NNLO Corrections Using Antenna Subtraction & Applications

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Subtracting Infrared Singularities Beyond NLO

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CERN

This talk

Part 1. **Antenna Subtraction Formalism**

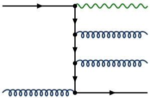
Part 2. **Example: Drell–Yan**

Part 3. **Going beyond...**

- ↪ Transverse Momentum Spectrum
- ↪ Projection-to-Born Method

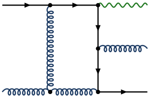
Anatomy of an NNLO calculation

$$\sigma_{\mathrm{NNLO}} = \int_{\Phi_{\mathrm{Z+3}}} \mathrm{d}\sigma^{\mathrm{RR}}_{\mathrm{NNLO}}$$



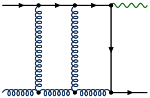
- single-unresolved
- double-unresolved

$$+ \int_{\Phi_{\mathrm{Z+2}}} \mathrm{d}\sigma^{\mathrm{RV}}_{\mathrm{NNLO}}$$



- single-unresolved
- $1/\epsilon^2$, $1/\epsilon$

$$+ \int_{\Phi_{\mathrm{Z+1}}} \mathrm{d}\sigma^{\mathrm{VV}}_{\mathrm{NNLO}}$$



- $1/\epsilon^4$, $1/\epsilon^3$, $1/\epsilon^2$, $1/\epsilon$

$\sum$ finite (Kinoshita–Lee–Nauenberg & factorization)

Non-trivial cancellation of infrared singularities

Anatomy of an NNLO calculation

$$\sigma_{\mathrm{NNLO}} = \int_{\Phi_{Z+3}} \mathrm{d}\sigma^{\mathrm{RR}}_{\mathrm{NNLO}}$$

$$+ \int_{\Phi_{Z+2}} \mathrm{d}\sigma^{\mathrm{RV}}_{\mathrm{NNLO}}$$

$$+ \int_{\Phi_{Z+1}} \mathrm{d}\sigma^{\mathrm{VV}}_{\mathrm{NNLO}}$$

$\sum$ finite

Different methods:

- Antenna subtraction .... [Gehrmann–De Ridder, Gehrmann, Glover '05]
- CoLorFul subtraction .... [Del Duca, Somogyi, Trocsanyi '05]
- q_{T} subtraction .... [Catani, Grazzini '07]
- Sector-improved residue subtraction ...... [Czakon '10], [Boughezal, Melnikov, Petriello '11]
- N-jettiness subtraction .... [Gaunt, Stahlhofen, Tackmann, Walsh '15] [Boughezal, Focke, Liu, Petriello '15]
- Projection-to-Born .... [Cacciari, et al. '15]
- Nested soft-collinear subtraction .... [Caola, Melnikov, Röntsch '17]

...

Approaches: subtraction, slicing

NNLO using Antenna

$$\sigma_{\rm NNLO} = \int_{\Phi_{Z+3}} \left(d\sigma^{RR}_{\rm NNLO} - d\sigma^{S}_{\rm NNLO} \right)$$

$$+ \int_{\Phi_{Z+2}} \left(d\sigma^{RV}_{\rm NNLO} - d\sigma^{T}_{\rm NNLO} \right)$$

$$+ \int_{\Phi_{Z+1}} \left(d\sigma^{VV}_{\rm NNLO} - d\sigma^{U}_{\rm NNLO} \right)$$

- $d\sigma^{S}_{\rm NNLO}$, $d\sigma^{T}_{\rm NNLO}$: mimic $d\sigma^{RR}_{\rm NNLO}$, $d\sigma^{RV}_{\rm NNLO}$ in unresolved limits
- $d\sigma^{T}_{\rm NNLO}$, $d\sigma^{U}_{\rm NNLO}$: *analytic* cancellation of poles in $d\sigma^{RV}_{\rm NNLO}$, $d\sigma^{VV}_{\rm NNLO}$

$\sum$ finite -0

$\Rightarrow$ each line suitable for numerical evaluation in $D = 4$

Antenna factorisation

- antenna formalism operates on *colour-ordered* amplitudes
- exploit universal factorisation properties in IR limits

$$\underbrace{|\mathcal{A}^0_{m+1}(\ldots,i,j,k,\ldots)|^2}_{\text{partial amplitude}} \xrightarrow{j \text{ unresolved}} \underbrace{X^0_3(i,j,k)}_{\text{antenna function}} \; \underbrace{|\mathcal{A}^0_m(\ldots,\widetilde{I},\widetilde{K},\ldots)|^2}_{\text{reduced ME}}$$

+ mapping
$\{p_i, p_j, p_k\} \to \{\widetilde{p}_I, \widetilde{p}_K\}$

- captures multiple limits* and smoothly interpolates between them

limit	$X^0_3(i,j,k)$	mapping
$p_j \to 0$	$\dfrac{2s_{ik}}{s_{ij}s_{jk}}$	$\widetilde{p}_I \to p_i,\;\; \widetilde{p}_K \to p_k$
$p_j \parallel p_i$	$\dfrac{1}{s_{ij}} P_{ij}(z)$	$\widetilde{p}_I \to (p_i + p_j),\;\; \widetilde{p}_K \to p_k$
$p_j \parallel p_k$	$\dfrac{1}{s_{jk}} P_{kj}(z)$	$\widetilde{p}_I \to p_i,\;\; \widetilde{p}_K \to (p_j + p_k)$

* c.f. dipoles: $X^0_3(i,j,k) \sim \mathcal{D}_{ij,k} + \mathcal{D}_{kj,i}$

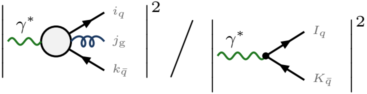
Antenna subtraction — building blocks

- $X(\ldots)$ based on physical matrix elements $\quad X = \overbrace{A,\ B,\ C}^{q\bar{q}},\overbrace{D,\ E}^{qg},\overbrace{F,\ G,\ H}^{gg}$

$$X_3^0(i,j,k) = \frac{|\mathcal{A}_3^0(i,j,k)|^2}{|\mathcal{A}_2^0(\widetilde{I},\widetilde{K})|^2}, \qquad X_4^0(i,j,k,l) = \frac{|\mathcal{A}_4^0(i,j,k,l)|^2}{|\mathcal{A}_2^0(\widetilde{I},\widetilde{L})|^2},$$

$$X_3^1(i,j,k) = \frac{|\mathcal{A}_3^1(i,j,k)|^2}{|\mathcal{A}_2^0(\widetilde{I},\widetilde{K})|^2} - X_3^0(i,j,k)\frac{|\mathcal{A}_2^1(\widetilde{I},\widetilde{K})|^2}{|\mathcal{A}_2^0(\widetilde{I},\widetilde{K})|^2},$$

