

# Recent developments in qT subtraction

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Subtracting Infrared Singularities Beyond NLO

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Universität  
Zürich<sup>UZH</sup>



SWISS NATIONAL SCIENCE FOUNDATION

# Outline

- Introduction
- $q_T$  subtraction
- The *MATRIX* project
- Most recent results:  $pp \rightarrow HH+X$  at NNLO
- Beyond colour singlet: heavy-quark production
- Summary & Outlook

# NNLO methods

Broadly speaking there are two approaches that we can follow:

- Organise the calculation from scratch so as to cancel all the singularities
  - sector decomposition T. Binoth, G.Heinrich (2000,2004)  
C.Anastasiou, K.Melnikov, F.Petriello (2004)
  - antenna subtraction A. & T. Gehrmann, N. Glover (2005)
  - “colourful” subtraction G, Somogyi, Z. Trocsanyi,  
V. Del Duca (2005, 2007)
  - join subtraction and sector decomposition M.Czakon (2010,2011)  
R.Boughezal, K.Melnikov, F.Petriello (2011)  
F.Caola, K.Melnikov, R.Rontsch (2017)
- Start from an inclusive NNLO calculation (sometimes obtained through resummation) and combine it with an NLO calculation for  $n+1$  parton process
  - $q_T$  subtraction S.Catani, MG (2007)
  - “N-jettiness” method R.Boughezal, C.Focke, X.Liu, F.Petriello (2015)  
F.Tackmann et al. (2015)
  - recently introduced “Born projection” method for VBF M.Cacciari, F.Dreyer, A.Karlberg, G.Salam, G.Zanderighi (2015)

...and then we need the relevant two-loop amplitudes !

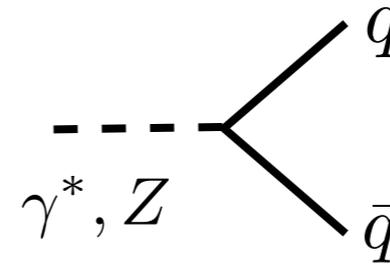
C.Anastasiou, F.Caola, M.Czakon, T.Gehrmann, N.Glover, M.Jaquier, A. Koukoutsakis  
C.Oleari, K.Melnikov, L.Tancredi, M.E. Tejeda-Yeomans, A. von Manteuffel and many others

# Our method

S. Catani, MG (2007)

To make the discussion simpler, let us consider  $e^+e^- \rightarrow 2 \text{ jets}$  as an example

 Same LO topology after crossing



Consider the two-jet rate:

$$R_{2\text{jets}}^{LO} = 1$$

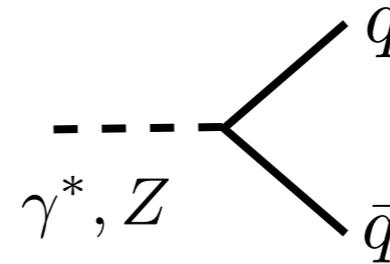
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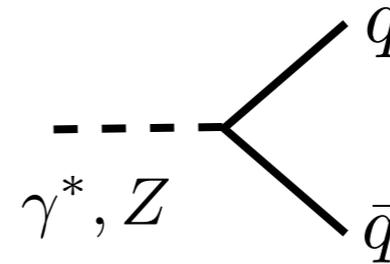
At NLO all events are two-jet like except those that contribute to the LO three-jet rate

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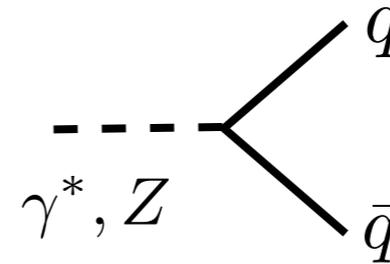
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At NNLO all events are two-jet like except those that contribute to the NLO three-jet rate and to the LO four jet rate

→  $R_{2\text{jets}}^{N^n LO}$  can be obtained through a  $N^{n-1} LO$  computation!

See e.g. NNLO calculation of  $A_{FB}$

S.Catani, M.H.Seymour (1999)

Can this idea be extended to hadron collisions ? It was !

## Computation of Higgs cross section with a jet veto up to NNLO

S. Catani, D. de Florian, MG (2001)

Jet veto: cut on jets with  $p_T > p_T^{\text{veto}}$

$$\sigma_{\text{veto}}^{LO} = \sigma_{\text{tot}}^{LO}$$

$$\sigma_{\text{veto}}^{NLO} = \sigma_{\text{tot}}^{NLO} - \sigma_{p_T^{\text{jet}} > p_T^{\text{veto}}}^{LO}$$

$$\sigma_{\text{veto}}^{NNLO} = \sigma_{\text{tot}}^{NNLO} - \sigma_{p_T^{\text{jet}} > p_T^{\text{veto}}}^{NLO}$$



(N)NLO computation can be done by having the (N)NLO total cross section and the (N)LO cross section for H+jet

Does it work for general cuts ? Yes !

Example:  $H \rightarrow \gamma\gamma$

$$p_T^\gamma > 20 \text{ GeV}$$

$$|y^\gamma| < 2.5 \text{ GeV}$$

$$E_T^{R=0.3} < 6 \text{ GeV}$$

photon isolation

$$d\sigma^{NNLO} = d\sigma_{\{p_T^\gamma > 20 \text{ GeV}, |y^\gamma| < 2.5\}}^{NNLO} - d\sigma_{\{p_T^\gamma > 20 \text{ GeV}, |y^\gamma| < 2.5; E_T^{R=0.3} > 6 \text{ GeV}\}}^{NLO}$$



**Inclusive cross section to be computed up to NNLO (no free lunch !)**



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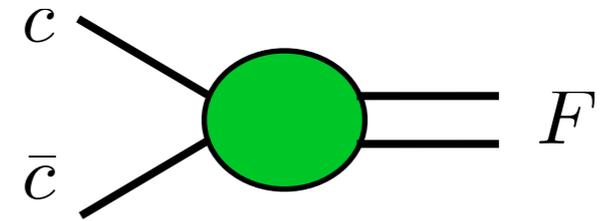
**H+jet cross section to be computed up to NLO**

# The $q_T$ subtraction method

S. Catani, MG (2007)

Let us consider a more general class of processes: the production of colourless high-mass systems  $F$  in hadron collisions ( $F$  may consist of lepton pairs, vector bosons, Higgs bosons.....)

At LO it starts with  $c\bar{c} \rightarrow F$



**Strategy:** start from NLO calculation of  $F+\text{jet}(s)$  and observe that as soon as the transverse momentum of the  $F$   $q_T \neq 0$  one can write:

$$d\sigma_{(N)NLO}^F|_{q_T \neq 0} = d\sigma_{(N)LO}^{F+\text{jets}}$$

Define a counterterm to deal with singular behaviour at  $q_T \rightarrow 0$

But.....

the singular behaviour of  $d\sigma_{(N)LO}^{F+\text{jets}}$  is well known from the resummation program of large logarithmic contributions at small transverse momenta

G. Parisi, R. Petronzio (1979)

J. Collins, D.E. Soper, G. Sterman (1985)

S. Catani, D. de Florian, MG (2000)

→ choose  $d\sigma^{CT} \sim d\sigma^{(LO)} \otimes \Sigma^F(q_T/Q)$

$$\text{where } \Sigma^F(q_T/Q) \sim \sum_{n=1}^{\infty} \left(\frac{\alpha_S}{\pi}\right)^n \sum_{k=1}^{2n} \Sigma^{F(n;k)} \frac{Q^2}{q_T^2} \ln^{k-1} \frac{Q^2}{q_T^2}$$

Then the calculation can be extended to include the  $q_T = 0$  contribution:

$$d\sigma_{(N)NLO}^F = \mathcal{H}_{(N)NLO}^F \otimes d\sigma_{LO}^F + \left[ d\sigma_{(N)LO}^{F+\text{jets}} - d\sigma_{(N)LO}^{CT} \right]$$

where I have subtracted the truncation of the counterterm at (N)LO and added a contribution at  $q_T = 0$  to restore the correct normalization

The function  $\mathcal{H}^F$  can be computed in QCD perturbation theory

$$\mathcal{H}^F = 1 + \left(\frac{\alpha_S}{\pi}\right) \mathcal{H}^{F(1)} + \left(\frac{\alpha_S}{\pi}\right)^2 \mathcal{H}^{F(2)} + \dots$$

## Let's focus on Higgs production (F=H)

The function  $\mathcal{H}^H$  can be computed in QCD perturbation theory as follows

$$\mathcal{H}^H = 1 + \left(\frac{\alpha_s}{\pi}\right) \mathcal{H}^{H(1)} + \left(\frac{\alpha_s}{\pi}\right)^2 \mathcal{H}^{H(2)} + \dots$$

S. Catani, MG (2011)

consider integral of  $q_T$   
distribution up to an  
arbitrary small  $Q_0$

$$\int_0^{Q_0^2} dq_T^2 \frac{d\hat{\sigma}_{H ab}}{dq_T^2}(q_T, M, \hat{s} = M^2/z) \equiv z\sigma_H^{(0)} \hat{R}_{ggab}^H(z, M/Q_0)$$

Up to  $\mathcal{O}(\alpha_s^2)$  the coefficients of the logarithmic expansion in  $l_0 = \ln M_H^2/Q_0^2$  are all known

D. de Florian, MG (2000)

$$\hat{R}_{gg\leftarrow ab}^{(1)}(z, M/Q_0) = l_0^2 \Sigma_{gg\leftarrow ab}^{H(1;2)}(z) + l_0 \Sigma_{gg\leftarrow ab}^{H(1;1)}(z) + \mathcal{H}_{gg\leftarrow ab}^{H(1)}(z) + \mathcal{O}(Q_0^2/M^2)$$

$$\hat{R}_{gg\leftarrow ab}^{(2)}(z, M/Q_0) = l_0^4 \Sigma_{gg\leftarrow ab}^{H(2;4)}(z) + l_0^3 \Sigma_{gg\leftarrow ab}^{H(2;3)}(z) + l_0^2 \Sigma_{gg\leftarrow ab}^{H(2;2)}(z)$$

$$+ l_0 \left( \Sigma_{ggab}^{H(2;1)}(z) - 16\zeta_3 \Sigma_{ggab}^{H(2;4)}(z) \right) + \left( \mathcal{H}_{ggab}^{H(2)}(z) - 4\zeta_3 \Sigma_{ggab}^{H(2;3)}(z) \right) + \mathcal{O}(Q_0^2/M^2)$$

The only missing one is  $\mathcal{H}_{gg\leftarrow ab}^{H(2)}(z)$

Total cross section

$q_T$  distribution

→ solve this equation to  
obtain the  $\mathcal{H}_{gg\leftarrow ab}^{H(2)}(z)$

$$\int_0^{Q_0^2} dq_T^2 \frac{d\hat{\sigma}_{H ab}}{dq_T^2}(q_T, M; z) = \hat{\sigma}_{ab}^H(z) - \int_{Q_0^2}^{\infty} dq_T^2 \frac{d\hat{\sigma}_{H ab}}{dq_T^2}(q_T, M; z)$$

Use available analytical NLO results

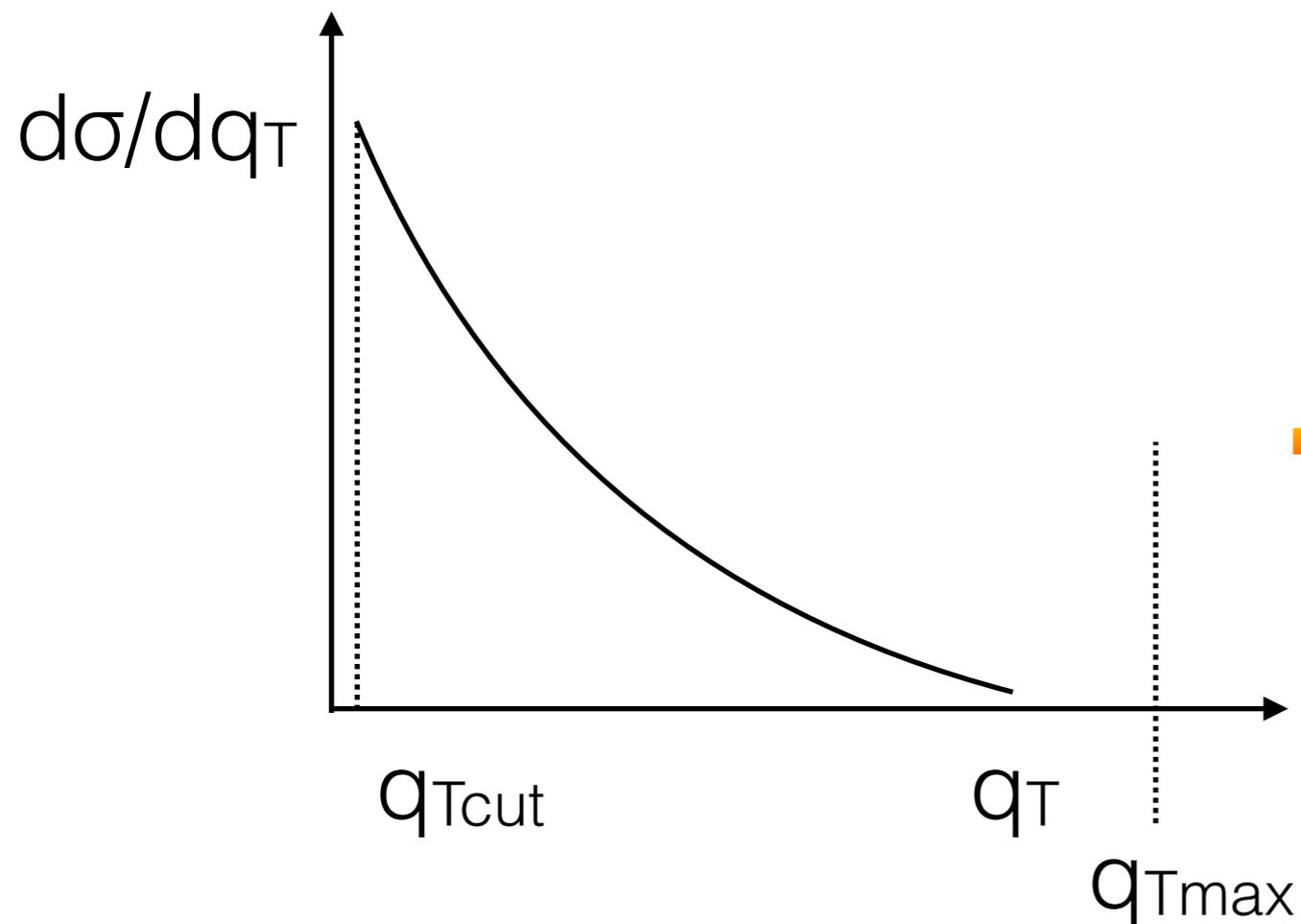
C.Glosser, C.Schmidt (2003)

For a generic  $pp \rightarrow F + X$  process:

- At NLO we need a LO calculation of  $d\sigma^{F+\text{jet}(s)}$  plus the knowledge of  $d\sigma_{LO}^{CT}$  and  $\mathcal{H}^{F(1)}$ 
  - the counterterm  $d\sigma_{LO}^{CT}$  requires the resummation coefficients  $A^{(1)}, B^{(1)}$  and the one loop anomalous dimensions
  - the general form of  $\mathcal{H}^{F(1)}$  is known D. de Florian, MG (2000)  
G. Bozzi, S. Catani, D. de Florian, MG (2005)
- At NNLO we need a NLO calculation of  $d\sigma^{F+\text{jet}(s)}$  plus the knowledge of  $d\sigma_{NLO}^{CT}$  and  $\mathcal{H}^{F(2)}$ 
  - the counterterm  $d\sigma_{NLO}^{CT}$  depends also on the resummation coefficients  $A^{(2)}, B^{(2)}$  and on the two loop anomalous dimensions
  - we have computed  $\mathcal{H}^{F(2)}$  for Higgs and vector boson production ! S. Catani, MG (2007)  
S. Catani, L. Cieri, G.Ferrera, D. de Florian, MG (2009)

 **this is enough to compute NNLO corrections for *any* process in this class provided F+jet is known up NLO and the two loop amplitude for  $c\bar{c} \rightarrow F$  is known**

# Subtraction or slicing ?



**Slicing:** integrate  $d\sigma_{CT}$  from  $q_{Tcut}$  to  $\infty$  (unitarity) (e.g. **MATRIX**)

→ Logarithmic terms in  $q_{Tcut}/Q$  that cancel those coming from the integral of the  $F+jet$  contribution

**(Non-local) subtraction:** map an event in the real contribution to a counter event at  $q_T=0$

(e.g. first versions of **DYNNLO** and **HNNLO**)

↙ **But:** the counter term is integrated only up to the kinematical boundary  $q_{Tmax}$  (practically irrelevant at the LHC but relevant for  $VH$  at the Tevatron !)

