Lepton flavor (universality) violation and kaon physics

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First forum on rare kaon decays

Higgs Centre for Theoretical Physics

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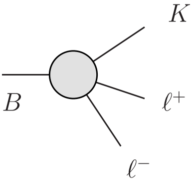
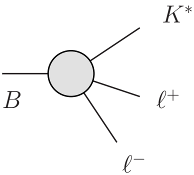
A. Crivellin, G. D'Ambrosio, MH, L. Tunstall, PRD (2016) 074038

V. Cirigliano, A. Crivellin, MH, 1712.06595, to appear in PRL

Flavor anomalies in B decays





- **Hints for LFUV** in $R(K)$ and $R(K^*)$ from LHCb 1406.6482, 1705.05802

$$R_K = \frac{\text{Br}[B \to K\mu^+\mu^-]}{\text{Br}[B \to Ke^+e^-]} \qquad R_{K^*} = \frac{\text{Br}[B \to K^*\mu^+\mu^-]}{\text{Br}[B \to K^*e^+e^-]}$$

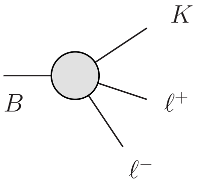
$$R_K^{[1.0,6.0]} = 0.745^{+0.090}_{-0.074} \pm 0.036 \qquad R_{K^*}^{[0.045,1.1]} = 0.66^{+0.11}_{-0.07} \pm 0.03 \qquad R_{K^*}^{[1.1,6.0]} = 0.69^{+0.11}_{-0.07} \pm 0.05$$

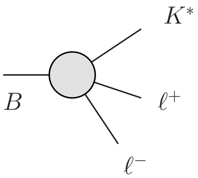
$\hookrightarrow$ tension of 2.6σ, 2.2σ, and 2.4σ

- Angular observables in $B \to K^*\mu^+\mu^-$ from LHCb 1308.1707, 1512.04442, Belle 1612.05014, CMS CMS-PAS-BPH-15-008, ATLAS ATLAS-CONF- 2017-023

- Branching ratio for $B_s \to \phi\mu^+\mu^-$ from LHCb 1506.08777 about 3σ below SM

Flavor anomalies in B decays





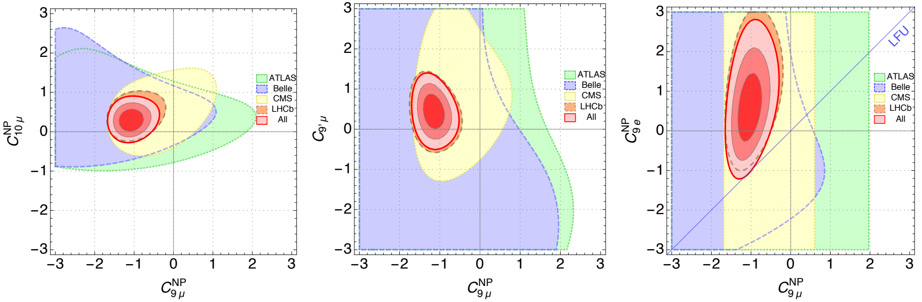
- All point towards BSM in muon channels $\Rightarrow$ **LFUV**
- EFT analysis: most relevant C_9, C_9', and C_{10}

$$Q_9 = \frac{e^2}{32\pi^2}\left[\bar{s}\gamma^\mu(1-\gamma_5)b\right]\sum_{\ell=e,\mu}\left[\bar{\ell}\gamma_\mu\ell\right] \qquad Q_{10} = \frac{e^2}{32\pi^2}\left[\bar{s}\gamma^\mu(1-\gamma_5)b\right]\sum_{\ell=e,\mu}\left[\bar{\ell}\gamma_\mu\gamma_5\ell\right]$$

$$Q_9' = \frac{e^2}{32\pi^2}\left[\bar{s}\gamma^\mu(1+\gamma_5)b\right]\sum_{\ell=e,\mu}\left[\bar{\ell}\gamma_\mu\ell\right]$$

