Karlsruhe Institute of Technology

Nested soft-collinear subtractions for NNLO calculations

Raoul Röntsch

KARLSRUHE INSTITUTE OF TECHNOLOGY

Subtracting Infrared Singularities Beyond NLO

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In collaboration with Fabrizio Caola and Kirill Melnikov
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Nested soft-collinear subtraction scheme

- Extension of FKS subtraction to NNLO.
- Independent subtraction of soft and collinear divergences **(color coherence)**.
- Use of sectors to separate overlapping ***collinear*** singularities.

  [Czakon ‘10, ‘11; Boughezal, Melnikov, Petriello ‘12; Czakon, Heymes ‘14].
- Clear physical origin of singularities.
- Combination of sectors leading to simplifications in integrated subtraction terms.
- Explicit (*almost* analytical) pole cancellation (independent of matrix element).
- Allows four-dimensional evaluation of matrix elements.
- Fully local.

FKS subtraction at NLO: Notation

Consider color singlet production $q(p_1)\bar{q}(p_2) \to V + g(p_4)$:

$$\mathrm{d}\sigma^{\mathrm{R}} = \frac{1}{2s}\int [\mathrm{d}g_4] F_{LM}(1,2,4) \equiv \langle F_{LM}(1,2,4)\rangle.$$

$$F_{LM}(1,2,4) = \mathrm{dLips}_V \, |\mathcal{M}(1,2,4,V)|^2 \, \mathcal{F}_{\mathrm{kin}}(1,2,4,V) \qquad [\mathrm{d}g_4] = \frac{\mathrm{d}^{d-1}p_4}{(2\pi)^d 2E_4}\theta(E_{\max} - E_4)$$

- dLips_V: Lorentz-inv. Phase space for V **(incl. delta-fn)**
- $|\mathcal{M}(1,2,4,V)|^2$: Matrix-element sq.
- $\mathcal{F}_{\mathrm{kin}}$: IR-safe observable
- $[\mathrm{d}g_4]$: Integration in partonic CoM frame
- $E_{\max}$: Arbitrarily large energy parameter

Define soft and collinear operators:

$$S_i A = \lim_{E_i \to 0} A \qquad C_{ij} A = \lim_{\rho_{ij} \to 0} A \qquad \rho_{ij} = 1 - \cos\theta_{ij}$$

FKS subtraction at NLO: Subtraction

Remove singular limits and add back as subtraction terms:

$$\begin{aligned}\langle F_{LM}(1,2,4)\rangle = & \langle (I - C_{41} - C_{42})(I - S_4) F_{LM}(1,2,4)\rangle + \\ & \langle S_4 F_{LM}(1,2,4)\rangle + \\ & \langle (C_{41} + C_{42})(I - S_4) F_{LM}(1,2,4)\rangle\end{aligned}$$

- First term: finite, can be integrated numerically in 4-dimensions.
- Second term: soft subtraction term – gluon decouples completely (need upper bound E_{max}).
- Third term: collinear and soft+collinear subtraction terms – gluon decouples partially or completely.
- Singularities made explicit by integrating subtraction terms over unresolved gluon.

FKS Subtraction at NLO: Poles

After integrating:

$$\hat{O}_{\mathrm{NLO}} \equiv (I - C_{41} - C_{42})(I - S_4)$$

$$2s \cdot \mathrm{d}\sigma^{\mathrm{R}} = 2[\alpha_s] s^{-\epsilon} \left(\frac{C_F}{\epsilon^2} + \frac{3C_F}{2\epsilon} \right) \frac{\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)} \langle F_{LM}(1,2) \rangle + \langle \hat{O}_{\mathrm{NLO}} F_{LM}(1,2,4) \rangle$$

$$- \frac{[\alpha_s] s^{-\epsilon}}{\epsilon} \frac{\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)} \int_0^1 \mathrm{d}z \mathcal{P}_{qq,R}(z) \left\langle \frac{F_{LM}(z \cdot 1, 2)}{z} + \frac{F_{LM}(1, z \cdot 2)}{z} \right\rangle.$$

- LO structures with and without boost, and regulated real emission:

$$\langle F_{LM}(1,2) \rangle \qquad \langle F_{LM}(z \cdot 1, 2)/z \rangle \qquad \langle F_{LM}(1, z \cdot 2)/z \rangle \qquad \langle \hat{O}_{\mathrm{NLO}} F_{LM}(1,2,4) \rangle$$

- Remove soft limits of splitting functions from collinear emission → Altarelli-Parisi kernels

$$\mathcal{P}_{qq,R}(z) = \hat{P}^{(0)}_{qq}(z) + \epsilon \mathcal{P}^{(\epsilon)}_{qq,R}(z)$$

- Poles in first term cancel with virtual, poles in second term cancel with pdf renorm.
- Cancellation occurs ***within each structure!***

FKS subtraction at NLO: finite result

After cancelling poles, we can take the $\epsilon \to 0$ limit and compute everything in four dimensions.

$$2s \cdot \mathrm{d}\hat{\sigma}^{\mathrm{NLO}} = \left\langle F_{LV}^{\mathrm{fin}}(1,2) + \frac{\alpha_s(\mu)}{2\pi}\left[\frac{2}{3}\pi^2 C_F F_{LM}(1,2)\right]\right\rangle + \langle \hat{O}_{\mathrm{NLO}} F_{LM}(1,2,4)\rangle +$$

$$+ \frac{\alpha_s(\mu)}{2\pi}\int_0^1 \mathrm{d}z \left[\ln\frac{s}{\mu^2}\hat{P}_{qq}^{(0)}(z) - \mathcal{P}_{qq,R}^{(\epsilon)}(z)\right]\left\langle \frac{F_{LM}(z\cdot 1,2)}{z} + \frac{F_{LM}(1,z\cdot 2)}{z}\right\rangle .$$

Sum of:

- **LO-like terms,** with or without convolutions with splitting functions.
- Real emission term, with singular configurations removed by iterated subtraction.
- Finite remainder of virtual corrections.

NNLO subtraction scheme

Aim to replicate NLO subtraction results *as much as possible:*

- Explicit (ideally analytical) cancellation of poles in each kinematic structure, *before* numerical implementation.
- Numerical implementation of finite result only: four-dimensional matrix elements.
- Finite result: (*relatively*) simple functions multiplying lower multiplicity structures – i.e. LO-like or NLO-like, with and without boosts – and regulated double-real term.

