BERNHARD MISTLBERGER

DOUBLE DIFFERENTIAL PREDICTIONS FOR HIGGS BOSON PRODUCTION

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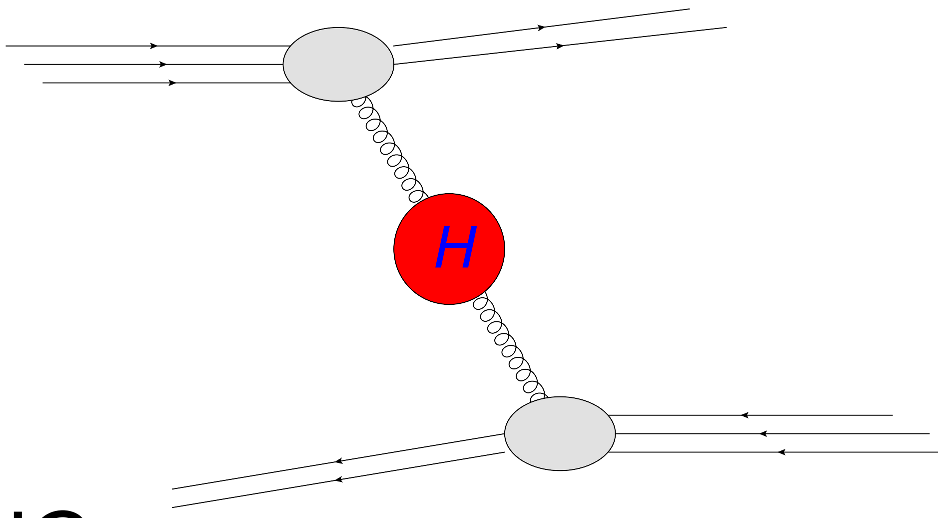
DEMAND FOR PRECISION ON THEORY SIDE



- Testing our understanding of nature: Compare experiments and theory!

- The key to theoretical predictions at the LHC:

$$\sigma \sim \int dxdy f(x) f(y) \hat{\sigma} + \mathcal{O}\left(\frac{\Lambda}{Q}\right)$$

$$\hat{\sigma} = \hat{\sigma}^{(0)} + \alpha_S^1 \hat{\sigma}^{(0)} + \alpha_S^2 \hat{\sigma}^{(2)} + \alpha_S^3 \hat{\sigma}^{(3)} \ldots$$

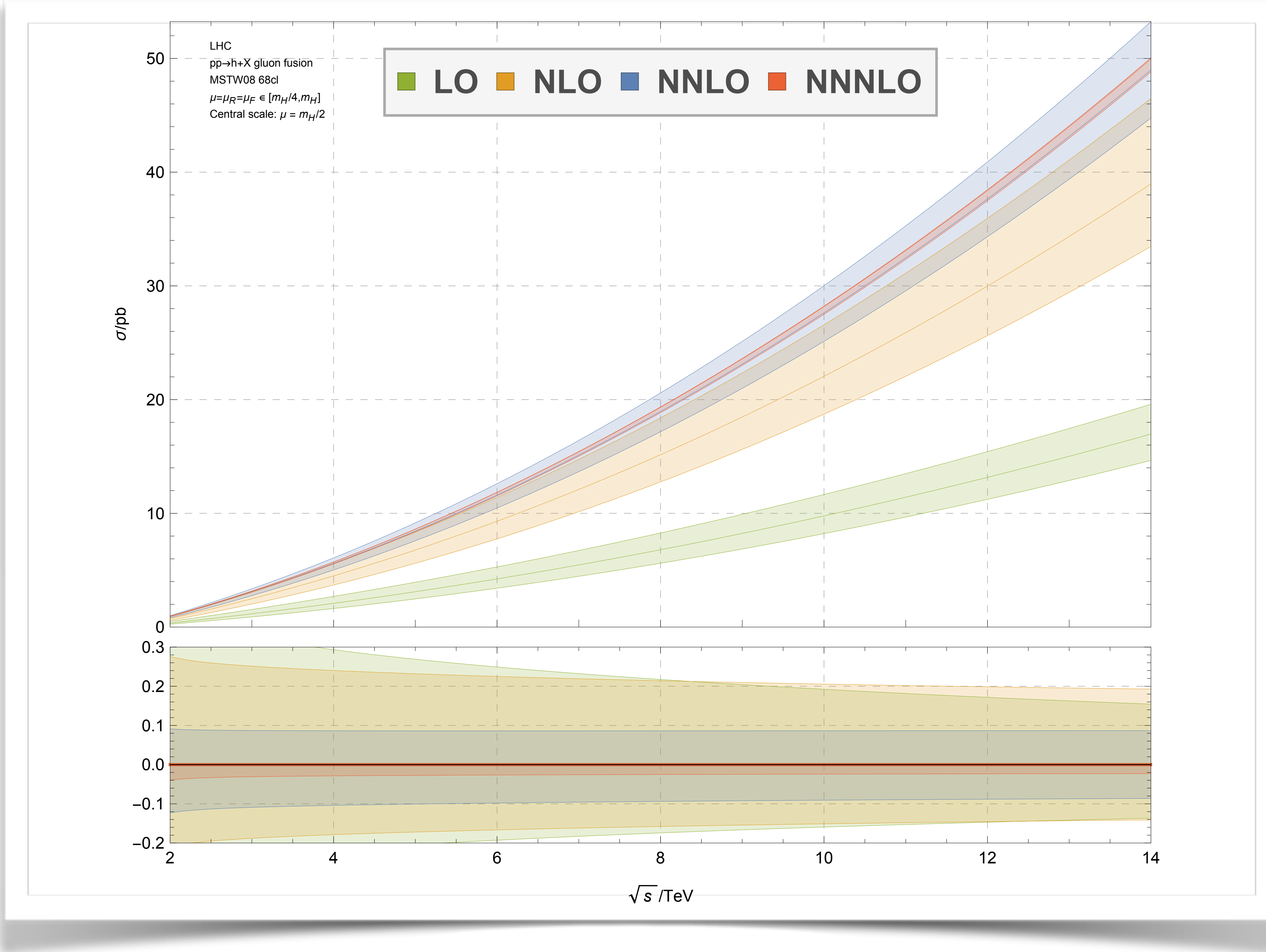
- Compute perturbative corrections from first principle QFT: Standard Model

- Allows for % - level predictions for - experimental precision will reach comparable levels!

THE HIGGS BOSON PRODUCTION CROSS SECTION

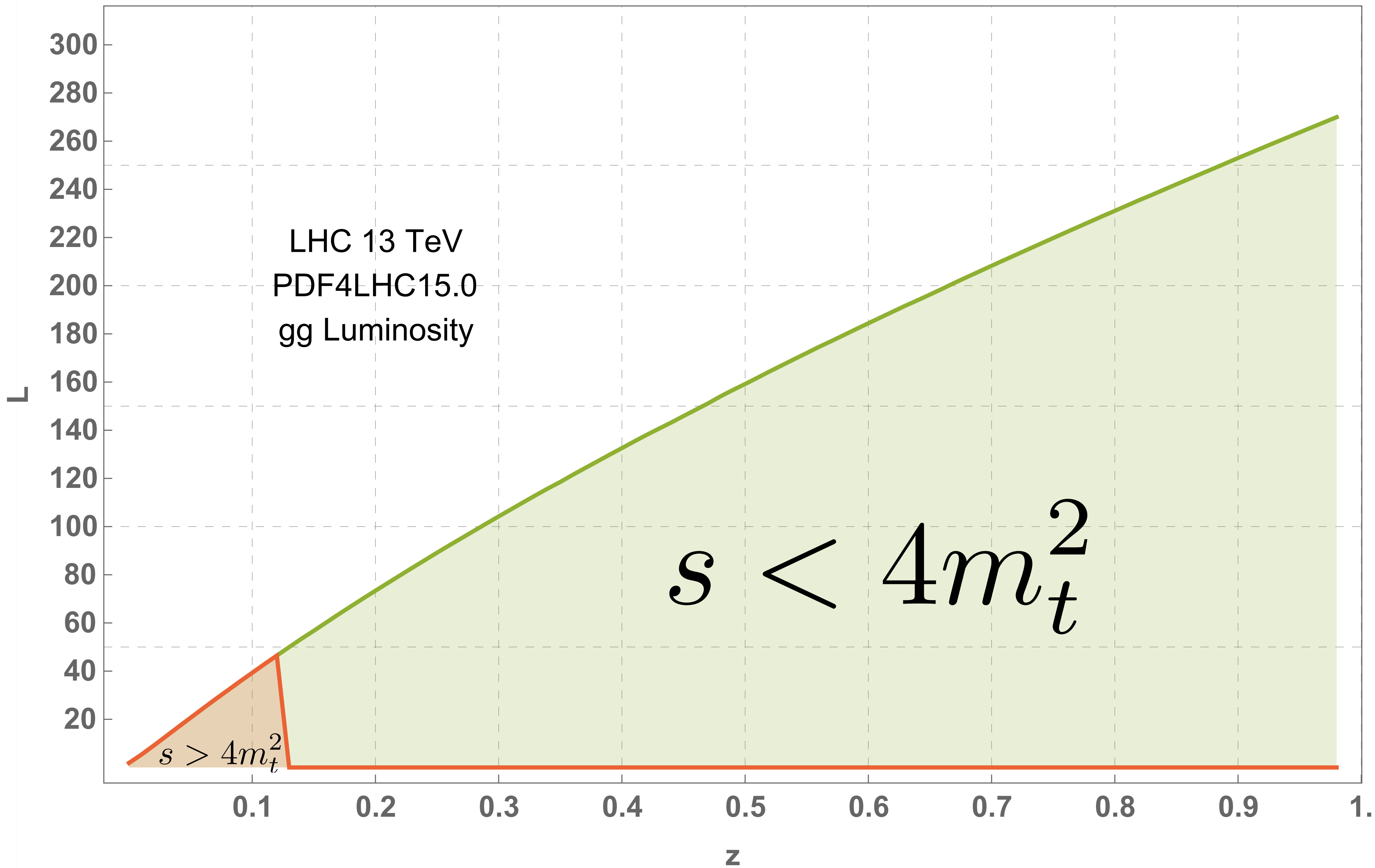
- Can be measured fairly precisely at the LHC! Lots of data still to come!

- Gluon Fusion!
- Suffers from large perturbative corrections.
- N3LO QCD corrections seem to stabilise the perturbative expansion



THE HIGGS BOSON PRODUCTION CROSS SECTION

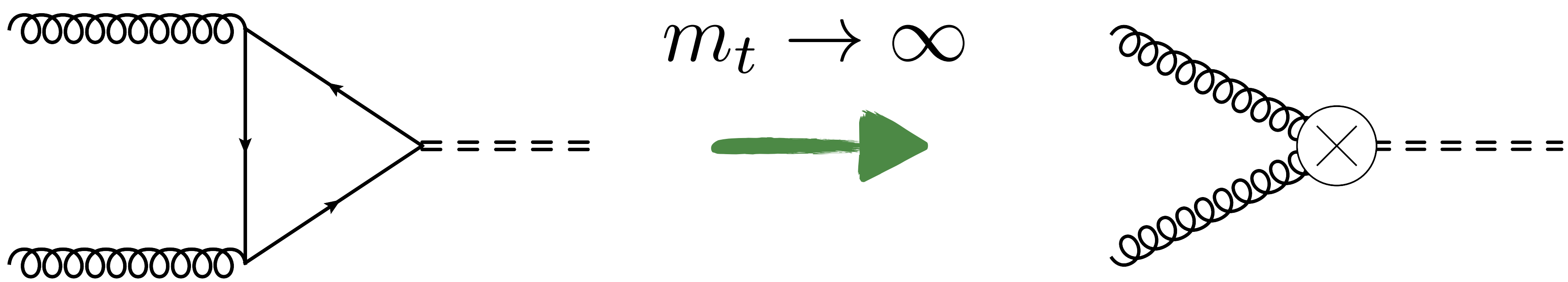
Probability to get two gluons out of the proton as a function of the partonic centre of mass energy:



COMPUTING HIGH ORDERS IS CHALLENGING

Simplifications:

- Work in an EFT



- Removes one loop!

- Excellent approximation: Captures dominant QCD effects.

$$\delta_t^{\mathrm{LO}} \sim 7\%$$

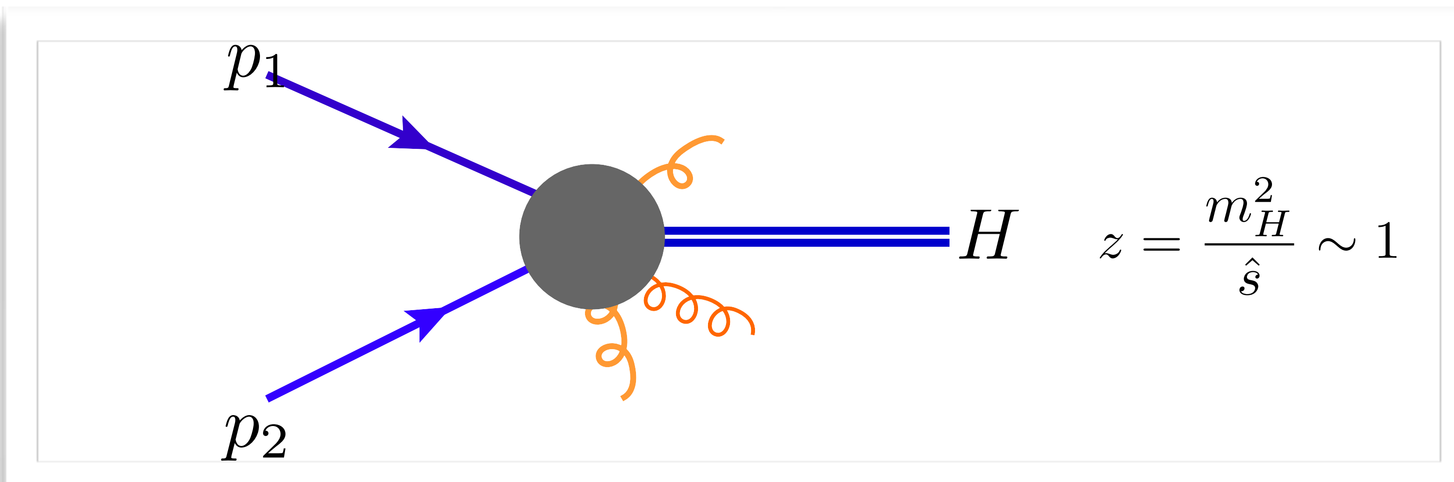
$$\delta_t^{\mathrm{NLO}} \sim 0.7\%$$

- Supplement with mass corrections, EWK corrections , etc.

