

NNLO Corrections Using Antenna Subtraction & Applications

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Subtracting Infrared Singularities Beyond NLO

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Part 1. Antenna Subtraction Formalism

Part 2. Example: Drell–Yan

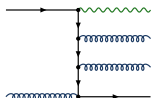
Part 3. Going beyond...

↪ Transverse Momentum Spectrum

↪ Projection-to-Born Method

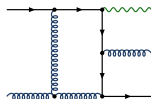
Anatomy of an NNLO calculation

$$\sigma_{\text{NNLO}} = \int_{\Phi_{Z+3}} d\sigma_{\text{NNLO}}^{\text{RR}}$$



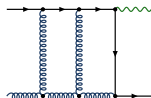
- ▶ single-unresolved
- ▶ double-unresolved

$$+ \int_{\Phi_{Z+2}} d\sigma_{\text{NNLO}}^{\text{RV}}$$



- ▶ single-unresolved
- ▶ $1/\epsilon^2, 1/\epsilon$

$$+ \int_{\Phi_{Z+1}} d\sigma_{\text{NNLO}}^{\text{VV}}$$



- ▶ $1/\epsilon^4, 1/\epsilon^3, 1/\epsilon^2, 1/\epsilon$

Σ

finite (Kinoshita–Lee–Nauenberg & factorization)

Non-trivial cancellation of infrared singularities

Anatomy of an NNLO calculation

$$\sigma_{\text{NNLO}} = \int_{\Phi_{Z+3}} d\sigma_{\text{NNLO}}^{\text{RR}}$$

$$+ \int_{\Phi_{Z+2}} d\sigma_{\text{NNLO}}^{\text{RV}}$$

$$+ \int_{\Phi_{Z+1}} d\sigma_{\text{NNLO}}^{\text{VV}}$$

Σ

finite

Different methods:

- ▶ Antenna subtraction
[Gehrmann–De Ridder, Gehrmann, Glover '05]
- ▶ CoLoRful subtraction
[Del Duca, Somogyi, Trocsanyi '05]
- ▶ q_T subtraction
[Catani, Grazzini '07]
- ▶ Sector-improved residue subtraction
[Czakon '10], [Boughezal, Melnikov, Petriello '11]
- ▶ N -jettiness subtraction
[Gaunt, Stahlhofen, Tackmann, Walsh '15]
[Boughezal, Focke, Liu, Petriello '15]
- ▶ Projection-to-Born
[Cacciari, et al. '15]
- ▶ Nested soft-collinear subtraction
[Caola, Melnikov, Rönsch '17]

...

Approaches: subtraction, slicing

NNLO using Antenna

$$\begin{aligned}\sigma_{\text{NNLO}} = & \int_{\Phi_{Z+3}} \left(d\sigma_{\text{NNLO}}^{\text{RR}} - d\sigma_{\text{NNLO}}^{\text{S}} \right) \\ & + \int_{\Phi_{Z+2}} \left(d\sigma_{\text{NNLO}}^{\text{RV}} - d\sigma_{\text{NNLO}}^{\text{T}} \right) \\ & + \int_{\Phi_{Z+1}} \left(d\sigma_{\text{NNLO}}^{\text{VV}} - d\sigma_{\text{NNLO}}^{\text{U}} \right)\end{aligned}$$

- ▶ $d\sigma_{\text{NNLO}}^{\text{S}}, d\sigma_{\text{NNLO}}^{\text{T}}$:
mimic $d\sigma_{\text{NNLO}}^{\text{RR}}, d\sigma_{\text{NNLO}}^{\text{RV}}$
in unresolved limits
- ▶ $d\sigma_{\text{NNLO}}^{\text{T}}, d\sigma_{\text{NNLO}}^{\text{U}}$:
analytic cancellation of
poles in $d\sigma_{\text{NNLO}}^{\text{RV}}, d\sigma_{\text{NNLO}}^{\text{VV}}$

Σ finite -0

\Rightarrow each line suitable for numerical evaluation in $D = 4$

Antenna factorisation

- ▶ antenna formalism operates on *colour-ordered* amplitudes
- ▶ exploit universal factorisation properties in IR limits

$$\underbrace{|\mathcal{A}_{m+1}^0(\dots, i, j, k, \dots)|^2}_{\text{partial amplitude}} \xrightarrow{j \text{ unresolved}} \underbrace{X_3^0(i, j, k)}_{\text{antenna function}} \underbrace{|\mathcal{A}_m^0(\dots, \tilde{I}, \tilde{K}, \dots)|^2}_{\text{reduced ME}}$$

+ mapping
 $\{p_i, p_j, p_k\} \rightarrow \{\tilde{p}_I, \tilde{p}_K\}$

- ▶ captures multiple limits* and smoothly interpolates between them

| limit | $X_3^0(i, j, k)$ | mapping |
|---------------------|--------------------------------|--|
| $p_j \rightarrow 0$ | $\frac{2s_{ik}}{s_{ij}s_{jk}}$ | $\tilde{p}_I \rightarrow p_i, \tilde{p}_K \rightarrow p_k$ |
| $p_j \parallel p_i$ | $\frac{1}{s_{ij}} P_{ij}(z)$ | $\tilde{p}_I \rightarrow (p_i + p_j), \tilde{p}_K \rightarrow p_k$ |
| $p_j \parallel p_k$ | $\frac{1}{s_{jk}} P_{kj}(z)$ | $\tilde{p}_I \rightarrow p_i, \tilde{p}_K \rightarrow (p_j + p_k)$ |

* c.f. dipoles: $X_3^0(i, j, k) \sim \mathcal{D}_{ij,k} + \mathcal{D}_{k,j,i}$

Antenna subtraction — building blocks

- $X(\dots)$ based on physical matrix elements $X = \overbrace{A, B, C}^{q\bar{q}}, \overbrace{D, E}^{qg}, \overbrace{F, G, H}^{gg}$

$$X_3^0(i, j, k) = \frac{|\mathcal{A}_3^0(i, j, k)|^2}{|\mathcal{A}_2^0(\tilde{I}, \tilde{K})|^2}, \quad X_4^0(i, j, k, l) = \frac{|\mathcal{A}_4^0(i, j, k, l)|^2}{|\mathcal{A}_2^0(\tilde{I}, \tilde{L})|^2},$$

$$X_3^1(i, j, k) = \frac{|\mathcal{A}_3^1(i, j, k)|^2}{|\mathcal{A}_2^0(\tilde{I}, \tilde{K})|^2} - X_3^0(i, j, k) \frac{|\mathcal{A}_2^1(\tilde{I}, \tilde{K})|^2}{|\mathcal{A}_2^0(\tilde{I}, \tilde{K})|^2},$$

$$A_3^0(i_q, j_g, k_{\bar{q}}) = \left| \begin{array}{c} \text{Diagram: A circle with a wavy green line entering from the left labeled } \gamma^*. \text{ Three lines exit to the right: a blue line labeled } j_g, \text{ and two black lines labeled } i_q \text{ and } k_{\bar{q}}. \end{array} \right|^2 \Big/ \left| \begin{array}{c} \text{Diagram: A wavy green line entering from the left labeled } \gamma^*. \text{ Two black lines exit to the right labeled } I_q \text{ and } K_{\bar{q}}. \end{array} \right|^2$$