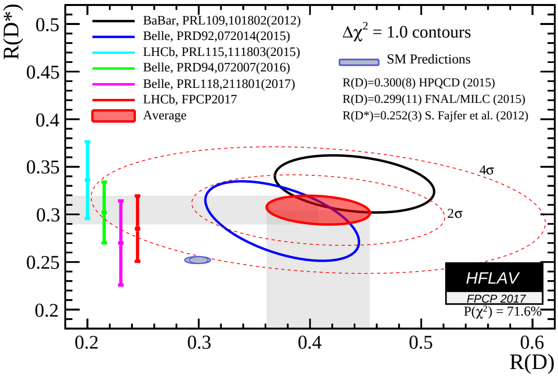
- Quantitative analysis depends on form factors especially in $B\to K^*\mu^+\mu^-$, but **global fits** consistently find BSM at 5σ or above Capdevila et al., Altmannshofer et al. 2017

Flavor anomalies in B decays



Capdevila et al. 2017

- BSM in C_9 required, in C_9', C_{10} possible
- BSM in $\mu\mu$ required, in *ee* possible
- BSM preferred $> 5\sigma$, LFUV by 3–4σ



Heavy flavor averaging group 2017

- More anomalies in $b \to c\tau\nu$

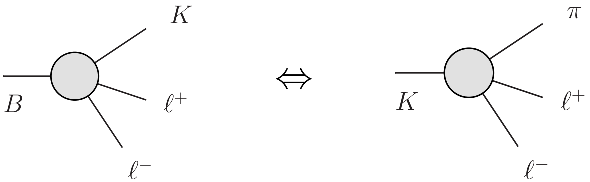
$$R_D = \frac{\text{Br}[B \to D\tau\nu_\tau]}{\text{Br}[B \to D\ell\nu_\ell]} \qquad R_{D^*} = \frac{\text{Br}[B \to D^*\tau\nu_\tau]}{\text{Br}[B \to D^*\ell\nu_\ell]} \qquad R_{J/\psi} = \frac{\text{Br}[B \to J/\psi\tau\nu_\tau]}{\text{Br}[B \to J/\psi\ell\nu_\ell]}$$

- In all cases: no single conclusive measurement, but deviations add up
- **What can we learn from kaon physics?**

Outline

1. Lepton flavor universality violation: $K \to \pi\ell^+\ell^-$
2. Lepton flavor universality violation: $K \to \ell^+\ell^-$
3. Lepton flavor violation: $K \to (\pi)\ell^+\ell'^-$
4. *CP* asymmetry in $\tau \to K_S\pi\nu_\tau$

Lepton flavor universality violation in B and K decays



- Effective $\Delta S = 1$ Lagrangian

$$\mathcal{H}_{\text{eff}}^{|\Delta B|=1} = -\frac{4G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_i C_i^B(\mu) Q_i^B(\mu) + \text{h.c.} \;\Leftrightarrow\; \mathcal{L}_{\text{eff}}^{|\Delta S|=1} = -\frac{G_F}{\sqrt{2}} V_{ud} V_{us}^* \sum_i C_i(\mu) Q_i(\mu) + \text{h.c.}$$

- $\boldsymbol{s \to d\ell^+\ell^-}$ instead of $\boldsymbol{b \to s\ell^+\ell^-}$

$$Q_{11} \equiv Q_{7V} = [\bar{s}\gamma^\mu(1-\gamma_5)d] \sum_{\ell=e,\mu} [\bar{\ell}\gamma_\mu\ell] \qquad Q_{12} \equiv Q_{7A} = [\bar{s}\gamma^\mu(1-\gamma_5)d] \sum_{\ell=e,\mu} [\bar{\ell}\gamma_\mu\gamma_5\ell]$$

$\hookrightarrow$ analogs of $Q_{9,10}^B$

- Assuming **Minimal Flavor Violation** $C_{9,10}^B$ and $C_{7V,7A}$ are correlated

  $\hookrightarrow$ probe $C_{9,10}^B = \mathcal{O}(1)$ in kaon decays?

