N-jettiness subtraction: overview, recent developments and applications

Frank Petriello

Subtracting Infrared Singularities Beyond NLO

April 11, 2018

NORTHWESTERN UNIVERSITY

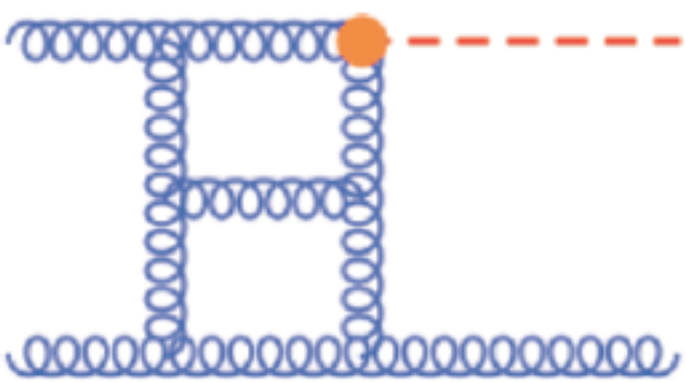
Argonne NATIONAL LABORATORY

Overview of N-jettiness subtraction

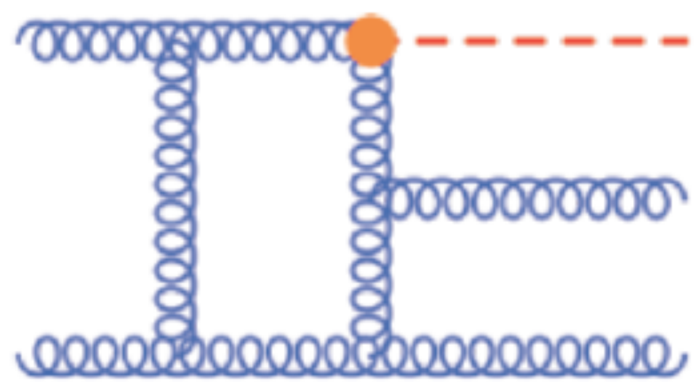
Fixed-order cross sections at NNLO

- Need the following ingredients for NNLO cross sections:

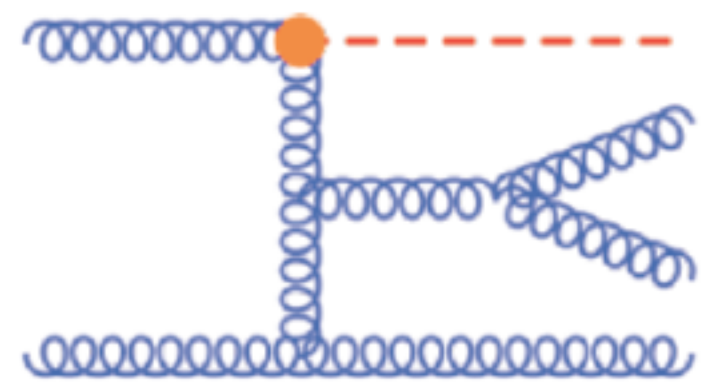
VV



RV



RR



$$\int \left[\frac{vv_4}{\epsilon^4} + \frac{vv_3}{\epsilon^3} + \frac{vv_2}{\epsilon^2} + \frac{vv_1}{\epsilon} + vv_0\right] \, d\Phi_2 \quad \int \left[\frac{rv_2}{\epsilon^2} + \frac{rv_1}{\epsilon} + rv_0\right] \, d\Phi_3 \quad \int [rr_0] \, d\Phi_4$$

- In principle this is straightforward: draw all diagrams and calculate. In practice, it is complicated by by the implicit poles in the real radiation corrections that only appear after integration over phase space; that's why we're here at this workshop!

Subtraction at NNLO

- This is typically dealt with using a subtraction scheme. The generic form of an NNLO subtraction scheme is the following:

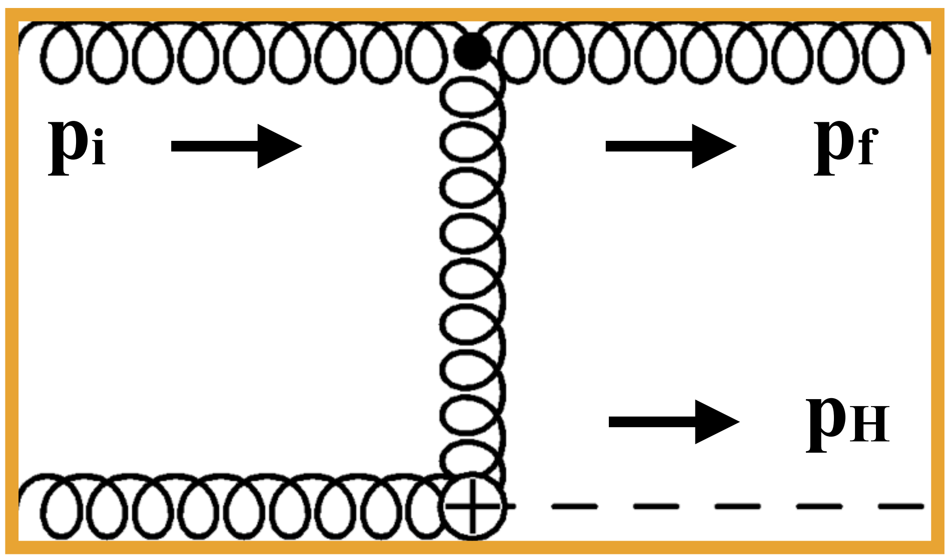
$$
\begin{aligned}
\mathrm{d}\sigma_{NNLO} = & \int_{\mathrm{d}\Phi_{m+2}} \left(\mathrm{d}\sigma^{R}_{NNLO} - \mathrm{d}\sigma^{S}_{NNLO}\right) \\
& + \int_{\mathrm{d}\Phi_{m+1}} \left(\mathrm{d}\sigma^{V,1}_{NNLO} - \mathrm{d}\sigma^{VS,1}_{NNLO}\right) \\
& + \int_{\mathrm{d}\Phi_{m+2}} \mathrm{d}\sigma^{S}_{NNLO} + \int_{\mathrm{d}\Phi_{m+1}} \mathrm{d}\sigma^{VS,1}_{NNLO} \\
& + \int_{\mathrm{d}\Phi_{m}} \mathrm{d}\sigma^{V,2}_{NNLO} \,,
\end{aligned}
$$

- Maximally singular configurations at NNLO can have two collinear, two soft singularities
- Subtraction terms must account for all of the many possible singular configurations: triple-collinear ($p_1||p_2||p_3$), double-collinear ($p_1||p_2,p_3||p_4$), double-soft, single-soft, soft +collinear, etc.

- There has been significant progress in developing subtraction schemes at NNLO over the past several years, which will extensively discussed at this workshop.

Regulating the IR with p_T

- To see the possibility of another approach, consider Higgs production at **NLO**, or **O(α_S)**, as an example. A real emission correction:



η=rapidity of jet

$$\frac{1}{2p_i \cdot p_f} = \frac{1}{2E_i p_{TH} e^{\eta}}$$

p_{TH}=transverse momentum of Higgs

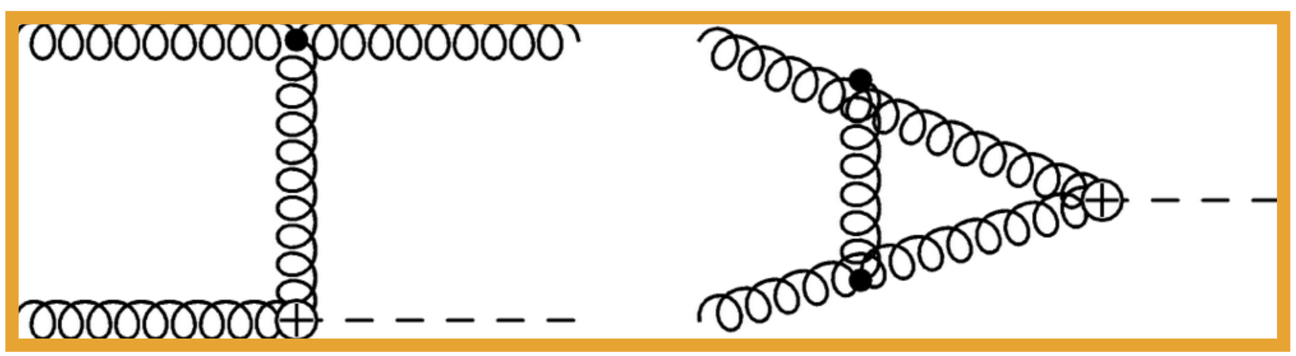
This propagator can't diverge for finite transverse momentum (note that η must be finite for non-vanishing p_{TH})

O(α_S) becomes a Born-level calculation with no singularities at finite p_{TH}

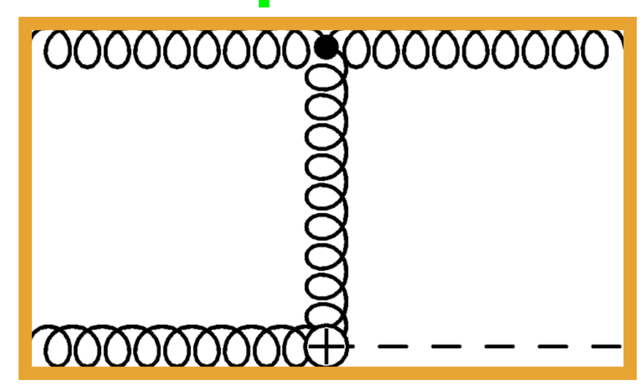
Regulating the IR with p_T

- This observation motivates the following partition of phase space for the differential cross section:

$$\sigma = \int dp_{TH} \frac{d\sigma}{dp_{TH}} \theta(p_{TH}^{cut} - p_{TH}) + \int dp_{TH} \frac{d\sigma}{dp_{TH}} \theta(p_{TH} - p_{TH}^{cut})$$



Singular regions of real emissions and virtual corrections go here



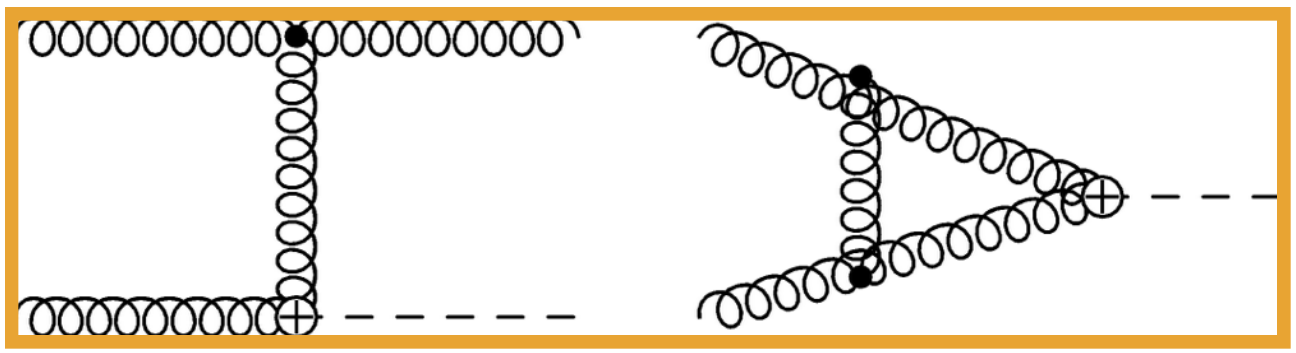
Finite regions of real emissions go here

This is a simple, finite tree-level calculation

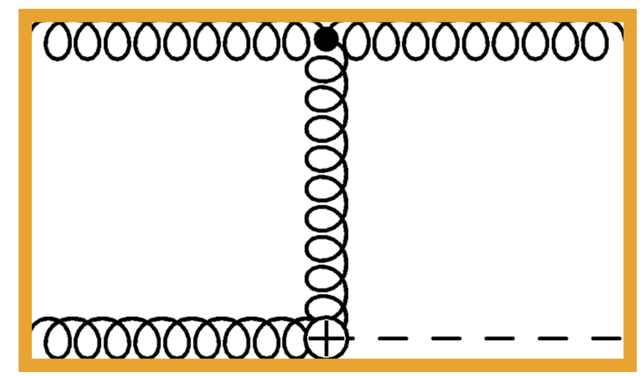
Regulating the IR with p_T

- This observation motivates the following partition of phase space for the differential cross section:

$$\sigma = \int dp_{TH} \frac{d\sigma}{dp_{TH}} \theta(p_{TH}^{cut} - p_{TH}) + \int dp_{TH} \frac{d\sigma}{dp_{TH}} \theta(p_{TH} - p_{TH}^{cut})$$



Singular regions of real emissions and



Finite regions of real emissions go here

This split is useful because there is a simpler way to derive the cross section below p_{cut}

This is a simple, finite tree-level calculation

Effective field theory for low p_{TH}

- ***Effective field theory*** can simplify the calculation when $p_{TH} \ll m_H$. It provides a systematic way of expanding the full differential cross section for small p_{TH}/m_H.

x_a, x_b=Bjorken-x for each beam

$$\frac{d\sigma}{dp_{TH}}(p_{TH} \ll m_H) \sim S(m_H, p_{TH}) \otimes C_a(p_{TH}, x_a) \otimes C_b(p_{TH}, x_b)$$

Universal function describing soft emissions

Functions which describe virtual corrections and collinear emissions

Collins, Soper, Sterman (1985)

This formula can be used at NNLO since S, C_i are known to $O(\alpha_S^2)$

It is a much simpler problem to calculate S and C_i than it is to cancel real and virtual singularities at NNLO for arbitrary observables!

Effective field theory for low p_{TH}

- ***Effective field theory*** can simplify the calculation when $p_{TH} \ll m_H$. It provides a systematic way of expanding the full differential cross section for small p_{TH}/m_H.

x_a, x_b=Bjorken-x for each beam

$$\frac{d\sigma}{dp_{TH}}(p_{TH} \ll m_H) \sim S(m_H, p_{TH}) \otimes C_a(p_{TH}, x_a) \otimes C_b(p_{TH}, x_b)$$

Universal function describing soft emissions

Functions which describe virtual corrections and collinear emissions

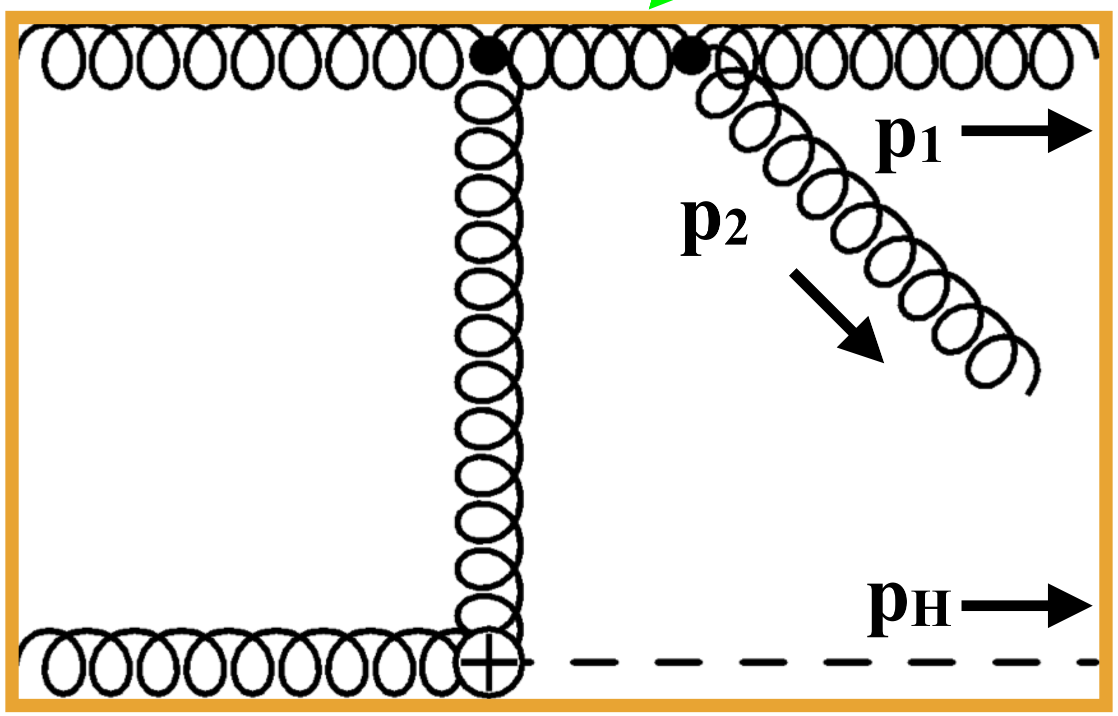
Collins, Soper, Sterman (1985)

For $p_{cut}/m_H \to 0$ this becomes an ***exact*** expression for the NNLO result. This is the idea behind q_T-subtraction. Catani, Grazzini (2007)

Jets at the LHC?

- A limitation of this approach is that it can only describe partonic processes with no final-state collinear singularities

Consider Higgs+jet at NLO, or Higgs at finite p_{TH}, as an example



$$\frac{1}{2p_1 \cdot p_2} = \frac{1}{2p_{T1}|\vec{p}_{TH} - \vec{p}_{T1}|} \times \frac{1}{\cosh(\Delta\eta) - \cos(\Delta\phi)}$$

This vanishes independently of p_{TH} for either p_{T1} or p_{T2} soft, or $p_1||p_2$

p_{TH} no longer resolves singularities in the presence of final-state collinear singularities

N-jettiness

- There is a resolution parameter suitable for final-state partons!

