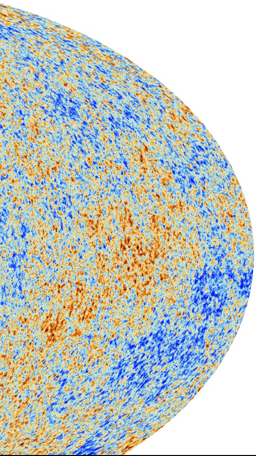
Lattice Holographic Cosmology



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Numerical approaches to holography, quantum gravity and cosmology
Edinburgh, 24 May 2018

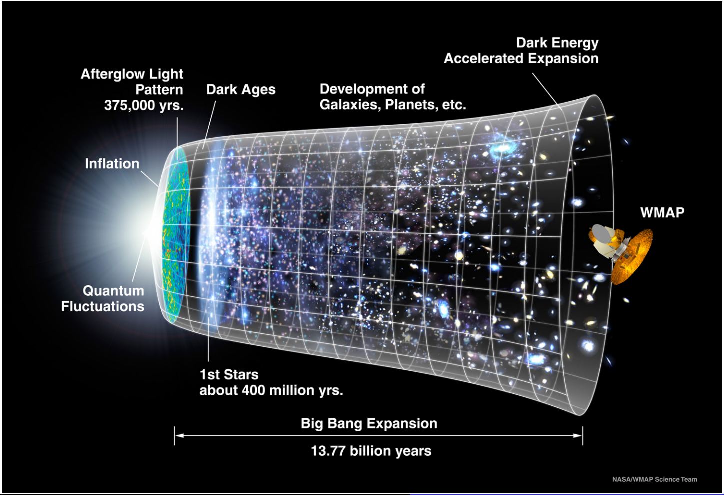
Outline

Introduction

- ➢ The physics of the very Early Universe is an example where we expect both General Relativity and Quantum Mechanics to be important.
- ➢ Cosmology provides a unique observational window to this era.

- ➢ One of the deepest insights about quantum gravity that emerged in recent times is that it is expected to be holographic.
- ➢ In this talk I will review a holographic approach to the physics of the very Early Universe, discuss its predictions and compare them against observations.

Timeline of the Universe



Why holographic cosmology?

Holographic cosmology is a new framework for cosmology.

- It can accommodate both conventional inflation (strong coupled dual QFT) and qualitatively new models for the very early Universe (weakly coupled QFT).
- The latest theoretical ideas about the nature of quantum gravity may be tested against observation data.
- The new models are falsifiable with current data.
- It gives new insight into conventional inflation.

Holographic cosmology in a nutshell

- Holography in general maps gravitational dynamics of $(d+1)$ dimensional theory to observables of a d-dimensional QFT with no gravity.

- In the context of early-times cosmology, it maps
- cosmological observable such as the power spectra and non-gaussianities
- to correlation functions of the energy momentum tensor of the dual QFT.

References

- A holographic framework for cosmology was put forward in works with Paul McFadden and Adam Bzowski (2009 - 2013).
- Related work:
  [Hull (1998)] ... (E-branes)
  [Witten (2001)] [Strominger (2001)] ... (dS/CFT correspondence)
  [Maldacena (2002)] ... (wavefunction of the universe)
  [Hartle, Hawking, Hertog (2012)] ... (quantum cosmology)

  [Trivedi et al][Garriga et al] [Coriano et al] [Maldacena, Pimentel] .... [Arkani-Hamed, Maldacena] ....

References

In this talk I will give an overview of the earlier work and discuss more recent and on-going work:

- N. Afshordi, C. Coriano, L. Delle Rose, E. Gould, KS, From Planck data to Planck era: Observational tests of Holographic Cosmology , Phys. Rev. Lett. 118 (2017), 041301, arXiv:1607.04878 [astro-ph.CO].
- Elizabeth Gould, Niayesh Afshordi, KS, Constraining holographic cosmology with Planck data, Phys. Rev. D95 (2017) no.12, 123505 , arXiv:1703.05385 [astro-ph.CO].
- G. Cossu, L. Del Debbio, E. Gould, A. Jüttner, M. Hanada, M. Mostert, A. Portelli, KS, P. Vranas, Lattice Holographic Cosmology, in progress.