- integrating the antennae \longleftrightarrow phase-space factorization

$$\begin{aligned} d\Phi_{m+1}(\dots, p_i, p_j, p_k, \dots) \\ = d\Phi_m(\dots, \tilde{p}_I, \tilde{p}_K, \dots) d\Phi_{X_{ijk}}(p_i, p_j, p_k; \tilde{p}_I + \tilde{p}_K) \end{aligned}$$

$$\mathcal{X}_3^{0,1}(i, j, k) = \int d\Phi_{X_{ijk}} X_3^{0,1}(i, j, k), \quad \mathcal{X}_4^0(i, j, k, l) = \int d\Phi_{X_{ijkl}} X_4^0(i, j, k, l)$$

- $X(\dots)$ based on physical matrix elements $X = \overbrace{A, B, C}^{q\bar{q}} \overbrace{D, E}^{qg} \overbrace{F, G, H}^{g\bar{g}}$

$$X_3^0(i, j, k) = \frac{|\mathcal{A}_3^0(i, j, k)|^2}{|\mathcal{A}_2^0(\tilde{I}, \tilde{K})|^2}, \quad X_4^0(i, j, k, l) = \frac{|\mathcal{A}_4^0(i, j, k, l)|^2}{|\mathcal{A}_2^0(\tilde{I}, \tilde{L})|^2},$$

All building blocks known!

X_3^0, X_4^0, X_3^1 and integrated counterparts $\mathcal{X}_3^0, \mathcal{X}_4^0, \mathcal{X}_3^1$

∇ configurations relevant at hadron colliders
 ↪ final-final, initial-final, initial-initial

- int

$$\begin{aligned} d\Phi_{m+1}(\dots, p_i, p_j, p_k, \dots) \\ = d\Phi_m(\dots, \tilde{p}_I, \tilde{p}_K, \dots) d\Phi_{X_{ijk}}(p_i, p_j, p_k; \tilde{p}_I + \tilde{p}_K) \end{aligned}$$

$$\mathcal{X}_3^{0,1}(i, j, k) = \int d\Phi_{X_{ijk}} X_3^{0,1}(i, j, k), \quad \mathcal{X}_4^0(i, j, k, l) = \int d\Phi_{X_{ijkl}} X_4^0(i, j, k, l)$$

Antenna subtraction

NLO

► real:

$$d\sigma^{\text{S,NLO}}$$

$$\sim d\Phi_{m+1}(\dots, p_i, p_j, p_k, \dots) X_3^0(i, j, k) |\mathcal{A}_m^0(\dots, \tilde{I}, \tilde{K}, \dots)|^2 \mathcal{J}(\tilde{p}_i)$$

$$\sim d\Phi_m(\dots, \tilde{p}_I, \tilde{p}_K, \dots) \underbrace{d\Phi_{X_{ijk}}}_{\text{integrate}} X_3^0(i, j, k) |\mathcal{A}_m^0(\dots, \tilde{I}, \tilde{K}, \dots)|^2 \mathcal{J}(\tilde{p}_i)$$

► virtual:

$$d\sigma^{\text{T,NLO}} \sim -d\Phi_m \mathcal{X}_3^0(s_{ij}) |\mathcal{A}_m^0(\dots, i, j, \dots)|^2 \mathcal{J}(p_i)$$

NNLO

► double real: $d\sigma^{\text{S}} \sim X_3^0 |\mathcal{A}_{m+1}^0|^2, \quad X_4^0 |\mathcal{A}_m^0|^2, \quad X_3^0 X_3^0 |\mathcal{A}_m^0|^2$

► real-virtual: $d\sigma^{\text{T}} \sim \mathcal{X}_3^0 |\mathcal{A}_{m+1}^0|^2, \quad X_3^0 |\mathcal{A}_m^1|^2, \quad X_3^1 |\mathcal{A}_m^0|^2$

► double virtual: $d\sigma^{\text{U}} = (\text{collect rest}) \sim \mathcal{X} |\mathcal{A}_m^{0,1}|^2$

What about those angular terms?

- ▶ Antenna subtraction: $X_n^l |\mathcal{A}_m|^2 \leftrightarrow$ spin averaged!
- ▶ angular terms in gluon splittings:

$$P_{g \rightarrow q\bar{q}} = \frac{2}{s_{ij}} \left[-g^{\mu\nu} + 4z(1-z) \frac{k_{\perp}^{\mu} k_{\perp}^{\nu}}{k_{\perp}^2} \right]$$

\hookrightarrow subtraction non-local in these limits!

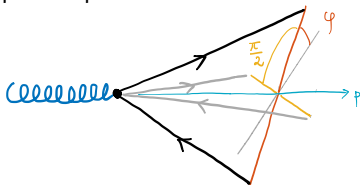
\hookrightarrow vanish upon azimuthal-angle (φ) average (\Rightarrow do not enter \mathcal{X})

sol. 1: supplement angular terms in the subtraction

sol. 2: exploit φ dependence & average in the phase space

$$\mathcal{A}_{\mu}^* \frac{k_{\perp}^{\mu} k_{\perp}^{\nu}}{k_{\perp}^2} \mathcal{A}_{\nu} \sim \cos(2\varphi + \varphi_0)$$

\Rightarrow add φ & $(\varphi + \pi/2)$!



$$\vec{r} \longrightarrow \text{PS}_{\text{gen.}} \longrightarrow \begin{bmatrix} \{p_i, & p_j, & \dots\} \\ \{p'_i, & p'_j, & \dots\} \end{bmatrix} \xrightarrow{(i||j)} \begin{bmatrix} \{p_i^{\varphi}, & p_j^{\varphi}, & \dots\} \\ \{p_i^{\varphi+\pi/2}, & p_j^{\varphi+\pi/2}, & \dots\} \end{bmatrix}$$