- **Experimental status**

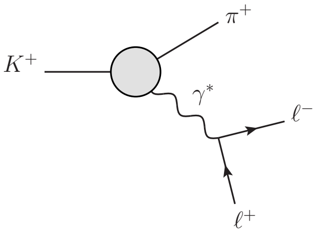
$$\mathrm{Br}[K^+ \to \pi^+ e^+ e^-] = 3.00(9) \times 10^{-7} \qquad \mathrm{Br}[K^+ \to \pi^+ \mu^+ \mu^-] = 9.4(6) \times 10^{-8}$$

$$\mathrm{Br}[K_S \to \pi^0 e^+ e^-]_{m_{ee} > 0.165\,\mathrm{GeV}} = 3.0^{+1.5}_{-1.2} \times 10^{-9} \qquad \mathrm{Br}[K_S \to \pi^0 \mu^+ \mu^-] = 2.9^{+1.5}_{-1.2} \times 10^{-9}$$

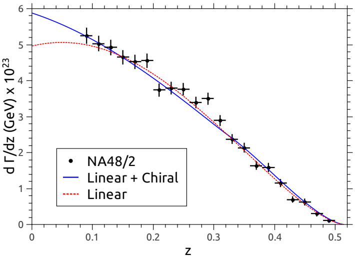
$$\mathrm{Br}[K_L \to \pi^0 e^+ e^-] < 2.8 \times 10^{-10} \qquad \mathrm{Br}[K_L \to \pi^0 \mu^+ \mu^-] < 3.8 \times 10^{-10}$$

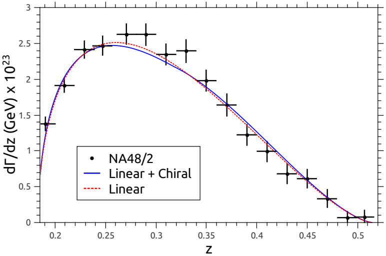
- Problem: **long-distance effects** in SM prediction
- Parameterize ignorance in **low-energy constants**
- In the future: lattice Christ et al. 2015, 2016
- Focus on $K^+ \to \pi^+\ell^+\ell^-$ in the following

  $\hookrightarrow$ in SM dominated by $K^\pm \to \pi^\pm\gamma^*$



Parameterization of vector form factor





figures from Cirigliano et al. 2011 NA48/2 *ee* NA48/2 $\mu\mu$

- **Spectrum** dominated by vector channel

$$\frac{d\Gamma}{dz} \propto |V_+(z)|^2 \qquad z = \frac{q^2}{M_K^2}$$

- ChPT implies representation D'Ambrosio et al. 1998

$$V_+(z) = a_+ + b_+ z + V_+^{\pi\pi}(z)$$

$\hookrightarrow$ a_+ and b_+ superposition of long- and short-distance physics

- Fits to E865 and NA48/2 spectra yield

$$a_+^{ee} = -0.584(8) \qquad a_+^{\mu\mu} = -0.575(39)$$

- **Key point**: LFU implies $a_+^{ee} = a_+^{\mu\mu}$

  $\hookrightarrow$ LFUV can be probed by taking difference $a_+^{ee} - a_+^{\mu\mu}$

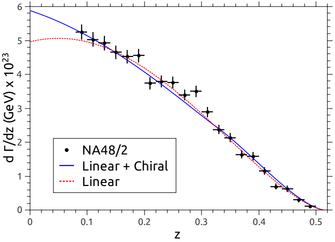
- Long-distance effects drop out in

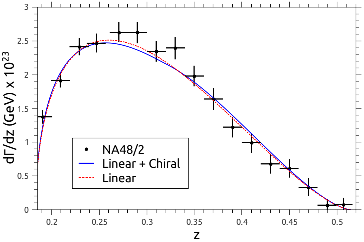
$$C_{7V}^{\mu\mu} - C_{7V}^{ee} = \alpha \frac{a_+^{\mu\mu} - a_+^{ee}}{2\pi\sqrt{2} V_{ud} V_{us}^*}$$

