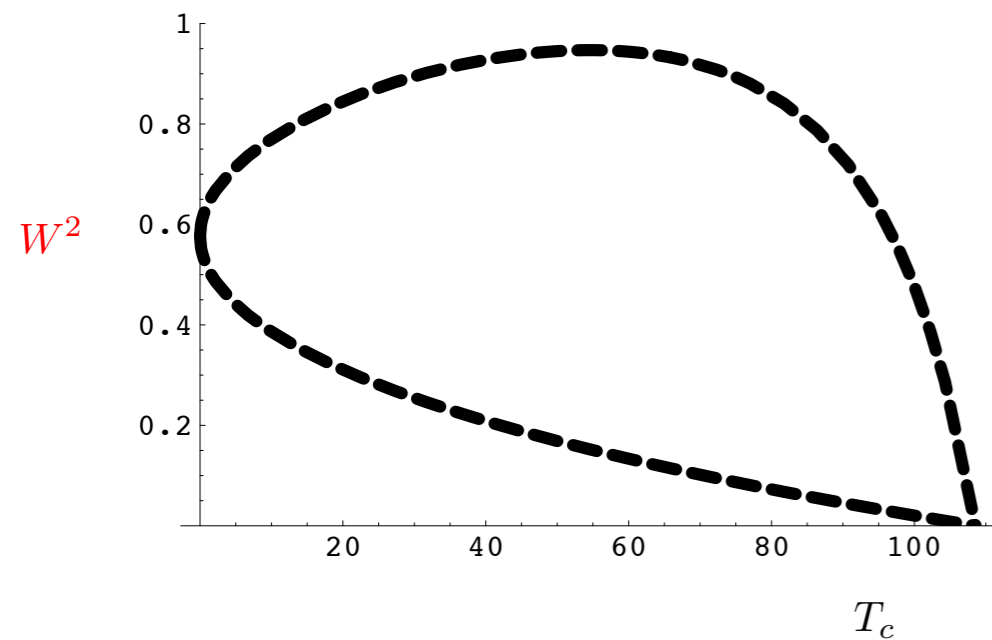
$$\frac{\partial^2 \Gamma}{\partial T_c^* \partial W^2} = \frac{\partial^2 \Gamma_{IB}}{\partial T_c^* \partial W^2} \left[1 + \frac{m_{\pi^+}^2}{m_K^2} 2 \operatorname{Re} \left(\frac{E1}{eA} \right) W^2 + \frac{m_{\pi^+}^4}{m_K^2} \left(\left| \frac{E1}{eA} \right|^2 + \left| \frac{M1}{eA} \right|^2 \right) W^4 \right]$$

$$W^2 = (q \cdot p_K)(q \cdot p_+) / (m_\pi^2 m_K^2)$$

$$A = A(K^+ \rightarrow \pi^+ \pi^0)$$

Departure from IB

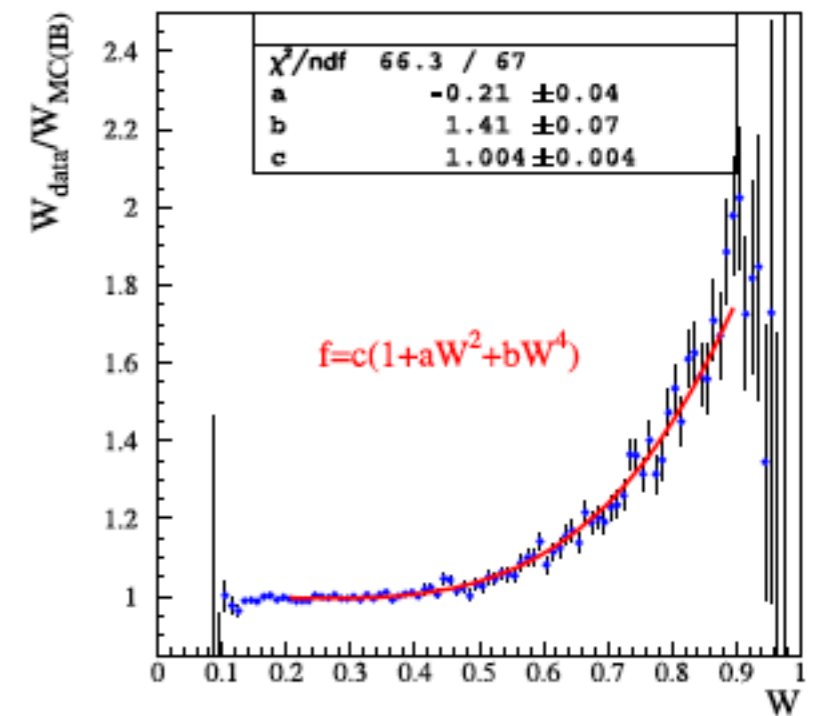
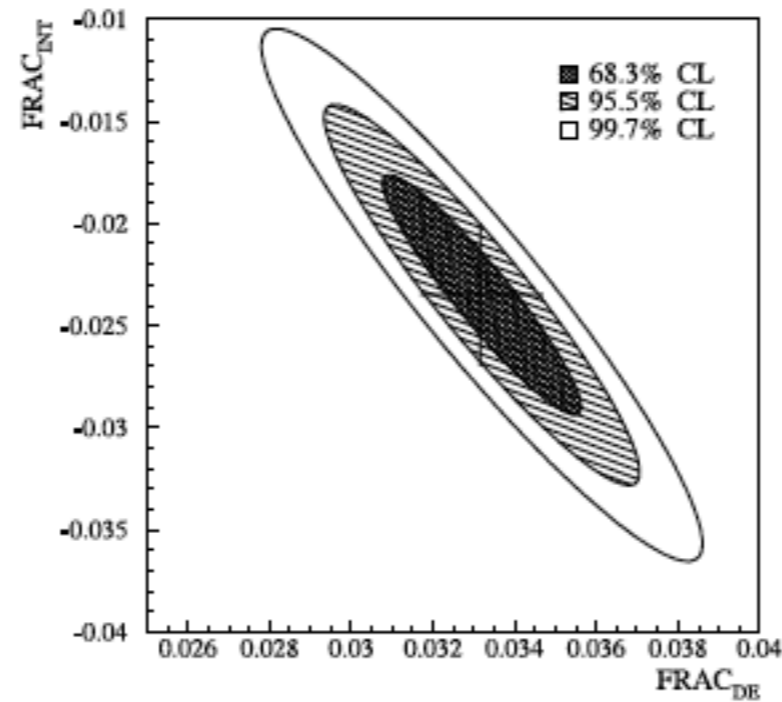
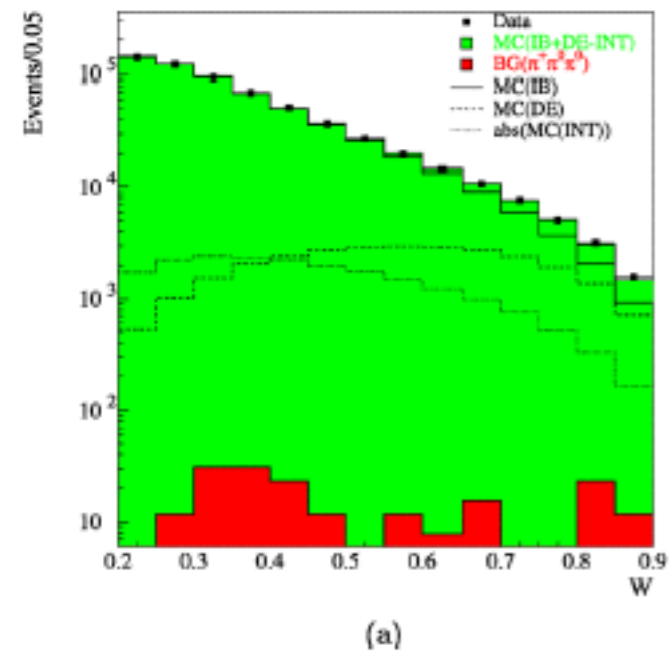
$$W^2 = (q \cdot p_K)(q \cdot p_+) / (m_\pi^2 m_K^2)$$



$$\frac{\partial^2 \Gamma}{\partial T_c^* \partial W^2} = \frac{\partial^2 \Gamma_{IB}}{\partial T_c^* \partial W^2} \left[1 + \frac{m_{\pi^+}^2}{m_K} 2 \operatorname{Re} \left(\frac{E1}{eA} \right) W^2 + \frac{m_{\pi^+}^4}{m_K^2} \left(\left| \frac{E1}{eA} \right|^2 + \left| \frac{M1}{eA} \right|^2 \right) W^4 \right]$$