NNLO: Real-real Corrections

Real-real corrections – process $q(p_1)\bar{q}(p_2) \to V + g(p_4)g(p_5)$.

$$2s \cdot \mathrm{d}\sigma^{\mathrm{RR}} = \frac{1}{2!} \int [\mathrm{d}g_4][\mathrm{d}g_5] F_{LM}(1,2,4,5).$$

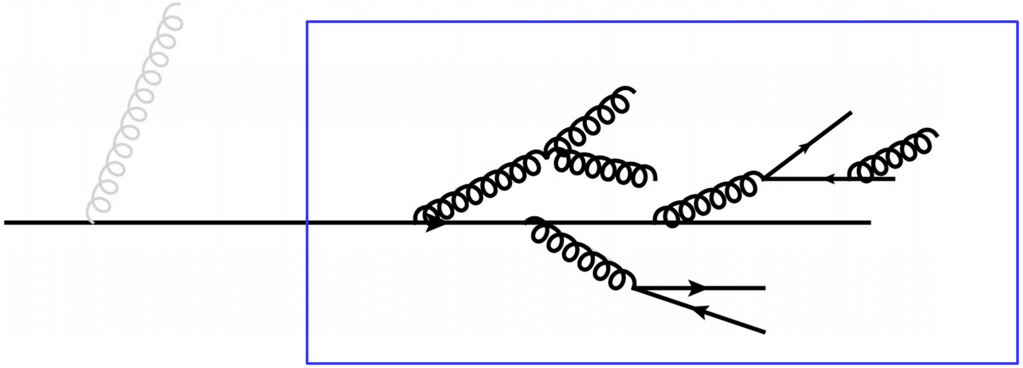
Singularity structure much more complicated:

- p_4 or $p_5 \to$ soft.
- p_4 or $p_5 \to$ collinear to initial state partons.
- p_4 or $p_5 \to$ collinear to each other.
- Combination of the above – can approach **each limit in different ways!**
- Need to integrate over **unresolved** phase space of each gluon while avoiding **overlapping** singularities

Separating the singularities is the name of the game!

Color coherence

- On-shell, gauge-invariant QCD scattering amplitudes : color coherence.
- Soft gluon cannot resolve details of later splittings; only sees total color charge.



Factorization of the amplitude in the soft limit is **insensitive** to any other radiation.

→ Soft emissions can be treated **independently** of collinear emissions:

- Regularize soft singularities first, then collinear singularities.
- Energies and angles can be independently parametrized – no need for energy-angle ordering.

Treatment of real-real singularities

- **Step 1:** New limit operators.

$$\mathbb{S}A = \lim_{E_4, E_5 \to 0} A, \text{ at fixed } E_5/E_4,$$

$$\mathbb{C}_i A = \lim_{\rho_{4i}, \rho_{5i} \to 0} A, \text{ with non vanishing } \rho_{4i}/\rho_{5i}, \rho_{45}/\rho_{4i}, \rho_{45}/\rho_{5i},$$

and recall $S_i A = \lim_{E_i \to 0} A \qquad C_{ij} A = \lim_{\rho_{ij} \to 0} A.$

- **Step 2:** Order gluon energies $E_4 > E_5$.

$$2\,\mathrm{s} \cdot \mathrm{d}\sigma^{\mathrm{RR}} = \int [\mathrm{d}g_4][\mathrm{d}g_5]\theta(E_4 - E_5) F_{LM}(1,2,4) \equiv \langle F_{LM}(1,2,4,5) \rangle.$$

- Gluon energies bounded by $E_{\max}$.
- Energies defined in CoM frame.
- Soft singularities: either **double soft** or ***p*₅ soft.**

Soft singularities

- **Step 3:** Regulate the soft singularities:

$$\left\langle F_{LM}(1,2,4,5)\right\rangle = \left\langle \mathbb{S} F_{LM}(1,2,4,5)\right\rangle + \left\langle S_5 (I-\mathbb{S}) F_{LM}(1,2,4,5)\right\rangle + \left\langle (I-S_5)(I-\mathbb{S}) F_{LM}(1,2,4,5)\right\rangle.$$

- First term: both p_4 and p_5 soft.
- Second term: p_5 soft, soft singularities in p_4 are regulated.
- Third term: regulated against all soft singularities.
- Final term still contains **overlapping** collinear singularities.

Phase-space partitioning

- **Step 4:** Introduce **phase-space partitions**

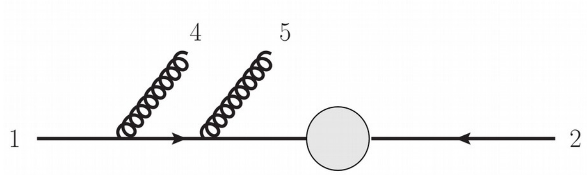
$$1 = w^{14,15} + w^{24,25} + w^{14,25} + w^{15,24}.$$

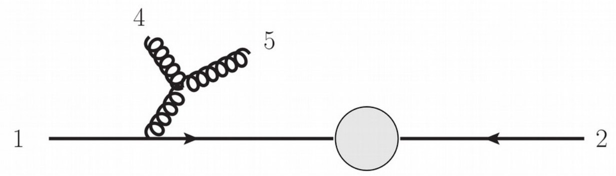
with

$$C_{42}w^{14,15} = C_{52}w^{14,15} = 0 \quad \Rightarrow \quad w^{14,15} \text{ contains } C_{41},\ C_{51},\ C_{45}$$

$$C_{41}w^{24,25} = C_{51}w^{24,25} = 0 \qquad w^{24,25} \text{ contains } C_{42},\ C_{52},\ C_{45}$$

Triple collinear partition



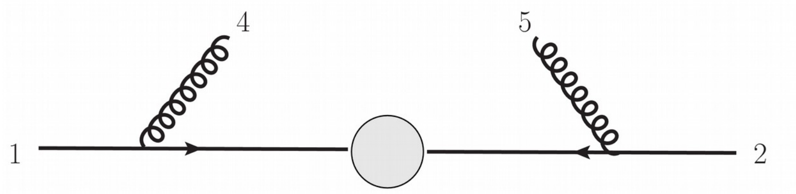


and

$$C_{42}w^{14,25} = C_{51}w^{14,25} = C_{45}w^{14,25} = 0 \quad \Rightarrow \quad w^{14,25} \text{ contains } C_{41},\ C_{52}$$

$$C_{41}w^{15,24} = C_{52}w^{15,24} = C_{45}w^{15,24} = 0 \qquad w^{15,24} \text{ contains } C_{42},\ C_{51}$$

