Imaging Fundamental Processes: The Story and Stories of Jets at Acceleraors

April 13, 2018

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1. Seeing the Unseen at Accelerators
2. From Short to Long Distances in Quantum Field Theory:
   What we can't compute and what we can
3. A Brief Biography of Particle Jets
4. Theory of Jets at Colliders

1. Seeing the Unseen at Colliders

(First, a few comments on the Triumph of the Standard Model at Accelerators)

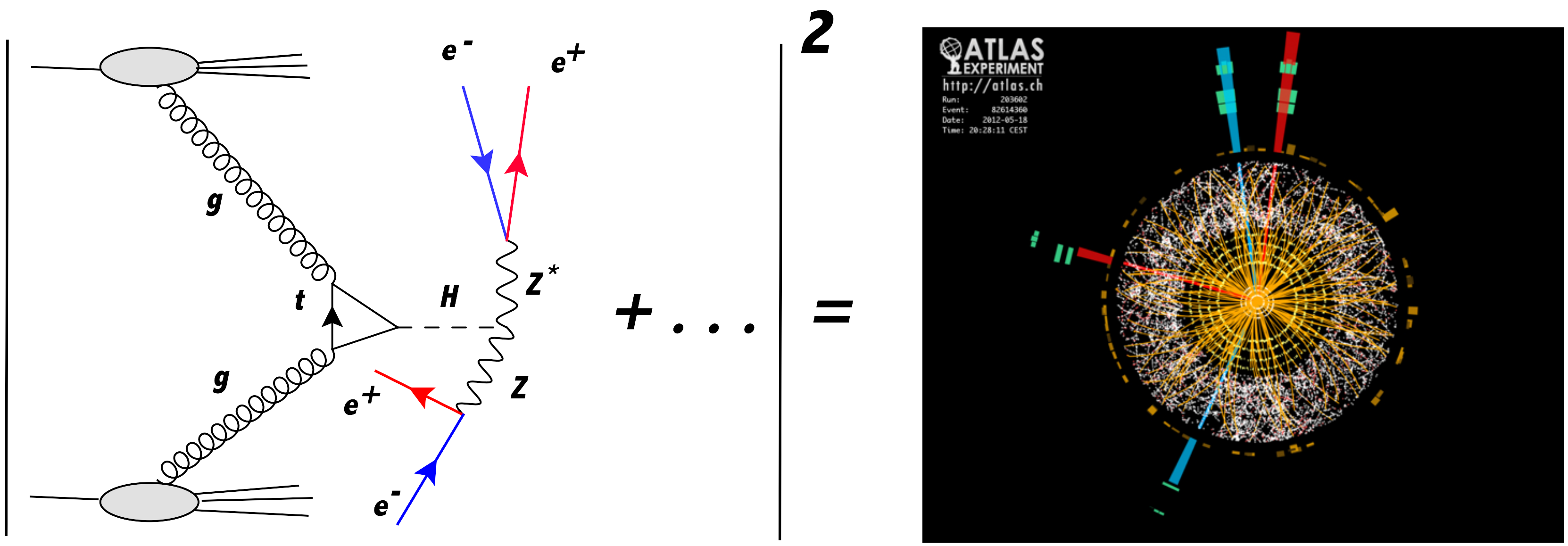
- **High energy accelerators offer the most direct window to short-lived quantum processes.**

- **The strategy of probing matter at short distances has resulted in the identification/discovery of the gauge and matter fields of the Standard Model**

- **Accelerator programs, however complex and costly, remain experiments following scientific canon. They are capable of design, replication and variation in response to the demands of nature and the imagination.**

- **I will review a little of how quantum field theory is applied in accelerator experiments, and how jets emerge in final states and what they tell us.**

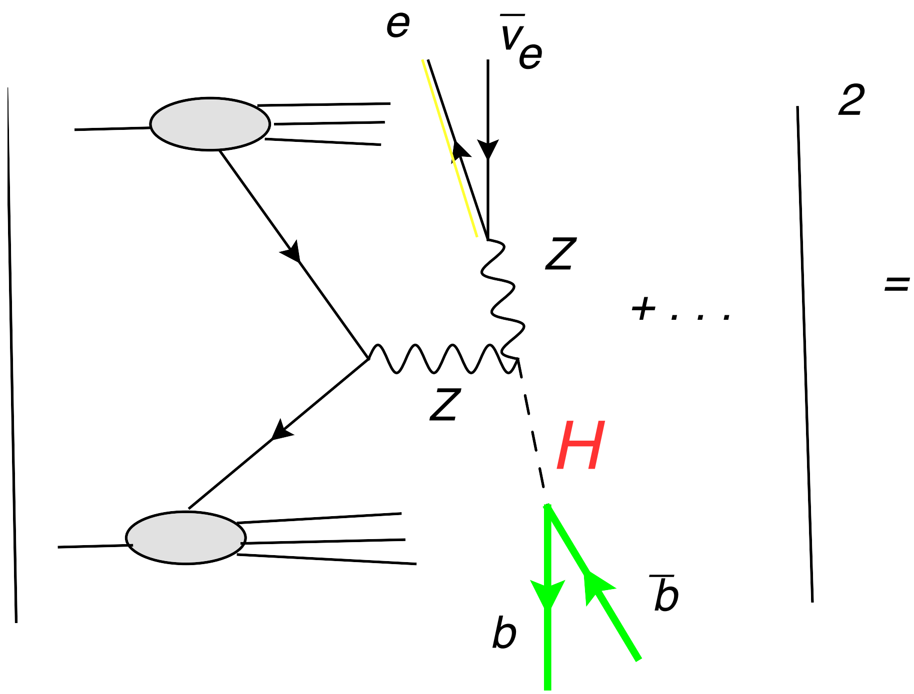
We can sum it up with a picture worth a thousand words:

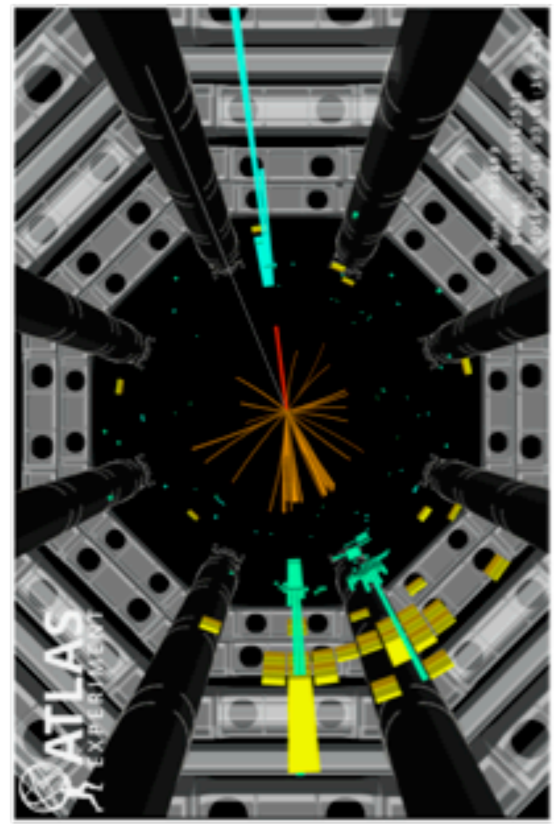


From $SU(3)$ color through the Higgs into $SU(2)_L \times U(1)$.

Every observed final state is the result of a quantum-mechanical set of stories, and so far the stories supplied by the Standard Model, built on an unbroken $SU(3)$ color gauge theory (very much like the original Yang-Mills Lagrangian) and a spontaneously-broken $SU(2)_L \times U(1)$, account for all observations at accelerators.

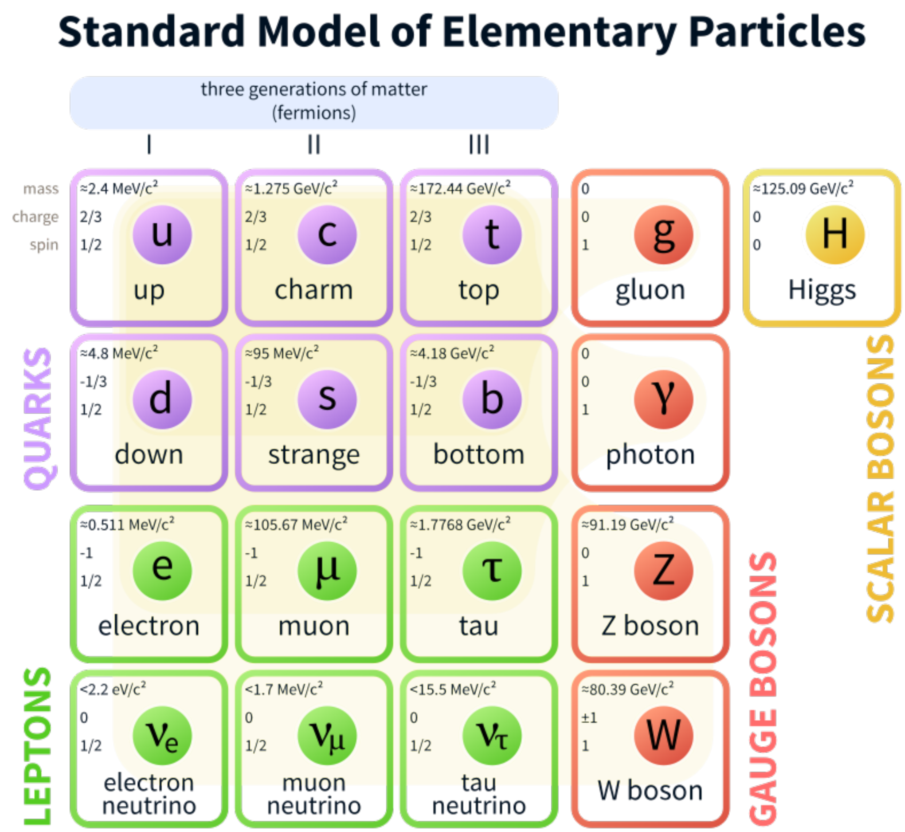
And recently, $Z + H \to b\bar{b}$ as revealed in boosted dijet decays:





- This could be the "end of the story", except:
  - Cosmological observations strongly suggest that there are other sources of gravitation in the universe: Dark Matter; Dark Energy, and (optionally) the mystery of flavor.
  - The mass of the Higgs particle in the Standard Model in isolation is unstable to overwhelming quantum corrections. So, what to expect?
  - This distress with the "hierarchy problem" of the Standard Model may be compared to 17th Century objections to action at a distance in Newtonian gravity. It comes from profound intuition, but does not immediately suggest a resolution.
  - Putting all this aside, as the progress of science put gravitational action at a distance aside until 1915, the success is extraordinary. And resolutions of the Standard Model's puzzles, and even of Dark Matter, may in the fulness of time come from theories with many or most of the Standard Model's properties, or generalization inspired by it..
  - Let's return to how we got to this stage, how we learned to recount the stories that lead to the Standard Model's successes, and the role of particle jets in these developments.