COMPUTING HIGH ORDERS IS CHALLENGING

Simplifications:

- Perform expansion around kinematic limit: **Production Threshold**

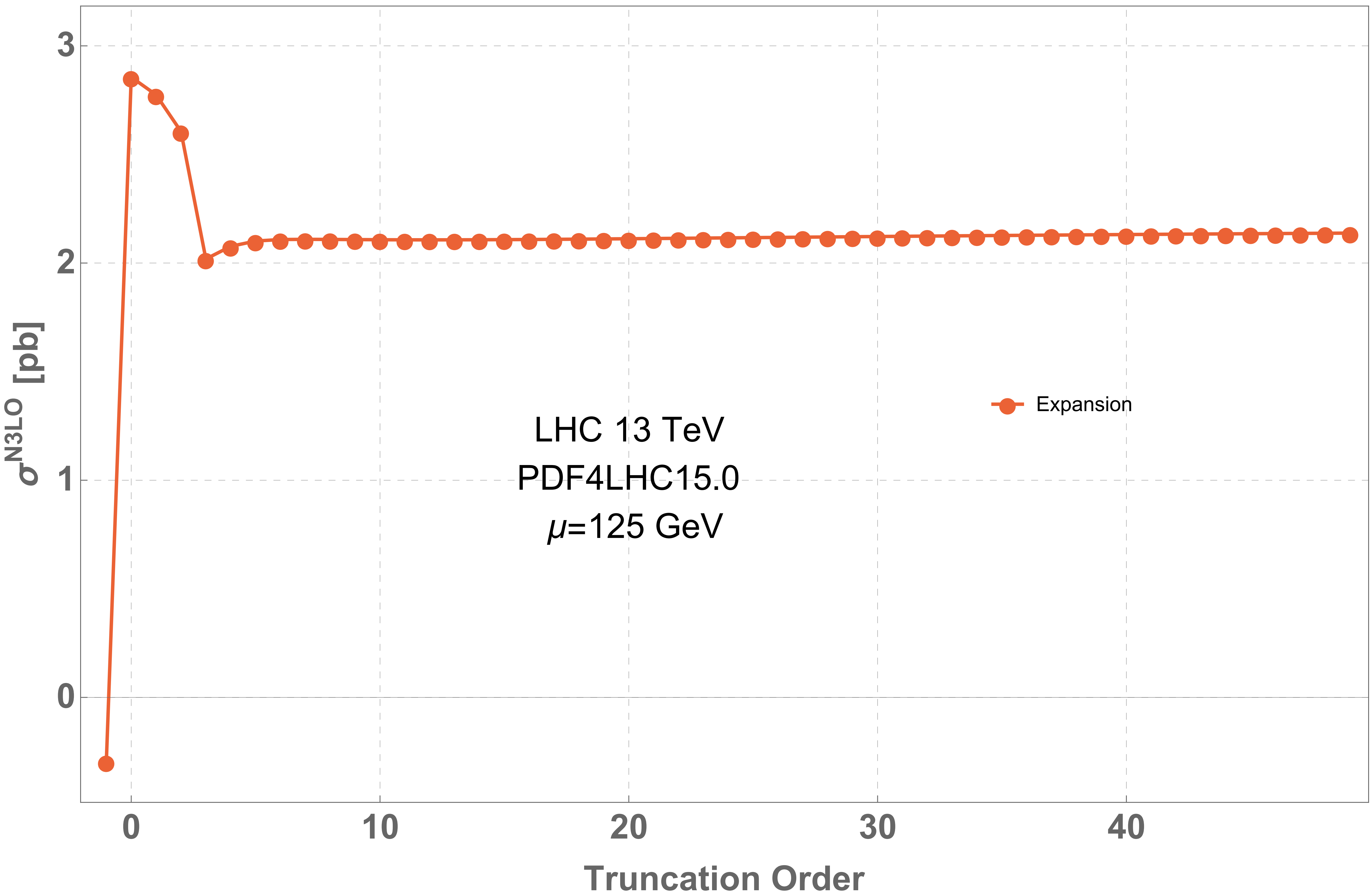


$$\bar{z} = 1 - z \quad \Rightarrow \quad \hat{\sigma}(\bar{z}) = \sigma^{SV} + \sigma^{(0)} + \bar{z}\sigma^{(1)} + \ldots$$

- Simplifies the analytic functions: Only numbers!

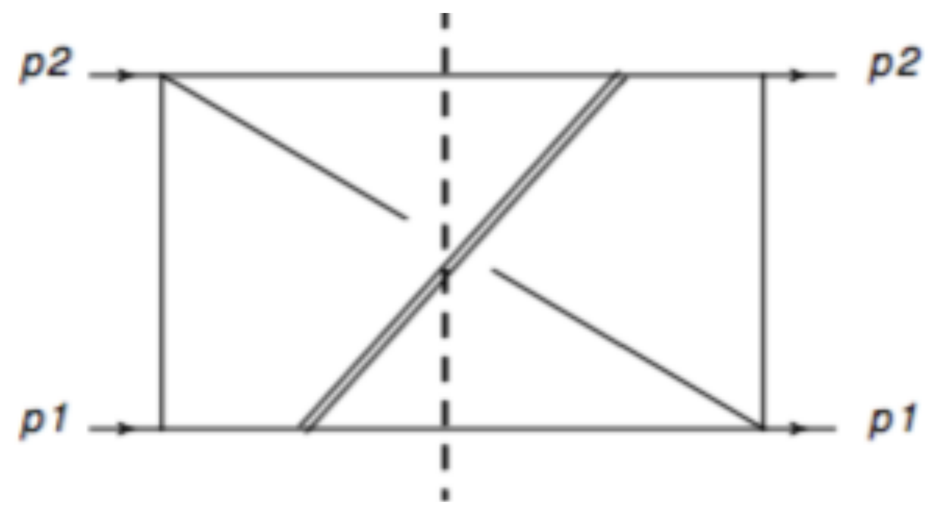
- Expand to sufficiently high order to ensure stable results.

THRESHOLD EXPANSION



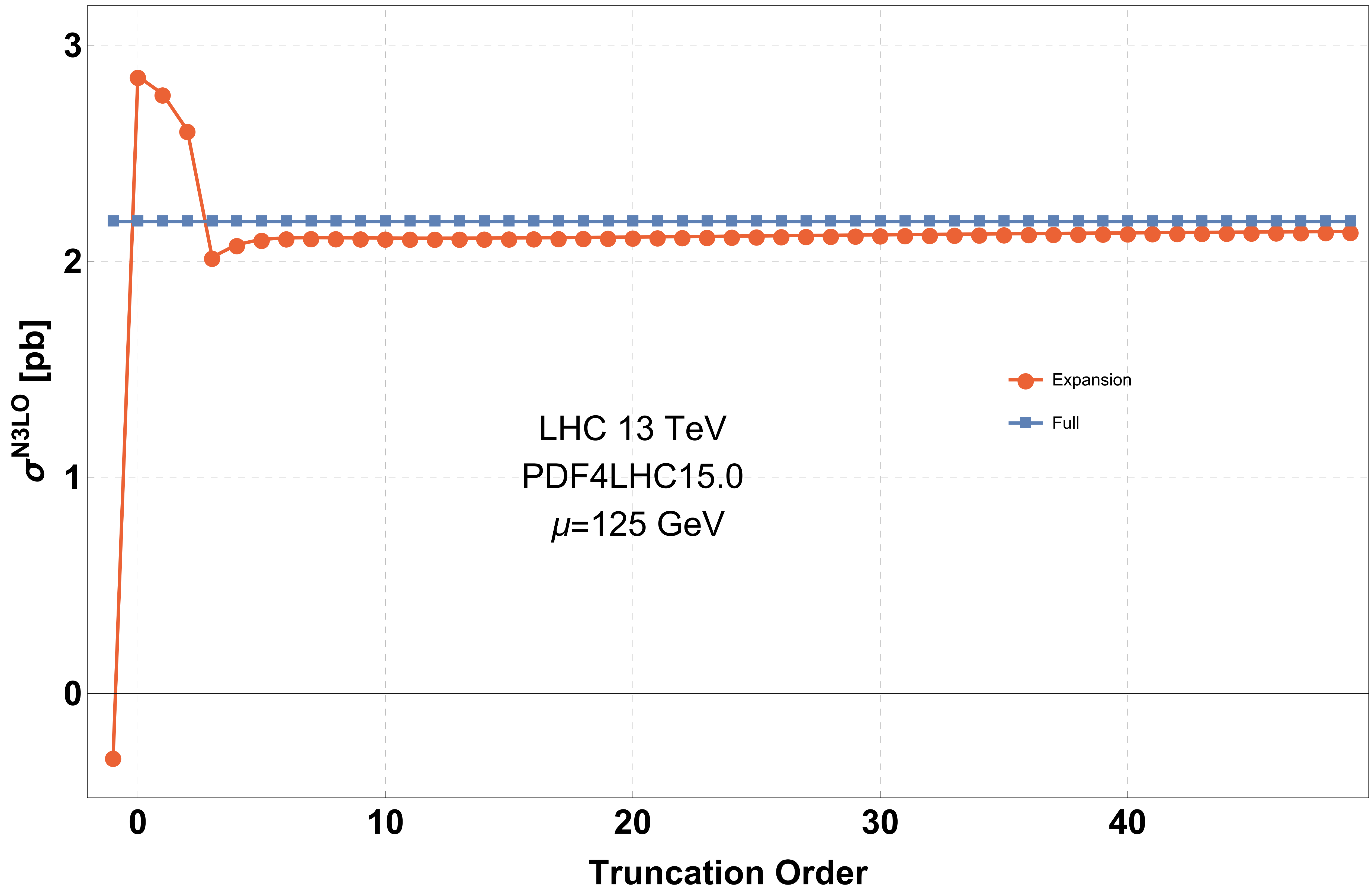
NEW: EXACT SOLUTION FOR GLUON FUSION @ N3LO

- Major analytic effort: 912 master integrals!
- Challenge of new analytic functions: Elliptic integrals!
- Elliptic integrals:
  We are at the beginning of understanding these functions!
  Analytic continuation, numerical evaluation, functional identities, …

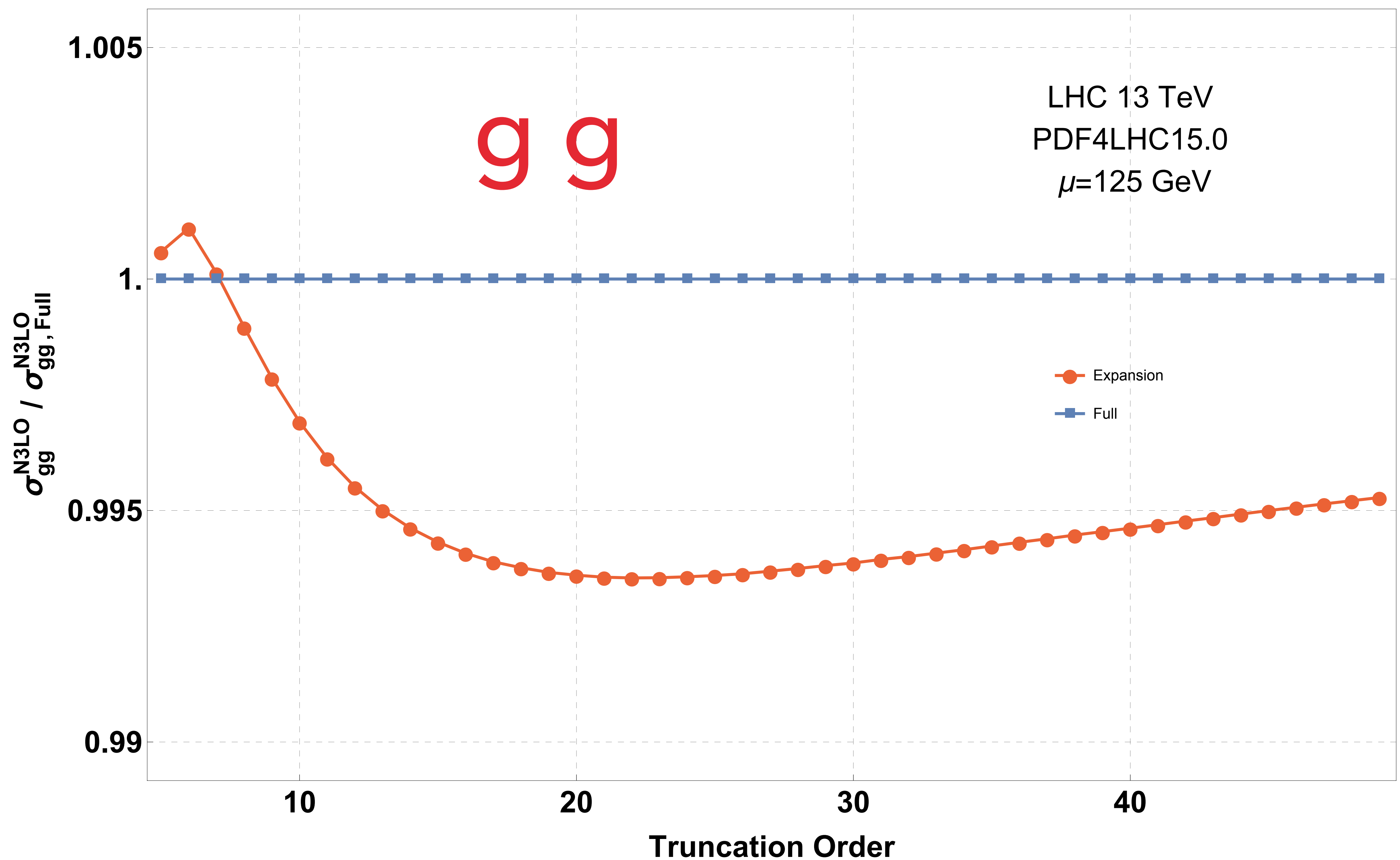


$$\text{Leading Singularity }(E_4) \sim \int dx \frac{\theta\left((x-z)\left(x^3-x^2z+2x^2+2xz+x-z\right)\right)}{\sqrt{(x-z)\left(x^3-x^2z+2x^2+2xz+x-z\right)}}.$$