Outline

Outline

Gauge/gravity duality: a primer

A conjectured equivalence:

Gravity in $(d+1)$ dimensions $\Leftrightarrow$ QFT in d dimensions

- In this talk we will focus exclusive on $d = 3$:
  a 4-dimensional universe and its dual $3d$ QFT.
- The idea of "holography" has its origins in black holes physics and concrete realisations appeared in string theory.
- No familiarity with black holes and string theory is needed to understand the duality.

QFT in a nutshell

- All information about a QFT is encoded in correlators functions.
- Textbooks usually discuss correlation functions of elementary fields.
- We will instead consider correlation functions of gauge invariant composite operators.
- For example, such operators are the energy-momentum tensor $T_{\mu\nu}$, or scalar operators such as $\mathrm{Tr} F_{\mu\nu} F^{\mu\nu}$, ($F^{\mu\nu}$ is the field strength of Yang-Mills field), $\bar{\psi}\psi$, etc.

Gauge/gravity duality in a nutshell

Gauge/gravity duality provides a way to

- obtain QFT correlation functions of gauge invariant operator by doing a gravitational computation, or conversely
- obtain the behaviour of a gravitational system using correlation functions of gauge invariant operators.

Weak/strong duality

- An important feature of the duality is that it is a weak/strong duality.

weakly coupled gravity $\Leftrightarrow$ strongly coupled QFT

- One can obtain QFT correlation functions of gauge invariant operators at strong coupling by solving Einstein equations.

Strongly coupled gravity $\Leftrightarrow$ weakly coupled QFT

- The extra dimension represents the energy scale at which we probe the theory.

Radial evolution is an RG flow.

Outline

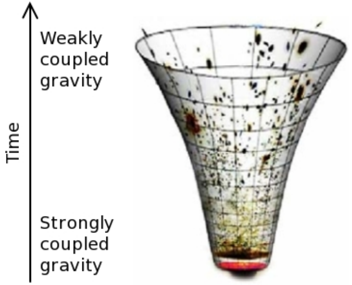
Holographic Universe

- In holographic cosmology:

  Cosmological evolution = inverse RG flow

The dual QFT should have:

- a strongly coupled UV fixed point corresponding to the current dark energy era.
- In the IR the theory should either flow to:
  - an IR fixed point (corresponding to de Sitter inflation), or
  - a phase governed by a super-renormalizable theory (corresponding to power-law inflation).



How do we make predictions?

- We need to provide formulae that relate cosmological observables with QFT correlation functions.
- The Cosmic Microwave Background (CMB) carries information about the very early Universe.
- Two of the main observables, currently measured by satellites (such as Planck) and other missions, are the power spectra and non-Gaussianities and there are explicit holographic formulae for them.
- Here I will focus on the power spectra.

Holographic formulae for power spectra [McFadden, KS]

- The 2-point function of the energy momentum tensor T_{ij} in momentum space has the form

$$\langle T_{ij}(q) T_{kl}(-q) \rangle = A(q^2)\Pi_{ijkl} + B(q^2)\pi_{ij}\pi_{kl},$$

where $\Pi_{ijkl} = \frac{1}{2}(\pi_{ik}\pi_{lj} + \pi_{il}\pi_{kj} - \pi_{ij}\pi_{kl}), \quad \pi_{ij} = \delta_{ij} - q_i q_j/q^2.$

- The power spectra are given by

$$\Delta^2_{\mathcal{R}}(q) = -\frac{q^3}{16\pi^2}\frac{1}{\mathrm{Im}\, B}, \quad \Delta^2_T(q) = -\frac{2q^3}{\pi^2}\frac{1}{\mathrm{Im}\, A},$$

where the imaginary part is taken after the analytic continuation,

$$q \to -iq, \qquad N \to -iN$$

- Non-gausianities are related with higher-point functions of T_{ij}.