THE PARTICLE CONTENT OF THE STANDARD MODEL: OBSERVED AND THE INFERRED



The six quarks in the upper left-hand corner are not seen in isolation, although five have lifetimes long enough to be "seen". The original three were inferred as an alphabet for bound states in the "quark model (Gell Mann & Zweig) from the mid-1960s.

- **The Standard Model developed through the latter half of the Twentieth Century in parallel with modern field-theoretic ideas of flow: couplings within theories (renormalization group) and between theories (Wilsonian).**

- **A primary theme of Twenty-first Century physics is strongly coupled theories with emergent degrees of freedom. This is part and parcel of the contemporary understanding of the strong interactions.**

- **The historic picture of strong interactions: nucleons, nuclei bound by meson exchange, with multiple excitations evolved into:**

- **THE QUARK MODEL, with (mostly) qqq' baryons and $q\bar{q}'$ mesons.**

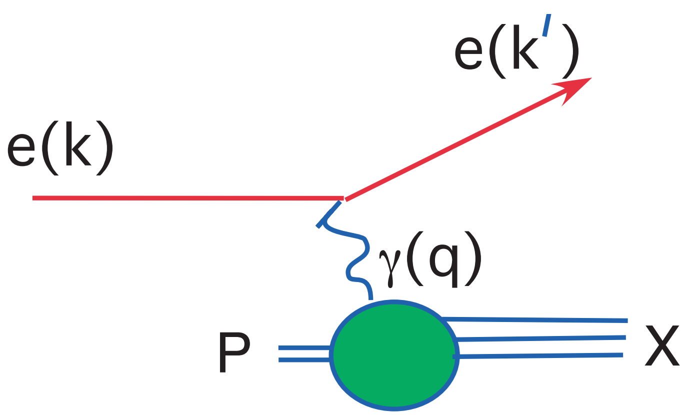
- **QUANTUM CHROMODYNAMICS a part of the Standard Model, is in some ways the exemplary QFT, still not fully understood, but illustrating the fundamental realization that quantum field theories are protean: manifesting themselves differently on different length scales, yet experimentally accessible at all scales.**

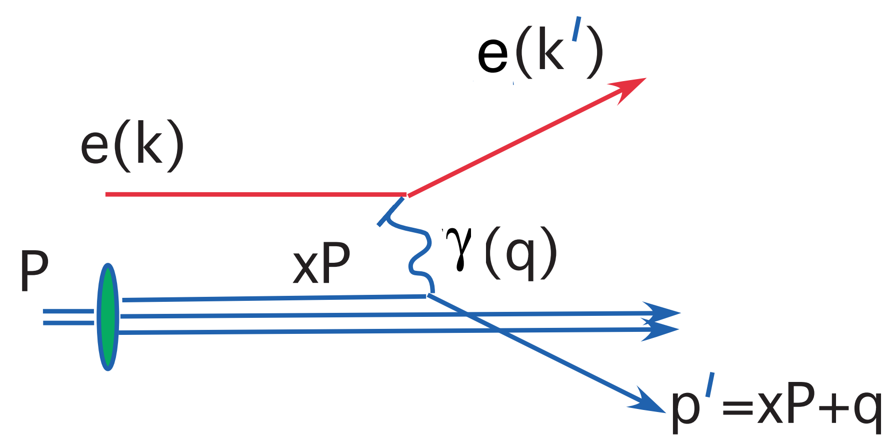
- To make a long story short: Quantum Chromodynamics (QCD) reconciled the irreconcilable. Here was the problem.

1. Quarks and gluons explain spectroscopy, but aren't seen directly – confinement.

2. In highly ("deep") inelastic, electron-proton scattering, the inclusive cross section was found to well-approximated by lowest-order elastic scattering of point-like (spin-1/2) particles (="partons" = quarks here) a result called "scaling":

$$\frac{d\sigma_{e+p}(Q, p\cdot q)}{dQ^2}\Big|_{\text{inclusive}} \propto F\left(x = \frac{Q^2}{2p\cdot q}\right)\frac{d\sigma^{\text{free}}_{e+\text{spin}\,\frac{1}{2}}}{dQ^2}\Big|_{\text{elastic}}$$





- If the "spin-$\frac{1}{2}$ is a quark, how can a confined quark scatter freely?

- This paradoxical combination of confined bound states at long distances and nearly free behavior at short distances was explained by asymptotic freedom: In QCD, the force between quarks behaves at short distances like

$$F(r) \sim \frac{\alpha_s(r)}{r^2}, \qquad \alpha_s(r^2) = \frac{4\pi}{\ln\left(\frac{1}{r^2\lambda^2}\right)}$$

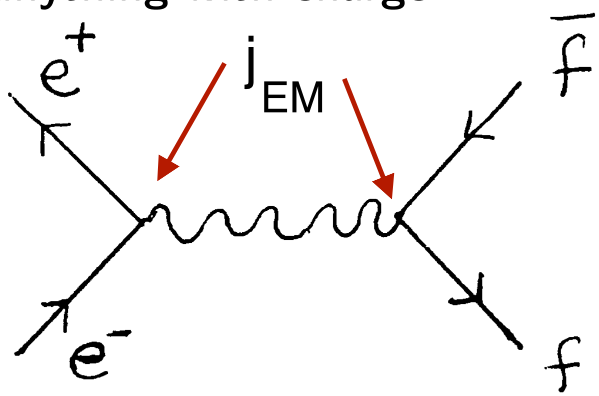
where $\Lambda \sim 0.2$ GeV. For distances much less than $1/(0.2GeV) \sim 10^{-8}$cm the force weakens. These are distances that began to be probed in deep inelastic scattering experiments at SLAC in the 1970s.

- The short explanation of DIS: Over the times $ct \leq \hbar/GeV$ it takes the electron to scatter from a quark-parton, the quark really does seem free. Later, the quark is eventually confined, but by then it's too late to change the probability for an event that has already happened.

- The function $F(x)$ is interpreted as the probability to find quark of momentum xP in a target of total momentum P – a *parton distribution*.

- **To explore further, SLAC used the quantum mechanical credo: anything that can happen, will.**

- **Quarks have electric charge, so if they are there to be produced, they will be. This can happen when colliding electron-positron pairs annihilate to a virtual photon, which ungratefully decays to just anything with charge**



- **But of course because of confinements its not that. But more generally, we** *believe* **that a virtual photon decays through a local operator:** $j_{\rm em}(x)$ **.**

- **This enables translating measurements into correlation functions . . . In fact, the cross section for electron-positron annihilation probes the vacuum with an electromagnetic current.**

- **On the one hand, all final states are familiar hadrons, with nothing special about them to tell the tale of QCD, $|N\rangle = |\text{pions, protons} \ldots\rangle$,**

$$\sigma_{\mathrm{e^+e^-\to\ hadrons}}(Q) \ \propto \ \sum_N |\langle 0|j^\mu_{\mathrm{em}}(0)|N\rangle|^2\, \delta^4(Q-p_N)$$

- **On the other hand, $\Sigma_N\, |N\rangle\langle N| = 1$, and using translation invariance this gives**

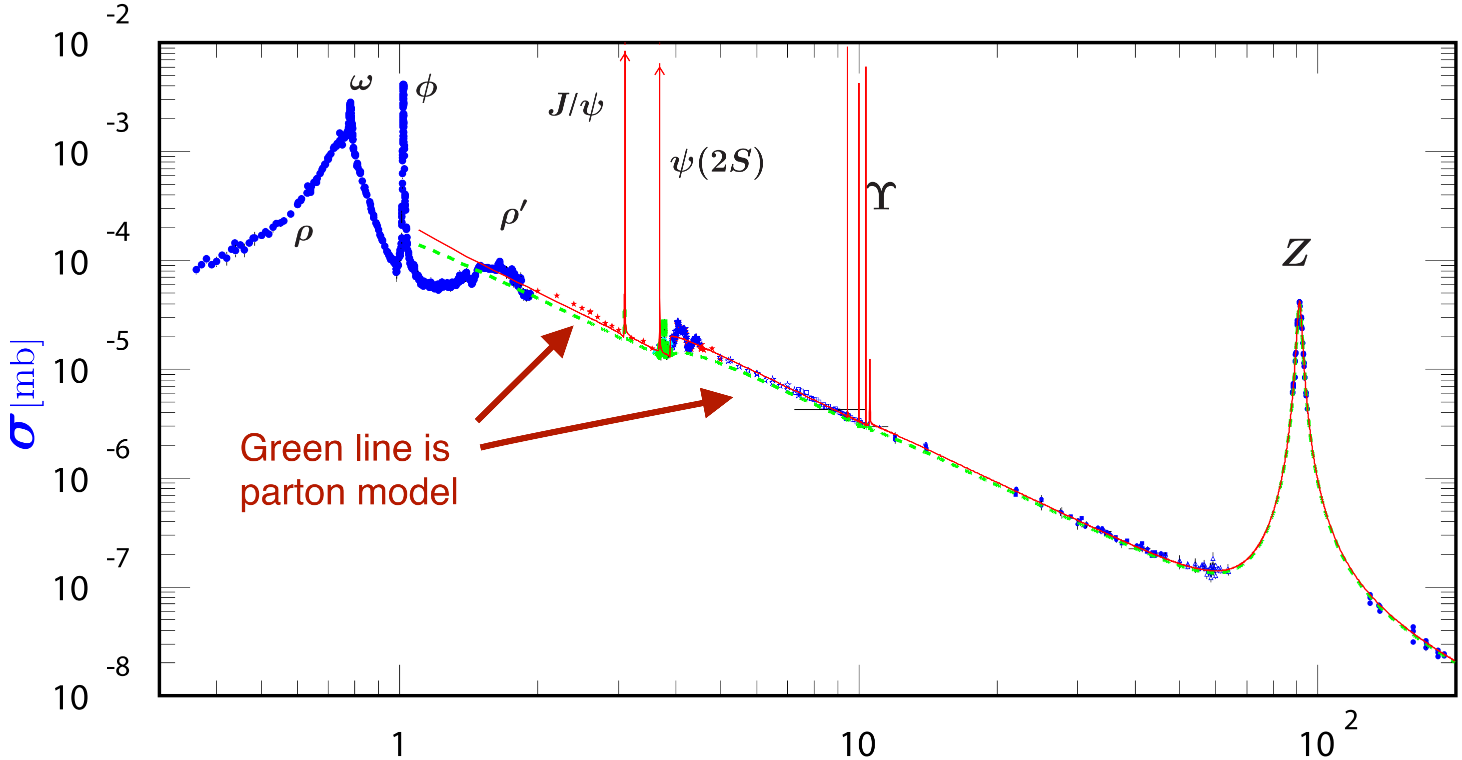
$$\sigma_{\mathrm{e^+e^-\to\ hadrons}}(Q) \ \propto \ \int d^4x\, e^{-iQ\cdot x}\ \langle 0|j^\mu_{\mathrm{em}}(0)\ j^\mu_{\mathrm{em}}(x)|0\rangle$$