IB from Low theorem

NA48/2 , 600 K candidates



$$\begin{array}{l}
 \text{NA48} \\
 \hline
 \text{Frac}(DE) = \\
 \text{Frac}(INT) =
 \end{array}
 \begin{array}{l}
 T_c^* \in [0, 80] \text{ MeV} \\
 (3.32 \pm 0.15 \pm 0.14) \times 10^{-2} \\
 (-2.35 \pm 0.35 \pm 0.39) \times 10^{-2}
 \end{array}$$

Frac(DE) ratio
to IB

Frac(INT) ratio
to IB

first experiment IB from theory

$$\begin{aligned}
N_{14} - N_{15} - N_{16} - N_{17} &\stackrel{FM}{\sim} -\frac{\kappa_f F_\pi^2}{2M_V^2} \sim -0.0025 \\
&\stackrel{\text{expt}}{=} +0.0022(7)
\end{aligned}$$

$$K^+ \rightarrow \pi^+ \pi^0 e e$$

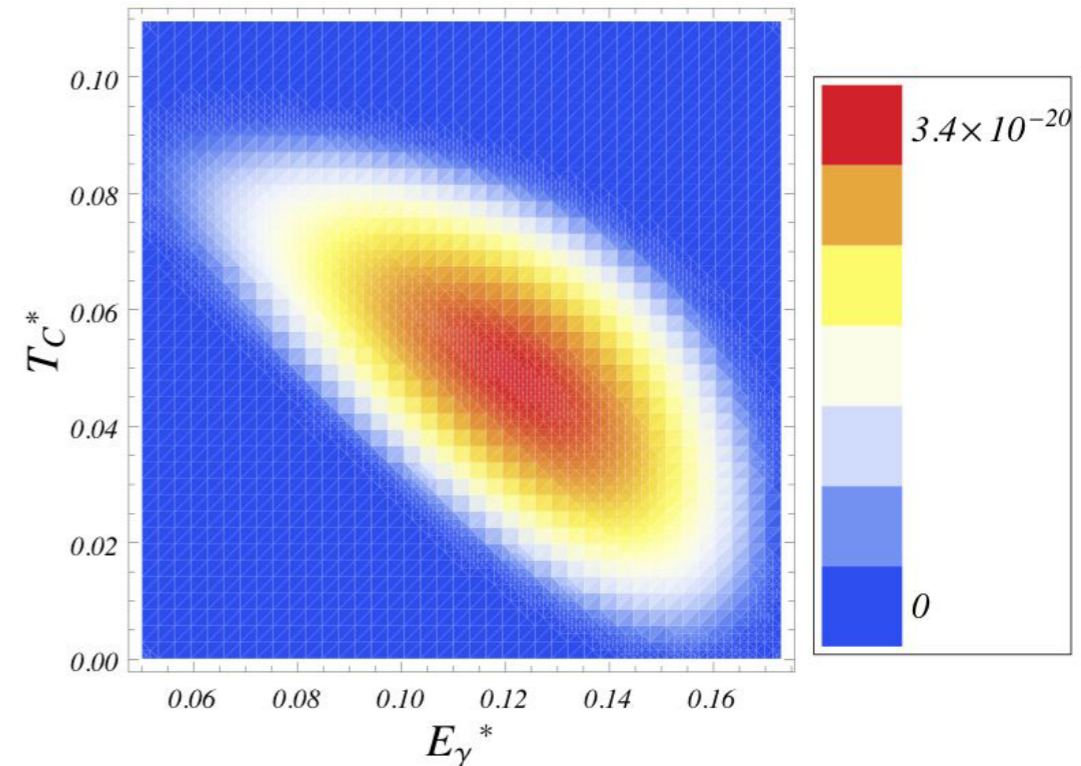
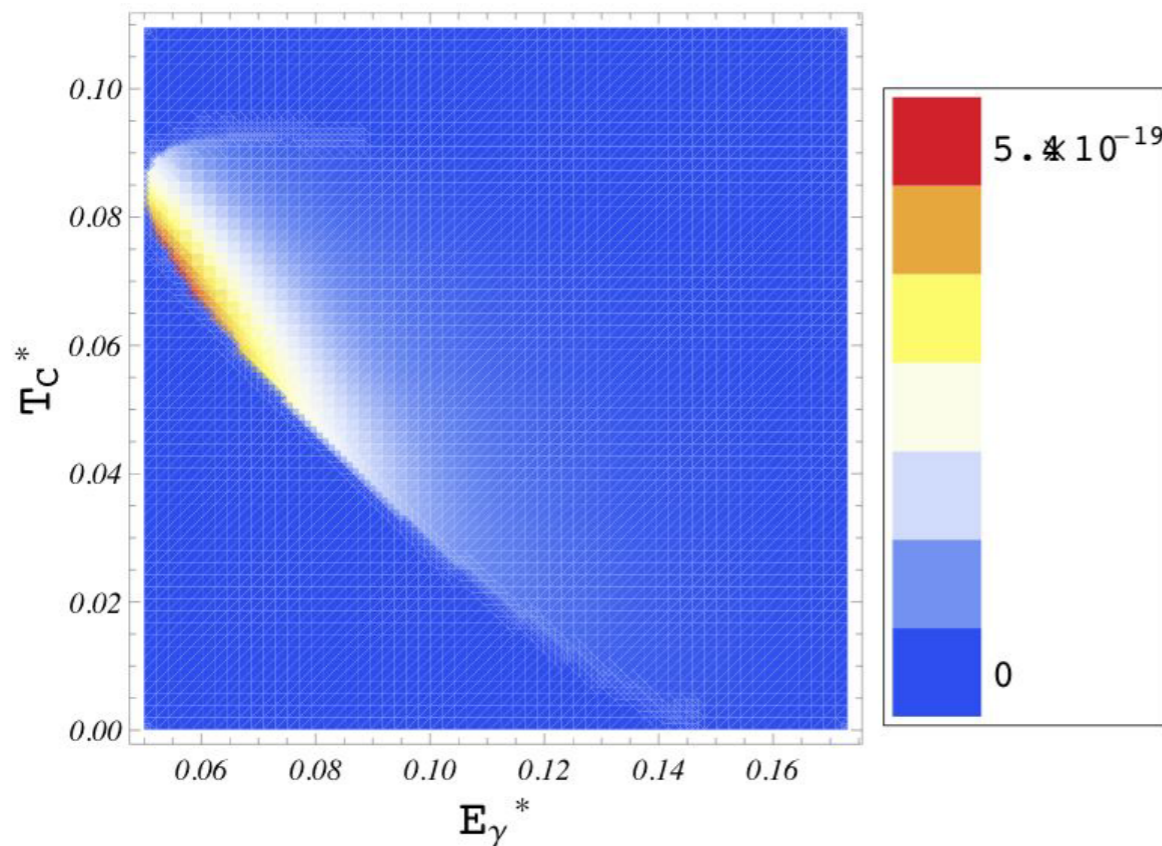
Cappiello, Cata, GD, Gao
Cappiello, Cata, GD

