Kaluza-Klein black holes and their role in holography

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Based on work with S. Catterall, R. Jha and D. Schaich; `arXiv:1709.07025`

Also forthcoming work with K. Cheamsawat

PRELUDE

- Arithmetic progression:

Year	Location
2009	London
2012	Santa Barbara
2015	Kyoto
	(2017 Kyoto)
2018	Edinburgh
$\vdots$	?

PRELUDE

- 0+1 SYM at large N and appropriately low temperature is dual to a quantum gravity (string) theory in a semiclassical regime.
- This allows for the first principle tests of *a theory of* quantum gravity.
- A little ‘rant’

  This is not *the* theory of quantum gravity.

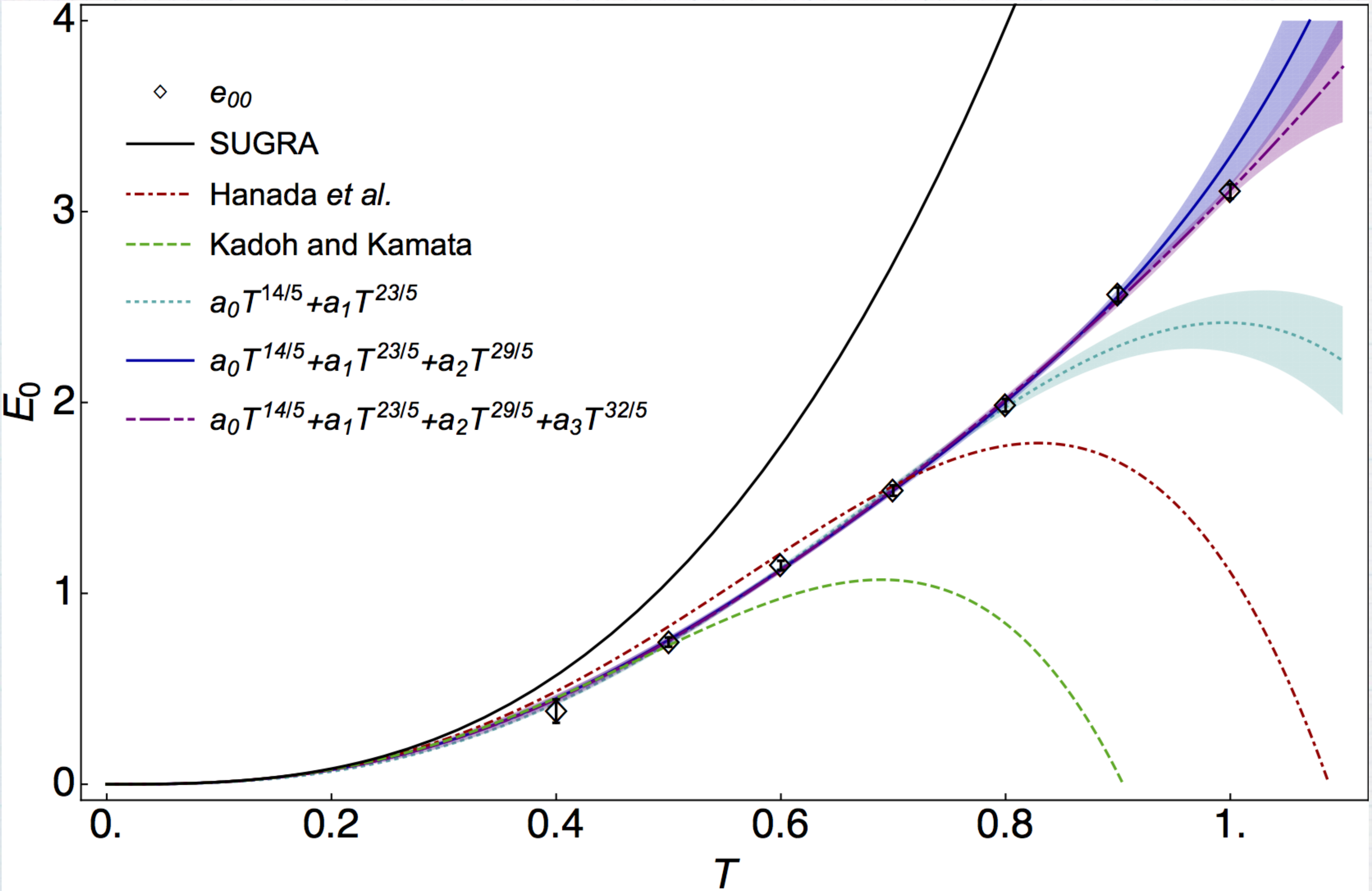
  However it is well defined and allows for computation.

  In principle any quantum gravity questions may be addressed – for example, emergence of spacetime (and black hole information loss)

PRELUDE

- Quenched simulations were performed in '04, then supersymmetry added in '07, '08.
- By now we have beautiful data confirming the thermodynamics of black holes is observed.