Summary

1. Choose a QFT.
2. Compute correlation functions of the energy momentum tensor.
3. Insert in holographic formula to obtain the holographic prediction.
4. Compare with cosmological observables.

Outline

Dual QFT

- The model is defined by providing the dual QFT, which we take to be

$$S = \frac{1}{g_{YM}^2} \int d^3x \mathrm{tr} \Big[\frac{1}{2} F_{ij} F^{ij} + \frac{1}{2} (D\phi^J)^2 + \bar{\psi}^K \not{D} \psi^K + \lambda_{J_1 J_2 J_3 J_4} \phi^{J_1} \phi^{J_2} \phi^{J_3} \phi^{J_4} + \mu_{J L_1 L_2} \phi^J \psi^{L_1} \psi^{L_2} \Big].$$

All fields are massless and in the adjoint of $SU(N)$, $\lambda_{J_1 J_2 J_3 J_4}$, $\mu_{J L_1 L_2}$ are dimensionless couplings while g_{YM}^2 has mass dimension 1.

Energy-momentum tensor

- For this class of theories, the 2-point function of the trace of T at large N takes the form,

$$\langle T(q)T(-q)\rangle = N^2 q^3 f(g_{\rm eff}^2),$$

where $g_{\rm eff}^2 = g_{\rm YM}^2 N/q$ is the effective dimensionless 't Hooft coupling and $f(g_{\rm eff}^2)$ is a general function of $g_{\rm eff}^2$.

- In perturbation theory and at 2-loops,

$$f(g_{\rm eff}^2) = f_0(1 - f_1 g_{\rm eff}^2 \log g_{\rm eff}^2 + f_2 g_{\rm eff}^2 + O[g_{\rm eff}^4]).$$

where f_0, f_1, f_2 are constants that depend on the field content etc.

Holographic power spectrum

- To compute the holographic scalar power spectrum we need to analytically continue $\langle T(q)T(-q)\rangle$:

$$\Delta^2_{\mathcal{R}}(q) = -\frac{q^3}{4\pi^2}\frac{1}{\mathrm{Im}\langle T(q)T(-q)\rangle} = \frac{1}{4\pi^2 N^2}\frac{1}{f(g^2_{\mathrm{eff}})}$$

- Thus, for this class of theories and in the perturbative regime there is a universal prediction:

$$\Delta^2_{\mathcal{R}}(q) = \left(\frac{1}{4\pi^2 N^2 f_0}\right)\frac{1}{1 - f_1 g^2_{\mathrm{eff}} \log g^2_{\mathrm{eff}} + f_2 g^2_{\mathrm{eff}}}$$

- This may be rewritten as

$$\Delta^2_{\mathcal{R}}(q) = \Delta_0 \frac{1}{1 + (gq_*/q)\ln|q/\beta g q_*|},$$

with $gq^* = f_1 g^2_{\mathrm{YM}} N$, $\beta = f_3/|f_1|$, $f_3 = \exp\left(-f_2/f_1\right)$, $q_* = 0.05\mathrm{Mpc}^{-1}$.

Outline

Main questions

1. Which of the two forms of power spectrum fits the data best:

$$\Delta^2_{\mathcal{R}}(q) = \Delta_0 \frac{1}{1 + (gq_*/q) \ln |q/\beta g q_*|},$$