NA48/2

extra kinematical variable to kill IB

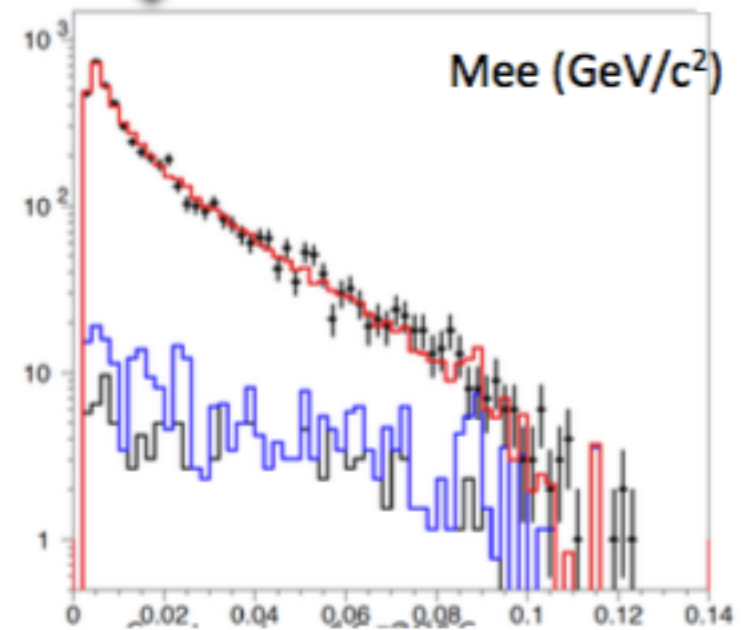
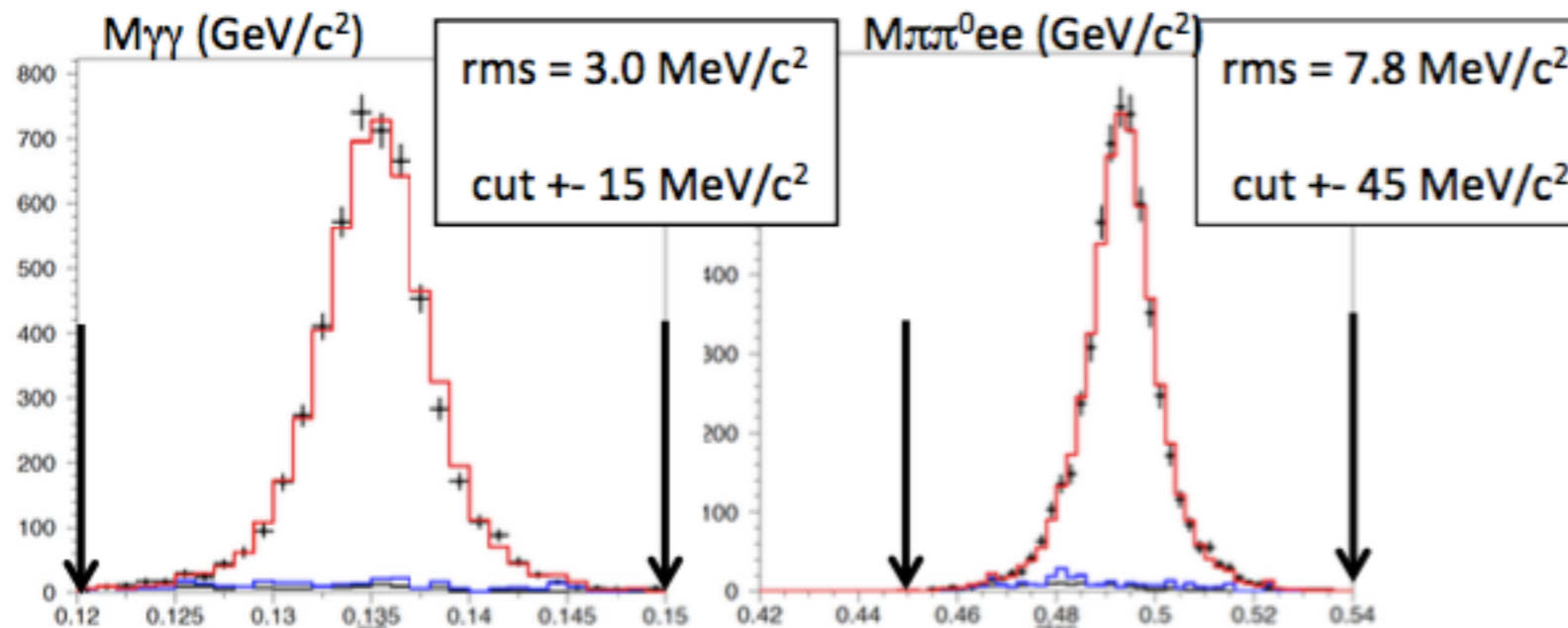
Starting from CP conserving IB, DE

q_c (MeV)	B [10^{-8}]	B/M	B/E	B/BE	B/BM
$2m_l$	418.27	71	4405	128	208
55	5.62	12	118	38	44
100	0.67	8	30	71	36
180	0.003	12	5	-19	44



NA48/2: $K^{+/-} \rightarrow \pi^{+/-} \pi^0 e^+ e^-$

Bloch-Devaux



New Result!

$$BR = (4.22 \pm 0.06_{\text{stat}} \pm 0.04_{\text{syst}} \pm 0.13_{\text{ext}}) 10^{-6}$$

dominated by external error on BR($\pi^0 D$)

In perfect agreement with

Theory : ChPT calculations EPJ C72 (2012)

IB + DE + INT

BR (IB) = $4.19 \cdot 10^{-6}$ no Rad Cor, No Isospin breaking Cor	Total $4.29 \cdot 10^{-6}$
BR (IB) = $4.10 \cdot 10^{-6}$ no Rad Cor, with Isospin breaking Cor**	Total $4.19 \cdot 10^{-6}$

π	2π	3π	N_i
$\pi^+\gamma^*$ $\pi^0\gamma^* (S)$ $\pi^+\gamma\gamma$ <hr style="border-top: 1px dashed black;"/>	$\pi^+\pi^0\gamma^*$ $\pi^0\pi^0\gamma^* (L)$ $\pi^+\pi^0\gamma\gamma$ $\pi^+\pi^-\gamma\gamma (S)$ $\pi^+\pi^0\gamma$ $\pi^+\pi^-\gamma (S)$ $\pi^+\pi^-\gamma^* (L)$ $\pi^+\pi^-\gamma^* (S)$ $\pi^+\pi^0\gamma^*$	$\pi^+\pi^+\pi^-\gamma$ $\pi^+\pi^0\pi^0\gamma$ $\pi^+\pi^-\pi^0\gamma (L)$ $\pi^+\pi^-\pi^0\gamma (S)$	$N_{14}^r - N_{15}^r$ $2N_{14}^r + N_{15}^r$ $N_{14} - N_{15} - 2N_{18}$ <hr style="border-top: 1px dashed black;"/> $N_{14} - N_{15} - N_{16} - N_{17}$ " " " " $7(N_{14}^r - N_{16}^r) + 5(N_{15}^r + N_{17}^r)$ $N_{14}^r - N_{15}^r - 3(N_{16}^r - N_{17}^r)$ $N_{14}^r - N_{15}^r - 3(N_{16}^r + N_{17}^r)$ $N_{14}^r + 2N_{15}^r - 3(N_{16}^r - N_{17}^r)$
	$\pi^+\pi^-\gamma (L)$ $\pi^+\pi^0\gamma$	$\pi^+\pi^-\pi^0\gamma (S)$ $\pi^+\pi^+\pi^-\gamma$ $\pi^+\pi^0\pi^0\gamma$ $\pi^+\pi^-\pi^0\gamma (S)$ $\pi^+\pi^-\pi^0\gamma (L)$	$N_{29} + N_{31}$ " " $3N_{29} - N_{30}$ $5N_{29} - N_{30} + 2N_{31}$ $6N_{28} + 3N_{29} - 5N_{30}$

$$K^\pm \rightarrow \pi^\pm \gamma^* :$$

$$a_+ = -0.578 \pm 0.016 [3, 4]$$

$$K_S \rightarrow \pi^0 \gamma^* :$$

$$a_S = (1.06_{-0.21}^{+0.26} \pm 0.07) [5, 6]$$

$$K^\pm \rightarrow \pi^\pm \pi^0 \gamma :$$

$$X_E = (-24 \pm 4 \pm 4) \text{ GeV}^{-4} [7]$$

$$K^+ \rightarrow \pi^+ \gamma \gamma :$$

$$\hat{c} = 1.56 \pm 0.23 \pm 0.11 [8] .$$

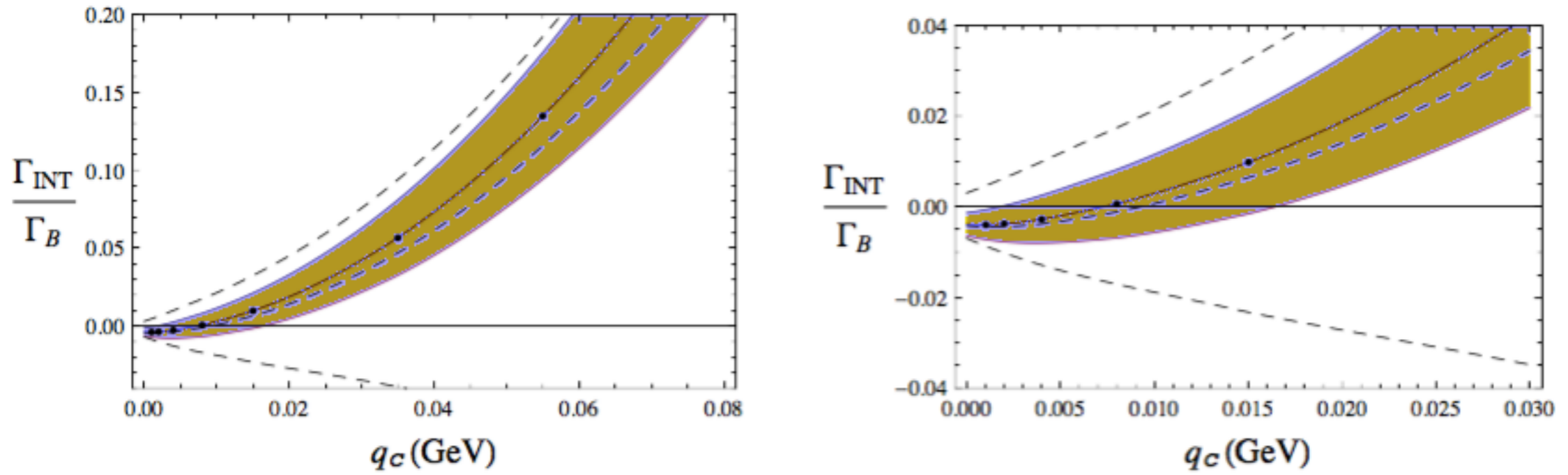
$$\mathcal{N}_E^{(1)} \equiv N_{14}^r - N_{15}^r = \frac{3}{64\pi^2} \left(\frac{1}{3} - \frac{G_F}{G_8} a_+ - \frac{1}{3} \log \frac{\mu^2}{m_K m_\pi} \right) - 3L_9^r ;$$