- **We are probing the vacuum at short distances, imposed by the Fourier transform as $Q\to\infty$. The currents are only a distance $1/Q$ apart.**

- **Asymptotic freedom suggests a "free" result: QCD at lowest order ("quark-parton model") at cm. energy Q and angle θ**

$$\sigma^{tot}_{e^+e^-\to\mathrm{hadrons}} \ = \ \frac{4\pi\alpha^2_{\mathrm{EM}}}{3Q^2}$$

- **This works for σ_{tot} to quite a good approximation (with calculable corrections)**



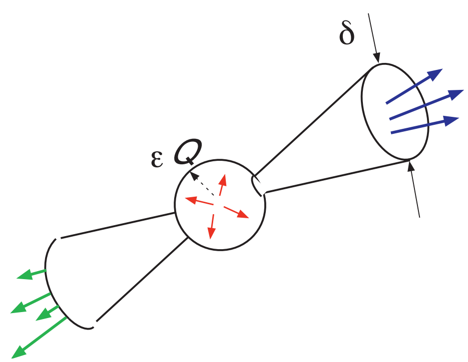
- **So the "free" theory again describes the inclusive sum over confined (nonperturbative) bound states – another "paradox".**

- **Is there an imprint on these states of their origin? Yes. What to look for? The spin of the quarks is imprinted in their angular distribution:**

$$\frac{d\sigma(Q)}{d\cos\theta} = \frac{\pi\alpha_{\mathrm{EM}}^2}{2Q^2}\left(1+\cos^2\theta\right)$$

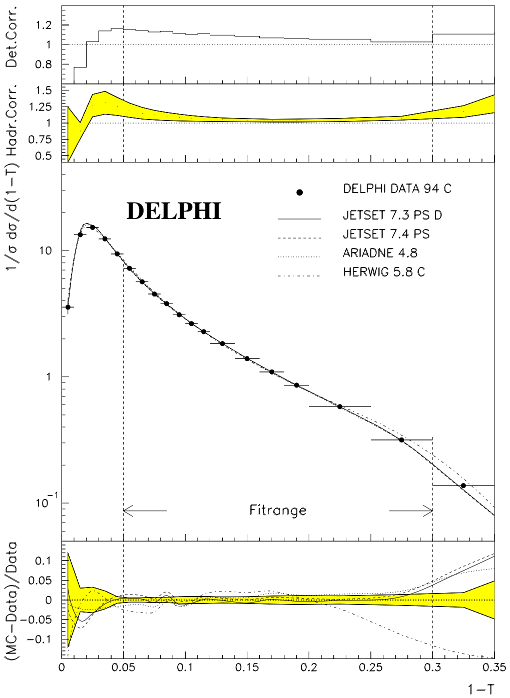
- **It's not quarks, but can look for a back to back flow of energy by finding an axis that maximizes the projection of particle momenta ("thrust") measuring a "jet-like" structure**

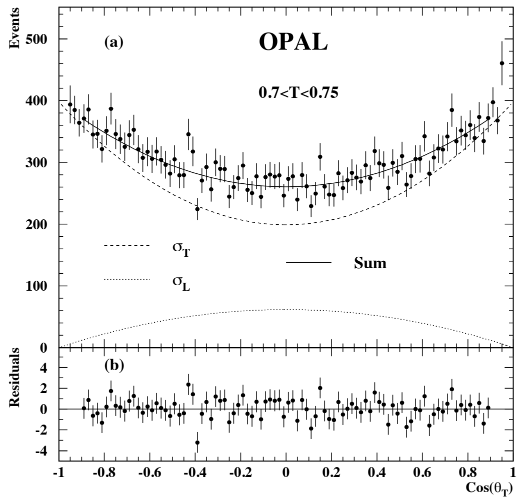
$$\frac{d\sigma_{e^+e^-\to\ \mathrm{hadrons}}(Q)}{dT} \propto \sum_N |\langle 0|j^{\mu}_{\mathrm{em}}(0)|N\rangle|^2\,\delta^4(Q-p_N)\ \delta\left(T-\frac{1}{Q}\max_{\hat{\mathrm{n}}}\sum_{i\in N}|\vec{p}_i\cdot\hat{\mathrm{n}}|\right)$$

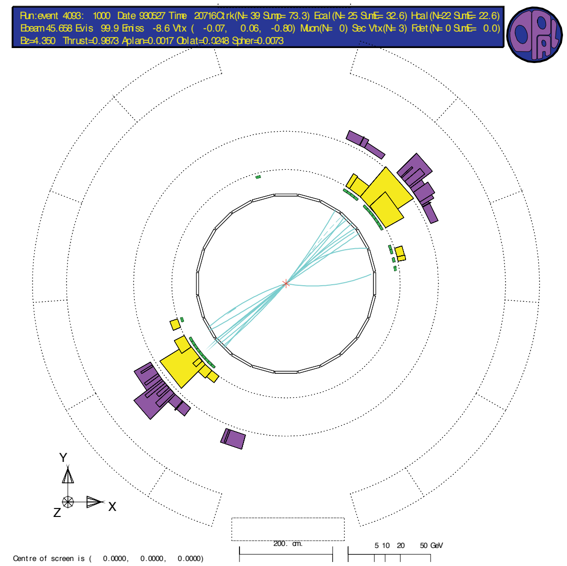


- **When the particles all line up $T\to 1$ (neglecting masses). So what happens?**

• **Here's what was found (from a little later, at LEP):**







• **Thrust is peaked near unity and follow the $1 + \cos^2\theta$ distribution – reflecting the production of spin $\frac{1}{2}$ particles – back-to-back. All this despite confinement. Quarks have been replaced by "jets" of hadrons. What could be better? But what's going on? How can we understand persistence of short-distance structure into the final state, evolving over many many orders of magnitude in time? This is the goal of the rest of the talk.**

2. From Short to Long Distances in Quantum Field Theory:
What we can't compute, and what we can

- **At the short distances accessible to accelerators, we can expand around the free field theory. The transitions between states *are* the stories that provide predictions.**

- **Perturbation theory really just follows from Schrödinger equation for mixing of free particle states (more on this later),**

$$i\hbar \frac{\partial}{\partial t}|\psi(t) >= \left(H^{(0)} + V\right)|\psi(t) >$$

Usually with free-state "IN" boundary condition :

$$|\psi(t = -\infty) >= |m_0 >= |p_1^{\rm IN}, p_2^{\rm IN}\rangle$$

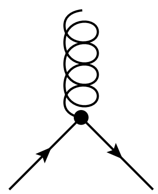
- **Notation :** $V_{ji} = \langle m_j|V|m_i\rangle$ **(vertices)**

- **Theories differ in their list of particles and their (hermitian)** V**s.**

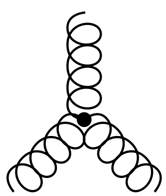
For QCD, the Lagrange density

$$\begin{aligned}\mathcal{L}_{\mathrm{QCD}} &= \overline{\psi}_i(i\gamma^\mu\partial_\mu - m)\psi_i - \tfrac{1}{4}F_a^{\mu\nu}F^a_{\mu\nu} - g_{\mathrm{s}}\,\overline{\psi}_i\lambda^a_{ij}\psi_j\,\gamma^\mu A^a_\mu \\ F_a^{\mu\nu} &= \partial^\mu A_a^\nu - \partial^\nu A_a^\mu - 2g_{\mathrm{s}}\,f_{abc}A_b^\mu A_c^\nu\end{aligned}$$

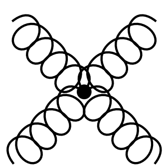
And vertices



$g_{\mathrm{s}}\,\bar{\psi}_i\lambda^a_{ij}\psi_j\gamma^\mu A^a_\mu$ quark-gluon vertex



$g_{\mathrm{s}}\,(\partial^\mu A_a^\nu - \partial^\nu A_a^\mu)f_{abc}A^b_\mu A^c_\nu$ 3-gluon vertex



$g_{\mathrm{s}}^2\,f_{abc}A_b^\mu A_c^\nu\,f_{ade}A^d_\mu A^e_\nu$ 4-gluon vertex

- **Solutions to the Schrödinger equation are sums of ordered time integrals. "Old-fashioned perturbation theory."**

$$\langle m_n | m_0 \rangle = \sum_{\tau\ \text{orders}} \int_{-\infty}^{\infty} d\tau_n \ldots \int_{-\infty}^{\tau_2} d\tau_1$$

$$\times \prod_{\text{loops}\ i} \int \frac{d^3\ell_i}{(2\pi)^3} \prod_{\text{lines}\ j} \frac{1}{2E_j} \times \prod_{\text{vertices}\ a} iV_{a\to a+1}$$

$$\times \exp\left[i \sum_{\text{states}\ m} \left(\sum_{j\ \text{in}\ m} E(\vec{p}_j) \right) (\tau_m - \tau_{m-1}) \right]$$

- **Perturbative QFT in a nutshell: integrals are divergent in QFT from:**

$\tau_i \to \tau_j$ **and** $\tau_i \to \infty$.

- **Coinciding times in . . .**

$$\langle m_n | m_0 \rangle = \sum_{\tau \text{ orders}} \int_{-\infty}^{\infty} d\tau_n \ldots \int_{-\infty}^{\tau_2} d\tau_1$$

$$\times \prod_{\text{loops } i} \int \frac{d^3\ell_i}{(2\pi)^3} \prod_{\text{lines } j} \frac{1}{2E_j} \times \prod_{\text{vertices } a} iV_{a\to a+1}$$

$$\times \exp\left[i \sum_{\text{states } m} \left(\sum_{j \text{ in } m} E(\vec{p}_j) \right) (\tau_m - \tau_{m-1}) \right] .$$

- **The "Ultraviolet=UV" problem from $\tau_i \to \tau_j$ is solved by renormalization, and results in scaling each term in V by an appropriate coupling constant $g(\mu)$, with**

$$(\tau_i - \tau_j)_{\text{min}} = 1/\mu.$$

In 4 dimensions only Yang-Mills theories have the property of asymptotic freedom,

$$g(\mu) \sim 1/\ln(\mu).$$

- **The couplings of the Standard Model are either asymptotically free, or are small enough to not change much over experimentally-accessible energies.**