N=number of jets

$$\tau_N = \sum_k \min\{n_i \cdot q_k\}$$

N-jettiness, an event shape variable (similar to thrust); first introduced in Stewart, Tackmann, Waalewijn (2009)

light-like directions of initial beams and final-state jets

momenta of final-state partons

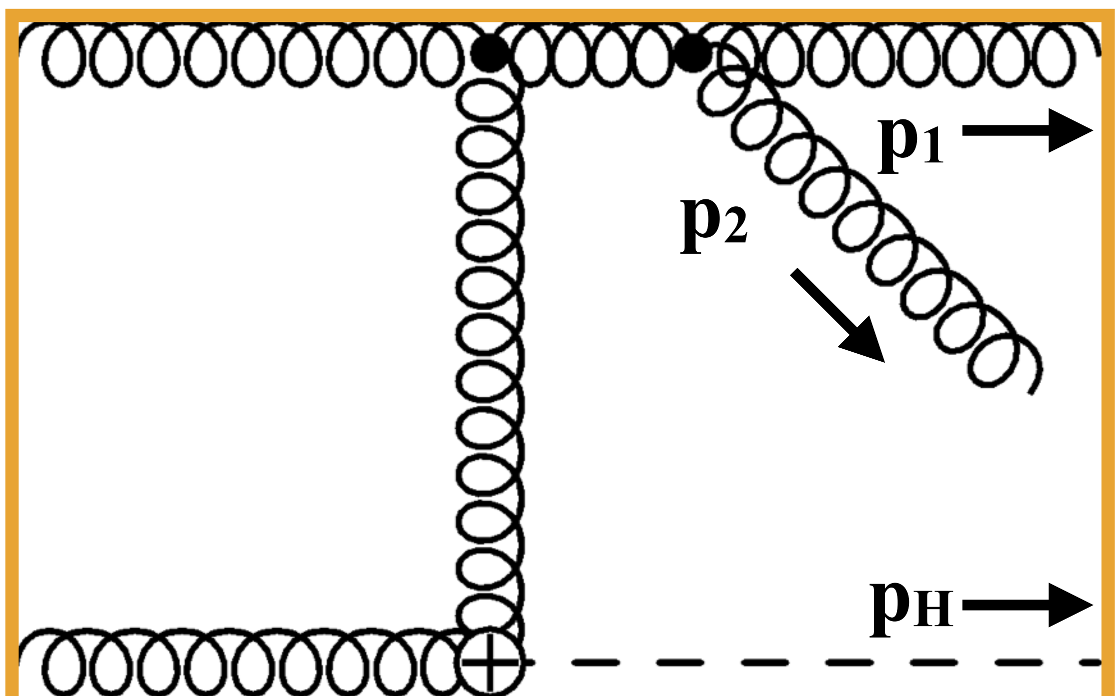
Intuition:

τ_N ~0: all radiation is either soft, or collinear to a beam/jet

τ_N>0: at least one additional jet beyond Born level is resolved

N-jettiness

- Go back and reconsider our Higgs+jet example using this variable, in the potentially singular kinematic limits $p_1 || p_2$ and $p_{1,2}$ soft:



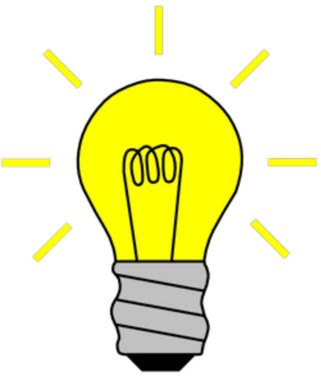
$$\frac{1}{2p_1 \cdot p_2} \approx \frac{1}{2E_J \tau_1}$$

final-state jet energy

1-jettiness, since our Born-level process has a single jet

All final-state singularities are regulated by τ_1!

N-jettiness subtraction



We can obtain NNLO predictions for arbitrary jet production processes using N-jettiness as a resolution parameter since we know the below-cut result already!

First derived in Stewart, Tackmann, Waalewijn (2009)

$$\frac{d\sigma}{d\tau_N}(\tau_N \ll Q) \sim H \otimes B_a \otimes B_b \otimes S \otimes \left[\prod_{n=1}^{N} J_n\right]$$

hard scales in the process (e.g., transverse momenta of jets)

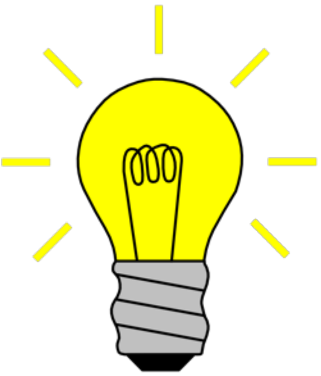
describes hard radiation

describes radiation collinear to initial-state beams; ***universal***

describes soft radiation; ***universal;*** depends on number of jets

describes radiation collinear to final-state jets; ***universal***

N-jettiness subtraction



We can obtain NNLO predictions for arbitrary jet production processes using N-jettiness as a resolution parameter since we know the below-cut result already!

First derived in Stewart, Tackmann, Waalewijn (2009)

$$\frac{d\sigma}{d\tau_N}(\tau_N \ll Q) \sim H \otimes B_a \otimes B_b \otimes S \otimes \left[\prod_{n=1}^{N} J_n\right]$$

hard scales in the process (e.g., transverse momen jets)

hard radiation

describes beams; *universal*

describes depends on number of jets

lescribes radiation ollinear to final-state jets; *universal*

Plus power corrections to be discussed later!

N-jettiness subtraction

- Practical application: Introduce τ_N^{cut} that separates the $\tau_N=0$ doubly-unresolved limit of phase space from the single-unresolved and hard regions

$$\sigma_{NNLO} = \int d\Phi_N \, |\mathcal{M}_N|^2 + \int d\Phi_{N+1} \, |\mathcal{M}_{N+1}|^2 \, \theta_N^<$$

$$+ \int d\Phi_{N+2} \, |\mathcal{M}_{N+2}|^2 \, \theta_N^< + \int d\Phi_{N+1} \, |\mathcal{M}_{N+1}|^2 \, \theta_N^>$$

$$+ \int d\Phi_{N+2} \, |\mathcal{M}_{N+2}|^2 \, \theta_N^>$$

$$\equiv \sigma_{NNLO}(\mathcal{T}_N < \mathcal{T}_N^{cut}) + \sigma_{NNLO}(\mathcal{T}_N > \mathcal{T}_N^{cut})$$

$$\theta_N^< = \theta(\tau_N^{cut} - \tau_N) \quad \text{and} \quad \theta_N^> = \theta(\tau_N - \tau_N^{cut})$$

N-jettiness subtraction

- Practical application: Introduce τ_N^{cut} that separates the $\tau_N=0$ doubly-unresolved limit of phase space from the single-unresolved and hard regions

$$\sigma_{NNLO} = \int d\Phi_N \, |\mathcal{M}_N|^2 + \int d\Phi_{N+1} \, |\mathcal{M}_{N+1}|^2 \, \theta_N^< + \int d\Phi_{N+2} \, |\mathcal{M}_{N+2}|^2 \, \theta_N^< + \int d\Phi_{N+1} \, |\mathcal{M}_{N+1}|^2 \, \theta_N^> + \int d\Phi_{N+2} \, |\mathcal{M}_{N+2}|^2 \, \theta_N^>$$

$$\equiv \sigma_{NNLO}(\mathcal{T}_N < \mathcal{T}_N^{cut}) + \sigma_{NNLO}(\mathcal{T}_N > \mathcal{T}_N^{cut})$$

- **For $\tau_N > \tau_N^{cut}$**: at least one of the two additional radiations that appear at NNLO is resolved; just the NLO correction to the N+1 jet process!
- **For $\tau_N < \tau_N^{cut}$**: use factorization theorem!

N-jettiness subtraction

- Only one more issue to address: what is known regarding the functions **H, B, S, J**? Do we known them to the requisite NNLO?

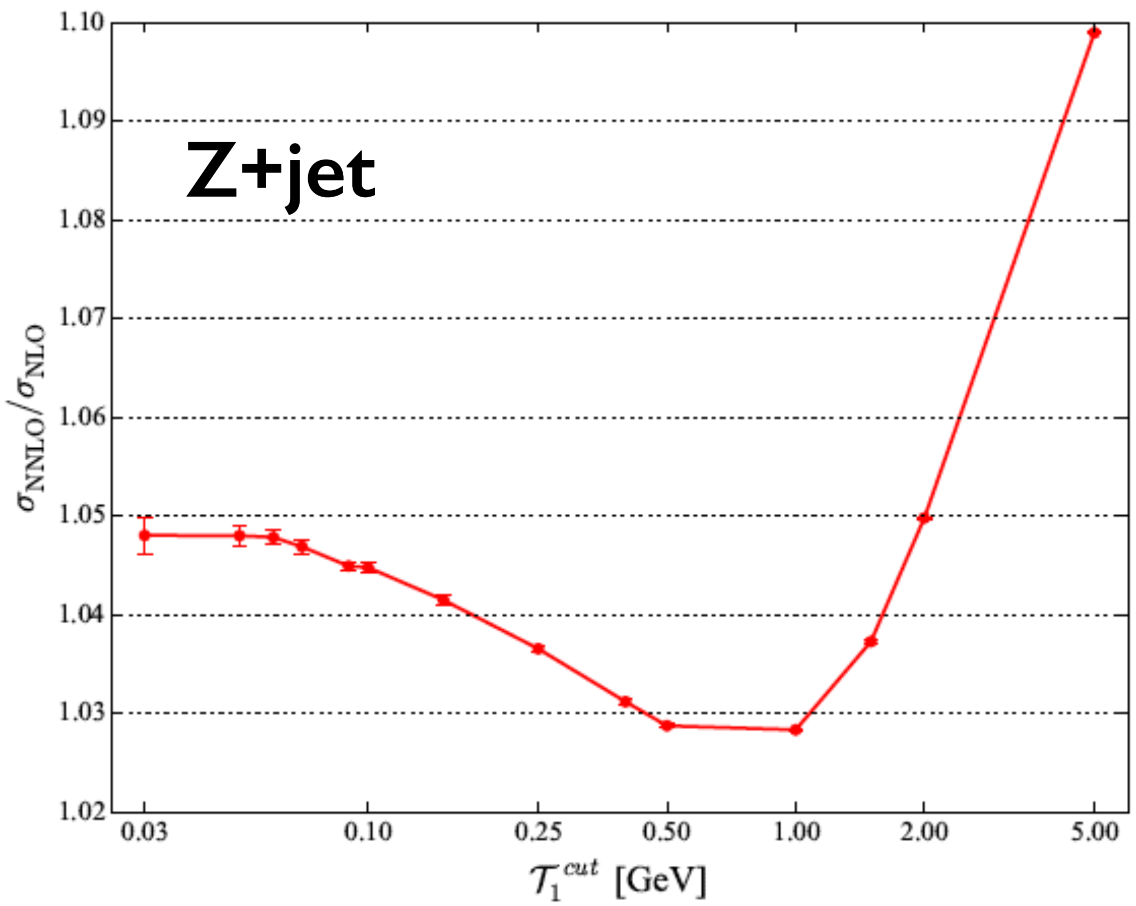
  - ***H@NNLO:*** for W/H+j, Gehrmann, Tancredi (2011); Gehrmann, Jaquier, Glover, Koukoutsakis (2011) (see also Becher, Bell, Lorentzen, Marti (2013))
  - ***B@NNLO:*** Gaunt, Stahlhofen, Tackmann (2014)
  - ***S@NNLO:*** Boughezal, Liu, FP PRD 91 (2015)
  - ***J@NNLO:*** Becher, Neubert (2006); Becher, Bell (2011)

Within the past few years all ingredients have become available to apply this idea to jet production at colliders!

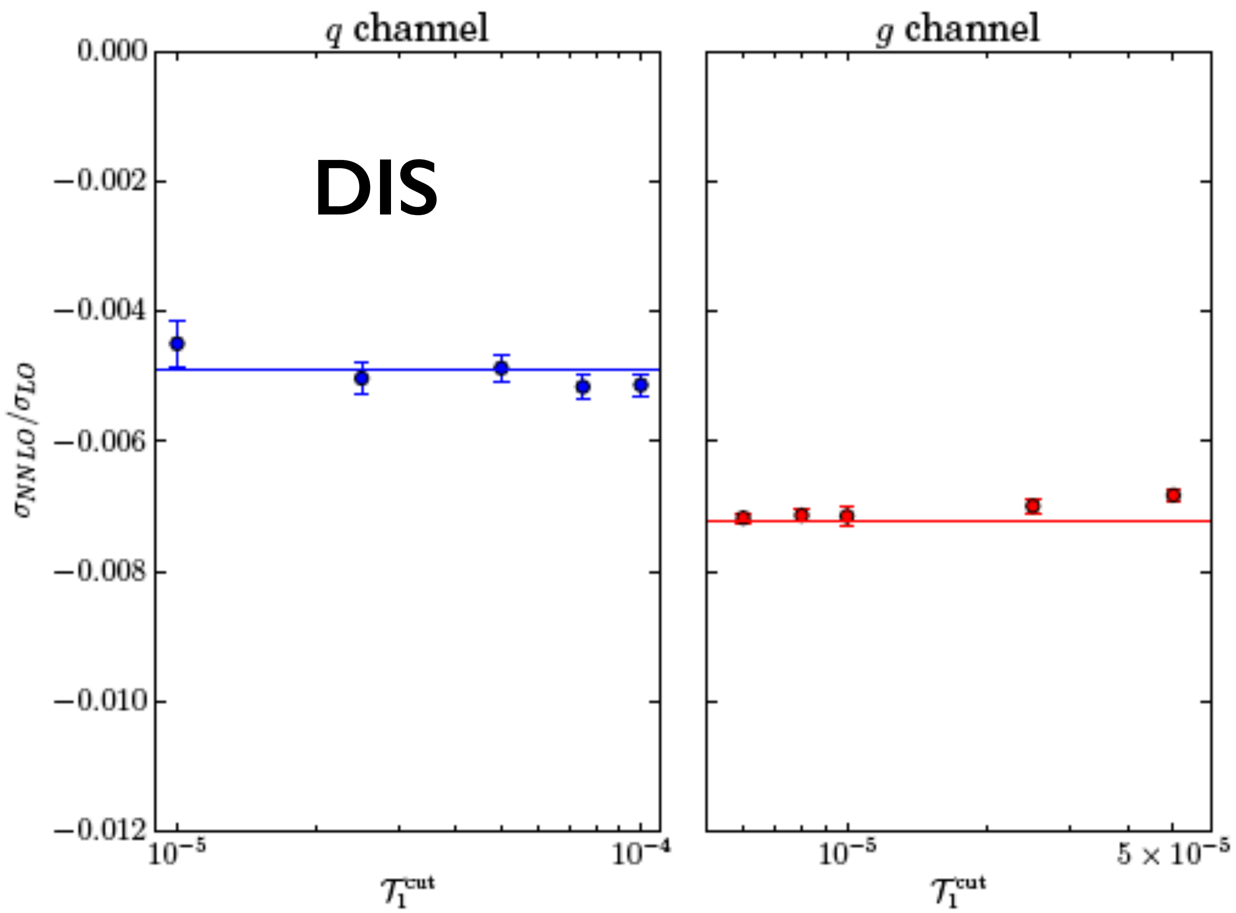
Recent developments: power corrections

Power corrections

- Primary numerical challenge is the impact of power corrections to the factorization formula



Boughezal et al, 1512.01291



Abelof, Boughezal, Liu, FP 1607.04921

Leading power corrections:

NLO: $\alpha_S \times \tau^{cut} \mathrm{Log}(\tau^{cut})$

NNLO: $\alpha_S^2 \times \tau^{cut} \mathrm{Log}^3(\tau^{cut})$

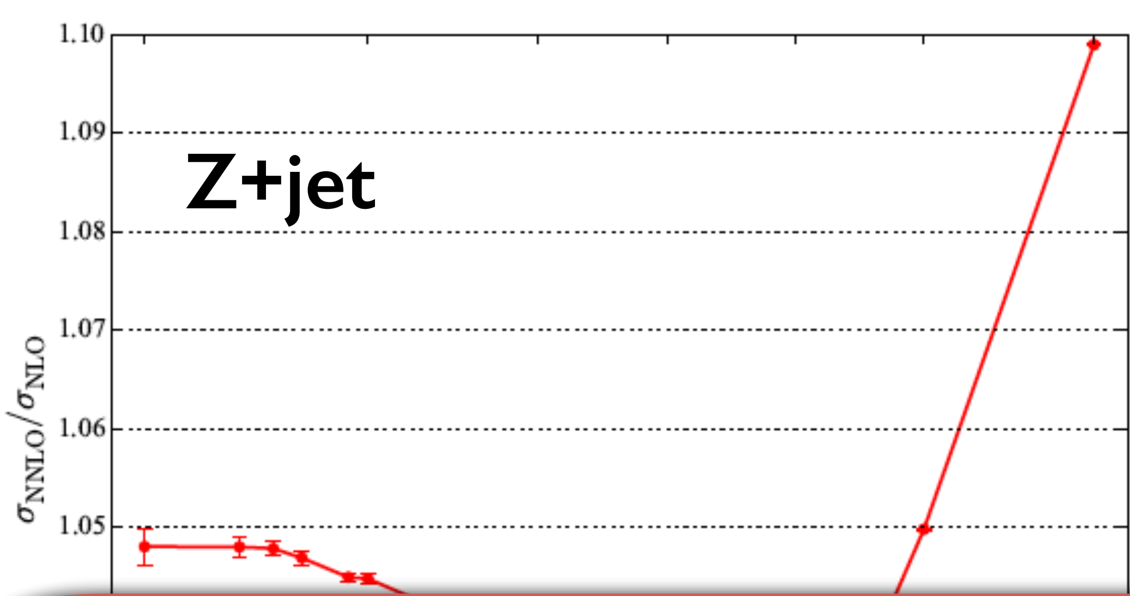
Behavior of below/above cut at leading power:

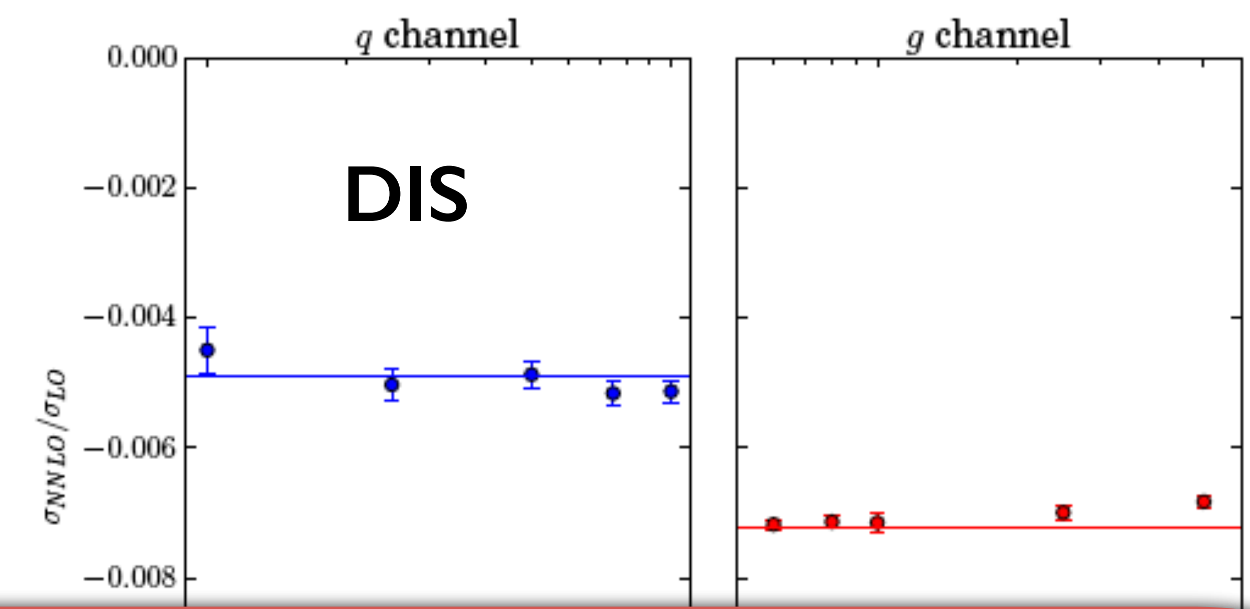
NLO: $\alpha_S \times \mathrm{Log}^2(\tau^{cut})$

NNLO: $\alpha_S^2 \times \mathrm{Log}^4(\tau^{cut})$

Power corrections

- Primary numerical challenge is the impact of power corrections to the factorization formula





Want to reduce τ^{cut} to minimize power corrections; this introduces numerical noise in the cancellation of logarithms between below and above cut contributions!

Leading power corrections:

NLO: $\alpha_S \times \tau^{cut} \mathrm{Log}(\tau^{cut})$

NNLO: $\alpha_S^2 \times \tau^{cut} \mathrm{Log}^3(\tau^{cut})$

Behavior of below/above cut at leading power:

NLO: $\alpha_S \times \mathrm{Log}^2(\tau^{cut})$

NNLO: $\alpha_S^2 \times \mathrm{Log}^4(\tau^{cut})$

Power corrections for color-singlet production

- Significant recent activity and progress in understanding power corrections for the simplest case of color-singlet production

  Moult et al, 1612.00450, 1710.03227; Boughezal et al, 1612.02911, 1802.00456

  our focus here

- Can consider the power corrections integrated up to τ^{cut}, and also at the unintegrated level
- Current status on next-to-leading power correction (NLP) from 1612.02911+1802.00456:

Integrated:	LL	NLL	Un-integrated:	LL	NLL
NLO	$\alpha_S \times \tau^{cut} \mathrm{Log}(\tau^{cut})$	$\alpha_S \times \tau^{cut}$	NLO	$\alpha_S \times \mathrm{Log}(\tau)$	α_S
NNLO	$\alpha_S^2 \times \tau^{cut} \mathrm{Log}^3(\tau^{cut})$	unknown	NNLO	$\alpha_S^2 \times \mathrm{Log}^2(\tau)$	unknown

Goals

- Calculate and include color-singlet power corrections where possible
- See what aspects of the calculation generalize beyond color-singlet production
- As a by-product of our analysis provide a map between direct QCD and SCET derivations of the N-jettiness spectrum
- Analyze different definitions of N-jettiness

$$\mathcal{T}_N = \sum_k \min_i \left\{ \frac{2q_i \cdot p_k}{Q_i} \right\}$$

Are there choices of the hardness measures that minimize power corrections; especially helpful when their calculation at NNLO is difficult!

Factorization of the phase space

- Use gluon-fusion Higgs production at NLO as an example

  Born process: $g(p_1)+g(p_2)\to H(p_H)$

  NLO real-emission correction: $g(p_1')+g(p_2')\to H(p_H)+g(p_3)$

- First step is to map NLO real emission events with fixed $\mathcal{T}$ to Born level:

$$\mathrm{PS_{Born}} = (2\pi)\int_0^1 \mathrm{d}x_a \int_0^1 \mathrm{d}x_b \,\frac{f_g(x_a)\, f_g(x_b)}{2 s x_a x_b}\,\delta(s x_a x_b - m_H^2)$$

$$\frac{\mathrm{dPS}^{(a)}_{\mathrm{NLO}}}{\mathrm{d}\mathcal{T}} = \frac{\mathcal{T}^{-\varepsilon}}{8\pi}\frac{(4\pi\mu_0^2)^{\varepsilon}}{\Gamma(1-\varepsilon)}\int_0^1 \mathrm{d}x_a \int_0^1 \mathrm{d}x_b \frac{f_g(x_b)}{2 s x_a x_b}\delta\left(s x_a x_b - m_H^2\right)\int_{x_a+\frac{Q_a\mathcal{T}}{m_H^2}}^{1-\frac{\mathcal{T}Q_b}{m_H^2}} \frac{\mathrm{d}z_a}{z_a}\,(Q_a)^{1-\varepsilon}\left(\frac{1-z_a}{z_a}\right)^{-\varepsilon}$$
$$\left\{ f_g\left(\frac{x_a}{z_a}\right) + \frac{\mathcal{T}}{m_H^2 z_a^2}\left[\left(Q_a' z_a x_a - Q_a z_a \varepsilon\right) f_g\left(\frac{x_a}{z_a}\right) + Q_a x_a f_g'\left(\frac{x_a}{z_a}\right)\right] + \mathcal{O}\left(\mathcal{T}^2\right)\right\}$$

Factorization of the phase space

- Use gluon-fusion Higgs production at NLO as an example

  Born process: $g(p_1)+g(p_2)\rightarrow H(p_H)$

  NLO real-emission correction: $g(p_1')+g(p_2')\rightarrow H(p_H)+g(p_3)$

- First step is to map NLO real emission events with fixed τ to Born level:

$$\mathrm{PS}_{\mathrm{Born}} = (2\pi)\int_0^1 \mathrm{d}x_a \int_0^1 \mathrm{d}x_b \, \frac{f_g(x_a)\, f_g(x_b)}{2 s x_a x_b}\, \delta(s x_a x_b - m_H^2)$$

Born phase space

$$\frac{\mathrm{dPS}_{\mathrm{NLO}}^{(a)}}{\mathrm{d}\mathcal{T}} = \frac{\mathcal{T}^{-\varepsilon}}{8\pi}\frac{(4\pi\mu_0^2)^{\varepsilon}}{\Gamma(1-\varepsilon)}\int_0^1 \mathrm{d}x_a \int_0^1 \mathrm{d}x_b \frac{f_g(x_b)}{2 s x_a x_b}\delta\left(s x_a x_b - m_H^2\right)\int_{x_a+\frac{Q_a\mathcal{T}}{m_H^2}}^{1-\frac{\mathcal{T}Q_b}{m_H^2}} \frac{\mathrm{d}z_a}{z_a}\left(Q_a\right)^{1-\varepsilon}\left(\frac{1-z_a}{z_a}\right)^{-\varepsilon}$$

$$\left\{ f_g\left(\frac{x_a}{z_a}\right) + \frac{\mathcal{T}}{m_H^2 z_a^2}\left[\left(Q_a' z_a x_a - Q_a z_a \varepsilon\right) f_g\left(\frac{x_a}{z_a}\right) + Q_a x_a f_g'\left(\frac{x_a}{z_a}\right)\right] + \mathcal{O}\left(\mathcal{T}^2\right)\right\}$$

Factorization of the phase space

- Use gluon-fusion Higgs production at NLO as an example

  Born process: $g(p_1)+g(p_2)\rightarrow H(p_H)$

  NLO real-emission correction: $g(p_1')+g(p_2')\rightarrow H(p_H)+g(p_3)$

- First step is to map NLO real emission events with fixed $\mathcal{T}$ to Born level:

$z_a=1$ corresponds to soft limit of p_3

$$\frac{\mathrm{dPS}^{(a)}_{\mathrm{NLO}}}{\mathrm{d}\mathcal{T}} = \frac{\mathcal{T}^{-\varepsilon}}{8\pi}\frac{(4\pi\mu_0^2)^{\varepsilon}}{\Gamma(1-\varepsilon)}\int_0^1 \mathrm{d}x_a \int_0^1 \mathrm{d}x_b \frac{f_g(x_b)}{2sx_ax_b}\delta\left(sx_ax_b - m_H^2\right)\int_{x_a+\frac{Q_a\mathcal{T}}{m_H^2}}^{1-\frac{\mathcal{T}Q_b}{m_H^2}} \frac{\mathrm{d}z_a}{z_a}\left(Q_a\right)^{1-\varepsilon}\left(\frac{1-z_a}{z_a}\right)^{-\varepsilon}$$
$$\left\{f_g\left(\frac{x_a}{z_a}\right) + \frac{\mathcal{T}}{m_H^2 z_a^2}\left[\left(Q_a' z_a x_a - Q_a z_a \varepsilon\right) f_g\left(\frac{x_a}{z_a}\right) + Q_a x_a f_g'\left(\frac{x_a}{z_a}\right)\right] + \mathcal{O}(\mathcal{T}^2)\right\}$$

Factorization of the phase space

- Use gluon-fusion Higgs production at NLO as an example

  Born process: $g(p_1)+g(p_2)\to H(p_H)$

  NLO real-emission correction: $g(p_1')+g(p_2')\to H(p_H)+g(p_3)$

- First step is to map NLO real emission events with fixed τ to Born level:

NLP correction from phase space

$$\frac{\mathrm{dPS}^{(a)}_{\mathrm{NLO}}}{\mathrm{d}\mathcal{T}} = \frac{\mathcal{T}^{-\varepsilon}}{8\pi}\frac{(4\pi\mu_0^2)^{\varepsilon}}{\Gamma(1-\varepsilon)}\int_0^1 \mathrm{d}x_a \int_0^1 \mathrm{d}x_b \frac{f_g(x_b)}{2sx_ax_b}\delta\left(sx_ax_b - m_H^2\right)\int_{x_a+\frac{Q_a\mathcal{T}}{m_H^2}}^{1-\frac{\mathcal{T}Q_b}{m_H^2}} \frac{\mathrm{d}z_a}{z_a}(Q_a)^{1-\varepsilon}\left(\frac{1-z_a}{z_a}\right)^{-\varepsilon}$$
$$\left\{f_g\left(\frac{x_a}{z_a}\right) + \frac{\mathcal{T}}{m_H^2 z_a^2}\left[\left(Q_a' z_a x_a - Q_a z_a \varepsilon\right) f_g\left(\frac{x_a}{z_a}\right) + Q_a x_a f_g'\left(\frac{x_a}{z_a}\right)\right] + \mathcal{O}\left(\mathcal{T}^2\right)\right\}$$

Factorization of the phase space

- Use gluon-fusion Higgs production at NLO as an example

  Born process: $g(p_1)+g(p_2)\rightarrow H(p_H)$

  NLO real-emission correction: $g(p_1')+g(p_2')\rightarrow H(p_H)+g(p_3)$

- First step is to map NLO real emission events with fixed τ to Born level (consider the region $\tau=n\cdot p_3$ as an example):

Derivative of PDF with respect to Bjorken-x

$$\frac{\mathrm{dPS}^{(a)}_{\mathrm{NLO}}}{\mathrm{d}\mathcal{T}} = \frac{\mathcal{T}^{-\varepsilon}}{8\pi}\frac{(4\pi\mu_0^2)^{\varepsilon}}{\Gamma(1-\varepsilon)}\int_0^1 \mathrm{d}x_a \int_0^1 \mathrm{d}x_b \frac{f_g(x_b)}{2s x_a x_b}\delta\left(s x_a x_b - m_H^2\right)\int_{x_a+\frac{Q_a\mathcal{T}}{m_H^2}}^{1-\frac{\mathcal{T}Q_b}{m_H^2}} \frac{\mathrm{d}z_a}{z_a}\left(Q_a\right)^{1-\varepsilon}\left(\frac{1-z_a}{z_a}\right)^{-\varepsilon}$$

$$\left\{ f_g\left(\frac{x_a}{z_a}\right) + \frac{\mathcal{T}}{m_H^2 z_a^2}\left[\left(Q_a' z_a x_a - Q_a z_a \varepsilon\right) f_g\left(\frac{x_a}{z_a}\right) + Q_a x_a f_g'\left(\frac{x_a}{z_a}\right)\right] + \mathcal{O}\left(\mathcal{T}^2\right)\right\}$$

Factorization of the phase space

- Use gluon-fusion Higgs production at NLO as an example

  Born process: $g(p_1)+g(p_2)\to H(p_H)$

  NLO real-emission correction: $g(p_1')+g(p_2')\to H(p_H)+g(p_3)$

- First step is to map NLO real emission events with fixed τ to Born level:

The steps leading to this form appear to be also applicable to jet production processes

$$\frac{\mathrm{dPS}^{(a)}_{\mathrm{NLO}}}{\mathrm{d}\mathcal{T}} = \frac{\mathcal{T}^{-\varepsilon}}{8\pi}\frac{(4\pi\mu_0^2)^{\varepsilon}}{\Gamma(1-\varepsilon)}\int_0^1 \mathrm{d}x_a \int_0^1 \mathrm{d}x_b \frac{f_g(x_b)}{2sx_ax_b}\delta\left(sx_ax_b - m_H^2\right)\int_{x_a+\frac{Q_a\mathcal{T}}{m_H^2}}^{1-\frac{\mathcal{T}Q_b}{m_H^2}} \frac{\mathrm{d}z_a}{z_a}\left(Q_a\right)^{1-\varepsilon}\left(\frac{1-z_a}{z_a}\right)^{-\varepsilon}$$
$$\left\{f_g\left(\frac{x_a}{z_a}\right) + \frac{\mathcal{T}}{m_H^2 z_a^2}\left[\left(Q_a' z_a x_a - Q_a z_a \varepsilon\right) f_g\left(\frac{x_a}{z_a}\right) + Q_a x_a f_g'\left(\frac{x_a}{z_a}\right)\right] + \mathcal{O}\left(\mathcal{T}^2\right)\right\}$$

Expansion of the matrix elements

- Straightforward to expand the matrix elements; consider the all-gluon channel as an example

Leading-power contribution

$$|\mathcal{M}(gg \to Hg)|^2 = |\mathcal{M}(gg \to H)|^2 \, (16 C_A \alpha_s \pi) \left\{ \frac{1}{Q_a \mathcal{T}} \frac{(1 - z_a + z_a^2)^2}{(1 - z_a) z_a} + \frac{1}{m_H^2} \left[5 - \frac{1}{1 - z_a} + \frac{1}{z_a^2} - \frac{1}{z_a} + z_a - \frac{2}{1 - \varepsilon} \right] + \mathcal{O}(\mathcal{T}) \right\}$$

Subleading-power contribution

Sub-leading soft limit; such terms in the matrix elements can lead to LL-NLP. Note that soft quarks can lead to such NLP soft structures (eg in qg->Hq)!