$$\mathcal{N}_S \equiv 2N_{14}^r + N_{15}^r = \frac{3}{32\pi^2} \left(\frac{1}{3} + \frac{G_F}{G_8} a_S - \frac{1}{3} \log \frac{\mu^2}{m_K^2} \right) ;$$

$$\mathcal{N}_E^{(0)} \equiv N_{14}^r - N_{15}^r - N_{16}^r - N_{17}^r = -\frac{|\mathcal{M}_K| f_\pi}{2G_8} X_E ;$$

$$\mathcal{N}_0 \equiv N_{14}^r - N_{15}^r - 2N_{18}^r = \frac{3}{128\pi^2} \hat{c} - 3(L_9^r + L_{10}^r) ,$$

Decay mode	counterterm combination	expt. value
$K^\pm \rightarrow \pi^\pm \gamma^*$	$N_{14} - N_{15}$	$-0.0167(13)$
$K_S \rightarrow \pi^0 \gamma^*$	$2N_{14} + N_{15}$	$+0.016(4)$
$K^\pm \rightarrow \pi^\pm \pi^0 \gamma$	$N_{14} - N_{15} - N_{16} - N_{17}$	$+0.0022(7)$
$K^\pm \rightarrow \pi^\pm \gamma \gamma$	$N_{14} - N_{15} - 2N_{18}$	$-0.0017(32)$



q cut in minimum dilepton

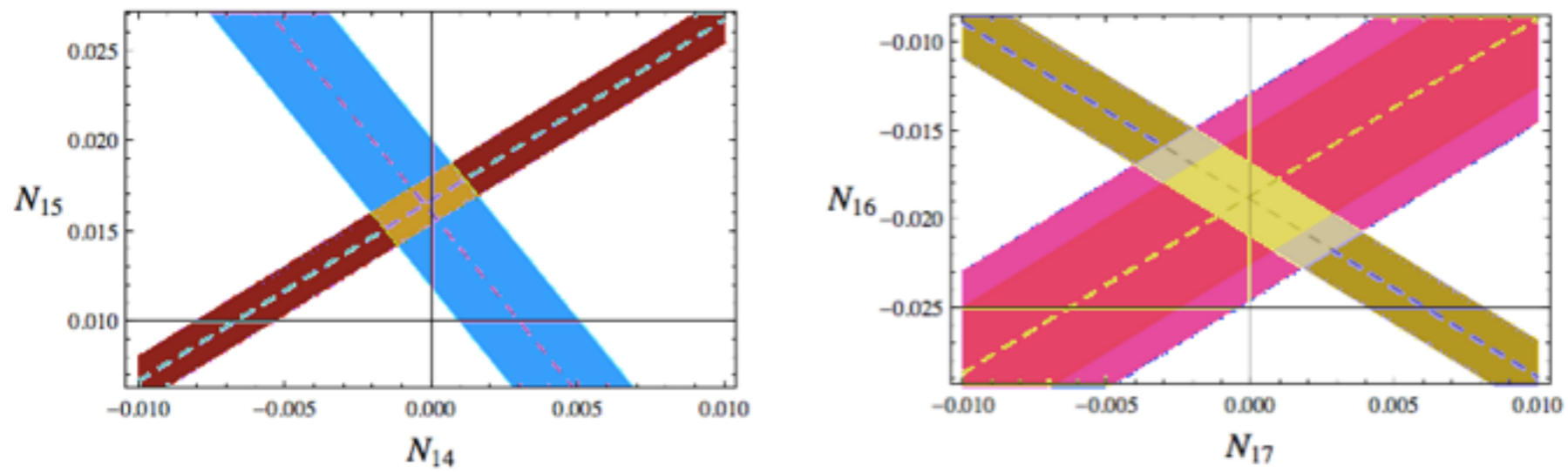


Figure 4: *Left panel: values of N_{14} and N_{15} as given by $K^\pm \rightarrow \pi^\pm \gamma^*$ (blue band) and $K_S \rightarrow \pi^0 \gamma^*$ (violet band). Right panel: values for N_{16} and N_{17} extracted from $K^\pm \rightarrow \pi^\pm \pi^0 \gamma$ (blue band) and $K^\pm \rightarrow \pi^\pm \pi^0 e^+ e^-$ (yellow band) measurements. The latter is an educated estimate (see main text).*

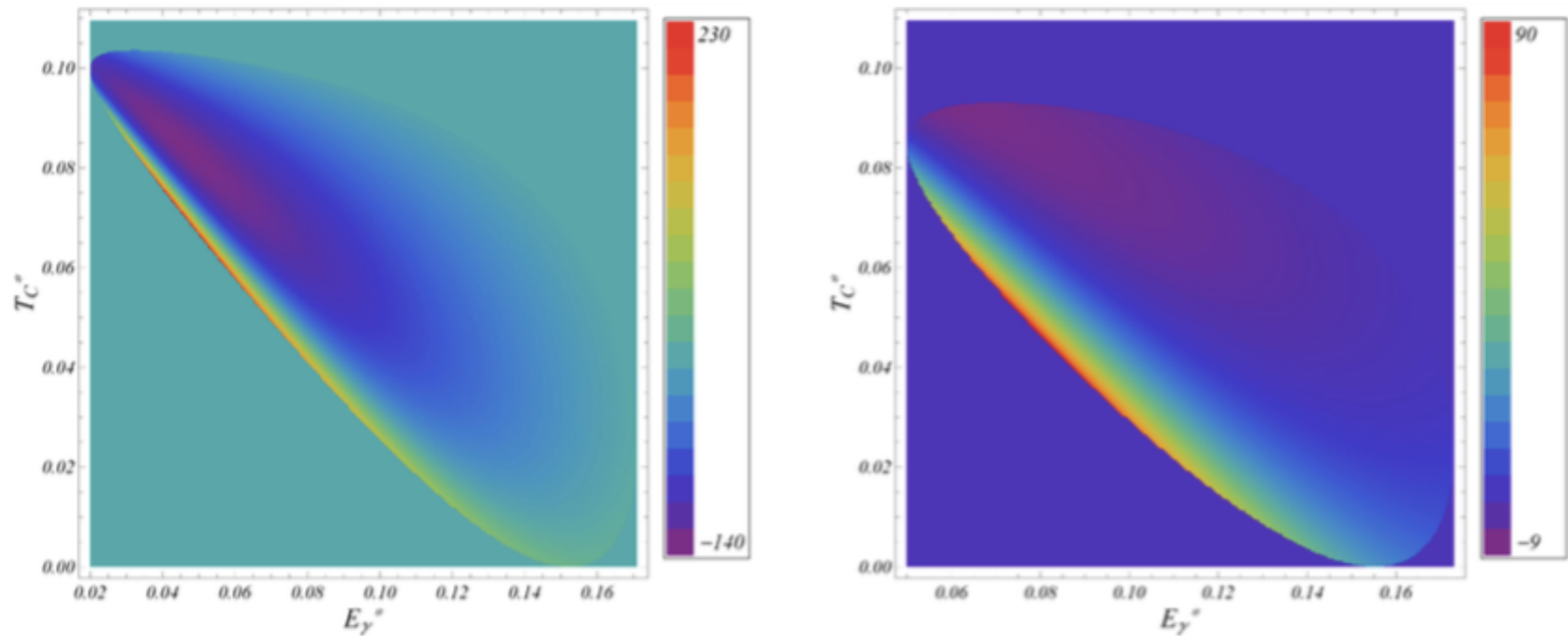


Figure 1: *Dalitz plots for the interference differential decay rate in the (E_γ, T_C) plane for $q = 20$ MeV (left panel) and $q = 50$ MeV (right panel). Numbers are given in units of $10^{-20} \text{ GeV}^{-1}$. The contour plot is 'spikier' the lower the q values are, a pattern mostly dictated by the structure of the Bremsstrahlung term.*