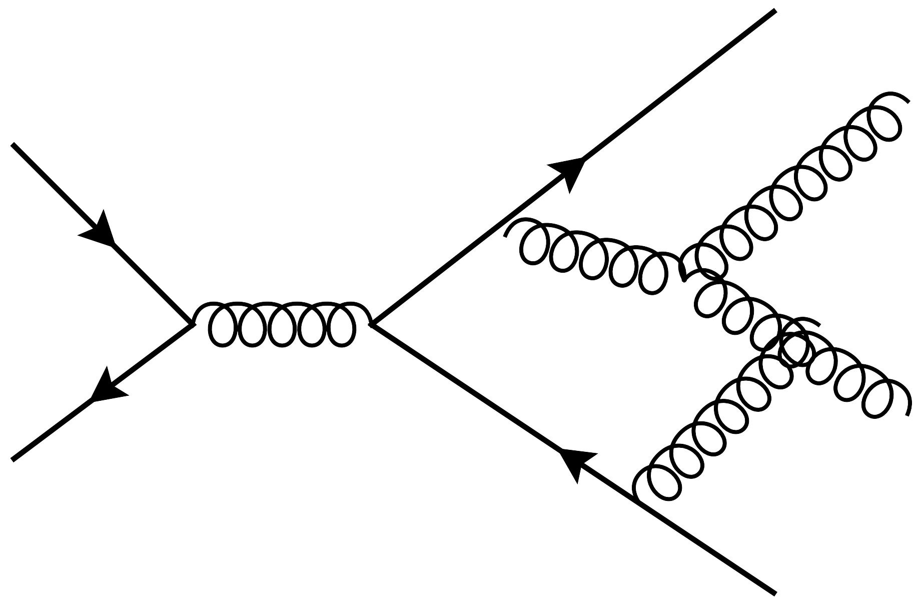
- **This makes an expansion in powers of $\alpha_s(\mu) = g^2(\mu)/4\pi$ plausible, at least in principle.**

- Large times in . . .

$$\langle m_n | m_0 \rangle = \sum_{\tau\ \text{orders}} \int_{-\infty}^{\infty} d\tau_n \ldots \int_{-\infty}^{\tau_2} d\tau_1$$
$$\times \prod_{\text{loops}\ i} \int \frac{d^3\ell_i}{(2\pi)^3} \prod_{\text{lines}\ j} \frac{1}{2E_j} \times \prod_{\text{vertices}\ a} iV_{a\to a+1}$$
$$\times \exp\left[i \sum_{\text{states}\ m} \left(\sum_{j\ \text{in}\ m} E(\vec{p}_j) \right) (\tau_m - \tau_{m-1}) \right]$$

- Divergences from $\tau_i \to \infty$ are **"Infrared=IR"**. In some sense, their "solution" is jets.

Each term in this expansion corresponds to a "time-ordered" diagram



Here the vertices are ordered. Sums of orderings give (topologically equivalent) "Feynman diagrams", which exhibit the Lorentz invariance of the manifestly.

- Once we do the expansion using renormalization, the form of an "ideal cross section" would be

- one with only a single kinematic scale, to which we can set μ:

$$Q^2\, \hat{\sigma}_{\rm SD}(Q^2, \mu^2, \alpha_s(\mu)) = \sum_n c_n(Q^2/\mu^2)\, {\alpha_s}^n(\mu) + \mathcal{O}\left(\frac{1}{Q^p}\right)$$
$$= \sum_n c_n(1)\, {\alpha_s}^n(Q) + \mathcal{O}\left(\frac{1}{Q^p}\right)$$

- The key is to find quantities that are observable, and for which the coefficients are well-behaved, and do not depend on scales μ for which the coupling is too large.

- Such quantities are commonly called "infrared safe"

- For proton accelerators or hadronic final states, the problem is that there are rather few examples.

- What is the problem?
- Mass-shell enhancements in perturbation theory
- Solutions to the Schrödinger equation are sums of ordered time integrals. "Old-fashioned perturbation theory."

$$\langle m_n | m_0 \rangle = \sum_{\tau \text{ orders}} \int_{-\infty}^{\infty} d\tau_n \ldots \int_{-\infty}^{\tau_2} d\tau_1$$

$$\times \prod_{\text{loops } i} \int \frac{d^3 \ell_i}{(2\pi)^3} \prod_{\text{lines } j} \frac{1}{2E_j} \times \prod_{\text{vertices } a} iV_{a \to a+1}$$

$$\times \exp \left[i \sum_{\text{states } m} \left(\sum_{j \text{ in } m} E(\vec{p}_j) \right) (\tau_m - \tau_{m-1}) \right]$$

- Time integrals extend to infinity, but usually oscillations damp them and answers are finite. Long-time, "infrared" divergences (logs) come about when phases vanish and the t integrals diverge.

- **When does this happen? Here's the phase:**

$$\exp\left[i \sum_{\text{states } m} \left(\sum_{j \text{ in } m} E(\vec{p}_j) \right) (\tau_m - \tau_{m-1}) \right] =$$

$$\exp\left[i \sum_{\text{vertices } m} \left(\sum_{j \text{ in } m} E(\vec{p}_j) \ - \sum_{j \text{ in } m-1} E(\vec{p}_j) \right) \ \tau_m \right]$$

- **Divergences for $\tau_i \to \infty$ requires two things:**

i) (RHS) the phase must vanish $\leftrightarrow$ "degenerate states"

$$\sum_{j \in m} E(\vec{p}_j) = \sum_{j \in m+1} E(\vec{p}_j) \,, \quad \text{and}$$

ii) (LHS) the phase must be stationary:

$$\frac{\partial}{\partial \ell_{i\mu}} \ [\,\text{phase}\,] = \sum_{\text{states } m} \ \sum_{j \text{ in } m} (\pm\beta_j^{\mu})(\tau_{m+1} - \tau_m) = 0$$

where the β_js are normal 4-velocities:

$$\beta_j = \pm\partial E_j / \partial \ell_i \,.$$

- **Condition of stationary phase:**

$$\sum_{\text{states}\, m} \sum_{j\, \text{in}\, m} (\pm\beta_j^\mu)(\tau_{m+1} - \tau_m) = 0$$

- $\beta^\mu \Delta\tau = x^\mu$ **is a classical translation. For IR divergences, there must be free, classical propagation as** $t \to \infty$**. Easy to satisfy if all the** β_j**'s are equal.**

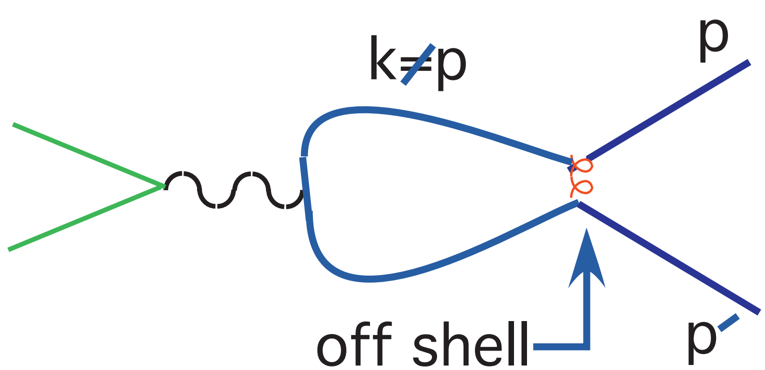
- **Whenever fast partons (quarks or gluons) emerge from the same point in space-time,**

  they will rescatter strongly with collinear partons.

  But note, all these states describe the same energy flow.

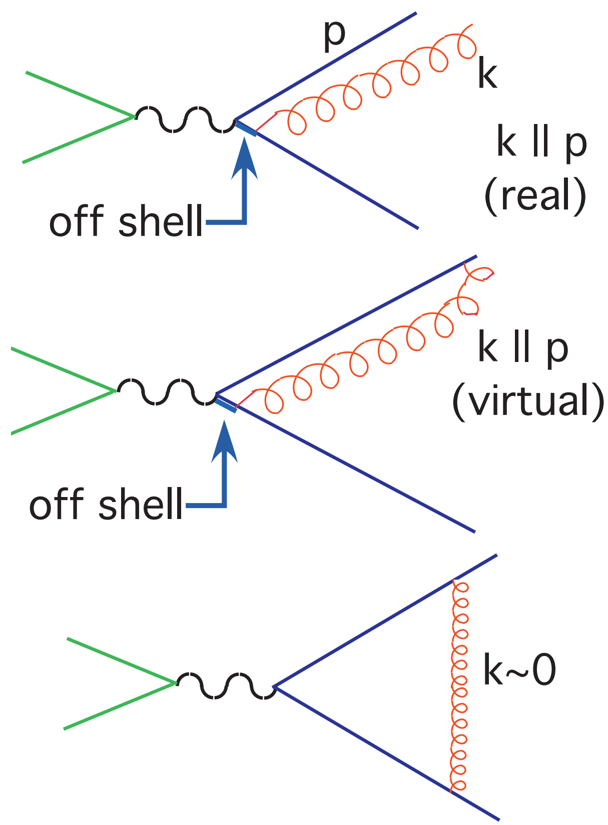
- **Let's illustrate the role of classical propagation.**

- **Example: degenerate states that cannot give long-time divergences:**



- **This makes identifying enhancements a lot simpler!**

- **RESULT: For particles emerging from a local scattering, (only) collinear or soft lines can give long-time behavior and enhancement. Example:**



- **This generalizes to any order, and any field theory, but gauge theories alone have soft ($k \to 0$) divergences.**

- **These are what we can't compute (as physical processes).**

- **But for e^+e^- annihilation, if we include all the states that can result from these collinear rescatterings, the $\tau \to \infty$ divergences are guaranteed to cancel, because the total probability for something to happen has to be one (unitarity).**

- **If we calculate detailed final states (how many quarks, how many gluons) we get totally unphysical answers, but if we sum over all possibilities so as to preserve energy flow, perturbation theory can give good answers.**

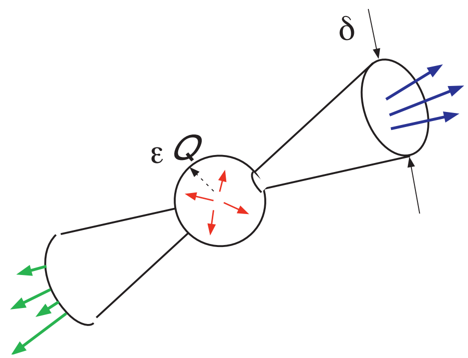
- **For example, you can use the optical theorem to show that the total cross section is IR safe (Appelquist, Georgi (1975))**

- **Once again, a sufficiently inclusive process that is nonperturbative at long distances can be described by the lowest order in the perturbative coupling, with calculable corrections.**

- The same applies jet cross sections (GS (1975), GS & Weinberg (1977)), if they are designed to respect the flow of energy

- These are what we can compute.

  (technically, all these singularities can be derived from rotationally non-invariant – but still hermitian – truncations of the QFT hamiltonian. see Soft-Collinear Effective Theory.)



- The smaller (larger) the “resolutions” ϵ and δ, the more (less) sensitivity to long times. We follow the story only to times like $1/Q\delta$.

ENERGY FLOW IS THE ORGANIZING PRINCIPLE OF THE CLASSICAL STORIES

3. A Brief Biolgraphy of Particle Jets

- Prehistory: the 1950's – 1960's
- First observations of high-energy collisions in cosmic ray 'jets'
- Particle jets in cosmic rays . . .