Checks of the calculation — unresolved limits

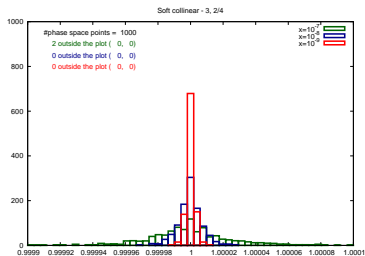
Double-real level

$$\blacktriangleright d\sigma^S \rightarrow d\sigma^{\text{RR}}$$

(single- & double-unresolved)

bin the ratio: $d\sigma^S/d\sigma^{\text{RR}} \xrightarrow{\text{unresolved}} 1$

$$q \bar{q} \rightarrow Z + g_3 g_4 g_5 \quad @ \text{ tree} \\ (g_3 \text{ soft} \ \& \ g_4 \parallel \bar{q})$$



(approach limit: $x_i = 10^{-7}, 10^{-8}, 10^{-9}$)

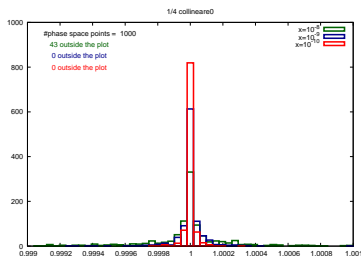
Real-virtual level

$$\blacktriangleright d\sigma^T \rightarrow d\sigma^{\text{RV}}$$

(single-unresolved)

bin the ratio: $d\sigma^T/d\sigma^{\text{RV}} \xrightarrow{\text{unresolved}} 1$

$$q \bar{q} \rightarrow Z + g_3 g_4 \quad @ \text{ 1-loop} \\ (g_4 \parallel q)$$



(approach limit: $x_i = 10^{-8}, 10^{-9}, 10^{-10}$)

Checks of the calculation — pole cancellation

DimReg: $D = 4 - 2\epsilon$

Double-virtual level

- Poles $(d\sigma^{VV} - d\sigma^U) = 0$
2-loop, $(1\text{-loop})^2 \rightsquigarrow 1/\epsilon^4, \dots, 1/\epsilon$

Real-virtual level

- Poles $(d\sigma^{RV} - d\sigma^T) = 0$
1-loop $\rightsquigarrow 1/\epsilon^2, 1/\epsilon$

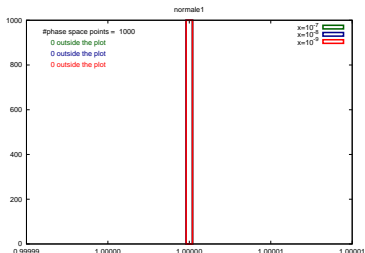
```
09:26:35 ...maple/process/Z
$ form autoqgB1g2ZgtoqU.frm
FORM 4.1 (Mar 13 2014) 64-bits
#-

poles = 0;

6.58 sec out of 6.64 sec
```

pole coefficient: $d\sigma^T/d\sigma^{RV} \equiv 1$

$q \bar{q} \rightarrow Z + g g$ @ 1-loop
($1/\epsilon$ coefficient)





X. Chen, J. Cruz-Martinez, J. Currie, R. Gauld, A. Gehrmann-De Ridder,
T. Gehrmann, E.W.N. Glover, AH, I. Majer, T. Morgan, J. Niehues, J. Pires,
D. Walker

Common framework for NNLO corrections using Antenna Subtraction

- ▶ parton-level event generator
- ▶ based on antenna subtraction
- ▶ test & validation framework
- ▶ APPLfast-NNLO interface
(Work in progress)
[Britzger, Gwenlan, AH, Morgan, Sutton, Rabbertz]
- ▶ ...

Processes:

- ▶ $pp \rightarrow V \rightarrow \bar{\ell}\ell + 0, 1 \text{ jets}$
- ▶ $pp \rightarrow H + 0, 1 \text{ jets}$
- ▶ $pp \rightarrow H + 2 \text{ jets (VBF)}$
- ▶ $pp \rightarrow \text{dijets}$
- ▶ $ep \rightarrow 1, 2 \text{ jets}$
- ▶ $e^+e^- \rightarrow 3 \text{ jets}$
- ▶ ...

Part 1. Antenna Subtraction Formalism

Part 2. **Example: Drell-Yan**

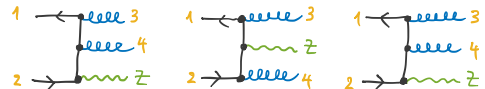
Part 3. Going beyond...

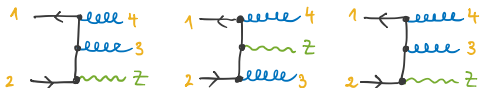
↔ Transverse Momentum Spectrum

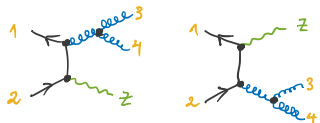
↔ Projection-to-Born Method

Colour Decomposition

(Example: $q\bar{q} \rightarrow ggZ$)

(a)  $(T^{a_3} T^{a_4})_{c_1 c_2}$

(b)  $(T^{a_4} T^{a_3})_{c_1 c_2}$

(c)  $T_{c_1 c_2}^a f^{a a_3 a_4} \sim [T^{a_3} T^{a_4} - T^{a_4} T^{a_3}]_{c_1 c_2}$

$$\Rightarrow \mathcal{M}_{q\bar{q} \rightarrow ggZ}^0 = (T^{a_3} T^{a_4})_{c_1 c_2} \mathcal{A}_4^0(1_q, 3_g, 4_g, 2_{\bar{q}}, Z) \leftrightarrow \text{"(a)+(c)"} \\ + (T^{a_4} T^{a_3})_{c_1 c_2} \mathcal{A}_4^0(1_q, 4_g, 3_g, 2_{\bar{q}}, Z) \leftrightarrow \text{"(b)-(c)"}$$