$A_3^0(i_q, j_g, k_{\bar{q}}) =$



- integrating the antennae $\longleftrightarrow$ phase-space factorization

$$\begin{aligned} &\mathrm{d}\Phi_{m+1}(\ldots,p_i,p_j,p_k,\ldots) \\ &\quad = \mathrm{d}\Phi_m(\ldots,\widetilde{p}_I,\widetilde{p}_K,\ldots)\mathrm{d}\Phi_{X_{ijk}}(p_i,p_j,p_k;\widetilde{p}_I+\widetilde{p}_K) \end{aligned}$$

$$\mathcal{X}_3^{0,1}(i,j,k) = \int \mathrm{d}\Phi_{X_{ijk}} X_3^{0,1}(i,j,k), \quad \mathcal{X}_4^0(i,j,k,l) = \int \mathrm{d}\Phi_{X_{ijkl}} X_4^0(i,j,k,l)$$

Antenna subtraction — building blocks

- $X(\ldots)$ based on physical matrix elements $\quad X = \overbrace{A,\ B,\ C}^{q\bar{q}}, \overbrace{D,\ E}^{qg}, \overbrace{F,\ G,\ H}^{gg}$

$$X_3^0(i,j,k) = \frac{|\mathcal{A}_3^0(i,j,k)|^2}{|\mathcal{A}_2^0(\widetilde{I},\widetilde{K})|^2}, \qquad X_4^0(i,j,k,l) = \frac{|\mathcal{A}_4^0(i,j,k,l)|^2}{|\mathcal{A}_2^0(\widetilde{I},\widetilde{L})|^2},$$

A_3^0

- int

All building blocks known!

X_3^0, X_4^0, X_3^1 and integrated counterparts $\mathcal{X}_3^0, \mathcal{X}_4^0, \mathcal{X}_3^1$

$\forall$ configurations relevant at hadron colliders
$\hookrightarrow$ final–final, initial–final, initial–initial

$$\mathrm{d}\Phi_{m+1}(\ldots,p_i,p_j,p_k,\ldots) = \mathrm{d}\Phi_m(\ldots,\widetilde{p}_I,\widetilde{p}_K,\ldots)\mathrm{d}\Phi_{X_{ijk}}(p_i,p_j,p_k;\widetilde{p}_I+\widetilde{p}_K)$$

$$\mathcal{X}_3^{0,1}(i,j,k) = \int \mathrm{d}\Phi_{X_{ijk}} X_3^{0,1}(i,j,k), \quad \mathcal{X}_4^0(i,j,k,l) = \int \mathrm{d}\Phi_{X_{ijkl}} X_4^0(i,j,k,l)$$

Antenna subtraction

NLO

- real:

$$\mathrm{d}\sigma^{\mathrm{S,NLO}}$$

$$\sim \mathrm{d}\Phi_{m+1}(\ldots, p_i, p_j, p_k, \ldots) \quad X_3^0(i,j,k)\, |\mathcal{A}_m^0(\ldots, \widetilde{I}, \widetilde{K}, \ldots)|^2\, \mathcal{J}(\widetilde{p}_i)$$

$$\sim \mathrm{d}\Phi_m(\ldots, \widetilde{p}_I, \widetilde{p}_K, \ldots) \underbrace{\mathrm{d}\Phi_{X_{ijk}}\, X_3^0(i,j,k)}_{\text{integrate}} |\mathcal{A}_m^0(\ldots, \widetilde{I}, \widetilde{K}, \ldots)|^2\, \mathcal{J}(\widetilde{p}_i)$$

- virtual:

$$\mathrm{d}\sigma^{\mathrm{T,NLO}} \sim -\mathrm{d}\Phi_m\, \mathcal{X}_3^0(s_{ij})\, |\mathcal{A}_m^0(\ldots, i, j, \ldots)|^2\, \mathcal{J}(p_i)$$

NNLO

- double real: $\mathrm{d}\sigma^{\mathrm{S}} \sim X_3^0\, |\mathcal{A}_{m+1}^0|^2, \quad X_4^0\, |\mathcal{A}_m^0|^2, \quad X_3^0\, X_3^0\, |\mathcal{A}_m^0|^2$
- real–virtual: $\mathrm{d}\sigma^{\mathrm{T}} \sim \mathcal{X}_3^0\, |\mathcal{A}_{m+1}^0|^2, \quad X_3^0\, |\mathcal{A}_m^1|^2, \quad X_3^1\, |\mathcal{A}_m^0|^2$
- double virtual: $\mathrm{d}\sigma^{\mathrm{U}} =$ (collect rest) $\sim \mathcal{X}\, |\mathcal{A}_m^{0,1}|^2$

What about those angular terms?

- Antenna subtraction: $X_n^l \; |\mathcal{A}_m|^2 \quad \leftrightarrow \quad$ spin averaged!
- angular terms in gluon splittings:

$$P_{g\to q\bar{q}} = \frac{2}{s_{ij}} \left[-g^{\mu\nu} + 4z(1-z)\, \frac{k_\perp^\mu k_\perp^\nu}{k_\perp^2} \right]$$

$\hookrightarrow$ subtraction non-local in these limits!