# $q_T$ subtraction vs Born projection

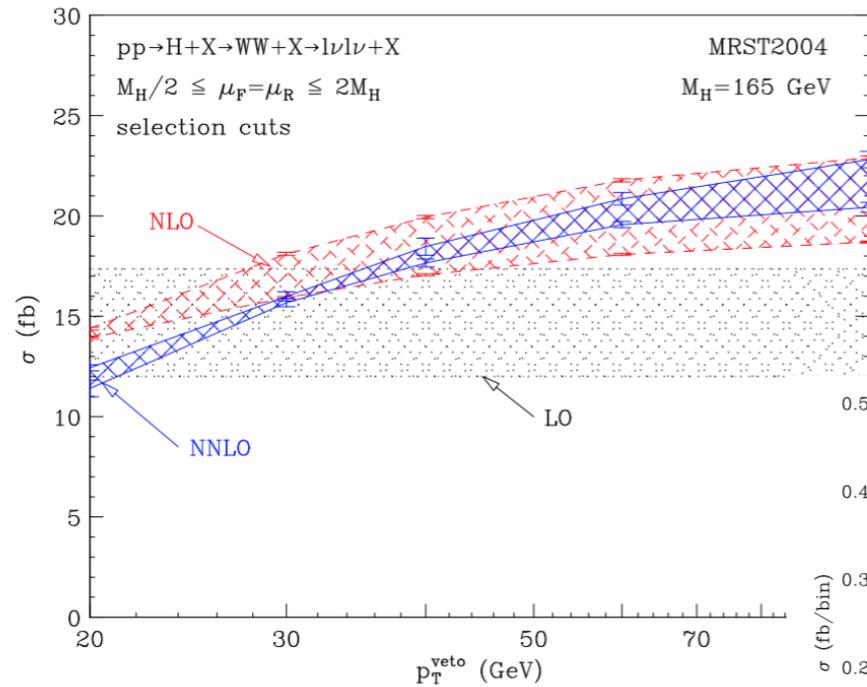
Suppose we have only one variable  $q_T$  (neglect rapidity....)

$$d\sigma \sim H\delta(q_T) + (d\sigma^R - d\sigma^{CT}) \quad \text{q}_T \text{ subtraction}$$

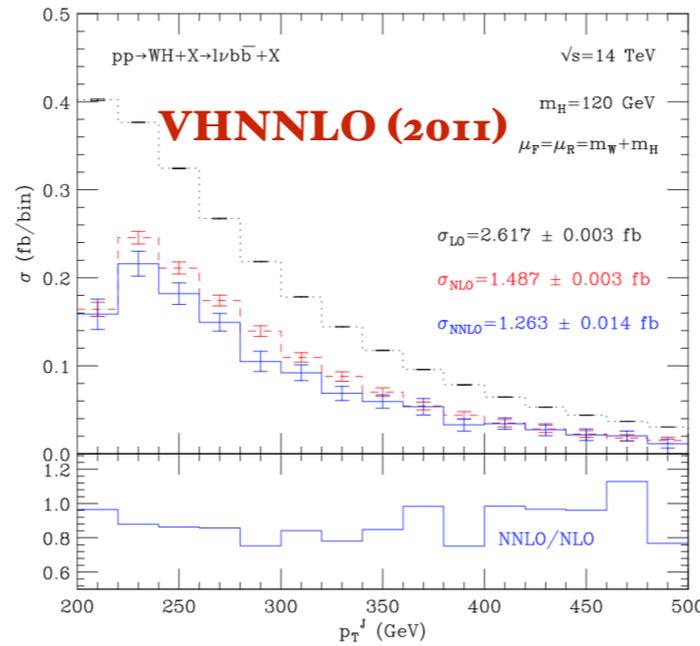
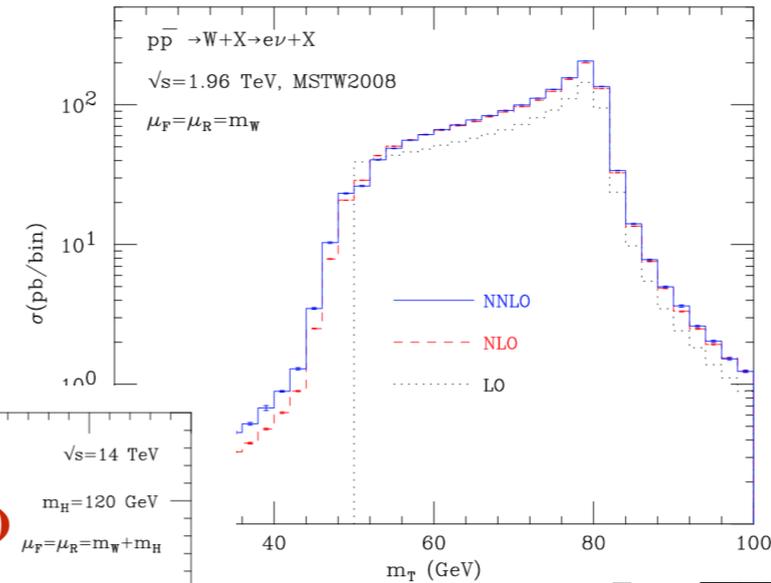
$$d\sigma \sim \sigma_{\text{tot}}\delta(q_T) + (d\sigma^R)_+ \quad \text{Born projection}$$

# Available implementations

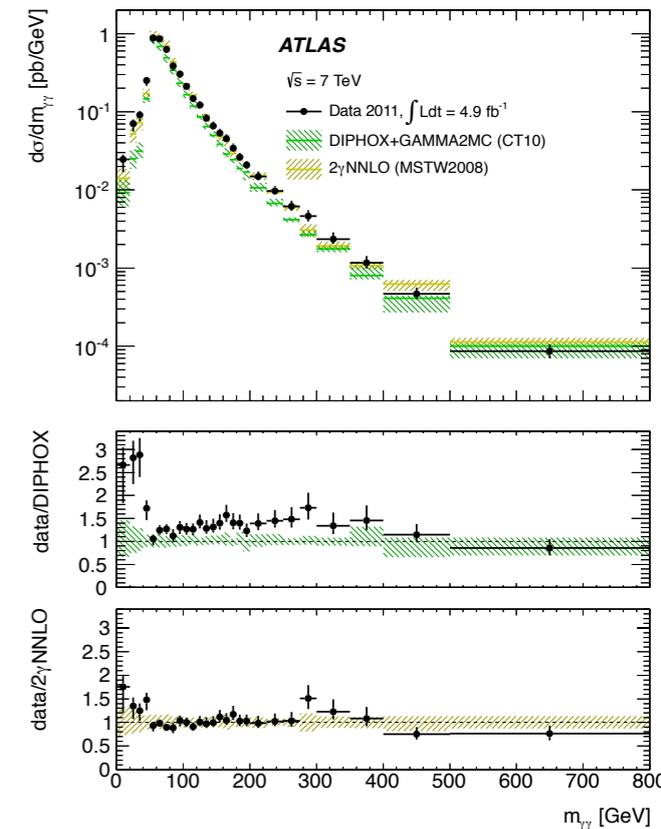
**HNNLO (2008)**



**DYNNLO (2009)**



**2gammaNNLO (2011)**

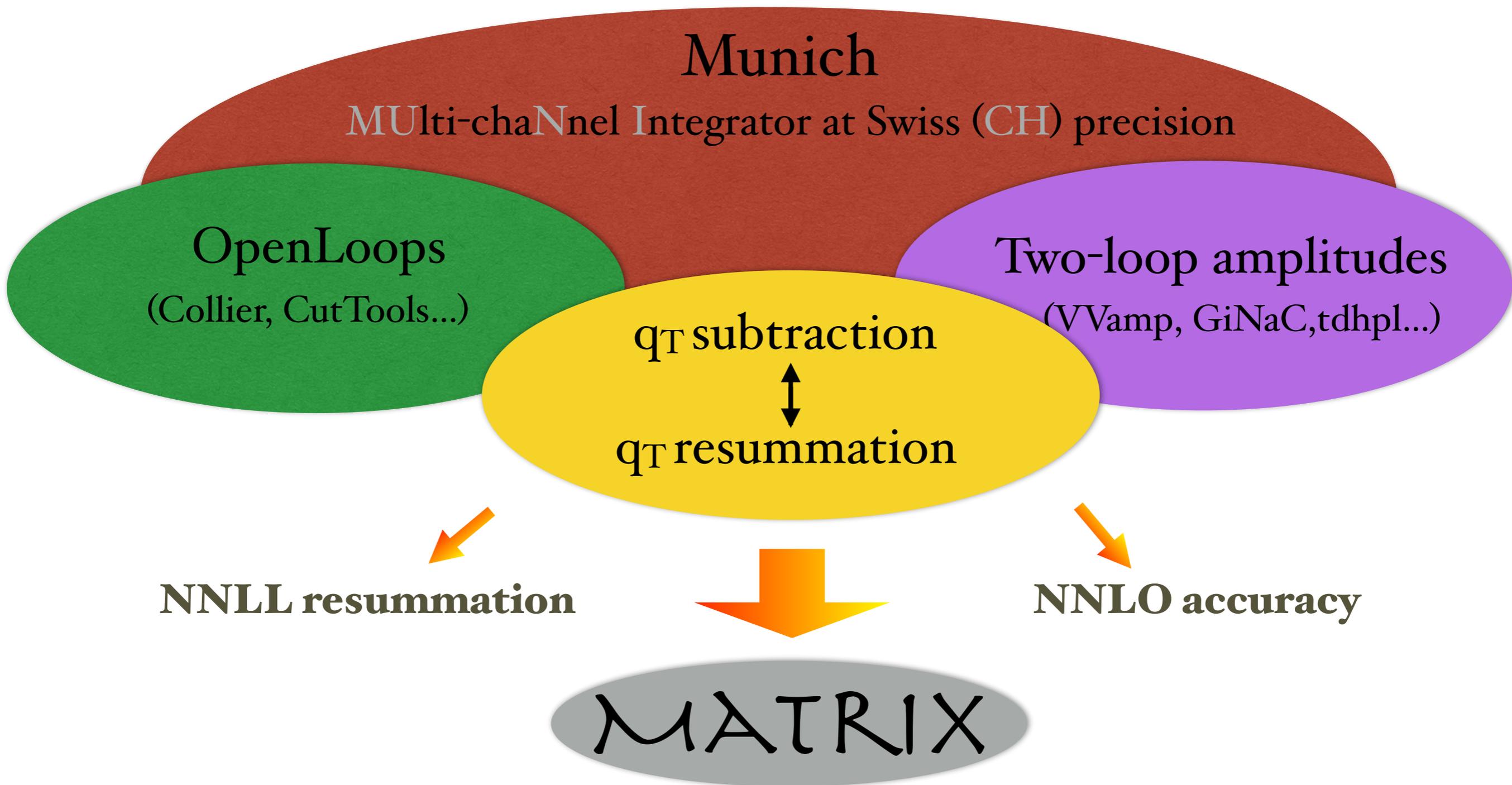


Up to now building up NNLO codes has been a craftsman work !

Generality of the method suggests that a single implementation in a general purpose program could be more efficient

# The MATRIX project

S.Kallweit, D.Rathlev, M.Wiesemann, MG + (H.Sargsyan, J.Mazzitelli....)



Munich Automates  $q_T$  subtraction and Resummation to Integrate  $X$ -sections

# Status

- $pp \rightarrow Z/\gamma^* (\rightarrow l^+l^-)$  ✓
- $pp \rightarrow W (\rightarrow lv)$  ✓
- $pp \rightarrow H$  ✓
- $pp \rightarrow \gamma\gamma$  ✓
- $pp \rightarrow W \gamma \rightarrow lv\gamma$  ✓
- $pp \rightarrow Z \gamma \rightarrow l^+l^-\gamma$  ✓
- $pp \rightarrow ZZ (\rightarrow 4l)$  ✓
- $pp \rightarrow WW \rightarrow (lv l'v')$  ✓
- $pp \rightarrow ZZ/WW \rightarrow llvv$  ✓
- $pp \rightarrow WZ \rightarrow lvll$  ✓
- $pp \rightarrow HH$  (✓) not in public release

First public release out  
in November 2017

S.Kallweit, M.Wiesemann, MG (2017)

# Stability of the subtraction procedure

$$d\sigma_{(N)NLO}^F = \mathcal{H}_{(N)NLO}^F \otimes d\sigma_{LO}^F + \left[ d\sigma_{(N)LO}^{F+jets} - d\sigma_{(N)LO}^{CT} \right]$$

The  $q_T$  subtraction counterterm is non-local  $\rightarrow$  the difference in the square bracket is evaluated with a cut-off  $r_{cut}$  on the ratio  $r = q_T/Q$

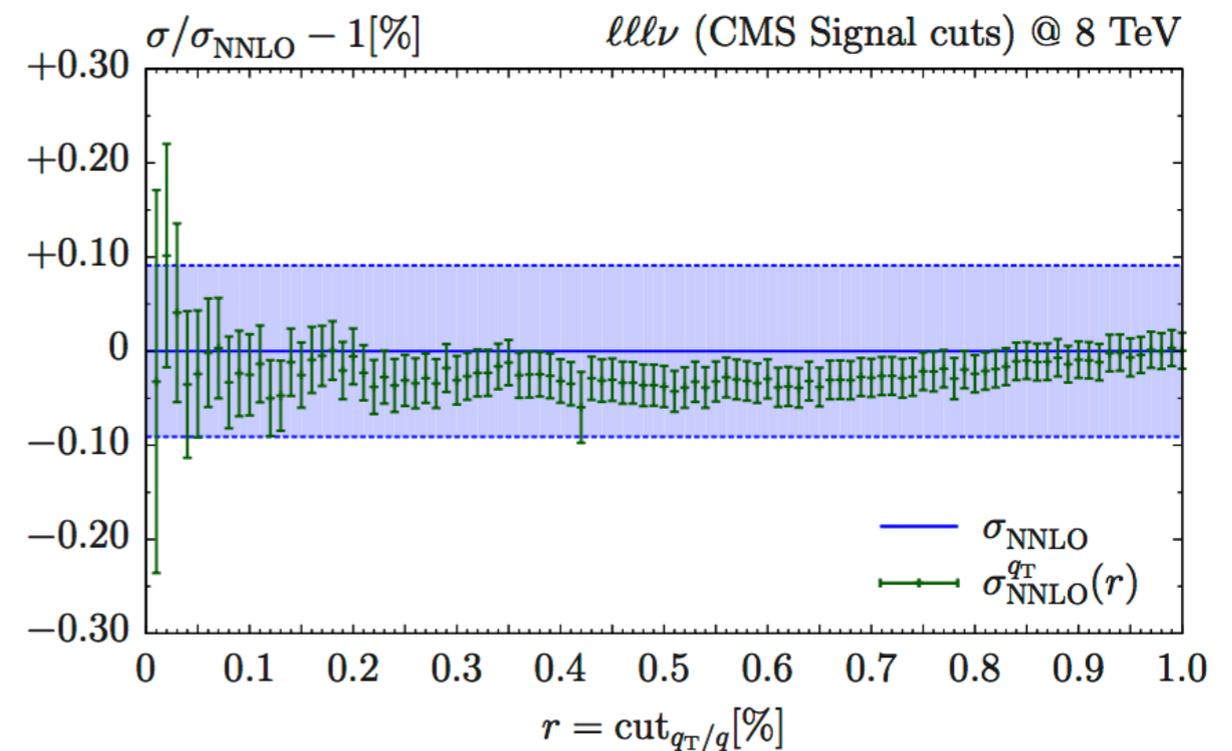
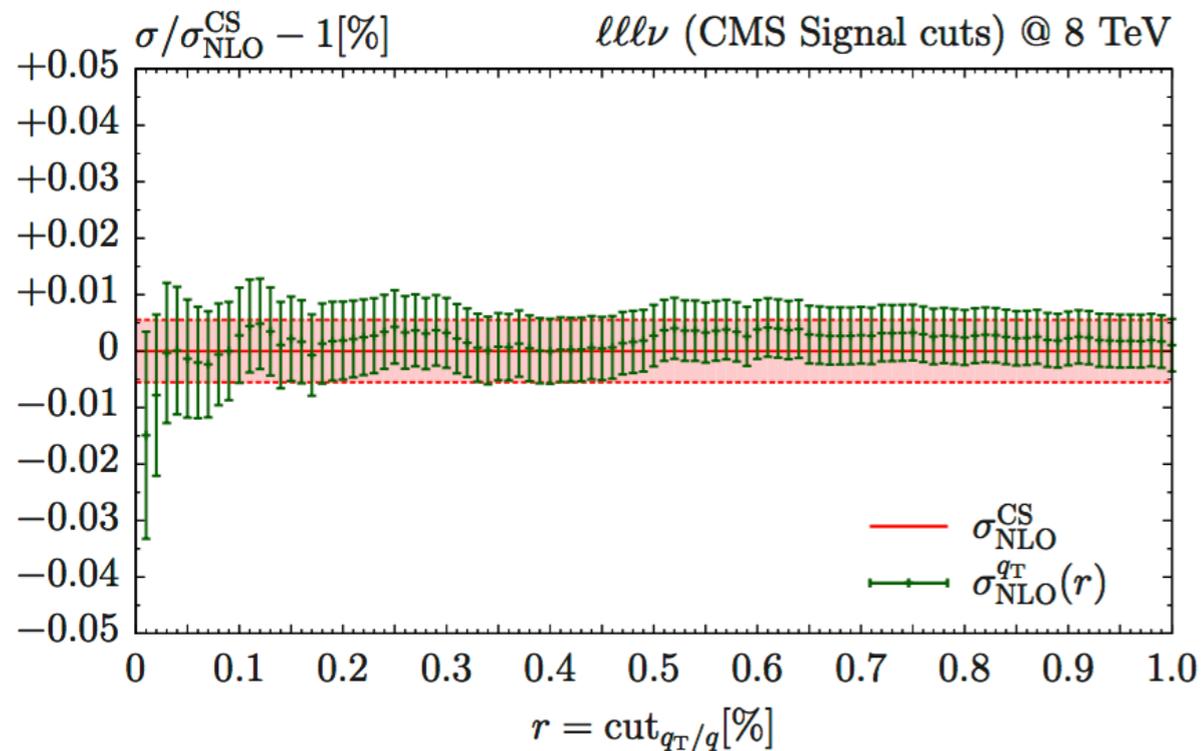
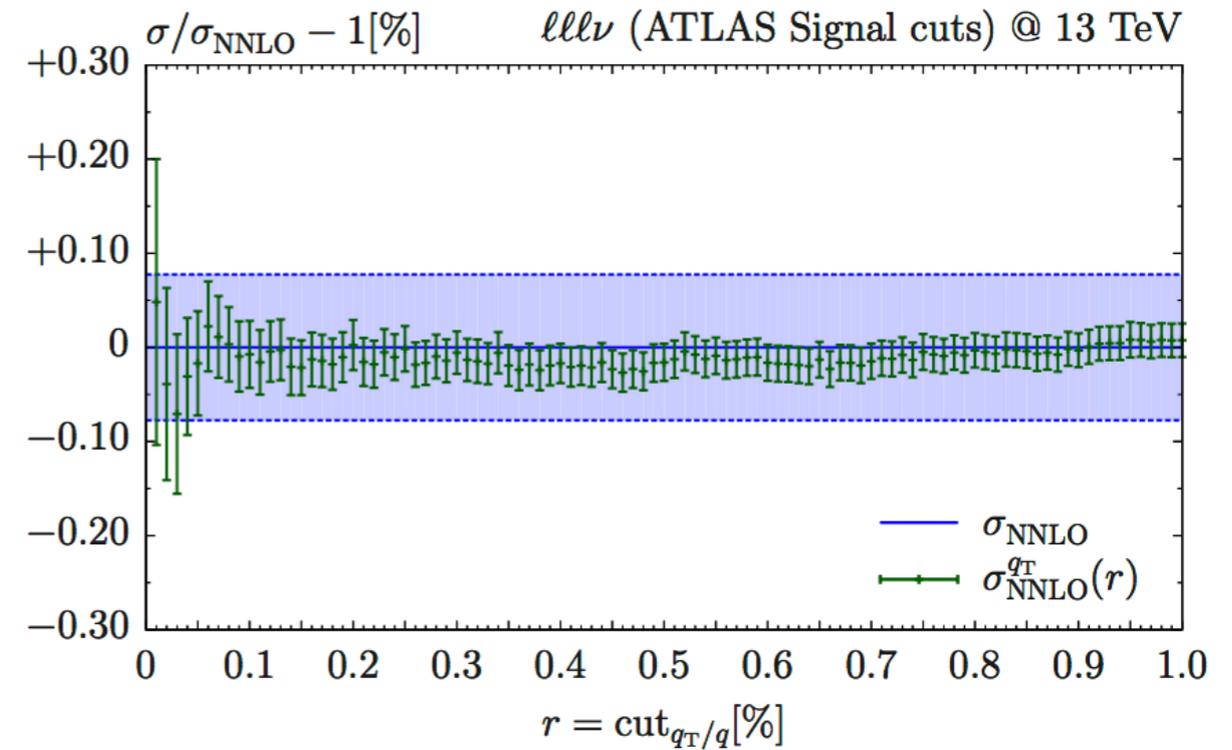
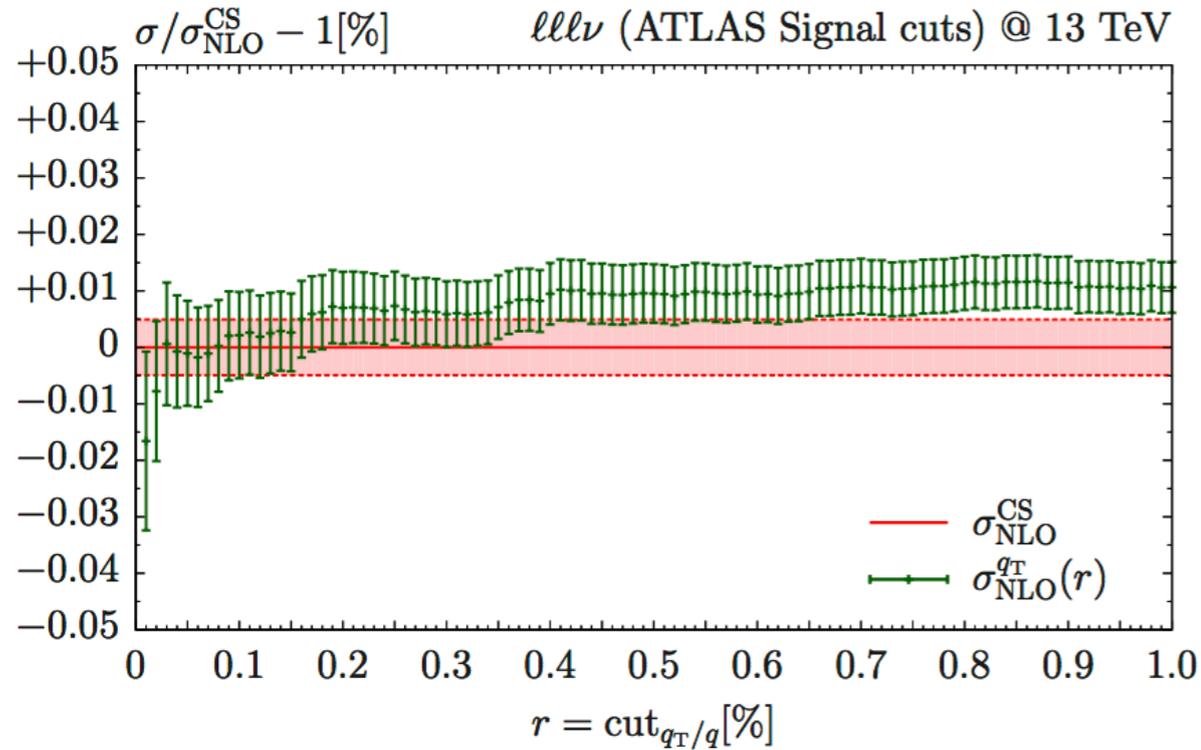
In MATRIX  $q_T$  subtraction indeed works as a slicing method