Drell-Yan: $q\bar{q} \rightarrow$ gluons channel

$$d\sigma^{RR} \sim \frac{1}{2!} \left[|\mathcal{A}_4^0(1q, 3g, 4g, 2\bar{q})|^2 + |\mathcal{A}_4^0(1q, 4g, 3g, 2\bar{q})|^2 - \frac{1}{N_c^2} |\tilde{\mathcal{A}}_4^0(1q, 3g, 4g, 2\bar{q})|^2 \right]$$

$$d\sigma^{RV} \sim |\mathcal{A}_3^1(1q, 3g, 2\bar{q})|^2 - \frac{1}{N_c^2} |\tilde{\mathcal{A}}_3^1(1q, 3g, 2\bar{q})|^2$$

$$d\sigma^{VV} \sim |\mathcal{A}_2^2(1q, 2\bar{q})|^2 - \frac{1}{N_c^2} |\tilde{\mathcal{A}}_2^2(1q, 2\bar{q})|^2$$

$$* 2 \operatorname{Re} \left\{ \mathcal{A}_4^0(1q, 3g, 4g, 2\bar{q}) \mathcal{A}_4^0(1q, 4g, 3g, 2\bar{q}) \right\} = |\tilde{\mathcal{A}}_4^0(1q, 3g, 4g, 2\bar{q})|^2 - |\mathcal{A}_4^0(1q, 3g, 4g, 2\bar{q})|^2 - |\mathcal{A}_4^0(1q, 4g, 3g, 2\bar{q})|^2 \quad \tilde{\mathcal{A}}_4^0(1q, 3g, 4g, 2\bar{q}) = \mathcal{A}_4^0(1q, 3g, 4g, 2\bar{q}) + \mathcal{A}_4^0(1q, 4g, 3g, 2\bar{q})$$

Subtraction Term for $|\mathcal{A}_4^0(1_q, 3_g, 4_g, 2_{\bar{q}})|^2$ RR

$$\left. \begin{aligned} &+ d_3^0(1_q, 3_g, 4_g) |\mathcal{A}_3^0(\tilde{1}_q, (\tilde{34})_g, 2_{\bar{q}})|^2 \\ &+ d_3^0(2_{\bar{q}}, 4_g, 3_g) |\mathcal{A}_3^0(1_q, (\tilde{34})_g, \tilde{2}_{\bar{q}})|^2 \end{aligned} \right\} \hat{=} \text{NLO Zjet subtr. term} \Rightarrow \begin{array}{l} \text{single: } 3 \\ \text{single: } 4 \end{array}$$

Subtraction Term for $|\mathcal{A}_4^{\circ}(1_q, 3_g, 4_g, 2_{\bar{q}})|^2$ RR

$$\begin{aligned}
 & + d_3^{\circ}(1_q, 3_g, 4_g) |\mathcal{A}_3^{\circ}(\tilde{1}_q, (\tilde{34})_g, 2_{\bar{q}})|^2 \\
 & + d_3^{\circ}(2_{\bar{q}}, 4_g, 3_g) |\mathcal{A}_3^{\circ}(1_q, (\tilde{34})_g, \tilde{2}_{\bar{q}})|^2 \\
 & + A_4^{\circ}(1_q, 3_g, 4_g, 2_{\bar{q}}) |\mathcal{A}_2^{\circ}(\tilde{1}_q, \tilde{2}_{\bar{q}})|^2
 \end{aligned}
 \left. \vphantom{\begin{aligned} & + d_3^{\circ}(1_q, 3_g, 4_g) |\mathcal{A}_3^{\circ}(\tilde{1}_q, (\tilde{34})_g, 2_{\bar{q}})|^2 \\ & + d_3^{\circ}(2_{\bar{q}}, 4_g, 3_g) |\mathcal{A}_3^{\circ}(1_q, (\tilde{34})_g, \tilde{2}_{\bar{q}})|^2 \end{aligned}} \right\} \hat{=} \text{NLO Zjet subtr. term} \Rightarrow \begin{array}{l} \text{single: } 3 \\ \text{single: } 4 \\ \\ \text{double: } 3\&4 \end{array}$$

Subtraction Term for $|\mathcal{A}_4^\circ(1_q, 3_g, 4_g, 2_{\bar{q}})|^2$ RR

$$\begin{aligned}
 & + d_3^\circ(1_q, 3_g, 4_g) |\mathcal{A}_3^\circ(\tilde{1}_q, \underline{(34)}_g, 2_{\bar{q}})|^2 \\
 & + d_3^\circ(2_{\bar{q}}, 4_g, 3_g) |\mathcal{A}_3^\circ(1_q, \underline{(34)}_g, \tilde{2}_{\bar{q}})|^2 \\
 & + A_4^\circ(1_q, \underline{3}_g, \underline{4}_g, 2_{\bar{q}}) |\mathcal{A}_2^\circ(\tilde{1}_q, \tilde{2}_{\bar{q}})|^2 \\
 & - d_3^\circ(1_q, 3_g, 4_g) A_3^\circ(\tilde{1}_q, \underline{(34)}_g, 2_{\bar{q}}) |\mathcal{A}_2^\circ(\tilde{1}_q, \tilde{2}_{\bar{q}})|^2 \\
 & - d_3^\circ(2_{\bar{q}}, 4_g, 3_g) A_3^\circ(1_q, \underline{(34)}_g, 2_{\bar{q}}) |\mathcal{A}_2^\circ(\tilde{1}_q, \tilde{2}_{\bar{q}})|^2
 \end{aligned}$$

NLO Zjet subtr. term \Rightarrow single: 3
 single: 4
 spurious singularities
 double: 3&4
 double: 3&4
 spurious singularities
 single: 3
 single: 4
 double: 3&4
 single: 3
 single: 4

DONE

Subtraction Term for $|\mathcal{A}_3^1(1_q, 3_g, 2_{\bar{q}})|^2$ RV

$$- \left[\frac{1}{2} \mathcal{D}_3^0(s_{13}) + \frac{1}{2} \mathcal{D}_3^0(s_{23}) \right] |\mathcal{A}_3^0(1_q, 3_g, 2_{\bar{q}})|^2 \left. \vphantom{\left[\frac{1}{2} \mathcal{D}_3^0(s_{13}) + \frac{1}{2} \mathcal{D}_3^0(s_{23}) \right]} \right\} \hat{=} \text{NLO ztjet subtr. term} \Rightarrow \text{poles}$$