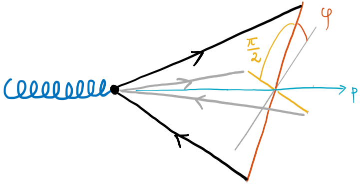
$\hookrightarrow$ vanish upon azimuthal-angle (φ) average ($\Rightarrow$ do not enter $\mathcal{X}$)

sol. 1: supplement angular terms in the subtraction

sol. 2: exploit φ dependence & average in the phase space

$$\mathcal{A}_\mu^* \, \frac{k_\perp^\mu k_\perp^\nu}{k_\perp^2} \, \mathcal{A}_\nu \sim \cos(2\varphi + \varphi_0)$$

$\Rightarrow$ add φ & $(\varphi + \pi/2)$!



$$\vec{r} \longrightarrow \text{PS}_{\text{gen.}} \longrightarrow \begin{bmatrix} \{p_i, & p_j, & \ldots\} \\ \{p_i', & p_j', & \ldots\} \end{bmatrix} \xrightarrow{(i\|j)} \begin{bmatrix} \{p_i^{\varphi}, & p_j^{\varphi}, & \ldots\} \\ \{p_i^{\varphi+\pi/2}, & p_j^{\varphi+\pi/2}, & \ldots\} \end{bmatrix}$$

Checks of the calculation — unresolved limits

Double-real level

- $d\sigma^{S} \rightarrow d\sigma^{RR}$

  (single- & double-unresolved)

Real–virtual level

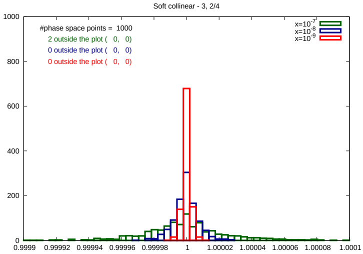
- $d\sigma^{T} \rightarrow d\sigma^{RV}$

  (single-unresolved)

bin the ratio: $d\sigma^{S}/d\sigma^{RR} \xrightarrow{\text{unresolved}} 1$

$q\,\bar{q} \rightarrow Z + g_3\; g_4\; g_5$ @ tree

(g_3 soft & $g_4 \parallel \bar{q}$)

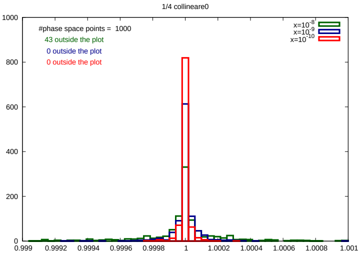


(approach limit: $x_i = 10^{-7}, 10^{-8}, 10^{-9}$)

bin the ratio: $d\sigma^{T}/d\sigma^{RV} \xrightarrow{\text{unresolved}} 1$

$q\,\bar{q} \rightarrow Z + g_3\; g_4$ @ 1-loop

($g_4 \parallel q$)



(approach limit: $x_i = 10^{-8}, 10^{-9}, 10^{-10}$)

Checks of the calculation — pole cancellation

DimReg: $D = 4 - 2\epsilon$

Double-virtual level

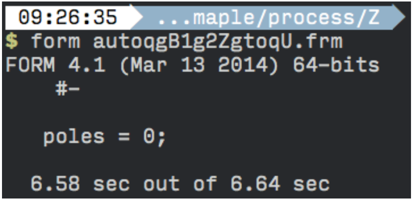
- Poles $\left(\mathrm{d}\sigma^{\mathrm{VV}} - \mathrm{d}\sigma^{\mathrm{U}}\right) = 0$

  2-loop, (1-loop)^2 $\rightsquigarrow$ $1/\epsilon^4, \ldots, 1/\epsilon$

Real–virtual level

- Poles $\left(\mathrm{d}\sigma^{\mathrm{RV}} - \mathrm{d}\sigma^{\mathrm{T}}\right) = 0$

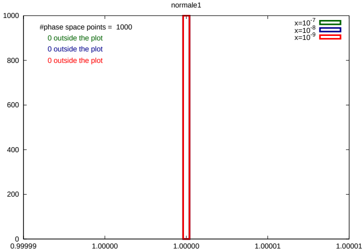
  1-loop $\rightsquigarrow$ $1/\epsilon^2, 1/\epsilon$



pole coefficient: $\mathrm{d}\sigma^{\mathrm{T}}/\mathrm{d}\sigma^{\mathrm{RV}} \equiv 1$

$q\,\bar{q} \to \mathrm{Z} + \mathrm{g}\ \mathrm{g}$ @ 1-loop

($1/\epsilon$ coefficient)



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```

X. Chen, J. Cruz-Martinez, J. Currie, R. Gauld, A. Gehrmann-De Ridder,
T. Gehrmann, E.W.N. Glover, AH, I. Majer, T. Morgan, J. Niehues, J. Pires,
D. Walker

Common framework for NNLO corrections using Antenna Subtraction

- parton-level event generator
- based on antenna subtraction
- test & validation framework
- `APPLfast-NNLO` interface
  (Work in progress)
  [Britzger, Gwenlan, AH, Morgan, Sutton, Rabbertz]
- …

Processes:

- $\mathrm{pp} \to V \to \bar{\ell}\ell \ + 0, 1$ jets
- $\mathrm{pp} \to \mathrm{H} \ + 0, 1$ jets
- $\mathrm{pp} \to \mathrm{H} \ + 2$ jets (VBF)
- $\mathrm{pp} \to$ dijets
- $\mathrm{ep} \to 1,\ 2$ jets
- $\mathrm{e}^+\mathrm{e}^- \ \to 3$ jets
- …

Part 1. Antenna Subtraction Formalism

Part 2. Example: Drell–Yan

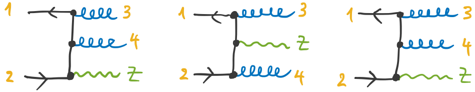
Part 3. Going beyond...

↪ Transverse Momentum Spectrum

↪ Projection-to-Born Method

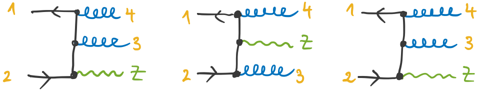
Colour Decomposition (Example: $q\bar{q} \to gg Z$)

(a)



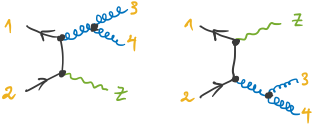
$$\left(T^{a_3}\, T^{a_4}\right)_{c_1 c_2}$$

(b)



$$\left(T^{a_4}\, T^{a_3}\right)_{c_1 c_2}$$

(c)



$$T^{a}_{c_1 c_2}\, f^{a a_3 a_4} \sim \left[T^{a_3} T^{a_4} - T^{a_4} T^{a_3}\right]_{c_1 c_2}$$

$$\Rightarrow \quad \mathcal{M}^{0}_{q\bar{q}\to ggZ} = \left(T^{a_3} T^{a_4}\right)_{c_1 c_2} \mathcal{A}_4^{0}(1_q, 3_g, 4_g, 2_{\bar{q}}, Z) \quad \longleftrightarrow \text{“(a)+(c)”}$$

$$+ \left(T^{a_4} T^{a_3}\right)_{c_1 c_2} \mathcal{A}_4^{0}(1_q, 4_g, 3_g, 2_{\bar{q}}, Z) \quad \longleftrightarrow \text{“(b)−(c)”}$$