$$BR(K_S \rightarrow \pi^+ \pi^- e^+ e^-) = \underbrace{4.74 \cdot 10^{-5}}_{\text{Brems.}} + \underbrace{4.39 \cdot 10^{-8}}_{\text{Int.}} + \underbrace{1.33 \cdot 10^{-10}}_{\text{DE}}$$

q_c (MeV)	$10^8 \times \Gamma_{\mathcal{B}}$	$\left[\frac{\Gamma_{\mathcal{E}}}{\Gamma_{\mathcal{B}}}\right]^{-1}$	$\left[\frac{\Gamma_{\text{INT}}}{\Gamma_{\mathcal{B}}}\right]_{(1,1,1)}^{-1}$	$\left[\frac{\Gamma_{\text{INT}}}{\Gamma_{\mathcal{B}}}\right]_{(1,0,1)}^{-1}$	$\left[\frac{\Gamma_{\text{INT}}}{\Gamma_{\mathcal{B}}}\right]_{(1,1,0)}^{-1}$	$\left[\frac{\Gamma_{\text{INT}}}{\Gamma_{\mathcal{B}}}\right]_{(0,1,1)}^{-1}$
$2m_l$	418.27	1100	-253	-225	-115	216
2	307.96	821	-265	-226	-98	159
4	194.74	529	-363	-264	-78	101
8	109.60	304	1587	-850	-59	58
15	56.12	161	102	156	-43	31
35	15.50	50	18	21	-26	11
55	5.62	22	7	9	-18	5
85	1.37	8	3	4	-13	3
100	0.67	5	2	3	-11	2
120	0.24	3	1.6	2	-10	1.4
140	0.04	2	1.0	1.1	-8	0.9
180	0.003	1	0.7	0.8	-7	0.7

Table 2: Branching ratios for the Bremsstrahlung and the relative weight of the electric and electric interference terms for different cuts in q , starting at q_{\min} (first row) and ending at 180 MeV. To highlight the role of the different counterterms, the last columns show how the interference term changes when they are switched off one at a time.

Rare Kaon decay program

Rare decay modes of the K mesons in gauge theories

M. K. Gaillard* and Benjamin W. Lee†

National Accelerator Laboratory, Batavia, Illinois 60510‡

(Received 4 March 1974)

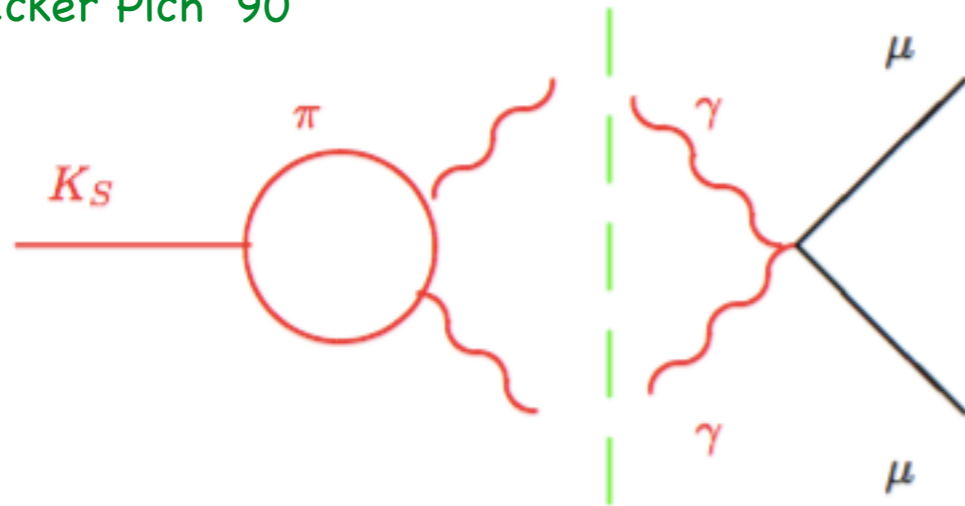
Rare decay modes of the kaons such as $K \rightarrow \mu\bar{\mu}$, $K \rightarrow \pi\nu\bar{\nu}$, $K \rightarrow \gamma\gamma$, $K \rightarrow \pi\gamma\gamma$, and $K \rightarrow \pi e\bar{e}$ are of theoretical interest since here we are observing higher-order weak and electromagnetic interactions. Recent advances in unified gauge theories of weak and electromagnetic interactions allow in principle unambiguous and finite predictions for these processes. The above processes, which are "induced" $|\Delta S|=1$ transitions, are a good testing ground for the cancellation mechanism first invented by Glashow, Iliopoulos, and Maiani (GIM) in order to banish $|\Delta S|=1$ neutral currents. The experimental suppression of $K_L \rightarrow \mu\bar{\mu}$ and nonsuppression of $K_L \rightarrow \gamma\gamma$ must find a natural explanation in the GIM mechanism which makes use of extra quark(s). The procedure we follow is the following: We deduce the effective interaction Lagrangian for $\lambda + \pi \rightarrow l + \bar{l}$ and $\lambda + \bar{\pi} \rightarrow \gamma + \gamma$ in the free-quark model; then the appropriate matrix elements of these operators between hadronic states are evaluated with the aid of the principles of conserved vector current and partially conserved axial-vector current. We focus our attention on the Weinberg-Salam model. In this model, $K \rightarrow \mu\bar{\mu}$ is suppressed due to a fortuitous cancellation. To explain the small $K_L - K_S$ mass difference and nonsuppression of $K_L \rightarrow \gamma\gamma$, it is found necessary to assume $m_{\phi}/m_{\phi'} \ll 1$, where m_{ϕ} is the mass of the proton quark and $m_{\phi'}$ the mass of the charmed quark, and $m_{\phi'} < 5$ GeV. We present a phenomenological argument which indicates that the average mass of charmed pseudoscalar states lies below 10 GeV. The effective interactions so constructed are then used to estimate the rates of other processes. Some of the results are the following: $K_S \rightarrow \gamma\gamma$ is suppressed; $K_S \rightarrow \pi\gamma\gamma$ proceeds at a normal rate, but $K_L \rightarrow \pi\gamma\gamma$ is suppressed; $K_L \rightarrow \pi\nu\bar{\nu}$ is very much forbidden, and $K^+ \rightarrow \pi^+\nu\bar{\nu}$ occurs with the branching ratio of $\sim 10^{-10}$; $K^+ \rightarrow \pi^+e\bar{e}$ has the branching ratio of $\sim 10^{-6}$, which is comparable to the presently available experimental upper bound. The predictions of other models are briefly discussed. Relevant renormalization

VALUE (10^{-9})	CL%	DOCUMENT ID	TECN
< 9	90	¹ AAIJ	2013G LHCb
••• We do not use the following data for averages, fits, limits, etc. •••			
$< 0.032 \times 10^4$	90	GJESDAL	1973 ASPK
$< 0.7 \times 10^4$	90	HYAMS	1969B OSPK

¹ AAIJ 2013G uses 1.0 fb^{-1} of pp collisions at $\sqrt{s} = 7$ TeV. They obtained $B(K_S^0 \rightarrow \mu^+\mu^-) < 11 \times 10^{-9}$ at 95% C.L.