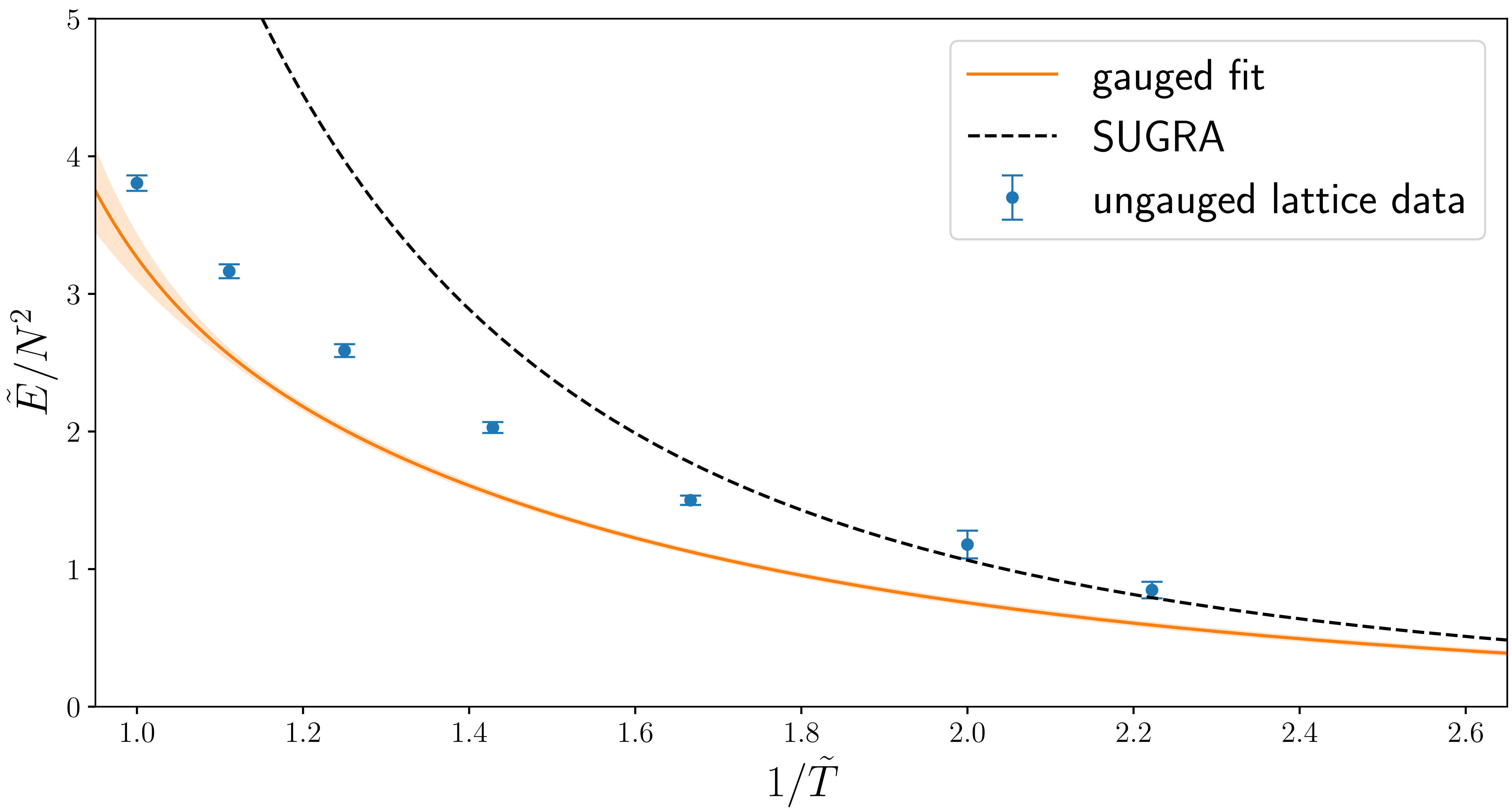
$$\frac{E}{\lambda^{\frac{1}{3}}} = 7.41 N^2 \left(\frac{T}{\lambda^{\frac{1}{3}}} \right)^{\frac{14}{5}}$$



Monte Carlo String/M-Theory Collaboration

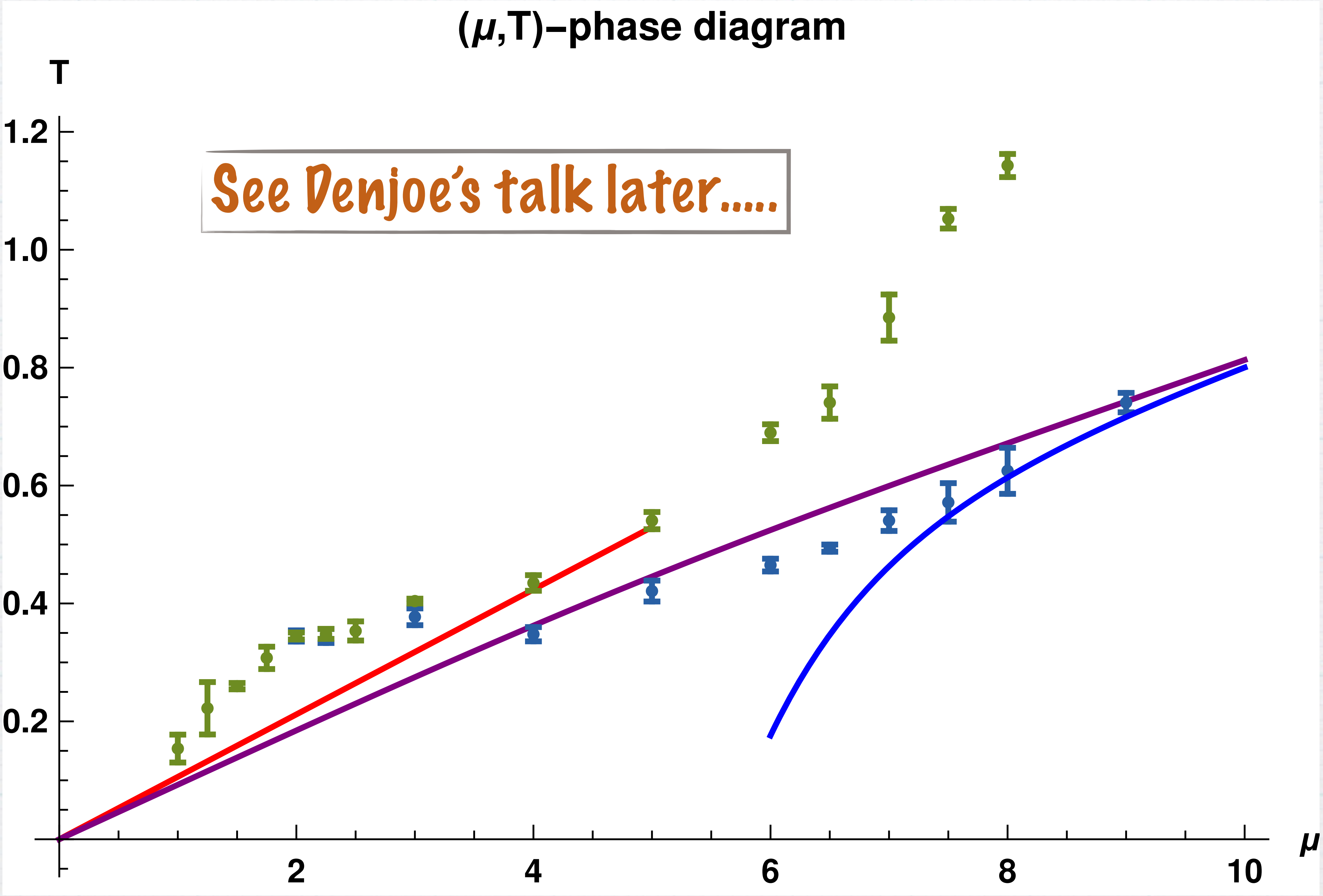
Berkowitz, Hanada, Ishiki, Rinaldi, Simasaki, Vranas '16

Gauged or ungauged? (Maldacena, Milekhin '18)



Berkowitz, Hanada, Rinaldi, Vranas '18

BMN deformation



Gravity: Costa, Greenspan, Penedones, Santos '14

Lattice: Asano, Filev, Kovacik, O'Connor '18

PRELUDE

- We may also consider 1+1 SYM (and 2+1 etc...)
- Again thermodynamics is predicted from dual black holes and may be tested
- From lattice perspective it is convenient to wrap the spatial direction into a circle – ie. the thermal 1+1 SYM lives on a Euclidean 2-torus.