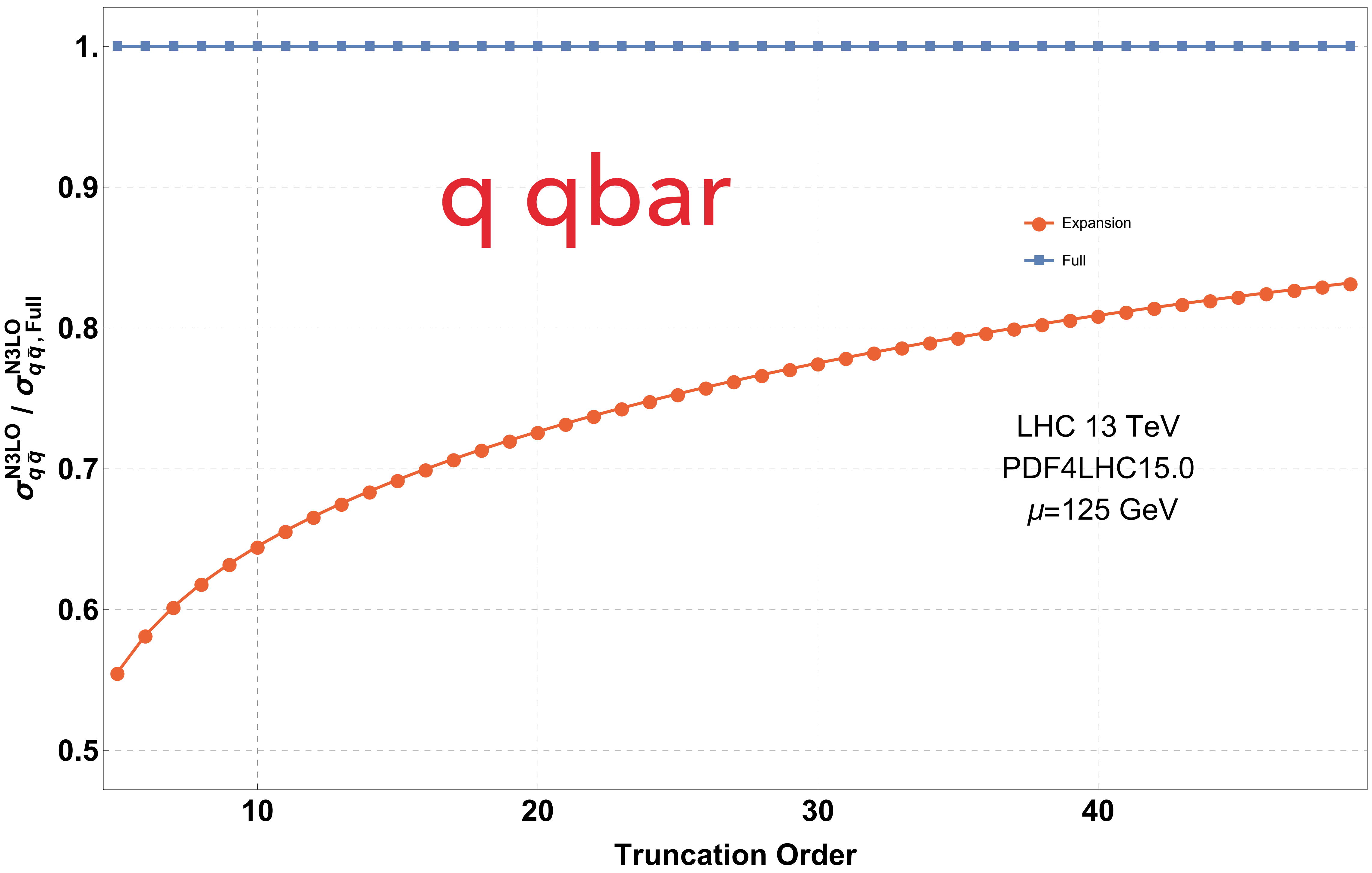
EXPANDED VS. EXACT



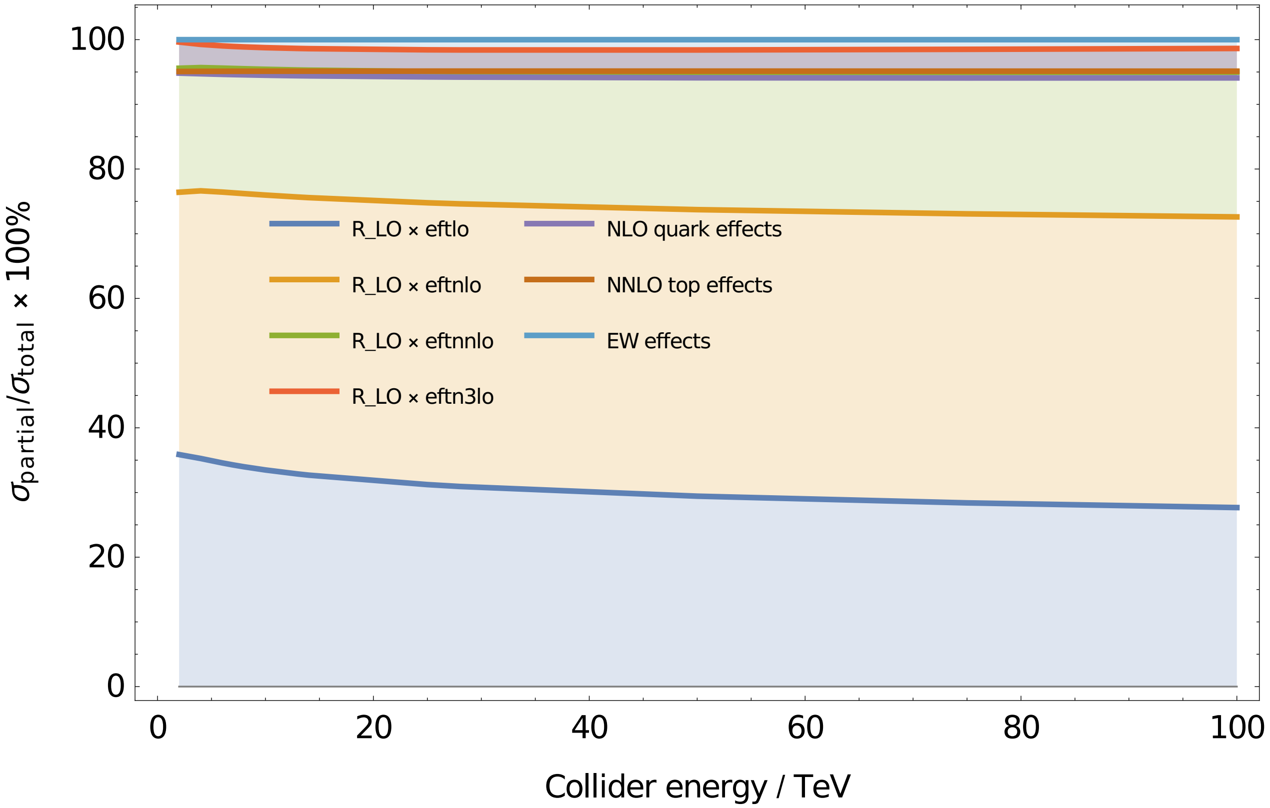
INDIVIDUAL INITIAL STATE CHANNELS



INDIVIDUAL INITIAL STATE CHANNELS

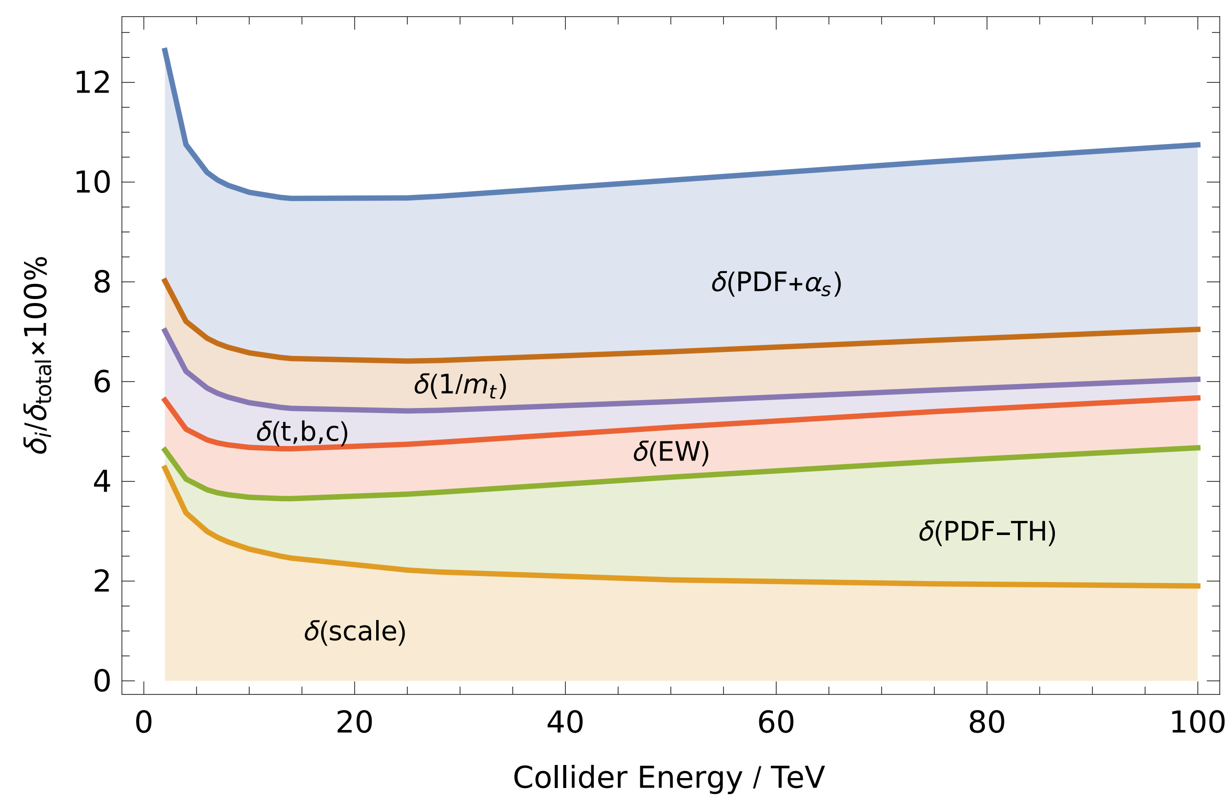


PHENOMENOLOGY: IT'S COMPLICATED

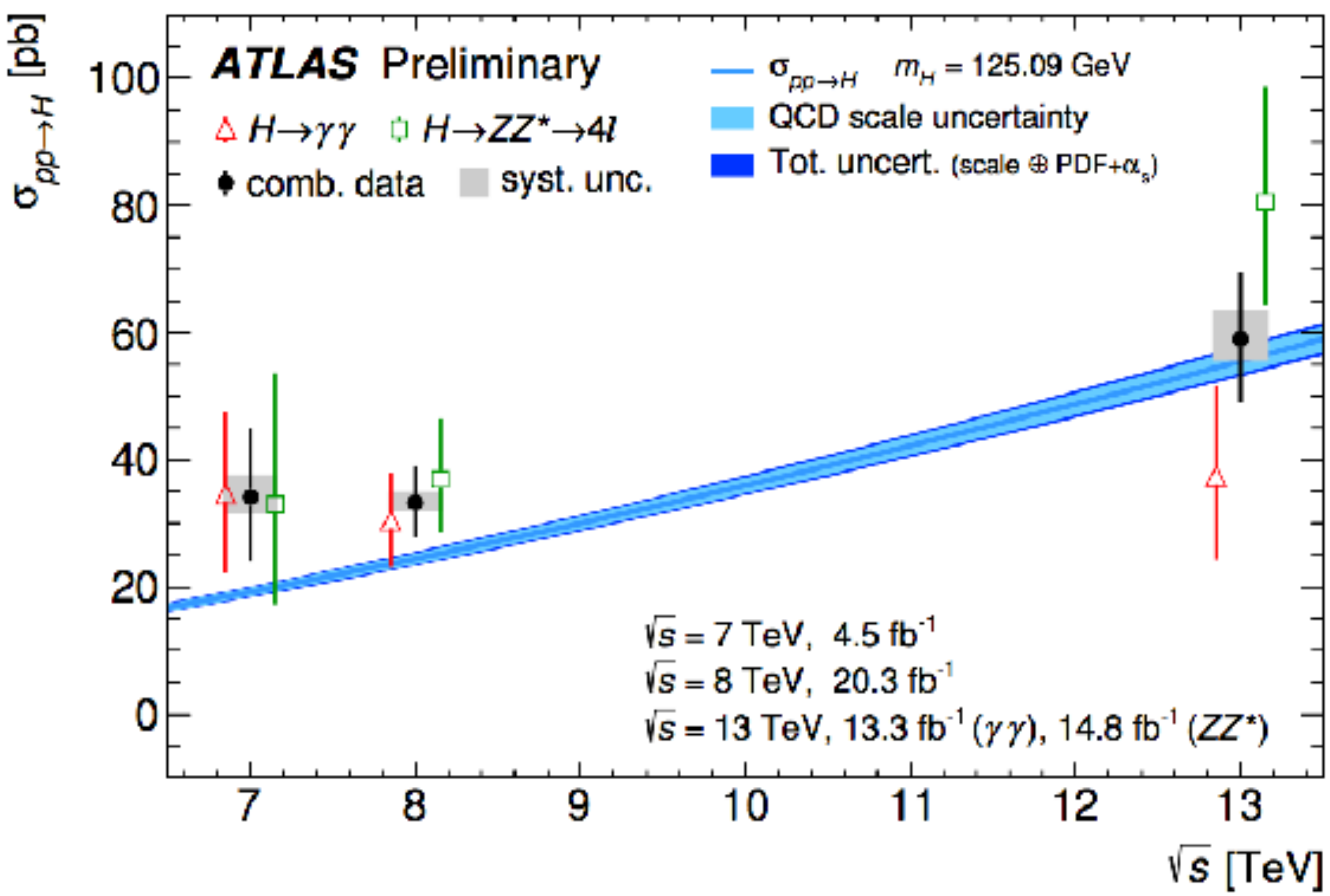


- Many contributions to be take into account: QCD, EWK, m_t, m_b, m_c …

- Many sources of uncertainty to estimate! Perturbative truncation, PDF, α_S …

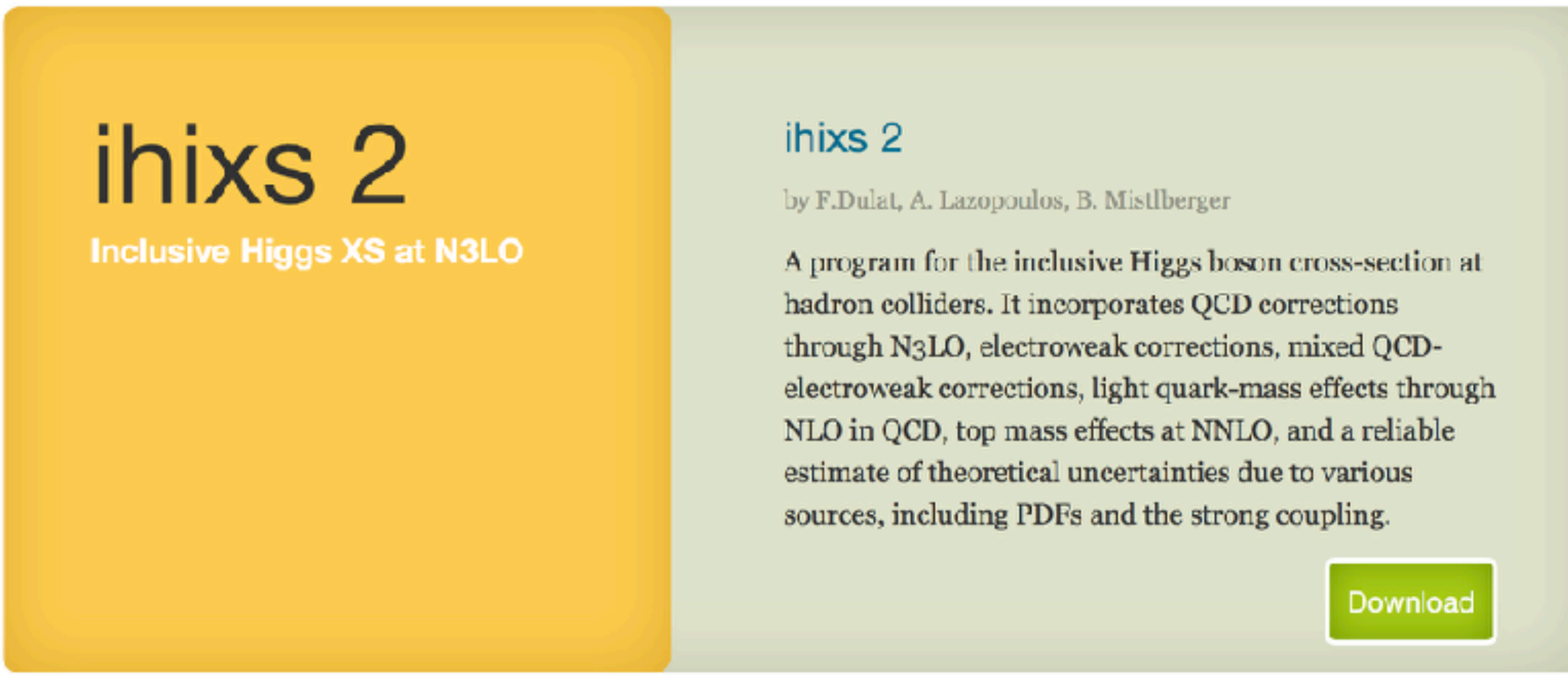


THE AGE OF PRECISION HIGGS PHYSICS



- Incredible agreement of data and theory
- Triumph of SM predictions
- Higgs production ~10 sigma observed

IHIXS 2 – NEW CODE!



https://people.phys.ethz.ch/~pheno/ihixs/

Differential Cross Sections

- Inclusive cross sections are idealised objects
  Important test of QFT, extraction of coupling constants, etc.
- Real life observables:
  Fiducial cross sections for realistic final states!
- Avoid extrapolation:
  Predict as close to experimental outcome as possible

CHALLENGES OF DIFFERENTIAL PREDICTIONS

- Analytic complexity of high order perturbative computation
  - Complicated mathematical structures: Elliptic / multiple polylogarithms, couple differential equations, algebraic complexity, …
- Numerical integration over complicated and “divergent” final state configurations:
  - Infrared subtraction at 2-loops and beyond.
  - Main challenge of the last couple of years.
  - Many methods available now.

- **Sector decomposition**
- **Non-Linear Mappings**
- **qT**
- **FKS+**
- **N-Jettiness** — H+J
- **Antenna**
- **Colourful**
- **Projection-To-Born** — VBF
- **…**

CHALLENGES OF DIFFERENTIAL PREDICTIONS

- Analytic complexity of high order perturbative computation
    - Complicated mathematical structures: Elliptic / multiple polylogarithms, couple differential equations, algebraic complexity, …
- Numerical integration over complicated and "divergent" final state configurations:

$2 \to 1$
Inclusive cross sections - analytic formulae: ~ seconds

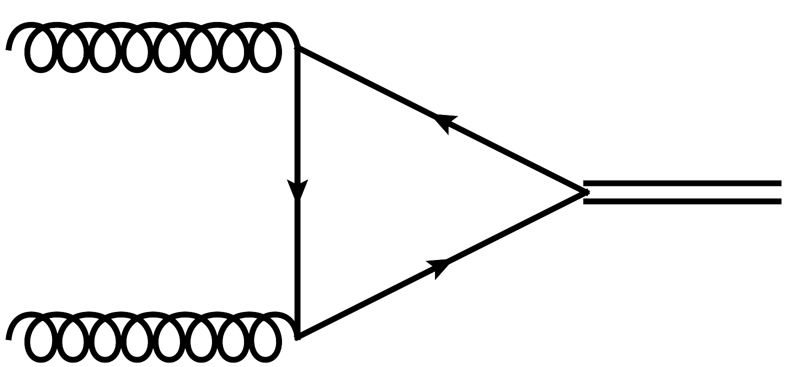
$2 \to 1$
Differential cross sections for Higgs boson final states: 10 - 100 CPU hours

$2 \to 2$
Differential cross sections for Higgs + J boson final states: 100000+ CPU hours

HIGGS – DIFFERENTIAL CROSS SECTIONS

- Introduce a framework that allows to compute differential cross sections at N3LO.
- Circumvent problems of NNLO infrared subtraction.
- Applicable for real-life observables at the LHC.