$K_S \rightarrow \mu\mu$

Ecker Pich '90



No CP conserving Short Distance due to Furry Theorem

Gaillard Lee

LD 5×10^{-12} 30% TH err

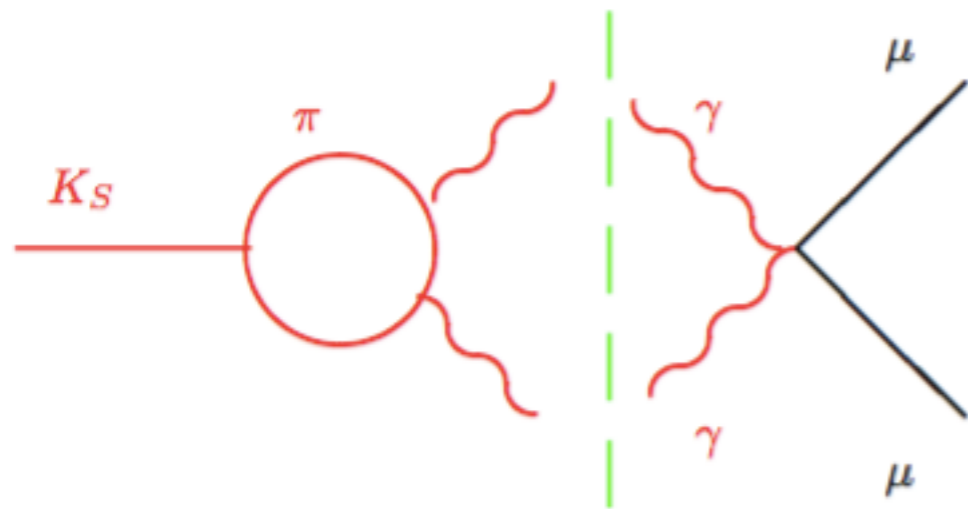
Short Distance

SM	$10^{-5} \Im(V_{ts}^* V_{td}) ^2 \sim 10^{-13}$
NP	few 10^{-11} allowed

LHCb

$< 8 \times 10^{-10}$ 90%CL

$K_S \rightarrow \mu\mu$: how to improve the THEORY error



Dispersive treatment of $K_S \rightarrow \gamma\gamma$ and $K_S \rightarrow \gamma l^+ l^-$

Gilberto Colangelo, Ramon Stucki, and Lewis C. Tunstall

LD 5×10^{-12} 20% TH err

$$K_S \rightarrow \gamma\mu\mu$$

$$K_S \rightarrow \mu\mu\mu\mu$$

$$K_S \rightarrow e e \mu\mu$$

$$K_S \rightarrow \gamma\gamma$$

$K_L \rightarrow \mu\mu$

$$\Gamma(K_L^0 \rightarrow \mu^+\mu^-) / \Gamma(K_L^0 \rightarrow \pi^+\pi^-)$$

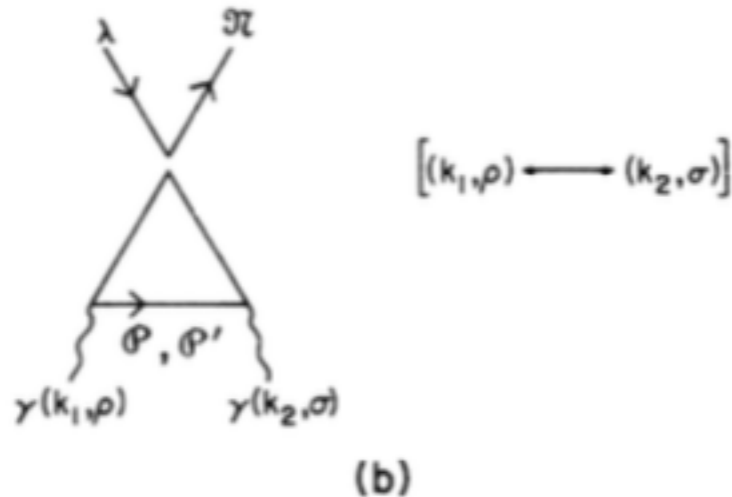
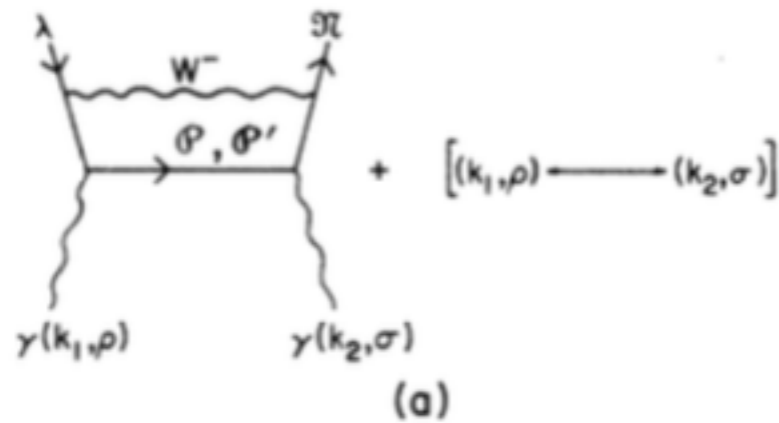


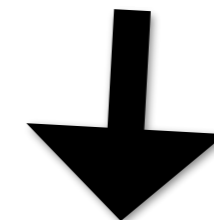
FIG. 7. Leading contributions to $\lambda + \bar{\pi} \rightarrow \gamma + \gamma$. To leading order in M_W^{-2} , the diagrams in (a) reduce to those of (b).

Gaillard Lee

VALUE (10^{-6})	EVTS	DOCUMENT ID	TECN	CO
3.48 ± 0.05	OUR AVERAGE			
3.474 ± 0.057	6210	AMBROSE	2000	B871
3.87 ± 0.30	179	¹ AKAGI	1995	SPEC
3.38 ± 0.17	707	HEINSON	1995	B791
... We do not use the following data for averages, fits, limits, etc. ...				
$3.9 \pm 0.3 \pm 0.1$	178	² AKAGI	1991B	SPEC

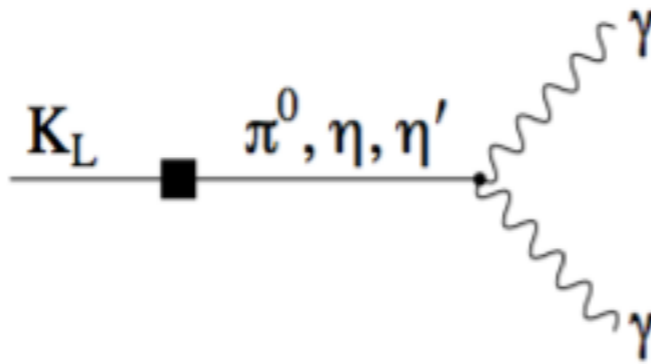
$$\mathcal{B}(K_L \rightarrow \mu^+\mu^-)_{\text{exp}} = (6.84 \pm 0.11) \times 10^{-9}$$

$K_L \rightarrow \gamma\gamma$ |_{exp} **known**



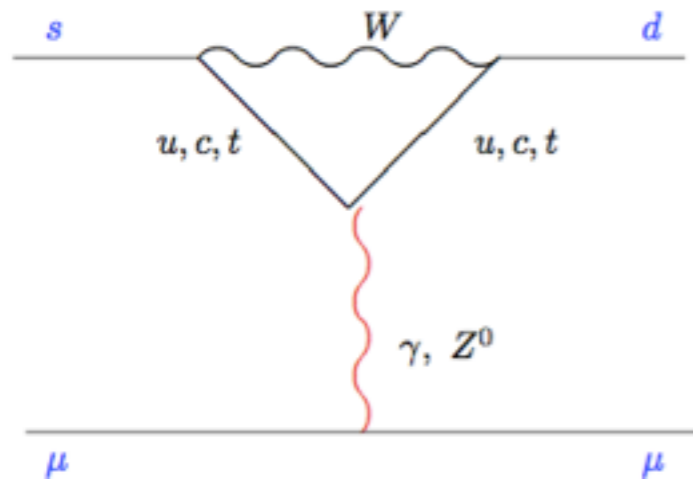
Dispersive calculation: **Re A**, Im A

We do not know the sign of $A(K_L \rightarrow \gamma\gamma)$

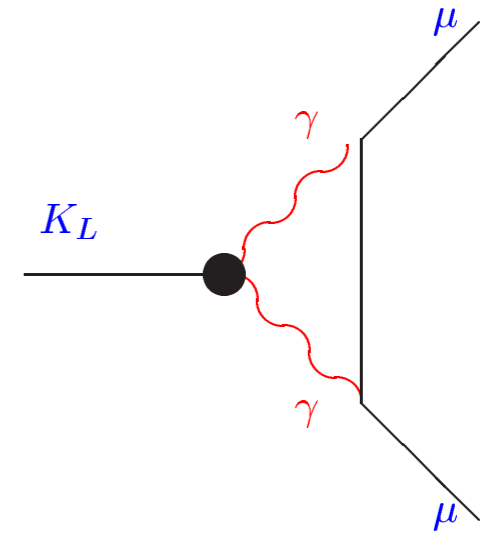


$$\begin{aligned}
 A(K_L \rightarrow 2\gamma_\perp)_{O(p^4)} &= A(K_L \rightarrow \pi^0 \rightarrow 2\gamma_\perp) + A(K_L \rightarrow \eta_8 \rightarrow 2\gamma_\perp) \\
 &= A(K_L \rightarrow \pi^0)A(\pi^0 \rightarrow 2\gamma_\perp) \left[\frac{1}{M_K^2 - M_\pi^2} + \frac{1}{3} \cdot \frac{1}{M_K^2 - M_8^2} \right] \simeq 0
 \end{aligned}$$