Mapping to SCET at LP

- We can establish a connection between out derivation and the SCET factorization theorem through the z_a integral in the following way:

$$\int_{x_a}^{1-\frac{\mathcal{T}Q_b}{m_H^2}} \mathrm{d}z_a = \int_{x_a}^{1} \mathrm{d}z_a - \int_{1-\frac{\mathcal{T}Q_b}{m_H^2}}^{1} \mathrm{d}z_a$$

At leading power gives exactly the beam function contribution from the SCET factorization theorem

At leading power gives exactly the soft function contribution from the SCET factorization theorem

Results for LL-NLP

- Consider the expression for the LL-NLO contribution (the full NLL-NLP can be derived as well):

$$\frac{\mathrm{d}\sigma_{\mathrm{LL}}^{\mathrm{NLP}}}{\mathrm{d}\mathcal{T}} = \left(\frac{C_A\alpha_s}{\pi}\right)\int_0^1 \mathrm{d}x_a \int_0^1 \mathrm{d}x_b \frac{(2\pi)}{2sx_ax_b}\delta(sx_ax_b - m_H^2)|\mathcal{M}(gg\to H)|^2$$
$$\left\{\frac{Q_a}{m_H^2}\log\left(\frac{\mathcal{T}}{Q_a}\right)f_g(x_b)\left[\left(1-\frac{Q_a' x_a}{Q_a}\right)f_g(x_a) - x_a f_g'(x_a)\right]\right.$$
$$\left.+\frac{Q_b}{m_H^2}\log\left(\frac{\mathcal{T}}{Q_b}\right)f_g(x_a)\left[\left(1-\frac{Q_b' x_b}{Q_b}\right)f_g(x_b) - x_b f_g'(x_b)\right]\right\}.$$

Results for LL-NLP

- Two results for $Q_{a,b}$ to consider: Hadronic: $Q_a = x_a\sqrt{s}, \quad Q_b = x_b\sqrt{s}$

$$\frac{d\sigma^{\rm NLP}_{\rm LL}}{d\mathcal{T}} = \left(\frac{C_A\alpha_s}{\pi}\right)\int_0^1 dx_a \int_0^1 dx_b \frac{(2\pi)}{2sx_ax_b}\delta(sx_ax_b - m_H^2)|\mathcal{M}(gg\to H)|^2$$
$$\left\{\frac{\sqrt{s}x_a}{m_H^2}\log\left(\frac{\mathcal{T}}{\sqrt{s}x_a}\right) f_g(x_b)\left[-x_a f_g'(x_a)\right] + \frac{\sqrt{s}x_b}{m_H^2}\log\left(\frac{\mathcal{T}}{\sqrt{s}x_b}\right) f_g(x_a)\left[-x_b f_g'(x_b)\right]\right\}.$$

$x_{a,b} \sim e^{\pm Y_H}$; strong rapidity dependence of the power corrections for hadronic 0-jettiness

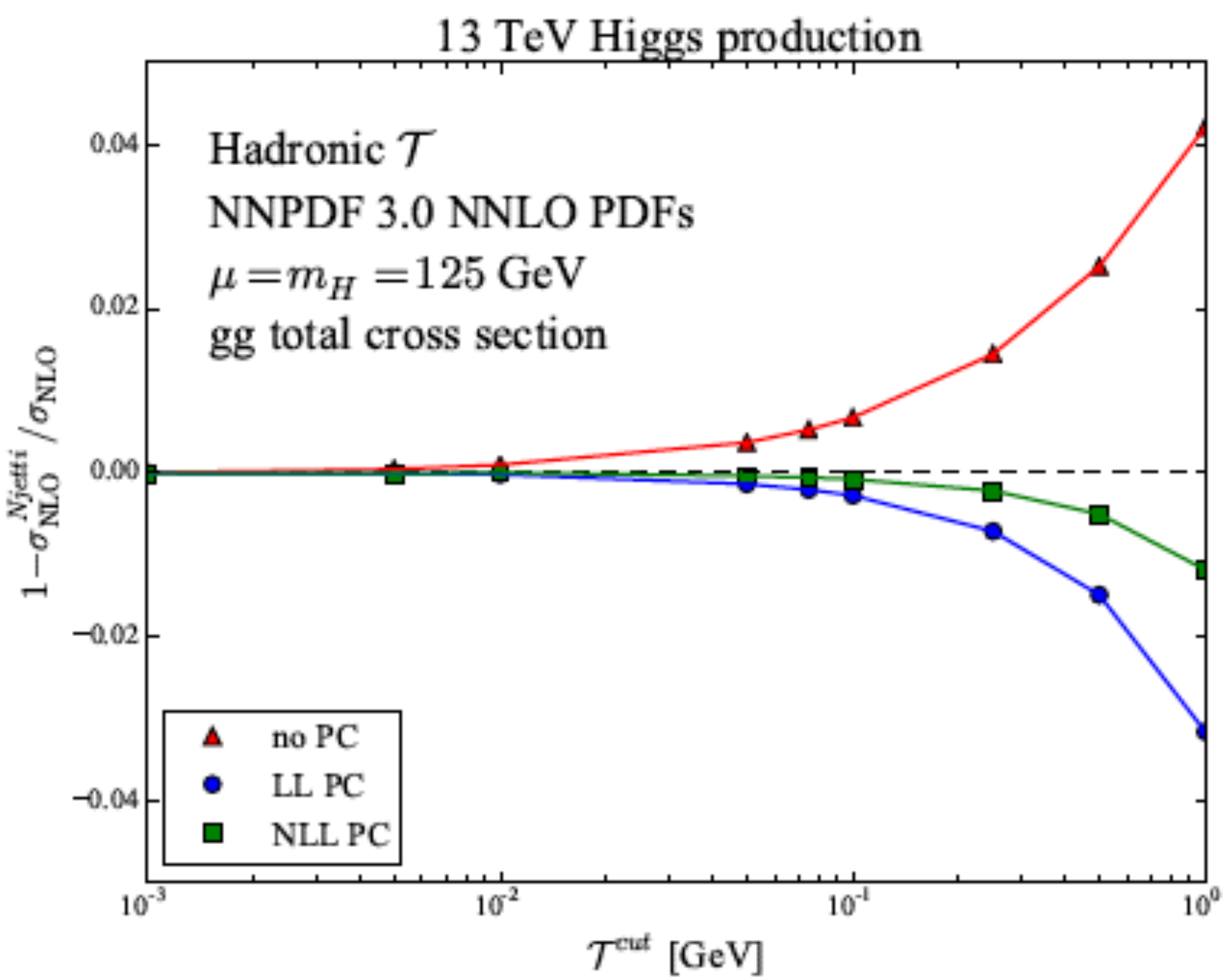
Results for LL-NLP

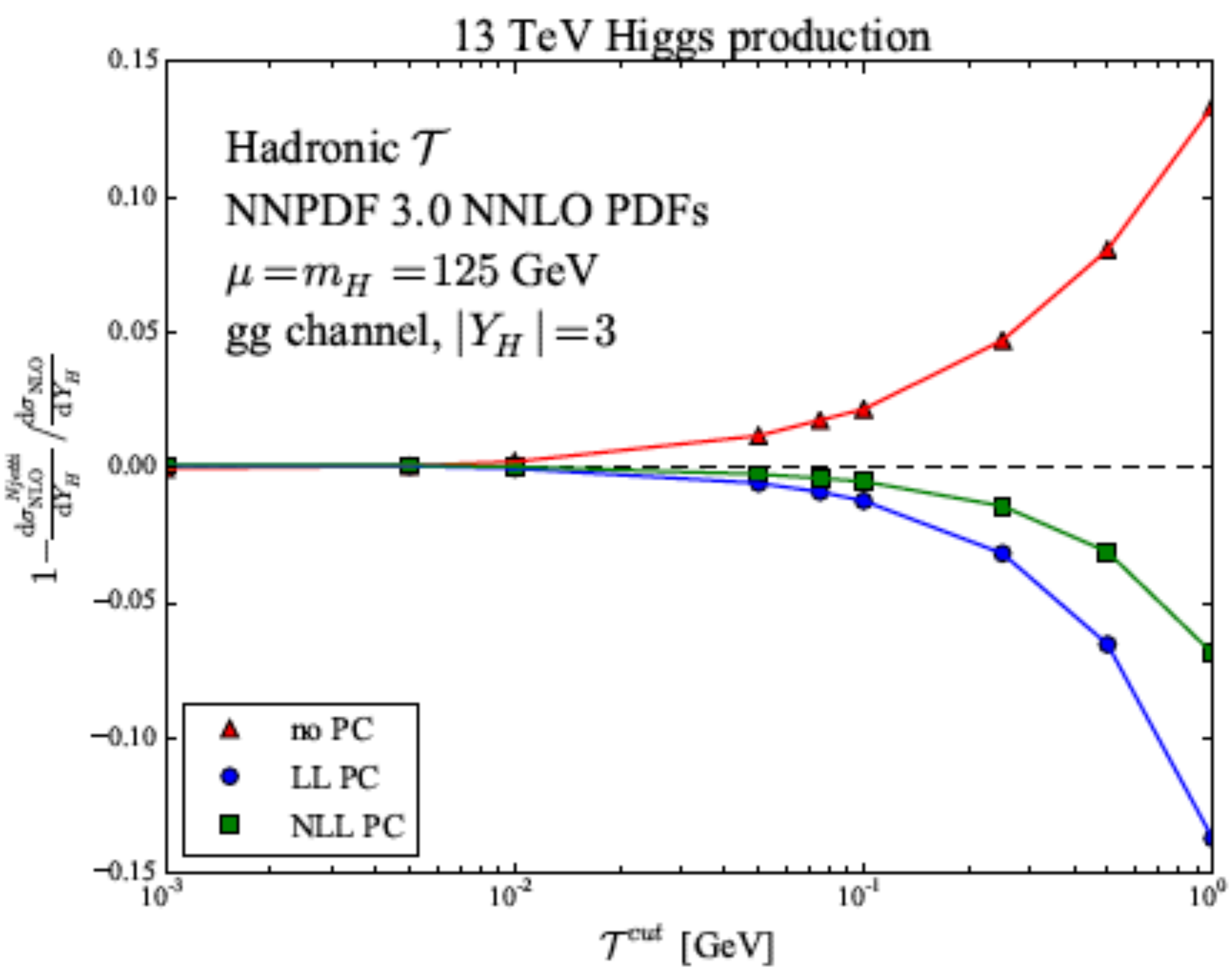
- Two results for $Q_{a,b}$ to consider: <u>Leptonic:</u> $Q_a = Q_b = Q$ ($Q=m_H$)

$$\frac{d\sigma^{\rm NLP}_{\rm LL}}{d\mathcal{T}} = \left(\frac{C_A\alpha_s}{\pi}\right)\int_0^1 dx_a \int_0^1 dx_b \frac{(2\pi)}{2sx_ax_b}\delta(sx_ax_b - m_H^2)|\mathcal{M}(gg\to H)|^2 \left\{\frac{1}{m_H}\log\left(\frac{\mathcal{T}}{m_H}\right)\left[2f_g(x_a)f_g(x_b) - x_af_g'(x_a)f_g(x_b) - x_bf_g(x_a)f_g'(x_b)\right]\right\}$$

No such strong rapidity dependence for leptonic 0-jettiness (first noted by Moult et al, 1612.00450)

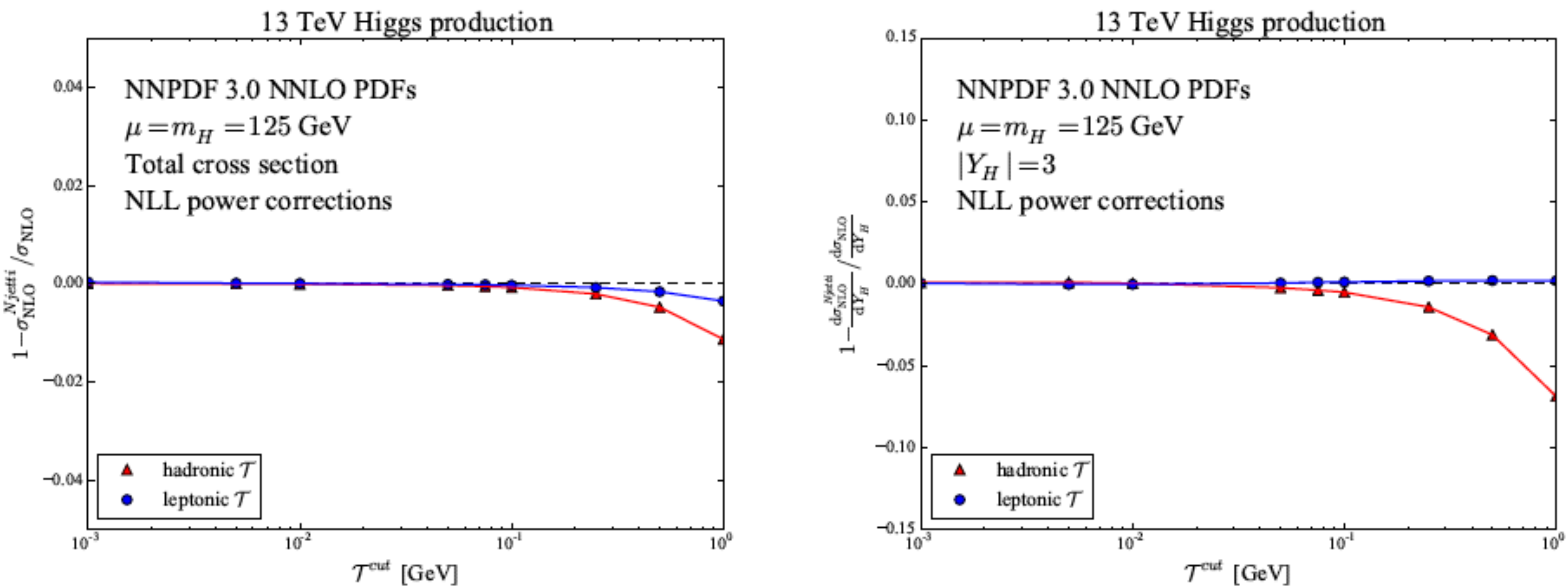
Numerics for hadronic 0-jettiness





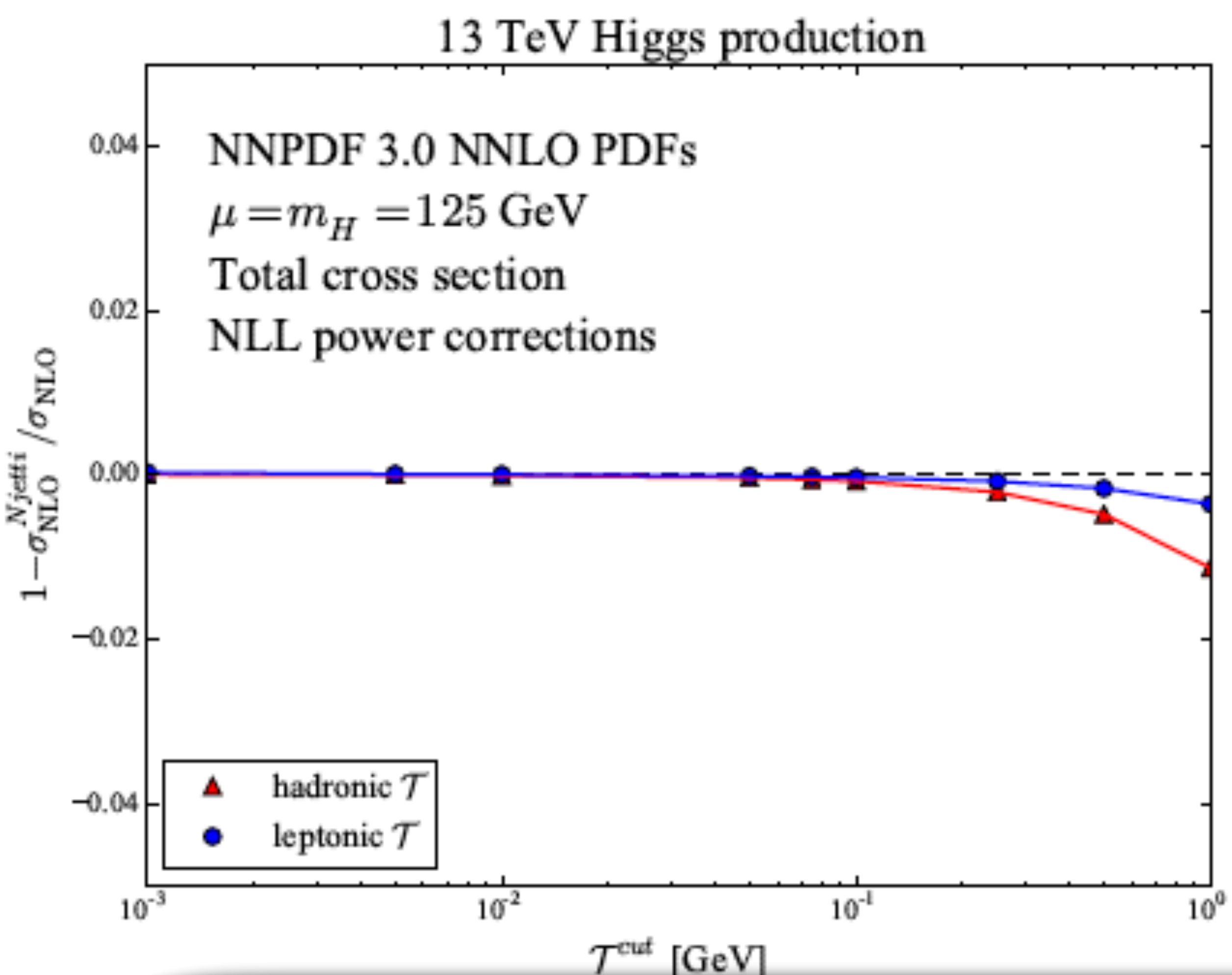
- Significant improvement upon including LL-NLP power corrections; even more observed when NLL-NLP is incorporated. Significant positive impact on numerics, much larger τ^{cut} can be chosen.