Subtraction Term for $|\mathcal{A}_3^1(1_q, 3_g, 2_{\bar{q}})|^2$ IRV

$$- \left[\frac{1}{2} \Delta_3^0(s_{13}) + \frac{1}{2} \Delta_3^0(s_{23}) \right] |\mathcal{A}_3^0(1_q, 3_g, 2_{\bar{q}})|^2 \left. \vphantom{\left[\frac{1}{2} \Delta_3^0(s_{13}) + \frac{1}{2} \Delta_3^0(s_{23}) \right]} \right\} \hat{=} \text{NLO z+jet subtr. term} \Rightarrow \text{poles}$$

$$+ A_3^0(1_q, 3_g, 2_{\bar{q}}) |\mathcal{A}_2^1(\tilde{1}_q, \tilde{2}_{\bar{q}})|^2 \left. \vphantom{A_3^0(1_q, 3_g, 2_{\bar{q}})} \right\} \begin{array}{l} \text{(tree) x (loop)} \\ \text{(loop) x (tree)} \end{array}$$

$$+ A_3^1(1_q, 3_g, 2_{\bar{q}}) |\mathcal{A}_2^0(\tilde{1}_q, \tilde{2}_{\bar{q}})|^2 \left. \vphantom{A_3^1(1_q, 3_g, 2_{\bar{q}})} \right\} \begin{array}{l} \text{(tree) x (loop)} \\ \text{(loop) x (tree)} \end{array}$$

single: 3

Subtraction Term for $|\mathcal{A}_3^1(1_q, 3_g, 2_{\bar{q}})|^2$ IRV

$$- \left[\frac{1}{2} \Delta_3^0(s_{13}) + \frac{1}{2} \Delta_3^0(s_{23}) \right] |\mathcal{A}_3^0(1_q, 3_g, 2_{\bar{q}})|^2 \left. \vphantom{\left[\frac{1}{2} \Delta_3^0(s_{13}) + \frac{1}{2} \Delta_3^0(s_{23}) \right]} \right\} \begin{array}{l} \hat{=} \text{NLO ztjet} \\ \text{subtr. term} \Rightarrow \text{poles} \\ \text{spurious} \\ \text{singularities} \end{array}$$

$$+ A_3^0(1_q, 3_g, 2_{\bar{q}}) |\mathcal{A}_2^1(\tilde{1}_q, \tilde{2}_{\bar{q}})|^2 \left. \vphantom{A_3^0(1_q, 3_g, 2_{\bar{q}})} \right\} \begin{array}{l} (\text{tree}) \times (\text{loop}) \\ (\text{loop}) \times (\text{tree}) \end{array}$$

$$+ \underline{A_3^1}(1_q, 3_g, 2_{\bar{q}}) |\mathcal{A}_2^0(\tilde{1}_q, \tilde{2}_{\bar{q}})|^2 \left. \vphantom{\underline{A_3^1}(1_q, 3_g, 2_{\bar{q}})} \right\} \begin{array}{l} \text{spurious} \\ \text{singularities} \end{array}$$

$$+ \left[\frac{1}{2} \Delta_3^0(s_{13}) + \frac{1}{2} \Delta_3^0(s_{23}) \right] A_3^0(1_q, 3_g, 2_{\bar{q}}) |\mathcal{A}_2^0(\tilde{1}_q, \tilde{2}_{\bar{q}})|^2 \left. \vphantom{\left[\frac{1}{2} \Delta_3^0(s_{13}) + \frac{1}{2} \Delta_3^0(s_{23}) \right]} \right\}$$

+ MF

DONE

Subtraction Term for $|A_2^2(1q, 2\bar{q})|^2$

UV

$$- A_4^0(s_{12}) |A_2^0(1q, 2\bar{q})|^2$$

from $A_4^0 @ RR$

$$- A_3^0(s_{12}) |A_2^1(1q, 2\bar{q})|^2$$

(tree)x(loop)

@ RV

$$- A_3^1(s_{12}) |A_2^0(1q, 2\bar{q})|^2$$

(loop)x(tree)

poles

+ MF

DONE

Part 1. Antenna Subtraction Formalism

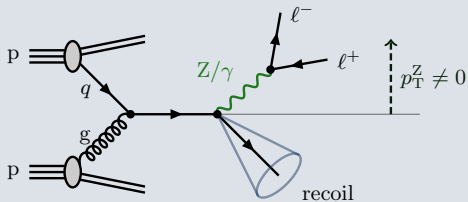
Part 2. Example: Drell-Yan

Part 3. **Going beyond...**

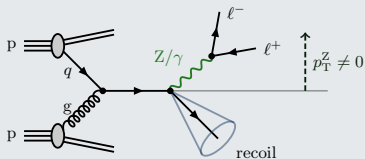
↪ Transverse Momentum Spectrum

↪ Projection-to-Born Method

Transverse Momentum Spectrum



Inclusive p_T spectrum from $X + \text{jet}$



$p p \rightarrow X + \text{recoil}$

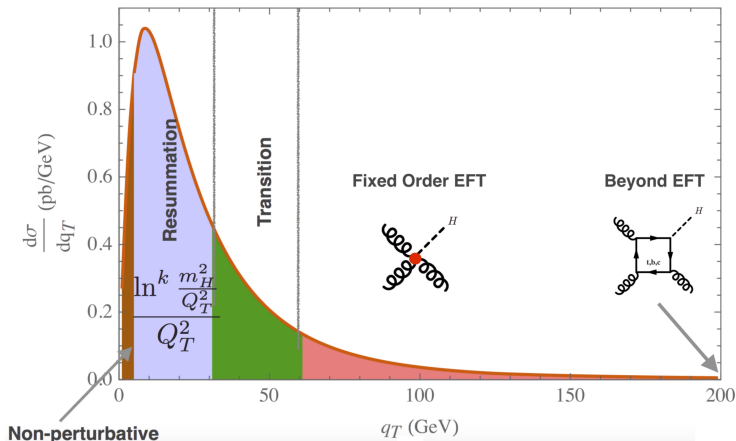
- ▶ fully inclusive in QCD emissions
 - ▶ require recoil: $p_T^X > p_{T, \text{cut}}^X$
- \Rightarrow can use $X + \text{jet}$ calculation

$X + \text{jet}$ @ NNLO

- ▶ $H + \text{jet}$ (Antenna, N -jettiness, Sector-improved R.S.)
- ▶ $W + \text{jet}$ (Antenna, N -jettiness)
- ▶ $Z + \text{jet}$ (Antenna, N -jettiness)
- ▶ $\gamma + \text{jet}$ (N -jettiness)