$K_L \rightarrow \mu\mu$



\ll



$$\frac{\Gamma(K_L \rightarrow \mu\bar{\mu})}{\Gamma(K_L \rightarrow \gamma\gamma)} \sim$$

$$|ReA|^2 + |ImA|^2$$

Absorptive calculation
model independent

27.14

Subtracting from expt. the Absorptive contribution

$$0.98 \pm 0.55 = |ReA|^2 = (\chi_{\gamma\gamma}(M_\rho) + \chi_{\text{short}} - 5.12)^2$$

$$|\chi_{\text{short}}^{\text{SM}}| = 1.96(1.11 - 0.92\bar{\rho})$$

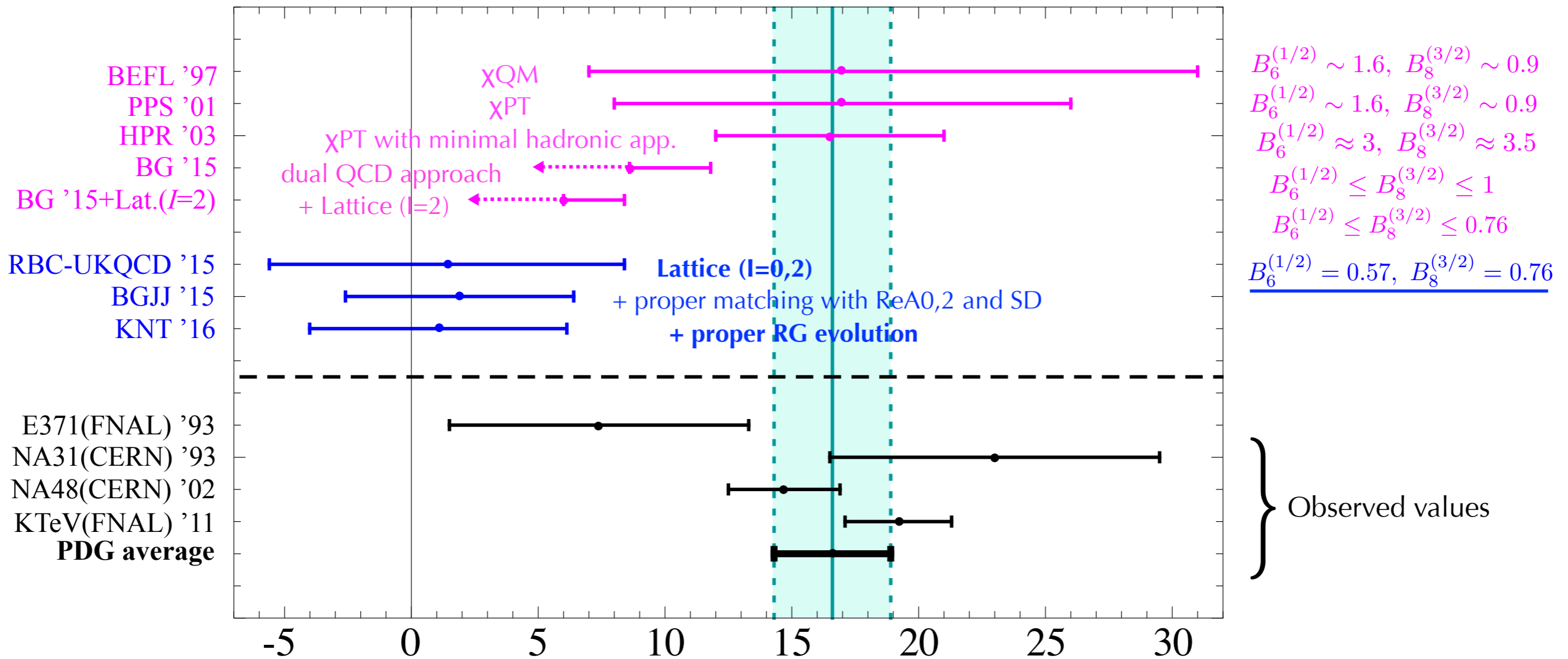
$$\varepsilon'/\varepsilon = (1.4 \pm 7.0) \cdot 10^{-4}$$

$$\left(\frac{\text{Re } A_0}{\text{Re } A_2} \right) = 31.0 \pm 6.6$$

$$\left(\varepsilon'/\varepsilon \right)_{\text{exp}} = (16.6 \pm 2.3) \cdot 10^{-4}$$

$$\left(\frac{\text{Re } A_0}{\text{Re } A_2} \right)_{\text{exp}} = 22.4$$

Current situation of $\epsilon'_K / \epsilon_K \propto \text{Im}A_0 - \left(\frac{\text{Re}A_0}{\text{Re}A_2}\right) \text{Im}A_2$



$B_6^{(1/2)} \sim 1.6, B_8^{(3/2)} \sim 0.9$
 $B_6^{(1/2)} \sim 1.6, B_8^{(3/2)} \sim 0.9$
 $B_6^{(1/2)} \approx 3, B_8^{(3/2)} \approx 3.5$
 $B_6^{(1/2)} \leq B_8^{(3/2)} \leq 1$
 $B_6^{(1/2)} \leq B_8^{(3/2)} \leq 0.76$
 $B_6^{(1/2)} = 0.57, B_8^{(3/2)} = 0.76$

large N limit (convention)

$$B_6^{(1/2)} = B_8^{(3/2)} = 1$$

dual QCD prediction

$$B_6^{(1/2)} \leq B_8^{(3/2)} < 1, B_8^{(3/2)} = 0.8$$

$$\text{Re } \epsilon'_K / \epsilon_K \times 10^4$$

	Exp.	χ PT	dual QCD	Lattice (I=0,2)
$\left(\frac{\text{Re}A_0}{\text{Re}A_2}\right)$	22.45 ± 0.05	~ 14	16.0 ± 1.5	31.0 ± 11.1

Models solving ϵ'/ϵ anomaly

- Several new physics models have been studied to explain ϵ'/ϵ anomaly

MSSM -- chargino Z penguin	<i>[M. Endo, S. Mishima, D. Ueda and KY, PLB762(2016)493]</i>
-- gluino Z penguin	<i>[M. Tanimoto and KY, PTEP(2016)no.12,123B02]</i>
-- gluino box	<i>[T.Kitahara, U.Nierste and P.Trempfer, PRL117(2016)no.9, 091802 A.Crivellin, G.D'Ambrosio, T.Kitahara and U.Nierste, 1703.05786]</i>
Vector-like quarks	<i>[C.Bobeth, A.J.Buras, A.Celis and M.Jung, JHEP1704(2017)079]</i>
Little Higgs Model with T-parity	<i>[M.Blanke, A.J.Buras and S.Recksiegel, EPJ.C76 (2016)no.4,182]</i>
331 model	<i>[A.J.Buras and F.De Fazio, JHEP1603(2016)010 & JHEP1608 (2016) 115]</i>
Right handed current	<i>[V.Cirigliano, W.Dekens, J.de Vries and E.Mereghetti, PLB 767 (2017) 1 S.Alioli, V.Cirigliano, W.Dekens, J.de Vries and E.Mereghetti, JHEP1705 (2017)086]</i>