It is important to monitor the dependence of our results on  $r_{cut}$

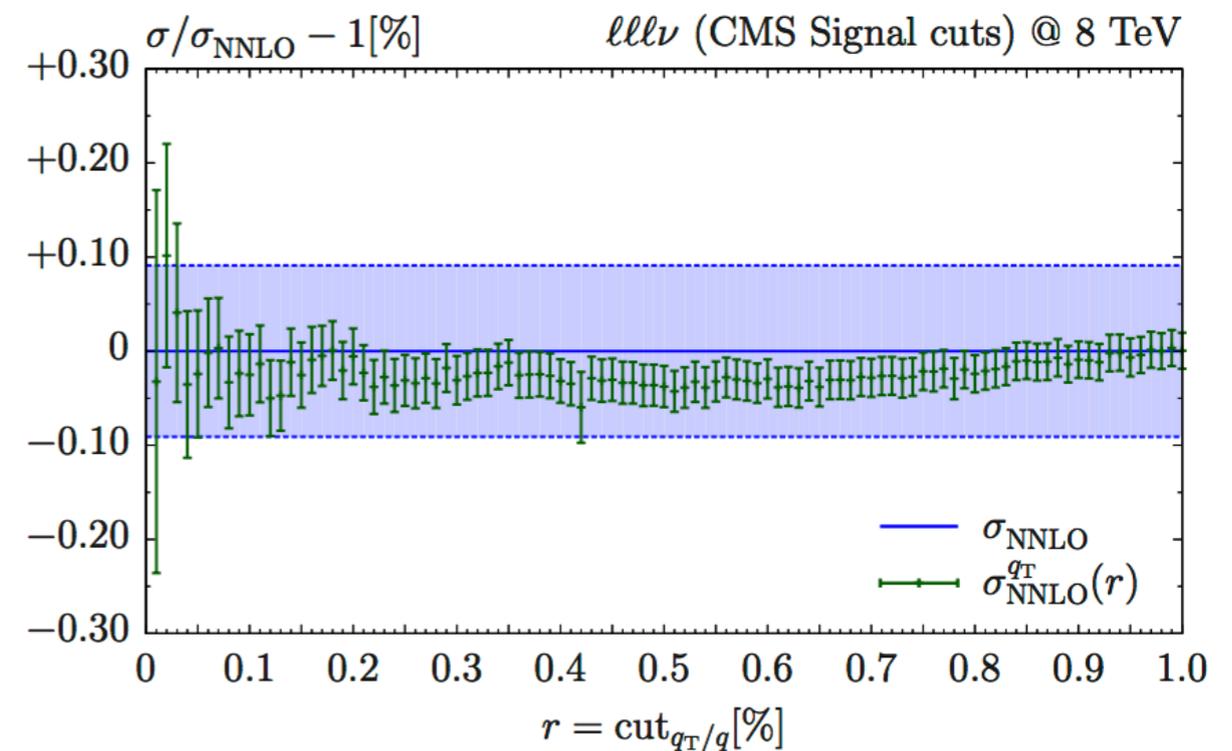
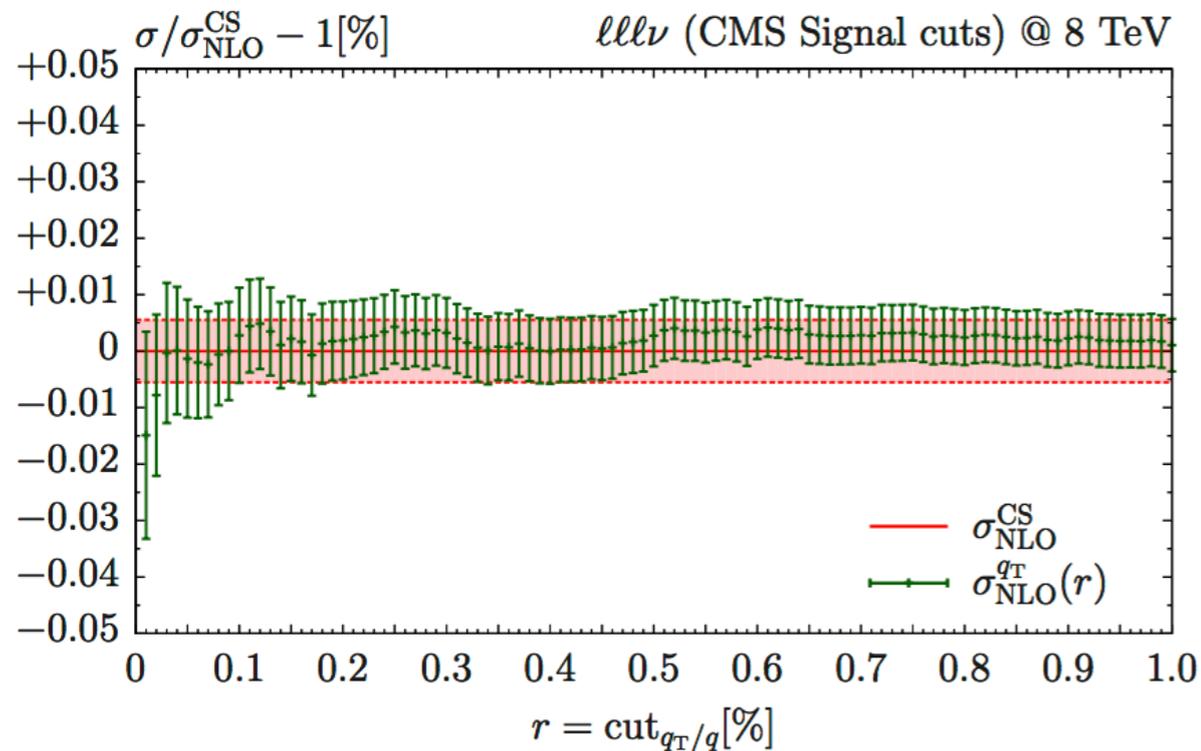
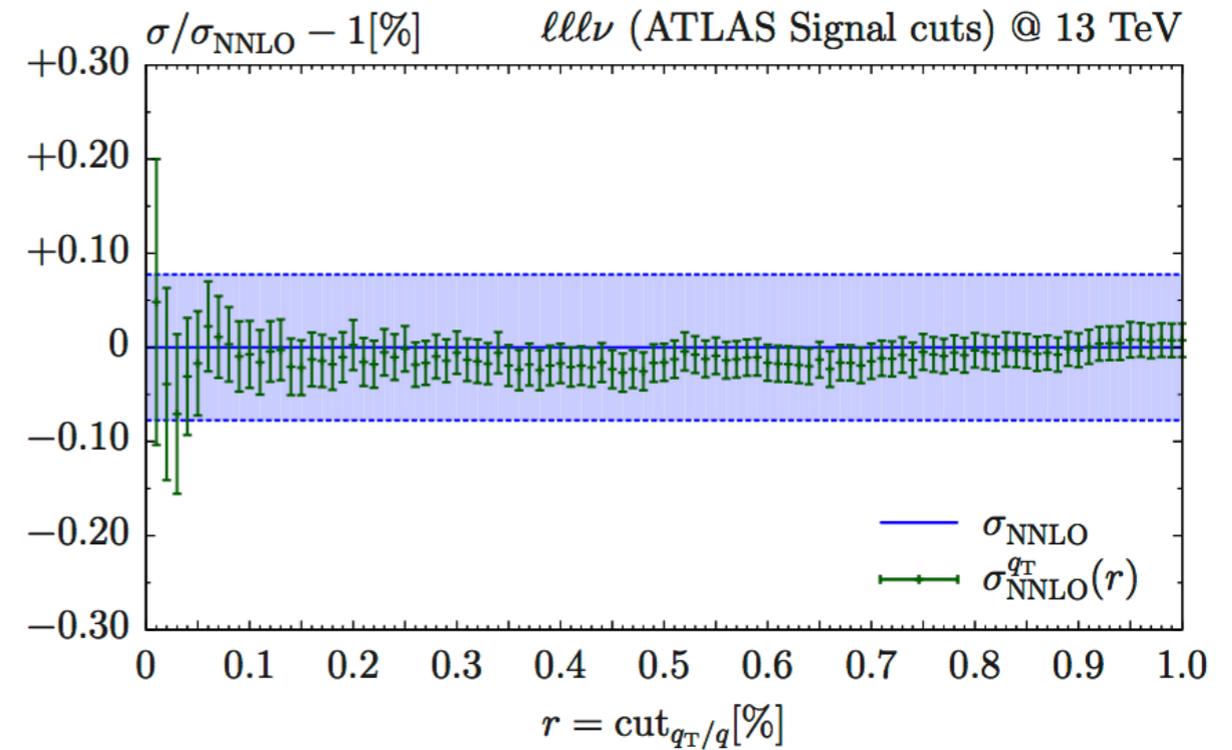
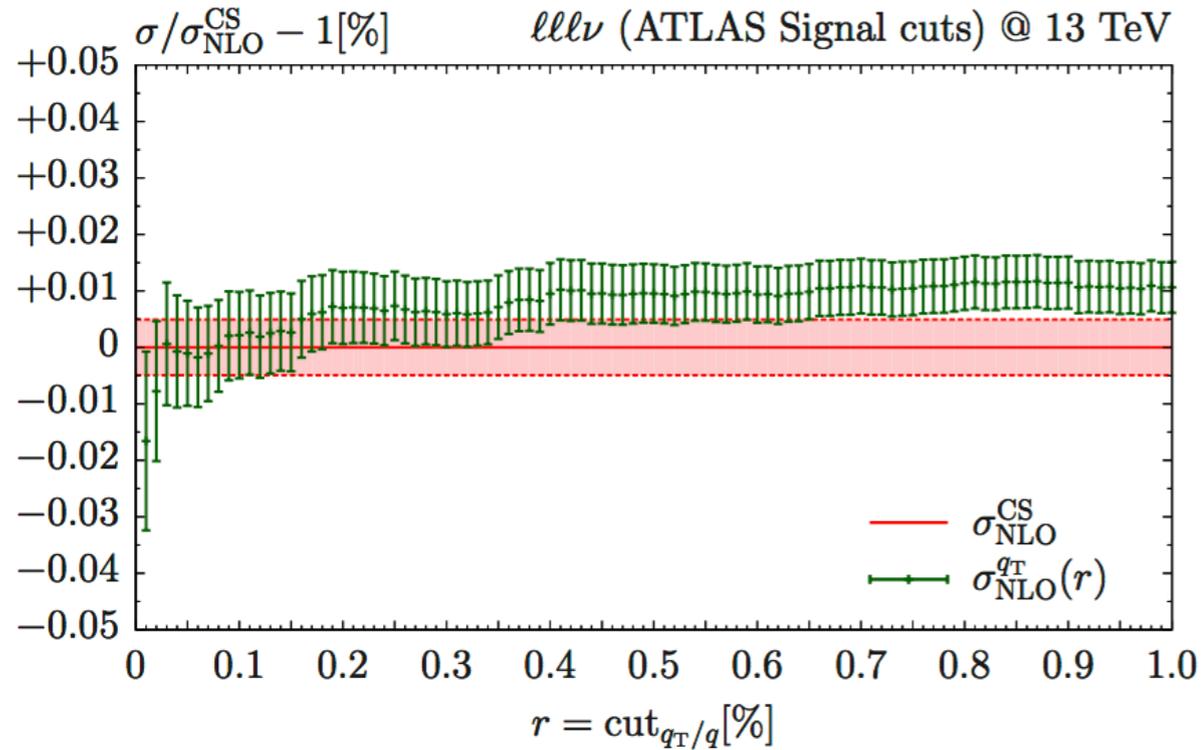
MATRIX allows for a simultaneous evaluation of the NNLO cross section for different values of  $r_{cut}$

The dependence on  $r_{cut}$  is used by the code to provide an estimate of the systematic uncertainty in any NNLO run

# Stability: the easy case



# Stability: the easy case



...but life is not always so easy !