- In **MFV**, putting in numbers,

$$C_9^{B,\mu\mu} - C_9^{B,ee} = -\frac{a_+^{\mu\mu} - a_+^{ee}}{\sqrt{2} V_{ts}^* V_{td}} = -20(80)$$

- Adding an **axial term** gives $|C_{10}^{B,\mu\mu} - C_{10}^{B,ee}| \lesssim 1000$





figures from Cirigliano et al. 2011 NA48/2 *ee* NA48/2 $\mu\mu$

- Fits to E865 and NA48/2 spectra yield

$$a_+^{ee} = -0.584(8) \qquad a_+^{\mu\mu} = -0.575(39)$$

$\hookrightarrow$ largest uncertainty in **muon mode**

- High statistics at NA62, radiative corrections Kubis, Schmidt 2010
- Factor 80 may be out of reach, but can still **rule out non-MFV models** that predict larger effects

LFUV and $K_L \to \ell^+\ell$

- Similar strategy for $K_L \to \ell^+\ell$, but complementary due to $C_{7V} \to C_{7A}$
- Decay rate

$$R_{\ell\ell} = \frac{\Gamma[K_L \to \ell^+\ell^-]}{\Gamma[K_L \to \gamma\gamma]} = 2\beta_\ell \left(\frac{\alpha}{\pi} r_\ell\right)^2 \left(|F_{\ell,\text{abs}}|^2 + |F_{\ell,\text{disp}}|^2\right) \qquad \beta_\ell = \sqrt{1 - \frac{4m_\ell^2}{M_K^2}}$$

$$F_{\ell,\text{abs}} = \frac{\pi}{2\beta_\ell} \log\left(\frac{1-\beta_\ell}{1+\beta_\ell}\right)$$

$$F_{\ell,\text{disp}} = \frac{1}{4\beta_\ell} \log^2\left(\frac{1-\beta_\ell}{1+\beta_\ell}\right) + \frac{1}{\beta_\ell}\text{Li}_2\left(\frac{\beta_\ell - 1}{\beta_\ell + 1}\right) + \frac{\pi^2}{12\beta_\ell} + 3\log\frac{m_\ell}{\mu} + \chi(\mu)$$

$\hookrightarrow$ again unknown low-energy constant

- **Short-distance contribution** from the difference

$$\text{Re}\, C_{7A}^{\mu\mu} - \text{Re}\, C_{7A}^{ee} = -\frac{\alpha}{F_K N_K}\left(\frac{2\Gamma[K_L \to \gamma\gamma]}{\pi M_K^3}\right)^{1/2}\left(\chi^{\mu\mu} - \chi^{ee}\right) \qquad N_K = G_F V_{ud} V_{us}^*$$

- In MFV

$$C_{10}^{B,\mu\mu} - C_{10}^{B,ee} = \frac{2\pi}{F_K G_F \lambda_t}\left(\frac{2\Gamma_{\gamma\gamma}}{\pi M_K^3}\right)^{1/2}\left(\chi^{\mu\mu} - \chi^{ee}\right) \qquad \lambda_t = V_{ts}^* V_{td}$$

LFUV and $K_L \to \ell^+\ell$: numbers

Channel	χ (Solution 1)	χ (Solution 2)
ee	$5.1^{+15.4}_{-10.3}$	$-(57.5^{+15.4}_{-10.3})$
$\mu\mu$	3.75(20)	1.52(20)

- **Experimental status**

$$R_{\mu\mu} = 1.25(2) \times 10^{-5} \qquad R_{ee} = 1.6^{+1.1}_{-0.7} \times 10^{-8}$$

- Two solutions for $\chi(M_\rho)$
- Suppose uncertainty could be reduced by a factor 10, then in MFV

$$\chi^{\mu\mu} - \chi^{ee} = 1.3(1.3) \qquad \Rightarrow \qquad C_{10}^{B,\mu\mu} - C_{10}^{B,ee} = 3.5(3.5)$$