Specifically: Differential Higgs Production in QCD



$$P\ P \to H + X \to \gamma\gamma + X$$

$$P\ P \to H + X \to 4l + X$$

- Today: Recent Progress, NNLO, Obstacles, Method

HIGGS – DIFFERENTIAL CROSS SECTIONS

- **Focus on the degrees of freedom of the Higgs boson:**

$$p_h = \begin{pmatrix} E \\ p_x \\ p_y \\ p_z \end{pmatrix} = \begin{pmatrix} \sqrt{p_T^2 + m_h^2}\cosh Y \\ p_T \cos\phi \\ p_T \sin\phi \\ \sqrt{p_T^2 + m_h^2}\sinh Y \end{pmatrix}$$

  - Trivial dependence on the azimuthal angle ϕ
  - Together with the Bjorken / PDF variables we have a 4 dimensional problem

$$\{x_1, x_2, p_T, Y\}$$

HIGGS – DIFFERENTIAL CROSS SECTIONS

- **Definition of the Higgs - Differential Cross Section**

$$\sigma_{PP\to H+X}\left[\mathcal{O}\right] = \sum_{i,j}\int_{-\infty}^{+\infty} dY \int_0^{\infty} dp_T^2 \int_0^{2\pi}\frac{d\phi}{2\pi}\int_0^1 dx_1 dx_2 f_i(x_1) f_j(x_2)$$

$$\times\frac{d^2\hat{\sigma}_{ij}}{dY\,dp_T^2}(S,x_1,x_2,m_h^2,Y,p_T^2)\mathcal{J}_{\mathcal{O}(Y,p_T^2,\phi,m_h^2)}$$

HIGGS – DIFFERENTIAL CROSS SECTIONS

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XS for observable O

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Integrate over
Higgs degrees of freedom

HIGGS – DIFFERENTIAL CROSS SECTIONS

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Partonic Higgs-differential cross section

HIGGS – DIFFERENTIAL CROSS SECTIONS

- **Definition of the Higgs - Differential Cross Section**

$$\sigma_{PP\to H+X}\left[\mathcal{O}\right] = \sum_{i,j} \int_{-\infty}^{+\infty} dY \int_0^{\infty} dp_T^2 \int_0^{2\pi} \frac{d\phi}{2\pi} \int_0^1 dx_1 dx_2 f_i(x_1) f_j(x_2)$$

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Definition of observable / measurement function

For example:

$$\mathcal{J}_{(Y,p_T^2,\phi,m_h^2)} = \theta(p_T > 20 GeV)$$

HIGGS – DIFFERENTIAL CROSS SECTIONS

- **Definition of the Higgs - Differential Cross Section**

$$\sigma_{PP\to H+X}\left[\mathcal{O}\right] = \sum_{i,j}\int_{-\infty}^{+\infty} dY \int_0^{\infty} dp_T^2 \int_0^{2\pi}\frac{d\phi}{2\pi}\int_0^1 dx_1 dx_2 f_i(x_1) f_j(x_2)$$

$$\times \frac{d^2\hat{\sigma}_{ij}}{dY\,dp_T^2}(S, x_1, x_2, m_h^2, Y, p_T^2)\mathcal{J}_{\mathcal{O}(Y,p_T^2,\phi,m_h^2)}$$

- Decays of the Higgs boson can be included multiplicatively,
- Phase space boundaries absorbed in the definition of the partonic cross section.

PARTONIC HIGGS – DIFFERENTIAL CROSS SECTIONS

- **How to compute a partonic Higgs - differential cross section:**

Inclusive:

$$\int d\Phi_{h+X} \sim \int d^d p_h \prod_i^n d^d p_i$$

Higgs - differential:

$$\int d\Phi_n \sim \int \cancel{d^d p_h} \prod_i^n d^d p_i$$

- **Partonic Higgs - differential cross section:**

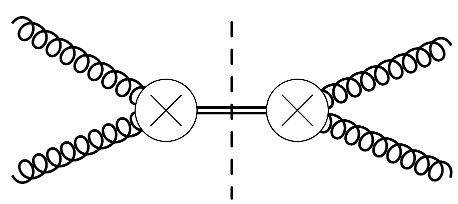
$$\frac{d^2\hat{\sigma}_{ij}}{dY\,dp_T^2} \sim \sum_X \int d\Phi_n \left|\mathcal{M}_{ij\to H+X}\right|^2$$

PARTONIC HIGGS – DIFFERENTIAL CROSS SECTIONS

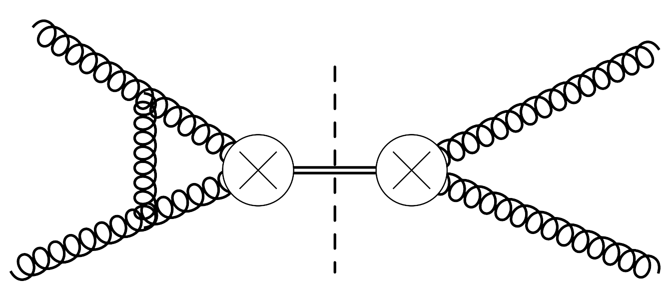
$$\frac{d^2\hat{\sigma}_{ij}}{dY\,dp_T^2} \sim \sum_X \int d\Phi_n \boxed{\left|\mathcal{M}_{ij\to H+X}\right|^2}$$

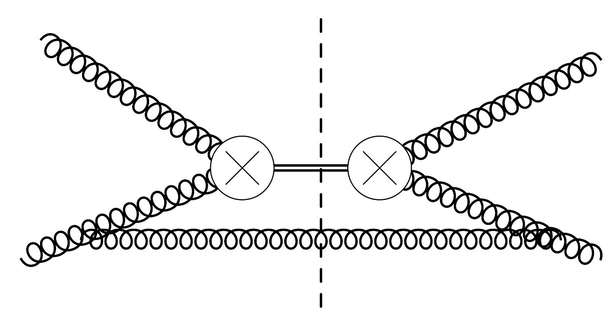
- Compute all required matrix elements of different final states X to a given order in perturbation theory.

LO:

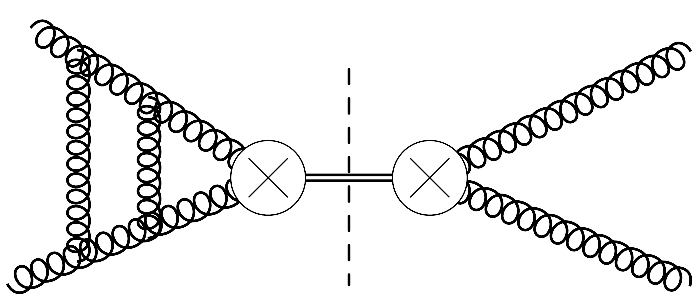


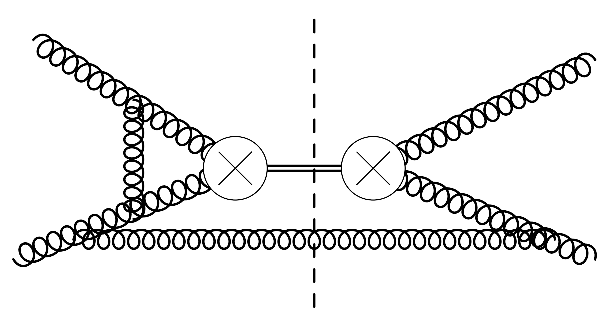
NLO:

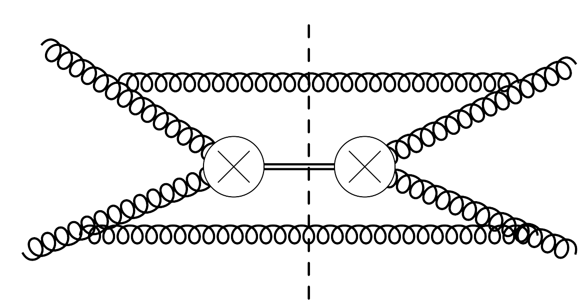




NNLO:

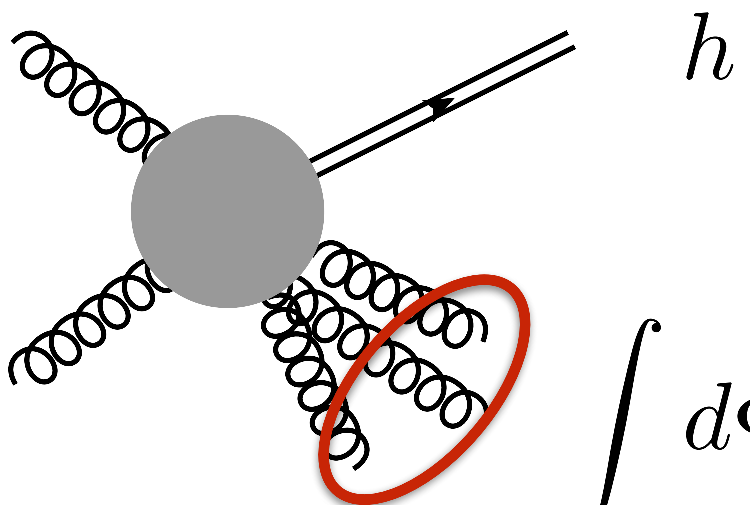






PARTONIC HIGGS – DIFFERENTIAL CROSS SECTIONS

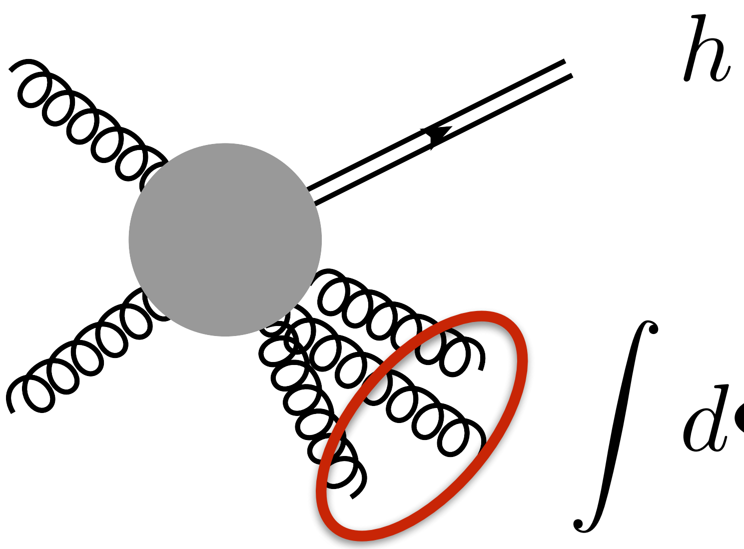
$$\frac{d^2\hat{\sigma}_{ij}}{dY\,dp_T^2} \sim \sum_X \int d\Phi_n \left|\mathcal{M}_{ij\to H+X}\right|^2$$



$$\int d\Phi_n$$

- Perform integration over parton phase space analytically
- Rely on tools to perform analytic computation learned from inclusive N3LO
- Make singularities of final state parton integrations manifest using dimensional regularisation. $d = 4 - 2\epsilon$

PARTONIC HIGGS – DIFFERENTIAL CROSS SECTIONS



$$\int d\Phi_n \sim \int \prod_{i=1}^{n} d^d p_i \delta_+(p_i^2)$$

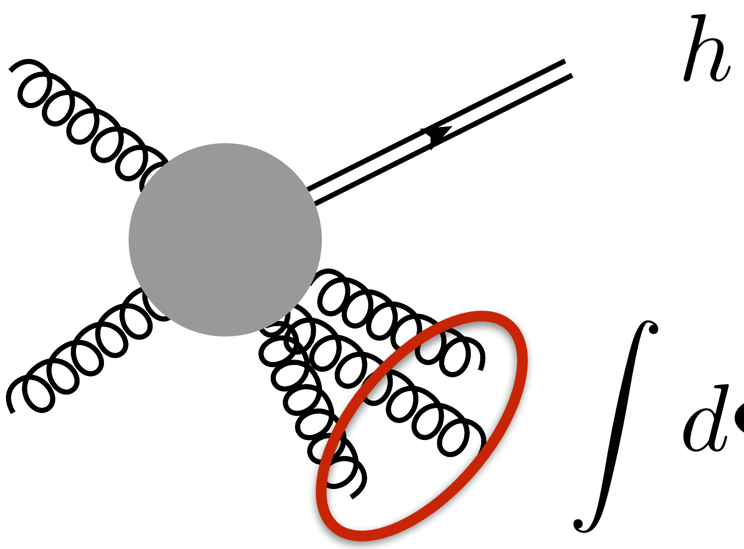
with $\delta_+(p_i^2) = \theta(E_i)\delta(p_i^2)$

REVERSE UNITARITY FRAMEWORK:

- Replace on-shell constraints with cut propagators

$$\delta_+(p_i^2) \sim \lim_{\delta\to 0}\left[\frac{1}{p_i^2 + i\delta} - \frac{1}{p_i^2 - i\delta}\right] = \left[\frac{1}{p_i^2}\right]_c$$

REVERSE UNITARITY FRAMEWORK:



$$\int d\Phi_n \sim \int \prod_{i=1}^{n} d^d p_i \left[\frac{1}{p_i^2}\right]_c$$

- Opens the door to large variety of loop integral technology!
    - IBPs + Differential equations
- Key observation: Cut propagators can be differentiated similar to usual propagators.

REVERSE UNITARITY FRAMEWORK:

$$\frac{d^2\hat{\sigma}_{ij}}{dY\,dp_T^2} \sim \sum_X \int d\Phi_n \left|\mathcal{M}_{ij\to H+X}\right|^2$$
$$= \sum_X \sum_i c_{X,\,i} F_{X,\,i}(S, p_T, Y, m_h^2)$$

- Coefficient: Rational function of remaining kinematic variables.
- Master Integral: Integrated Feynman integrals: Polylogarithms, rational functions of remaining kinematic variables.
- Explicit Laurent series in dimensional regulator.

PARTONIC HIGGS – DIFFERENTIAL CROSS SECTIONS

- Our choice of variables for the partonic cross sections:

$$\{\bar{z}, \lambda, x\}$$

$$Y = \frac{1}{2}\log\left(\frac{x_2}{x_1}\frac{1-\bar{z}\lambda}{1-\bar{z}\frac{\bar{\lambda}}{(1-\bar{z}\lambda x)}}\right) \qquad \frac{p_T^2}{m_h^2} = \frac{\bar{z}^2}{z}\bar{x}\frac{\lambda\bar{\lambda}}{1-\bar{z}\lambda x}$$

- Nice integration bounds: [0,1] $(\bar{y} = 1 - y)$

- Remaining singularities (initial collinear, pT=0) are located at the edges of phase space

$$\bar{z} = 0$$

$$\lambda = \{0, 1\}$$

$$x = \{0, 1\}$$

Singularities take for example to form of :

$$\lambda^{-1-\epsilon}$$

PARTONIC HIGGS – DIFFERENTIAL CROSS SECTIONS

- Our choice of variables for the partonic cross sections:

$$\{\bar{z}, \lambda, x\}$$

- Nice properties of x:
  - x=0:
    Virtuality of radiation vanishes: NLO type kinematics
  - x=1
    Transverse momentum vanishes

$$x = \frac{p_g^2 s}{(2p_g p_1)(2p_g p_2)} = \frac{p_g^2 S}{(2p_g P_1)(2p_g P_2)}$$