or

$$\Delta^2_{\mathcal{R}}(q) = \Delta_0 q^{n_s - 1}.$$

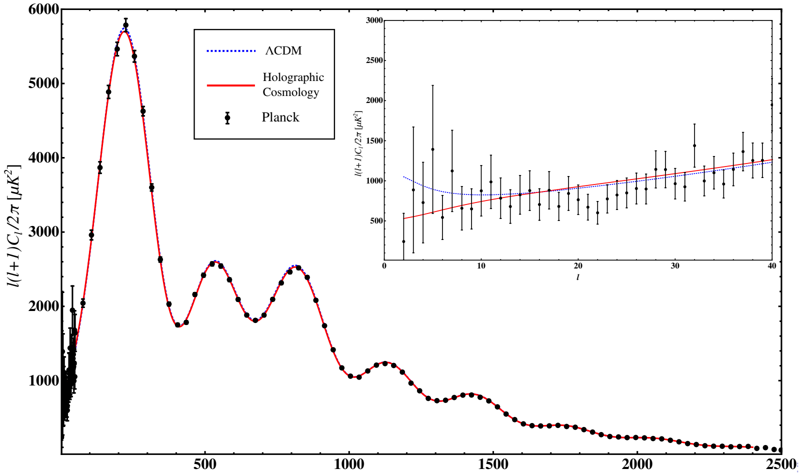
2. Given the observational values of

$$\Delta_0, \quad g, \quad \ln \beta$$

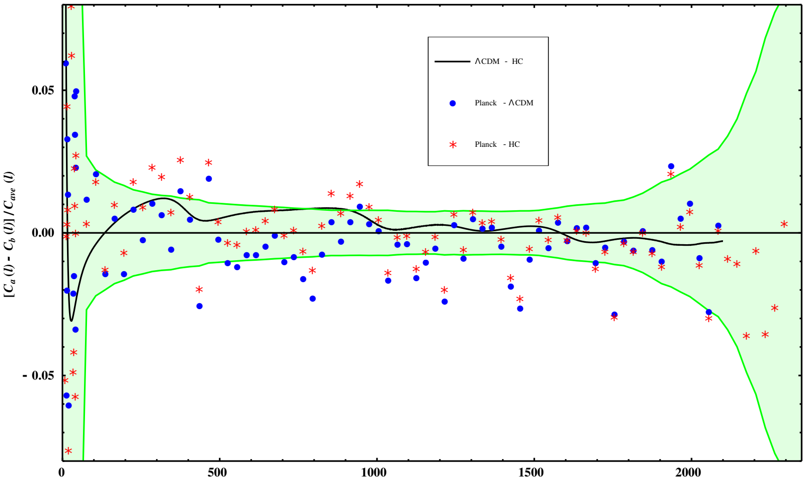
what can we say about the dual QFT?

Is the data constraining enough to rule out/in theories?

Planck 2015 vs ΛCDM vs holographic model (TT)



Planck 2015 vs ΛCDM vs holographic model (TT)



Results

- The fit to data implies that $g_{eff}^2 = g_{YM}^2 N/q$ is very small for all scales seen in CMB, except at very low multipoles, justifying *a posteriori* the use of perturbation theory.

- For $l < 30$ the model becomes non-perturbative and one cannot trust the perturbative prediction.

- Goodness of fit ($l > 30$)

	HC	ΛCDM
χ^2	824.0	824.5

The difference in χ^2 indicate that the models are less than 1σ apart.

Outline

Selecting the holographic model

We now want to use the observations to select the holographic model. We need to satisfy:

1. Observational constraints
2. Theoretical constraints

Observational constraints

1. The ratio of scalars-to-tensors should be small:

$$r < 0.123$$

2. The amplitude should come out to be

$$\Delta_0 = 2.044 \times 10^{-9}$$

3. The model should reproduce the value of $\log \beta$,

$$\log \beta = 1.014$$

Theoretical constraints

1. Large N limit should be valid, i.e. N^2 should bigger than any other parameter of the model.
2. The model should be perturbative at all scales probed by Planck
   - Effective coupling constant must be small

$$g_{\rm eff}^2 = (1/f_1)(gq_*/q) \ll 1$$

$\Rightarrow$ If $g_{\rm eff}^2$ is not small for all relevant momenta, one should use non-perturbative methods.

A model that is ruled out

- A gauge field coupled to fermions only has

$$\log \beta \geq 6.09$$

- This class of models is ruled out by the data.