$\hookrightarrow$ also at least an order of magnitude missing to MFV

- In practice: two-loop corrections likely important, estimate from m_ℓ-dependent terms $\Delta\chi^{\mu\mu} - \Delta\chi^{ee} = -2.8$

LFUV and $K_S \to \ell^+ \ell$

- **Experimental status** LHCb 1706.00758

$$\text{Br}[K_S \to \mu^+\mu^-] < 0.8 \times 10^{-9} \text{ at } 90\% \text{ c.l.}$$

- Standard model Ecker, Pich 1991: $\text{Br}[K_S \to \mu^+\mu^-] \approx 5 \times 10^{-12}$
- Decay rate

$$\Gamma[K_S \to \ell^+\ell^-] = \frac{M_K}{8\pi}\beta_\ell\Big(\beta_\ell^2|B|^2 + |C|^2\Big) \qquad A[K \to \ell^+\ell^-] = \bar{u}(k_-)\big(iB + C\gamma_5\big)v(k_+)$$

- **Short-distance contribution**

$$\text{Im}\, C = -2\sqrt{2}G_F F_K m_\ell \text{Im}\left(V_{ud}V_{us}^* C_{7A}\right)$$

- No (CP-conserving) **long-distance** contribution to C amplitude Isidori, Unterdorfer 2003, thus LHCb limit implies

$$|\text{Im}\, C_{10}^{B,\mu\mu}| \leq \frac{2\pi}{\alpha F_K G_F \lambda_t m_\mu}\sqrt{\frac{\pi\Gamma[K_S \to \ell^+\ell^-]}{M_K\beta_\mu}} < 116$$

LFV: $K_L \to \mu^\pm e^\mp$, $K_L \to \pi^0\mu^\pm e^\mp$, $K^+ \to \pi^+\mu^\pm e^\mp$

- **LFV**: no long-distance contributions
- Decays sensitive to different combinations of Wilson coefficients
  - $K_L \to \mu^\pm e^\mp$ ($\text{Br}[K_L \to \mu^\pm e^\mp] < 4.7 \times 10^{-12}$):

$$\sqrt{\big|C_{7V}^{\mu e} + (C_{7V}^{e\mu})^*\big|^2 + \big|C_{7A}^{\mu e} + (C_{7A}^{\mu e})^*\big|^2} < 1.9 \times 10^{-6}$$

  - $K_L \to \pi^0\mu^\pm e^\mp$ ($\text{Br}[K_L \to \pi^0\mu^\pm e^\mp] < 7.6 \times 10^{-11}$):

$$\sqrt{\big|C_{7V}^{\mu e} - (C_{7V}^{e\mu})^*\big|^2 + \big|C_{7A}^{\mu e} - (C_{7A}^{\mu e})^*\big|^2} < 3.5 \times 10^{-5}$$

  - $K^+ \to \pi^+\mu^\pm e^\mp$ ($\text{Br}[K^+ \to \pi^+\mu^+ e^-] < 1.3 \times 10^{-11}$, $\text{Br}[K^+ \to \pi^+\mu^- e^+] < 5.2 \times 10^{-10}$):

$$\sqrt{\big|C_{7V}^{e\mu}\big|^2 + \big|C_{7A}^{e\mu}\big|^2} < 2.2 \times 10^{-5}$$

$$\sqrt{\big|C_{7V}^{\mu e}\big|^2 + \big|C_{7A}^{\mu e}\big|^2} < 1.4 \times 10^{-4}$$

- Can again be related to B system via MFV

LFUV and $K^{\pm} \to \pi^{\pm}\ell^{+}\ell$: conclusions

- $C_9^{B,\mu\mu} - C_9^{B,ee} = -20(80)$ vs. $C_9^{B,\mu\mu} = \mathcal{O}(1)$ for B anomalies