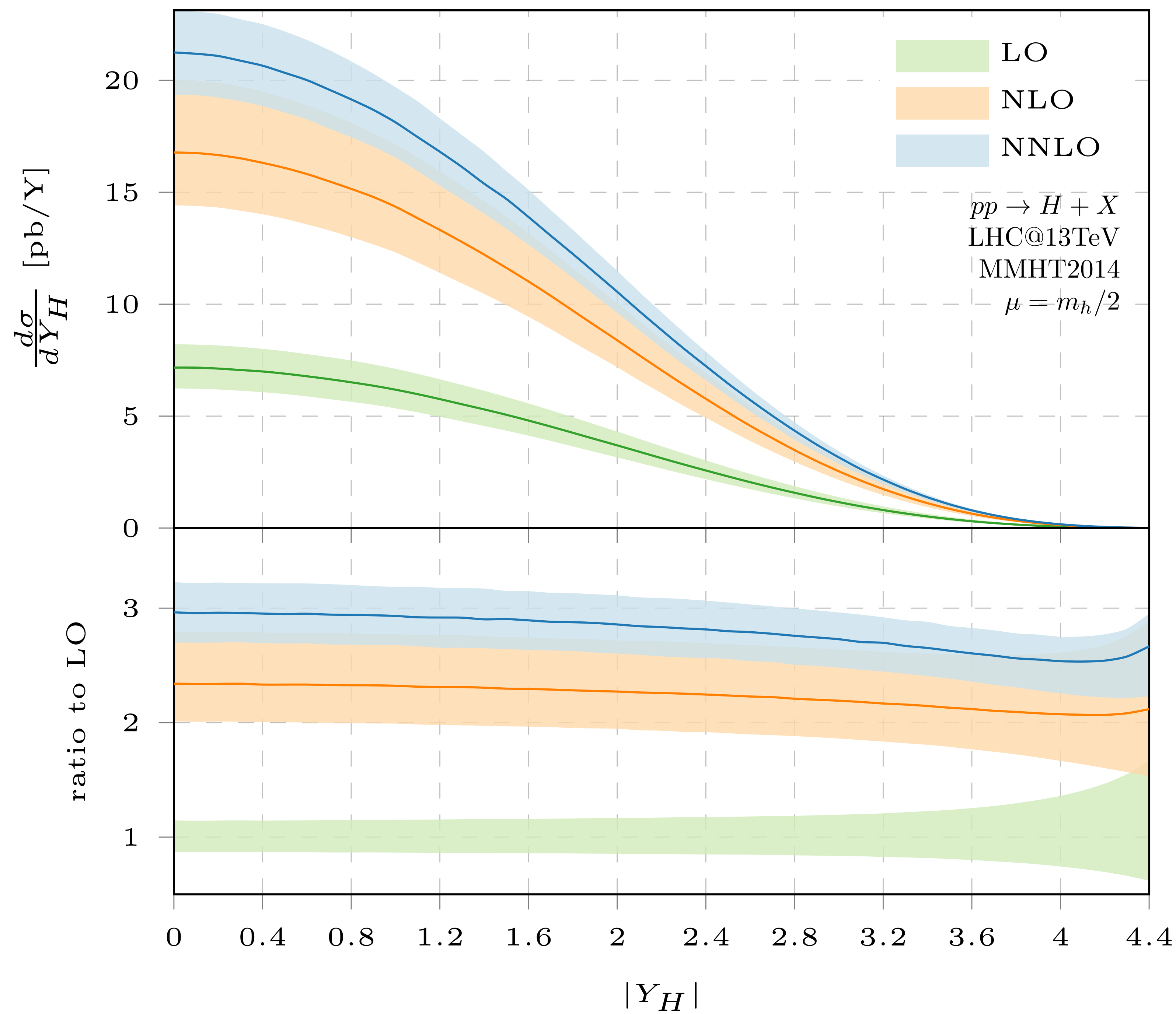
p_g sum of all final state parton momenta

- x is invariant under separate rescaling of $\{p_g, p_1, p_2\}$

HIGGS – DIFFERENTIAL CROSS SECTIONS: PROOF OF PRINCIPLE

NNLO:

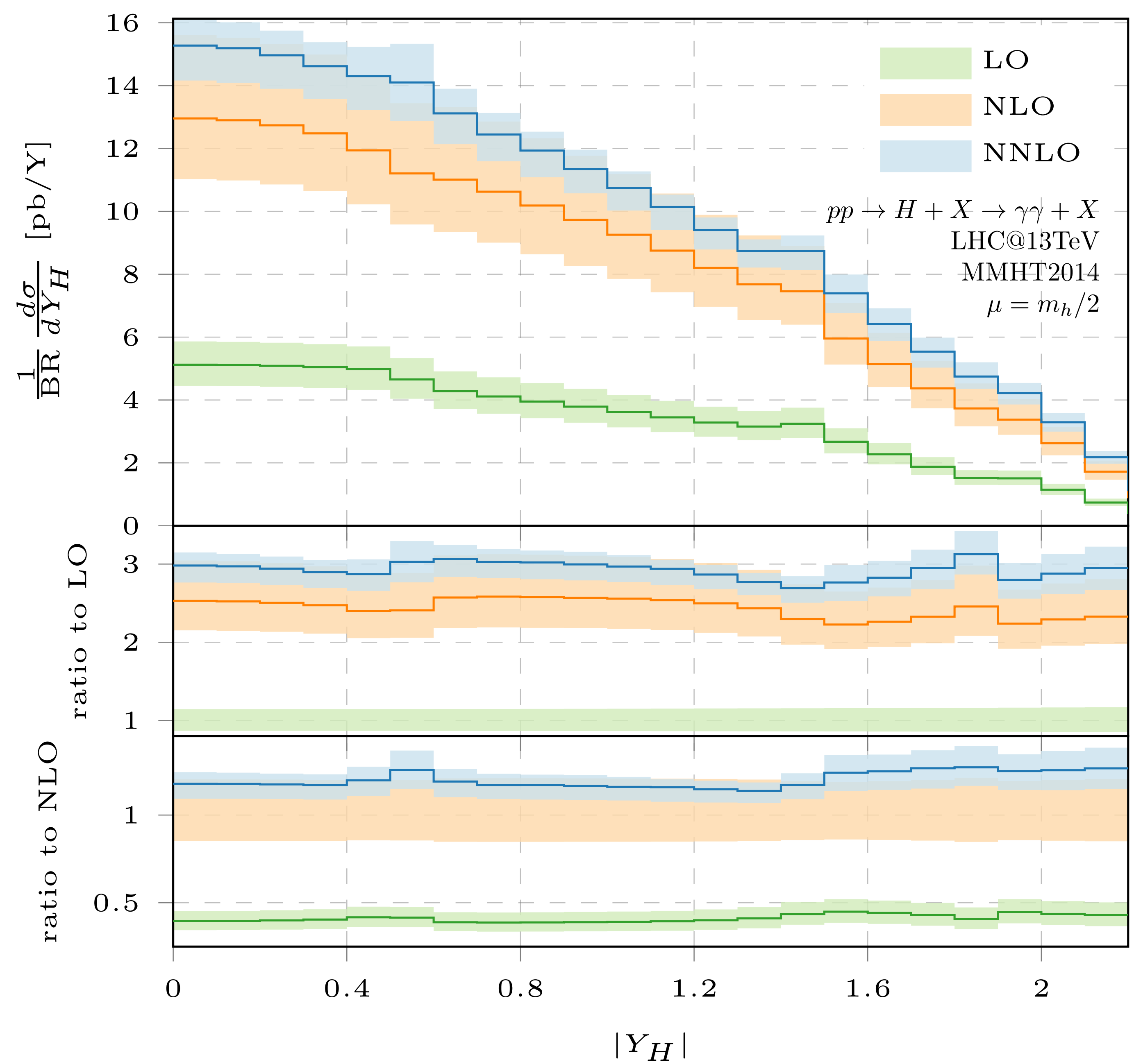
- Inclusive rapidity distribution
- Large K-factors



HIGGS – DIFFERENTIAL CROSS SECTIONS: FIDUCIAL XS (ATLAS)

$$P\ P \to H + X \to \gamma\gamma + X$$

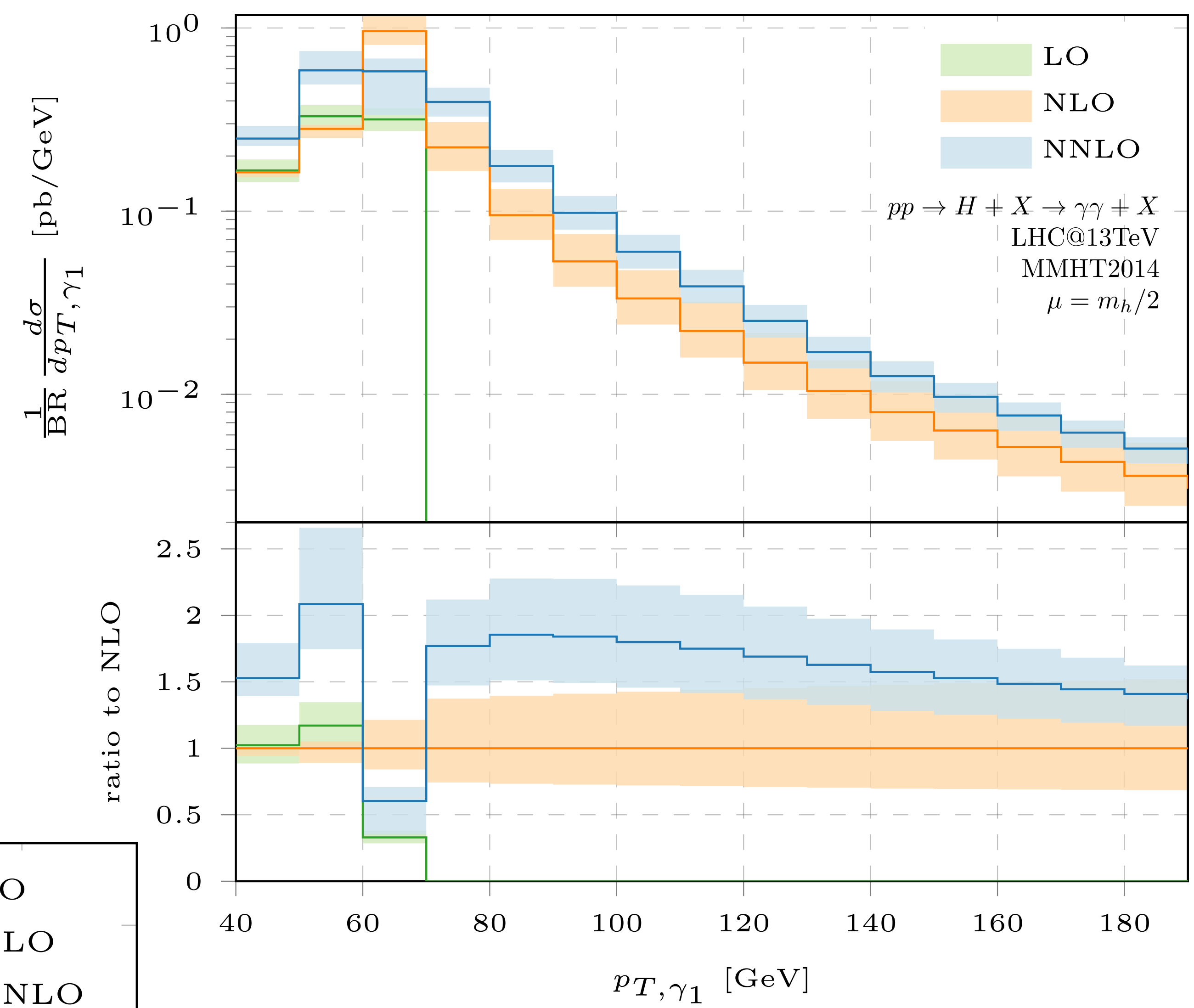
- Fiducial rapidity distribution.
- Non-trivial features due to
  selection criteria.
- Relatively flat K-factors
- Similar perturbative behaviour as inclusive distribution

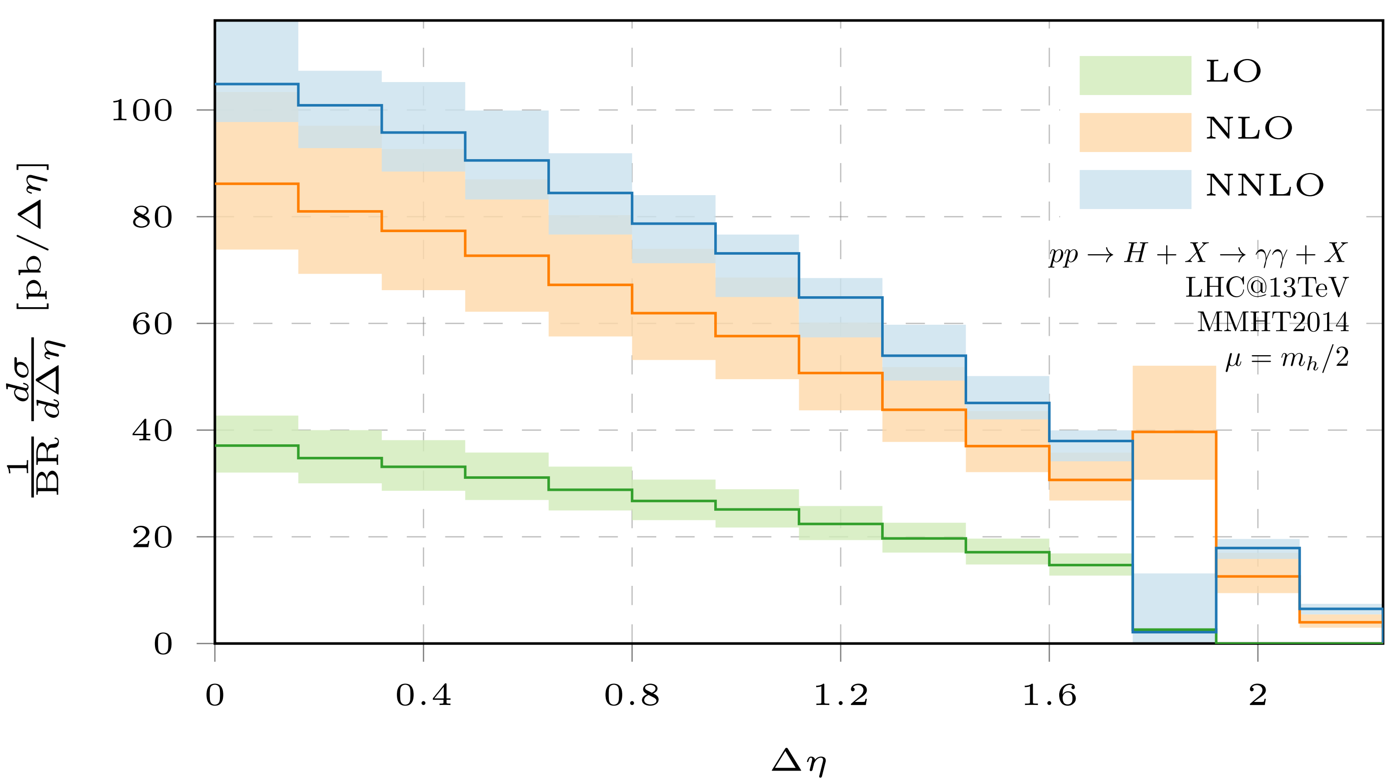


HIGGS – DIFFERENTIAL CROSS SECTIONS: FIDUCIAL XS

$$P\ P \to H + X \to \gamma\gamma + X$$

- Distributions of the photon momenta:
  - Leading Photon pT
  - Pseudo - rapidity difference





$$\Delta\eta = |\eta_{\gamma_1} - \eta_{\gamma_2}|$$

BEYOND NNLO

WHAT DID WE LEARN FROM NNLO

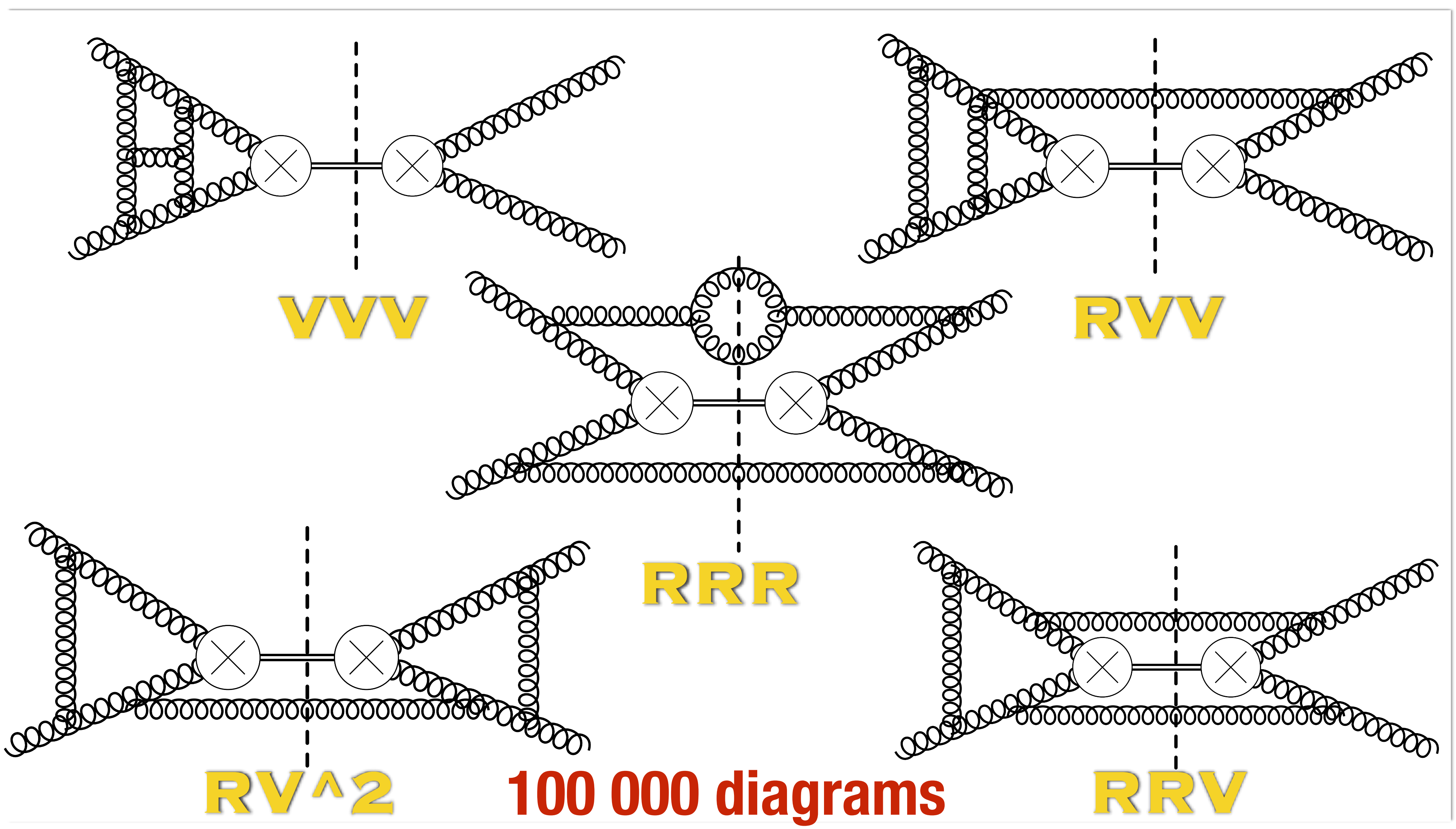
- Higgs-differential cross sections: fast and stable framework for fiducial cross sections.
- Analytic computation at NNLO comparably simple.

