Hierarchies, dynamical supersymmetry breaking & holographic duality

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Talk based on work done with Tony Gherghetta some time ago

Stabilizing the hierarchy

- Hierarchies in warped compactifications can be supported by RR and NS fluxes [Giddings, Kachru & Polchinski 2001]
- This is a stringy realization of the Goldberger & Wise (1999) phenomenological mechanism for stabilizing the radion.
- In string theory this is a moduli field, and so it is not surprising that fluxes can stabilize it.

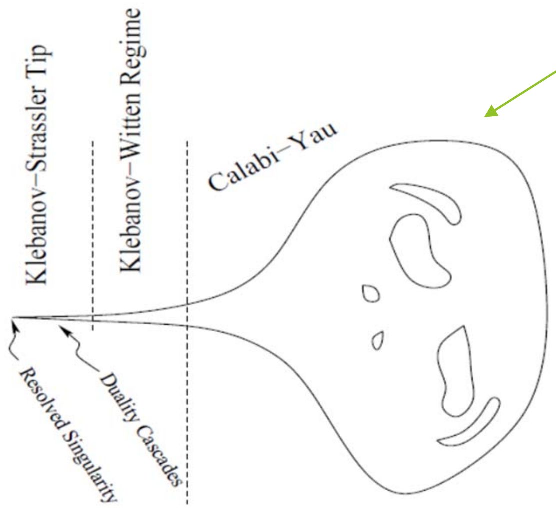
Dual picture of the hierarchy

- In the dual gauge theory, this is the usual story of a hierarchy due to dimensional transmutation with a weak coupling at the high scale.

$$\Lambda = a^{-1} e^{-1/bg^2}$$

- Planck brane, negative tension brane, needed for RSI
- Stringy: O3 planes, wrapped D7 branes

Refinement of RS1:
Follow viewpoint of GKP



Replaces Planck (UV) brane

Replaces TeV (IR) brane

Gherghetta & JG 2006

- Conifold: present as embedding into 4-complex-dimensional space.

$$ds^2 = |dz_1|^2 + |dz_2|^2 + |dz_3|^3 + |dz_4|^2$$

$$z_1 z_2 - z_3 z_4 = 0$$

Satisfying the embedding condition

- The Gaussian coordinates (or at least one possible choice) are given by:

$$z_1 = r^{3/2} e^{\frac{i}{2}(\psi - \phi_1 - \phi_2)} \sin\frac{\theta_1}{2} \sin\frac{\theta_2}{2}$$

$$z_2 = r^{3/2} e^{\frac{i}{2}(\psi + \phi_1 + \phi_2)} \cos\frac{\theta_1}{2} \cos\frac{\theta_2}{2}$$

$$z_3 = r^{3/2} e^{\frac{i}{2}(\psi + \phi_1 - \phi_2)} \cos\frac{\theta_1}{2} \sin\frac{\theta_2}{2}$$

$$z_4 = r^{3/2} e^{\frac{i}{2}(\psi - \phi_1 + \phi_2)} \sin\frac{\theta_1}{2} \cos\frac{\theta_2}{2}$$

- Then the metric for the conifold becomes:

$$ds_6^2 = dr^2 + r^2 ds_{T^{1,1}}^2$$

$$ds_{T^{1,1}}^2 = \frac{1}{9}\left(d\psi + \sum_{i=1}^{2} \cos\theta_i d\phi_i\right)^2 + \frac{1}{6}\sum_{i=1}^{2}\left(d\theta_i^2 + \sin^2\theta_i d\phi_i^2\right)$$

- The full 10d metric is given by:

$$ds_{10}^2 = H^{-1/2}(r)(-dt^2 + d\vec{x}^2) + H^{1/2}(r)ds_6^2$$

$$H(r) = 1 + \frac{L^4}{r^4}$$

$$L^4 = 4\pi g_s N(\alpha')^2$$

- N D3 branes at the conical singularity r=0.