# The extrapolation

We introduce an automatic extrapolation procedure to obtain the best result for the NNLO cross section with a solid estimate of its systematic uncertainty

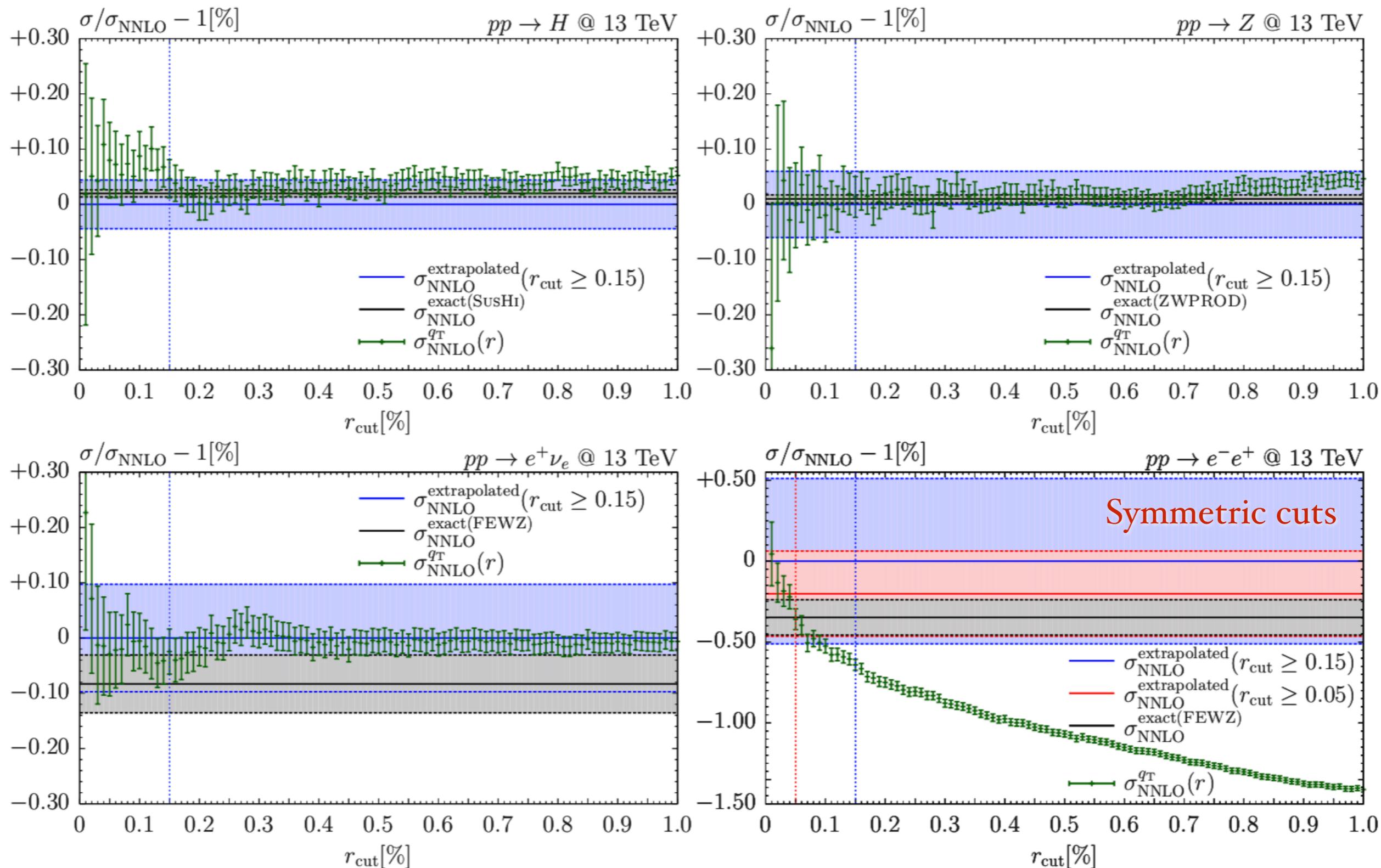
We use a simple quadratic least  $\chi^2$  fit

**Two options:** start from a minimum  $r_{\text{cut}}=0.15\%$  (default) and from  $r_{\text{cut}}=0.05\%$

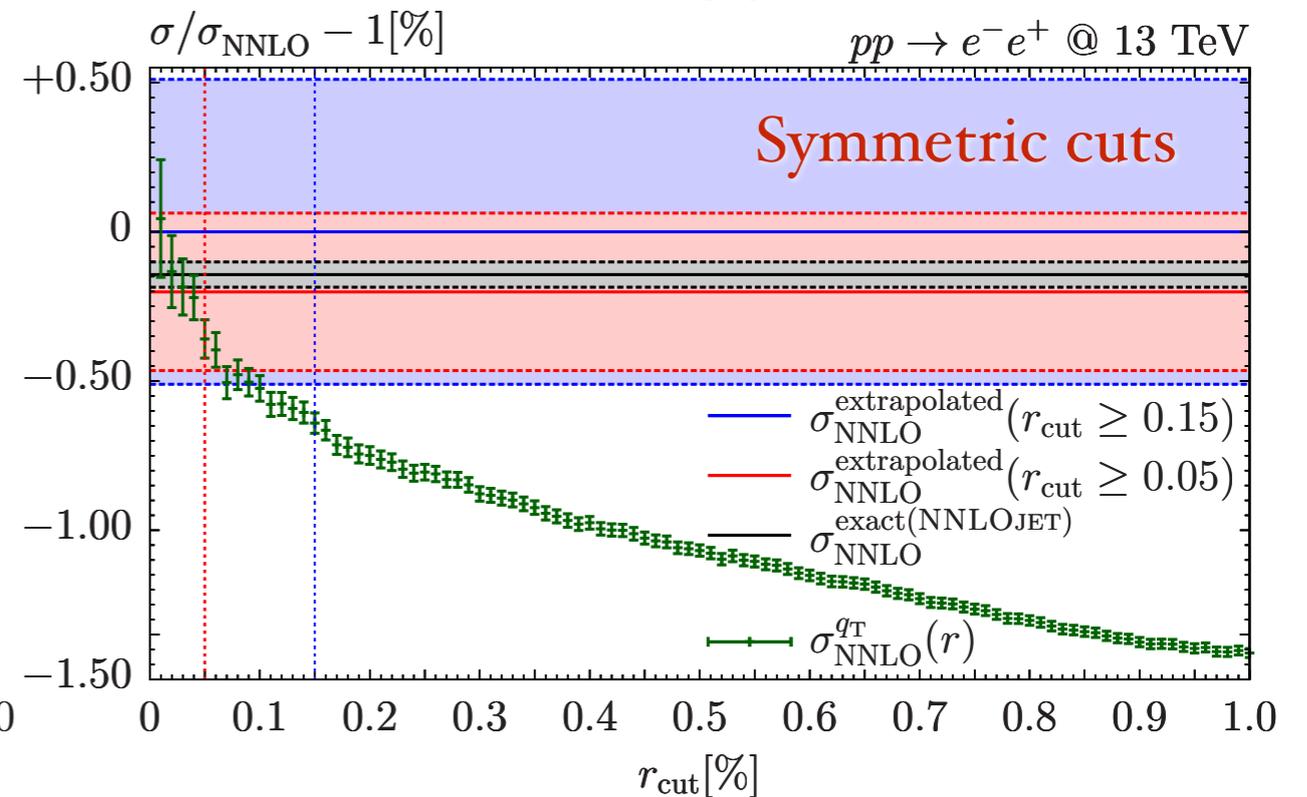
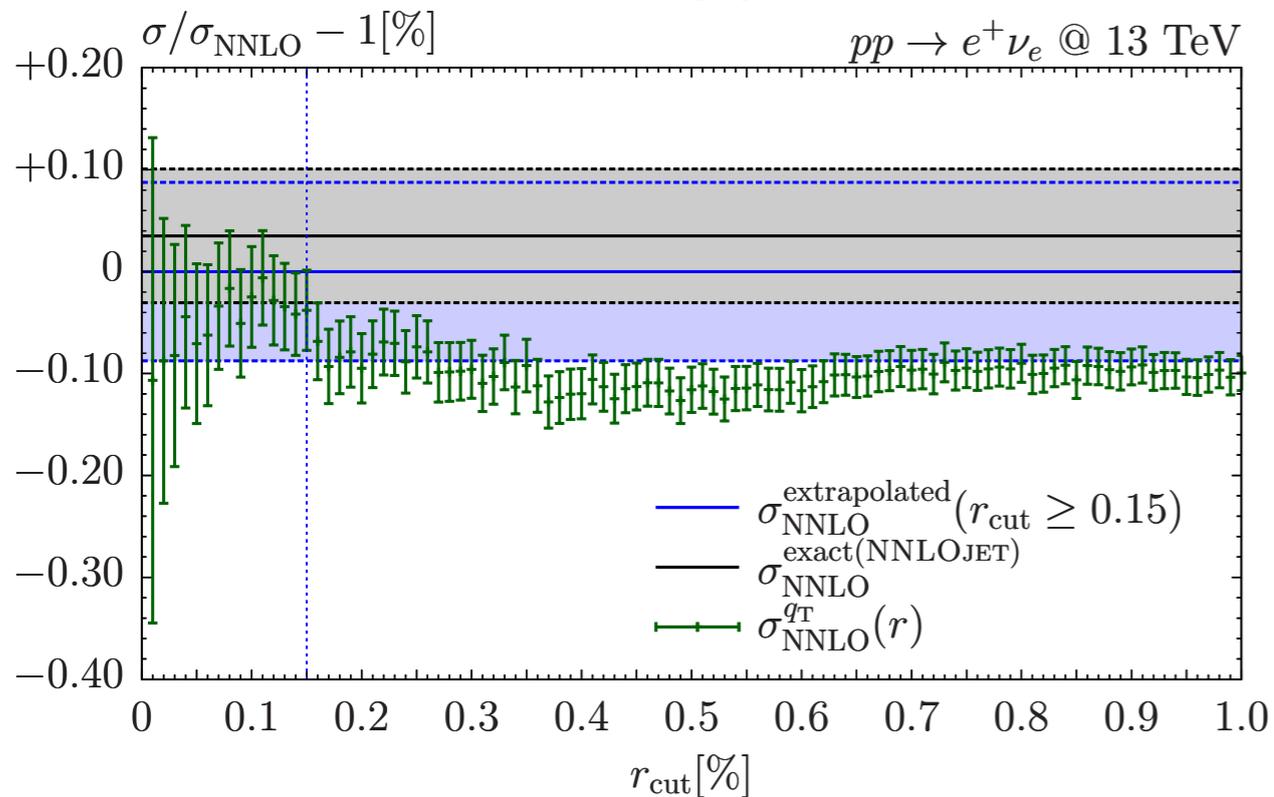
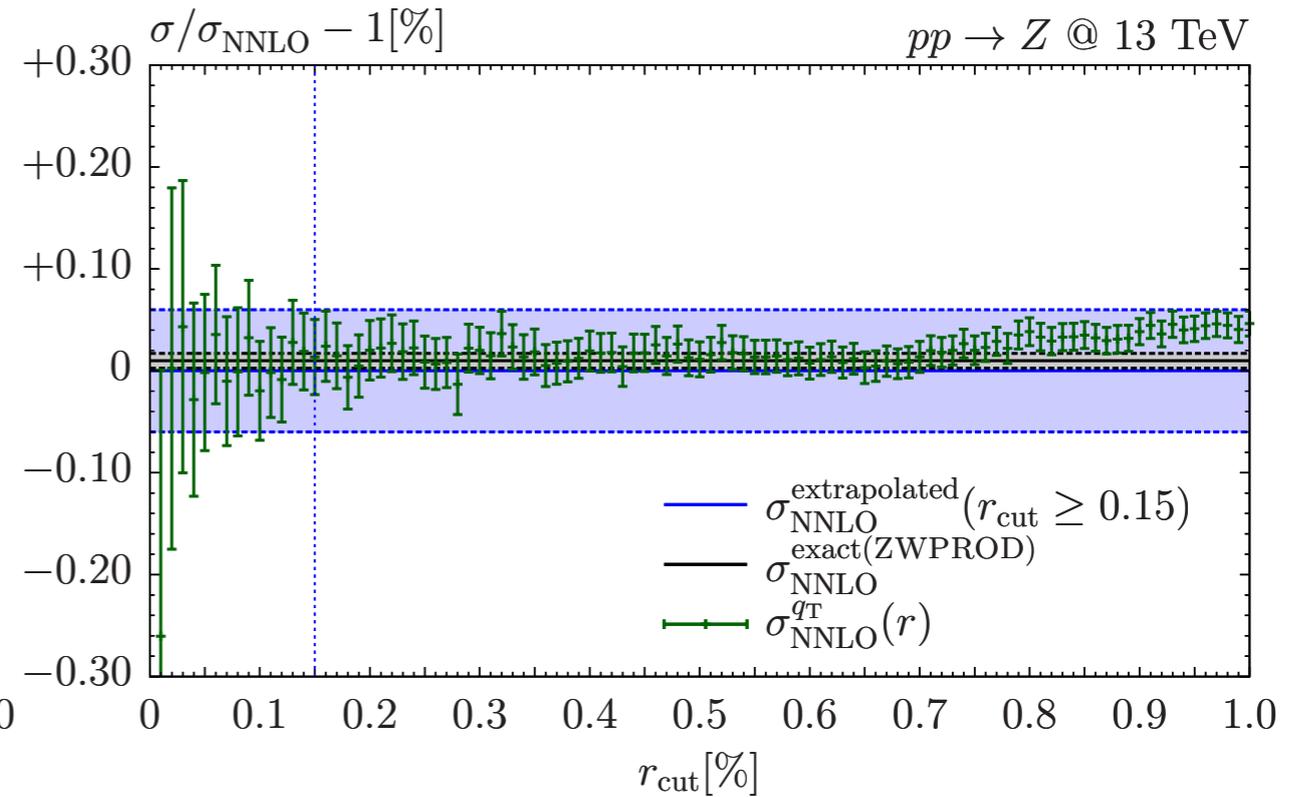
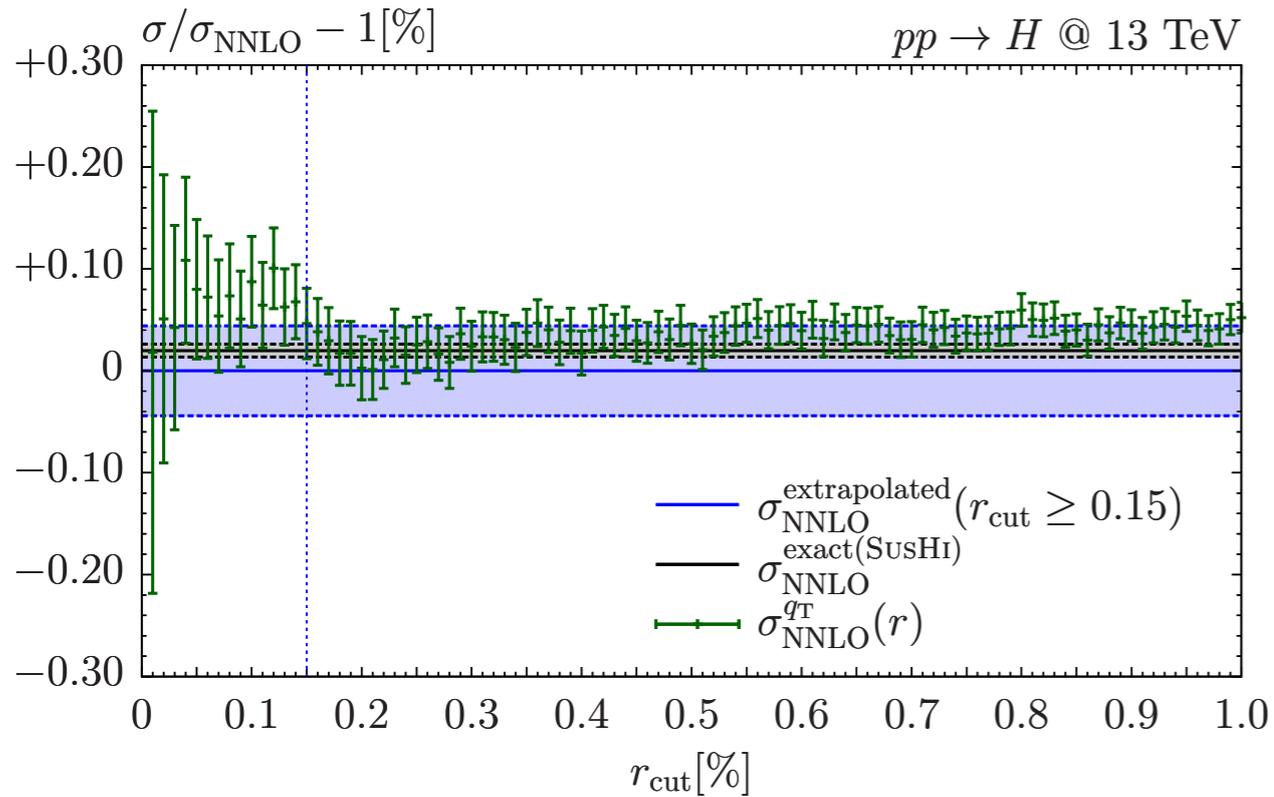
Repeat the fit by varying the upper limit of  $r_{\text{cut}}$  and assign an uncertainty by comparing the difference between the results

Introduce a lower limit to the uncertainty as half of the difference between the best fit and the cross section evaluated at the minimum  $r_{\text{cut}}=0.15\%$  ( $0.05\%$ )