A model that is ruled in

- A gauge field coupled to N_ϕ non-minimal scalars with ϕ^4 self-interaction.
- Non-minimal means that the energy momentum tensor contains an improvement term:

$$T_{ij} = T_{ij}(A) + \mathrm{Tr}\left(\partial_i\phi\partial_j\phi - \delta_{ij}(\frac{1}{2}(\partial\phi)^2 + \frac{1}{4!}\phi^4) + \xi(\delta_{ij}\Box - \partial_i\partial_j)\phi^2\right)$$

$\xi = 1/8$ yields conformal scalars.

A model that is ruled in

- All observational constraints are satisfied by the following values:

$$\xi = 0.133, \quad N = 2995, \quad N_\phi = 23255$$

- Theoretical constaints:
  1. Large N validity:

$$N^2 \sim 8 \times 10^6 > N_\phi \sim 2 \times 10^4$$

  2. Perturbation theory:
     - Effective coupling constant,

$$3.3 \times 10^{-4} < g^2_{eff}(q) < 0.41$$

       for all momenta q seen by Planck.
     - 2-loop approximation breaks down for

$$l < 35$$

Outline

Holographic Lattice Cosmology

- Perturbative QFT yields new interesting models for the very Early Universe.
- Comparing with data suggests that we may need to go beyond leading order/need non-perturbative information.
- QFT at intermediate coupling may provide yet more interesting models.

➠ Use Lattice to compute the relevant QFT observables.

Toy Model

- A non-minimally coupled massless scalar field in the adjoint of $SU(N)$ with ϕ^4 self-interaction

$$S = \frac{1}{\lambda}\int d^3x \mathrm{Tr}\left(\frac{1}{2}(\partial_\mu\phi)^2 + \frac{1}{4!}\phi^4\right),$$

and energy momentum tensor

$$T_{ij} = \frac{1}{\lambda}\mathrm{Tr}\left(\partial_i\phi\partial_j\phi - \delta_{ij}(\frac{1}{2}(\partial\phi)^2 + \frac{1}{4!}\phi^4) + \xi(\delta_{ij}\Box - \partial_i\partial_j)\phi^2\right)$$

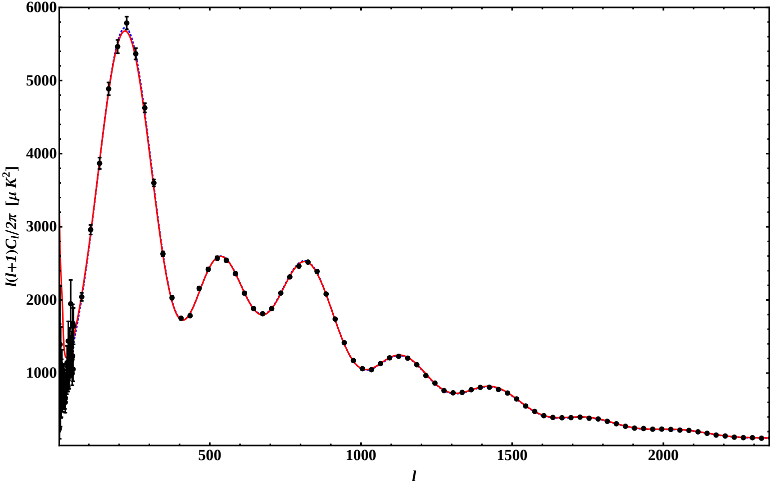
- The perturbative answer to 2-loops is [Coriano, Delle Rose, KS (to appear)]