Double collinear partition

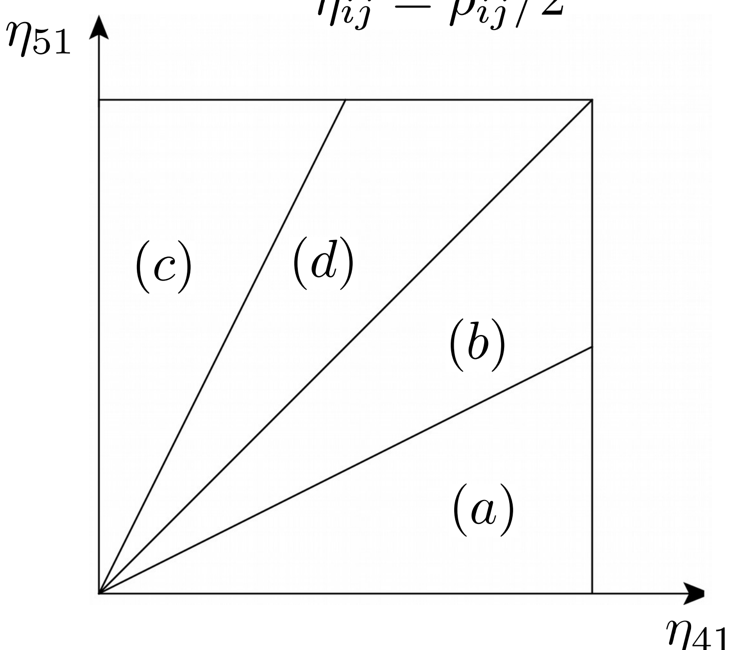


Sector Decomposition

- **Step 5: Sector decomposition:**
- Triple collinear sectors still have **overlapping** singularities.
- Define angular ordering to separate singularities.

$$1 = \theta\left(\eta_{51} < \frac{\eta_{41}}{2}\right) + \theta\left(\frac{\eta_{41}}{2} < \eta_{51} < \eta_{41}\right) + \theta\left(\eta_{41} < \frac{\eta_{51}}{2}\right) + \theta\left(\frac{\eta_{51}}{2} < \eta_{41} < \eta_{51}\right) \equiv \theta^{(a)} + \theta^{(b)} + \theta^{(c)} + \theta^{(d)}.$$

$\eta_{ij} = \rho_{ij}/2$



- Thus the limits are

$$\theta^{(a)} : C_{51} \qquad \theta^{(b)} : C_{45}$$
$$\theta^{(c)} : C_{41} \qquad \theta^{(d)} : C_{45}$$

- Each sector/partition has only one collinear singularity – **no overlaps!**
- Sectors *a,c* and *b,d* same to *4* ↔ *5*, but recall energy ordering.

Removing collinear singularities (I)

We can write **soft regulated** term as

$$\Big\langle (I-S_5)(I-\mathbb{S})F_{LM}(1,2,4,5)\Big\rangle = \Big\langle F_{LM}^{s_r c_s}(1,2,4,5)\Big\rangle + \Big\langle F_{LM}^{s_r c_t}(1,2,4,5)\Big\rangle + \Big\langle F_{LM}^{s_r c_r}(1,2,4,5)\Big\rangle,$$

with

$$\langle F_{LM}^{s_r c_r}(1,2,4,5)\rangle = \sum_{(ij)\in dc} \Bigg\langle [I-\mathbb{S}]\,[I-S_5]\Big[(I-C_{5j})(I-C_{4i})\Big][\mathrm{d}g_4][\mathrm{d}g_5]w^{i4,j5}F_{LM}(1,2,4,5)\Bigg\rangle$$
$$+\sum_{i\in tc}\Bigg\langle [I-\mathbb{S}]\,[I-S_5]\Big[\theta^{(a)}\,[I-\mathbb{C}_i]\,[I-C_{5i}] + \theta^{(b)}\,[I-\mathbb{C}_i]\,[I-C_{45}]$$
$$+\theta^{(c)}\,[I-\mathbb{C}_i]\,[I-C_{4i}] + \theta^{(d)}\,[I-\mathbb{C}_i]\,[I-C_{45}]\Big][\mathrm{d}g_4][\mathrm{d}g_5]w^{i4,i5}$$
$$\times F_{LM}(1,2,4,5)\Bigg\rangle.$$

- All singularities removed through **nested subtractions** of soft and collinear divergences – evaluated in four dimensions.
- Only term involving fully-resolved matrix element $F_{LM}(1,2,4,5)$.

Removing collinear singularities (II)

$$\left\langle (I - S_5)(I - \mathbb{S}) F_{LM}(1,2,4,5) \right\rangle = \left\langle F_{LM}^{s_r c_s}(1,2,4,5) \right\rangle + \left\langle F_{LM}^{s_r c_t}(1,2,4,5) \right\rangle + \left\langle F_{LM}^{s_r c_r}(1,2,4,5) \right\rangle .$$

Remaining two terms contain singularities:

$\left\langle F_{LM}^{s_r c_s}(1,2,4,5) \right\rangle$

- Soft-regulated single-collinear subtraction.
- Partitioning factors and sectors: **one collinear singularity** in each term.

$\left\langle F_{LM}^{s_r c_t}(1,2,4,5) \right\rangle$

- Triple-collinear subtraction – all other singularities regulated.

Treating singular limits

We have four singular subtraction terms:

$$\left\langle \mathbb{S} F_{LM}(1,2,4,5) \right\rangle \quad \left\langle S_5 (I - \mathbb{S}) F_{LM}(1,2,4,5) \right\rangle \quad \left\langle F_{LM}^{s_r c_s}(1,2,4,5) \right\rangle \quad \left\langle F_{LM}^{s_r c_t}(1,2,4,5) \right\rangle$$

We know how to treat them:

- Gluon(s) decouple partially or completely.
- Decouple completely:
  - Integrate over gluonic angles and energy.
- Decouple partially:
  - Integrate over gluonic angles.
  - Integral(s) over energy → integrals over splitting function in *z.*
- Results in **lower particle multiplicity terms** convoluted with (new) splitting functions.

Soft subtraction terms

Double soft subtraction: $\langle S F_{LM}(1,2,4,5) \rangle$

- Both gluons decouple: $S F_{LM}(1,2,4,5) = g_{s,b}^4 \mathrm{Eik}_2(1,2,4,5) F_{LM}(1,2).$

Double eikonal function

- Overall energy factorizes → integrand *independent* of partonic energy.
- Integral is **constant** for color-singlet production.

$$\langle S F_{LM}(1,2,4,5) \rangle = [\alpha_s]^2 \langle E_{\max}^{-4\epsilon} F_{LM}(1,2) \rangle \left(\frac{c_4}{\epsilon^4} + \frac{c_3}{\epsilon^3} + \frac{c_2}{\epsilon^2} + \frac{c_1}{\epsilon} + c_0 \right).$$

c_i : Constants for color-singlet production

- Abelian contribution: product of NLO structures.
- Non-abelian: more complicated:
  - Initially integrate over relative energies and over gluonic angles *numerically*.
  - Integrals now done *analytically*.