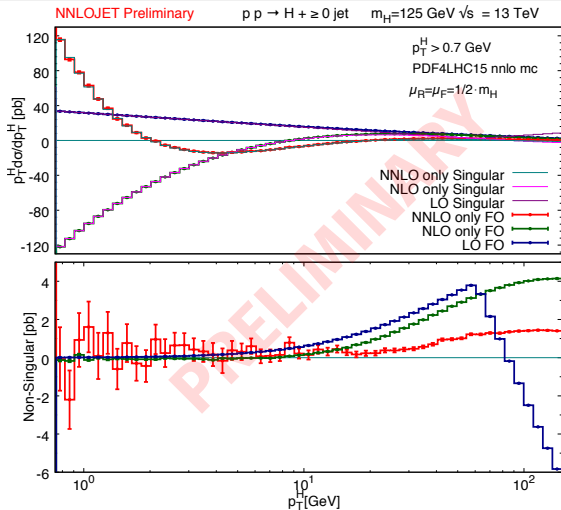
\leadsto validation & opportunity for benchmarks

The p_T spectrum



- ▶ fixed-order prediction *diverges* for $p_T \rightarrow 0$
 - ▶ large logarithms: $\ln^k(p_T/M)/p_T^2 \sim$ all-order resummation needed!
- \Rightarrow matching: $d\sigma_{\text{matched}} = d\sigma_{\text{f.o.}} + d\sigma_{\text{res.}} - d\sigma_{\text{res.}}|_{\text{exp.}}$

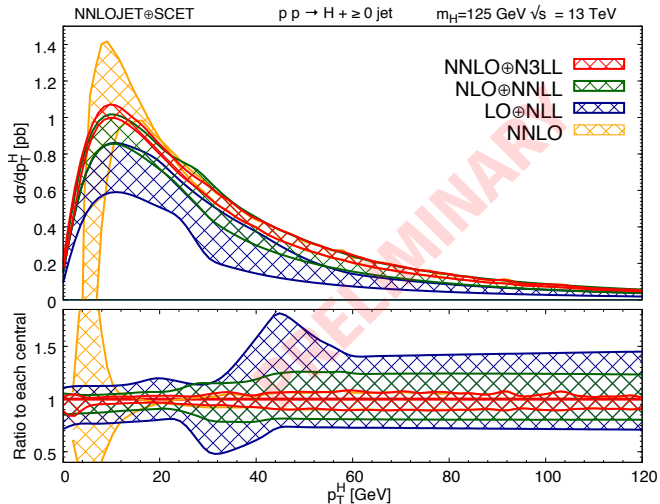
Compare the logs — fixed-order vs. resummation



[Chen, Gehrmann, Glover, AH, Li, Neill, Schulze, Stewart, Zhu]

- ▶ excellent agreement within stat. errors $\sim 1\%$
- ▶ predictions down to $p_T^H = 0.7 \text{ GeV}$
- ▶ important *cross check* \rightsquigarrow matching of NNLO and $N^3\text{LL}$

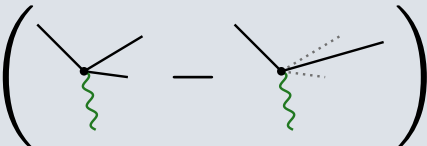
Matched p_T spectrum



[Chen, Gehrmann, Glover, AH, Li, Neill, Schulze, Stewart, Zhu]

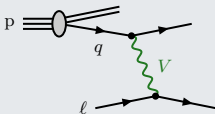
- ▶ **NNLO** & **NNLO** ⊕ **N³LL** start to deviate @ $p_T \lesssim 30 \text{ GeV}$
- ▶ reduction of uncertainties by more than a factor of two
- ▶ **NLO** ⊕ **NLL** → **NNLO** ⊕ **N³LL**: large impact in peak region

Projection-to-Born Method

$$\int \left(\text{Diagram 1} - \text{Diagram 2} \right)$$


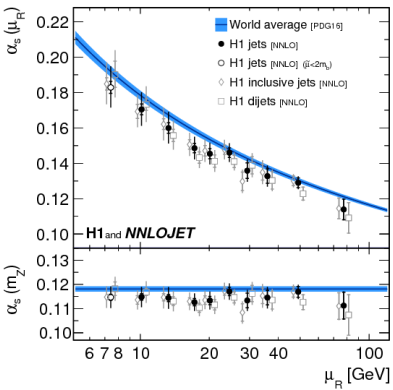
The diagram shows two Feynman diagrams enclosed in large parentheses, with a minus sign between them. The first diagram on the left shows a central vertex with three solid lines extending from it: one to the upper left, one to the upper right, and one to the right. A green wavy line extends downwards from the vertex. The second diagram on the right shows a similar setup, but the line extending to the right is dashed, and there are several small grey dots scattered around the vertex area, representing a projection or approximation.

Deep Inelastic Scattering



precise probe to resolve the inner structure of the proton

- ▶ PDF constraints
- ▶ α_s extraction (+ running)



[Eur.Phys.J. C77 (2017) no.11, 791]

- ▶ DIS 2 jet @ NNLO

[Currie, Gehrmann, Niehues '16]

[Currie, Gehrmann, AH, Niehues '17]

⇐ precise α_s determination

- ▶ DIS 1 jet @ N³LO

[Currie, Gehrmann, Glover, AH, Niehues, Vogt. '18]

DIS 2 jet
@ NNLO

[Currie, Gehrmann, Niehues '16]
[Currie, Gehrmann, AH, Niehues '17]

Projection-to-Born



[Cacciari, et al. '15]

DIS structure
function
@ N³LO

[Moch, Vermaseren, Vogt '05]

=

DIS fully
differential @ N³LO

Projection-to-Born

inclusive DIS (structure function)

$$\sigma_{\text{NLO}}^{\text{incl.}} = \text{[Born diagram]} + \int_{1, \text{incl.}} \text{[NLO diagram]}$$

The diagram on the left is a Born-level diagram showing a quark line with a gluon emission from the vertex, labeled with α_s . The diagram on the right is a Next-to-Leading Order (NLO) diagram showing a quark line with a gluon emission from the vertex, with a shaded region indicating the inclusive integration over the gluon's phase space.

Born kinematics: $Q^2 = -q^2 > 0$, $x = \frac{-Q^2}{2P \cdot q}$

Projection-to-Born

differential DIS (w/o IR treatment)

$$\sigma_{\text{NLO}}^{\text{diff.}} = \text{[Diagram: circle with } \alpha_s \text{ and a wavy line]} + \int_1^{\text{diff.}} \text{[Diagram: vertex with a wavy line]}$$

Born kinematics: $Q^2 = -q^2 > 0$, $x = \frac{-Q^2}{2P \cdot q}$

Projection-to-Born

differential DIS (w/ IR treatment)

$$\sigma_{\text{NLO}}^{\text{diff.}} = \text{Born} + \int_{1, \text{incl.}} \text{Virtual} + \int_{1, \text{diff.}} \left(\text{Virtual} - \text{Soft} \right)$$

The equation shows the NLO differential cross-section as a sum of three terms. The first term is the Born approximation, represented by a circle with α_s and a wavy line. The second term is an integral over the inclusive region of a virtual correction diagram. The third term is an integral over the differential region of the difference between a virtual correction diagram and a soft emission diagram.