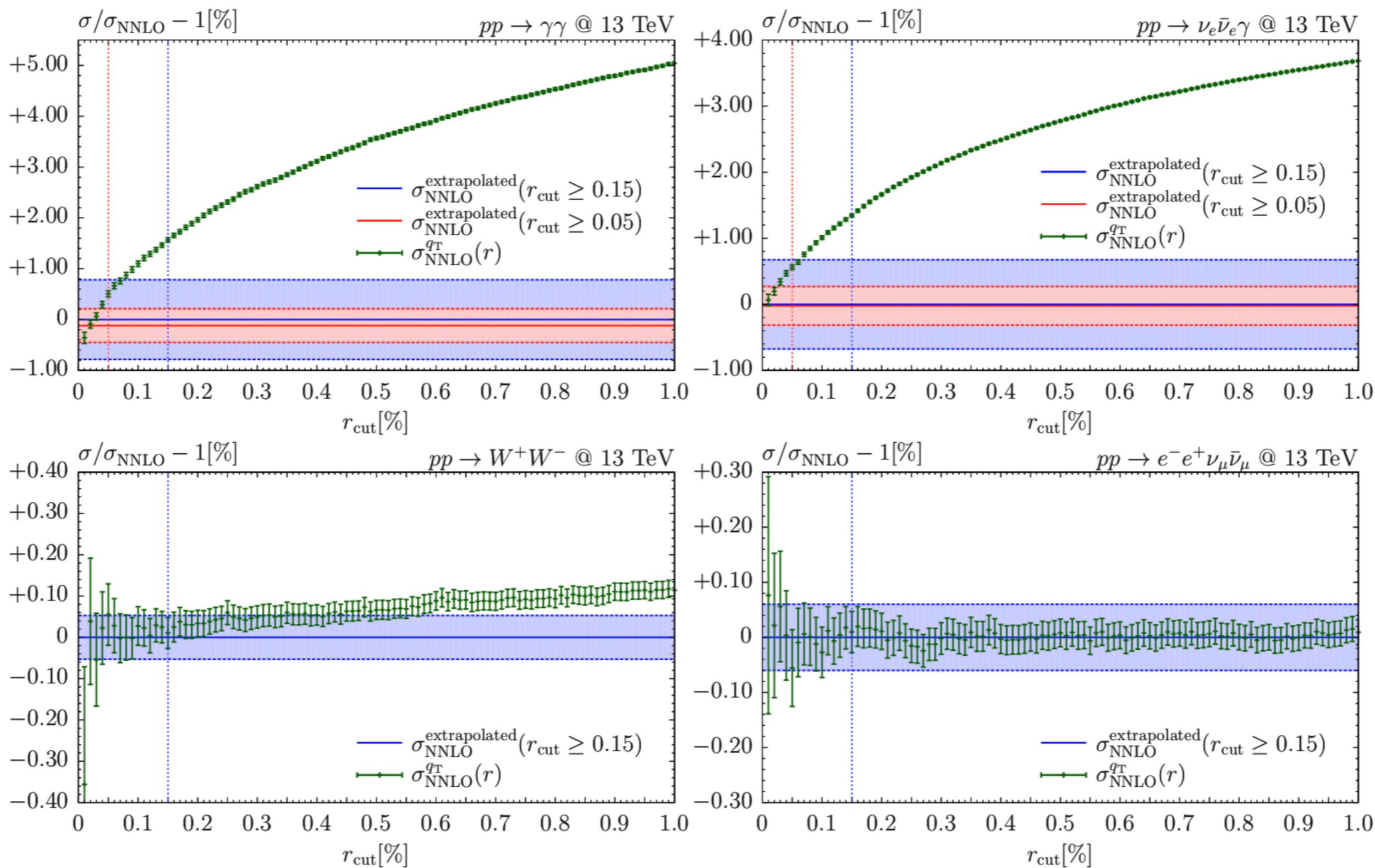
# Stability plots



# Stability plots

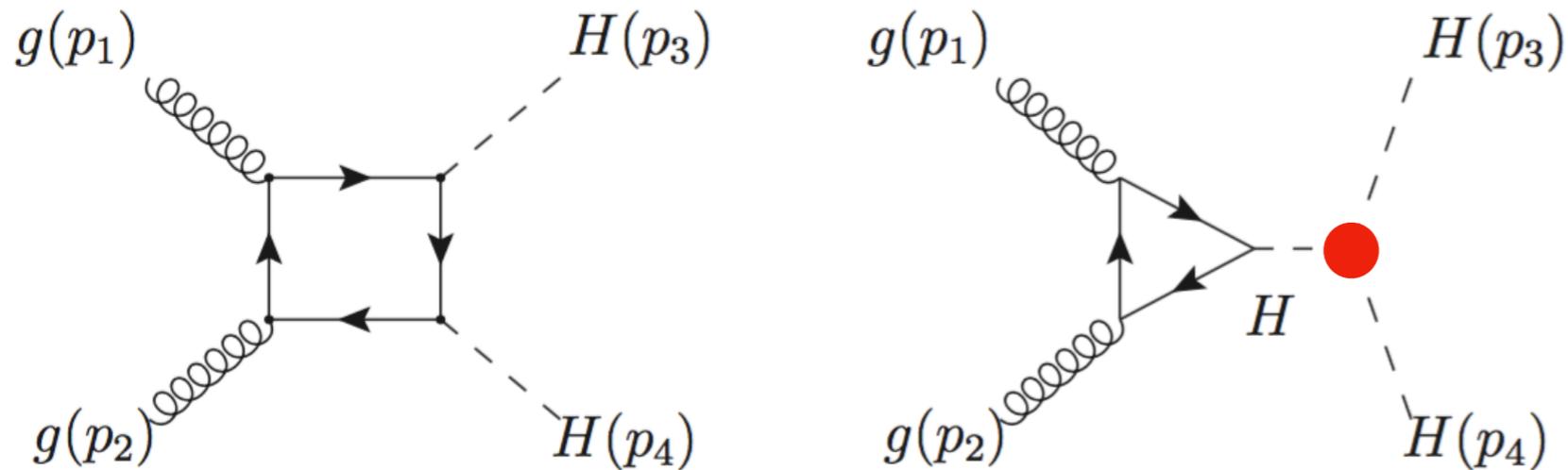


# Stability plots



**Most recent results: HH**

# HH



Large cancellations and small available phase space makes rate very small

It is the process that gives direct access to the Higgs self coupling  $\lambda$

Up to very recently QCD corrections at NLO and NNLO known only in the large- $m_{\text{top}}$  approximation

S.Dawson,S.Dittmaier,M.Spira (1998)  
D. de Florian, J.Mazzitelli (2013)

NNLL resummation available

D. de Florian, J.Mazzitelli (2015)

Main issue: large- $m_{\text{top}}$  approximation known not to work so well

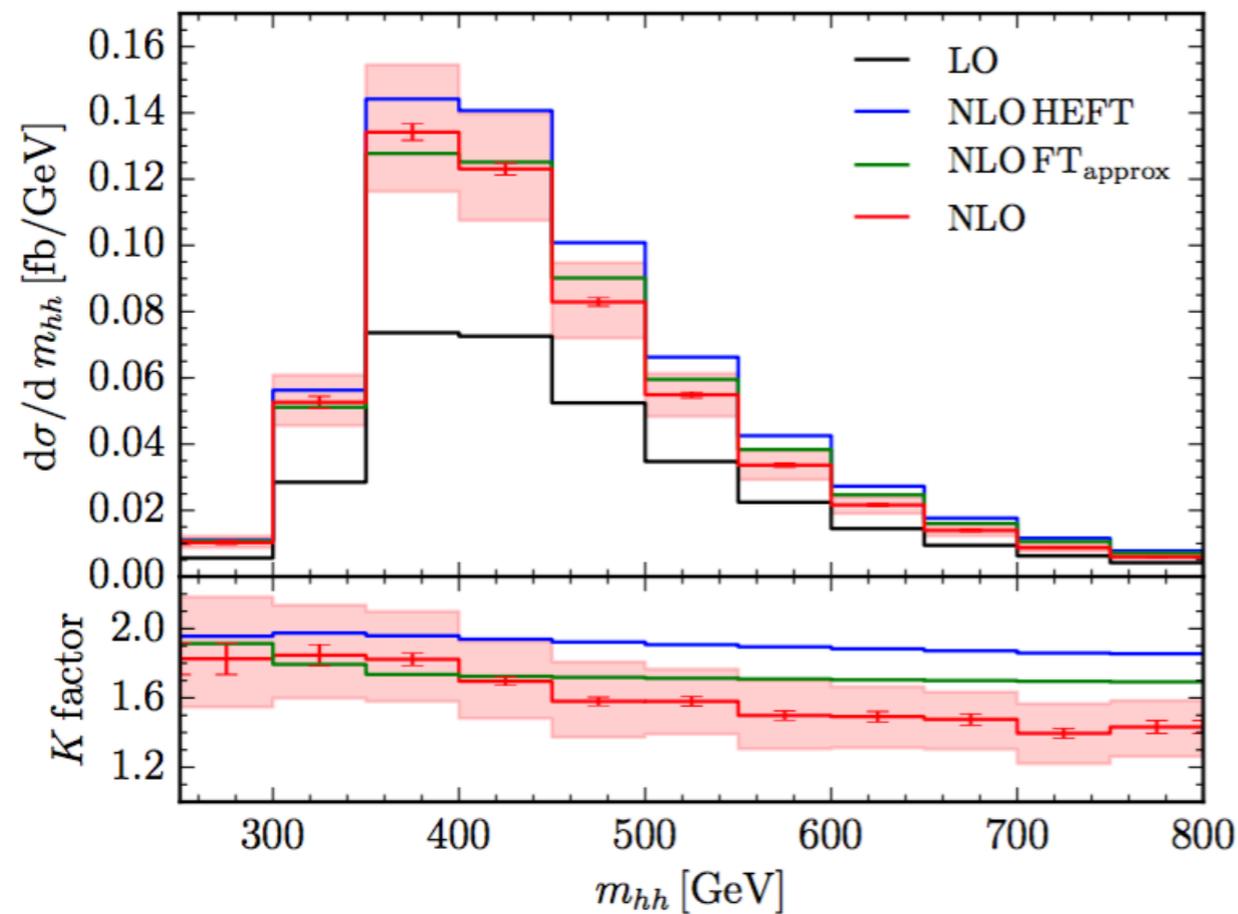
J.Grigo et al. (2013)

# HH

Recent breakthrough: exact NLO calculation completed

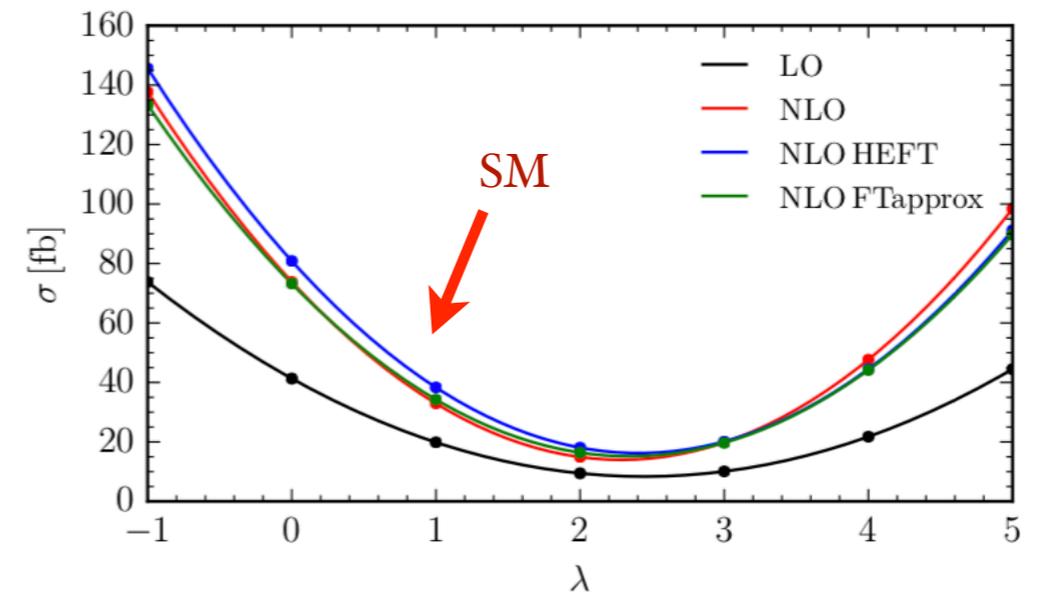
S.Borowka et al (2016)

Multi scale two-loop integrals evaluated numerically



Accurate predictions must account for exact NLO

Promising for other important multi scale NLO calculations



# HH

G.Heinrich, S.Jones, S.Kallweit, M.Kerner, J.Lindert, MG (2018)

Approximate NNLO calculation recently presented combining the most advanced perturbative information available at present

**NNLO<sub>FTapprox</sub>** Combine exact double real emission amplitudes with suitably reweighted single real and double virtual contributions

- Start from exact NLO
- At NNLO:
  - use exact double-real one loop amplitudes
  - use real-virtual and double virtual HEFT amplitudes reweighed with

$$\mathcal{R}(ij \rightarrow HH + X) = \frac{\mathcal{A}_{\text{Full}}^{\text{Born}}(ij \rightarrow HH + X)}{\mathcal{A}_{\text{HEFT}}^{(0)}(ij \rightarrow HH + X)}$$

# HH

G.Heinrich, S.Jones, S.Kallweit, M.Kerner, J.Lindert, MG (2018)

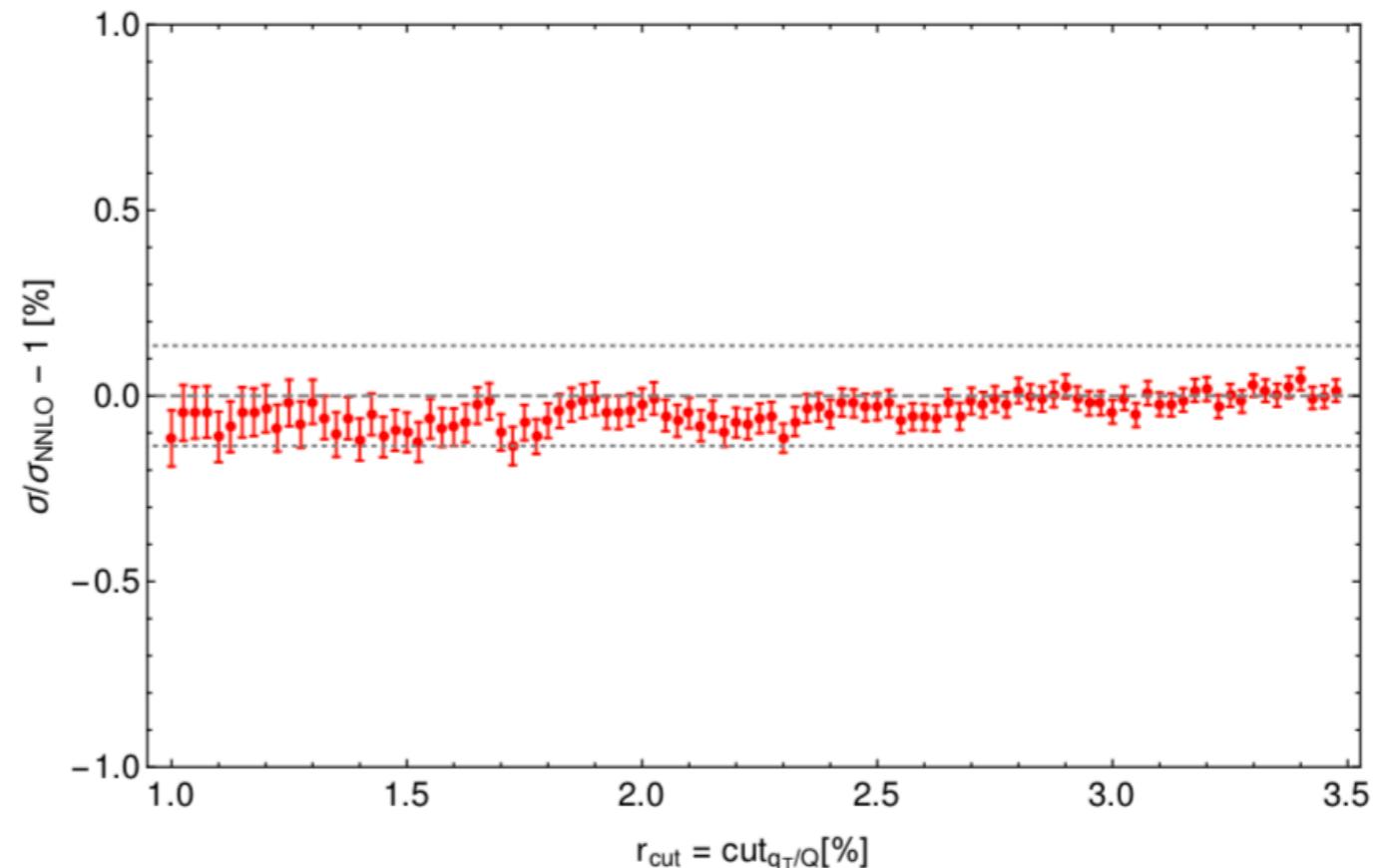
Numerical stability: the double real contribution requires  $gg \rightarrow HHgg$  six point integrals to be evaluated in the unresolved region

Quadrupole precision prohibitive (10s/phase space point)

→ Switch to (reweighted) HEFT amplitudes below a given  $\alpha_{\text{cut}}$

Set  $\alpha_{\text{cut}}=10^{-4}$  but check the independence of the results by varying  $\alpha_{\text{cut}}$   $10^{-3}$  and  $10^{-5}$

$r_{\text{cut}}$  stability at the few per mille level



# HH

G.Heinrich, S.Jones, S.Kallweit, M.Kerner, J.Lindert, MG (2018)

$\sqrt{s}$	13 TeV	14 TeV	27 TeV	100 TeV
NLO [fb]	27.78 $^{+13.8\%}_{-12.8\%}$	32.88 $^{+13.5\%}_{-12.5\%}$	127.7 $^{+11.5\%}_{-10.4\%}$	1147 $^{+10.7\%}_{-9.9\%}$
NLO <sub>FTapprox</sub> [fb]	28.91 $^{+15.0\%}_{-13.4\%}$	34.25 $^{+14.7\%}_{-13.2\%}$	134.1 $^{+12.7\%}_{-11.1\%}$	1220 $^{+11.9\%}_{-10.6\%}$
NNLO <sub>NLO-i</sub> [fb]	32.69 $^{+5.3\%}_{-7.7\%}$	38.66 $^{+5.3\%}_{-7.7\%}$	149.3 $^{+4.8\%}_{-6.7\%}$	1337 $^{+4.1\%}_{-5.4\%}$
NNLO <sub>B-proj</sub> [fb]	33.42 $^{+1.5\%}_{-4.8\%}$	39.58 $^{+1.4\%}_{-4.7\%}$	154.2 $^{+0.7\%}_{-3.8\%}$	1406 $^{+0.5\%}_{-2.8\%}$
NNLO <sub>FTapprox</sub> [fb]	31.05 $^{+2.2\%}_{-5.0\%}$	36.69 $^{+2.1\%}_{-4.9\%}$	139.9 $^{+1.3\%}_{-3.9\%}$	1224 $^{+0.9\%}_{-3.2\%}$
$M_t$ unc. NNLO <sub>FTapprox</sub>	$\pm 2.6\%$	$\pm 2.7\%$	$\pm 3.4\%$	$\pm 4.6\%$
NNLO <sub>FTapprox</sub> /NLO	1.118	1.116	1.096	1.067