  There is now a rich phase structure connected to a very interesting gravitation system, **Kaluza-Klein** theory, which is in the topic of this talk....

PLAN

- Some review;
  - Black holes in Kaluza-Klein theory
  - Black holes dual to 1+1 SYM
- Recent developments
- Adding string charge in Kaluza-Klein theory
- New predictions for 1+1 SYM

BLACK HOLES IN KALUZA-KLEIN THEORY

- Consider Kaluza-Klein theory – pure gravity in d+1 dimensions with asymptotics $M^{1,d-1} \times S^1$, there the circle has size L.

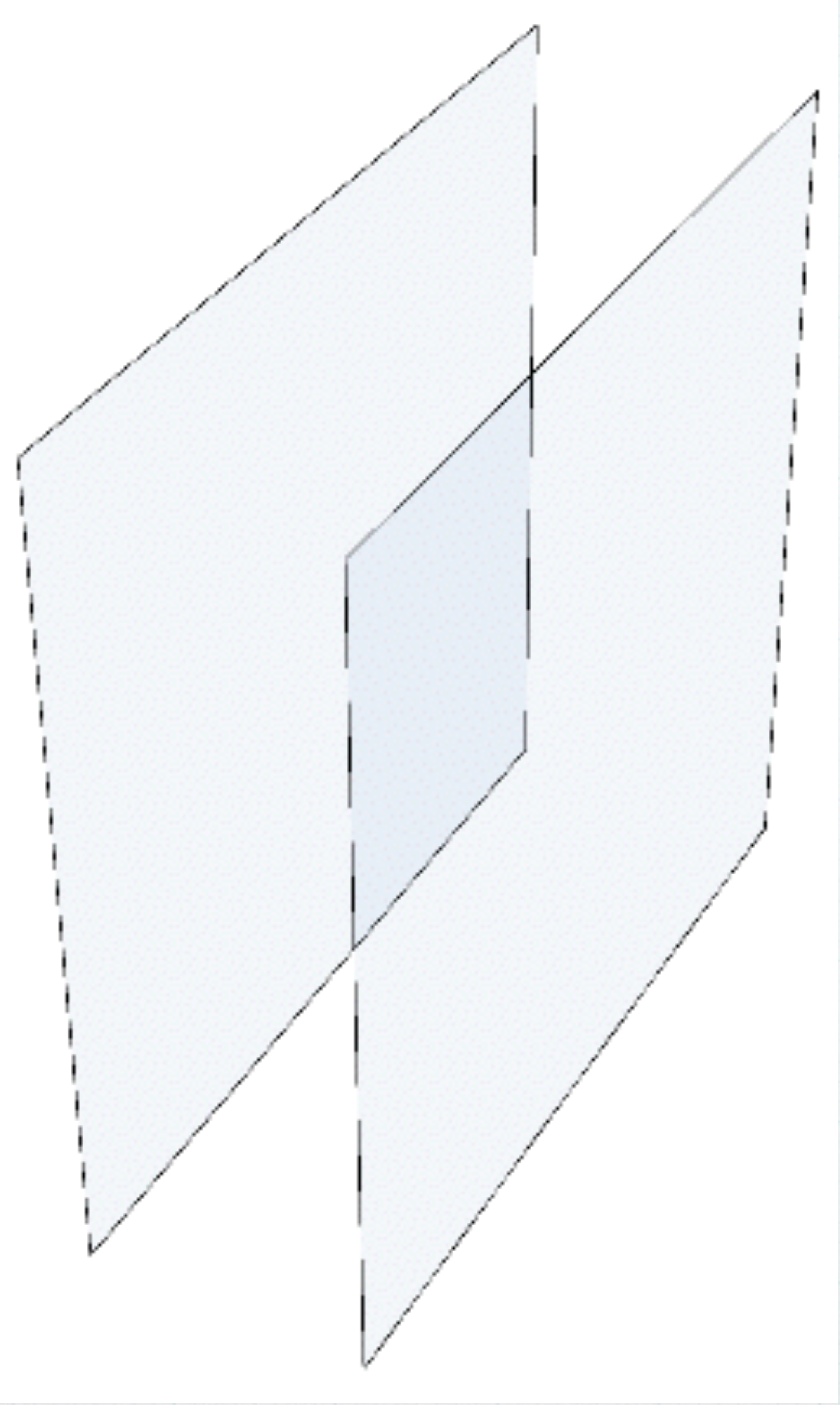
  For now take $d = 4$.