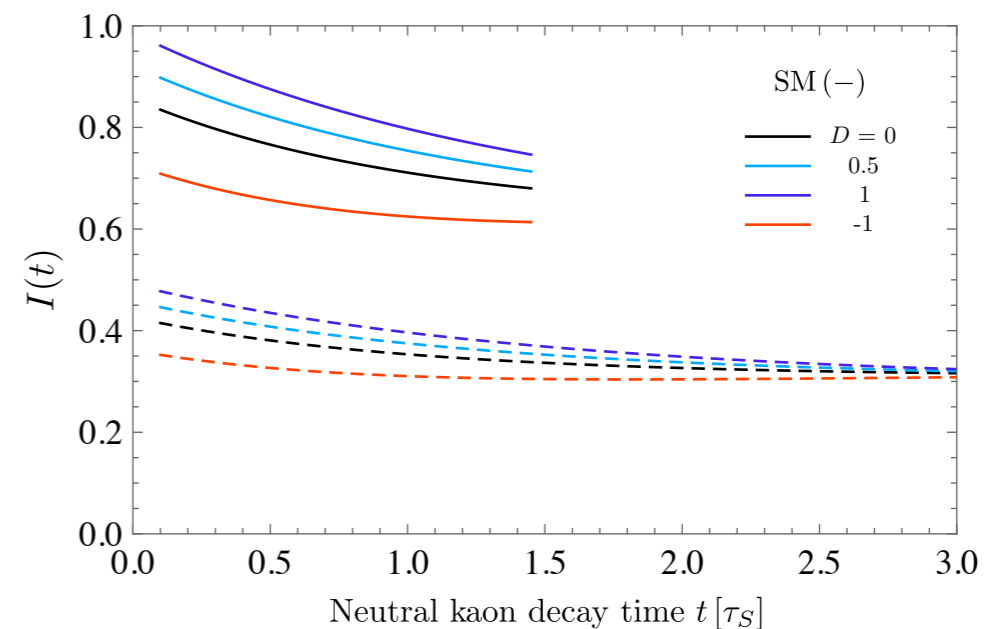
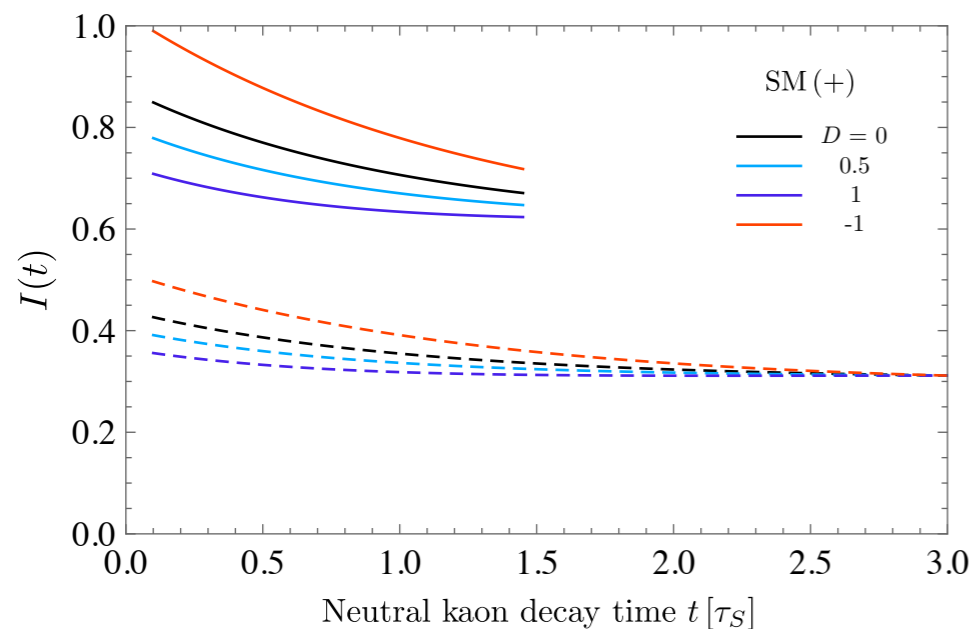
- Different implications (correlations & predictions) for other observables appear depending on models \Rightarrow Possibility of model discriminations

Can we study $K^0(t)$?

GD, Kitahara
1707.06999 PRL

$$pp \rightarrow K^0 K^- X$$

$$pp \rightarrow K^{*+} X \rightarrow K^0 \pi^+ X$$



$$|\bar{K}^0(t)\rangle = \frac{1}{\sqrt{2}(1 \pm \bar{\epsilon})} \left[e^{-iH_S t} (|K_1\rangle + \bar{\epsilon}|K_2\rangle) \right. \\ \left. \pm e^{-iH_L t} (|K_2\rangle + \bar{\epsilon}|K_1\rangle) \right]$$

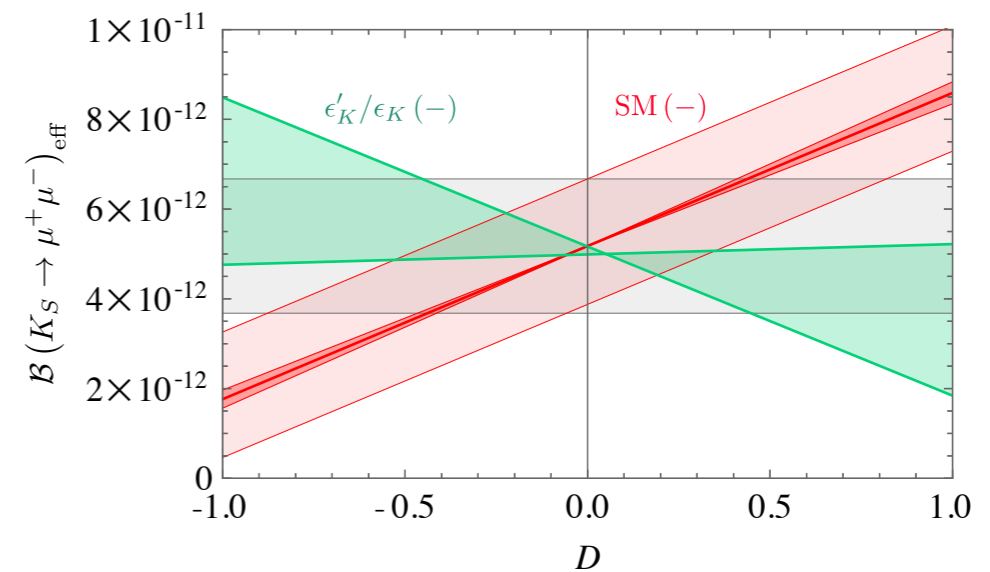
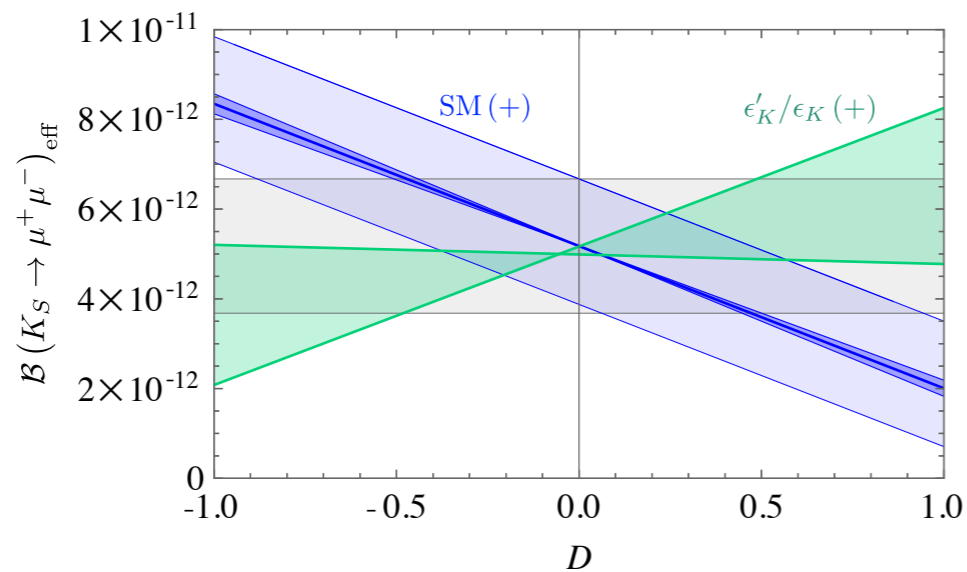
$$D = \frac{K^0 - \bar{K}^0}{K^0 + \bar{K}^0}$$

- **Short distance interfering** with Large CP conserving LD contribution !
- We may be able to study the time evolution of K^0 by tracking the associated particles (K^-)

$$\sum_{\text{spin}} \mathcal{A}(K_1 \rightarrow \mu^+ \mu^-)^* \mathcal{A}(K_2 \rightarrow \mu^+ \mu^-)$$

$$\sim \text{Im}[\lambda_t] y'_{7A} \left\{ A_{L\gamma\gamma}^\mu - 2\pi \sin^2 \theta_W (\text{Re}[\lambda_t] y'_{7A} + \text{Re}[\lambda_c] y_c) \right\}$$

Short distance window



$$\begin{aligned} & \mathcal{B}(K_S \rightarrow \mu^+ \mu^-)_{\text{eff}} \\ &= \tau_S \left[\int_{t_{\min}}^{t_{\max}} dt \left(\Gamma(K_1) e^{-\Gamma_S t} + \frac{D}{8\pi M_K} \sqrt{1 - \frac{4m_\mu^2}{M_K^2}} \sum_{\text{spin}} \text{Re} \left[e^{-i\Delta M_K t} \mathcal{A}(K_1)^* \mathcal{A}(K_2) \right] e^{-\frac{\Gamma_S + \Gamma_L}{2} t} \right) \varepsilon(t) \right] \\ & \times \left(\int_{t_{\min}}^{t_{\max}} dt e^{-\Gamma_S t} \varepsilon(t) \right)^{-1}, \end{aligned}$$

Conclusions

- QCD tools prepared for the uncover all secrets of flavor including
- Many rare channels to study
- $K \rightarrow \pi l l$ work in progress: hope results soon
- collaboration is crucial