MAIN CHALLENGES FOR N3LO

- Rapid growth in analytic complexity: Many more integrals to compute, large rational expressions as a result
- Numerical stability vs. speed in evaluation of analytic coefficients.

FIXED ORDER MATRIX ELEMENTS

Rapid growth in complexity

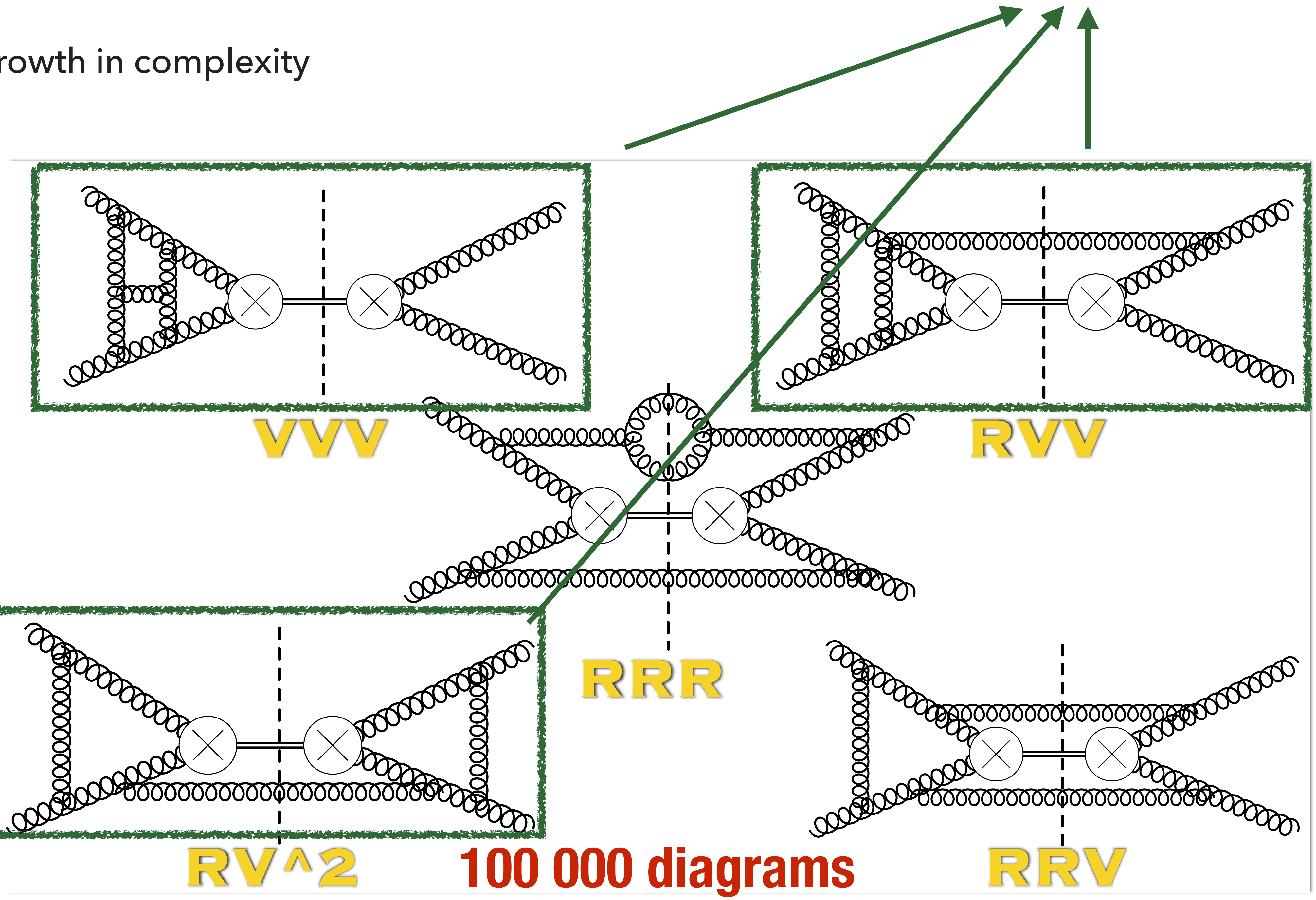


1000 @ NNLO

FIXED ORDER MATRIX ELEMENTS

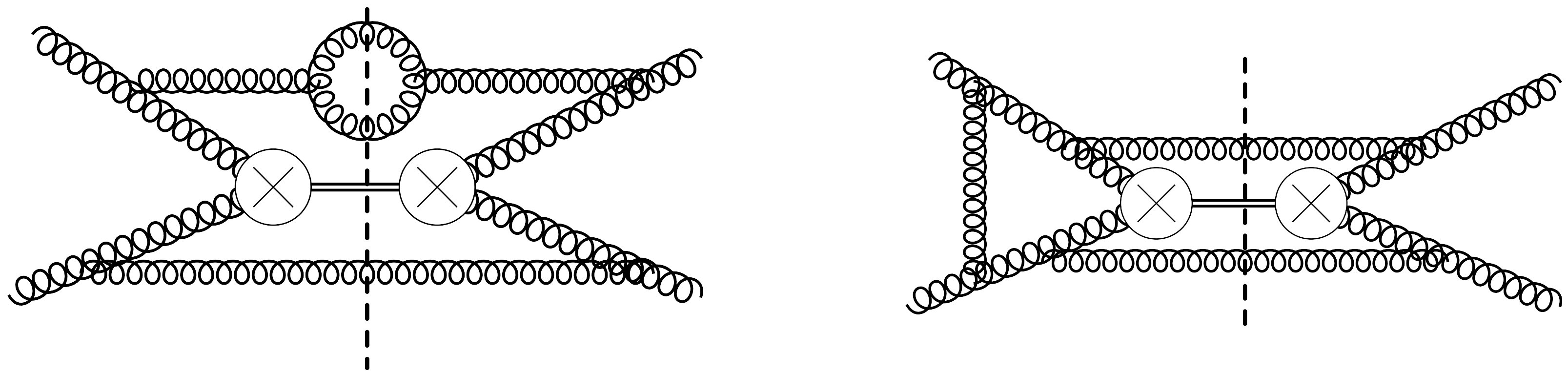
Known already! (Inclusive / H+J @NNLO)

Rapid growth in complexity



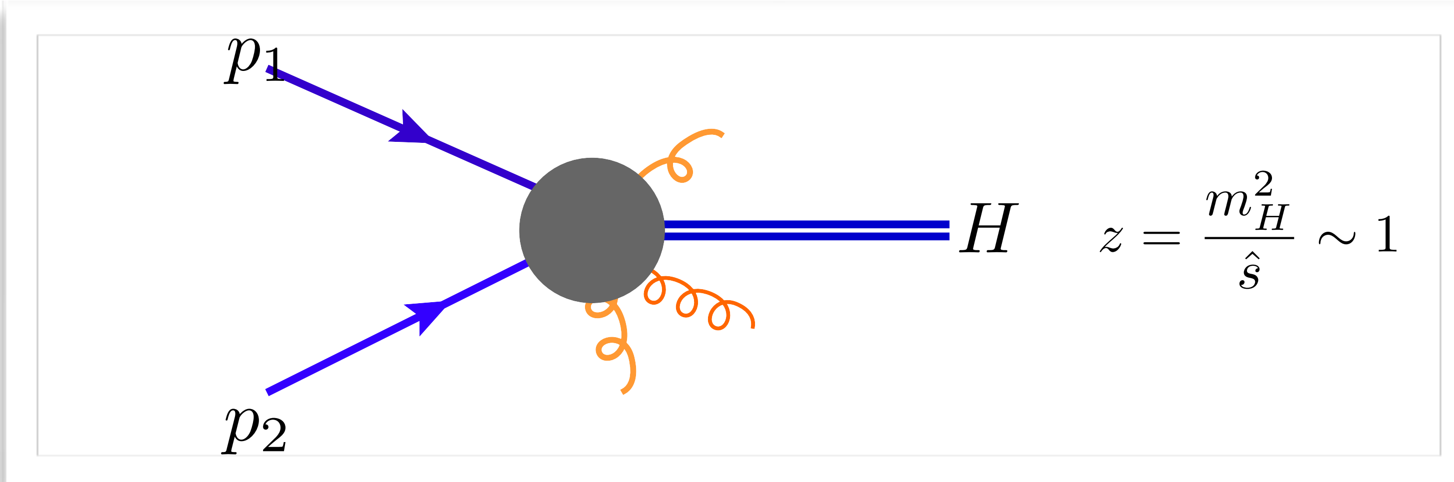
1000 @ NNLO

FIXED ORDER MATRIX ELEMENTS



- Missing matrix elements with 2 or 3 final state partons.
- Same strategy as for NNLO: Analytic computation using reverse unitarity, master integrals and differential equations.
- Number of master integrals required: 100 x NNLO.
- Solving differential equations for master integrals: Need boundary conditions = Master integrals evaluated at one single point.

THRESHOLD EXPANSION FOR DIFFERENTIAL CROSS SECTIONS ???

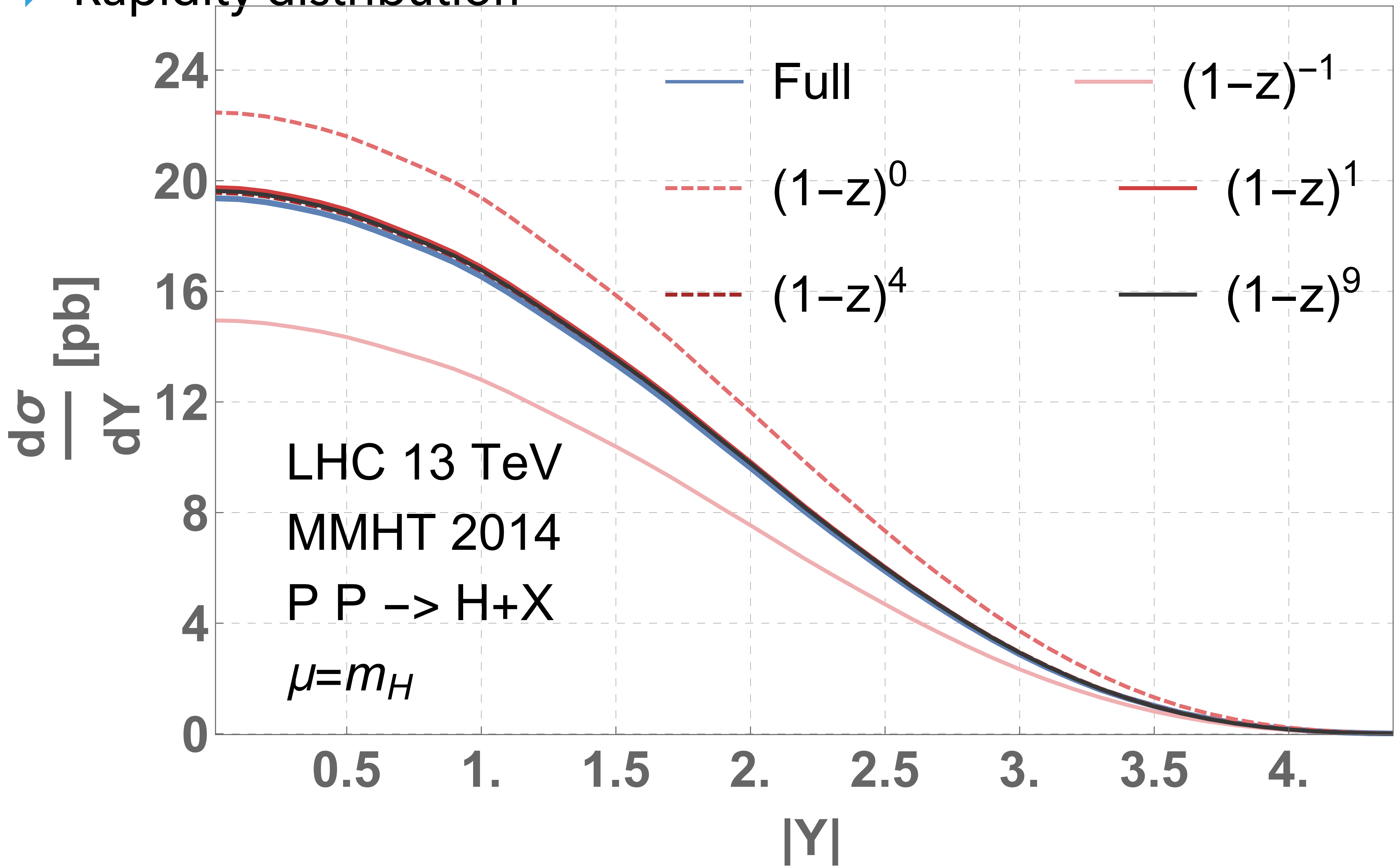


$$\bar{z} = 1 - z \quad \Rightarrow \quad \hat{\sigma}(\bar{z}) = \sigma^{SV} + \sigma^{(0)} + \bar{z}\sigma^{(1)} + \ldots$$

- Excellent approximation for inclusive cross section.
- **Reason Nr.1**:
  Crucial analytic information a full calculation relies on - boundary conditions.
  + checks, testing ground for technology, etc.
- **Reason Nr. 2**: Can we use it for phenomenology?