- Here, the 5d base of the cone is the well-known quotient manifold $T^{1,1}$, which is among those classified by Romans as having reduced supersymmetry (fewer than the maximal number of Killing spinors).

$$T^{1,1} \simeq \frac{SU(2) \times SU(2)}{U(1)} \simeq \frac{S^3 \times S^3}{S^1}$$

$$Q = T_1^3 \oplus T_2^3$$

- Others in this class are $T^{p,q}$

$$Q = pT_1^3 \oplus qT_2^3$$

- The isometry group is easier to see with a change of coordinates (isn't that what GR is all about?):

$$z_1 = w_1 + iw_2, \quad z_2 = w_1 - iw_2$$

$$z_3 = w_3 + iw_4, \quad z_4 = -(w_3 - iw_4)$$

- Then the conifold is described by the more transparent equation

$$\sum_i w_i^2 = \det(w_4 \mathbb{I}_2 + i\sigma_a w_a) = 0$$

Symmetries and intersections

- This clearly has an $SU(2)\times SU(2)\times U(1)$ invariance:

$$w_4\mathbb{I}_2 + i\sigma_a w_a \to e^{i\alpha} U(w_4\mathbb{I}_2 + i\sigma_a w_a)V$$

- The $T^{1,1}$ base is the intersection of this with the 7-sphere:

$$2\sum_i |w_i|^2 = \mathrm{Tr}(w_4\mathbb{I}_2 + i\sigma_a w_a)(w_4\mathbb{I}_2 + i\sigma_a w_a)^\dagger = 2r^3$$

- It also clearly has the $SU(2)\times SU(2)\times U(1)$ invariance.

- We follow Levi & Ouyang (2005) and add probe D7 branes to introduce "quark" flavors.
- Builds on Klebanov-Witten: less SUSY
- D7 branes give rise to a "meson" spectrum that L&O analyze, including numerically.
- Tony & I wanted to see if we could relate this to Randall-Sundrum type phenomenology.
- To do this, we had to derive an effective 5d action from the L&O setup.

- The D7 embedding generally looks like

$$X^M(\xi), \quad \xi = (\xi^0, \ldots, \xi^7), \quad M = 0, \ldots, 9$$

- But we take

$$X^M = (x^\mu, r, \theta_1, \theta_2, \phi_1, \phi_2, \psi)$$

with each of these eight components depending on ξ

- Then we further take for the “background embedding” (i.e., ignoring fluctuations)

$$\xi^\mu \equiv x^\mu, \quad \mu = 0, \dots, 3$$

$$\xi^4 = \theta_1, \quad \xi^5 = \theta_2, \quad \xi^6 = \phi_1, \quad \xi^7 = \phi_2$$

$$r = r_0(\theta_1, \theta_2) \quad \text{s.t.} \quad \mu = r_0^{3/2} \sin\frac{\theta_1}{2} \sin\frac{\theta_2}{2}$$

$$\psi = \psi_0(\phi_1, \phi_2) \quad \text{s.t.} \quad \psi_0 = \phi_1 + \phi_2$$

- The action for the D7 probe brane is given by DBI:

$$S_{\text{DBI}} = -\tau_7 \int d^4x \; d^2\theta \; d^2\phi \; \sqrt{\varphi^*(g) + \varphi^*(B) + 2\pi\alpha' F}$$

- The pullback of the metric (which we need for our scalar analysis) is

$$\varphi^*(g) = (g_0)_{ab} = \frac{\partial X^M}{\partial \xi^a}\frac{\partial X^N}{\partial \xi^b} g_{MN}$$

$$(g_0)_{ab} = \text{diag}(r_0^2 \eta_{\mu\nu}, g_{\theta_i\theta_j}, g_{\phi_i\phi_j})$$

- …and in detail…

$$g_{\theta_i\theta_j} = \begin{pmatrix} \frac{1}{6} + \frac{1}{9}\cot^2\frac{\theta_1}{2} & \frac{1}{9}\cot\frac{\theta_1}{2}\cot\frac{\theta_2}{2} \\ \frac{1}{9}\cot\frac{\theta_1}{2}\cot\frac{\theta_2}{2} & \frac{1}{6} + \frac{1}{9}\cot^2\frac{\theta_2}{2} \end{pmatrix}$$

$$g_{\phi_i\phi_j} = \begin{pmatrix} \frac{1}{6}\sin^2\theta_1 + \frac{1}{9}(1+\cos\theta_1)^2 & \frac{1}{9}(1+\cos\theta_1)(1+\cos\theta_2) \\ \frac{1}{9}(1+\cos\theta_1)(1+\cos\theta_2) & \frac{1}{6}\sin^2\theta_2 + \frac{1}{9}(1+\cos\theta_2)^2 \end{pmatrix}$$