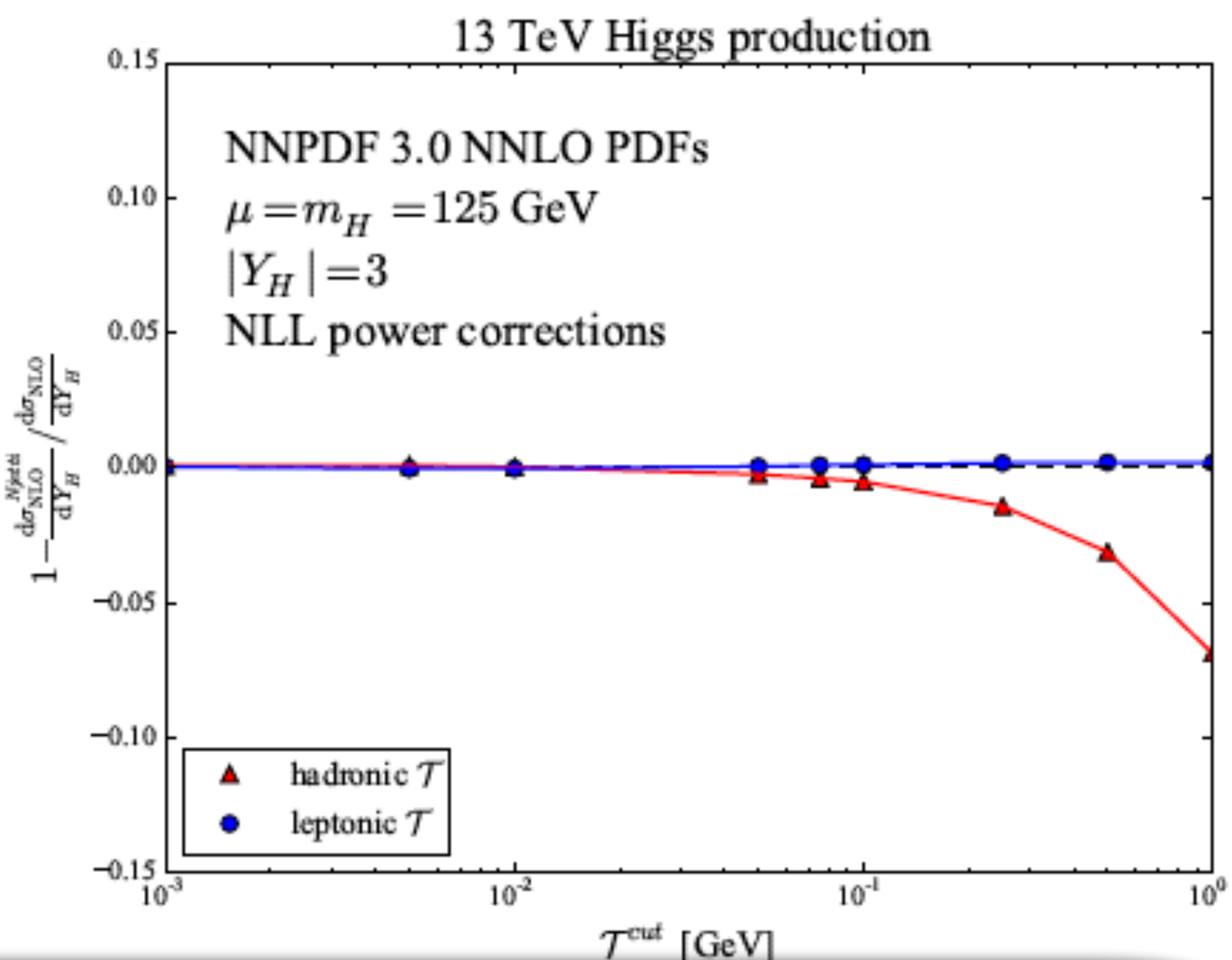
Numerics for leptonic 0-jettiness



- Almost no deviation from dipole subtraction observed when NLL-NLP corrections are included for leptonic 0-jettiness.

Numerics for leptonic 0-jettiness



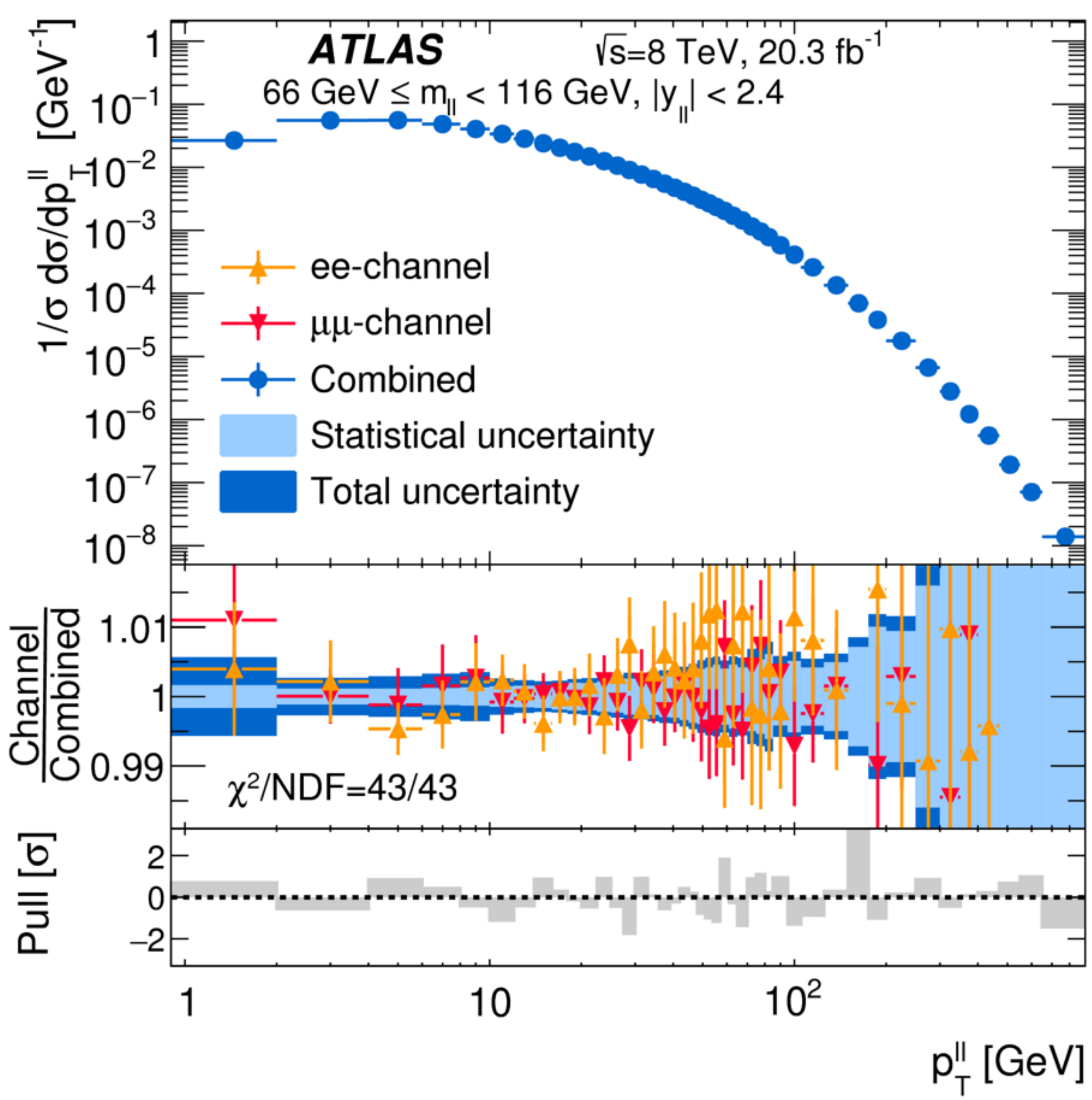


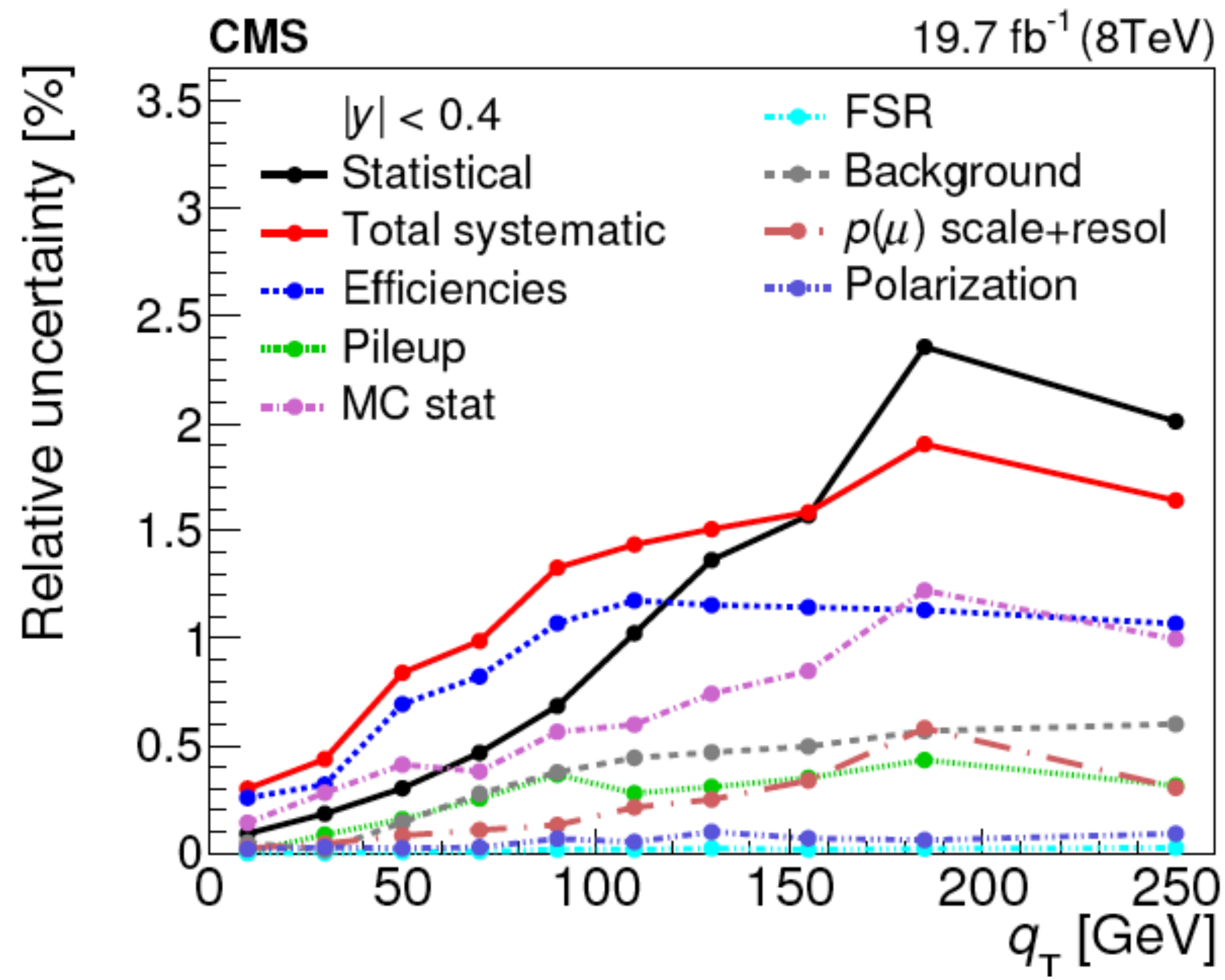
Including full NLL-NLP power corrections, and a clever choice of 0-jettiness, almost completely removes power corrections for 0-jet processes; we're very hopeful that similar conclusions for jet production will hold as well!

Selected recent applications

The Z-boson transverse momentum

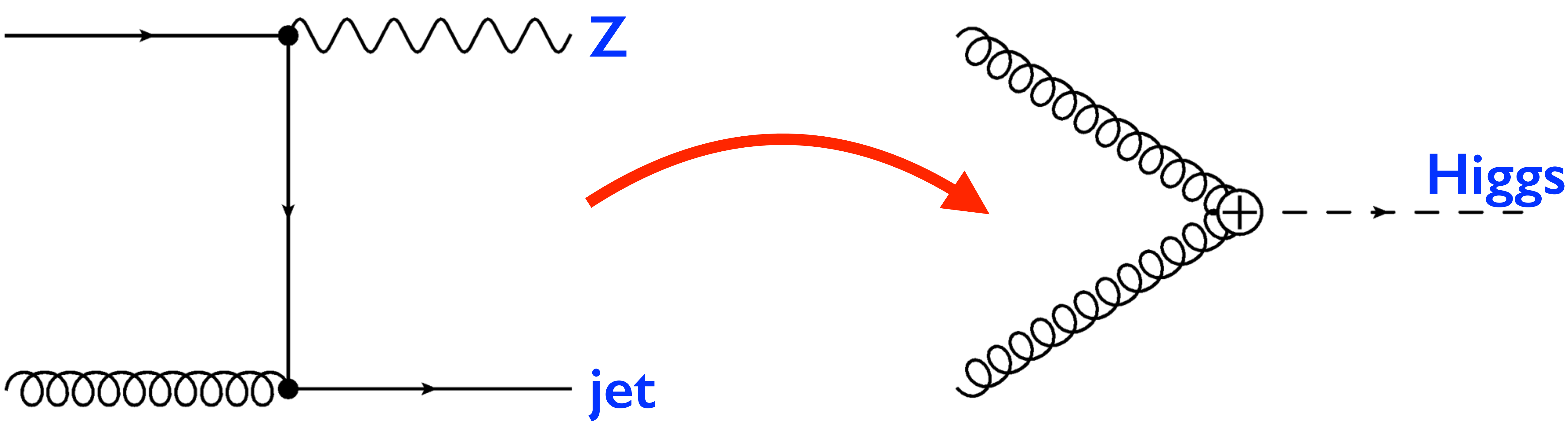
- The Z-boson transverse momentum spectrum measurement has reached a remarkable precision at the LHC, with errors below 1% over a large range





The Z-boson transverse momentum

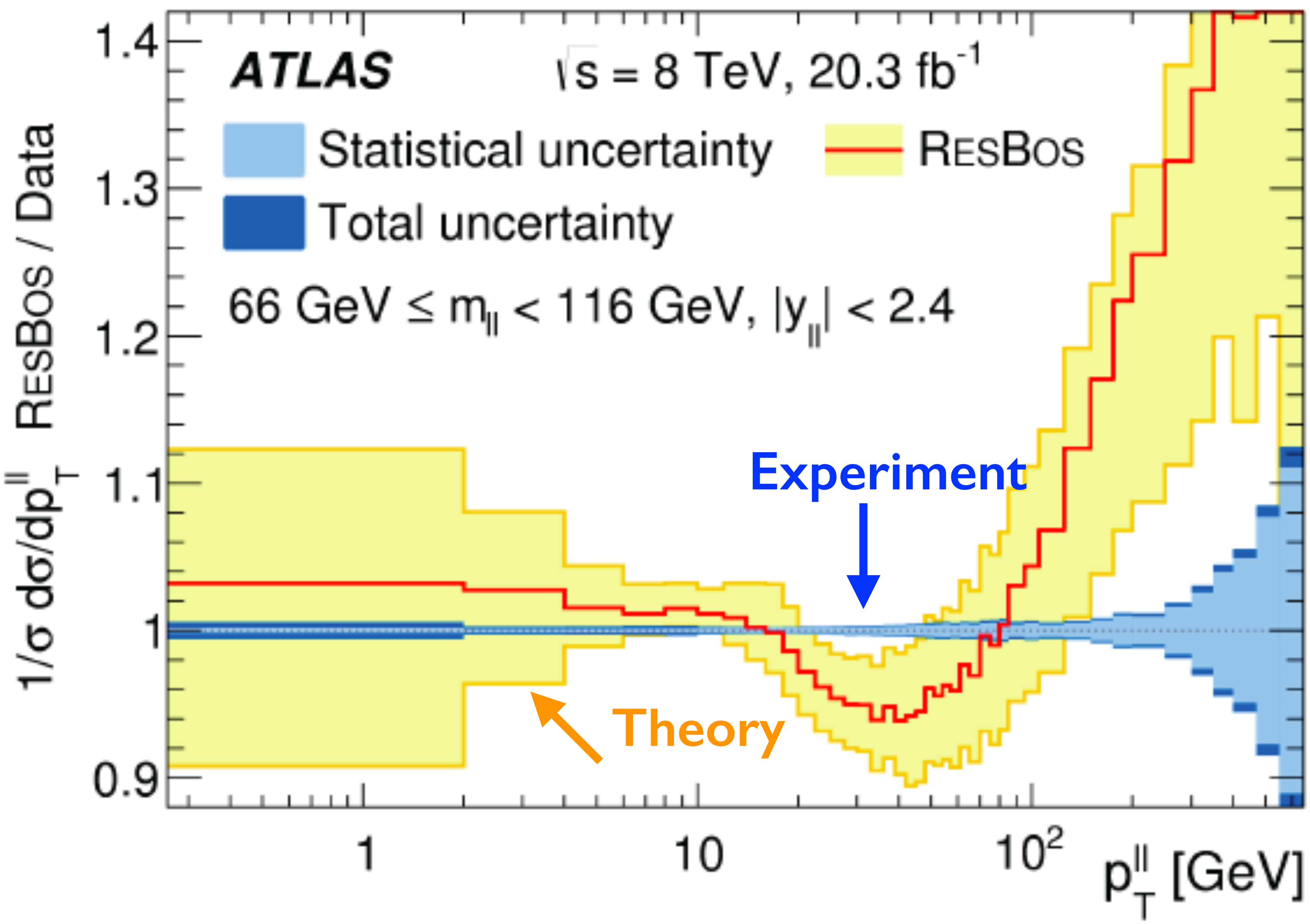
- The Z-boson transverse momentum spectrum measurement has reached a remarkable precision at the LHC, with errors below 1% over a large range