Uncertainty from finite  $m_{\text{top}}$  effects down to the few percent level

# Beyond colour singlets

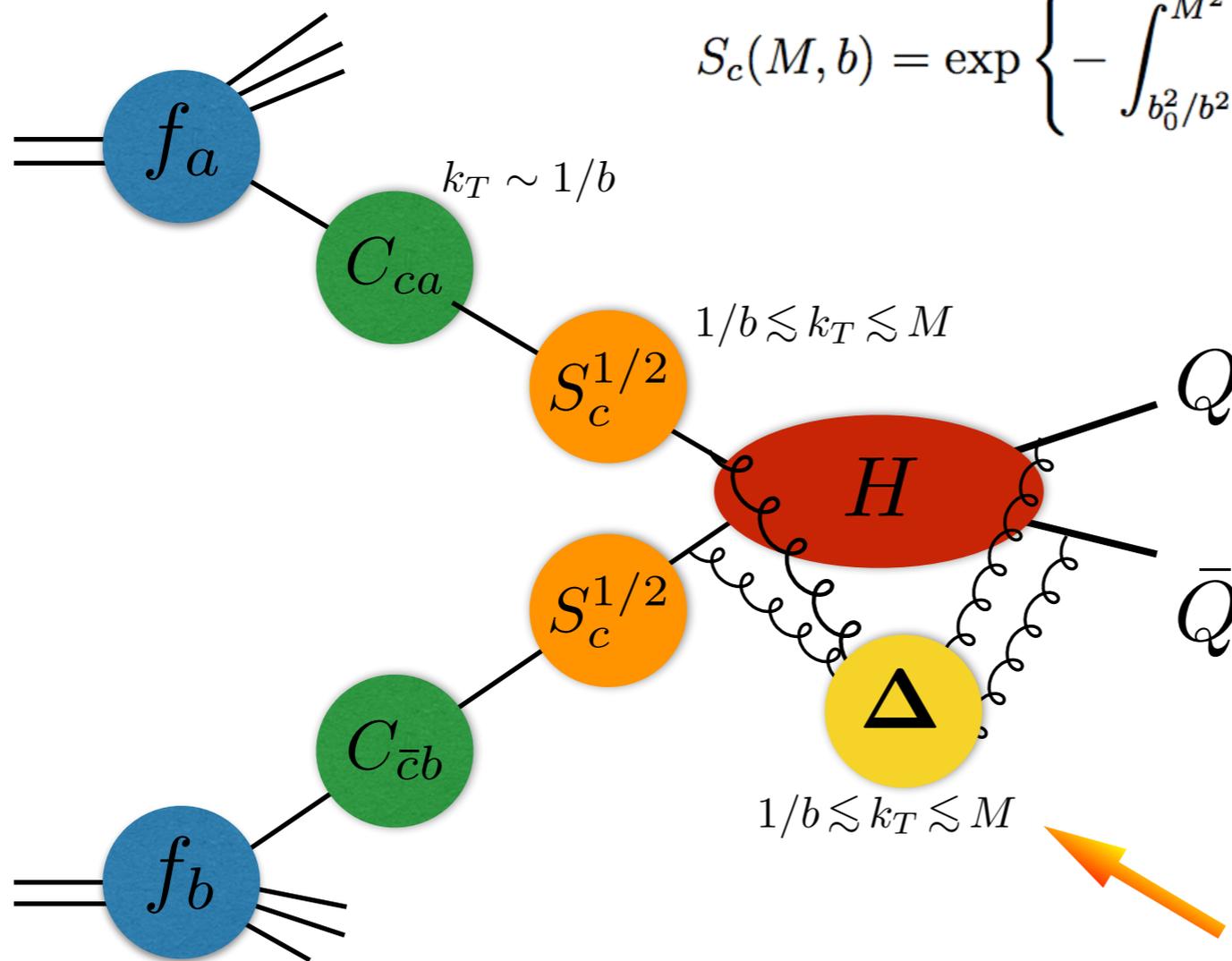
# The case of heavy-quark production

S.Catani, A.Torre, MG (2014)

$$\frac{d\sigma^{(\text{sing})}(P_1, P_2; \mathbf{q}_T, M, y, \Omega)}{d^2\mathbf{q}_T dM^2 dy d\Omega} = \frac{M^2}{2P_1 \cdot P_2} \sum_{c=q, \bar{q}, g} \left[ d\sigma_{c\bar{c}}^{(0)} \right] \int \frac{d^2\mathbf{b}}{(2\pi)^2} e^{i\mathbf{b} \cdot \mathbf{q}_T} S_c(M, b)$$

$$\times \sum_{a_1, a_2} \int_{x_1}^1 \frac{dz_1}{z_1} \int_{x_2}^1 \frac{dz_2}{z_2} [(\mathbf{H} \Delta) C_1 C_2]_{c\bar{c}; a_1 a_2} f_{a_1/h_1}(x_1/z_1, b_0^2/b^2) f_{a_2/h_2}(x_2/z_2, b_0^2/b^2)$$

$$S_c(M, b) = \exp \left\{ - \int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2} \left[ A_c(\alpha_S(q^2)) \ln \frac{M^2}{q^2} + B_c(\alpha_S(q^2)) \right] \right\}$$



$C$  coefficients embody collinear radiation at scale  $1/b$

$S_c$  embodies soft and flavour conserving collinear radiation in the region  $1/b < k_T < M$

$H^F$  includes hard radiation at scales  $k_T \sim M$

Additional radiative factor of purely soft origin (starts to contribute at NLL)

# The case of heavy-quark production

S.Catani, A.Torre, MG (2014)

$$(\mathbf{H} \Delta)_{c\bar{c}} = \frac{\langle \widetilde{\mathcal{M}}_{c\bar{c} \rightarrow Q\bar{Q}} | \Delta | \widetilde{\mathcal{M}}_{c\bar{c} \rightarrow Q\bar{Q}} \rangle}{\alpha_S^2(M^2) |\mathcal{M}_{c\bar{c} \rightarrow Q\bar{Q}}^{(0)}(p_1, p_2; p_3, p_4)|^2} \quad | \widetilde{\mathcal{M}}_{c\bar{c} \rightarrow Q\bar{Q}} \rangle \quad \text{subtracted virtual amplitude}$$

$$\Delta(\mathbf{b}, M; y_{34}, \phi_3) = \mathbf{V}^\dagger(b, M; y_{34}) \mathbf{D}(\alpha_S(b_0^2/b^2); \phi_{3b}, y_{34}) \mathbf{V}(b, M; y_{34})$$

$$\mathbf{V}(b, M; y_{34}) = \bar{P}_q \exp \left\{ - \int_{b_0^2/b^2}^{M^2} \frac{dq^2}{q^2} \Gamma_t(\alpha_S(q^2); y_{34}) \right\} \quad \alpha_S^n L^m \text{ terms} \quad n \geq m$$



soft anomalous dimension

$\Gamma_t^{(1)}$  and  $\Gamma_t^{(2)}$  directly related to singular structure of  $|\mathcal{M}_{c\bar{c} \rightarrow Q\bar{Q}}\rangle$

M.Neubert et al. (2009)

$\mathbf{D}(\alpha_S; \phi_{3b}, y_{34})$  embodies azimuthal correlations at scale  $1/b$   $\langle \mathbf{D}(\alpha_S; \phi_{3b}, y_{34}) \rangle_{\text{av.}} = 1$

# Beyond colour singlets: top-quark production

R.Bonciani, S.Catani, H.Sargsyan and A.Torre , MG (2015)

The  $q_T$  subtraction method can be extended to heavy-quark production

We have used this method to compute  $t\bar{t}$  production at NLO and to include all the off-diagonal partonic channels at NNLO

$\sigma(\text{pb})$	NLO	$O(\alpha_s^4)_{qg}$	$O(\alpha_s^4)_{qq+qq'}$
$q_T$ subtraction	226.2(1)	-2.25(5)	1.51(3)
Top++	226.3	-2.253	1.48

pp, 8 TeV

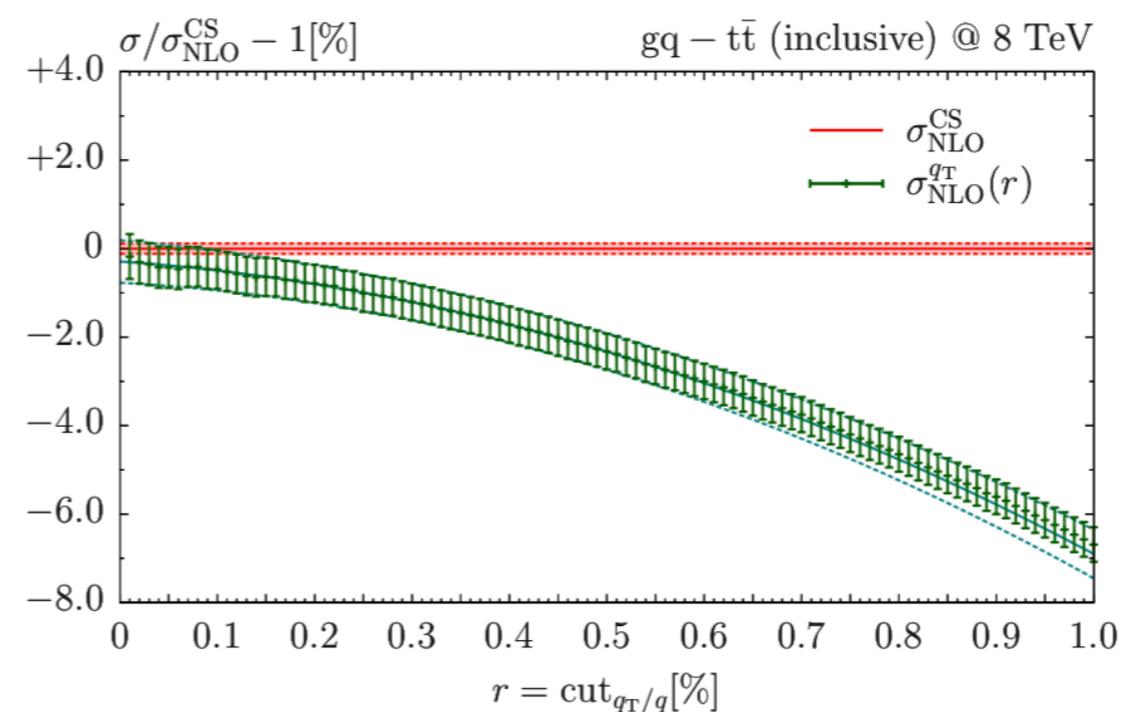
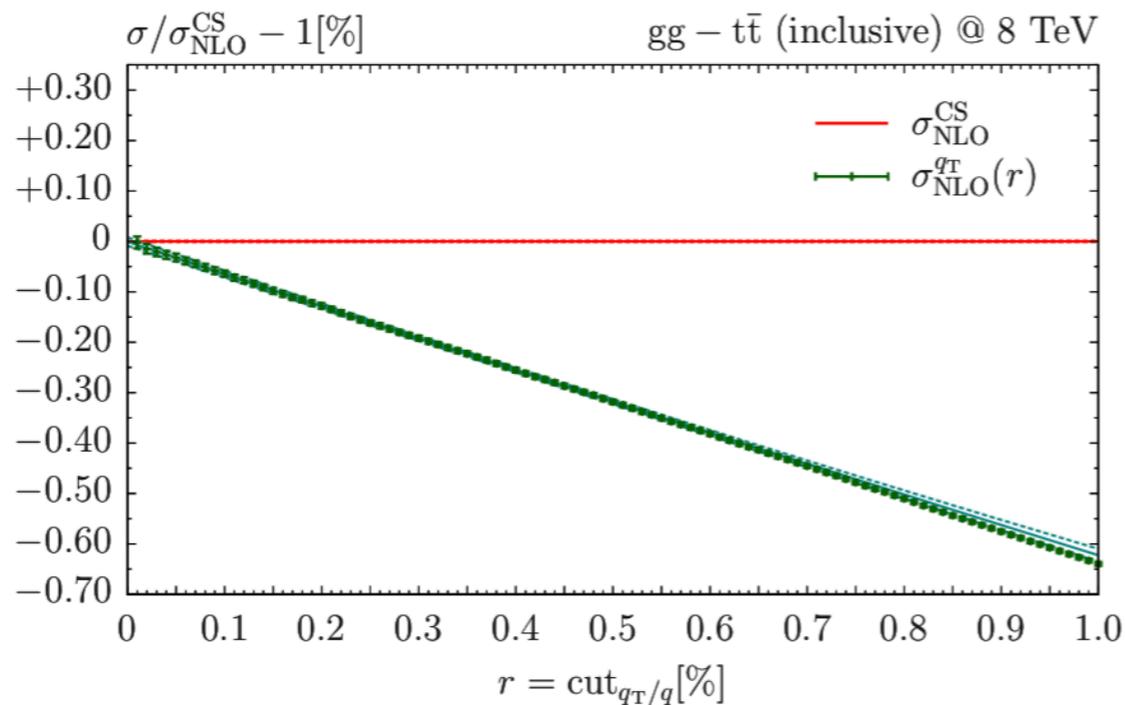
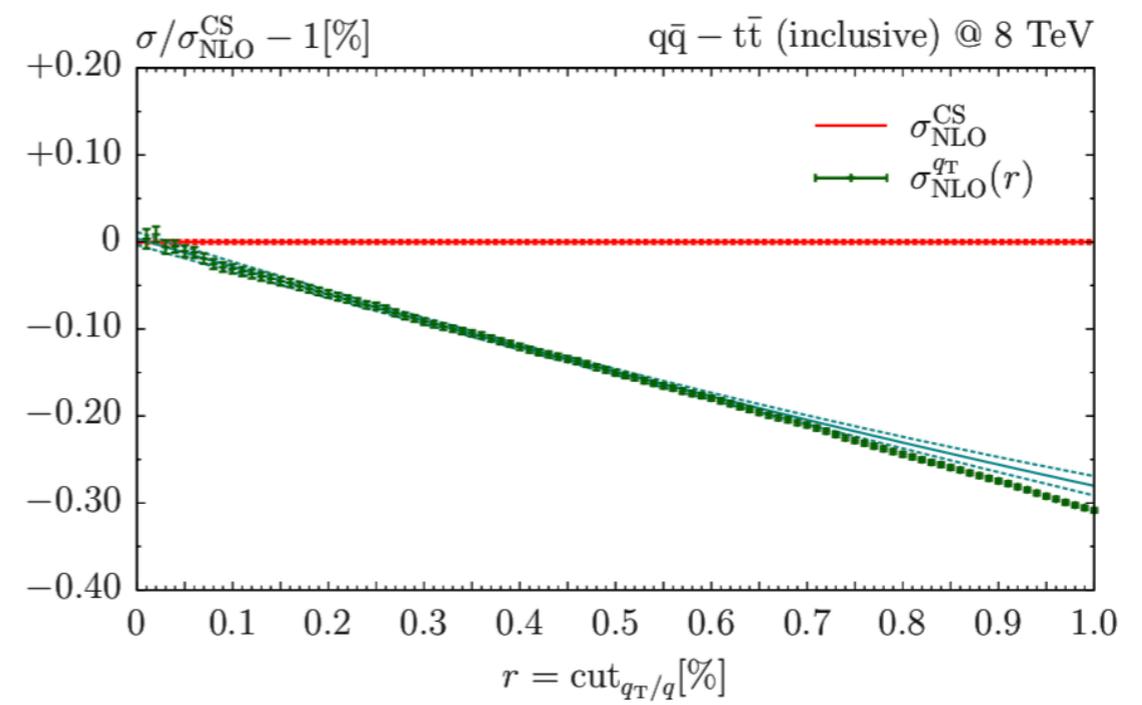
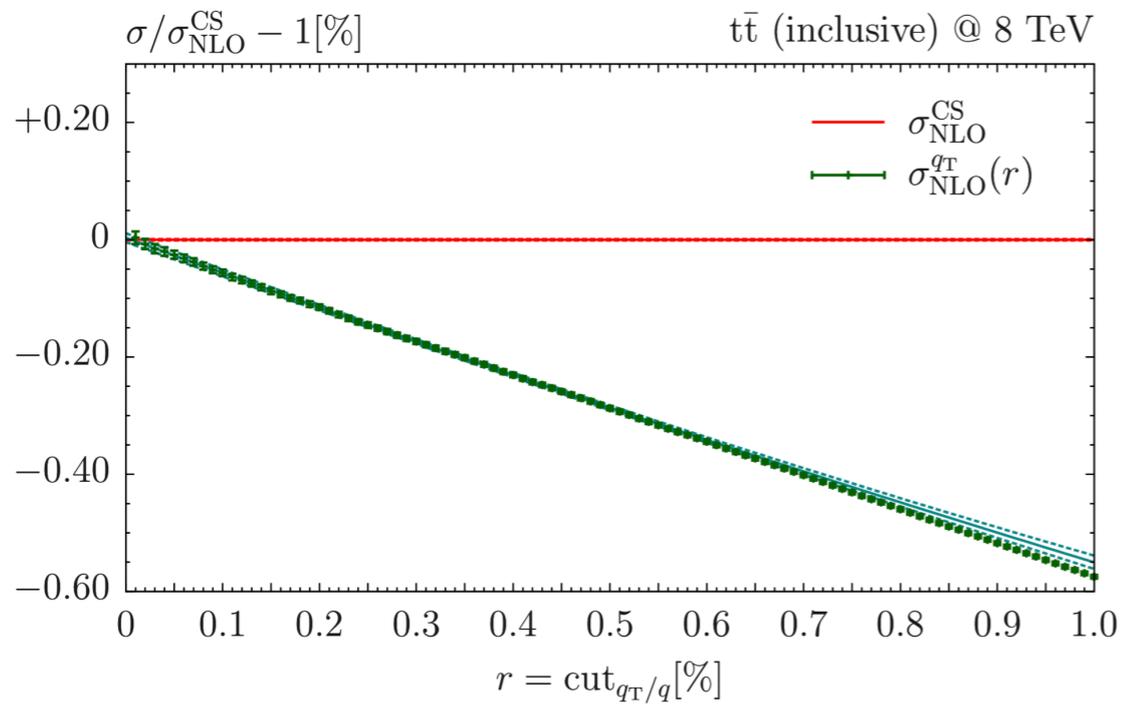
$\sigma(\text{fb})$	NLO	$O(\alpha_s^4)_{qg}$	$O(\alpha_s^4)_{qq+qq'}$
$q_T$ subtraction	7083(3)	-61.5(5)	1.33(1)
Top++	7086	-61.53	1.33