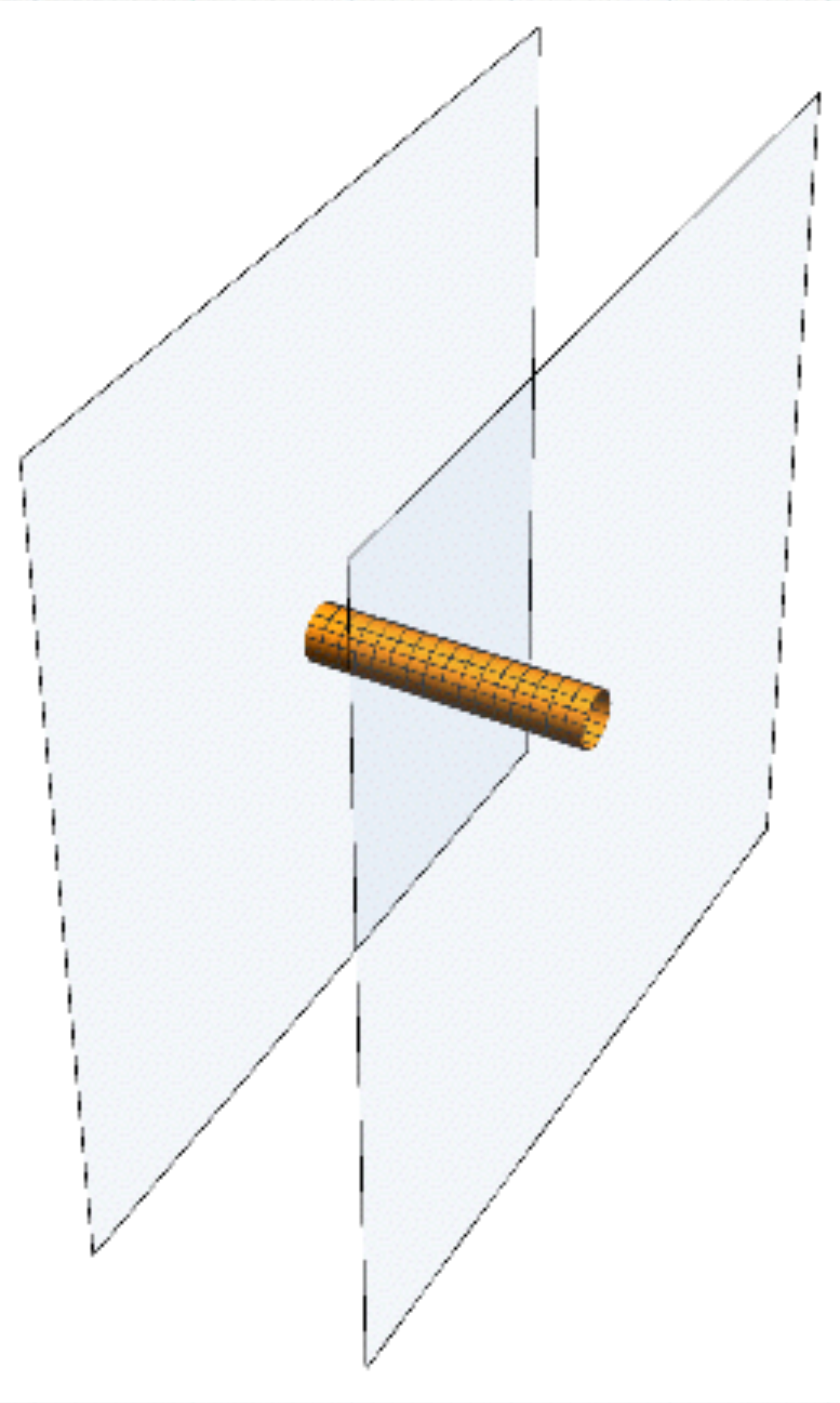
- For large black holes, radius $\gg L$, we expect black holes to behave as 4-d ones; an *exact* solution is the **homogeneous black string** (/uniform black string);

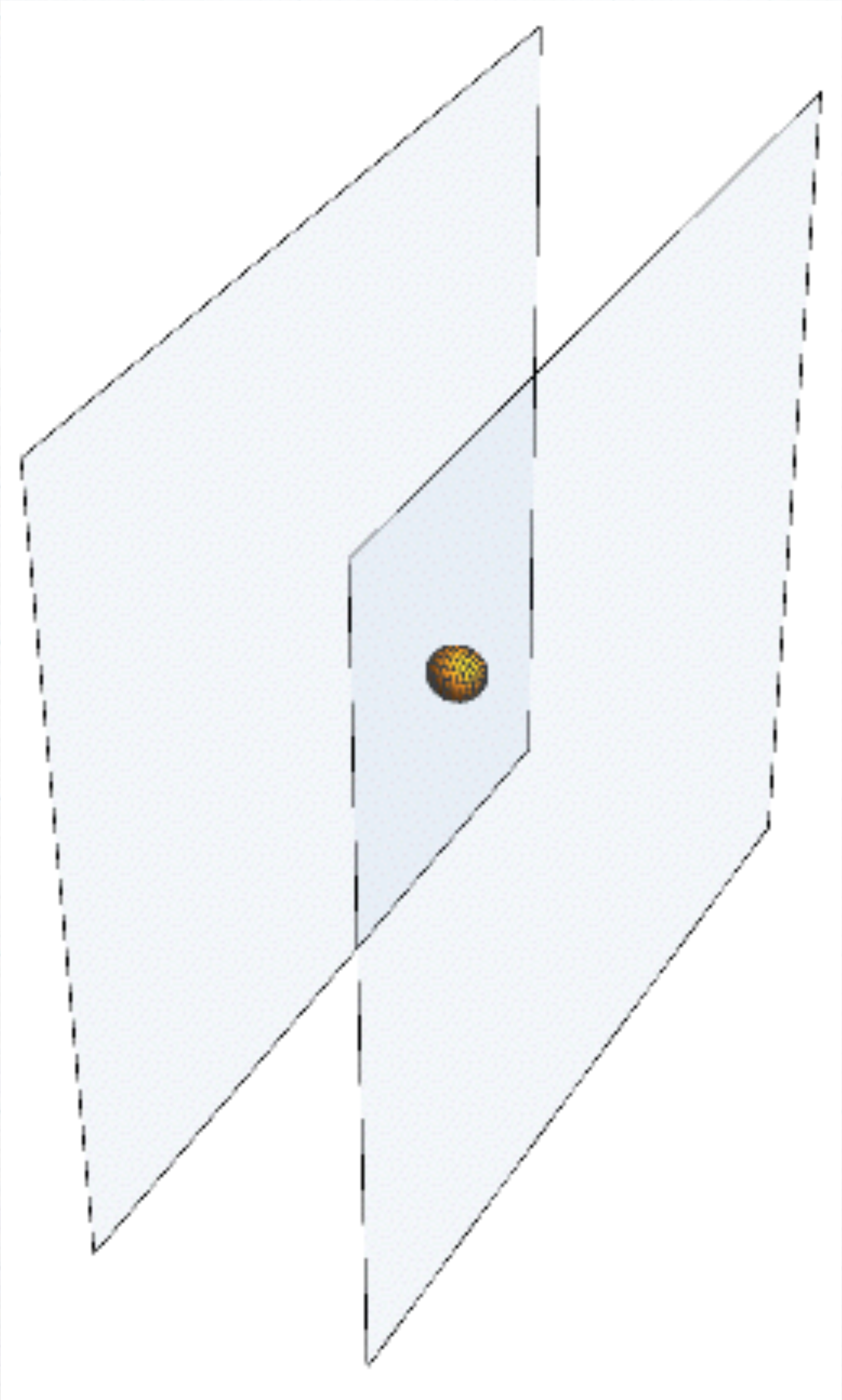
$$ds^2 = -\left(1 - \frac{r_0}{r}\right) dt^2 + \left(1 - \frac{r_0}{r}\right)^{-1} dr^2 + r^2 d\Omega^2_{(2)} + dz^2$$

- For small black holes, radius $\ll L$, we expect black holes to behave as 5-d ones; an *approximate* solution is the **localized black hole**;

$$ds^2 \simeq -\left(1 - \frac{\rho_0^2}{\rho^2}\right) dt^2 + \left(1 - \frac{\rho_0^2}{\rho^2}\right)^{-1} d\rho^2 + \rho^2 d\Omega^2_{(3)}$$







BLACK HOLES IN KALUZA-KLEIN THEORY

- Gregory and Laflamme showed that **homogeneous black strings** become linearly unstable if their radius is small compared to L, ie. $r_0 < 0.876 \frac{L}{2\pi}$.