Drell-Yan: $q\bar{q} \to$ gluons channel

$$d\sigma^{RR} \sim \frac{1}{2!}\left[\left|\mathcal{A}_4^0(1_q,3_g,4_g,2_{\bar{q}})\right|^2 + \left|\mathcal{A}_4^0(1_q,4_g,3_g,2_{\bar{q}})\right|^2 - \frac{1}{N_c^2}\left|\tilde{\mathcal{A}}_4^0(1_q,3_g,4_g,2_{\bar{q}})\right|^2 \right]$$

$$d\sigma^{RV} \sim \left|\mathcal{A}_3^1(1_q,3_g,2_{\bar{q}})\right|^2 - \frac{1}{N_c^2}\left|\tilde{\mathcal{A}}_3^1(1_q,3_g,2_{\bar{q}})\right|^2$$

$$d\sigma^{VV} \sim \left|\mathcal{A}_2^2(1_q,2_{\bar{q}})\right|^2 - \frac{1}{N_c^2}\left|\tilde{\mathcal{A}}_2^2(1_q,2_{\bar{q}})\right|^2$$

$* \; 2\,\mathrm{Re}\left\{\mathcal{A}_4^0(1_q,3_g,4_g,2_{\bar{q}})^* \mathcal{A}_4^0(1_q,4_g,3_g,2_{\bar{q}})\right\} = \left|\tilde{\mathcal{A}}_4^0(1_q,3_g,4_g,2_{\bar{q}})\right|^2 - \left|\mathcal{A}_4^0(1_q,3_g,4_g,2_{\bar{q}})\right|^2 - \left|\mathcal{A}_4^0(1_q,4_g,3_g,2_{\bar{q}})\right|^2$

$\tilde{\mathcal{A}}_4^0(1_q,3_g,4_g,2_{\bar{q}}) = \mathcal{A}_4^0(1_q,3_g,4_g,2_{\bar{q}}) + \mathcal{A}_4^0(1_q,4_g,3_g,2_{\bar{q}})$

Subtraction Term for $|\mathcal{A}_4^0(1_q, 3_g, 4_g, 2_{\bar{q}})|^2$ RR

$$+ \; d_3^0(1_q, 3_g, 4_g) \; |\mathcal{A}_3^0(\tilde{1}_q, (\widetilde{34})_g, 2_{\bar{q}})|^2$$

$$+ \; d_3^0(2_{\bar{q}}, 4_g, 3_g) \; |\mathcal{A}_3^0(1_q, (\widetilde{34})_g, \tilde{2}_{\bar{q}})|^2$$

$\hat{=}$ NLO Z+jet subtr. term $\Rightarrow$ single: 3, single: 4

Subtraction Term for $|\mathcal{A}_4^0(1_q, 3_g, 4_g, 2_{\bar{q}})|^2$ RR

$$+ \; d_3^0(1_q, 3_g, 4_g) \; |\mathcal{A}_3^0(\tilde{1}_q, \widetilde{(34)}_g, 2_{\bar{q}})|^2$$

$$+ \; d_3^0(2_{\bar{q}}, 4_g, 3_g) \; |\mathcal{A}_3^0(1_q, \widetilde{(34)}_g, \tilde{2}_{\bar{q}})|^2$$

$\hat{=}$ NLO Z+jet subtr. term $\Rightarrow$ single: 3, single: 4

$$+ \; A_4^0(1_q, 3_g, 4_g, 2_{\bar{q}}) \; |\mathcal{A}_2^0(\tilde{1}_q, \tilde{2}_{\bar{q}})|^2$$

double: 3 & 4

Subtraction Term for $|\mathcal{A}_4^0(1_q, 3_g, 4_g, 2_{\bar{q}})|^2$ LRR

$$+\, d_3^0(1_q, 3_g, 4_g)\, |\mathcal{A}_3^0(\tilde{1}_q, \widetilde{(34)}_g, 2_{\bar{q}})|^2$$

$$+\, d_3^0(2_{\bar{q}}, 4_g, 3_g)\, |\mathcal{A}_3^0(1_q, \widetilde{(34)}_g, \tilde{2}_{\bar{q}})|^2$$

$\hat{=}$ NLO Z+jet subtr. term $\Rightarrow$ single: 3, single: 4

spurious singularities — double: 3&4

$$+\, A_4^0(1_q, 3_g, 4_g, 2_{\bar{q}})\, |\mathcal{A}_2^0(\tilde{1}_q, \tilde{2}_{\bar{q}})|^2$$

spurious singularities — double: 3&4, single: 3, single: 4

$$-\, d_3^0(1_q, 3_g, 4_g)\, A_3^0(\tilde{1}_q, \widetilde{(34)}_g, 2_{\bar{q}})\, |\mathcal{A}_2^0(\tilde{\tilde{1}}_q, \tilde{2}_{\bar{q}})|^2$$

$$-\, d_3^0(2_{\bar{q}}, 4_g, 3_g)\, A_3^0(1_q, \widetilde{(34)}_g, 2_{\bar{q}})\, |\mathcal{A}_2^0(\tilde{1}_q, \tilde{\tilde{2}}_{\bar{q}})|^2$$

double: 3&4, single: 3, single: 4

DONE

Subtraction Term for $|\mathcal{A}_3^1(1_q, 3_g, 2_{\bar{q}})|^2$

IRV

$$-\left[\frac{1}{2}\Delta_3^0(s_{13}) + \frac{1}{2}\Delta_3^0(s_{23})\right]|\mathcal{A}_3^0(1_q, 3_g, 2_{\bar{q}})|^2 \quad \Bigg\} \;\hat{=}$$

NLO Z+jet subtr. term $\Rightarrow$ poles

Subtraction Term for $|\mathcal{A}_3^1(1_q, 3_g, 2_{\bar{q}})|^2$ RV

$$- \left[\frac{1}{2} \mathcal{D}_3^0(s_{13}) + \frac{1}{2} \mathcal{D}_3^0(s_{23}) \right] |\mathcal{A}_3^0(1_q, 3_g, 2_{\bar{q}})|^2$$

$\hat{=}$ NLO Z+jet subtr. term $\Rightarrow$ poles

$$+ A_3^0(1_q, 3_g, 2_{\bar{q}}) \; |\mathcal{A}_2^1(\tilde{1}_q, \tilde{2}_{\bar{q}})|^2$$

$$+ A_3^1(1_q, 3_g, 2_{\bar{q}}) \; |\mathcal{A}_2^0(\tilde{1}_q, \tilde{2}_{\bar{q}})|^2$$