  $\hookrightarrow$ need at least an order of magnitude improvement
- Currently largest uncertainty in $\mu^{+}\mu^{-}$ channel
- Similar strategy in $\boldsymbol{K_L \to \ell^{+}\ell^{-}}$: sensitive to $C_{7A} \leftrightarrow C_{10}^{B}$, but in MFV also more than a factor 10 missing
- Limits from LFV modes $K \to \pi\mu e$, $K \to \mu e$
- Can probe models in which effects in kaon physics are larger than in MFV, but: no reason why BSM should be MFV

  $\hookrightarrow$ **potentially large impact of forthcoming kaon experiments**!

- Consider CP asymmetry

$$A_{CP}^{\tau} = \frac{\Gamma[\tau^+ \to \pi^+ K_S \bar{\nu}_\tau] - \Gamma[\tau^- \to \pi^- K_S \nu_\tau]}{\Gamma[\tau^+ \to \pi^+ K_S \bar{\nu}_\tau] + \Gamma[\tau^- \to \pi^- K_S \nu_\tau]}$$

- In SM dominated by indirect CP violation

$$A_{CP}^{\tau,\text{indirect}} = A_L = \frac{\Gamma[K_L \to \pi^- \ell^+ \nu_\ell] - \Gamma[K_L \to \pi^+ \ell^- \bar{\nu}_\ell]}{\Gamma[K_L \to \pi^- \ell^+ \nu_\ell] + \Gamma[K_L \to \pi^+ \ell^- \bar{\nu}_\ell]} = 3.32(6) \times 10^{-3}$$

- BaBar 2011 measurement

$$A_{CP}^{\tau,\text{exp}} = -3.6(2.3)(1.1) \times 10^{-3} \qquad \text{vs.} \qquad A_{CP}^{\tau,\text{SM}} = 3.6(1) \times 10^{-3}$$

$A_{CP}^{\tau,\text{SM}}$ includes corrections due to K_S reconstruction Grossman, Nir 2012

- **2.8σ tension**

- How is this related to kaon physics? Via $K_{\ell 3}$ form factors!

BSM contributions?

$$\frac{d\Gamma}{ds} = G_F^2 |V_{us}|^2 S_{\text{EW}} \frac{\lambda_{\pi K}^{1/2}(s)(m_\tau^2 - s)^2 (M_K^2 - M_\pi^2)^2}{1024\pi^3 m_\tau s^3}$$

$$\times \left[\xi(s) \left(|V(s)|^2 + |A(s)|^2 + \frac{4(m_\tau^2 - s)^2}{9 s m_\tau^2} |T(s)|^2 \right) + |S(s)|^2 + |P(s)|^2 \right]$$

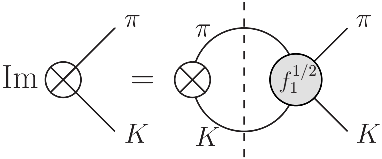
$$V(s) = f_+(s) c_V - T(s) \qquad S(s) = f_0(s) \left(c_V + \frac{s}{m_\tau (m_s - m_u)} c_S \right) \qquad T(s) = \frac{3s}{m_\tau^2 + 2s} \frac{m_\tau}{M_K} c_T B_T(s)$$

- Can direct *CP* violation from BSM physics explain the measurement?
- **Vector–scalar** interference has same form factor $f_0(s)$

  $\hookrightarrow$ strong phase vanishes
- Leaves **vector–tensor** interference Devi, Dhargyal, Sinha 2014

  $\hookrightarrow$ strong phase from relative phase of $f_+(s)$ and $B_T(s)$

What do we know about the tensor form factor?