  "The average transverse momentum resulting from our measurements is p_T=0.5 BeV/c for pions . . . Table 1 gives a summary of jet events observed to date . . ." (B. Edwards et al, Phil. Mag. 3, 237 (1957))

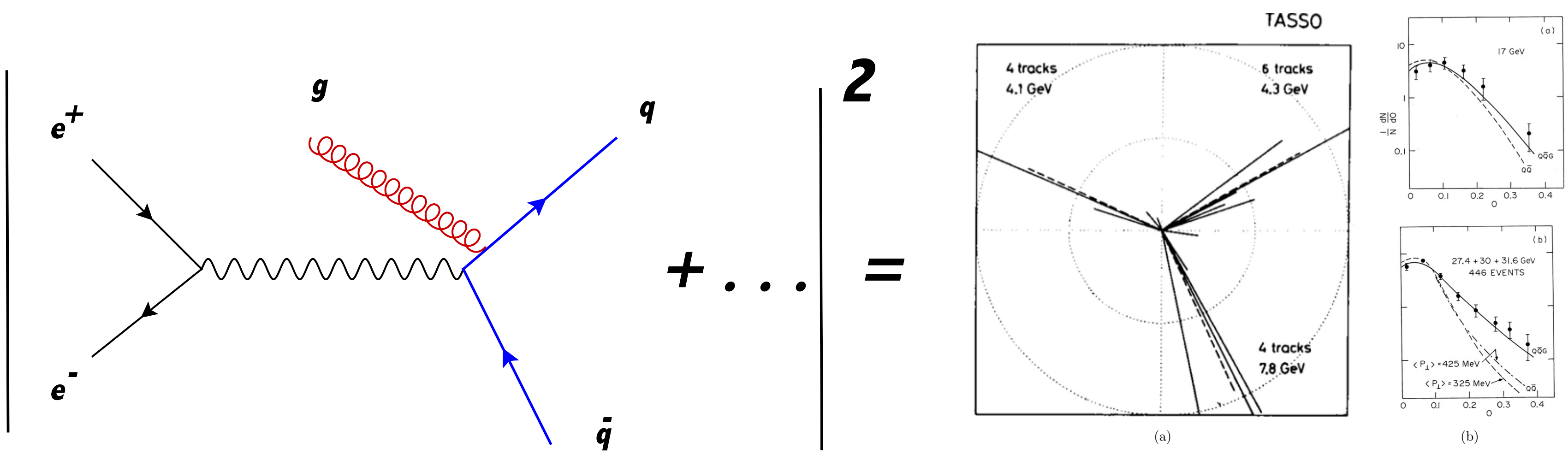
- The era of high energy physics and the discovery of the Standard Model

  Once asymptotic freedom explained scaling (Feynman, Bjorken)

$$\sigma^{\rm incl}_{e\,{\rm proton}}\left(Q, x = \frac{Q^2}{2p\cdot q}\right) \to \sigma^{\rm excl}_{e\,{\rm parton}}(Q) \times F_{\rm proton}(x)\,, \qquad (1)$$

- this is when the question arose: what happens to partons in the final state?
  (Feynman, Bjorken & Paschos, Drell, Levy & Yan, 1969)
  Do "the hadrons 'remember' the directions along which the bare constituents were emitted? . . . "the observation of such 'jets' in colliding beam processes would be most spectacular." (Bjorken & Brodsky, 1969) Or does confinement forbid a it?

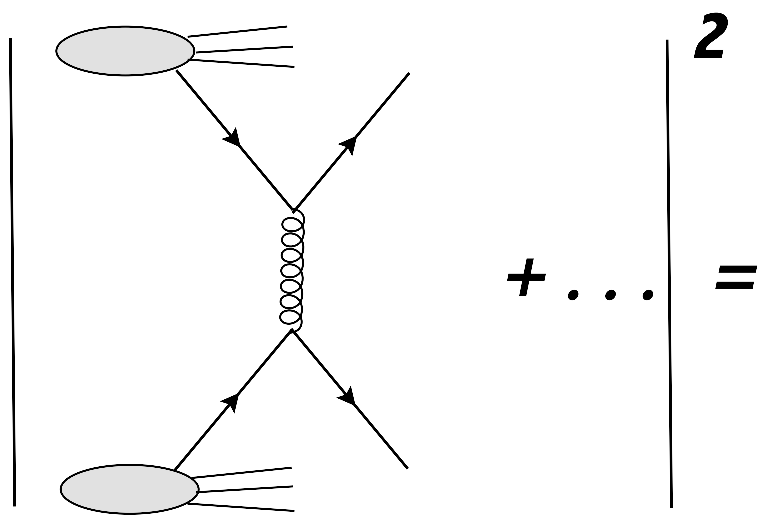
- **1975 -1980: the first quark and gluon jets**
- **As we've seen: in electron-positron annihilation to hadrons, the angular distribution for energy flow follows the lowest-order ("Born") cross section for the creation of spin-1/2 pairs of quarks and antiquarks** (As first seen by Hanson *et al*, at SLAC in 1975)
- **Jets are "rare" because the high momentum transfer scattering of partons is rare (but calculable), but in e^+e^- annihilation to hadrons the "rarity" is in the likelihood of annihilation. Once that takes places, jets are nearly always produced.**
- **And then (**Ellis, Gaillard, Ross (1976) Ellis, Karliner (1979)**): hints of three gluons in Upsilon decay, and then unequivocal gluon jets at Petra (1979)** (S.L. Wu (1984))

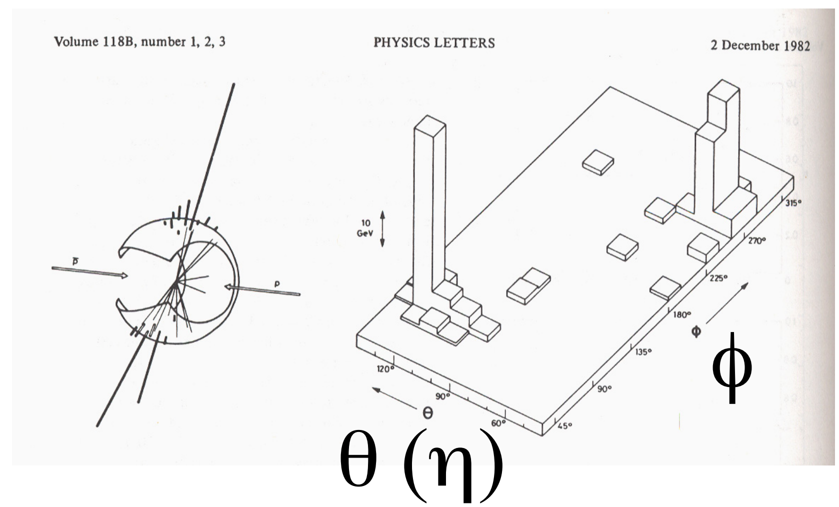


(On the right, O is oblateness, which measures the spread of energy in a plane.)

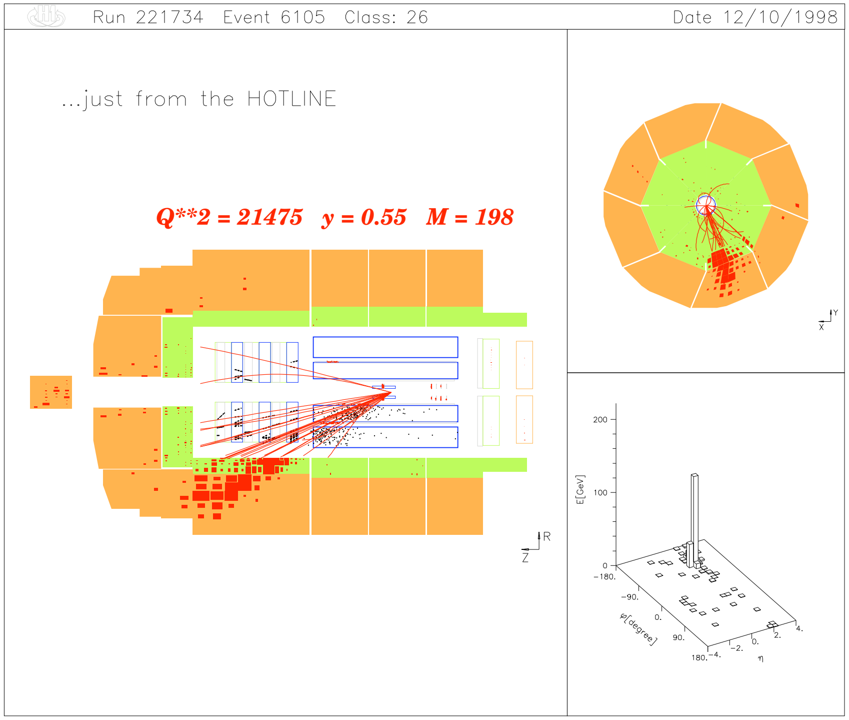
- **confirmed color as a dynamical variable.**

- **Jets at hadron colliders . . .**
- **80's: direct and indirect 'sightings' of scattered parton jets at Fermilab and the ISR, often in the context of single-particle spectra. Overall, however, an unsettled period until the SPS large angular coverage makes possible (UA2) 'lego plots' in terms of energy flow, and leads to the unequivocal observation of high-p_T jet pairs that represent scattered partons.**



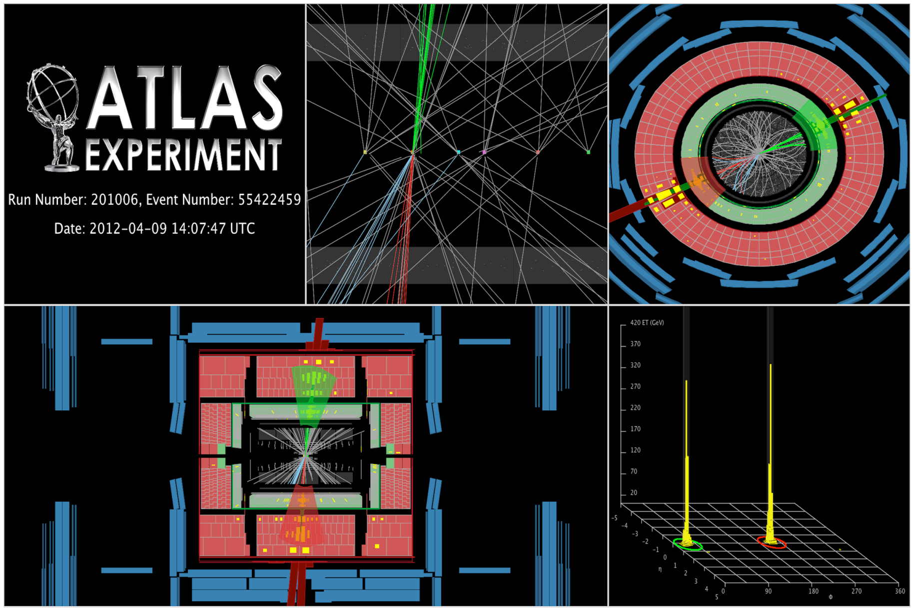


- **1990's – 2005: The great Standard Model machines: HERA, the Tevatron Run I, and LEP I and II provided jet cross sections over multiple orders of magnitude. The scattered quark appears.**