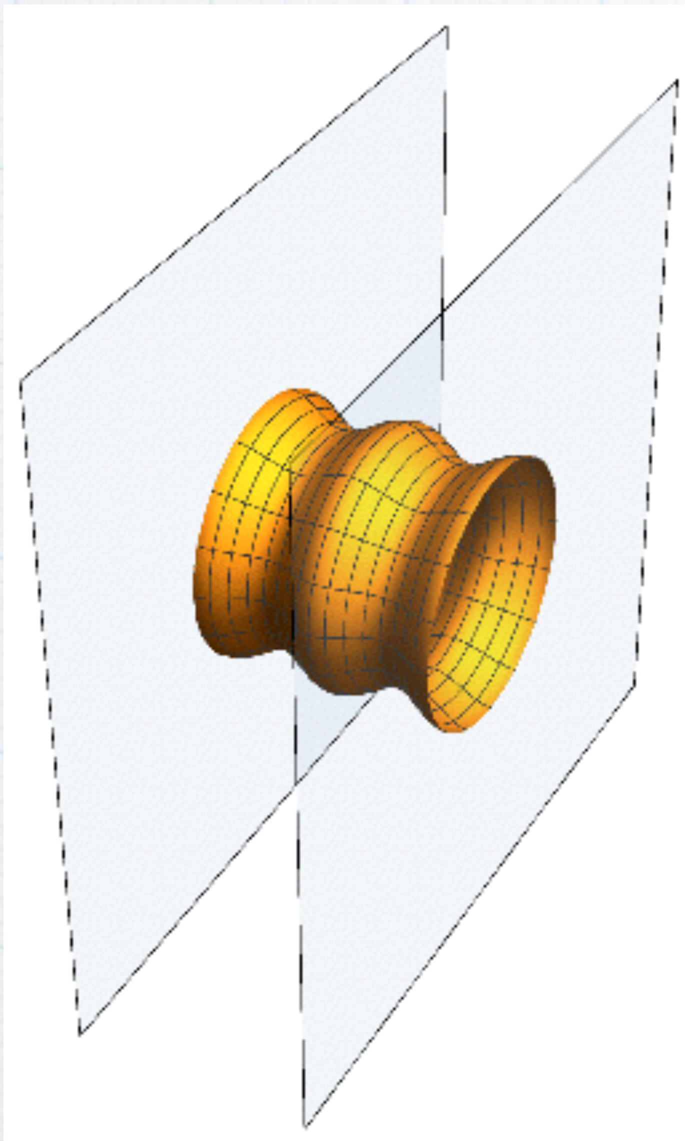
- The metric perturbation takes the form; $\delta g_{\mu\nu}(t, r, z) \sim h_{\mu\nu}(r) e^{\Omega(k)t} \cos(kz)$

  $\Omega(k) > 0$, **real** for all $k\, r_0 < 0.876$

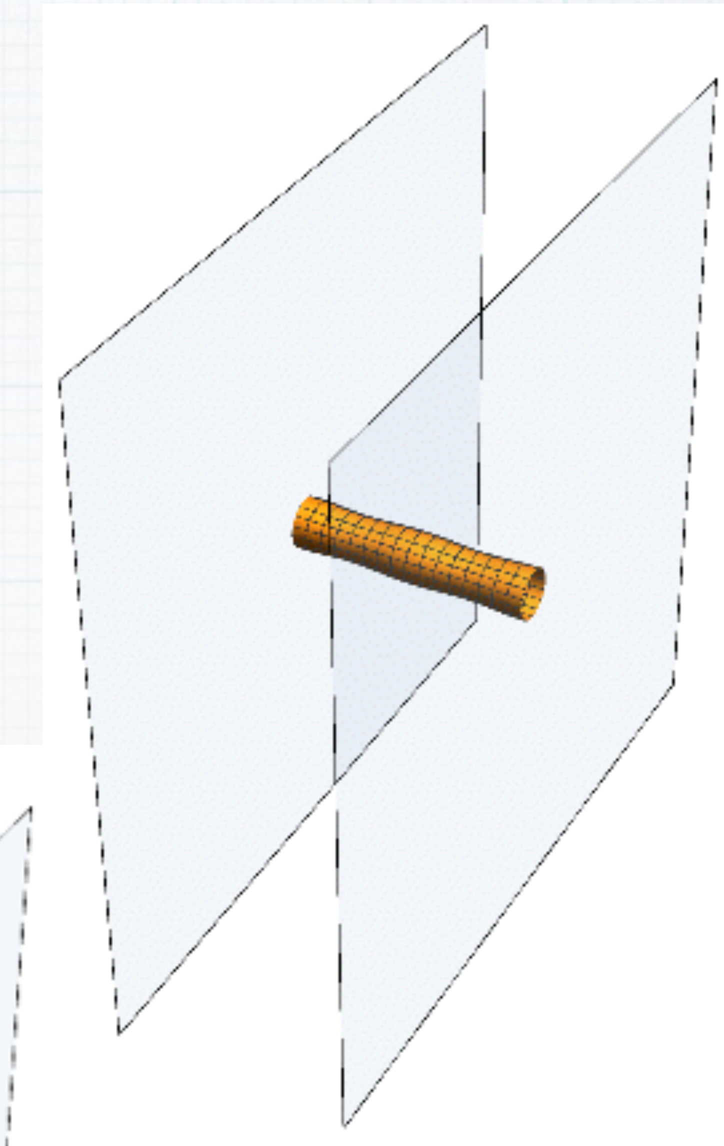
  Hence it is associated to breaking translation invariance on the circle.