ppbar, 2 TeV

But: the  $r_{\text{cut}}$  dependence is larger in this case

# Beyond colour singlets: top-quark production

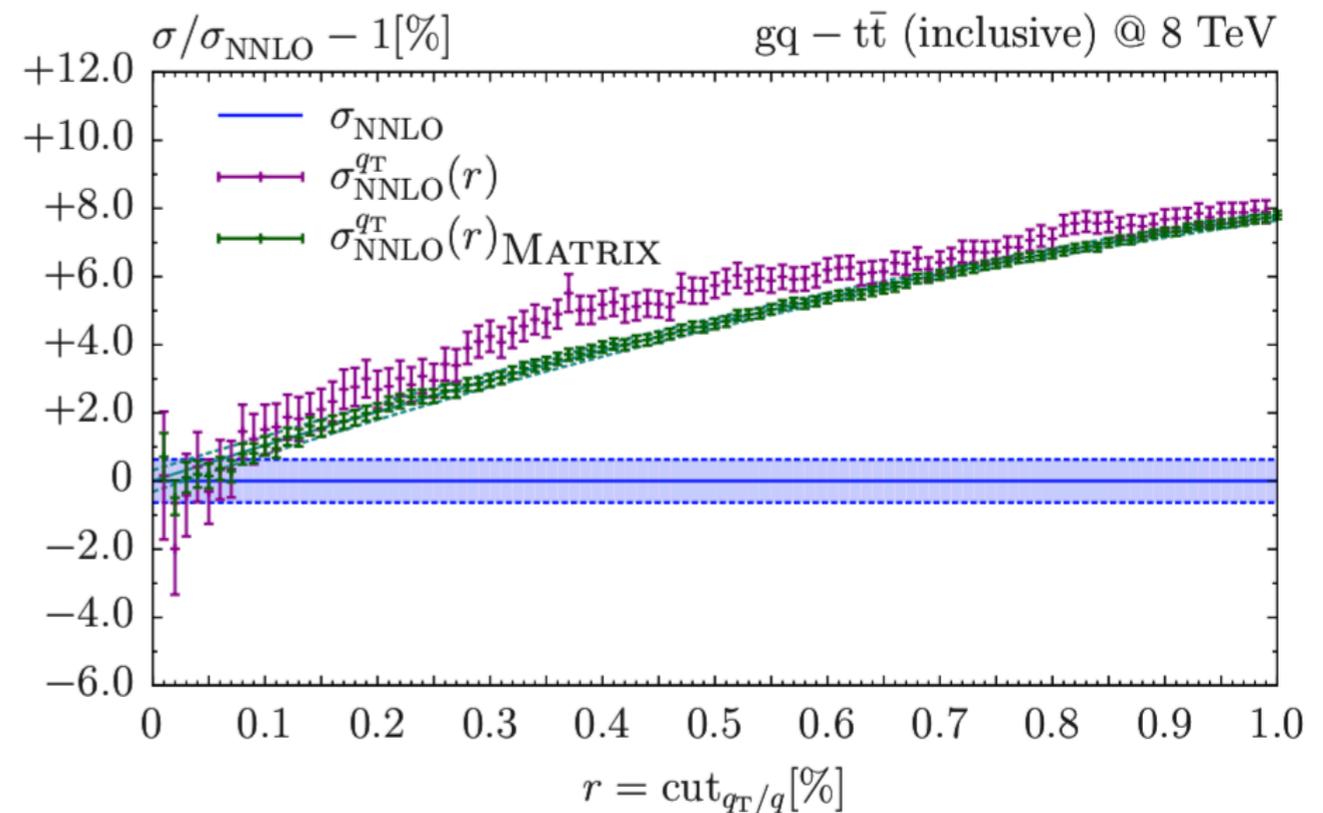
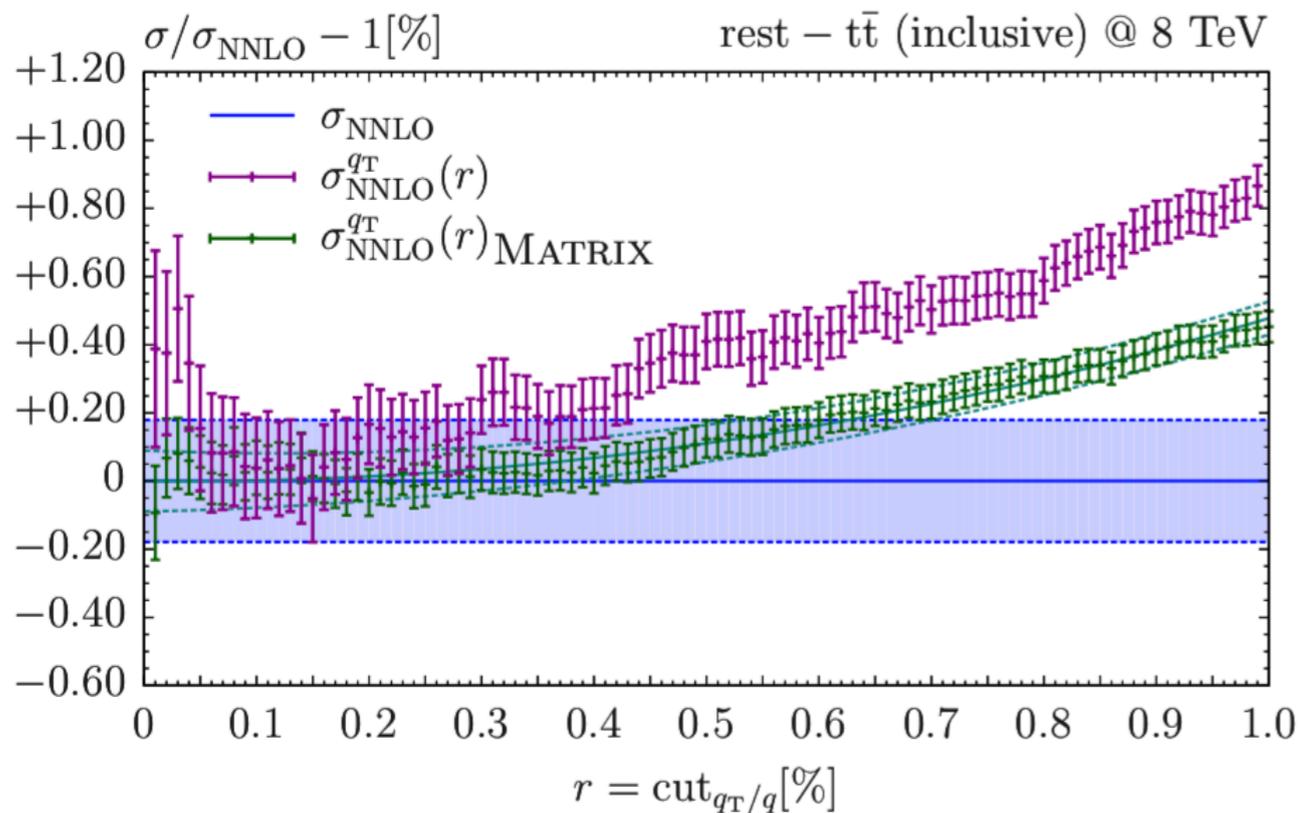
S.Catani, S.Devoto, J.Mazzitelli, S.Kallweit, H.Sargsyan, MG (in progress)



# Beyond colour singlets: top-quark production

S.Catani, S.Devoto, J.Mazzitelli, S.Kallweit, H.Sargsyan, MG (in progress)

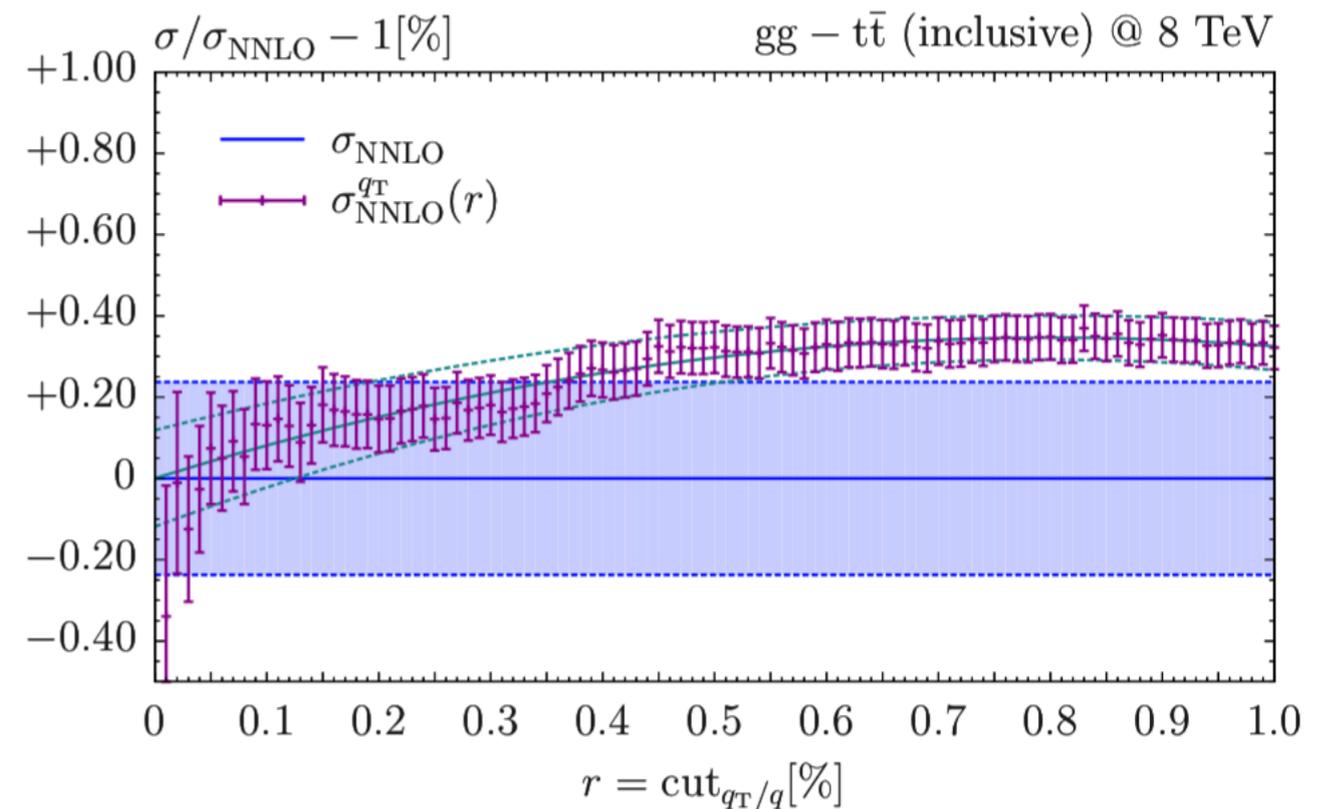
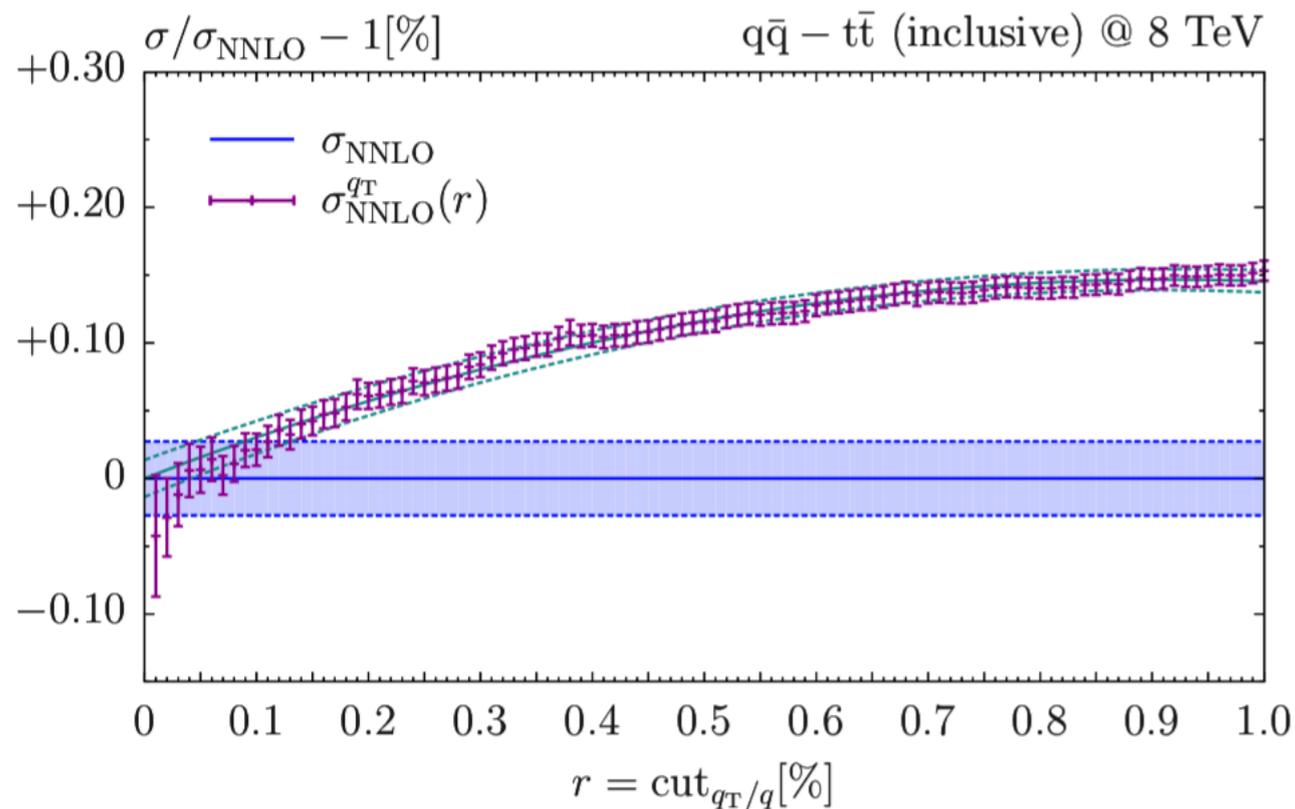
$r_{\text{cut}}$  stability: off-diagonal channels



# Beyond colour singlets: top-quark production

S.Catani, S.Devoto, J.Mazzitelli, S.Kallweit, H.Sargsyan, MG (in progress)

$r_{\text{cut}}$  stability: diagonal channels



# Summary & Outlook

- The  $q_T$  subtraction method has been used to perform a number of important NNLO calculations where a coloured singlet final state is produced in hadron collisions
- The calculations were implemented in numerical codes which are to a large extent independent from each other
- We provide a new NNLO parton level generator which implements all these calculations in a unique framework and includes all the vector-boson pair production processes



MATRIX

- The program combines the MUNICH Monte Carlo framework with amplitudes from Openloops and  $q_T$  subtraction and will eventually include transverse-momentum resummation at NNLL

# Summary & Outlook

- MATRIX encompasses all the previous codes in a single general framework
- First public version including single vector and Higgs boson production and all the diboson processes has been released
- Non-trivial applications to loop induced processes: HH production
- **Some items on our to do list:**
  - NLO gg in WW and ZZ
  - inclusion of processes with a heavy-quark pair
  - Include EW corrections
  - Include anomalous couplings