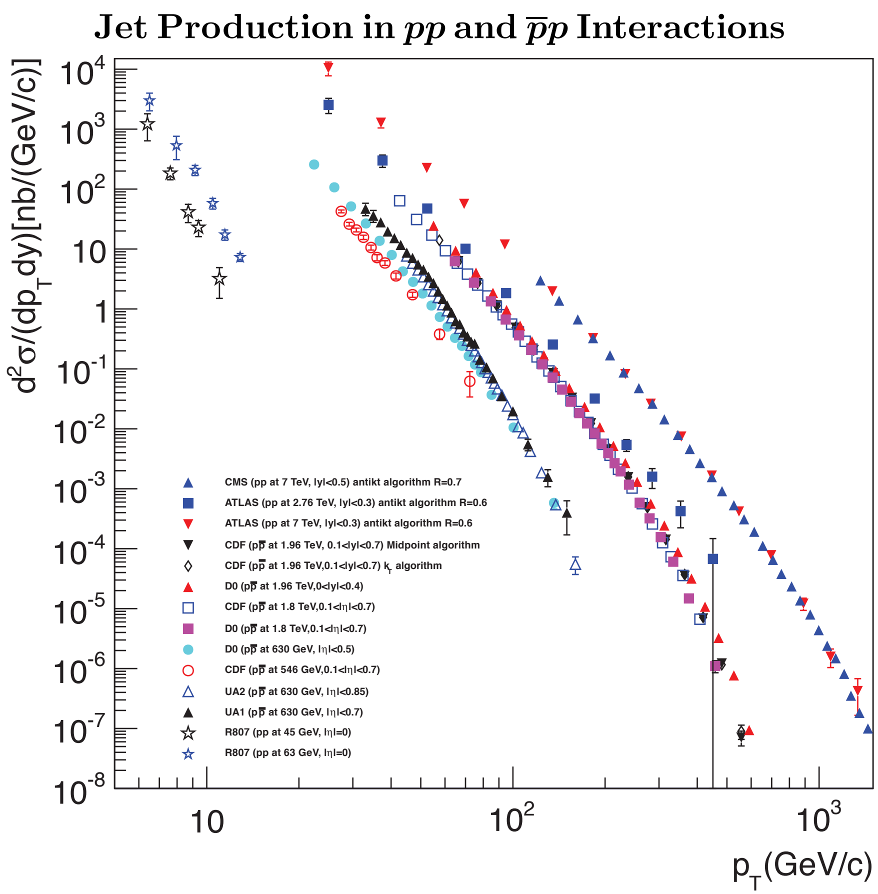


- **And now . . . a new era of jets at the anticipated limits of the SM, ushered in by Tevatron Run II, on to the LHC: 2 $\to$ 7 $\to$ 8 $\to$ 13 TeV .**

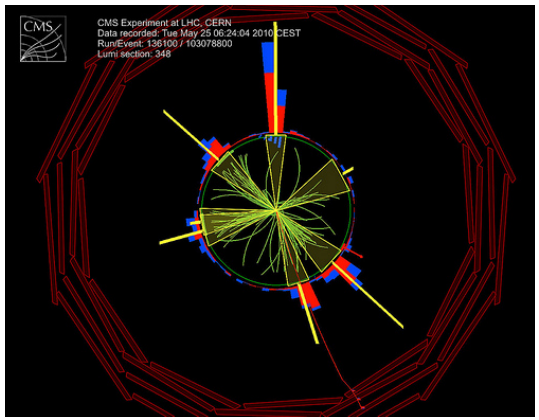
- **Events at the scale** $\delta x \sim \frac{\hbar}{1\ \mathrm{TeV}} \sim 2 \times 10^{-19}$ **meters . . . observed about 10 meters away.**

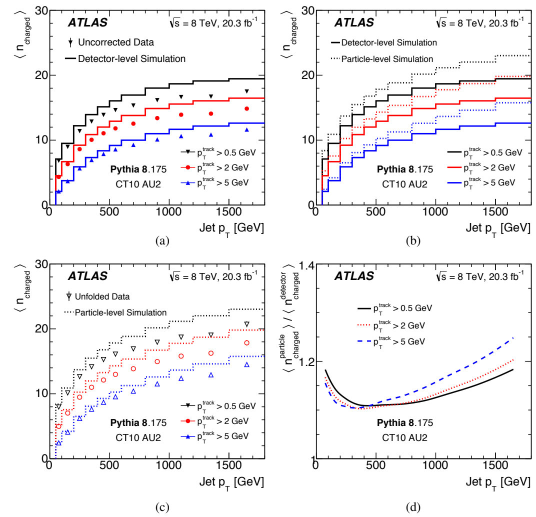


“REVIEW OF PARTICLE PROPERTIES” FIGURE: TEV JETS AND BEYOND

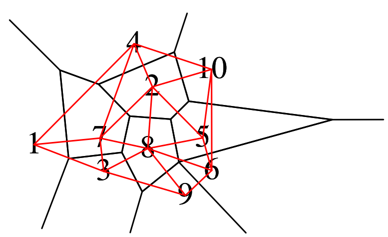


A NEW AGE OF JET IDENTIFICATION INSPIRED BY THE LHC
THE NEED TO DEAL WITH VERY COMPLEX FINAL STATES





Which requires numerical sophtication and computing power (Cacciari, Salam, Soyez, 2006)



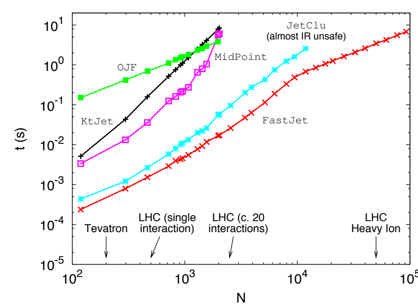


Figure 1: Left: the Voronoi diagram (black lines) of ten points in a plane, numbered 1...10. Superimposed, in red, is the Delaunay triangulation. Right: CPU time taken to cluster N particles for various jet-finders. `FastJet` is available at `http://www.lpthe.jussieu.fr/~salam/fastjet`.

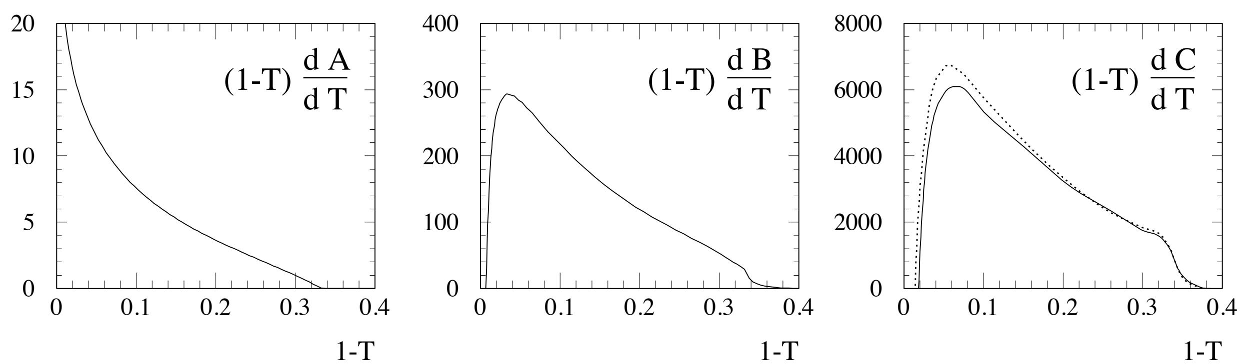
4. The Theory of Jets at Colliders

- **What we do: For e^+e^- collisions, we compute jet cross sections directly in perturbative QCD as though the final state consisted of quarks and gluons**

- **Because they depend only on (relatively) short distances (lack of pinches!)**

- **In this case, we simply compute the cross section in perturbative QFT, with partons in the final state. It seemed strange at first, knowing that quarks and gluons are confined. The theory gives a prediction, and the theory will tell us when this prediction is not self-consistent. What we get . . .**

- **For two-jet cross sections, the "thrust", coefficients of α_s/π, $(\alpha_s/\pi)^2$ and $(\alpha_s/\pi)^3$:** Gehrmann De Ridder et al., 0711.4711



- **Machines with hadrons involve the scattering of "pre-existing" quarks and gluons from hadrons, whose interactions extend back to nucleosythesis, requiring:**

  Factorization: Following the New Stories into the Final State

  The essence of predictions for Std. Model and proposed theories:

$$Q^2\sigma_{\rm phys}(Q,m,f) \;=\; \hat\sigma(Q/\mu,\alpha_s(\mu),f) \otimes f_{\rm LD}(\mu,m) \;+\; \mathcal{O}\left(\frac{1}{Q^p}\right)$$

  μ = factorization scale; m= IR scale **(m *may* be perturbative)**

  This is a "first this and then that" multiplication of probabilities – the essence of factorization. It requires a "sufficiently" inclusive cross section, much as in the calculation of jets in e^+e^- annihilation.

- **Newly-minted jets and possible "new physics" are in $\hat\sigma$; $f_{\rm LD}$ "universal"**

- **Again, a factorized cross section:**

$$Q^2\sigma_{\rm phys}(Q,m,f) \;=\; \hat{\sigma}(Q/\mu,\alpha_s(\mu),f) \otimes f_{\rm LD}(\mu,m) \;+\; \mathcal{O}\left(\frac{1}{Q^p}\right)$$

- **What we do:**
  - **Compute σ and $f_{\rm LD}$ in an IR-regulated variant of QCD, where we can prove the factorization explicitly, then extract $\hat{\sigma}$, assuming it is the same in true QCD as in its IR-regulated version.**
  - **We compare the formula with unknown physical parton distributions to a suite of data and do a "global fit" for the $f(x,\mu)$ for different quarks and the gluon.**
- **What we get: absolute predictions for the creation of jets and heavy particles from QCD, and for new degrees of freedom in BSM hypotheses.**
  - **The process is a "bootstrap", resulting in feedback between parton distributions, predictions and measurements.**

The range of these predictions is greatly extended by Evolution & Resummation: If we have factorization, we can automatically extrapolate from one energy scale to another.