Born kinematics: $Q^2 = -q^2 > 0$, $x = \frac{-Q^2}{2P \cdot q}$

Projection-to-Born

differential DIS (w/ IR treatment)

$$\sigma_{\text{NLO}}^{\text{diff.}} = \underbrace{\left(\text{tree} + \int_{1, \text{incl.}} \text{1-loop} \right)}_{\text{DIS structure function @ NLO}} + \underbrace{\left(\int_{1, \text{diff.}} \left(\text{2-loop} - \text{1-loop} \right) \right)}_{\text{DIS 2 jet @ LO}}$$

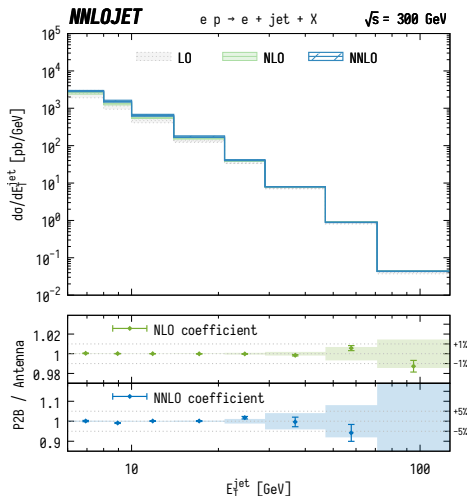
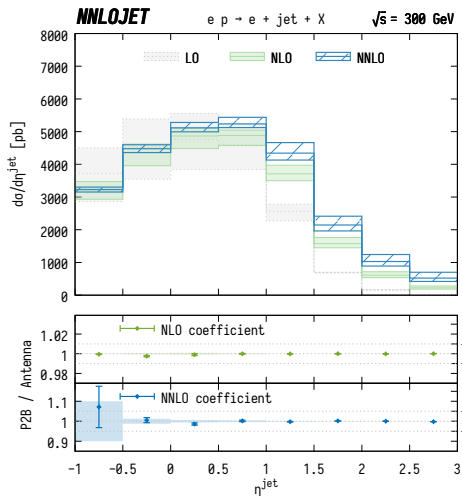
The diagram shows the decomposition of the differential NLO cross-section. The first part, labeled 'DIS structure function @ NLO', consists of a tree-level diagram with a vertex labeled α_s and a wavy gluon line, plus an integral over a 1-loop diagram with a gluon jet. The second part, labeled 'DIS 2 jet @ LO', consists of an integral over the difference between two diagrams: a 2-loop diagram with a gluon jet and a 1-loop diagram with a gluon jet.

only “special” processes *but not restricted to any order*

$$+ \left. \begin{array}{l} \text{inclusive } X \\ X + \text{jet} \end{array} \right\} \begin{array}{l} @ N^n \text{LO} \\ @ N^{n-1} \text{LO} \end{array} \sim X @ N^n \text{LO}$$

$$\text{Born kinematics: } Q^2 = -q^2 > 0, \quad x = \frac{-Q^2}{2P \cdot q}$$

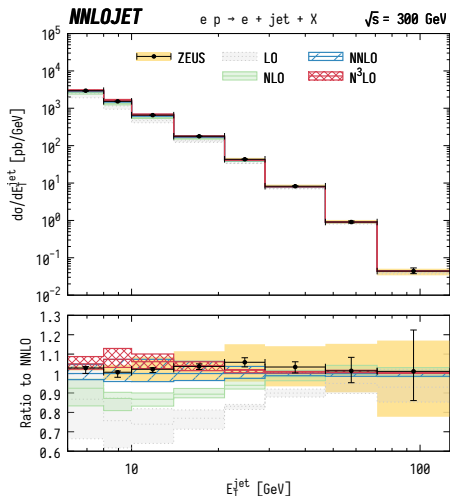
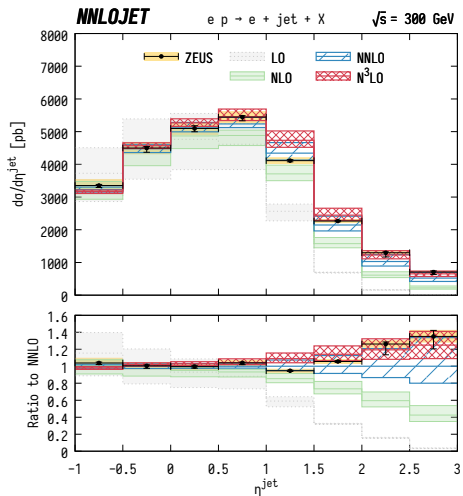
Validation up to NNLO — Antenna vs. P2B



NLO coefficient: $\lesssim 5\%$
 NNLO coefficient: $\lesssim 2\%$

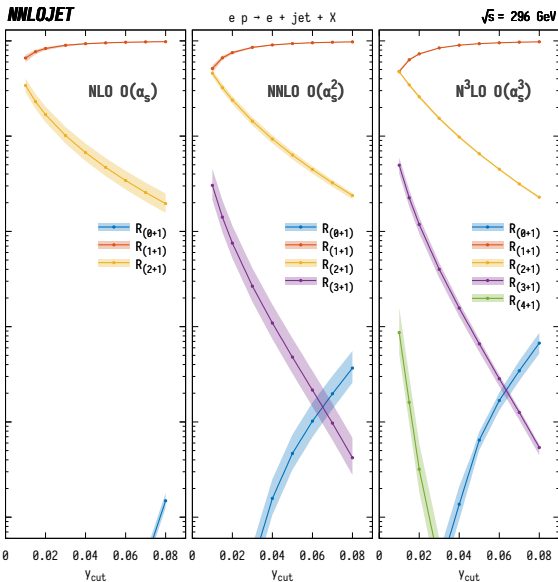
} \rightsquigarrow agreement @ full NNLO: $\lesssim 1\%$

Differential distributions at N³LO



- ▶ for the first time: *overlapping* scale bands agreement with data
- ▶ reduction of scale uncertainties