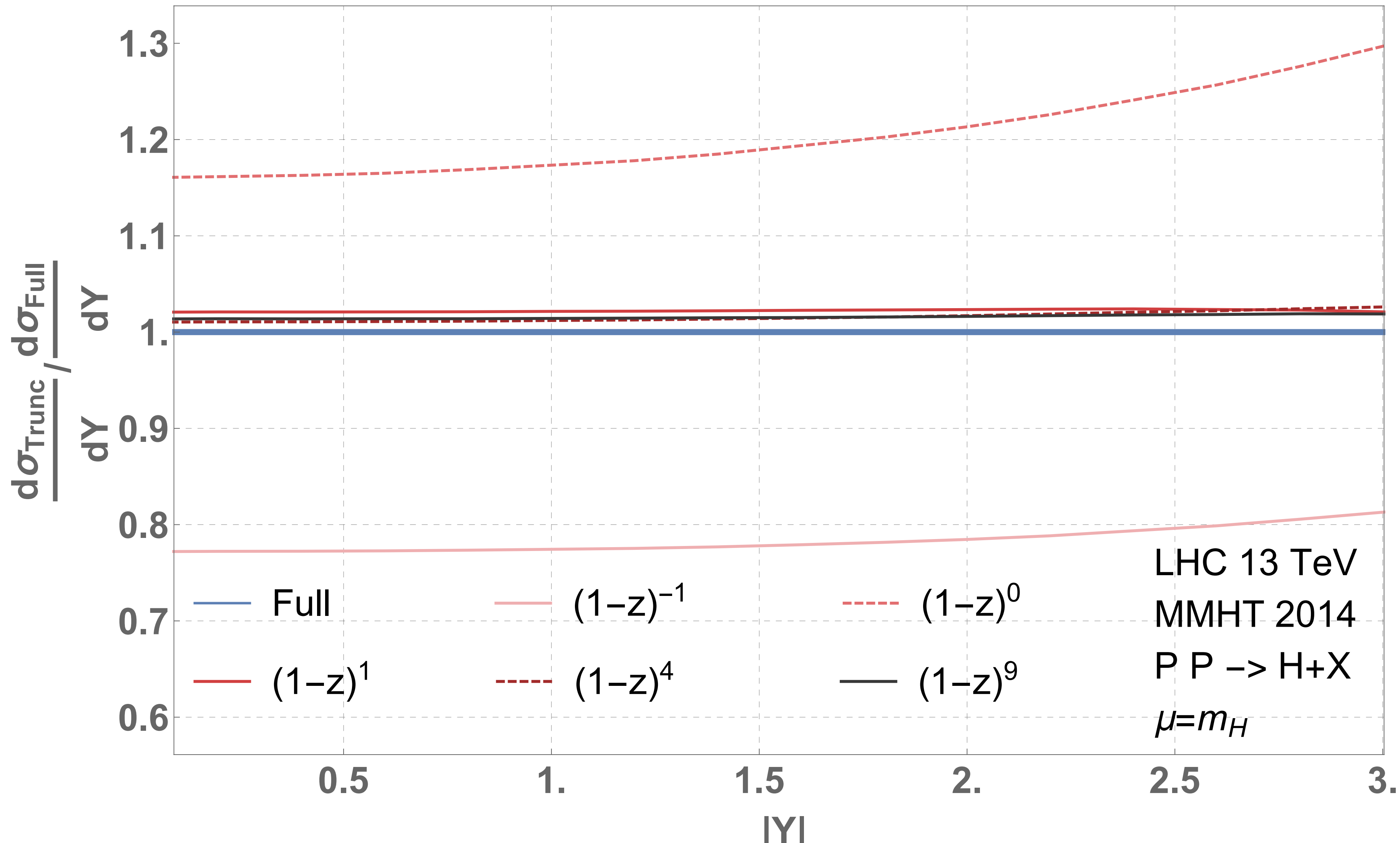
THRESHOLD EXPANSION @ NNLO DIFFERENTIAL

- Rapidity distribution



THRESHOLD EXPANSION @ NNLO DIFFERENTIAL

- Rapidity distribution normalised to true value.



- Bulk of XS is described well with a couple of terms

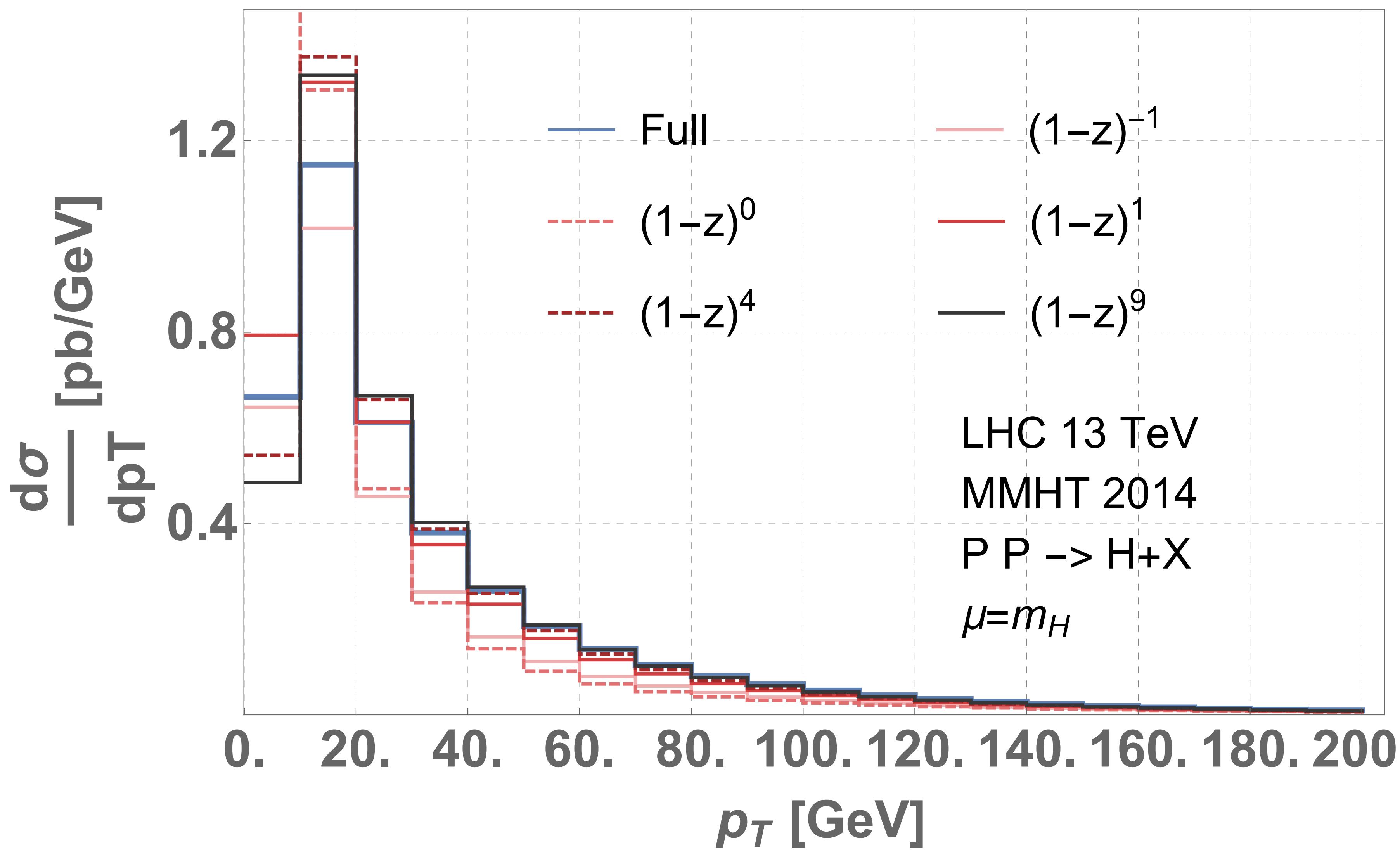
- Systematic improvement possible

THRESHOLD EXPANSION @ NNLO DIFFERENTIAL

- PT distribution

- Bad convergence at low pT

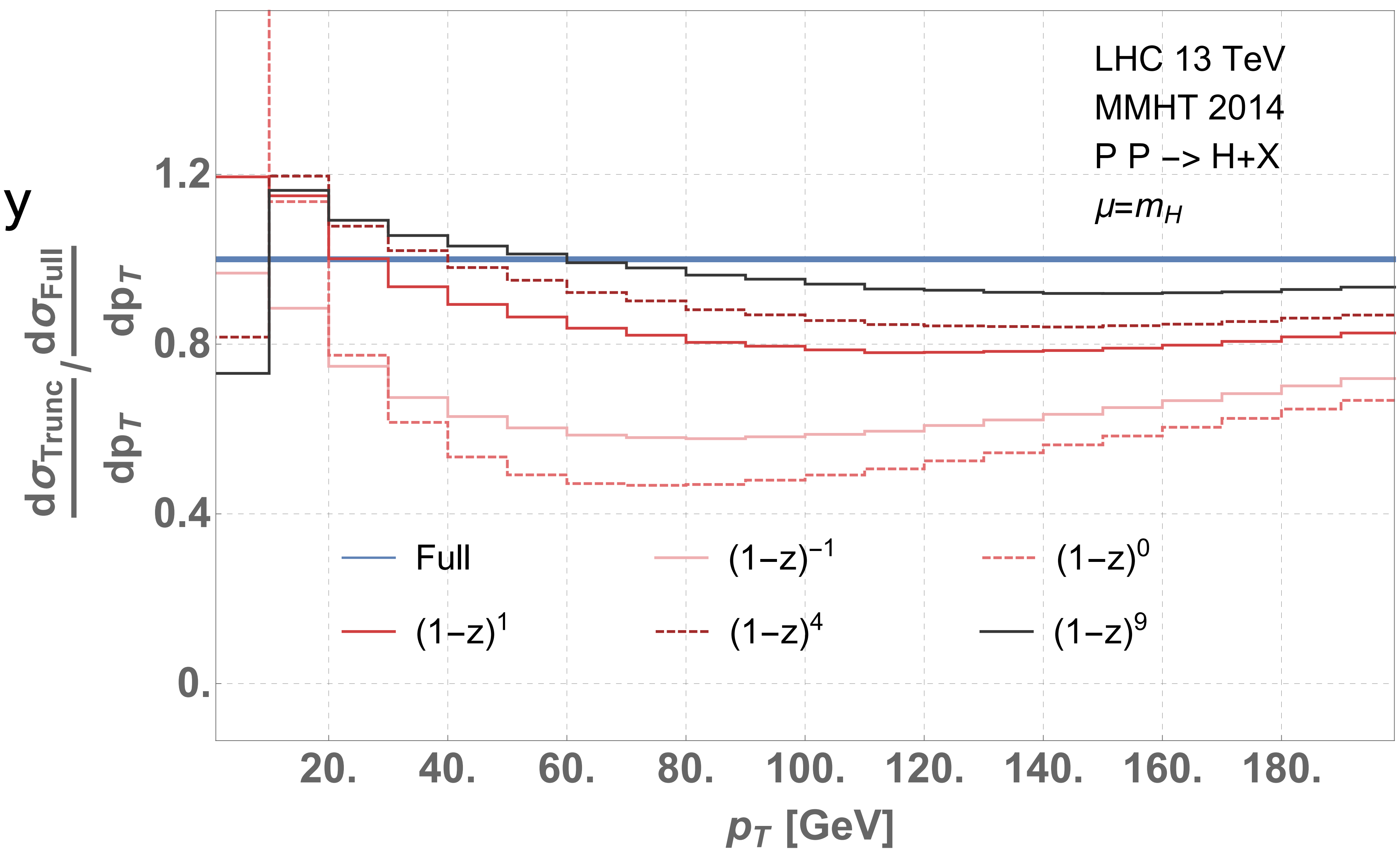
- On-set of distribution at NLO while threshold limit is tree level.



THRESHOLD EXPANSION @ NNLO DIFFERENTIAL

- PT distribution normalised to true value.

- Even with 10 terms marginally within 20%
- Quality of expansion is subject to observable: Threshold sensitivity



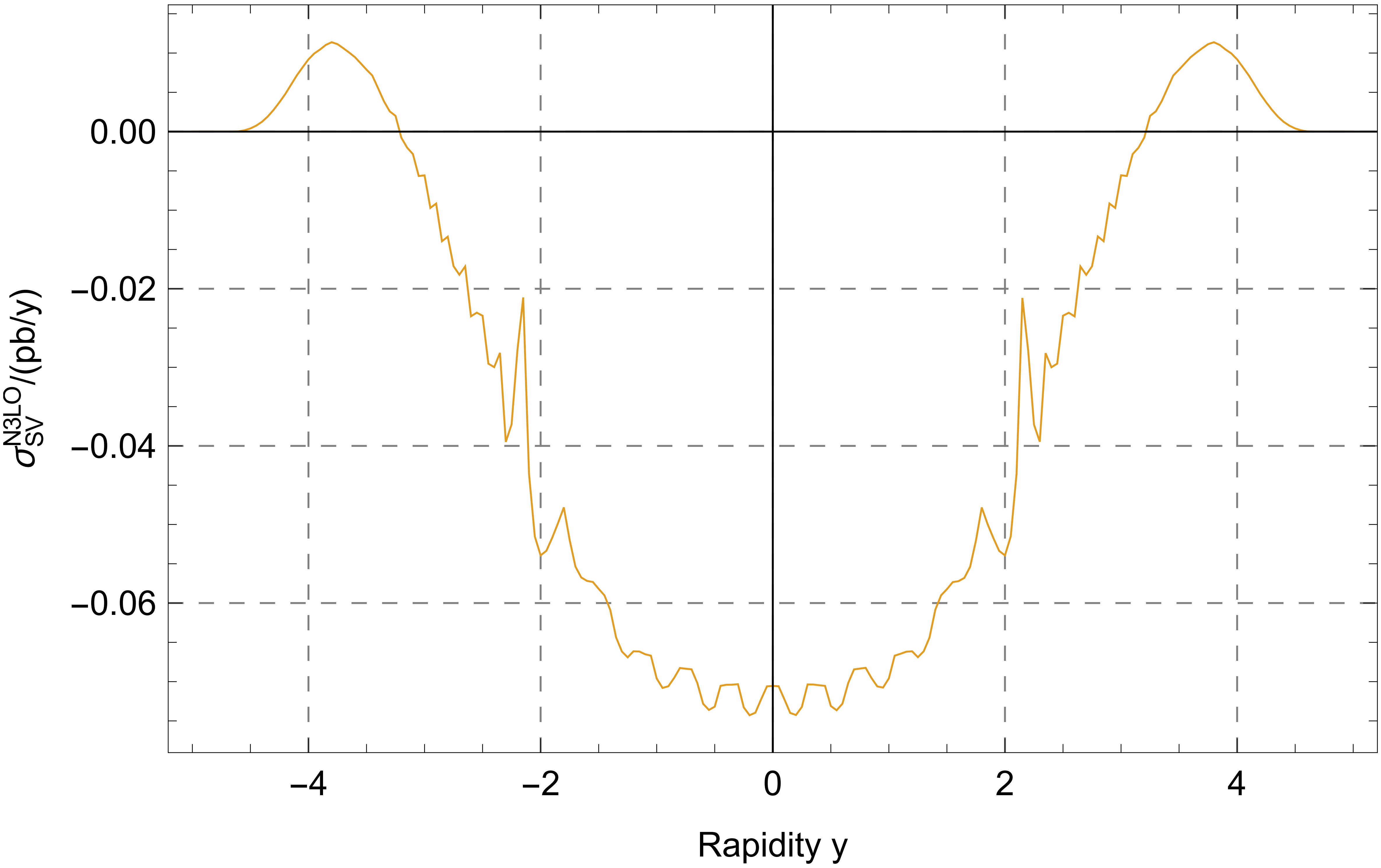
THRESHOLD EXPANSION

- Systematically improvable approximation.
- Soft expansion gives the opportunity to study differential distribution
- Doing phenomenology in this approximation requires careful case by case analysis to see if the approximation is valid!