$$f_0 = \frac{(1-8\xi)^2}{256}, \qquad f_1 = 0, \qquad f_2 = -\frac{1}{24}$$

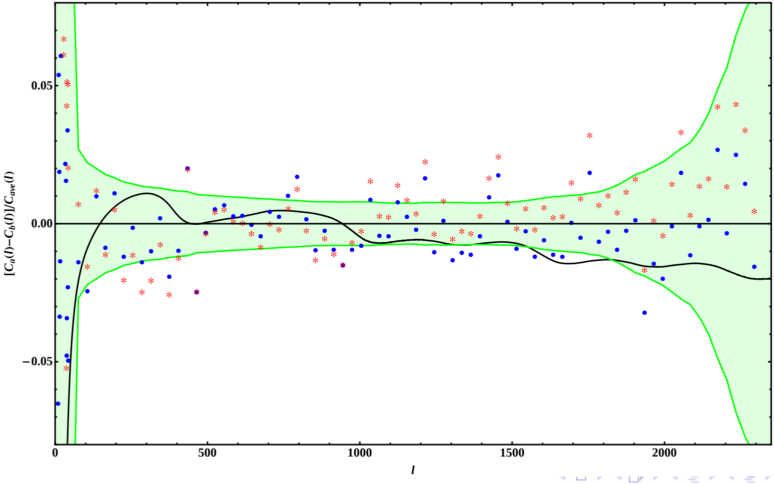
Is this just a toy model?

- If this model fits the data, it would be the most economic description.
- λ can be scaled away, so the model contains two parameters: the rank of the gauge group N and the non-minimality parameter ξ.
- N is related with the smallness of the amplitude of the primordial perturbations.
- ξ is related with the tensor-to-scalar ratio.

Fit to Planck



Fit to Planck



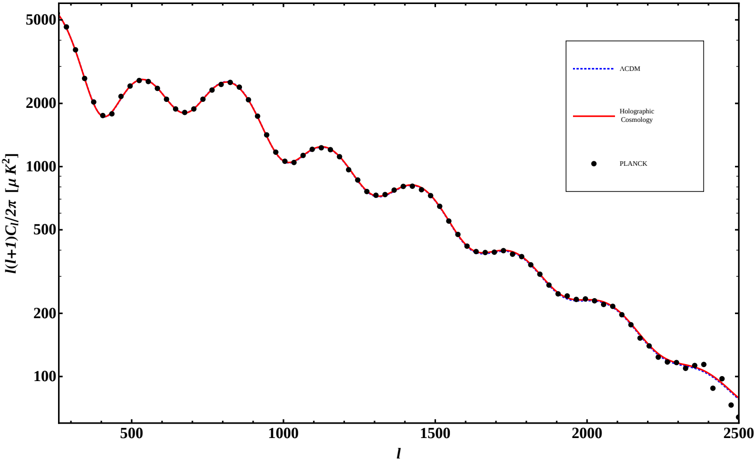
Is this model perturbative?

- Fit-to-data gives $|g| = 0.0156$, which implies that perturbation theory break down at

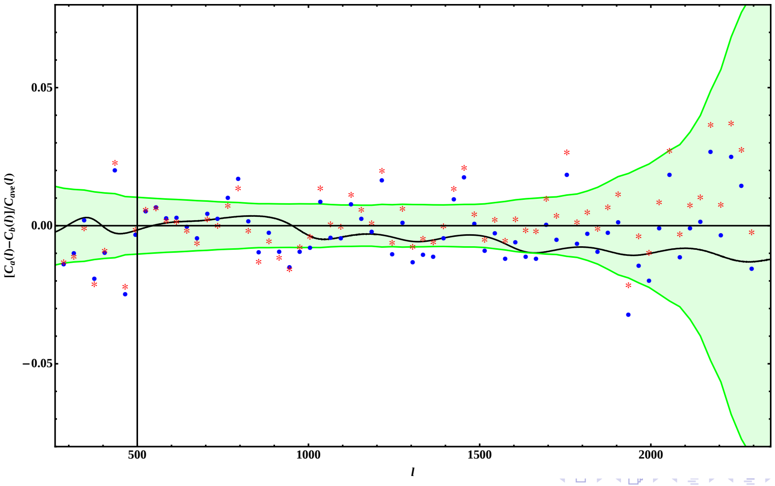
$$g_{eff}^2 \geq 1 \qquad \Rightarrow \qquad l < 260$$

- We cannot trust the prediction of perturbation theory below $l = 260$.