Single-soft subtraction term

Single-soft subtraction: $\langle (I - \mathbb{S}) S_5 F_{LM}(1,2,4,5) \rangle$

- **Gluon 5** decouples – integrate over it:

$$\langle (I - \mathbb{S}) S_5 F_{LM}(1,2,4,5) \rangle = \frac{[\alpha_s]}{\epsilon^2} \langle E_4^{-2\epsilon} f(\rho_{12}, \rho_{14}, \rho_{24}, \epsilon) \, (I - S_4) F_{LM}(1,2,4) \rangle.$$

- **Not integrable:** contains NLO-like collinear singularities.
- Regulate the singularities as done at NLO:

$$\left\langle \left[I - \mathbb{S} \right] S_5 F_{LM}(1,2,4,5) \right\rangle = \frac{[\alpha_s]}{\epsilon^2} \langle E_4^{-2\epsilon} f(\rho_{12}, \rho_{14}, \rho_{24}, \epsilon) \times \left[I - C_{41} - C_{42} \right] \left[I - S_4 \right] F_{LM}(1,2,4) \rangle - \frac{[\alpha_s]^2 s^{-2\epsilon}}{\epsilon^3} f(\epsilon) \int_0^1 \mathrm{d}z \, \mathcal{P}_{qq,RR_1}(z) \left\langle \frac{F_{LM}(z \cdot 1, 2) + F_{LM}(1, z \cdot 2)}{z} \right\rangle.$$

- First term: Regulated through nested subtractions.
- Second term: LO matrix element convoluted with splitting function.

Collinear subtraction terms

- General structure: splitting functions with **explicit poles** convoluted with lower multiplicity terms:

$$F_{LM}(z\cdot 1,2) \qquad F_{LM}(1,z\cdot 2) \qquad F_{LM}(1,2)$$

$$F_{LM}(z\cdot 1,2,4) \quad F_{LM}(1,z\cdot 2,4) \quad F_{LM}(1,2,4)$$

- Further singularities regulated →

$$\left\langle \hat{\mathcal{O}}_{NLO} F_{LM}(z\cdot 1,2,4)\right\rangle \qquad \left\langle \hat{\mathcal{O}}_{NLO} F_{LM}(1,z\cdot 2,4)\right\rangle \qquad \left\langle \hat{\mathcal{O}}_{NLO} F_{LM}(1,2,4)\right\rangle$$

$$\langle F_{LM}(z\cdot 1,\bar{z}\cdot 2)\rangle \qquad \langle F_{LM}(z\cdot 1,2)\rangle \qquad \langle F_{LM}(1,z\cdot 2)\rangle \qquad \langle F_{LM}(1,2)\rangle$$

- At NLO, pole cancellation achieved in *each structure.*

➡ Recombine structures from different sectors/partitions.

Double-collinear partition

In single-collinear subtraction:

$$DC = \Big\langle [I - \mathbb{S}]\,[I - S_5]\Big[\big(C_{41}[\mathrm{d}g_4] + C_{52}[\mathrm{d}g_5]\big)\, w^{14,25} + \big(C_{42}[\mathrm{d}g_4] + C_{51}[\mathrm{d}g_5]\big)\, w^{24,15}\Big] \times F_{LM}(1,2,4,5)\Big\rangle.$$

Collinear limit acts on phase space!

Consider fourth term:

$$\big\langle [I - \mathbb{S}]\,[I - S_5]\,C_{51}[\mathrm{d}g_5] w^{24,15} F_{LM}(1,2,4,5)\big\rangle$$

$$= -\frac{[\alpha_s] s^{-\epsilon}}{\epsilon} \int\limits_{z_{\min}(E_4)}^{1} \frac{\mathrm{d}z}{(1-z)^{1+2\epsilon}} \hat{\mathcal{P}}_{qq}^{(-)}(z) \big\langle \tilde{w}_{5||1}^{24,15} F_{LM}(z \cdot 1, 2, 4)\big\rangle.$$

$$z_{\min}(E_4) = 1 - E_4/E_1$$

Ideally: integral on [0:1]

Consider first term:

$$z_{\max}(E_5) \equiv 1 - E_5/E_1$$

$$\big\langle [I - \mathbb{S}]\,[I - S_5]\,C_{41}[\mathrm{d}g_4] w^{14,25} F_{LM}(1,2,4,5)\big\rangle$$

$$= -\frac{[\alpha_s] s^{-\epsilon}}{\epsilon} \int\limits_{0}^{z_{\max}(E_5)} \frac{\mathrm{d}z}{(1-z)^{1+2\epsilon}} \mathcal{P}_{qq}(z) \big\langle \tilde{w}_{4||1}^{14,25} [I - S_5] F_{LM}(z \cdot 1, 2, 5)\big\rangle.$$

Combining partitions

Rename the resolved gluon 4 in the first term and combine:

$$z_{\max}(E_4) \equiv 1 - E_4/E_1 = z_{\min}(E_4)$$

$$\left\langle [I - \mathbb{S}]\,[I - S_5]\,[C_{41}[\mathrm{d}g_4]w^{14,25} + C_{51}[\mathrm{d}g_4]w^{15,24} F_{LM}(1,2,4,5)\right\rangle$$

$$= -\frac{[\alpha_s]s^{-\epsilon}}{\epsilon}\int_0^1 \frac{\mathrm{d}z}{(1-z)^{1+2\epsilon}}\left\langle \tilde{w}^{15,24}_{5||1}\left(\hat{\mathcal{P}}^{(-)}_{qq}(z)\,[I - S_4]\,F_{LM}(z\cdot 1,2,4)+\right.\right.$$

$$\left.\left.\theta(z_4 - z)2C_F\,[I - S_4]\,F_{LM}(1,2,4) + \theta(z_4 - z)\hat{\mathcal{P}}^{(-)}_{qq}(z)S_4 F_{LM}(z\cdot 1,2,4)\right)\right\rangle.$$

- **Simplifications** after combining sectors.
- Different splitting functions in two terms → restrictions on *z.*
- Similar simplifications on combining terms from **double** & **triple** collinear partitions.
- Regulate against NLO-like singularities → **no restrictions** on *z* in final result!

Collinear subtraction terms

Terms with double-unresolved collinear limits: $\langle F_{LM}^{s_r c_t}(1,2,4,5)\rangle$

- For **triple-collinear partitions**, limits involve complicated *triple-collinear splitting function:*
  - Integration is non-trivial.
  - Expand the *integrand* in ϵ.
  - Evaluate numerically (analytic evaluation should also be possible).
  - Produces $1/\epsilon$ pole & finite term.