THE ROAD TO N3LO VIA THRESHOLD EXPANSIONS

- Extend analytic techniques
  for automatic soft amplitude expansions.
- Apply reverse unitarity, differential equations,
  Multiple PolyLog, IBPs, symbol tools, ….
- Compute 110 new double differential soft master integrals.
- Compute the first terms (Soft-Virtual SV) at N3LO
- Put into code and look at the N3LO corrections to the rapidity distribution and …

N3LO CORRECTIONS TO THE RAPIDITY DISTRIBUTION



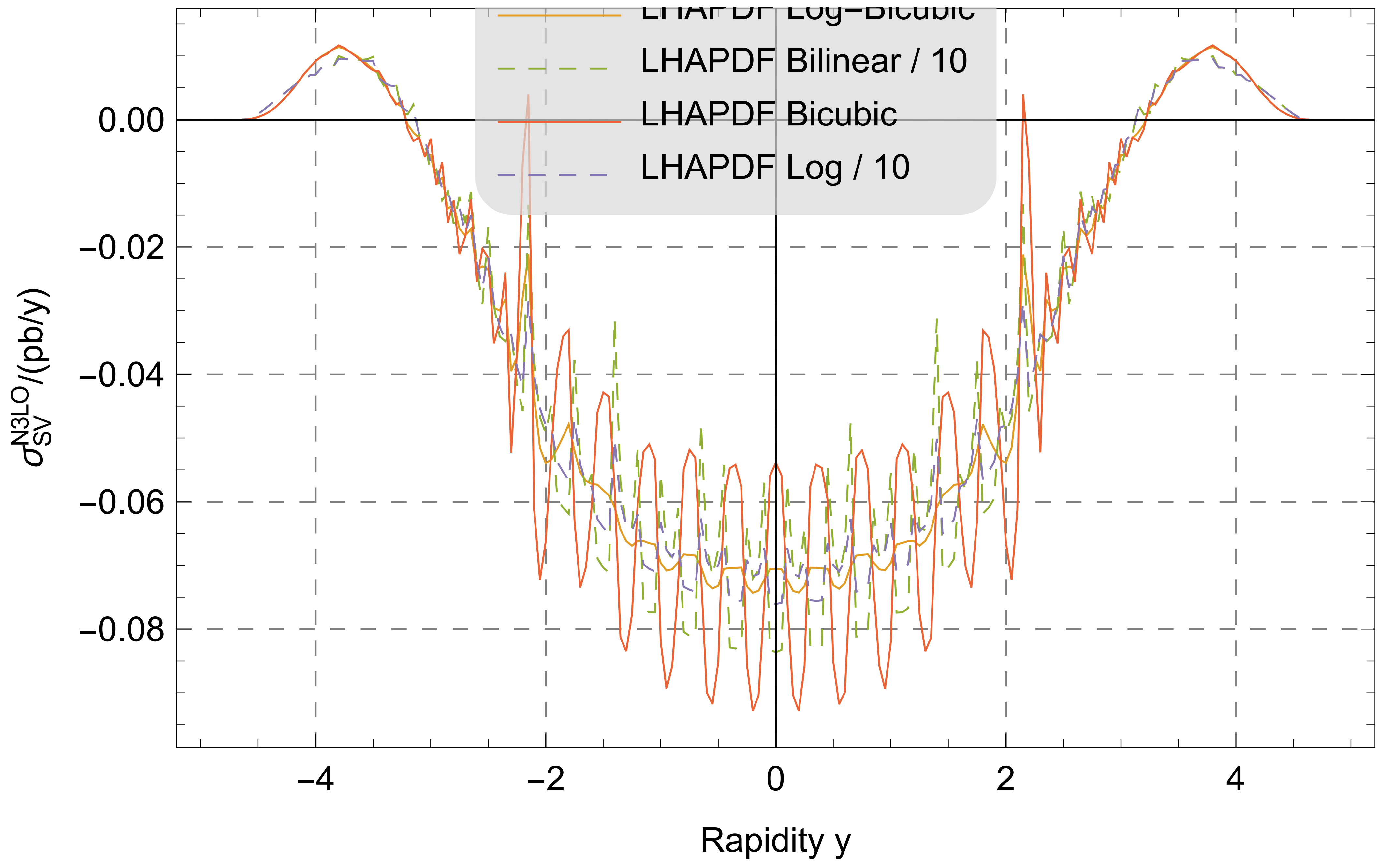
BE CAREFUL WHEN YOU DO SOMETHING NEW

$$\sigma \sim \int dz \mathcal{L}_{gg}(z) \left[\frac{log^{5}(1-z)}{1-z} \right]_{+}$$

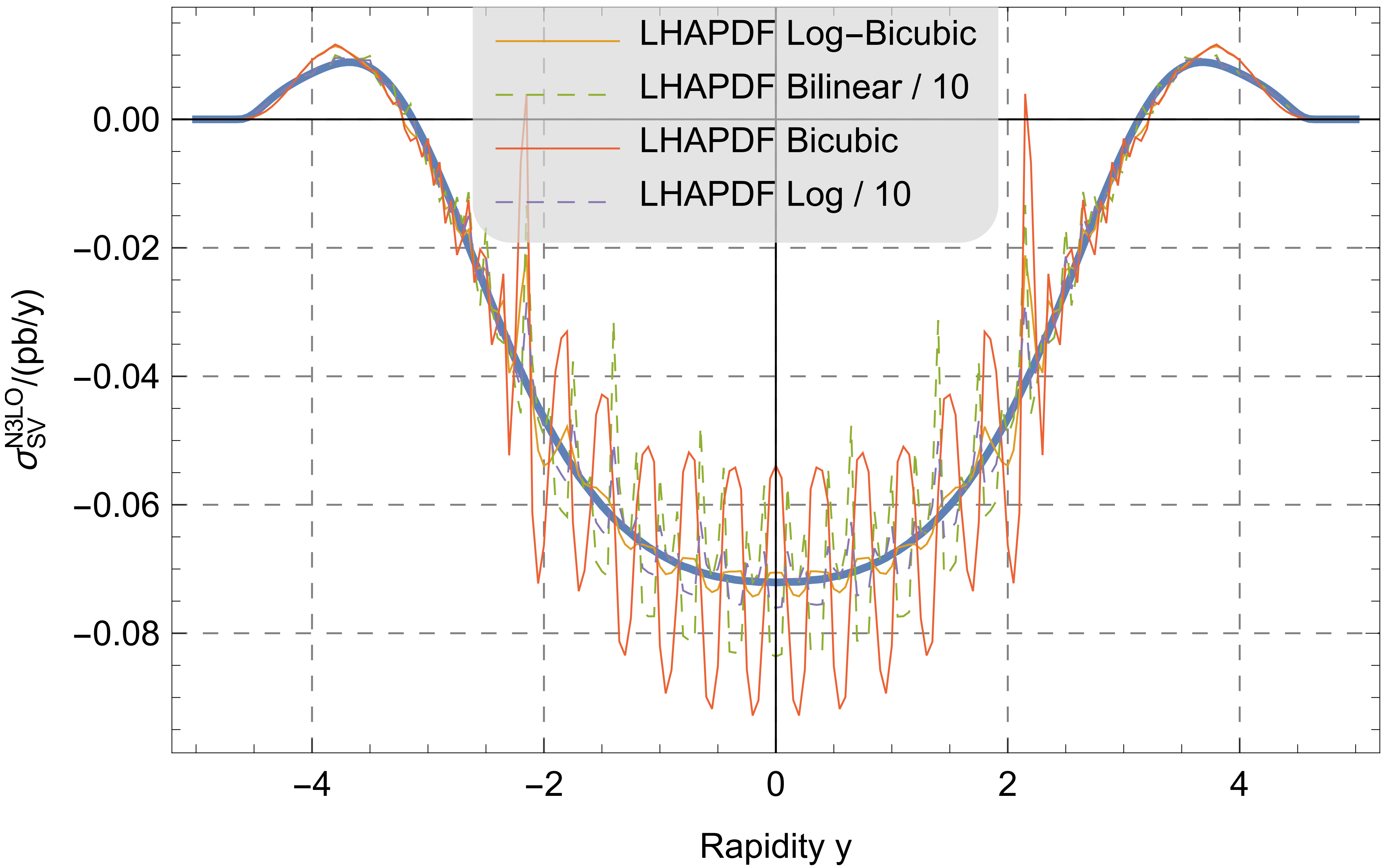
LHAPDF

- LHAPDF: Grid of points for PDFs in x and Q
- Interpolation between points with certain precision
- Not meant to be precise enough for N3LO plus distributions yet ….
- Improvements required: New interpolator, evolve from smooth PDF …. ?

N3LO CORRECTIONS TO THE RAPIDITY DISTRIBUTION



N3LO CORRECTIONS TO THE RAPIDITY DISTRIBUTION

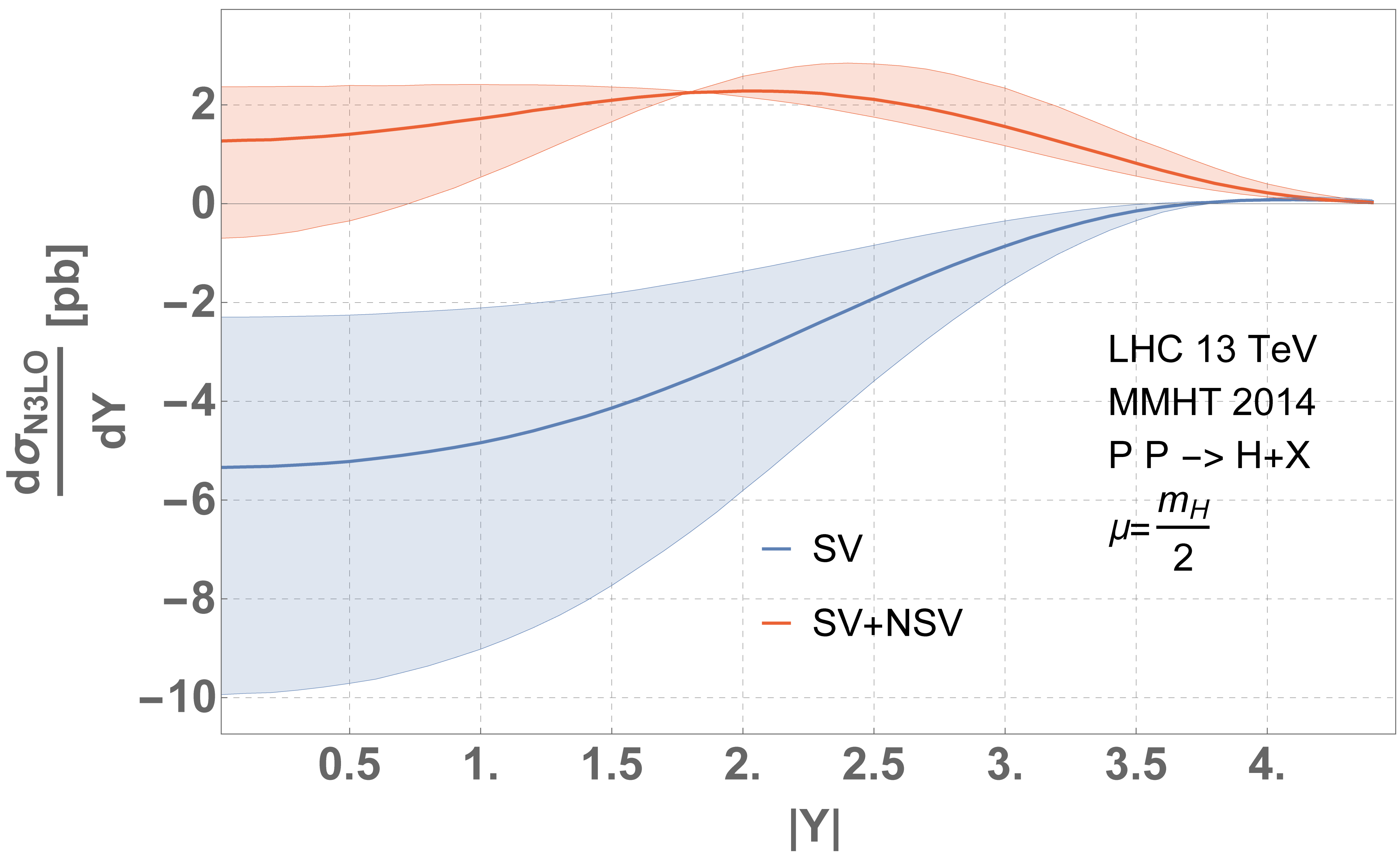


Use Log^12 Interpolator or fit!

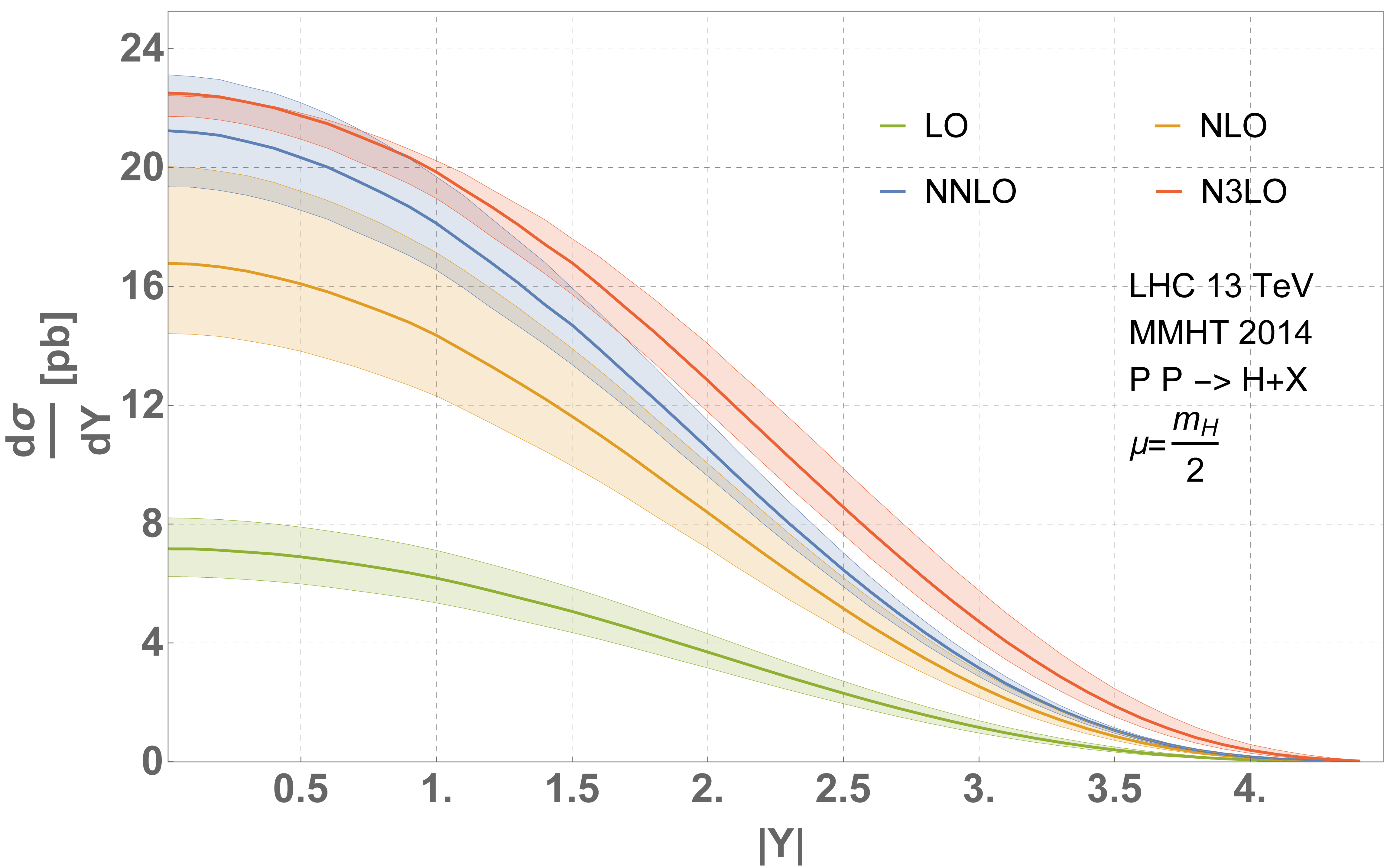
FIRST RESULTS

- So far: We computed the first two terms in the threshold expansion.
- Important step:
  All required analytical boundary information for full computation obtained.
- Remember: 2 terms in the threshold expansion are not enough!
- However ….

FIRST RESULTS: N3LO CORRECTIONS ON RAPIDITY DISTRIBUTION



FIRST RESULTS: FIRST COMBINATION WITH LOWER ORDERS



CONCLUSIONS

- Exact computation of
  N3LO inclusive Higgs production cross section.
- Higgs-differential cross sections:
  Promising framework for realistic final state observables.
- Threshold expansions provide a key ingredient for analytic computation.
- Threshold expansion can be used at the differential level to approximate differential cross section predictions.
- Many interesting things to be encountered when going to higher order.

Thank you!

HIGH TIME FOR HIGH ORDERS!

https://indico.mitp.uni-mainz.de/event/126/

- Workshop at MITP in Mainz
- 13.8. - 24.8. 2018
- Analytic computation, treatment of real radiation singularities,
  analytic resummation / parton showers and a lot more!

mitp Mainz Institute for Theoretical Physics

UV RENORMALISATION AND IR FACTORISATION

- To derive UV counter terms and IR subtraction terms we require NNLO cross sections computed beyond the finite term in ϵ

- Allow to derive complete N3LO scale variation from DGLAP

$$\hat{\sigma}^{(3)} = \hat{\sigma}_0^{(3)} + \hat{\sigma}_1^{(3)} \log\left(\frac{m_h^2}{\mu^2}\right) + \hat{\sigma}_2^{(3)} \log^2\left(\frac{m_h^2}{\mu^2}\right) + \hat{\sigma}_3^{(3)} \log^3\left(\frac{m_h^2}{\mu^2}\right)$$