- We can carry out a geometric of analysis for the D7 embedding. Note that in terms of the original conifold coordinates it looks like (Y_4):

$$\mu z_2 = z_3 z_4$$

- This has a $U(1) \times U(1)$ invariance with charges $(2,1,1)$ and $(0,1,-1)$ for the three complex coordinates under each U(1).
- It also has a scaling symmetry $\Gamma : \quad z_2 \to \lambda^2 z_2, \quad z_{3,4} \to \lambda z_{3,4}$
- The base of the cone Y_4 is therefore given by $X_3 = Y_4/\Gamma$

- We can parameterize Y_4 by:

$$z_3 = \rho e^{i\alpha} \cos\frac{\gamma}{2}, \quad z_4 = \rho e^{i\beta} \sin\frac{\gamma}{2}$$

- The base just corresponds to the intersection with the S^3 of radius ρ, or the equation:

$$|z_3|^2 + |z_4|^2 = \rho^2$$

- Consider the homeomorphism

$$\mu z_2 = (1-s) z_3 z_4, \quad s \in [0,1]$$

- At s=1, we have $z_2 = 0; \quad z_3, z_4$ **arbitrary** $\Rightarrow \quad \mathbb{R}_+ \times S^3$
- As s=0 we recover our D7 embedding.

- Thus our D7 embedding is homeomorphic to $\mathbb{R}_+ \times S^3$
- From this we conclude that the base X_3 is topologically equivalent to S^3

- On this basis we take the lowest mass states of the KK decomposition of D7 embedding fluctuations to be constant modes of angular coordinates; i.e., the lowest hyperspherical harmonic.

- It is of interest to relate Y^4 to the conifold geometry, particularly the coordinate r.

$$r^3 = \sum_{i=1}^{4} |z_i|^2 = \mu^2 + \rho^2 + \frac{1}{4\mu^2}\rho^4 \sin^2 \gamma$$

- First, note that as $r \to \mu^{2/3}$, the $X_3 \simeq S^3$ radius ρ shrinks to zero. This shows in detail how the D7 branes “end” in the AdS_5 radial direction.

- Thus, since we have identified the eight worldvolume parameters with eight of the coordinates of the underlying 10d geometry, the fluctuations in the D7 embedding will correspond to changes in the two functions r_0 and ψ_0

$$r = r_0(1+\chi)$$

$$\psi = \psi_0 + 3\eta$$

- Then one substitutes these into the pullback of the metric and expands the DBI action:

$$\varphi^*(g) = g_0 + \delta g_0(\chi, \eta) \qquad \sqrt{\det(g_0 + \delta g_0)} = \exp \frac{1}{2} \mathrm{Tr}(\ln g_0 + \ln(1 + g_0^{-1} \delta g_0))$$

good stuff!

- Straightforward but tedious to get the quadratic action:

$$
\begin{aligned}
S \;=\; & -\tau_7 \int d^4x \; d^2\theta \; d^2\phi \left\{ \sqrt{-g_0} \left[\frac{g_0^{ab}}{2C} (\partial_a \chi \partial_b \chi + \partial_a \eta \partial_b \eta) \right. \right. \\
& \left. + \frac{4}{C} (\sin^2 \frac{\theta_i}{2})^{-1} \chi \partial_{\phi_i} \eta - \frac{2}{C^2} (\sin^2 \frac{\theta_i}{2})^{-1} \cot \frac{\theta_j}{2} \partial_{\theta_j} (\chi \partial_{\phi_i} \eta) \right] \\
& \left. - \partial_{\theta_i} \left[\frac{\sqrt{-g_0}}{C} \cot \frac{\theta_i}{2} (3\chi^2 + 2\chi) \right] \right\}
\end{aligned}
$$

$$
C = 1 + \frac{2}{3} \cot^2 \frac{\theta_1}{2} + \frac{2}{3} \cot^2 \frac{\theta_2}{2}
$$