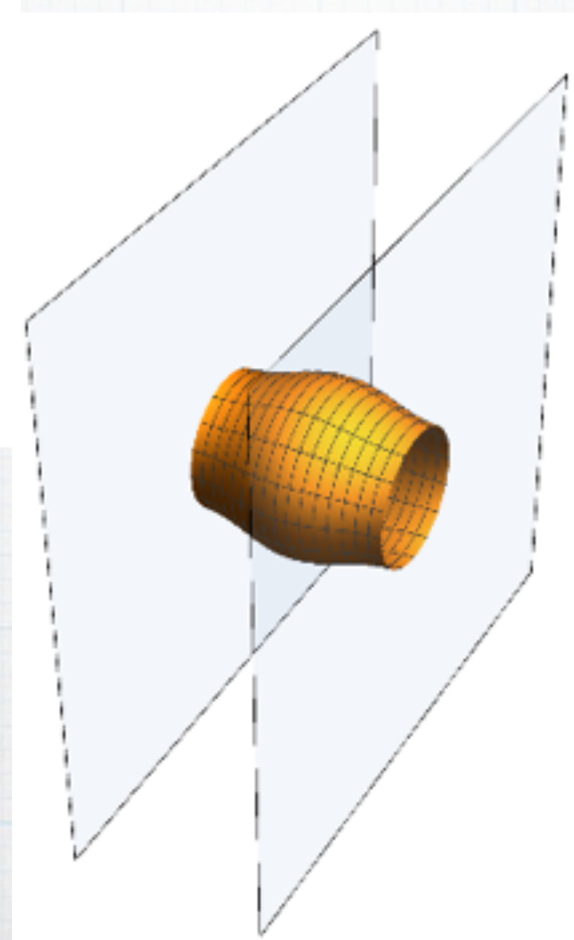
- The onset of the instability implies an exactly marginal (ie. static) linear perturbation – this lifts to the non-linear theory, and generates a new branch of **inhomogeneous black strings** (/non-uniform black string).