Jet Rates



Jet rates:

$$R_{(n+1)} = N_{(n+1)} / N_{\text{tot}}$$

JADE algorithm

\hookrightarrow cluster partons if:

$$\frac{2E_i E_j (1 - \cos \theta_{ij})}{W^2} < y_{\text{cut}}$$

Projection-to-Born — an “antenna” view

Consider the real-emission subtraction in the antenna subtraction formalism for $H + 0\text{jet}$ (@ LC):

$$\begin{aligned} & \int \left\{ d\sigma_{H+0\text{jet}}^{\text{R}} - d\sigma_{H+0\text{jet}}^{\text{SNLO}} \right\} \\ &= \int d\Phi_{H+1} \left\{ A3g0H(1_g, 2_g, 3_g, H) \mathcal{J}(\Phi_{H+1}) \right. \\ & \quad \left. - F_3^0(1_g, 2_g, 3_g) A2g0H(\tilde{1}_g, \tilde{2}_g, H) \mathcal{J}(\tilde{\Phi}_{H+0}) \right\} \end{aligned}$$

Projection-to-Born — an “antenna” view

Consider the real-emission subtraction in the antenna subtraction formalism for $H + 0\text{jet}$ (@ LC):

$$\begin{aligned} & \int \left\{ d\sigma_{H+0\text{jet}}^{\text{R}} - d\sigma_{H+0\text{jet}}^{\text{SNLO}} \right\} \\ &= \int d\Phi_{H+1} \left\{ A3g0H(1_g, 2_g, 3_g, H) \mathcal{J}(\Phi_{H+1}) \right. \\ & \quad \left. - F_3^0(1_g, 2_g, 3_g) A2g0H(\tilde{1}_g, \tilde{2}_g, H) \mathcal{J}(\tilde{\Phi}_{H+0}) \right\} \end{aligned}$$

Antennae = ratios of physical Matrix Elements:

$$F_3^0(i_g, j_g, k_g) \equiv \frac{A3g0H(i_g, j_g, k_g, H)}{A2g0H(\tilde{i}_g, \tilde{k}_g, H)}$$

Projection-to-Born — an “antenna” view

Consider the real-emission subtraction in the antenna subtraction formalism for $H + 0\text{jet}$ (@ LC):

$$\begin{aligned} & \int \left\{ d\sigma_{H+0\text{jet}}^{\text{R}} - d\sigma_{H+0\text{jet}}^{\text{SNLO}} \right\} \\ &= \int d\Phi_{H+1} \left\{ \mathbf{A3g0H}(1_g, 2_g, 3_g, H) \mathcal{J}(\Phi_{H+1}) \right. \\ & \quad \left. - F_3^0(1_g, 2_g, 3_g) \mathbf{A2g0H}(\tilde{1}_g, \tilde{2}_g, H) \mathcal{J}(\tilde{\Phi}_{H+0}) \right\} \\ &= \int d\Phi_{H+1} \mathbf{A3g0H}(1_g, 2_g, 3_g, H) \left\{ \mathcal{J}(\Phi_{H+1}) - \mathcal{J}(\tilde{\Phi}_{H+0}) \right\} \end{aligned}$$

Projection-to-Born — an “antenna” view

Consider the real-emission subtraction in the antenna subtraction formalism for $H + 0\text{jet}$ (@ LC):

$$\begin{aligned} & \int \left\{ d\sigma_{H+0\text{jet}}^R - d\sigma_{H+0\text{jet}}^{\text{SNLO}} \right\} \\ &= \int d\Phi_{H+1} \left\{ \mathbf{A3g0H}(1_g, 2_g, 3_g, H) \mathcal{J}(\Phi_{H+1}) \right. \\ & \quad \left. - F_3^0(1_g, 2_g, 3_g) \mathbf{A2g0H}(\tilde{1}_g, \tilde{2}_g, H) \mathcal{J}(\tilde{\Phi}_{H+0}) \right\} \\ &= \int d\Phi_{H+1} \mathbf{A3g0H}(1_g, 2_g, 3_g, H) \left\{ \mathcal{J}(\Phi_{H+1}) - \mathcal{J}(\tilde{\Phi}_{H+0}) \right\} \end{aligned}$$

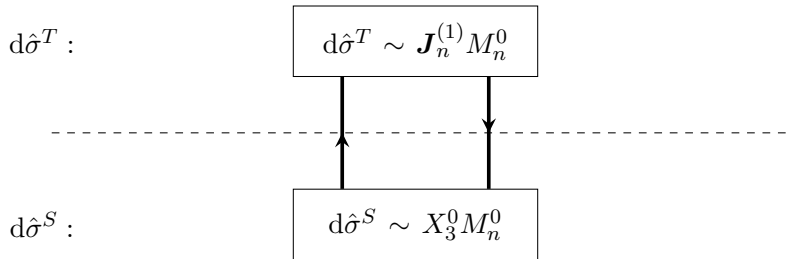
⇒ Simple processes where antenna \simeq real-emission Matrix Element
↔ **Projection-to-Born**

Similarly at NNLO: X_4^0 & $X_3^0 \times X_3^0$ are “projections” of RR ME & NLO(+jet) subtraction term.

- ▶ Antenna subtraction successfully applied to many important processes:
 - ↪ $pp \rightarrow X + 0, 1 \text{ jets}$ ($X = H, Z, W$)
 - ↪ $pp \rightarrow H + 2 \text{ jets}$ (VBF)
 - ↪ $pp \rightarrow \text{dijets}$ ($N_c^2, N_c N_F, N_F^2$)
 - ↪ $ep \rightarrow 1, 2 \text{ jets}$
 - ↪ $e^+e^- \rightarrow 3 \text{ jets}$
- ⇒ subtraction set up for: $pp \rightarrow \text{“colour neutral”} + 0, 1, 2 \text{ jets}$
- ▶ inclusive p_T^H spectrum: NNLO prediction matched to N³LL
 - ↪ fixed order: stable predictions down to $p_T^H = 0.7 \text{ GeV}$
 - ↪ is it good enough for q_T -subtraction @ N³LO?!
- ▶ Projection-to-Born method \oplus Antenna subtraction
 - ↪ first fully differential N³LO prediction: inclusive jets in DIS
 - ↪ method also applicable for colour-neutral final states pp in collisions

Thank you

Backup Slides



Antenna subtraction @ NNLO

[J. Currie, E.W.N. Glover, S. Wells '13]