- Embedding of K D7 branes:

$$z_1 = \mu = r^{3/2} \sin\frac{\theta_1}{2} \sin\frac{\theta_2}{2}$$

- We make a change of variables

$$\theta_{\pm} = \frac{1}{2}(\theta_1 \pm \theta_2)$$

- This allows us to eliminate θ_+ in favor of $\hat{r} = r/\mu^{2/3}$

$$2\mu = r^{3/2}(\cos\theta_- - \cos\theta_+)$$

- Then we have integration over r instead of the angle θ_+, which is what we want. I.e., the RS model has an integration over the 5th dimension r.
- We still have to deal with the integral over θ_-. The domain of integration depends on r.

$$\theta_- \in [-\frac{\pi}{2} + \sin^{-1}\frac{\mu}{r^{3/2}}, \frac{\pi}{2} - \sin^{-1}\frac{\mu}{r^{3/2}}]$$

- This introduces another dependence on r that must be kept track of.

- Thus reducing to the zeromodes of X3 we have

$$S = -24\pi^2\mu^{8/3}\tau_7 \int d^4x \int_1^\infty d\hat{r} \left\{ \frac{1}{2}\mu^{-4/3}\hat{r}^{-9/2}F_1(\hat{r})\eta^{\mu\nu}(\partial_\mu\chi\partial_\nu\chi + \partial_\mu\eta\partial_\nu\eta) + \frac{1}{2}\hat{r}^{-1/2}\tilde{F}_1(\hat{r})[(\partial_{\hat{r}}\chi)^2 + (\partial_{\hat{r}}\eta)^2] + \text{t.d.} \right\}$$

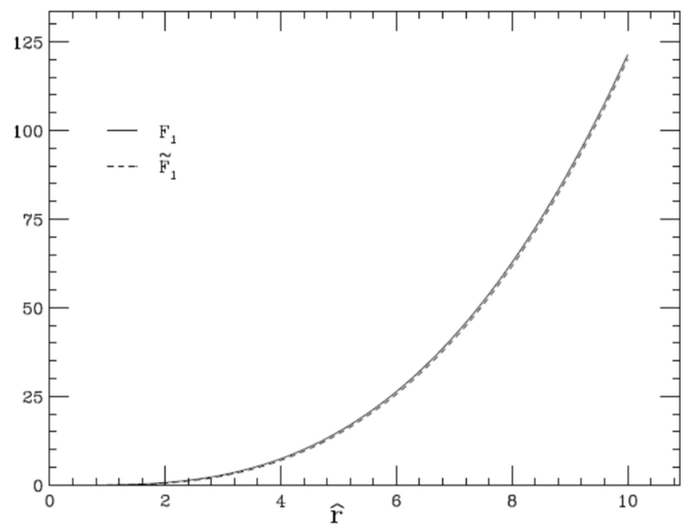
$$F_1(\hat{r}) = \int_{-\theta_0(\hat{r})}^{\theta_0(\hat{r})} d\theta_- \frac{\sqrt{-g}}{\sin\theta_+ C}(\hat{r}, \theta_-)$$

$$\tilde{F}_1(\hat{r}) = \int_{-\theta_0(\hat{r})}^{\theta_0(\hat{r})} d\theta_- \frac{\sqrt{-g}(C-1)}{\sin\theta_+ C^2}(\hat{r}, \theta_-)$$

- Careful study yields an approximate answer:

$$F_1(\hat{r}) \approx \frac{1}{6}\hat{r}^{5/2}\ln\hat{r}$$

- It’s good to about five decimal places.
- Numerically exact results are shown in the accompanying plot (next page):



125
100
75
50
25
0
0
2
4
6
8
10
F1
F̃1
r̂

- It can be seen that the tilde cousin is quite close in absolute value:

$$\tilde{F}_1(\hat{r}) \approx F_1(\hat{r})$$

- However, the approximation becomes *relatively* poor for $\hat{r} = \mathcal{O}(1)$

- I.e., the relative error $\dfrac{\tilde{F}_1(\hat{r}) - F_1(\hat{r})}{F_1(\hat{r})}$ is not small.