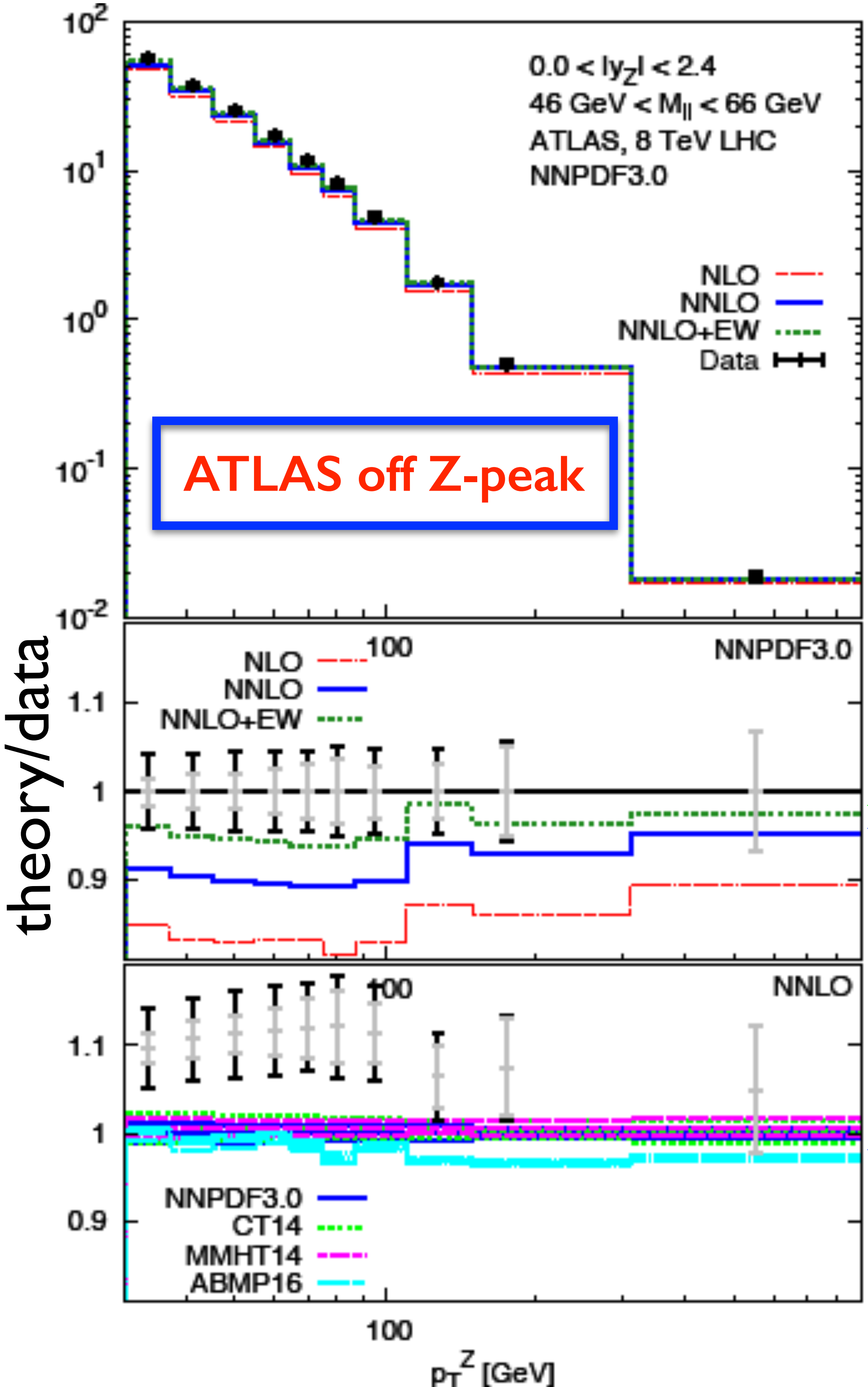
Can learn about the gluon distribution entering Higgs production from this data!

Comparison with NLO theory

- NLO theory errors more than an order of magnitude larger than experimental ones; can't use this data to measure the gluon without NNLO!



Comparison with NNLO theory



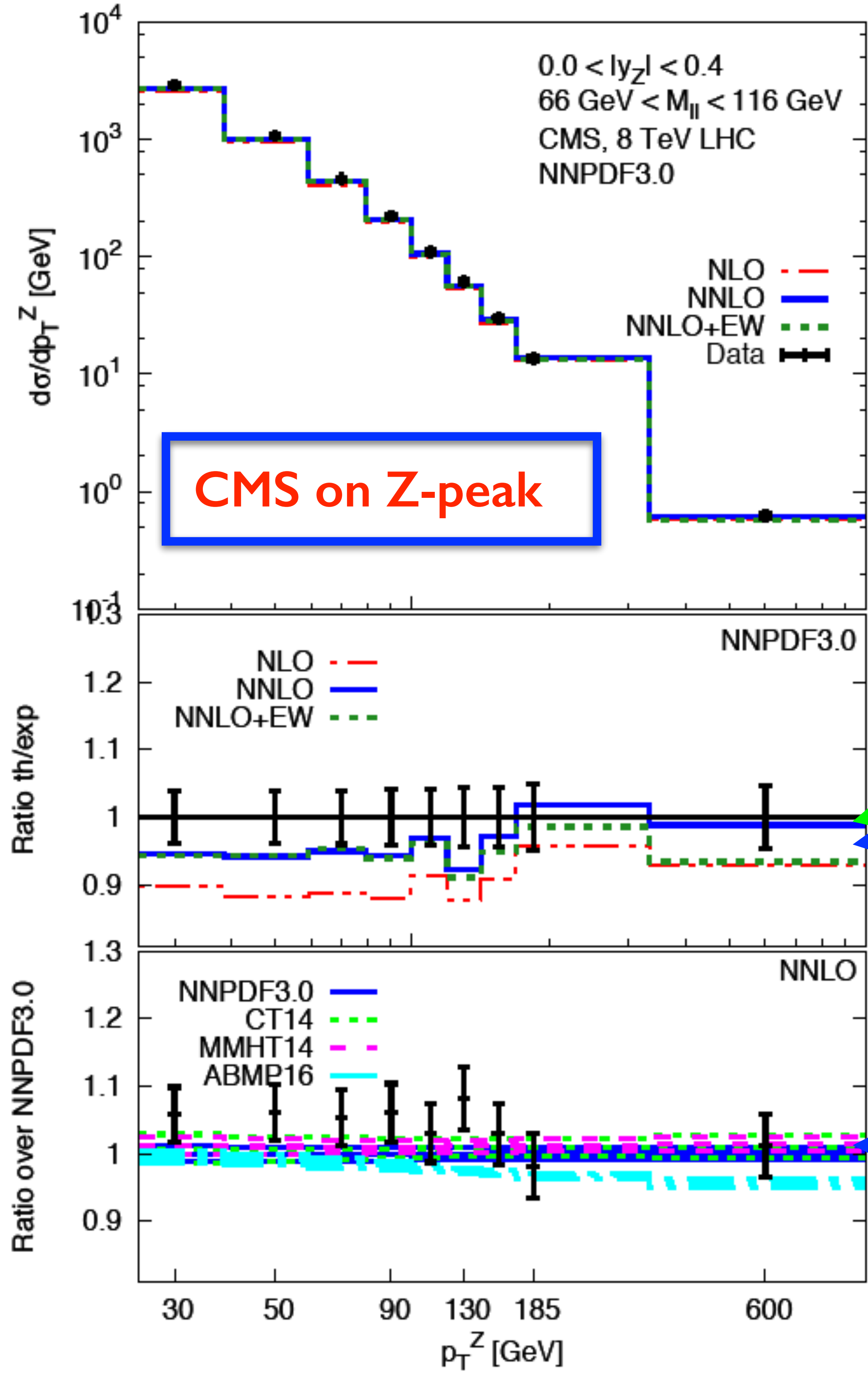
- We have performed an NNLO QCD calculation using N-jettiness subtraction and extensively compared with ATLAS and CMS (see also talk of A. Huss for another calculation of this quantity)
- We have combined NNLO QCD and NLO electroweak corrections for this prediction

Note the importance of NNLO QCD+NLO EW as compared to just NNLO QCD in the off-peak data

No current PDF set describes this well; feed this information back into the PDF fit!

Boughezal, Guffanti, FP, Ubiali JHEP 1707 (2017)

Comparison with NNLO theory



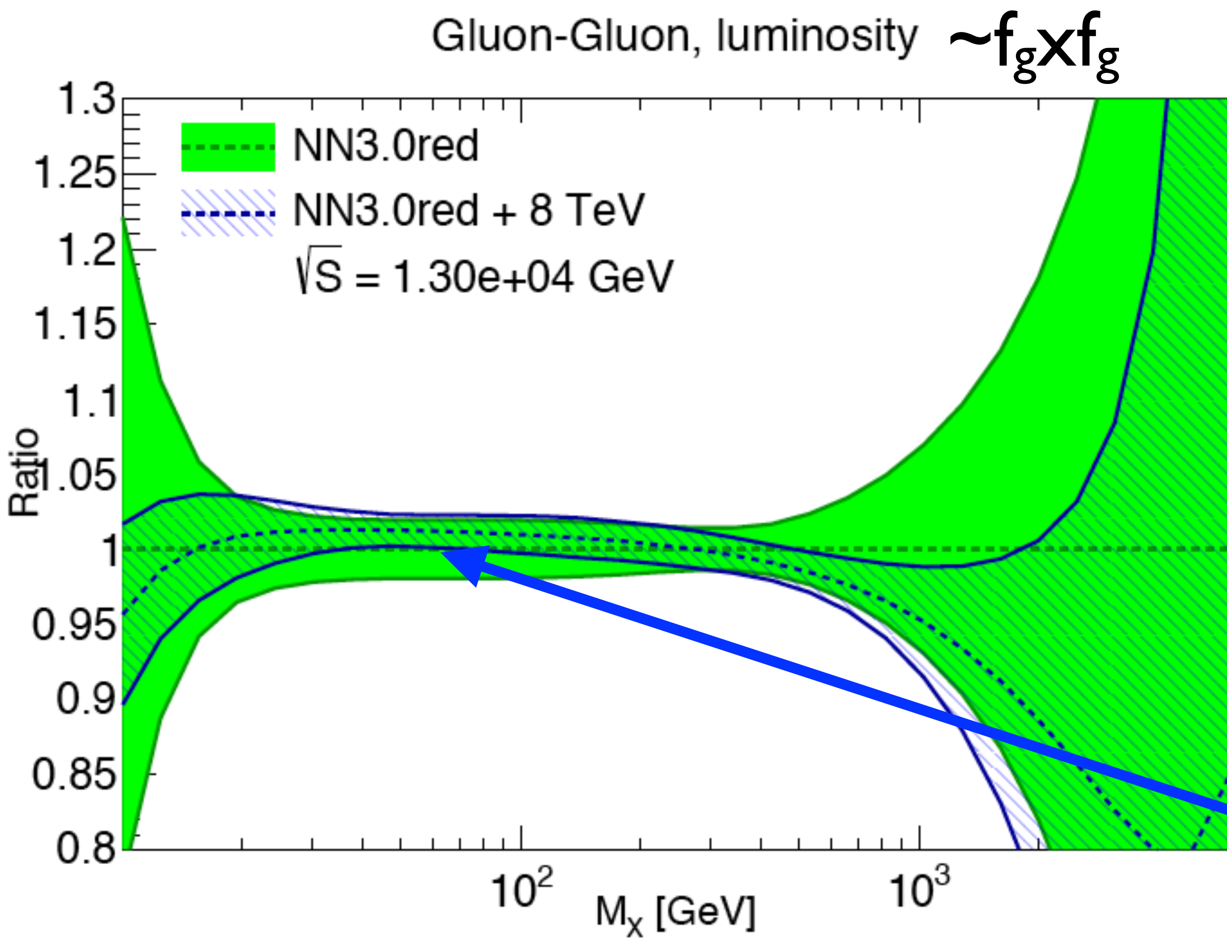
- We have performed an NNLO QCD calculation using N-jettiness subtraction and extensively compared with ATLAS and CMS (see also talk of A. Huss for another calculation of this quantity)
- We have combined NNLO QCD and NLO electroweak corrections for this prediction

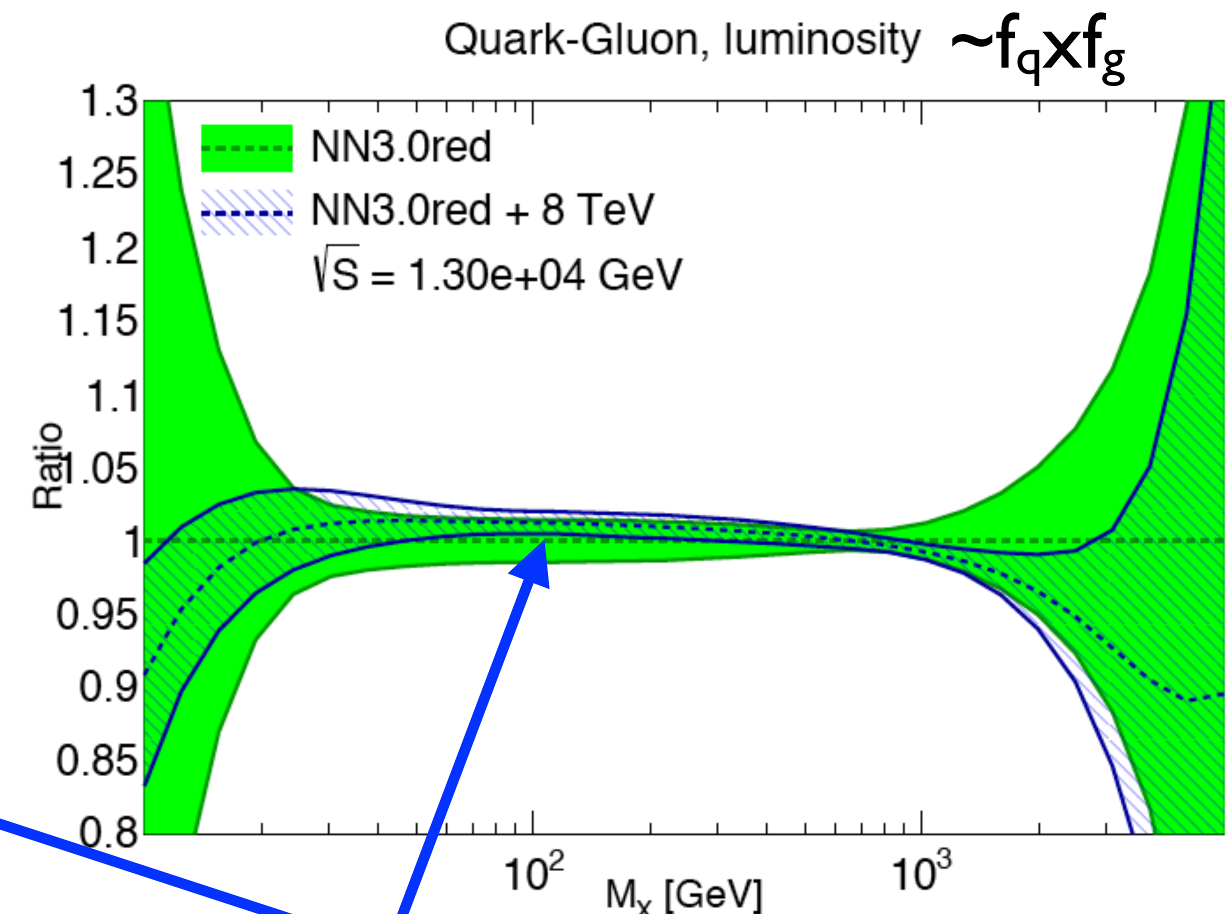
NLO EW as not as important on-peak; NNLO QCD leads to a much improved description

Better than off-peak, but still no current PDF set describes this well; feed this information back into the PDF fit!

Boughezal, Guffanti, FP, Ubiali JHEP 1707 (2017)

Impact on PDFs





Gluon-gluon and quark-gluon luminosity errors reduced right near $M_X \sim m_H$=125 GeV!

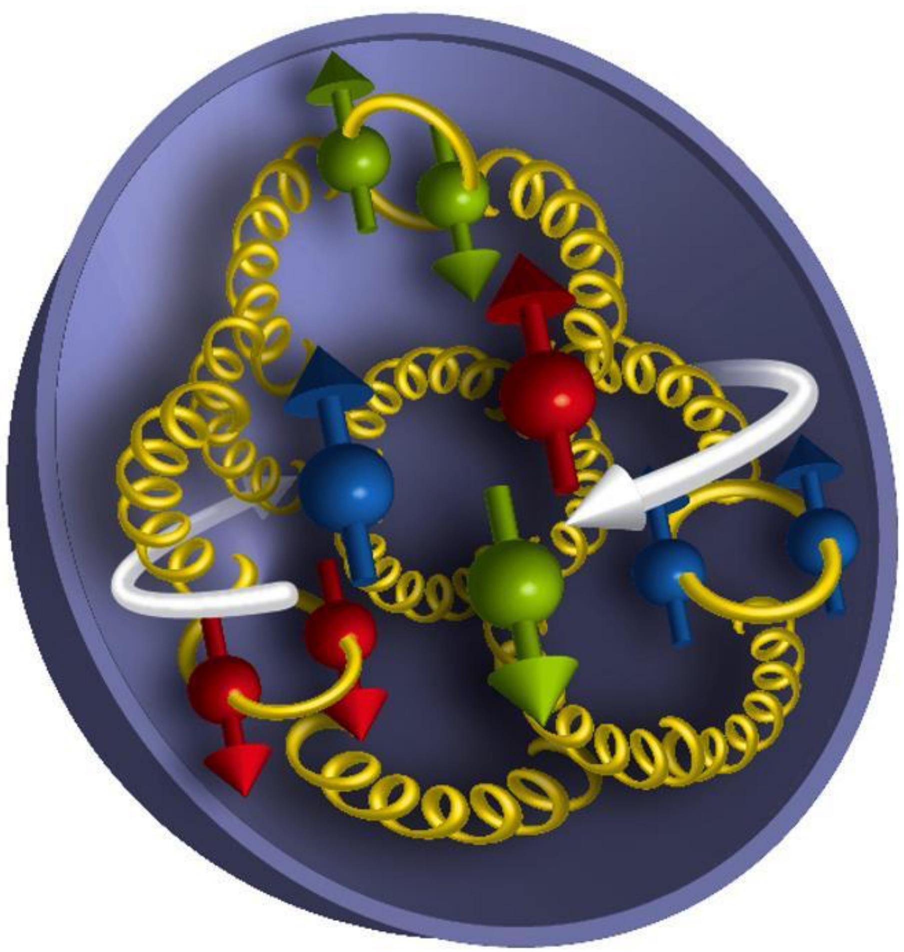
	Before p_T^Z data	After p_T^Z data
$\sigma_{gg\to H}$ [pb]	48.22 ± 0.89 (1.8%)	48.61 ± 0.61 (1.3%)
$\sigma_{\rm VBF}$ [pb]	3.92 ± 0.06 (1.5%)	3.96 ± 0.04 (1.0%)

PDF error on Higgs cross sections reduced!