Fit to Planck (l>260)



Fit to Planck



Goodness of fit ($l > 260$)

	HC	ΛCDM	Toy Model
χ^2	824.0	824.5	823.5

- All three models are within 1σ.

Lattice Holographic Cosmology

- Compute the QFT correlators on the lattice.
- This would allow us to model the low-l region ...

- ... and also investigate the implications of the singularity resolution.

Plan

- Discretize the continuum model.
- Find the massless point.
- Find the energy-momentum tensor.
- Compute its 2-point function.
- Compare with Planck data.

Massless point

- We need to simulate a massless theory.
- This requires introducing a bare mass δ_m^2 and fine tune its value so that the theory becomes massless in the continuum limit.
- In perturbation theory δ_m^2 can be computed order by order:

$$\delta_m^2 = -\lambda \frac{Z_0}{a}\left(2N - \frac{3}{N}\right) + \lambda^2 \left(N^2 - 6 + \frac{18}{N^2}\right) D + \mathcal{O}(\lambda^3)$$

where $Z_0 = 0.252728(6)$ and D is a 2-loop integral.

Massless point: perturbation theory

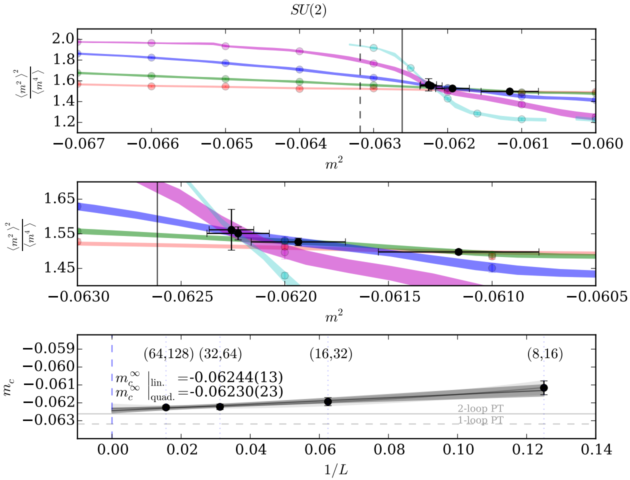
- The 2-loop integral however is IR divergent.
- It was argued in [Jackiw,Templeton (1981)][Appelquist, Pisarski(1981)] that these type of theories are non-perturbatively IR finite: the dimensionful coupling constant effectively acts as an IR regulator.
- As we will see, our results led support to this general claim.

- In our case we are interested in the non-perturbative evaluation of the 2-point function, so we need to know how to find the massless point non-perturbatively.

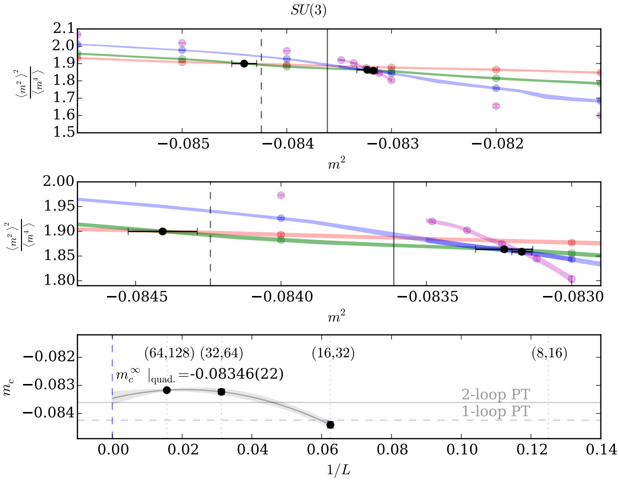
Massless point: non-perturbative

- If the mass in the continuum limit is positive then $\langle M^n \rangle = 0$ for any n, where $M = \sum_{\vec{n}} \phi_{\vec{n}}$.
- If the mass in the continuum limit is negative we are in the spontaneously broken phase, $\langle \mathrm{Tr}\, M^n \rangle \neq 0$.
- To find the massless point one may compute the Binder Cumulant $\langle (TrM^2) \rangle^2 / \langle TrM^4 \rangle$ for different lattice sizes and find the intersection point.