- To obtain a 5d action with the usual normalizations, we have to rescale the scalar fluctuation fields:

$$\chi = \hat{r}^{3/2}(\ln \hat{r})^{-1/2}\chi'$$

$$\eta = \hat{r}^{3/2}(\ln \hat{r})^{-1/2}\eta'$$

- Then we obtain, after some manipulations:

$$S(\chi') \approx -2\pi^2\mu^{8/3}\tau_7 \int d^4x \int_{\hat{R}}^{\infty} d\hat{r}\Big\{\frac{\hat{r}}{\mu^{4/3}}\eta^{\mu\nu}\partial_\mu\chi'\partial_\nu\chi'$$

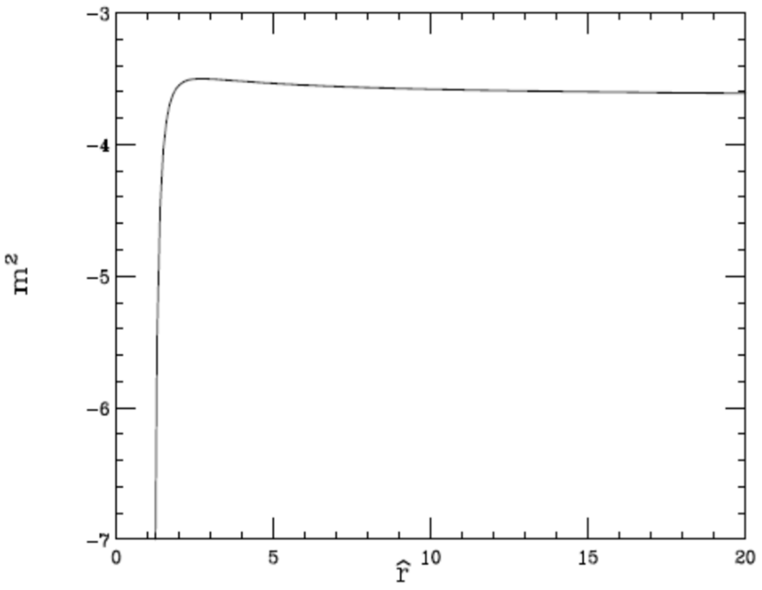
$$f(\hat{r})[\hat{r}^5(\partial_{\hat{r}}\chi')^2 + \hat{r}^3 m^2(\hat{r})\chi'^2]\Big\}$$

$$f(\hat{r}) = \frac{\tilde{F}_1(\hat{r})}{F_1(\hat{r})} \approx 1 \quad \text{for} \quad \hat{r} \gg 1$$

- The $\hat{r}$ dependent mass is

$$m^2(\hat{r}) = -\frac{15}{4} + \frac{1}{2\ln\hat{r}} - \frac{1}{4(\ln\hat{r})^2}$$

- It looks like this:



- The large negative mass-squared implies Dirichlet BCs

$$\lim_{\hat{r}\to 1} \chi'(\hat{r}) = \lim_{\hat{r}\to 1} \eta'(\hat{r}) = 0$$

Ignoring logs, we get conformally coupled scalar

- Reintroducing the AdS radius L

$$S(\chi') \approx -2\pi^2 L^{-5}\tau_7 \int d^4x \int_R^\infty dr \left\{ \frac{r}{L}\eta^{\mu\nu}\partial_\mu\chi'\partial_\nu\chi' + \frac{r^5}{L^5}(\partial_r\chi')^2 - \frac{15}{4L^2}\frac{r^3}{L^3}\chi'^2 \right\}$$

Dual gauge theory

Klebanov & Witten 1998

- Superpotential of KW dual theory

$$W = \lambda \epsilon_{ik} \epsilon_{jl} \mathrm{Tr}(A_i B_j A_k B_l)$$

- $SU(N) \times SU(N)$ gauge theory with bifundamentals A_1, A_2, B_1, B_2

- NSVZ beta function: *N=1 SCFT* at IRFP with known anomalous dimensions.

- F-term condition $\partial W/\partial \phi_i = 0$ implies conifold condition if we take

$$z_1 = A_1 B_1, \quad z_2 = A_2 B_2$$

$$z_3 = A_1 B_2, \quad z_4 = A_2 B_1$$

$$\Rightarrow \quad z_1 z_2 - z_3 z_4 = 0$$

- So the geometry of Y_6 is the moduli space. Similar to *N=4* case but more interesting.