Double-real cross section: recap

We have now written complete double-real cross section as:

- Splitting functions convoluted with LO matrix elements – including explicit $1/\epsilon^4$ (and lower) poles.

$$\langle F_{LM}(z\cdot 1, \bar{z}\cdot 2)\rangle\,,\ \langle F_{LM}(z\cdot 1, 2)\rangle\,,\ \langle F_{LM}(1, z\cdot 2)\rangle\,,\ \langle F_{LM}(1,2)\rangle$$

- Splitting functions convoluted with NLO matrix elements, regulated by iterative subtraction – including explicit $1/\epsilon^2$ (and lower) poles.

$$\langle \mathcal{O}_{NLO} F_{LM}(z\cdot 1, 2, 4)\rangle\,,\ \langle \mathcal{O}_{NLO} F_{LM}(1, z\cdot 2, 4)\rangle\,,\ \langle \mathcal{O}_{NLO} F_{LM}(1, 2, 4)\rangle$$

- NNLO matrix elements, regulated by iterative subtraction – **finite.**
- All singularities made explicit.
- Evaluate in **four dimensions.**

Pole cancellation

- Combine poles from real-real, real-virtual, virtual-virtual, pdf renormalization.
- Poles *must* cancel for each structure F_{LM} :
  - ✔ $\langle \mathcal{O}_{NLO} F_{LM}(z\cdot 1,2,4)\rangle$, $\langle \mathcal{O}_{NLO} F_{LM}(1,z\cdot 2,4)\rangle$, $\langle \mathcal{O}_{NLO} F_{LM}(1,2,4)\rangle$ cancel *analytically*.
  - ✔ $\left\langle F_{LV}^{\text{fin}}(1,2)\right\rangle$ cancels *analytically*.
  - ✔ $\langle F_{LM}(z\cdot 1,\bar{z}\cdot 2)\rangle$, $\langle F_{LM}(z\cdot 1,2)\rangle$, $\langle F_{LM}(1,z\cdot 2)\rangle$, $\langle F_{LM}(1,2)\rangle$ cancels *analytically* to $\mathcal{O}(1/\epsilon^2)$, *numerically* at $\mathcal{O}(1/\epsilon)$ (triple collinear limit integrated numerically).

Finite remainders

- **Relatively compact** expressions for finite remainders for each *lower-multiplicity structure.*
- Extension of NLO calculation to NNLO:
  - Boosted LO and NLO results multiplied by known functions.
  - Nested subtraction for real-real contribution.

$$
\begin{aligned}
&\mathrm{d}\hat{\sigma}^{\mathrm{NNLO}}_{F_{LM}(z\cdot 1,2)}(\mu^2 = s) = \\
&\left[\frac{\alpha_s(\mu)}{2\pi}\right]^2 \int_0^1 \mathrm{d}z \Bigg\{ C_F^2 \Bigg[8\tilde{\mathcal{D}}_3(z) + 4\tilde{\mathcal{D}}_1(z)(1+\ln 2) + 4\tilde{\mathcal{D}}_0(z)\left[\frac{\pi^2}{3}\ln 2 + 4\zeta_3\right] \\
&+ \frac{5z-7}{2} + \frac{5-11z}{2}\ln z + (1-3z)\ln 2 \ln z + \ln(1-z)\left[\frac{3}{2}z - (5+11z)\ln z\right] \\
&\quad + 2(1-3z)\mathrm{Li}_2(1-z) \\
&+ (1-z)\left[\frac{4}{3}\pi^2 + \frac{7}{2}\ln^2 2 - 2\ln^2(1-z) + \ln 2\left[4\ln(1-z) - 6\right] + \ln^2 z \right. \\
&\quad \left. + \mathrm{Li}_2(1-z)\right] + (1+z)\left[-\frac{\pi^2}{3}\ln z - \frac{7}{4}\ln^2 2 \ln z - 2\ln 2 \ln(1-z)\ln z \right. \\
&\quad \left. + 4\ln^2(1-z)\ln z - \frac{\ln^3 z}{3} + \left[4\ln(1-z) - 2\ln 2\right]\mathrm{Li}_2(1-z)\right] \\
&\quad + \left[\frac{1+z^2}{1-z}\right]\ln(1-z)\left[3\mathrm{Li}_2(1-z) - 2\ln^2 z\right] - \frac{5-3z^2}{1-z}\mathrm{Li}_3(1-z) \\
&\quad + \frac{\ln z}{(1-z)}\left[12\ln(1-z) - \frac{3-5z^2}{2}\ln^2(1-z) - \frac{7+z^2}{2}\ln 2 \ln z\right]\Bigg] \\
&\quad + C_A C_F \Bigg[-\frac{22}{3}\tilde{\mathcal{D}}_2(z) + \left(\frac{134}{9} - \frac{2}{3}\pi^2\right)\tilde{\mathcal{D}}_1(z) + \left[-\frac{802}{27} + \frac{11}{18}\pi^2 \right. \\
&\quad \left. + (2\pi^2 - 1)\frac{\ln 2}{3} + 11\ln^2 2 + 16\zeta_3\right]\tilde{\mathcal{D}}_0(z) + \frac{37-28z}{9} + \frac{1-4z}{3}\ln 2 \\
&\quad - \left(\frac{61}{9} + \frac{161}{18}z\right)\ln(1-z) + (1+z)\ln(1-z)\left[\frac{\pi^2}{3} - \frac{22}{3}\ln 2\right] \\
&\quad - (1-z)\left[\frac{\pi^2}{6} + \mathrm{Li}_2(1-z)\right] - \frac{2+11z^2}{3(1-z)}\ln 2 \ln z - \frac{1+z^2}{1-z}\mathrm{Li}_2(1-z)\times \\
&\quad \times \left[2\ln 2 + 3\ln(1-z)\right]\Bigg] + R_+^{(c)}\mathcal{D}_0(z) + R^{(c)}(z)\Bigg\}\left\langle \frac{F_{LM}(z\cdot 1,2)}{z}\right\rangle.
\end{aligned}
$$

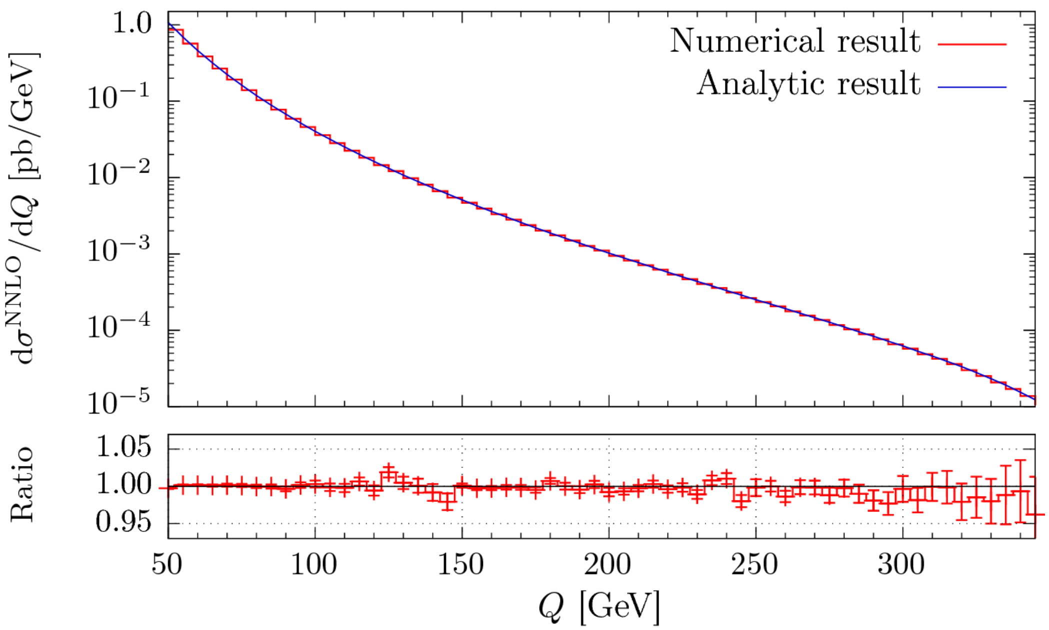
Proof-of-principle

- Calculate $pp \to \gamma^* + X \to e^+e^- + X$ to NNLO
- Lepton pairs with invariant mass $50 \text{ GeV} \leq Q \leq 350 \text{ GeV}.$
- Extract results from [Hamberg, Matsuura, van Neerven '91] to compare (analytic in *Q*).
- NNLO *contributions* for $q\bar{q} \to \gamma^* + gg$ channel:

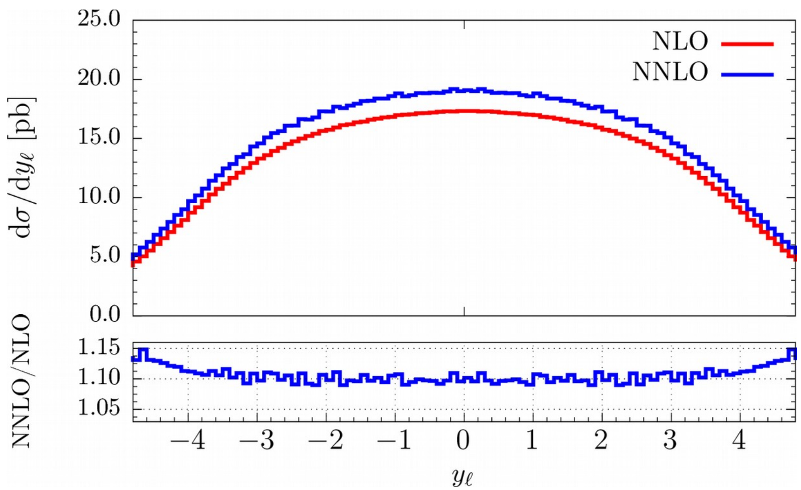
$$\mathrm{d}\sigma^{\mathrm{NNLO}} = 14.471(4)\ \mathrm{pb}$$

$$\mathrm{d}\sigma^{\mathrm{NNLO}}_{\mathrm{analytic}} = 14.470\ \mathrm{pb}$$

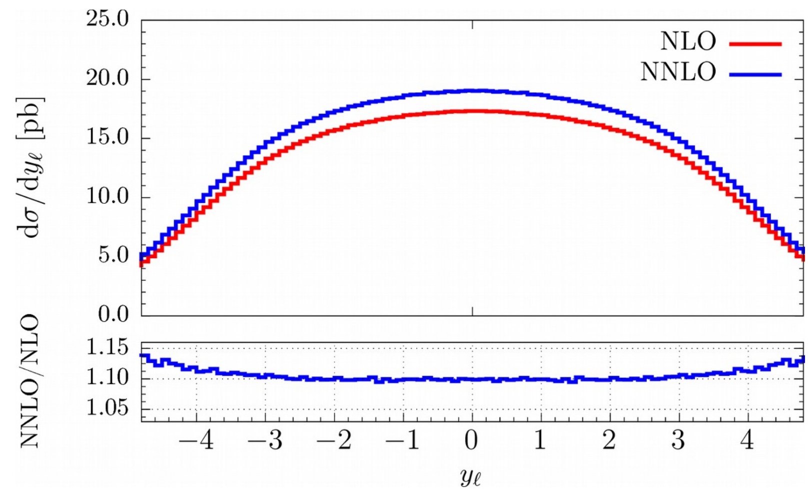
- Sub per-mille agreement in cross sections.
- Per-mille to percent agreement across **5 orders of magnitude** in *Q*.



Differential distributions (I)



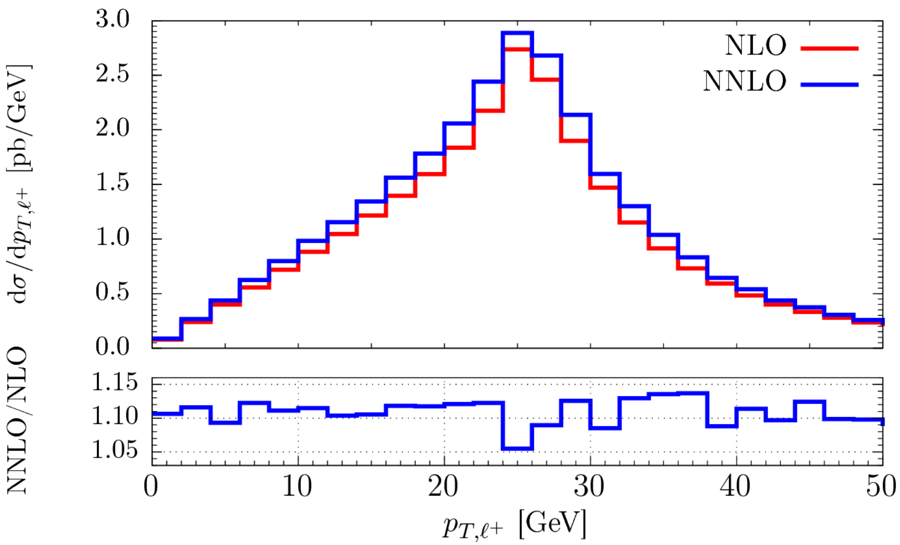
O(10 CPU hours) runtime



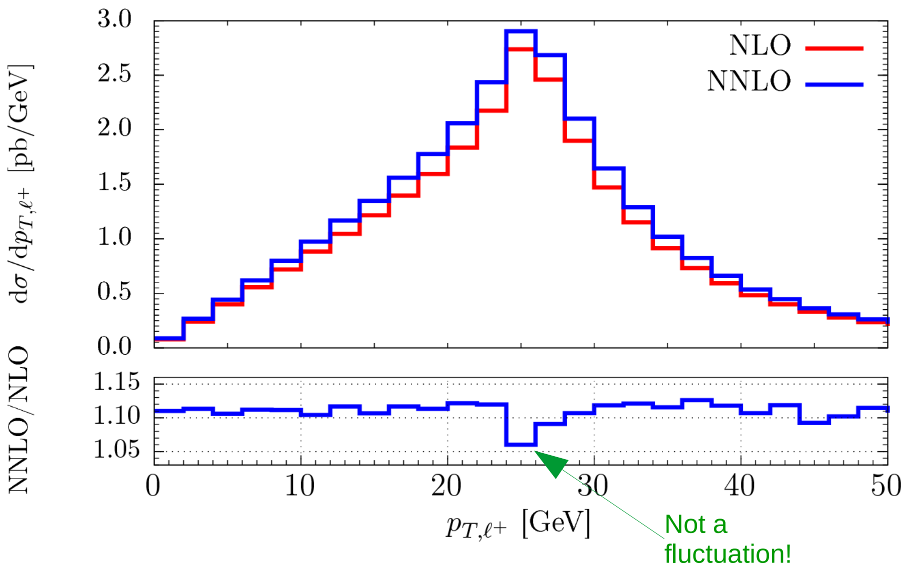
O(100 CPU hours) runtime

- Lepton rapidity.
- O(10 CPU hours): percent-level bin-to-bin fluctuations.
- O(100 CPU hours): per-mille bin-to-bin fluctuations.