(tree) x (loop)

(loop) x (tree)

single: 3

Subtraction Term for $|\mathcal{A}_3^1(1_q, 3_g, 2_{\bar{q}})|^2$ IRV

$$-\left[\frac{1}{2}\Delta_3^0(s_{13}) + \frac{1}{2}\Delta_3^0(s_{23})\right]|\mathcal{A}_3^0(1_q, 3_g, 2_{\bar{q}})|^2$$

≙ NLO Z+jet subtr. term ⇒ poles

spurious singularities

single: 3

$$+ A_3^0(1_q, 3_g, 2_{\bar{q}}) \quad |\mathcal{A}_2^1(\tilde{1}_q, \tilde{2}_{\bar{q}})|^2$$

$$+ A_3^1(1_q, 3_g, 2_{\bar{q}}) \quad |\mathcal{A}_2^0(\tilde{1}_q, \tilde{2}_{\bar{q}})|^2$$

(tree) x (loop)

(loop) x (tree)

spurious singularities

single: 3

poles

$$+\left[\frac{1}{2}\Delta_3^0(s_{13}) + \frac{1}{2}\Delta_3^0(s_{23})\right] A_3^0(1_q, 3_g, 2_{\bar{q}}) \, |\mathcal{A}_2^0(\tilde{1}_q, \tilde{2}_{\bar{q}})|^2$$

poles

single: 3

+ MF

DONE

Subtraction Term for $|\mathcal{A}_2^2(1_q, 2_{\bar{q}})|^2$

LVV

$$-\mathcal{A}_4^0(s_{12})\ |\mathcal{A}_2^0(1_q,2_{\bar{q}})|^2$$

} from A_4^0 @ RR

$$-\mathcal{A}_3^0(s_{12})\ |\mathcal{A}_2^1(1_q,2_{\bar{q}})|^2$$

$$-\mathcal{A}_3^1(s_{12})\ |\mathcal{A}_2^0(1_q,2_{\bar{q}})|^2$$

} (tree)x(loop) / (loop)x(tree) @ RV

} poles

+ MF

DONE

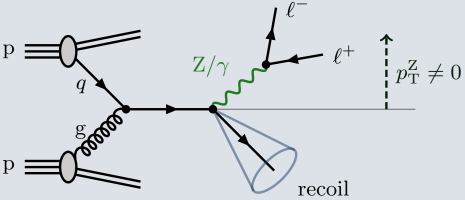
Part 1. Antenna Subtraction Formalism

Part 2. Example: Drell–Yan

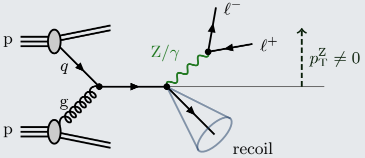
Part 3. Going beyond...

- ↪ Transverse Momentum Spectrum
- ↪ Projection-to-Born Method

Transverse Momentum Spectrum



Inclusive p_{T} spectrum from $X + \mathrm{jet}$



$\mathrm{p\,p} \;\to\; X + $ recoil

- fully inclusive in QCD emissions
- require recoil: $p_{\mathrm{T}}^{X} > p_{\mathrm{T,\,cut}}^{X}$

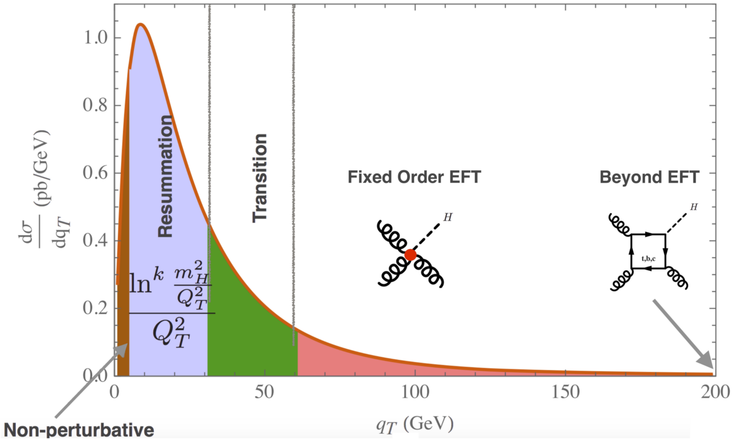
$\Rightarrow$ can use $X + \mathrm{jet}$ calculation

$X + \mathrm{jet}$ @ NNLO

- $\mathbf{H + jet}$ .... (Antenna, N-jettiness, Sector-improved R.S.)
- $\mathbf{W + jet}$ .... (Antenna, N-jettiness)
- $\mathbf{Z + jet}$ .... (Antenna, N-jettiness)
- $\boldsymbol{\gamma} + \mathbf{jet}$ .... (N-jettiness)

$\rightsquigarrow$ validation & opportunity for benchmarks

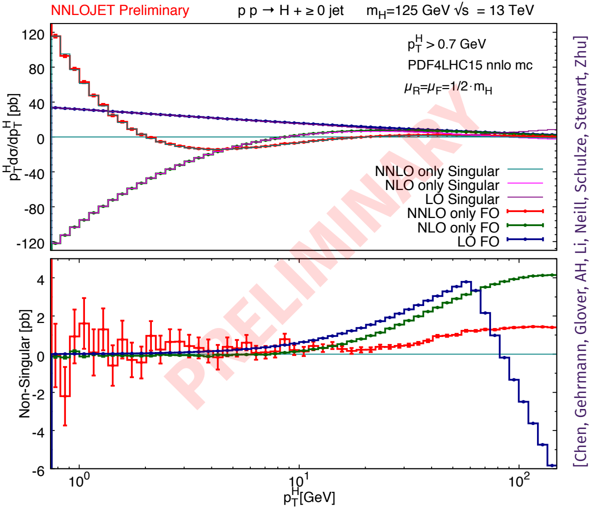
The p_{T} spectrum



- fixed-order prediction *diverges* for $p_{\mathrm{T}} \to 0$
- large logarithms: $\ln^k(p_{\mathrm{T}}/M)/p_{\mathrm{T}}^2 \quad \rightsquigarrow$ all-order resummation needed!