– Whenever there is factorization, there is evolution

$$0 = \mu \frac{d}{d\mu} \ln \sigma_{\rm phys}(Q, m)$$

$$\mu \frac{d \ln f}{d\mu} = -P(\alpha_s(\mu)) = -\mu \frac{d \ln \hat{\sigma}}{d\mu}$$

– **We can calculate P because we can calculate $\hat{\sigma}$.**
(Dokshitzer, Gribov, LIpatov, Altarelli, Parisi)

– Wherever there is evolution there is resummation,

$$\sigma_{\rm phys}(Q, m) = \sigma_{\rm phys}(q, m) \ \otimes \ \exp\left\{\int_q^Q \frac{d\mu'}{\mu'} P\left(\alpha_s(\mu')\right)\right\}$$

– **For example:** $\sigma_{\rm phys} \equiv \tilde{F}_2(Q^2, N) = \int_0^1 dx \; x^{N-1} F(Q^2, x)$, **a moment in ep deep-inelastic scattering.**

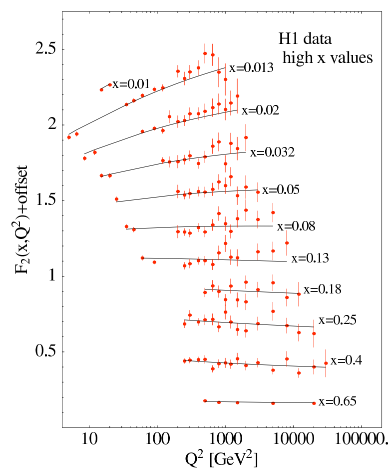
– **& then we know $\tilde{P}(N,\alpha_s) = \gamma_N = \gamma_N^{(1)}(\alpha_s/\pi) + \ldots$, and we get**

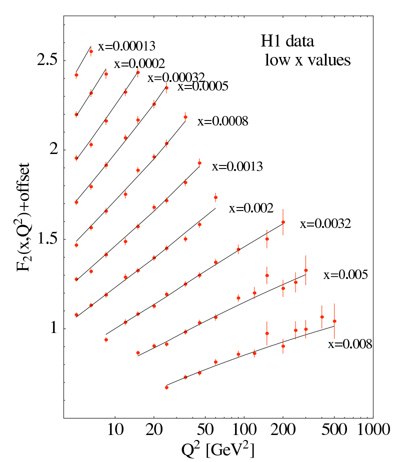
$$\tilde{F}_2(N,\mu) = \tilde{F}_2(N,\mu_0)\ \exp\left[-\frac{1}{2} \int_{\mu_0^2}^{\mu^2} \frac{d\mu'^2}{\mu'^2}\ \gamma(N,\alpha_s(\mu')) \right]$$

– **and with $\alpha_s(\mu) = 4\pi/b_0 \ln(\mu^2/\Lambda^2_{\rm QCD})$, this is**

$$\tilde{F}_2(N,Q)\ = \tilde{F}_2 q/H(N,Q_0) \left(\frac{\ln(Q^2/\Lambda^2_{\rm QCD})}{\ln(Q_0^2/\Lambda^2_{\rm QCD})} \right)^{-2\gamma_N^{(1)}/b_0}$$

It works really well. *Approximate scaling at moderate x, pronounced evolution for smaller x:*





For hadron-hadron scattering

– General relation for hadron-hadron scattering for a hard, inclusive process with momentum transfer M to produce final state $F + X$:

$$d\sigma_{\mathrm{H_1H_2}}(p_1, p_2, M) = \sum_{a,b} \int_0^1 d\xi_a \, d\xi_b \, d\hat{\sigma}_{ab \to F+X} \left(\xi_a p_1, \xi_b p_2, M, \mu\right) \times \phi_{a/H_1}(\xi_a, \mu) \, \phi_{b/H_2}(\xi_b, \mu),$$

– "Factorization proofs: justifying the "universality" of the parton distributions. At the bottom, this is just the observation that the long-distance, classical pictures associated with outgoing jets cannot interfere with those associated with the incoming hadrons, or with each other. Thus we can organize them separately into probability-like functions.

It gets a little complicated in gauge theories (and of course, QCD is a gauge thoery) because of classical long range forces, but at the end these are mutually Lorentz contracted, and don't spoil the factorization if the cross section is inclusive enough.

An enormous amount of (well spent) time has been put into these calculations, often at the boundary of contemporary mathematics.

Just for example (Anastasiou, Duhr, Dulat, Fulan, Gehrmann, Herzog and Mistlberger (1403.4616, Phys. Lett.)) part of $\hat{\sigma}$ for inclusive H production **. . . (taking $\mu = M_H$, a small part of it is)**

$$\hat{\sigma}_{ij}(m_H^2, \hat{s}) = \frac{\pi\, C(\mu^2)^2}{v^2\, V^2} \sum_{k=0}^{\infty} \left(\frac{\alpha_s}{\pi}\right)^k \eta_{ij}^{(k)}(z)$$

$$\begin{aligned}
\hat{\eta}^{(3)}(z) = \delta(1-z) \Bigg\{ & C_A^3 \left(-\frac{2003}{48}\zeta_6 + \frac{413}{6}\zeta_3^2 - \frac{7579}{144}\zeta_5 + \frac{979}{24}\zeta_2\zeta_3 - \frac{15257}{864}\zeta_4 - \frac{819}{16}\zeta_3 + \frac{16151}{1296}\zeta_2 + \frac{215131}{5184}\right) \\
& + N_F \Bigg[C_A^2 \left(\frac{869}{72}\zeta_5 - \frac{125}{12}\zeta_3\zeta_2 + \frac{2629}{432}\zeta_4 + \frac{1231}{216}\zeta_3 - \frac{70}{81}\zeta_2 - \frac{98059}{5184}\right) \\
& \quad + C_A C_F \left(\frac{5}{2}\zeta_5 + 3\zeta_3\zeta_2 + \frac{11}{72}\zeta_4 + \frac{13}{2}\zeta_3 - \frac{71}{36}\zeta_2 - \frac{63991}{5184}\right) + C_F^2 \left(-5\zeta_5 + \frac{37}{12}\zeta_3 + \frac{19}{18}\right) \Bigg] \\
& + N_F^2 \left[C_A \left(-\frac{19}{36}\zeta_4 + \frac{43}{108}\zeta_3 - \frac{133}{324}\zeta_2 + \frac{2515}{1728}\right) + C_F \left(-\frac{1}{36}\zeta_4 - \frac{7}{6}\zeta_3 - \frac{23}{72}\zeta_2 + \frac{4481}{2592}\right) \right] \Bigg\} \\
+ \left[\frac{1}{1-z}\right]_+ \Bigg\{ & C_A^3 \left(186\zeta_5 - \frac{725}{6}\zeta_3\zeta_2 + \frac{253}{24}\zeta_4 + \frac{8941}{108}\zeta_3 + \frac{8563}{324}\zeta_2 - \frac{297029}{23328}\right) + N_F^2 C_A \left(\frac{5}{27}\zeta_3 + \frac{10}{27}\zeta_2 - \frac{58}{729}\right) \\
& + N_F \left[C_A^2 \left(-\frac{17}{12}\zeta_4 - \frac{475}{36}\zeta_3 - \frac{2173}{324}\zeta_2 + \frac{31313}{11664}\right) + C_A C_F \left(-\frac{1}{2}\zeta_4 - \frac{19}{18}\zeta_3 - \frac{1}{2}\zeta_2 + \frac{1711}{864}\right) \right] \Bigg\} \\
+ \left[\frac{\log(1-z)}{1-z}\right]_+ \Bigg\{ & C_A^3 \left(-77\zeta_4 - \frac{352}{3}\zeta_3 - \frac{152}{3}\zeta_2 + \frac{30569}{648}\right) + N_F^2 C_A \left(-\frac{4}{9}\zeta_2 + \frac{25}{81}\right) \\
& + N_F \left[C_A^2 \left(\frac{46}{3}\zeta_3 + \frac{94}{9}\zeta_2 - \frac{4211}{324}\right) + C_A C_F \left(6\zeta_3 - \frac{63}{8}\right) \right] \Bigg\} \\
+ \left[\frac{\log^2(1-z)}{1-z}\right]_+ & \left\{ C_A^3 \left(181\zeta_3 + \frac{187}{3}\zeta_2 - \frac{1051}{27}\right) + N_F \left[C_A^2 \left(-\frac{34}{3}\zeta_2 + \frac{457}{54}\right) + \frac{1}{2} C_A C_F \right] - \frac{10}{27} N_F^2 C_A \right\} \\
+ \left[\frac{\log^3(1-z)}{1-z}\right]_+ & \left\{ C_A^3 \left(-56\zeta_2 + \frac{925}{27}\right) - \frac{164}{27} N_F C_A^2 + \frac{4}{27} N_F^2 C_A \right\} \\
+ \left[\frac{\log^4(1-z)}{1-z}\right]_+ & \left(\frac{20}{9} N_F C_A^2 - \frac{110}{9} C_A^3\right) + \left[\frac{\log^5(1-z)}{1-z}\right]_+ 8 C_A^3 .
\end{aligned}$$

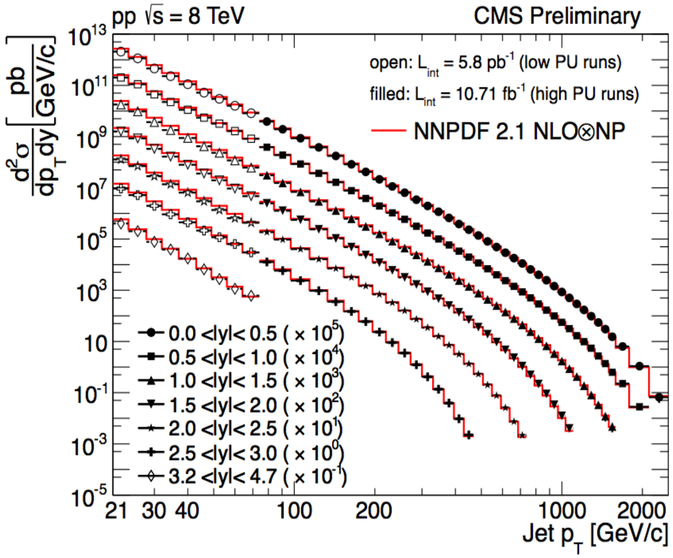
Computing jet cross sections

- **Factorized jet cross sections look like this:** **(Amati, Petronzio, Veneziano; Ellis, Machachek, Efremov, Radyushkin; Politzer, Ross: Libby, GS (1979); Bodwin; Collins Soper, GS (1985,1988))**