Differential distributions (II)



O(10 CPU hours) runtime



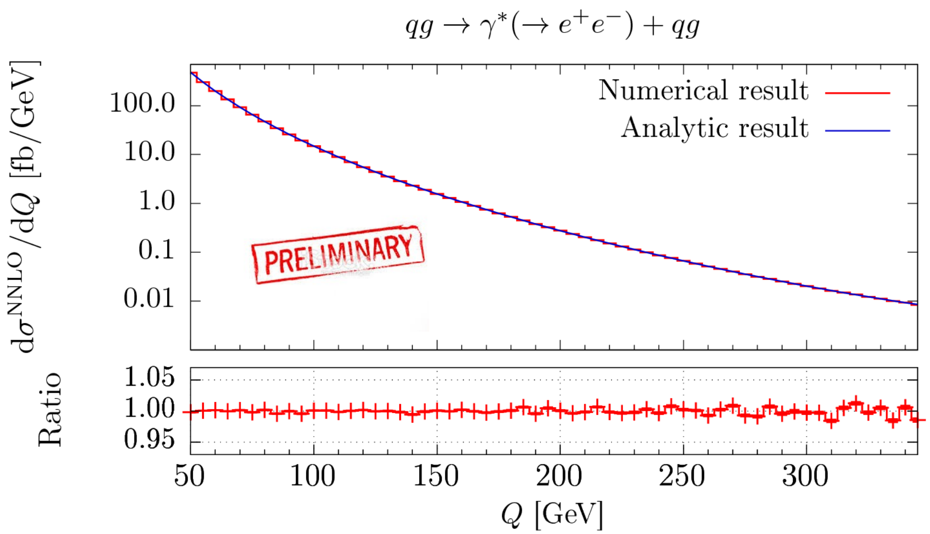
O(100 CPU hours) runtime

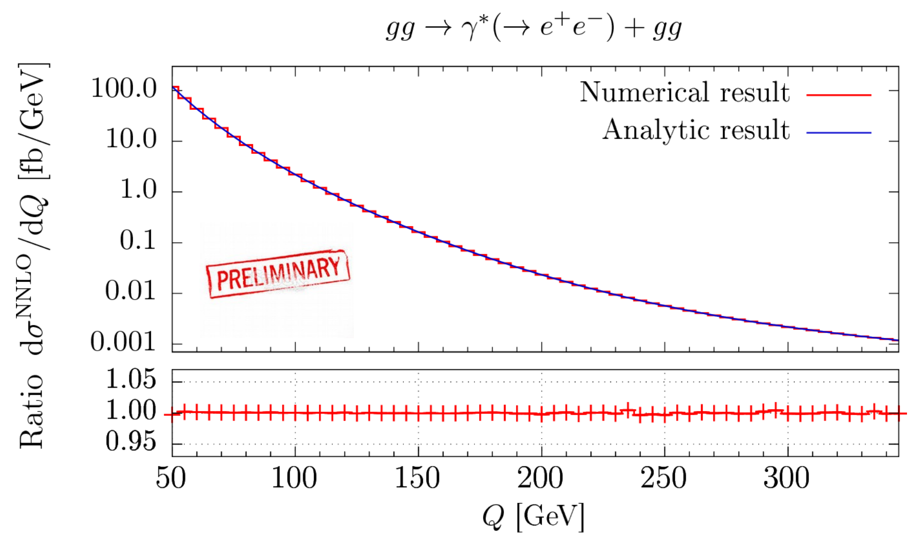
- Lepton transverse momentum.
- O(100 CPU hours): percent-level bin-to-bin fluctuations.
- Delicate observable: receives contributions from large range of invariant masses.
  - Improves once introduce Z boson propagator.
  - Comparison with other NNLO codes?

KIT

Other partonic channels

- Other partonic channels (*qg, gq, gg, qq→qq)* follow same strategy, but fewer limits.
- All calculated within this approach for DY and W production.
  - Similar agreement with analytic results, including for numerically tiny channels.





- Independent of matrix element – can be used for any $q\bar{q}$ color singlet (gg a work in progress).

$H \to bb$ decay

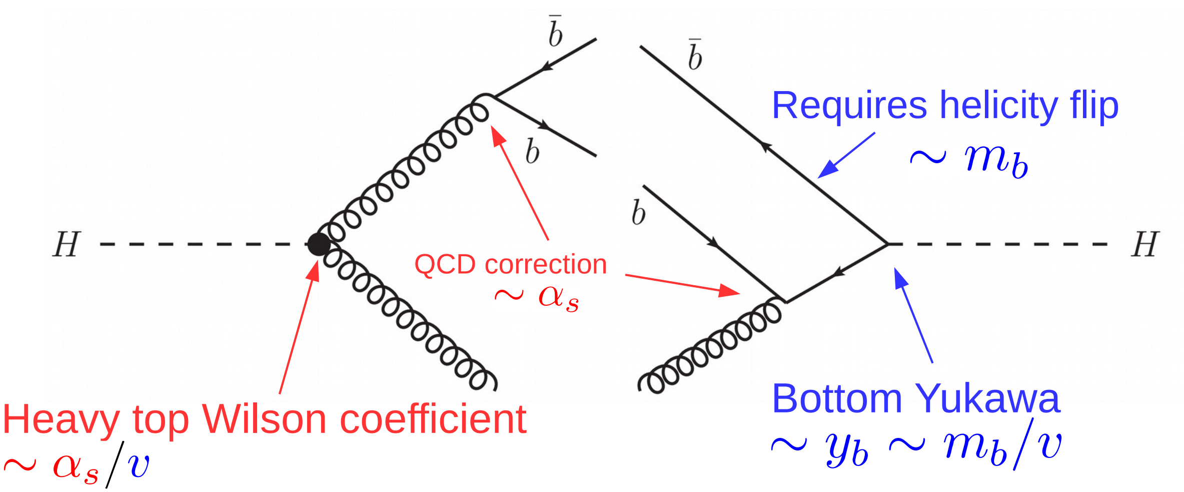
- Applied **nested soft-collinear subtraction** to $H \to bb$ decay at NNLO.
- Applicable to $q\bar{q}$ production from color singlet, e.g. $e^+e^- \to q\bar{q}$.
- Emissions collinear to final state *b*-quark can be absorbed into momentum of *b*-quark
  - **No need** for "boosted" structures (obviously since no pdfs).
- Structures are then