- Normalization from lattice QCD Baum et al. 2011: $B_T(0)/f_+(0) = 0.676(27)$
- **Optical theorem**: elastic unitarity relations from πK intermediate states

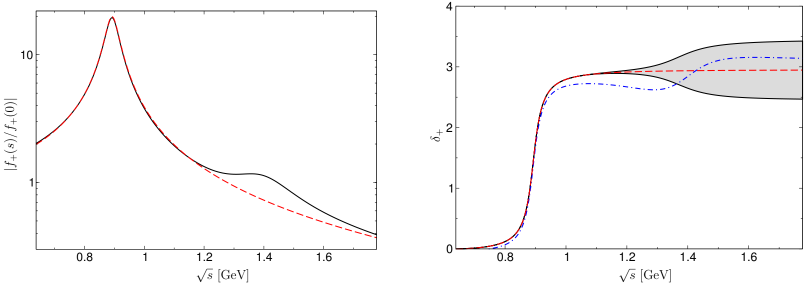
$$\text{Im}\, B_T(s) = \frac{\lambda_{\pi K}^{1/2}(s)}{s} B_T(s) \big(f_1^{1/2}(s)\big)^*$$

$$\text{Im}\, f_+(s) = \frac{\lambda_{\pi K}^{1/2}(s)}{s} f_+(s) \big(f_1^{1/2}(s)\big)^*$$

$\hookrightarrow$ Watson's final-state theorem: $\arg B_T(s) = \arg f_+(s) = \delta_1^{1/2}(s)$

- **Vector–tensor** interference vanishes up to inelastic corrections

Estimating inelastic corrections

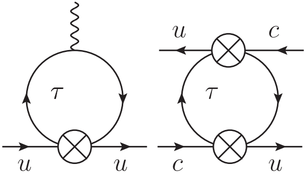


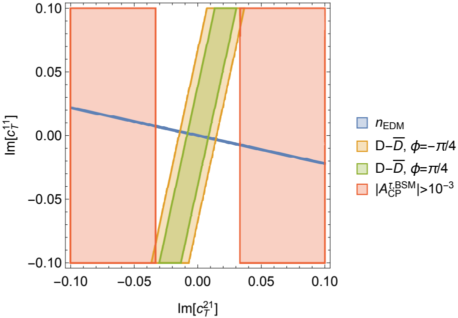
- $f_+(s)$ dominated by elastic ***K**(892) resonance**
- Some inelastic corrections around ***K**(1410)**
- Our estimate

$$\left|A_{CP}^{\tau,\mathrm{BSM}}\right| \lesssim 0.03|\mathrm{Im}\, c_T|$$

Constraints on tensor operator

- ***SU*(2) invariance** relates tensor operator to $(\bar{\tau}_L\sigma_{\mu\nu}\tau_R)(\bar{u}_L\sigma^{\mu\nu}u_R)$
  $\hookrightarrow$ contribution to **neutron EDM**
- Similar operator with $\bar{c}_L\sigma^{\mu\nu}u_R$ (c_T^{11})
  $\hookrightarrow$ contribution to ***D*–*D̄* mixing**
- Need intricate cancellations to satisfy both constraints, for neutron EDM at $\mathcal{O}(10^{-4})$!





CP asymmetry in $\tau \to K_S \pi \nu_\tau$: conclusions

- BaBar measurement disagrees with SM by 2.8σ
- Only BSM contribution from **tensor operator**, but strongly suppressed due to **Watson's theorem**
- Evading constraints from **neutron EDM** and ***D*–*D̄* mixing** requires strong fine-tuning

  $\hookrightarrow$ explanation involving BSM in the UV extremely difficult
- If confirmed at Belle-II: light BSM physics?

QED corrections

- QED corrections Antonelli et al. 2013 produce non-vanishing **vector–scalar** interference
- Suppressed by
  - $f_0(s)$ vs. $f_+(s)$
  - Kinematics
  - $\mathcal{O}(\alpha/\pi)$
- Final estimate

$$|A_{CP}^{\tau,\text{BSM}}| \lesssim 10^{-4}|\text{Im}\, c_S|$$

- From $\tau \to K_S\pi\nu_\tau$ spectrum: $|\text{Im}\, c_S| \lesssim 1$

  $\hookrightarrow$ phenomenologically irrelevant