Adding fundamental flavors

Ouyang 2003; Levi & Ouyang 2005

- Add to the superpotential

$$\Delta W = h\tilde{q}A_1Q + gqB_1\tilde{Q} + \mu_1 q\tilde{q} + \mu_2 Q\tilde{Q}$$

- Then demanding a massless mode gives:

$$A_1B_1 = \frac{\mu_1\mu_2}{gh} \quad \Leftrightarrow \quad z_1 = \mu$$

- Levi & Ouyang go on to determine the dimensions of operators from the "meson" spectra (coming from the D7 brane embedding).
- It has the interpretation:

$$q(AB)\cdots(AB)\tilde{q}$$

n insertions of (AB)

$$\Delta \sim \frac{3}{2}n$$

- Dimensions follow from KW superpotential being marginal (dim=3).

SUSY breaking from 5d geometry

- Supersymmetry breaking is introduced by a Type IIB supergravity motivated deformation [Kuperstein & Sonnenschein, hep-th/0309011] of the Klebanov-Strassler background:

$$ds^2 = A^2(z)(-dt^2 + d\vec{x}^2 + dz^2)$$

$$A^2(z) = \frac{1}{(kz)^2}\left[1 - \epsilon\left(\frac{z}{z_1}\right)^4\right]$$

- According to holographic duality, this is supposed to be dual to soft SUSY breaking gaugino masses in the strongly coupled gauge theory.

- This is the 5d deformation that we extracted from the solution of KuSo, following techniques similar to those in the first part of this talk (KW).
- The KuSo background is 10d, and is a non-SUSY background that solved Type IIB SUGRA EOM following techniques of Borokhov & Gubser (2002).
- In that work, SUSY breaking deformations of KS were worked out.

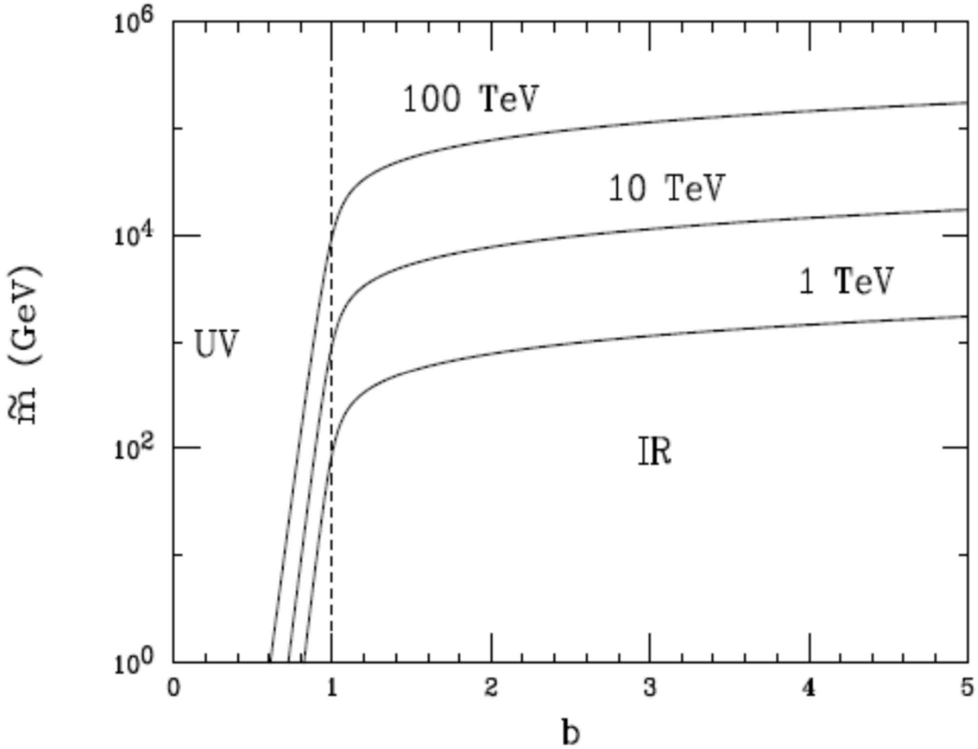
- Because of partial compositeness of fields in the bulk, this is a single-sector SUSY breaking model.