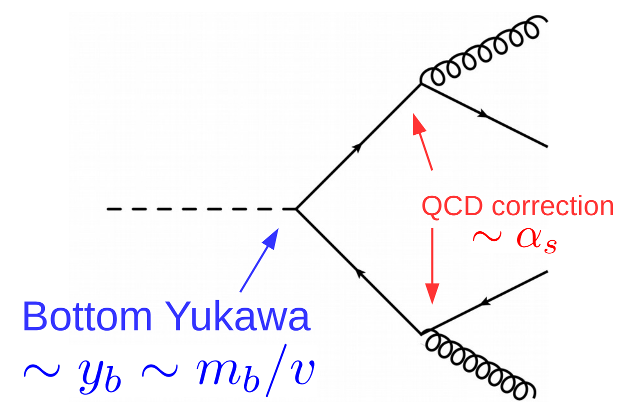
$$\langle \mathcal{O}_{NLO} F_{LM}(1,2,4) \rangle \qquad \langle \mathcal{O}_{NLO} F_{LM}^{\mu\nu}(1,2,4) \rangle \qquad \langle F_{LM}(1,2) \rangle$$

- Poles cancel within each structure.
- Expressions for finite result extremely compact.

Bottom mass effects

However, $H \to bb$ **NOT WELL DEFINED** for **massless** *b*-quarks beyond NLO!





Top-loop interference contribution

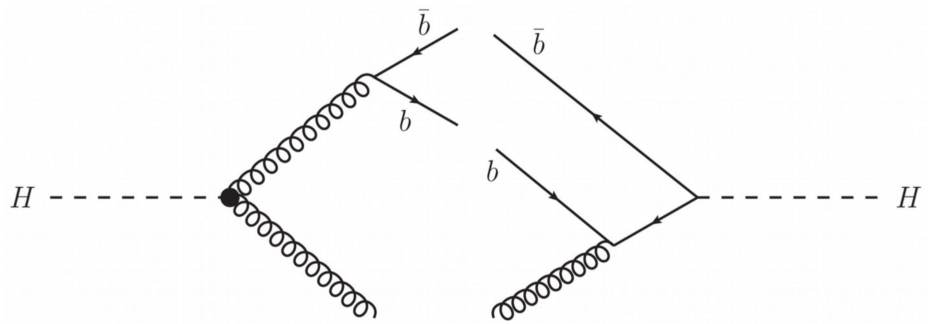
$\sim \alpha_s^2 m_b^2/v^2 \sim \alpha_s^2 y_b^2$

"Regular" contribution.

Squared $\sim \alpha_s^2 m_b^2/v^2 \sim \alpha_s^2 y_b^2$

Interference contribution has **identical parametric scaling** to other NNLO corrections.

Bottom mass interference



Obvious strategy: factor out one power of m_b and then take $m_b = 0$.
BUT:

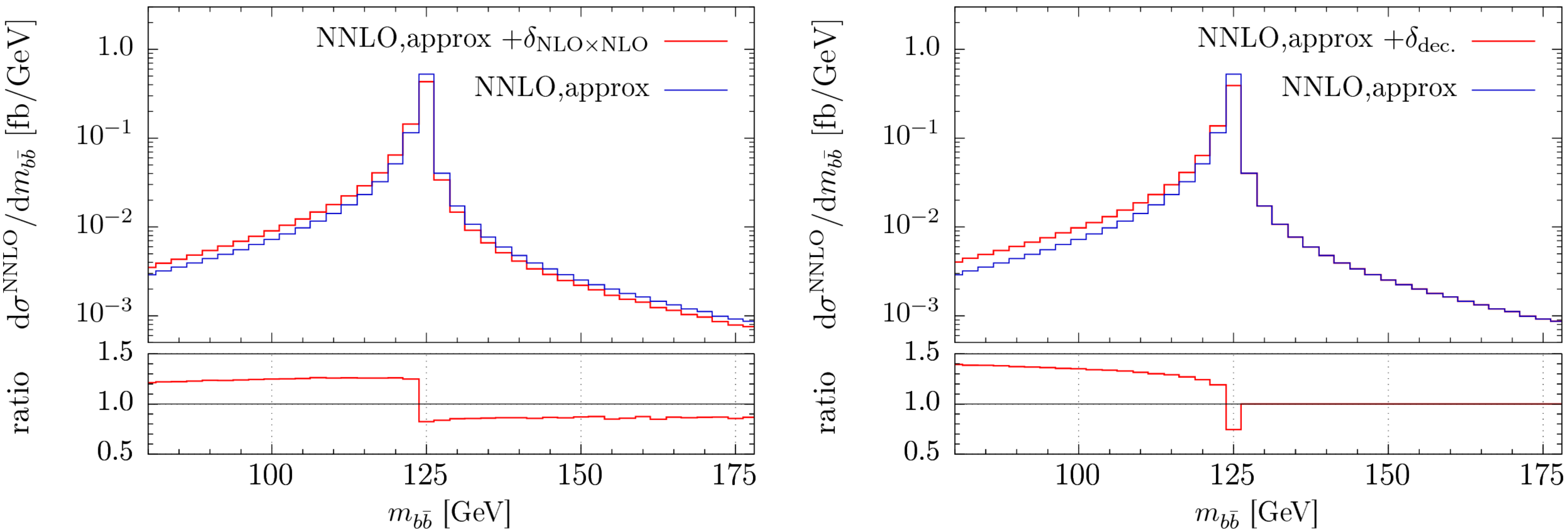
- Reduced matrix elements have unusual IR behaviour, e.g. **soft singularities from quarks!**
- $\log(m_b)$ **don't cancel** between real and virtual interference terms – inclusive definition not possible in massless limit!
- **Cannot** be regulated using flavor-kT algorithm (doesn't regulate soft quark singularity).
- Calculation in double-log approx: **~ 30%** of NNLO corrections to *H* → *bb* decay.

⇒ NNLO calculation of *H*→*bb* to massive bottom quarks required.

Associated production with $H \to bb$

Considered $pp \to W^-(\to e^-\bar{\nu})H(\to b\bar{b})$ to NNLO in both production and decay.

(cf. Ferrera, Somogyi, Tramontano, hep-ph/1705.10304)



Summary

- Nested soft-collinear subtraction method for NNLO calculations – based on FKS + sector decomposition.
- Characterized by decoupling of soft and collinear limits.
- Develop iterative subtraction procedure:
  - Manifestly regulated finite term.
  - Integrated subtraction terms: convolutions of splitting function with explicit poles with lower multiplicity processes.
  - Pole cancellation independent of matrix elements.
- **Process independent** (colorless final state).
- Tested in DY and *W* production *for all partonic channels; H* → *bb* decay
  - **Excellent agreement** with analytic results.

Future work:

- **Extension to colored final states.**
- **Efficient implementation in numerical integration.**
- Better parametrization of phase space of radiated partons?
- Improved definition/understanding of partition functions?
- Include alpha-parameters.
- …

THANK YOU!

QUESTIONS?