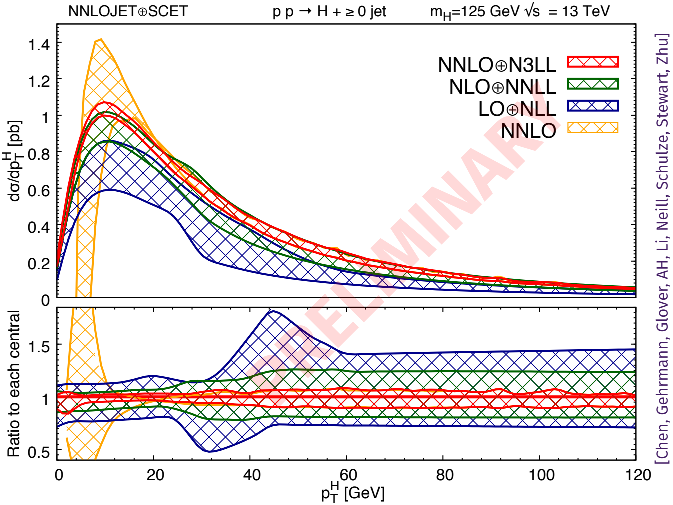
$\Rightarrow$ matching: $\quad \mathrm{d}\sigma_{\text{matched}} = \mathrm{d}\sigma_{\text{f.o.}} + \mathrm{d}\sigma_{\text{res.}} - \mathrm{d}\sigma_{\text{res.}}\big|_{\text{exp.}}$

Compare the logs — fixed-order vs. resummation



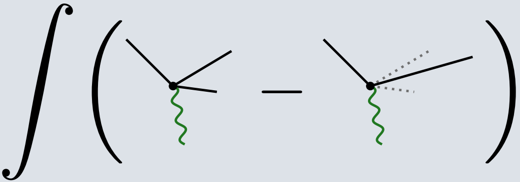
- excellent agreement within stat. errors $\sim 1\%$
- predictions down to $p_{\mathrm{T}}^{\mathrm{H}} = 0.7\ \mathrm{GeV}$
- important *cross check* $\rightsquigarrow$ matching of NNLO and N^{3}LL

Matched p_T spectrum

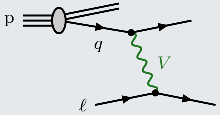


- NNLO & NNLO ⊕ N^{3}LL start to deviate @ $p_T \lesssim 30$ GeV
- reduction of uncertainties by more than a factor of two
- NLO ⊕ NLL $\longrightarrow$ NNLO ⊕ N^{3}LL: large impact in peak region

Projection-to-Born Method

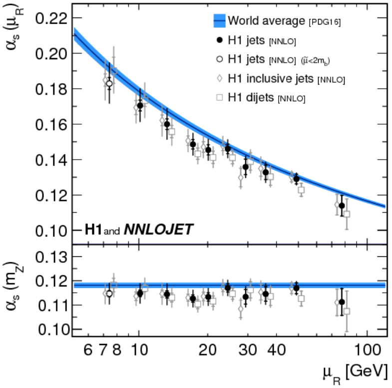


Deep Inelastic Scattering



precise probe to resolve the inner structure of the proton

- PDF constraints
- α_s extraction (+ running)



[Eur.Phys.J. C77 (2017) no.11, 791]

- DIS 2 jet @ NNLO

  [Currie, Gehrmann, Niehues '16]

  [Currie, Gehrmann, AH, Niehues '17]

$\Leftarrow$ precise α_s determination

- DIS 1 jet @ N^{3}LO

  [Currie, Gehrmann, Glover, AH, Niehues, Vogt. '18]

Fully Differential N^3LO

DIS 2 jet

@ NNLO

[Currie, Gehrmann, Niehues '16]
[Currie, Gehrmann, AH, Niehues '17]

Projection-to-Born

$\oplus$

[Cacciari, et al. '15]

DIS structure function

@ N^3LO

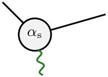
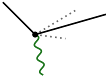
[Moch, Vermaseren, Vogt '05]

$=$

DIS fully differential @ N^3LO

Projection-to-Born

inclusive DIS (structure function)

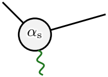
$\sigma^{\text{incl.}}_{\text{NLO}} =$  $+ \int_{1,\,\text{incl.}}$ 

Born kinematics: $Q^2 = -q^2 > 0, \quad x = \dfrac{-Q^2}{2P\cdot q}$

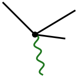
Projection-to-Born

differential DIS (w/o IR treatment)

$$\sigma^{\text{diff.}}_{\text{NLO}} =$$



$$+ \int_{1,\,\text{diff.}}$$

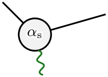


Born kinematics: $Q^2 = -q^2 > 0, \quad x = \frac{-Q^2}{2P \cdot q}$

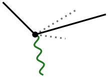
Projection-to-Born

differential DIS (w/ IR treatment)

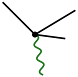
$\sigma_{\mathrm{NLO}}^{\mathrm{diff.}} =$



$+ \int_{1,\,\mathrm{incl.}}$



$+ \int_{1,\,\mathrm{diff.}} \Bigg($



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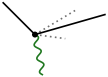
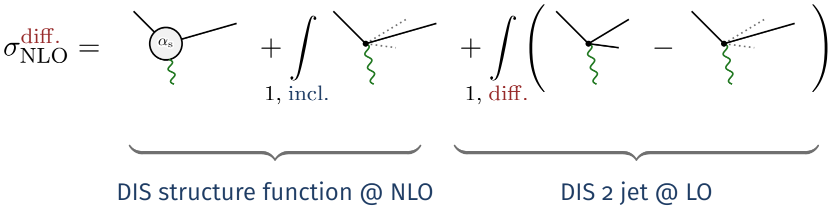


$\Bigg)$

Born kinematics: $Q^2 = -q^2 > 0, \quad x = \dfrac{-Q^2}{2P \cdot q}$

Projection-to-Born

differential DIS (w/ IR treatment)