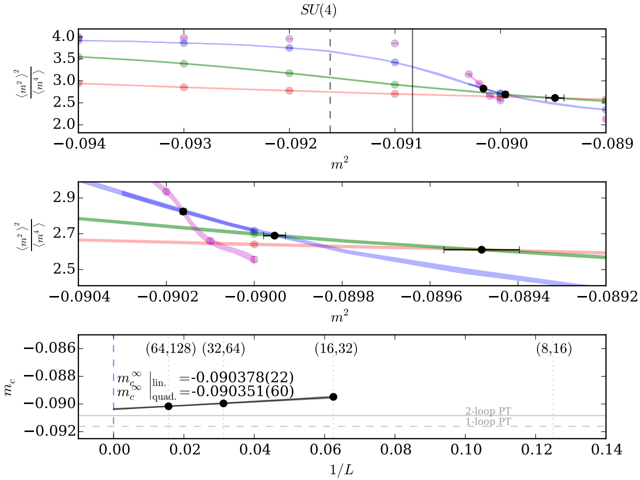
Binder Cumulant $SU(2)$ [PRELIMINARY]



Binder Cumulant $SU(3)$ *[PRELIMINARY]*



Binder Cumulant $SU(4)$ *[PRELIMINARY]*



Energy momentum tensor

- Since the lattice breaks Poincaré invariance, the energy momentum tensor is not automatically conserved and may mix with other operators.
- In our case, the energy momentum tensor can mix with 5 operators.
- The correct operator is found by imposing the conservation Ward identity:

$$\langle \partial^\mu T_{\mu\nu}(x)\phi(x_1)\cdots\phi(x_k)\rangle = \sum_{i=1}^{k} \delta(x-x_i)\cdot\frac{\partial}{\partial x_i^\nu}\langle\phi(x_1)\cdots\phi(x_k)\rangle\,,$$

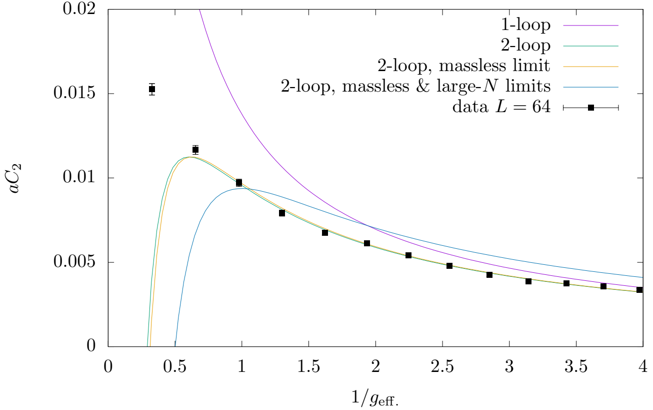
- Still in progress ....

2-point functions

- Compute $\langle TT \rangle$ for different values of N to extract the large N behavior.
- Consider $g^2_{eff} = \lambda N/q \ll 1$ and check with perturbative results.
- Consider $g^2_{eff} \sim 1$ and compare with Planck data.

Preliminary results

- Compute $C_2 = \langle \mathrm{Tr}\phi^2(p)\mathrm{Tr}\phi^2(-p)\rangle$:



Outline

Conclusions

- Holography offers a unified framework for discussing the very Early Universe:
  - Strongly couple QFT: conventional inflation.
  - Perturbative QFT: new non-geometric models.

Conclusions

- Perturbative holographic models provide an excellent fit to Planck data and are competitive to ΛCDM.
- The data rules out mostly fermionic theories.
- The data selects YM theory coupled to non-minimal scalars with quartic potential as the dual QFT.

Outlook

- At very low q we need to go beyond leading order/need non-perturbative information.
- Use Lattice to compute the relevant QFT observables.
- ... on-going ... for a toy model without the YM field.
- If successful, this "toy model" would provide an incredibly simple model for the very early Universe.