Backup

process (\$\{process\_id\})	$\sigma_{\text{LO}}$	$\sigma_{\text{NLO}}$	$\sigma_{\text{loop}}^{\text{loop}}$ ( $\sigma_{\text{loop}}/\Delta\sigma_{\text{NNLO}}^{\text{ext}}$ )	$\sigma_{\text{NNLO}}^{\text{r-cut}}$	$\sigma_{\text{NNLO}}^{\text{extrapolated}}$	$K_{\text{NLO}}$	$K_{\text{NNLO}}$
$pp \rightarrow H$ (pph21)	15.42(0) $^{+22\%}_{-17\%}$ pb	30.26(1) $^{+20\%}_{-15\%}$ pb	—	39.93(3) $^{+11\%}_{-10\%}$ pb	39.93(3) $^{+11\%}_{-10\%}$ pb	+96.2%	+32.0%
$pp \rightarrow Z$ (ppz01)	43.32(0) $^{+12\%}_{-13\%}$ nb	54.20(1) $^{+3.1\%}_{-4.9\%}$ nb	—	56.01(3) $^{+0.84\%}_{-1.1\%}$ nb	55.99(3) $^{+0.84\%}_{-1.1\%}$ nb	+25.1%	+3.31%
$pp \rightarrow W^-$ (ppw01)	60.15(0) $^{+13\%}_{-14\%}$ nb	75.95(2) $^{+3.3\%}_{-5.3\%}$ nb	—	78.36(3) $^{+0.98\%}_{-1.2\%}$ nb	78.33(8) $^{+0.98\%}_{-1.2\%}$ nb	+26.3%	+3.14%
$pp \rightarrow W^+$ (ppwx01)	81.28(1) $^{+13\%}_{-14\%}$ nb	102.2(0) $^{+3.4\%}_{-5.3\%}$ nb	—	105.8(1) $^{+0.93\%}_{-1.3\%}$ nb	105.8(1) $^{+0.93\%}_{-1.3\%}$ nb	+25.7%	+3.52%
$pp \rightarrow e^-e^+$ (ppeex02)	592.8(1) $^{+14\%}_{-14\%}$ pb	699.7(2) $^{+2.9\%}_{-4.5\%}$ pb	—	728.4(3) $^{+0.48\%}_{-0.72\%}$ pb	732.7(3.4) $^{+0.43\%}_{-0.79\%}$ pb	+18.0%	+4.72%
$pp \rightarrow \nu_e\bar{\nu}_e$ (ppnenex02)	2876(0) $^{+12\%}_{-13\%}$ pb	3585(1) $^{+3.0\%}_{-4.9\%}$ pb	—	3705(2) $^{+0.86\%}_{-1.1\%}$ pb	3710(2) $^{+0.85\%}_{-1.1\%}$ pb	+24.6%	+3.48%
$pp \rightarrow e^-\bar{\nu}_e$ (ppenex02)	2972(0) $^{+14\%}_{-15\%}$ pb	3674(1) $^{+3.1\%}_{-5.2\%}$ pb	—	3772(2) $^{+0.89\%}_{-0.94\%}$ pb	3768(3) $^{+0.90\%}_{-0.93\%}$ pb	+23.6%	+2.57%
$pp \rightarrow e^+\nu_e$ (ppexne02)	3964(0) $^{+14\%}_{-14\%}$ pb	4855(1) $^{+3.0\%}_{-5.1\%}$ pb	—	4986(2) $^{+0.88\%}_{-0.95\%}$ pb	4986(3) $^{+0.88\%}_{-0.95\%}$ pb	+22.5%	+2.70%
$pp \rightarrow \gamma\gamma$ (ppaa02)	5.592(1) $^{+10\%}_{-11\%}$ pb	25.75(1) $^{+8.8\%}_{-7.5\%}$ pb	2.534(1) $^{+24\%}_{-17\%}$ pb (17.4%)	40.86(2) $^{+8.7\%}_{-7.2\%}$ pb	40.28(30) $^{+8.7\%}_{-7.0\%}$ pb	+361%	+56.4%
$pp \rightarrow e^-e^+\gamma$ (ppeexa03)	1469(0) $^{+12\%}_{-12\%}$ fb	2119(1) $^{+2.9\%}_{-4.6\%}$ fb	16.02(1) $^{+24\%}_{-18\%}$ fb (8.14%)	2326(1) $^{+1.2\%}_{-1.3\%}$ fb	2316(5) $^{+1.1\%}_{-1.2\%}$ fb	+44.3%	+9.29%
$pp \rightarrow \nu_e\bar{\nu}_e\gamma$ (ppnenexa03)	63.61(1) $^{+2.7\%}_{-3.5\%}$ fb	98.75(2) $^{+3.3\%}_{-2.7\%}$ fb	2.559(2) $^{+26\%}_{-19\%}$ fb (17.3%)	114.7(1) $^{+3.2\%}_{-2.6\%}$ fb	113.5(6) $^{+2.9\%}_{-2.4\%}$ fb	+55.2%	+15.0%
$pp \rightarrow e^-\bar{\nu}_e\gamma$ (ppenexa03)	726.1(1) $^{+11\%}_{-12\%}$ fb	1850(1) $^{+6.6\%}_{-5.3\%}$ fb	—	2286(1) $^{+4.0\%}_{-3.7\%}$ fb	2256(15) $^{+3.7\%}_{-3.5\%}$ fb	+155%	+22.0%
$pp \rightarrow e^+\nu_e\gamma$ (ppexnea03)	861.7(1) $^{+10\%}_{-11\%}$ fb	2187(1) $^{+6.6\%}_{-5.3\%}$ fb	—	2707(3) $^{+4.1\%}_{-3.8\%}$ fb	2671(35) $^{+3.8\%}_{-3.6\%}$ fb	+154%	+22.1%
$pp \rightarrow ZZ$ (ppzz02)	9.845(1) $^{+5.2\%}_{-6.3\%}$ pb	14.10(0) $^{+2.9\%}_{-2.4\%}$ pb	1.361(1) $^{+25\%}_{-19\%}$ pb (52.9%)	16.68(1) $^{+3.2\%}_{-2.6\%}$ pb	16.67(1) $^{+3.2\%}_{-2.6\%}$ pb	+43.3%	+18.2%
$pp \rightarrow W^+W^-$ (ppwxw02)	66.64(1) $^{+5.7\%}_{-6.7\%}$ pb	103.2(0) $^{+3.9\%}_{-3.1\%}$ pb	4.091(3) $^{+27\%}_{-19\%}$ pb (29.5%)	117.1(1) $^{+2.5\%}_{-2.2\%}$ pb	117.1(1) $^{+2.5\%}_{-2.2\%}$ pb	+54.9%	+13.4%
$pp \rightarrow e^-\mu^-e^+\mu^+$ (ppemexmx04)	11.34(0) $^{+6.3\%}_{-7.3\%}$ fb	16.87(0) $^{+3.0\%}_{-2.5\%}$ fb	1.971(1) $^{+25\%}_{-18\%}$ fb (57.6%)	20.30(1) $^{+3.5\%}_{-2.9\%}$ fb	20.30(1) $^{+3.5\%}_{-2.9\%}$ fb	+48.8%	+20.3%
$pp \rightarrow e^-e^-e^+e^+$ (ppeexex04)	5.781(1) $^{+6.3\%}_{-7.4\%}$ fb	8.623(3) $^{+3.1\%}_{-2.5\%}$ fb	0.9941(4) $^{+25\%}_{-18\%}$ fb (56.9%)	10.37(1) $^{+3.5\%}_{-3.0\%}$ fb	10.37(1) $^{+3.5\%}_{-3.0\%}$ fb	+49.2%	+20.2%
$pp \rightarrow e^-e^+\nu_\mu\bar{\nu}_\mu$ (ppeexnmnx04)	22.34(0) $^{+5.3\%}_{-6.4\%}$ fb	33.90(1) $^{+3.3\%}_{-2.7\%}$ fb	3.212(1) $^{+25\%}_{-19\%}$ fb (49.6%)	40.39(2) $^{+3.5\%}_{-2.8\%}$ fb	40.38(2) $^{+3.5\%}_{-2.8\%}$ fb	+51.7%	+19.1%
$pp \rightarrow e^-\mu^+\nu_\mu\bar{\nu}_e$ (ppemxnmnex04)	232.9(0) $^{+6.6\%}_{-7.6\%}$ fb	236.1(1) $^{+2.8\%}_{-2.4\%}$ fb	26.93(1) $^{+27\%}_{-19\%}$ fb (94.3%)	264.7(1) $^{+2.2\%}_{-1.4\%}$ fb	264.6(2) $^{+2.2\%}_{-1.4\%}$ fb	+1.34%	+12.1%
$pp \rightarrow e^-e^+\nu_e\bar{\nu}_e$ (ppeexnenex04)	115.0(0) $^{+6.3\%}_{-7.3\%}$ fb	203.4(1) $^{+4.7\%}_{-3.8\%}$ fb	12.62(1) $^{+26\%}_{-19\%}$ fb (33.8%)	240.8(1) $^{+3.4\%}_{-3.0\%}$ fb	240.7(1) $^{+3.4\%}_{-3.0\%}$ fb	+76.9%	+18.4%
$pp \rightarrow e^-\mu^-e^+\bar{\nu}_\mu$ (ppemexnmx04)	11.50(0) $^{+5.7\%}_{-6.8\%}$ fb	23.55(1) $^{+5.5\%}_{-4.5\%}$ fb	—	26.17(1) $^{+2.2\%}_{-2.1\%}$ fb	26.17(2) $^{+2.2\%}_{-2.1\%}$ fb	+105%	+11.1%
$pp \rightarrow e^-e^-e^+\bar{\nu}_e$ (ppeexnrex04)	11.53(0) $^{+5.7\%}_{-6.8\%}$ fb	23.63(1) $^{+5.5\%}_{-4.5\%}$ fb	—	26.27(1) $^{+2.3\%}_{-2.1\%}$ fb	26.25(2) $^{+2.3\%}_{-2.1\%}$ fb	+105%	+11.1%
$pp \rightarrow e^-e^+\mu^+\nu_\mu$ (ppeexmxnm04)	17.33(0) $^{+5.3\%}_{-6.3\%}$ fb	34.14(1) $^{+5.3\%}_{-4.3\%}$ fb	—	37.74(2) $^{+2.2\%}_{-2.0\%}$ fb	37.74(4) $^{+2.2\%}_{-2.0\%}$ fb	+97.0%	+10.6%
$pp \rightarrow e^-e^+e^+\nu_e$ (ppeexexne04)	17.37(0) $^{+5.3\%}_{-6.3\%}$ fb	34.21(2) $^{+5.3\%}_{-4.3\%}$ fb	—	37.85(2) $^{+2.3\%}_{-2.0\%}$ fb	37.84(3) $^{+2.3\%}_{-2.0\%}$ fb	+96.9%	+10.6%

process ({process_id})	LO runtime (relative uncertainty)	NLO runtime (relative uncertainty)	NNLO runtime (relative uncertainty)	NNLO runtime estimate for $10^{-3}$ uncertainty
$pp \rightarrow H$ (pph21)	0 d 0 h 2 m ( $1.5 \cdot 10^{-4}$ )	0 d 0 h 12 m ( $2.7 \cdot 10^{-4}$ )	35 d 23 h 23 m ( $7.2 \cdot 10^{-4}$ )	19 CPU days
$pp \rightarrow Z$ (ppz01)	0 d 0 h 10 m ( $8.2 \cdot 10^{-5}$ )	0 d 0 h 16 m ( $2.6 \cdot 10^{-4}$ )	53 d 15 h 31 m ( $4.6 \cdot 10^{-4}$ )	11 CPU days
$pp \rightarrow W^-$ (ppw01)	0 d 0 h 7 m ( $8.1 \cdot 10^{-5}$ )	0 d 0 h 22 m ( $2.6 \cdot 10^{-4}$ )	50 d 17 h 29 m ( $4.4 \cdot 10^{-4}$ )	10 CPU days
$pp \rightarrow W^+$ (ppwx01)	0 d 0 h 14 m ( $8.1 \cdot 10^{-5}$ )	0 d 0 h 24 m ( $2.6 \cdot 10^{-4}$ )	47 d 7 h 46 m ( $4.9 \cdot 10^{-4}$ )	11 CPU days
$pp \rightarrow e^- e^+$ (ppeex02)	0 d 0 h 48 m ( $1.0 \cdot 10^{-4}$ )	0 d 2 h 24 m ( $2.8 \cdot 10^{-4}$ )	173 d 20 h 36 m ( $3.6 \cdot 10^{-4}$ )	22 CPU days
$pp \rightarrow \nu_e \bar{\nu}_e$ (ppnenex02)	0 d 1 h 31 m ( $8.2 \cdot 10^{-5}$ )	0 d 1 h 0 m ( $2.5 \cdot 10^{-4}$ )	89 d 18 h 17 m ( $4.5 \cdot 10^{-4}$ )	18 CPU days
$pp \rightarrow e^- \bar{\nu}_e$ (ppenex02)	0 d 1 h 46 m ( $8.7 \cdot 10^{-5}$ )	0 d 5 h 21 m ( $2.2 \cdot 10^{-4}$ )	114 d 2 h 18 m ( $4.3 \cdot 10^{-4}$ )	21 CPU days
$pp \rightarrow e^+ \nu_e$ (ppexne02)	0 d 1 h 56 m ( $8.5 \cdot 10^{-5}$ )	0 d 3 h 43 m ( $2.6 \cdot 10^{-4}$ )	114 d 6 h 18 m ( $4.6 \cdot 10^{-4}$ )	24 CPU days
$pp \rightarrow \gamma\gamma$ (ppaa02)	0 d 1 h 13 m ( $9.8 \cdot 10^{-5}$ )	0 d 4 h 11 m ( $2.8 \cdot 10^{-4}$ )	27 d 17 h 7 m ( $4.6 \cdot 10^{-4}$ )	6 CPU days
$pp \rightarrow e^- e^+ \gamma$ (ppeexa03)	0 d 17 h 55 m ( $9.2 \cdot 10^{-5}$ )	1 d 19 h 48 m ( $2.8 \cdot 10^{-4}$ )	1276 d 12 h 47 m ( $3.6 \cdot 10^{-4}$ )	167 CPU days
$pp \rightarrow \nu_e \bar{\nu}_e \gamma$ (ppnenexa03)	0 d 2 h 50 m ( $8.7 \cdot 10^{-5}$ )	0 d 8 h 59 m ( $2.5 \cdot 10^{-4}$ )	75 d 9 h 6 m ( $4.7 \cdot 10^{-4}$ )	17 CPU days
$pp \rightarrow e^- \bar{\nu}_e \gamma$ (ppenexa03)	0 d 22 h 18 m ( $1.0 \cdot 10^{-4}$ )	3 d 16 h 59 m ( $3.2 \cdot 10^{-4}$ )	1484 d 16 h 50 m ( $4.0 \cdot 10^{-4}$ )	232 CPU days
$pp \rightarrow e^+ \nu_e \gamma$ (ppexnea03)	1 d 7 h 8 m ( $9.6 \cdot 10^{-5}$ )	6 d 8 h 7 m ( $3.0 \cdot 10^{-4}$ )	428 d 7 h 1 m ( $1.0 \cdot 10^{-3}$ )	443 CPU days
$pp \rightarrow ZZ$ (ppzz02)	0 d 1 h 44 m ( $8.2 \cdot 10^{-5}$ )	0 d 1 h 6 m ( $2.4 \cdot 10^{-4}$ )	132 d 19 h 37 m ( $4.4 \cdot 10^{-4}$ )	25 CPU days
$pp \rightarrow W^+ W^-$ (ppwxw02)	0 d 1 h 23 m ( $8.2 \cdot 10^{-5}$ )	0 d 0 h 48 m ( $2.5 \cdot 10^{-4}$ )	69 d 20 h 49 m ( $4.3 \cdot 10^{-4}$ )	13 CPU days
$pp \rightarrow e^- \mu^- e^+ \mu^+$ (ppemexmx04)	0 d 5 h 43 m ( $8.2 \cdot 10^{-5}$ )	0 d 4 h 32 m ( $2.7 \cdot 10^{-4}$ )	219 d 16 h 33 m ( $4.5 \cdot 10^{-4}$ )	45 CPU days
$pp \rightarrow e^- e^- e^+ e^+$ (ppeexex04)	0 d 11 h 34 m ( $9.0 \cdot 10^{-5}$ )	0 d 12 h 8 m ( $3.4 \cdot 10^{-4}$ )	742 d 13 h 37 m ( $5.1 \cdot 10^{-4}$ )	193 CPU days
$pp \rightarrow e^- e^+ \nu_\mu \bar{\nu}_\mu$ (ppeexnmnm04)	0 d 6 h 33 m ( $9.4 \cdot 10^{-5}$ )	0 d 6 h 36 m ( $2.7 \cdot 10^{-4}$ )	158 d 13 h 40 m ( $4.4 \cdot 10^{-4}$ )	31 CPU days
$pp \rightarrow e^- \mu^+ \nu_\mu \bar{\nu}_e$ (ppemxnmnex04)	0 d 13 h 33 m ( $9.2 \cdot 10^{-5}$ )	1 d 22 h 9 m ( $2.7 \cdot 10^{-4}$ )	521 d 2 h 20 m ( $4.8 \cdot 10^{-4}$ )	119 CPU days
$pp \rightarrow e^- e^+ \nu_e \bar{\nu}_e$ (ppeexnenex04)	0 d 23 h 36 m ( $8.2 \cdot 10^{-5}$ )	0 d 17 h 46 m ( $4.8 \cdot 10^{-4}$ )	270 d 6 h 59 m ( $4.4 \cdot 10^{-4}$ )	52 CPU days
$pp \rightarrow e^- \mu^- e^+ \bar{\nu}_\mu$ (ppemexnm04)	0 d 5 h 18 m ( $1.0 \cdot 10^{-4}$ )	0 d 5 h 15 m ( $2.9 \cdot 10^{-4}$ )	104 d 16 h 46 m ( $4.3 \cdot 10^{-4}$ )	19 CPU days
$pp \rightarrow e^- e^- e^+ \bar{\nu}_e$ (ppeexn04)	0 d 14 h 19 m ( $8.3 \cdot 10^{-5}$ )	0 d 14 h 56 m ( $2.7 \cdot 10^{-4}$ )	179 d 14 h 6 m ( $4.7 \cdot 10^{-4}$ )	39 CPU days
$pp \rightarrow e^- e^+ \mu^+ \nu_\mu$ (ppeexmxnm04)	0 d 10 h 32 m ( $8.1 \cdot 10^{-5}$ )	0 d 8 h 18 m ( $2.6 \cdot 10^{-4}$ )	104 d 17 h 58 m ( $4.5 \cdot 10^{-4}$ )	21 CPU days
$pp \rightarrow e^- e^+ e^+ \nu_e$ (ppeexexne04)	0 d 9 h 19 m ( $1.0 \cdot 10^{-4}$ )	0 d 13 h 11 m ( $4.6 \cdot 10^{-4}$ )	167 d 6 h 49 m ( $5.1 \cdot 10^{-4}$ )	44 CPU days