Stable



Unstable

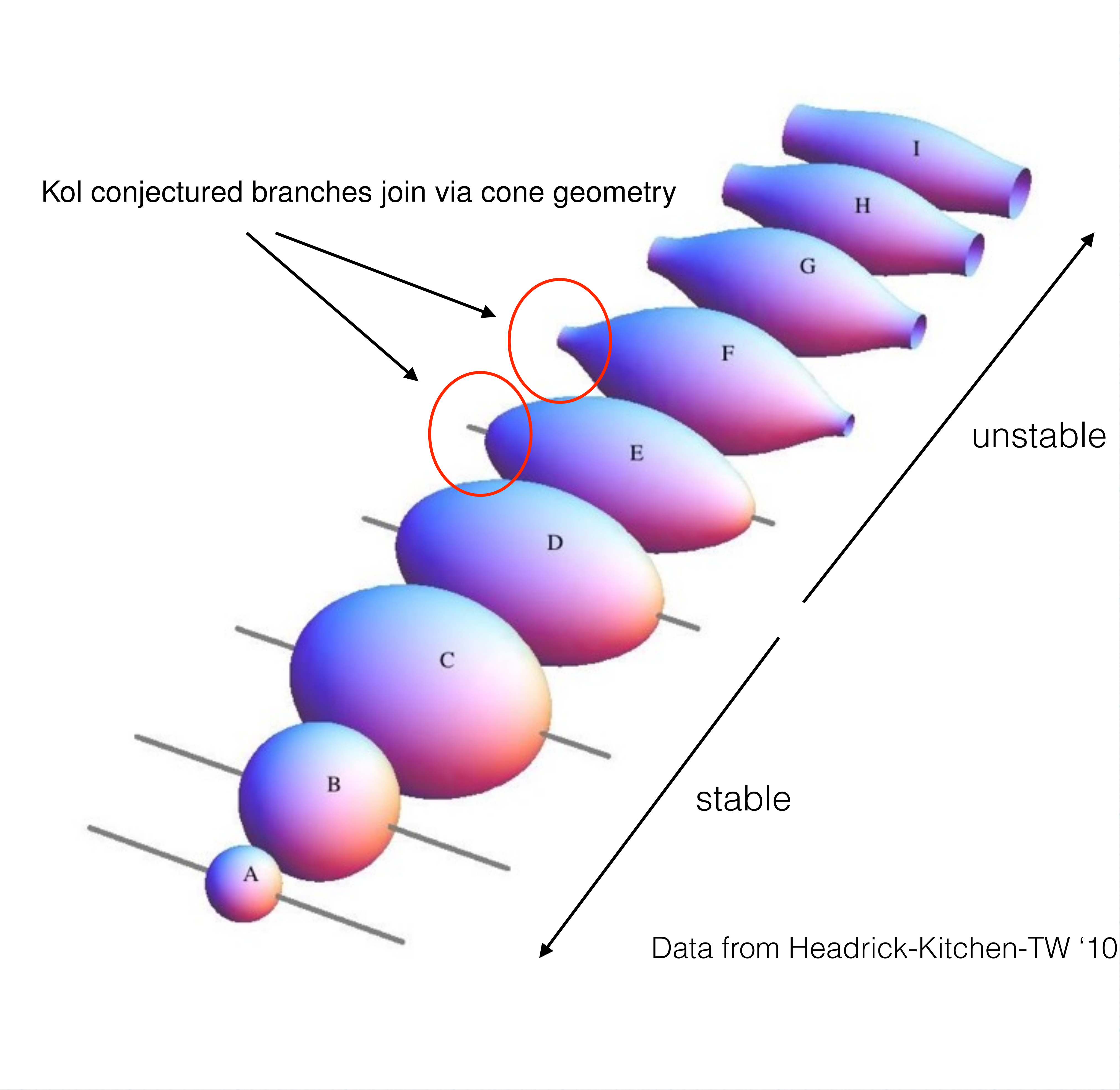




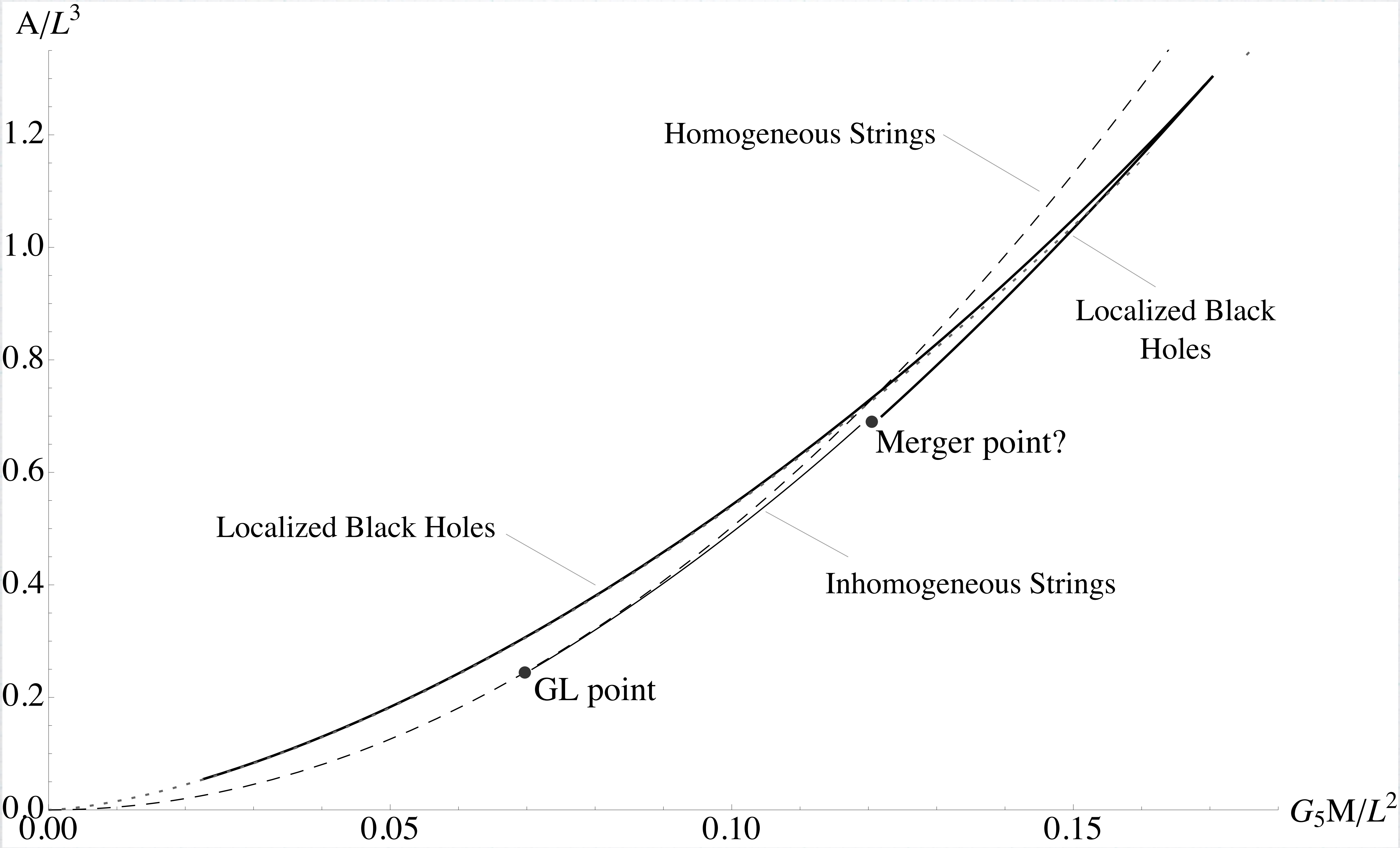
Marginal

BLACK HOLES IN KALUZA-KLEIN THEORY

- Using numerical methods the interplay between these 3 distinct classes of solution has been elucidated;



Kol conjectured branches join via cone geometry
I
H
G
F
E
D
C
B
A
unstable
stable
Data from Headrick-Kitchen-TW ‘10



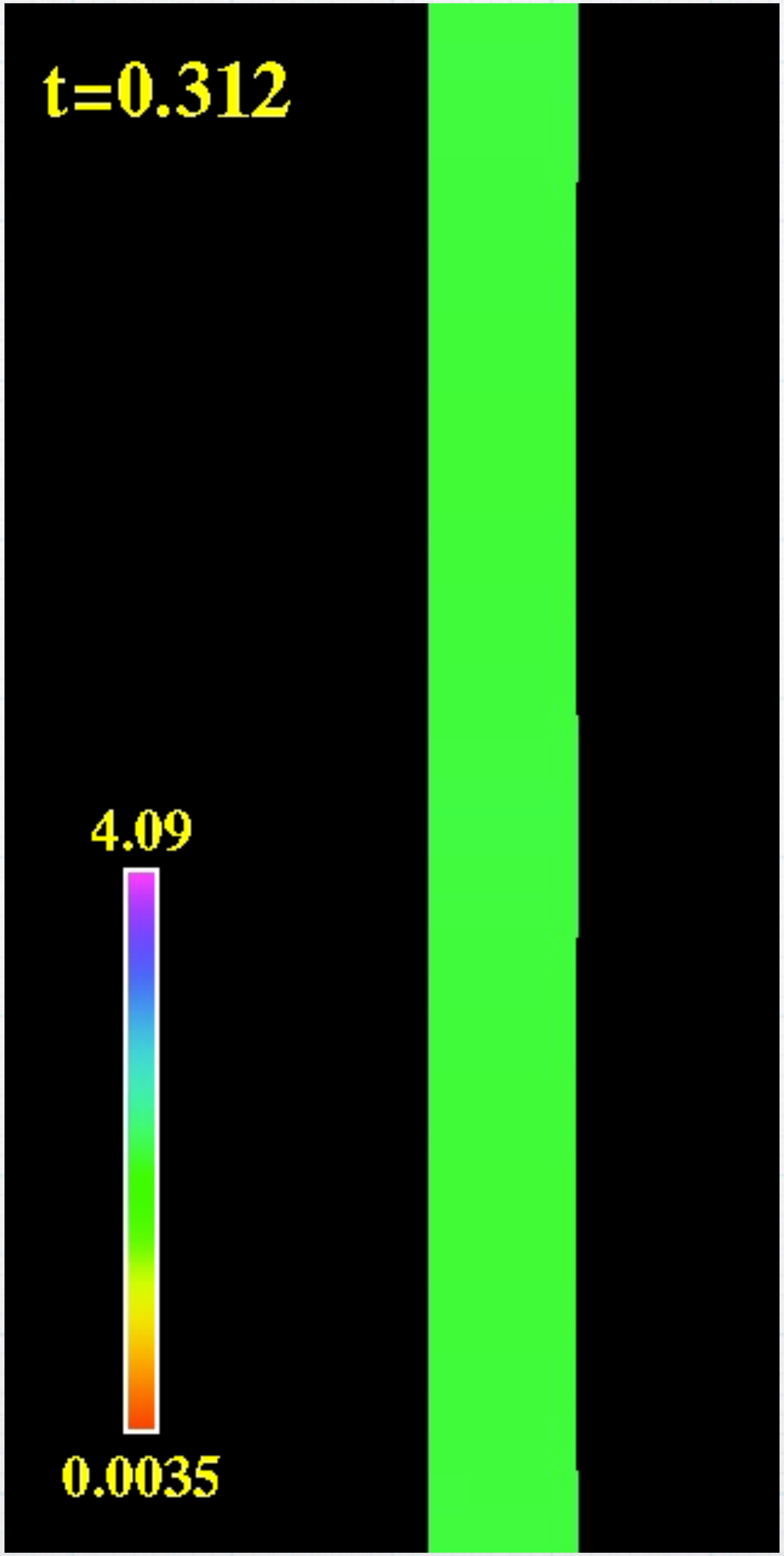
A/L^3
1.2
1.0
0.8
0.6
0.4
0.2
0.0
0.00
0.05
0.10
0.15
G_5M/L^2
Homogeneous Strings
Localized Black Holes
Merger point?
Localized Black Holes
Inhomogeneous Strings
GL point

BLACK HOLES IN KALUZA-KLEIN THEORY

- Since the inhomogeneous black holes have lower area that the unstable uniform ones, they cannot be an endpoint of the GL instability.
- The endpoint is believed to be the localized black holes, but this appears to involve a topology changing dynamics which violated cosmic censorship.