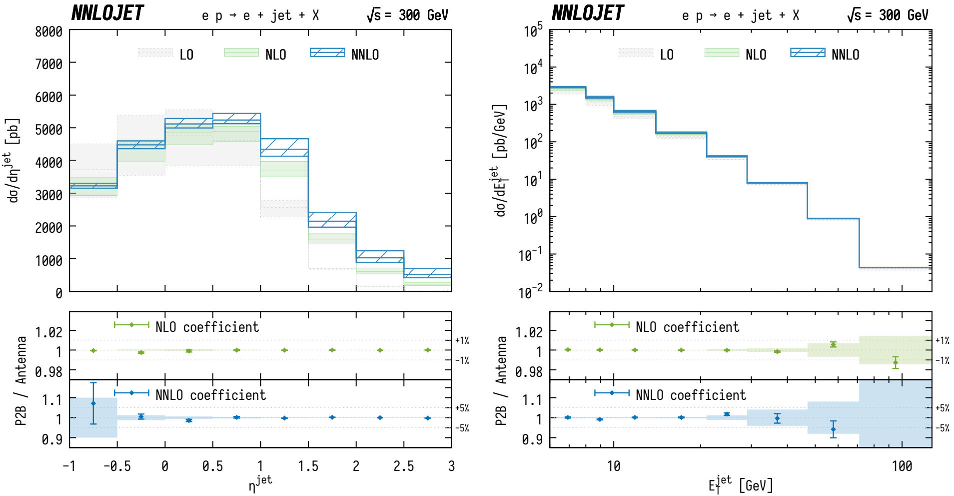


only “special” processes *but* not restricted to any order

$$\left.\begin{array}{lll} & \text{inclusive } X & @\ \text{N}^{n}\text{LO} \\ + & X + \text{jet} & @\ \text{N}^{n-1}\text{LO} \end{array}\right\} \rightsquigarrow \quad X\ @\ \text{N}^{n}\text{LO}$$

Born kinematics: $Q^2 = -q^2 > 0, \quad x = \dfrac{-Q^2}{2P \cdot q}$

Validation up to NNLO — Antenna vs. P2B

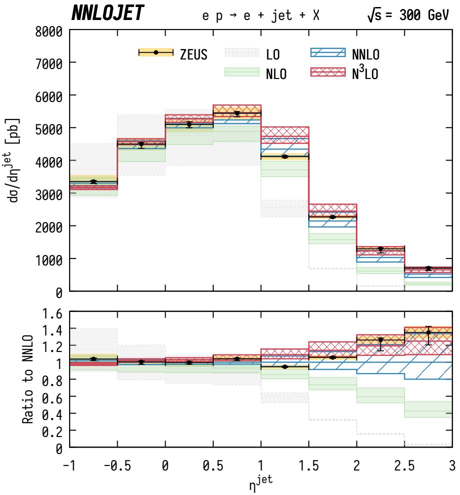


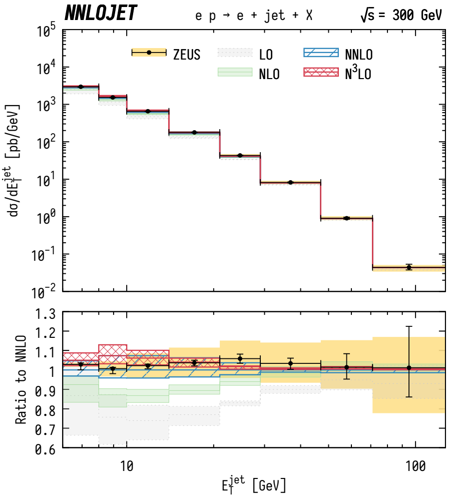
NLO *coefficient*: $\lesssim$ 5‰

NNLO *coefficient*: $\lesssim$ 2%

⇝ agreement @ full NNLO: $\lesssim$ 1‰

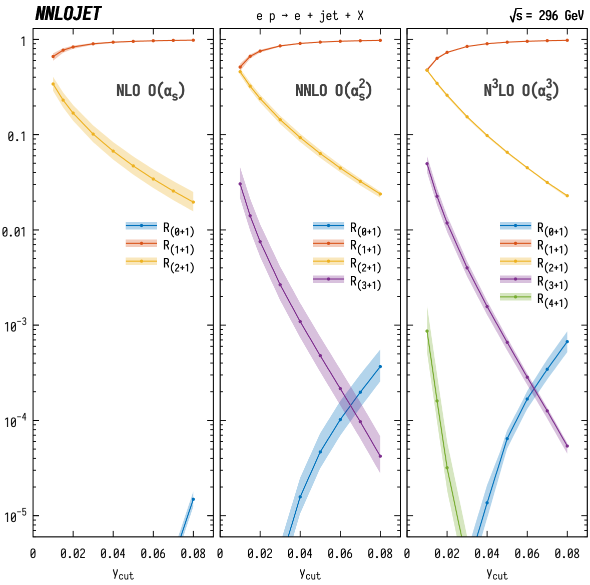
Differential distributions at N^{3}LO





- for the first time: *overlapping* scale bands agreement with data
- reduction of scale uncertainties

Jet Rates



Jet rates:

$$R_{(n+1)} = N_{(n+1)} / N_{\text{tot}}$$

JADE algorithm

$\hookrightarrow$ cluster partons if:

$$\frac{2E_iE_j(1-\cos\theta_{ij})}{W^2} < y_{\text{cut}}$$

Projection-to-Born — an "antenna" view

Consider the real-emission subtraction in the antenna subtraction formalism for $\mathrm{H}+0\mathrm{jet}$ (@ LC):

$$\int \left\{ \mathrm{d}\sigma^{\mathrm{R}}_{\mathrm{H}+0\mathrm{jet}} - \mathrm{d}\sigma^{\mathrm{SNLO}}_{\mathrm{H}+0\mathrm{jet}} \right\}$$

$$= \int \mathrm{d}\Phi_{\mathrm{H}+1} \Big\{ \texttt{A3g0H}(1_{\mathrm{g}}, 2_{\mathrm{g}}, 3_{\mathrm{g}}, \mathrm{H})\, \mathcal{J}(\Phi_{\mathrm{H}+1})$$

$$- F_3^0(1_{\mathrm{g}}, 2_{\mathrm{g}}, 3_{\mathrm{g}})\, \texttt{A2g0H}(\tilde{1}_{\mathrm{g}}, \tilde{2}_{\mathrm{g}}, \mathrm{H})\, \mathcal{J}(\widetilde{\Phi}_{\mathrm{H}+0}) \Big\}$$