Provides gravity dual of older pheno models:
Arkani-Hamed, Luty & Terning 1997
Luty & Terning 1998
Cohen, Kaplan & Nelson 1996

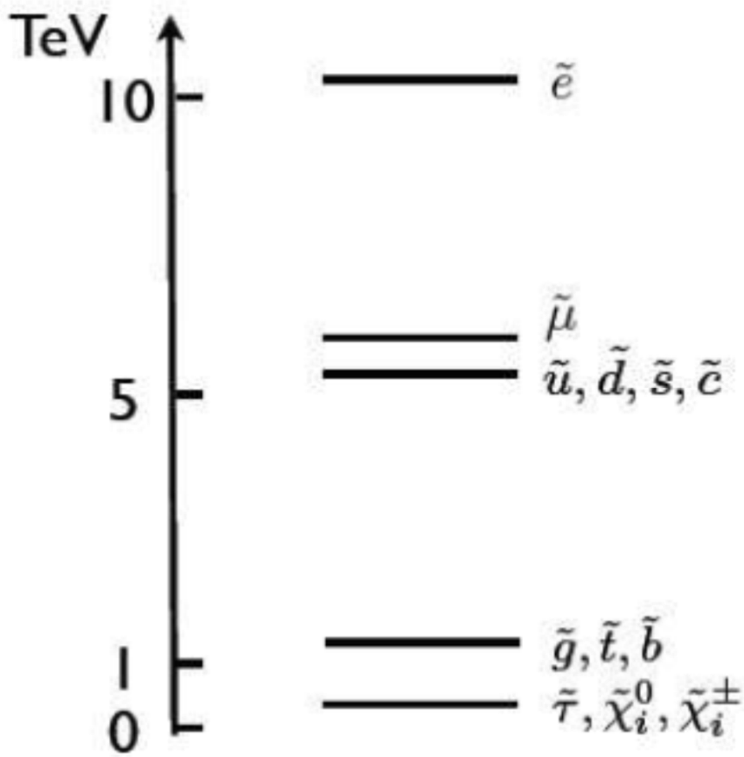
- Soft scalar masses are determined in terms of the deformation parameter and the localization parameter b:

$$\tilde{m}^2 \approx \epsilon k^2 \frac{(1-b)(b+10)}{(kz_1)^4} \frac{(kz_1)^{1+b} - (kz_1)^{1-b}}{(kz_1)^{1-b} - (kz_1)^{b-1}}$$

- Also have predictions for gaugino soft masses, etc.
- [Recall that susy pheno is determined by soft lagrangian.]
- In contrast to usual WED models, Higgs is confined to UV brane. Mass protected by SUSY in UV.