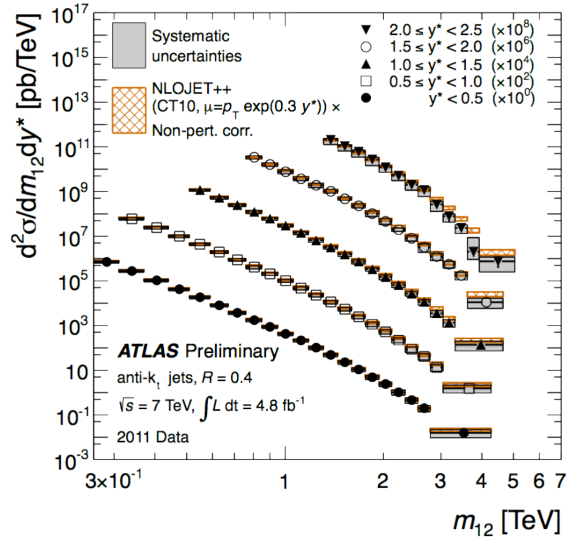
$$
\begin{aligned}
& d\sigma(A+B\to\{p_i\}) \\
& \quad = \int dx_a dx_b\, f_{a/A}(x_a,\mu_F)\, f_{b/B}(x_b,\mu_F) \\
& \quad\quad \times\; C\left(x_a p_A, x_b p_B, \frac{Q}{\mu_F}, \frac{p_i\cdot p_j}{p_k\cdot p_l}\right)_{ab\to c_1\dots c_{N_{\rm jets}}+X} \\
& \quad\quad \times d\left[\prod_{i=1}^{N_{\rm jets}} J_{c_i}(p_i,\mu_F)\right]
\end{aligned} \tag{2}
$$

- **Parton distributions, short distance "coefficients" and functions of the jet momenta tell a story of autonomous correlated on-shell propagations punctuated by a single short-distance interaction.**

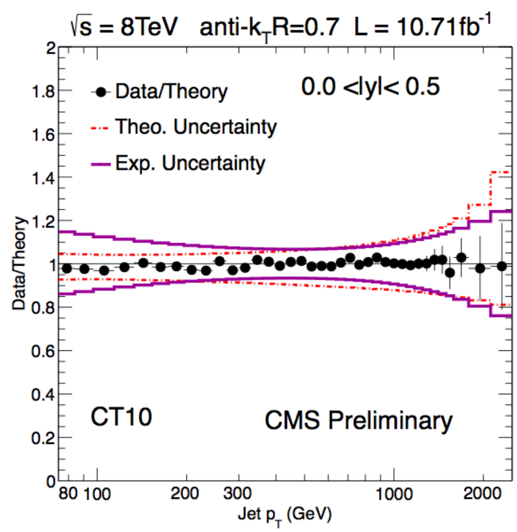
Correlated and "autonomous" dynamics. The data confront calculations . . .

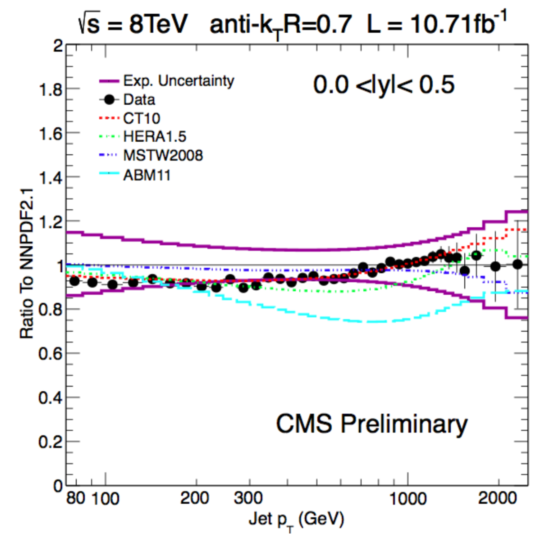


(CMS-PAS-SMP-12-012)



(ATLAS-CONF-2012-021)





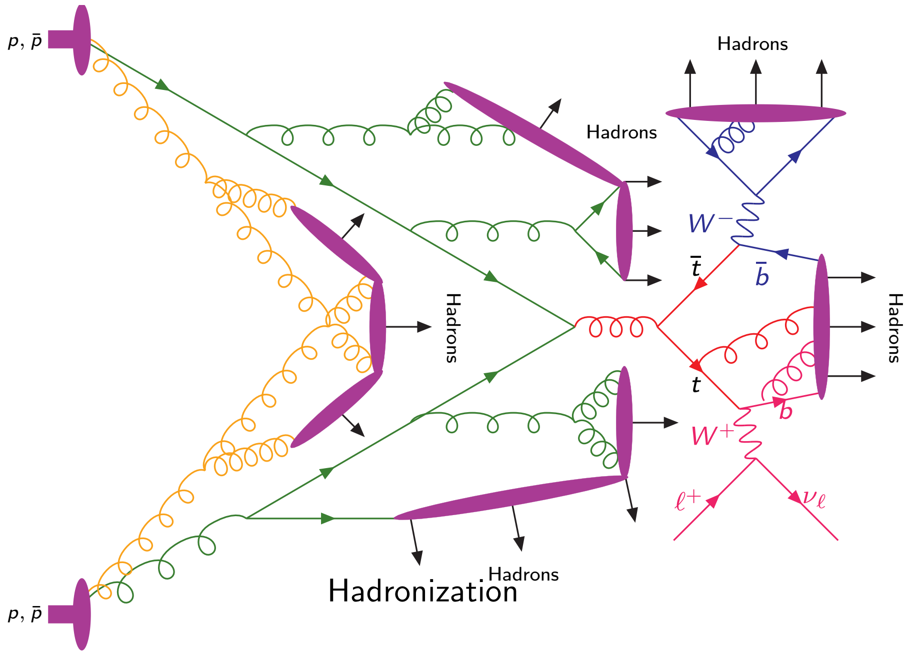
- **We have seen that enhancement of particle correlations is built into QFT, and mutual autonomy is a feature of classical pictures. Different jets follow different paths.**

- **The same factorization $\rightarrow$ evolution step applies to our jets, and they "evolve"**

$$J(\text{scale } \mu_2) \ \sim \ J(\text{scale } \mu_1) \exp\left[\int_{\mu_1}^{\mu_2} \frac{d\mu'}{\mu'} \ \int dx \ P(x, \alpha_s(\mu'))\right]$$

- **Each term in the exponent corresponds to the potential emission of a new "subjet", which factors from the remaining jet and evolves nearly autonomously into the final state, branching further subjets along the way.**

- **This is exploited systematically to build event generators (PYTHIA, Herwig . . .), which simulate the details of events by probabilistic steps specified in detail by the calculable "spitting functions" $P(x, \alpha_s)$.**



Here's a representation of an Event generated by Herwig. Although it looks like an amplitude, each step is probabilistic, and given by splitting functions as above.
(P. Richardson, 2015)

- **Which brings us full circle.** **To model "real" final states, the step has to be made between perturbative jets given by gluons and quarks, and hadrons. Modern event generators exploit the calculable momentum and quantum number distributions provided by perturbation theory to make the final step: hadronization, shown here between final-state partons that are "close enough" in phase space.**
  It is close to here that the tide of our theory reaches its current high water mark.

Conclusions

Accelerators have confirmed the fundamental degrees of freedom in the gauge theories of the Standard Model directly, relying on methods of infrared safety, factorization and evolution to complement and motivate the extraordinary technology.

QCD, however, transforms its degrees of freedom on length scales beyond nucleon scales. For the most part observations are designed for identifying partonic states, in an effort to detect and reject QCD backgrounds.

The history of QCD jets and the evolution of partons into hadrons is there for the reading if only we can learn the language.

Appendix: Factorization in gauge theories

- Think of classical fields seen by scattered charges.

x frame (everything else) $\quad$ x' frame (jet)

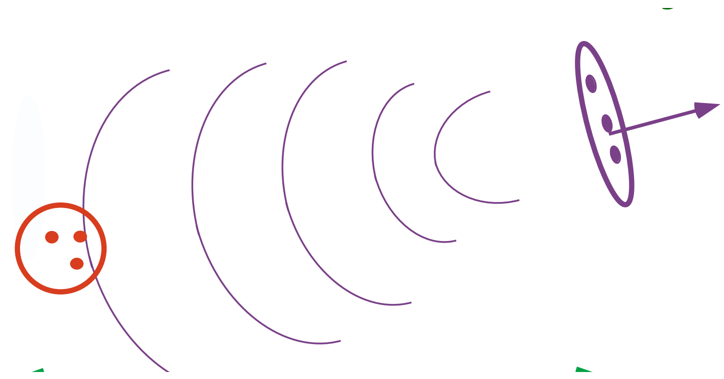
$\Delta \equiv x'_3 + \beta c t'$

- Why a classical picture isn't so far-fetched . . .

The correspondence principle is the key to IR divergences.

An accelerated charge must produce classical radiation, and an infinite numbers of soft gluons are required to make a classical field.

Again . . .

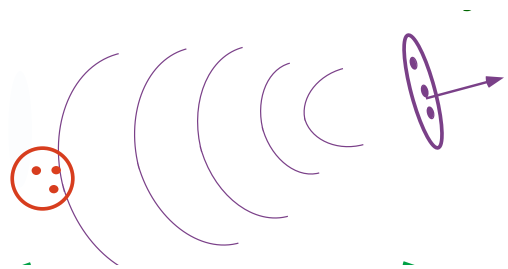


- Our charges move in the x'_3 direction, in the field of "everything else" which also has some abelian charge q in it.

Lorentz transformation to the rest frame of the charge q:

$$x_3 = \gamma(x'_3 \ + \ vt') \ = \ \gamma\Delta.$$

The "collision" is at $\Delta = 0$, i.e. $t' = -\ \frac{1}{v}x'_3$.



Electric fields in the x and x' frame:

$$\boldsymbol{E_3(x)} = \frac{\boldsymbol{q}}{|\vec{\boldsymbol{x}}|^2} \qquad \boldsymbol{E'_3(x')} = \frac{\boldsymbol{q\gamma\Delta}}{(\boldsymbol{x_T^2} + \gamma^2\boldsymbol{\Delta}^2)^{3/2}} \sim \frac{1}{\gamma^2}\frac{\boldsymbol{q}}{\boldsymbol{\Delta}} \tag{3}$$

- **The electric, $\vec{\mathrm{E}}$ field seen by the receeding particles is highly contracted, falling off as $1/\gamma^2$ once it passes by.**

- In contrast, the vector potential, A^μ is uncontracted, but is mostly a total derivative as seen in the x' frame:

$$A^\mu = q\frac{\partial}{\partial x'_\mu} \ \ln\left(\Delta(t', x'_3)\right) + \mathcal{O}(1-\beta)$$

- The "large" part of A^μ can be removed by a gauge transformation. Implementing this freedom makes proofs of factorization challenging in gauge theories.

- The residual "drag" forces are corrections to the total derivative:

$$1-\beta \ \sim \ \frac{1}{2}\ \left[\sqrt{1-\beta^2}\right]^2 \ \sim \ \frac{m^2}{2E^2}$$

Corrections to the autonomous = factorized description of high energy processes are power suppressed in momentum transfer.

- **How it works in QCD: for k collinear to p, with q, r and s the rest of the $2 \rightarrow 2$ collision, all diagrams contribute, but:**