Projection-to-Born — an "antenna" view

Consider the real-emission subtraction in the antenna subtraction formalism for $\mathrm{H}+0\mathrm{jet}$ (@ LC):

$$\int \left\{ \mathrm{d}\sigma^{\mathrm{R}}_{\mathrm{H+0jet}} - \mathrm{d}\sigma^{\mathrm{SNLO}}_{\mathrm{H+0jet}} \right\}$$

$$= \int \mathrm{d}\Phi_{\mathrm{H+1}} \Big\{ \mathtt{A3g0H}(1_{\mathrm{g}}, 2_{\mathrm{g}}, 3_{\mathrm{g}}, \mathrm{H})\; \mathcal{J}(\Phi_{\mathrm{H+1}})$$

$$- F_3^0(1_{\mathrm{g}}, 2_{\mathrm{g}}, 3_{\mathrm{g}})\; \mathtt{A2g0H}(\tilde{1}_{\mathrm{g}}, \tilde{2}_{\mathrm{g}}, \mathrm{H})\; \mathcal{J}(\widetilde{\Phi}_{\mathrm{H+0}}) \Big\}$$

Antennae = ratios of *physical* Matrix Elements:

$$F_3^0(i_{\mathrm{g}}, j_{\mathrm{g}}, k_{\mathrm{g}}) \equiv \frac{\mathtt{A3g0H}(i_{\mathrm{g}}, j_{\mathrm{g}}, k_{\mathrm{g}}, \mathrm{H})}{\mathtt{A2g0H}(\tilde{i}_{\mathrm{g}}, \tilde{k}_{\mathrm{g}}, \mathrm{H})}$$

Projection-to-Born — an "antenna" view

Consider the real-emission subtraction in the antenna subtraction formalism for $\mathrm{H}+0\mathrm{jet}$ (@ LC):

$$\int \left\{ \mathrm{d}\sigma^{\mathrm{R}}_{\mathrm{H+0jet}} - \mathrm{d}\sigma^{\mathrm{SNLO}}_{\mathrm{H+0jet}} \right\}$$

$$= \int \mathrm{d}\Phi_{\mathrm{H+1}} \Big\{ \mathtt{A3g0H}(1_{\mathrm{g}}, 2_{\mathrm{g}}, 3_{\mathrm{g}}, \mathrm{H})\, \mathcal{J}(\Phi_{\mathrm{H+1}}) - F_3^0(1_{\mathrm{g}}, 2_{\mathrm{g}}, 3_{\mathrm{g}})\, \mathtt{A2g0H}(\tilde{1}_{\mathrm{g}}, \tilde{2}_{\mathrm{g}}, \mathrm{H})\, \mathcal{J}(\widetilde{\Phi}_{\mathrm{H+0}}) \Big\}$$

$$= \int \mathrm{d}\Phi_{\mathrm{H+1}}\, \mathtt{A3g0H}(1_{\mathrm{g}}, 2_{\mathrm{g}}, 3_{\mathrm{g}}, \mathrm{H}) \left\{ \mathcal{J}(\Phi_{\mathrm{H+1}}) - \mathcal{J}(\widetilde{\Phi}_{\mathrm{H+0}}) \right\}$$

Projection-to-Born — an "antenna" view

Consider the real-emission subtraction in the antenna subtraction formalism for $\mathrm{H}+0\mathrm{jet}$ (@ LC):

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$$= \int \mathrm{d}\Phi_{\mathrm{H+1}} \Big\{ \texttt{A3g0H}(1_{\mathrm{g}}, 2_{\mathrm{g}}, 3_{\mathrm{g}}, \mathrm{H})\, \mathcal{J}(\Phi_{\mathrm{H+1}})$$

$$- F_3^0(1_{\mathrm{g}}, 2_{\mathrm{g}}, 3_{\mathrm{g}})\, \texttt{A2g0H}(\tilde{1}_{\mathrm{g}}, \tilde{2}_{\mathrm{g}}, \mathrm{H})\, \mathcal{J}(\widetilde{\Phi}_{\mathrm{H+0}}) \Big\}$$

$$= \int \mathrm{d}\Phi_{\mathrm{H+1}}\, \texttt{A3g0H}(1_{\mathrm{g}}, 2_{\mathrm{g}}, 3_{\mathrm{g}}, \mathrm{H}) \left\{ \mathcal{J}(\Phi_{\mathrm{H+1}}) - \mathcal{J}(\widetilde{\Phi}_{\mathrm{H+0}}) \right\}$$

$\Rightarrow$ Simple processes where antenna $\simeq$ real-emission Matrix Element
$\leftrightsquigarrow$ Projection-to-Born

Similarly at NNLO: X_4^0 & $X_3^0 \times X_3^0$ are "projections" of RR ME & NLO(+jet) subtraction term.

Summary & Outlook — tl;dr

- Antenna subtraction successfully applied to many important processes:
  - $\hookrightarrow$ $\mathrm{pp} \to X \ + 0, 1$ jets $\quad (X = \mathrm{H},\ \mathrm{Z},\ \mathrm{W})$
  - $\hookrightarrow$ $\mathrm{pp} \to \mathrm{H} \ + 2$ jets (VBF)
  - $\hookrightarrow$ $\mathrm{pp} \to$ dijets $\quad (N_c^2,\ N_c N_F,\ N_F^2)$
  - $\hookrightarrow$ $\mathrm{ep} \to 1,\ 2$ jets
  - $\hookrightarrow$ $\mathrm{e^+e^-} \to 3$ jets

$\Rightarrow$ subtraction set up for: $\mathrm{pp} \to$ "colour neutral" $+ 0, 1, 2$ jets

- inclusive p_T^H spectrum: NNLO prediction matched to N^{3}LL
  - $\hookrightarrow$ fixed order: stable predictions down to $p_\mathrm{T}^\mathrm{H} = 0.7\ \mathrm{GeV}$
  - $\rightsquigarrow$ is it good enough for q_T-subtraction @ N^{3}LO?!

- Projection-to-Born method $\oplus$ Antenna subtraction
  - $\hookrightarrow$ first fully differential N^{3}LO prediction: inclusive jets in DIS
  - $\hookrightarrow$ method also applicable for colour-neutral final states pp in collisions

Thank you

Backup Slides

Antenna subtraction @ NLO

[J. Currie , E.W.N. Glover, S. Wells '13]

$\mathrm{d}\hat{\sigma}^T$:

$$\mathrm{d}\hat{\sigma}^T \sim \boldsymbol{J}_n^{(1)} M_n^0$$

$\mathrm{d}\hat{\sigma}^S$:

$$\mathrm{d}\hat{\sigma}^S \sim X_3^0 M_n^0$$

Antenna subtraction @ NNLO

[J. Currie , E.W.N. Glover, S. Wells ’13]