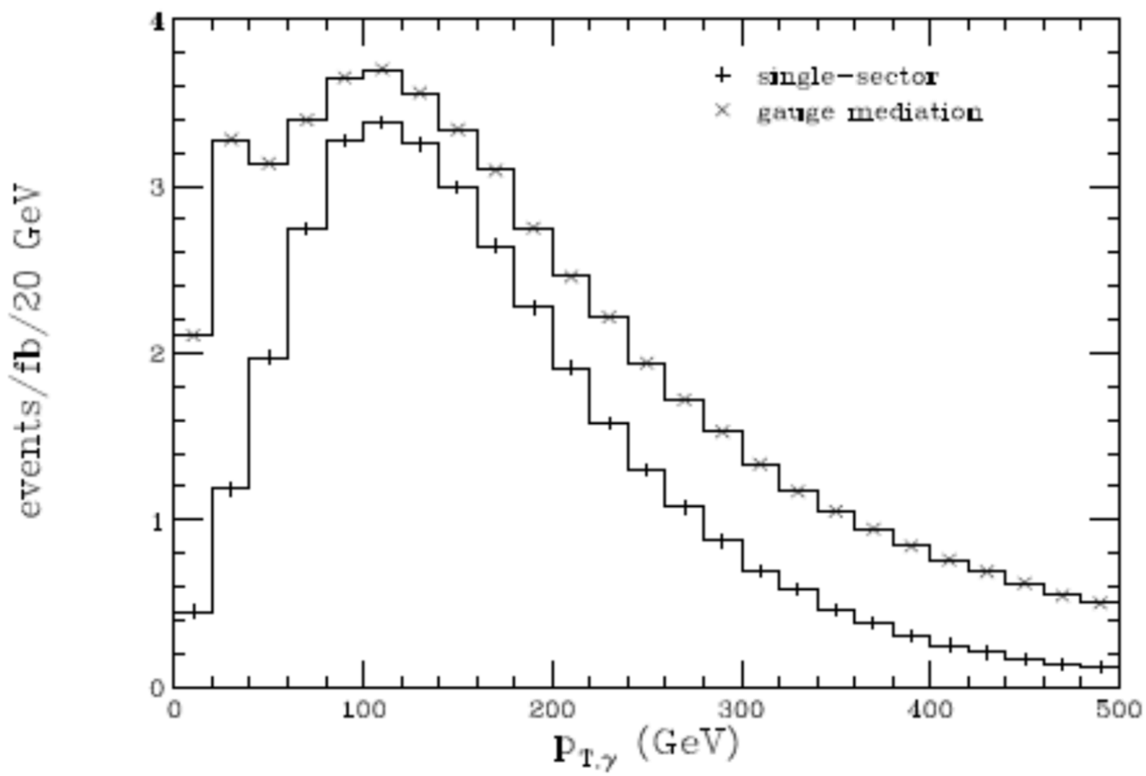
$$z_1^{-1} = 1, 10, 100 \text{ TeV}$$



TeV
10
5
1
0
$\tilde{e}$
$\tilde{\mu}$
$\tilde{u}, \tilde{d}, \tilde{s}, \tilde{c}$
$\tilde{g}, \tilde{t}, \tilde{b}$
$\tilde{\tau}, \tilde{\chi}_i^0, \tilde{\chi}_i^\pm$

$\tilde{e}_L$, $\tilde{e}_R$, $\tilde{\nu}_{eL}$	10160, 10150, 10160 GeV
$\tilde{\mu}_L$, $\tilde{\mu}_R$, $\tilde{\nu}_{\mu L}$	5145, 5130, 5145 GeV
$\tilde{d}_L$, $\tilde{d}_R$, $\tilde{u}_L$, $\tilde{u}_R$	5905, 5885, 5970, 5890 GeV
$\tilde{s}_L$, $\tilde{s}_R$, $\tilde{c}_L$, $\tilde{c}_R$	5905, 5885, 5970, 5890 GeV
$\tilde{g}$	1615 GeV
$\tilde{b}_1$, $\tilde{b}_2$, $\tilde{t}_1$, $\tilde{t}_2$	1354, 1369, 1253, 1369 GeV
$\tilde{\tau}_1$, $\tilde{\tau}_2$, $\tilde{\nu}_{\tau L}$	511, 630, 633 GeV
$\tilde{\chi}_1^\pm$, $\tilde{\chi}_2^\pm$	478, 593 GeV
$\tilde{\chi}_1^0$, $\tilde{\chi}_2^0$, $\tilde{\chi}_3^0$, $\tilde{\chi}_4^0$	288, 480, 511, 598 GeV
h^0, A^0, H^0, $H^\pm$	115, 646, 646, 651 GeV
$\tilde{G}$	2.35 eV

Diphoton signal (ruled out in 1st month of LHC running)



Conclusions

- Warped geometries, a hierarchy of scales, and dynamical SUSY breaking can be accommodated in a string/D-brane construction.
- Randall-Sundrum type set-up, with fields in the bulk (compositeness) can be extracted from probe D7 branes in the Klebanov-Witten and Klebanov-Strassler geometries.
- This provides a calculable approach to theories that are strongly coupled in the gauge theory dual.
- Realistic phenomenology can be derived in this set-up, but with stringy origins and modifications of the usual pheno assumptions.

Lattice connection

- This KW gauge theory would be nice to simulate.
- Holographic duality is well established.
- Something other than N=4.
- Close to SQCD…so new challenges we've discussed in white papers.
- Exact results to aim for…marginality of "baryon" operator [see Intriligator & Seiberg]
- Holographic duality with IRFP, rather than finite theory.
- Moduli space is maybe a little sexier.
- A lot is known about the CFT at the IRFP, b/c of SUSY.