Boughezal, Guffanti, FP, Ubiali JHEP 1707 (2017)

The emergent proton spin

- Our efforts to understand QCD are not limited to questions arising from the LHC… Even after four decades of study, basic aspects of QCD still surprise us



How is the proton spin formed from its microscopic constituents?

Quark spin — Gluon spin — Orbital

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_{G+q}$$

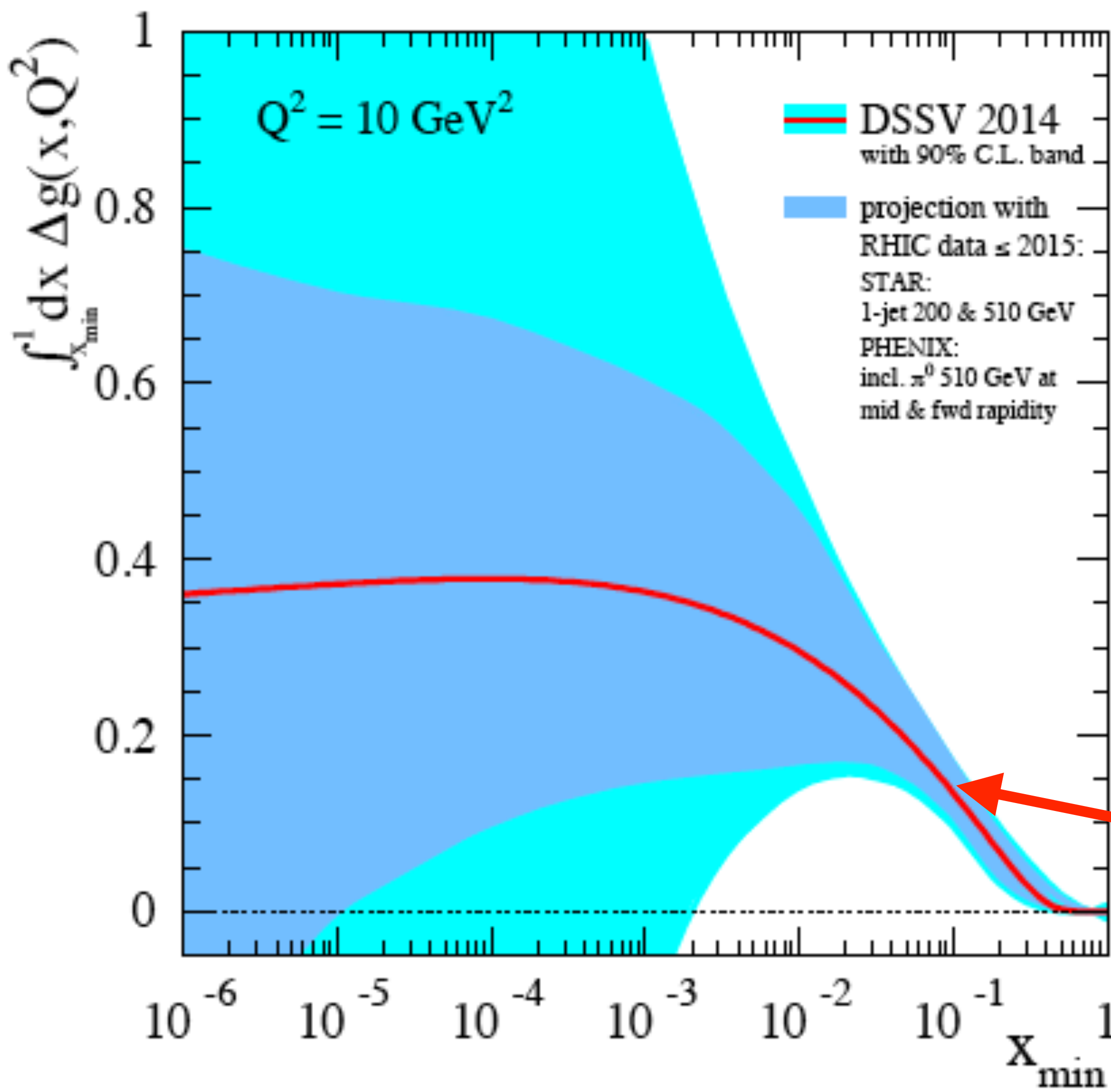
Only ~30%

?

Lattice suggest that this is not 70%

The emergent proton spin

- Our efforts to understand QCD are not limited to questions arising from the LHC… Even after four decades of study, basic aspects of QCD still surprise us



momentum fraction of
proton carried by gluon

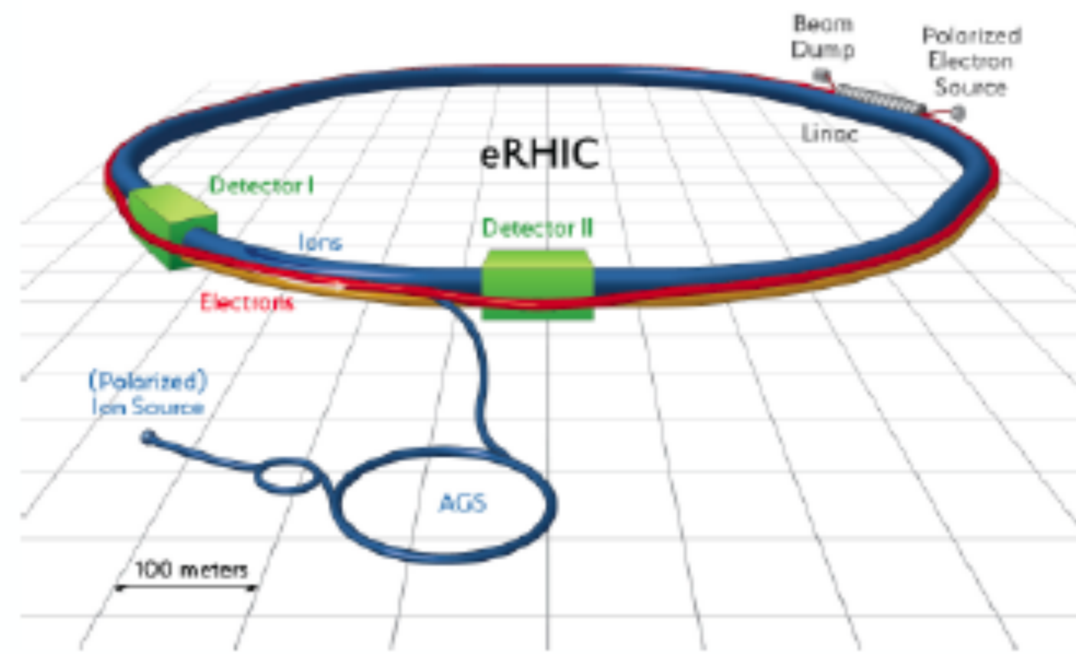
How is the proton spin formed from its microscopic constituents?

$$\frac{1}{2} = \frac{1}{2}\Delta\Sigma + \Delta G + L_{G+q}$$

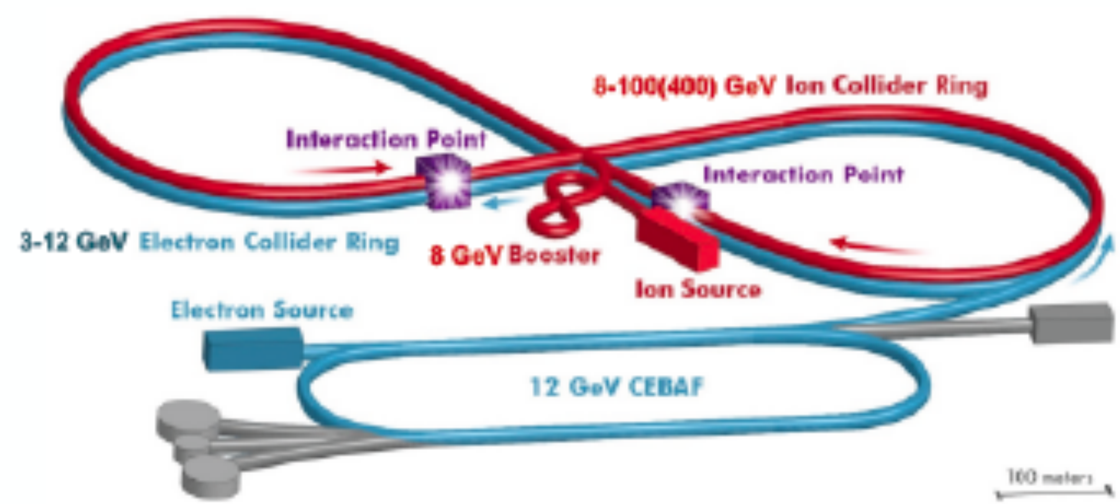
First glimpses of the gluon spin contribution are being provided by RHIC; large errors still!

A definitive answer to this questions will require a future electron-ion collider (EIC), a top priority for DOE nuclear physics

- **eRHIC (BNL)**
  - Add e Rings to RHIC facility: Ring-Ring (alt. recirculating Linac-Ring)
  - Electrons up to 18 GeV
  - Protons up to 275 GeV
  - $\sqrt{s}=30\text{-}140\ \sqrt{(Z/A)}$ GeV
  - $L \approx 1\times10^{34}\ \text{cm}^{-2}\text{s}^{-1}$ at $\sqrt{s}=105$ GeV

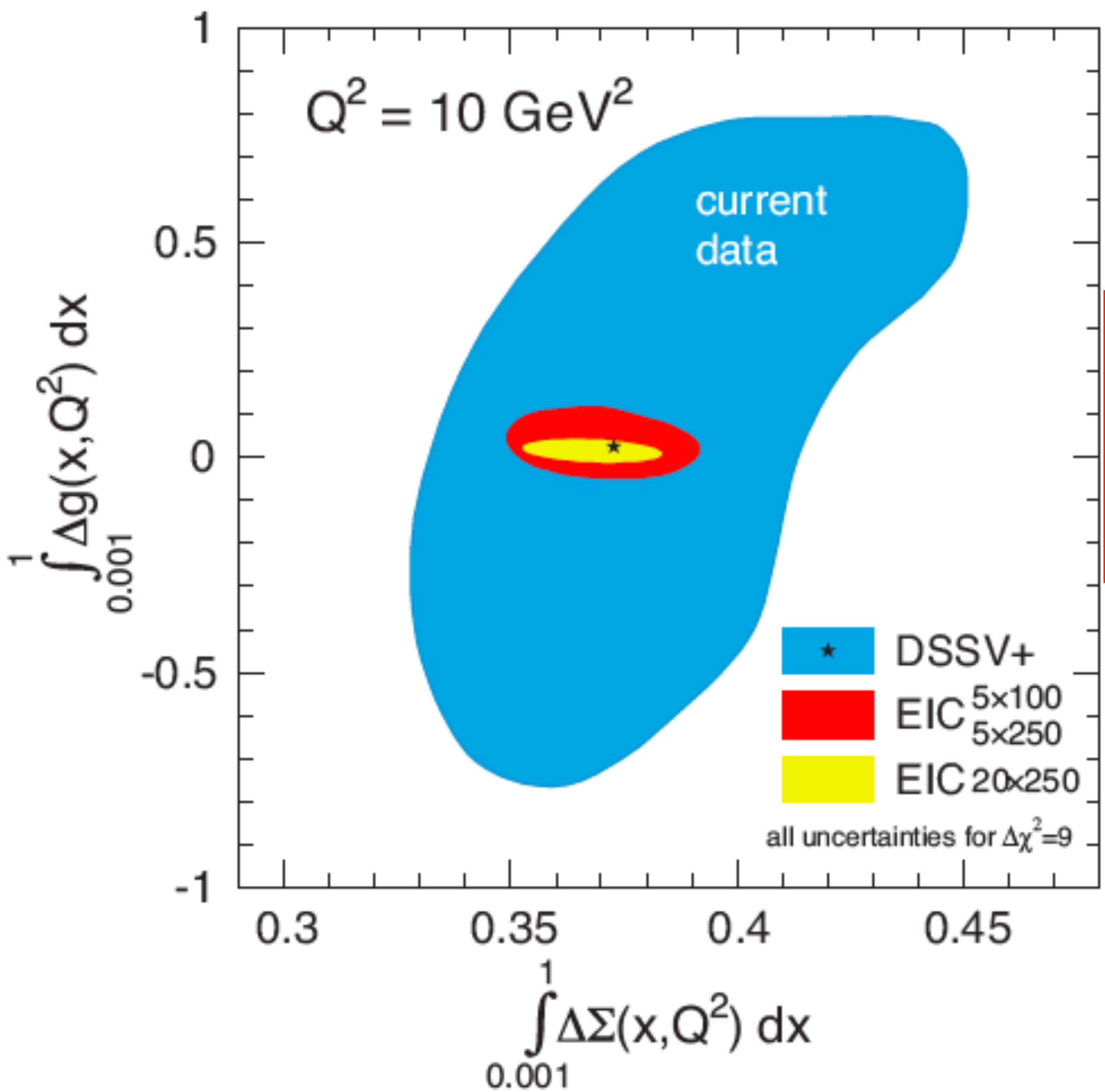


- **JLEIC (JLab)**
  - Figure-8 Ring-Ring Collider, use of CEBAF as injector
  - Electrons 3-10 GeV
  - Protons 20-100 GeV
  - e+A up to $\sqrt{s}=40$ GeV/u
  - e+p up to $\sqrt{s}=64$ GeV
  - $L \approx 2\times10^{34}\ \text{cm}^{-2}\ \text{s}^{-1}$ at $\sqrt{s}=45$ GeV



eRHIC: arXiv:1409.1633, JLEIC: arXiv:1504.07961

A definitive answer to this questions will require a future electron-ion collider (EIC), a top priority for DOE nuclear physics

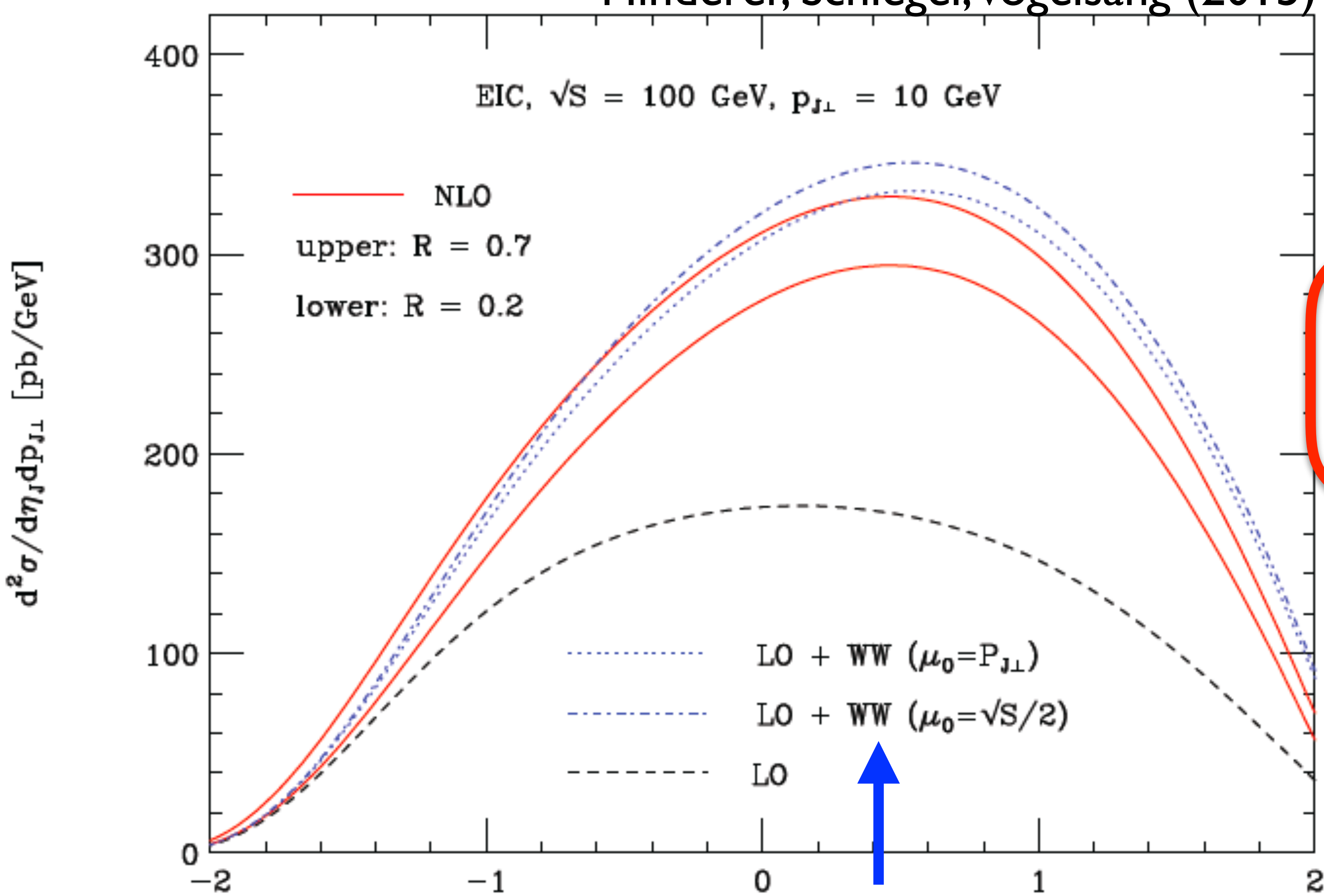


Percent-level probes of the proton spin structure are possible at an EIC!

Jet physics at an Electron-Ion Collider

- Proton structure studies will be a central aspect of a future EIC. Jets will play an important role these probes, just as at the LHC.

Hinderer, Schlegel, Vogelsang (2015)



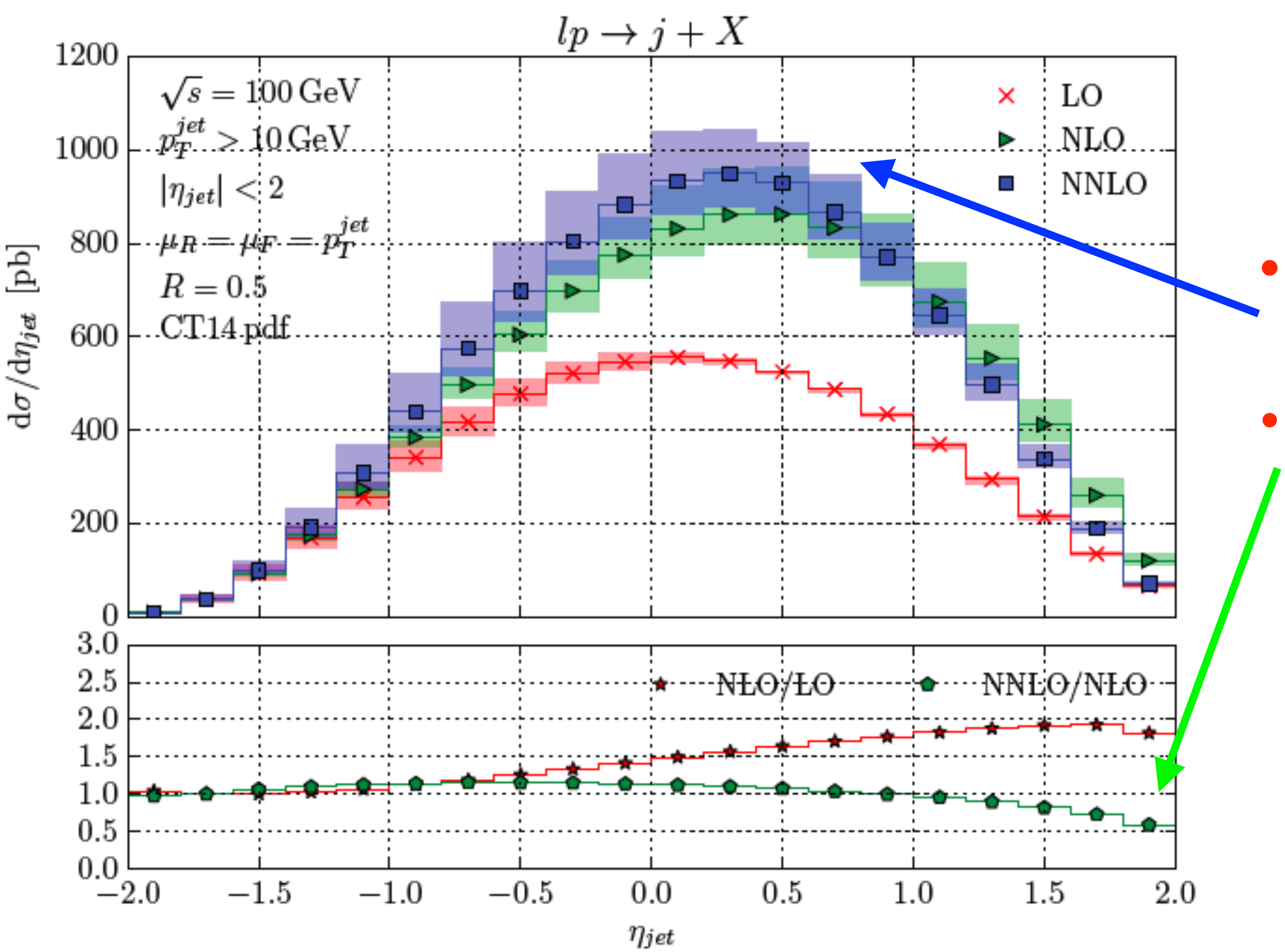
It is a challenge to theory to match this precision!

γ+p→jet+X contribution

η_J = jet pseudorapidity

EIC jet production at NNLO

- N-jettiness subtraction allows for a NNLO calculation of EIC jet production!

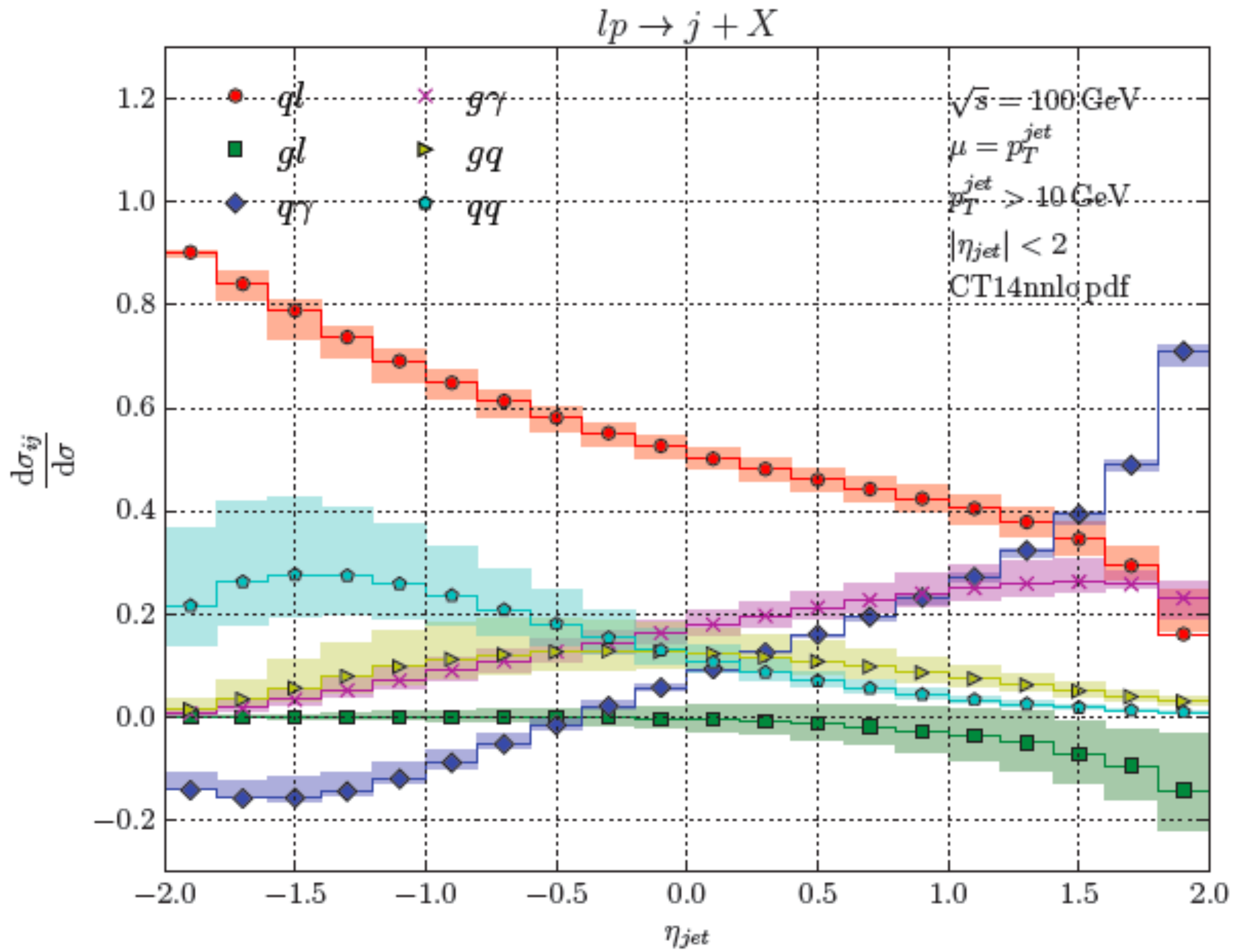


- **Perturbation theory stabilizes at NNLO!**
- Large corrections in the forward region; don't want to confuse this with PDF x dependence!

Abelof, Boughezal, Liu, FP, PLB 763 (2016)

EIC jet production at NNLO

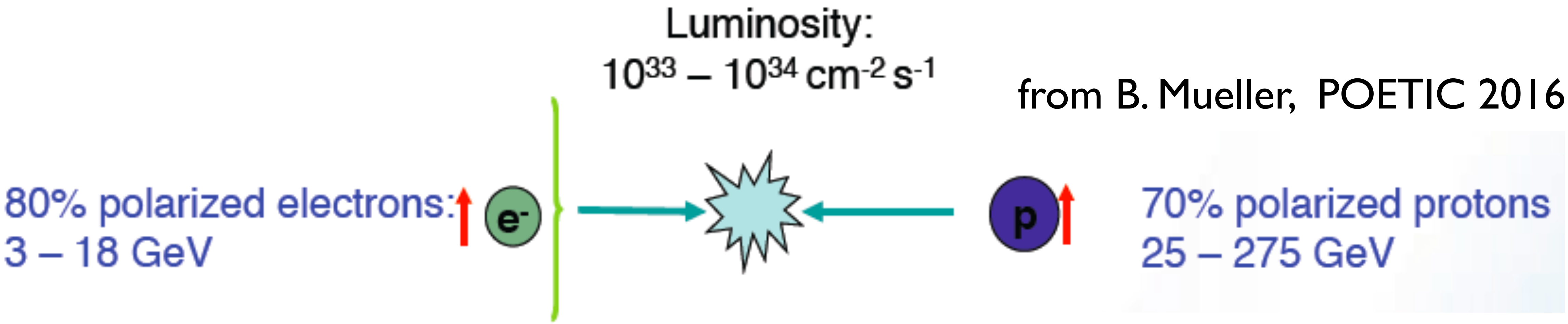
- Jet distributions at the EIC are an excellent probe of PDFs; no single channel dominates over all of phase space, indicating that different kinematic regions provide access to different partonic luminosities.



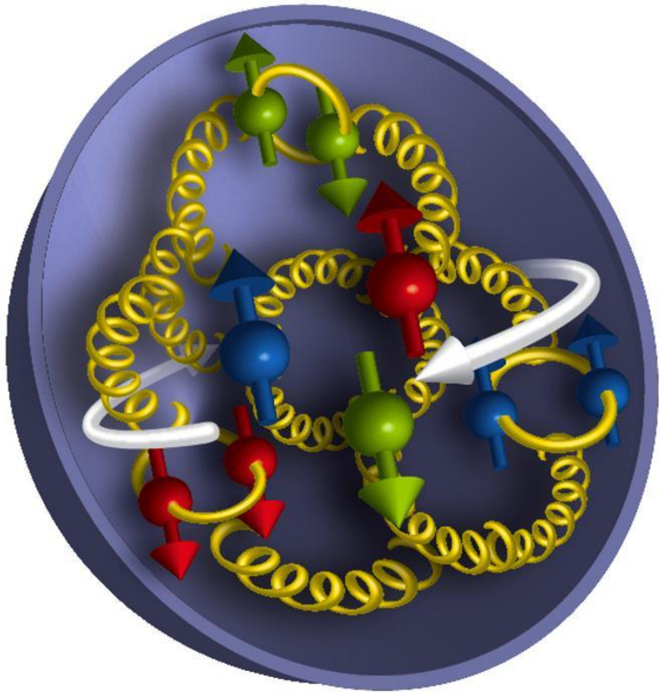
Abelof, Boughezal, Liu, FP, PLB 763 (2016)

Polarized jet production

- We are also interested in polarized collisions at the EIC.



from B. Mueller, POETIC 2016



Need to formulate N-jettiness subtraction to handle polarized collisions!

Extending to polarized collisions

- Schematic form of factorization theorem for unpolarized and longitudinally polarized collisions (Δ denotes the different between right-handed and left-handed polarizations):

unpolarized: $d\sigma/d\tau \sim H \otimes B \otimes J \otimes S$

polarized: $d\Delta\sigma/d\tau \sim \Delta H \otimes \Delta B \otimes J \otimes S$

jet and soft functions are unchanged

known helicity-dependent 2-loop virtual corrections

two-loop helicity-dependent beam function; **we have recently calculated this unknown quantity!**

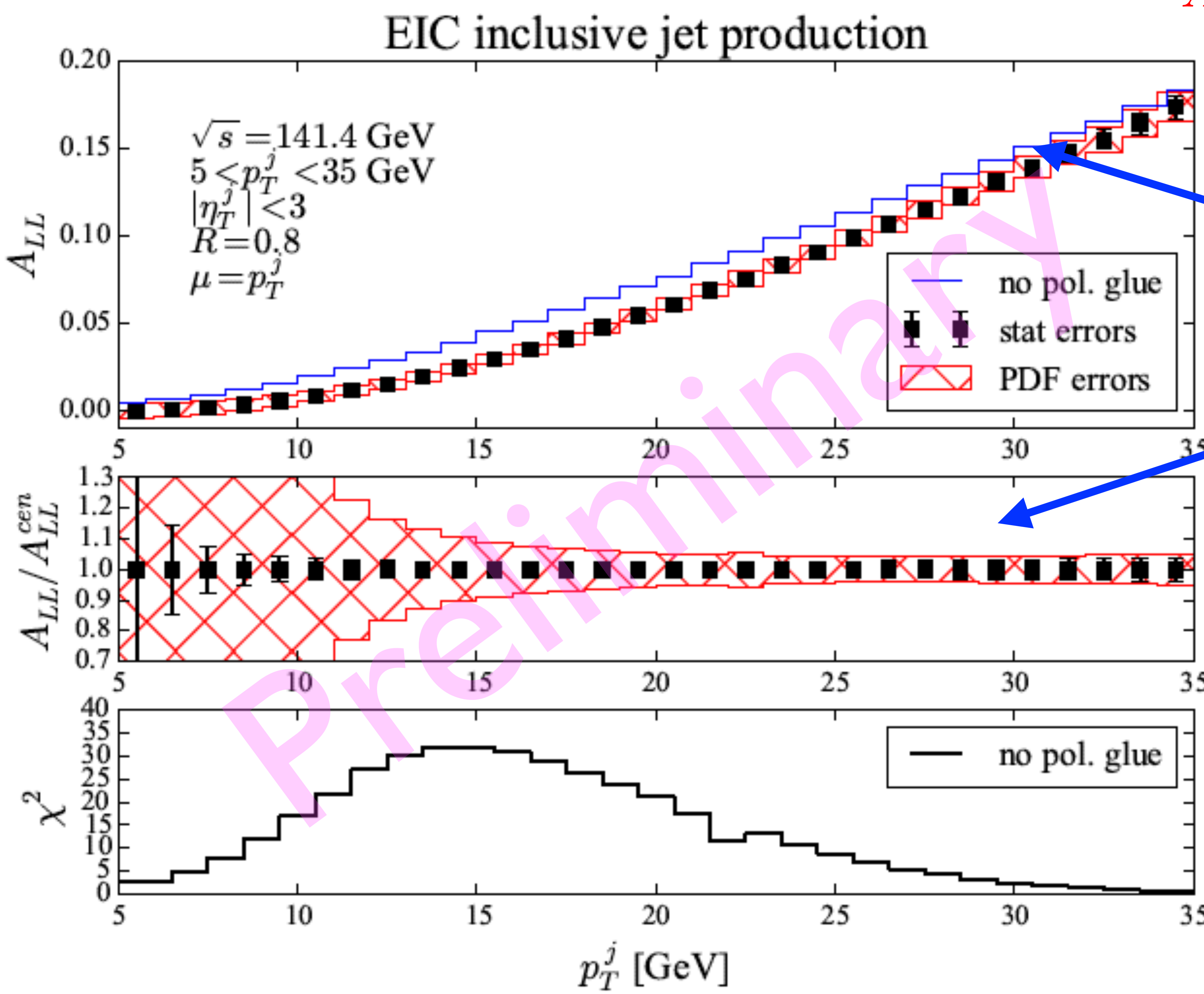
All ingredients now known!

Boughezal, FP, Schubert, Xing PRD 96 (2017)

Polarized PDFs at the EIC

- Polarization asymmetries in EIC jet production are a powerful probe of gluon and quark distributions!

$$A_{LL} = \frac{\sigma_{LL} + \sigma_{RR} - \sigma_{LR} - \sigma_{RL}}{\sigma_{LL} + \sigma_{RR} + \sigma_{LR} + \sigma_{RL}}$$



results with polarized gluon turned off

PDF errors larger than expected statistical errors over much of phase space

Can learn about polarized PDFs from jet measurements!

Boughezal, FP, Xing, in progress