(Also occurs in asymptotically flat settings for black rings – see Pau Figueras's talk)

Lehner, Pretorius '11



BLACK HOLES DUAL TO 1+1 SYM

- Consider 1+1 SYM with coupling g_{YM}. Put this at temperature $T = 1/\beta$.
- At large N we work with $\lambda = N g_{YM}^2$ and define a (dimensionless) coupling; $r_\beta = \beta\sqrt{\lambda}$
- Holography states;

  1+1 SYM is dual to IIB string theory with N units of $D1$ brane charge and appropriate asymptotics (also at temperature T).

- Decoupling limit;

  For $N \to \infty$ with $r_\beta \gg 1$ this is described by supergravity a black hole.

BLACK HOLES DUAL TO 1+1 SYM

- Now in addition put the theory on a circle size L_{YM}.
- We may now define two (dimensionless) couplings;

$$r_L = L_{YM}\sqrt{\lambda} \quad \text{and} \quad r_\beta = \beta\sqrt{\lambda}$$

- The dual spacetime has a spatial direction wrapped on an S^1 of size L_{YM}.
- The decoupling limit again requires; $N \to \infty$ with $r_\beta \gg 1$
- However to be described by IIB gravity we also need $r_\beta \ll r_L^2$

  This ensures strings winding the circle are heavy.
- However (via T-duality) there is another (partially overlapping) different IIA gravity description if $r_L \ll r_\beta$...

BLACK HOLES DUAL TO 1+1 SYM

- IIA gravity description;

  For $N \to \infty$ with $r_\beta \gg 1$ with $r_L \ll r_\beta$ we have 10-d supergravity black holes.

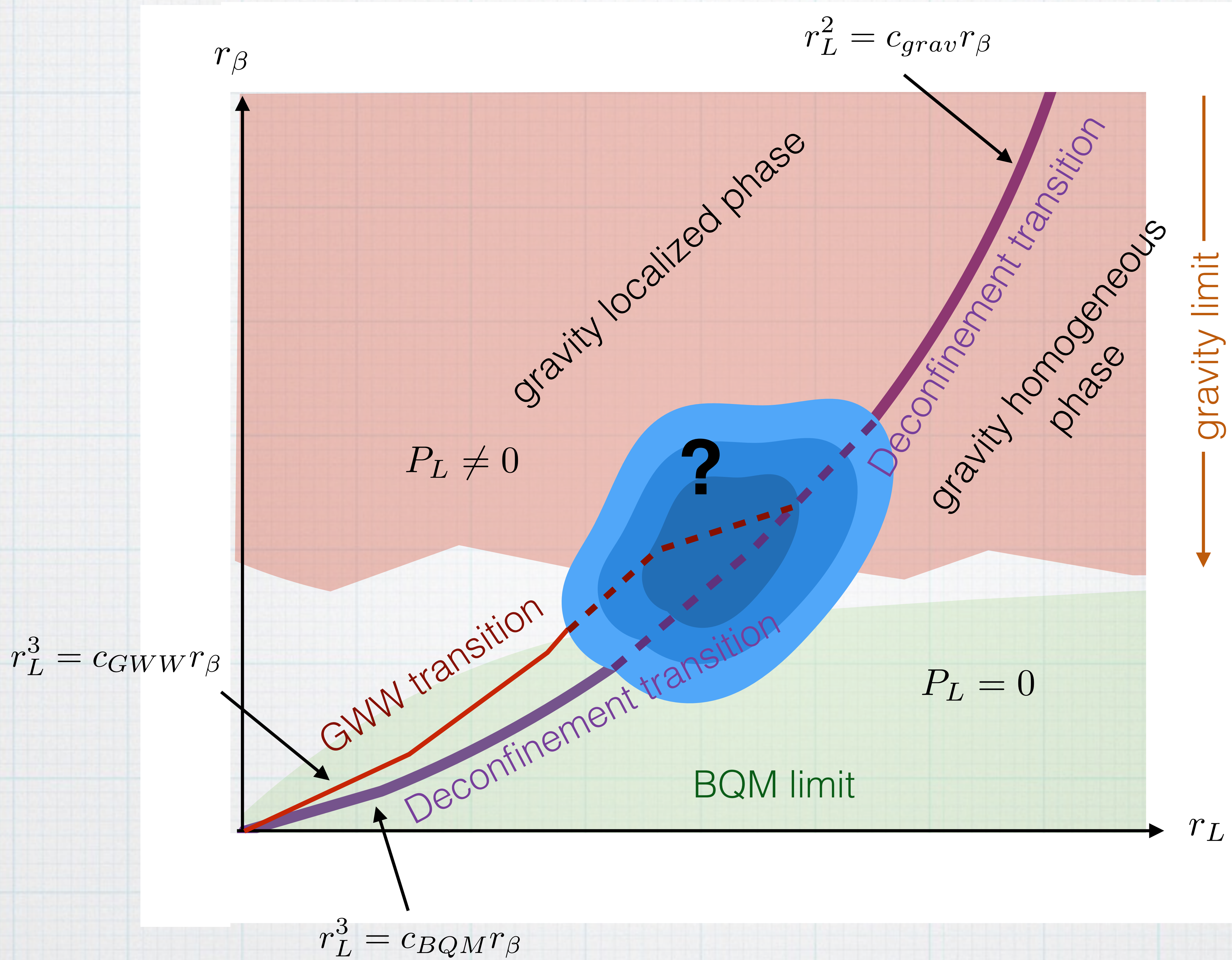
- Remarkably by a simple solution generating technique in fact these black holes are given by (9+1)-dimensional Kaluza-Klein black holes with a circle $L \sim \frac{\alpha'}{L_{YM}}$.

- The KK black hole solutions now are transformed to near extremal black holes, with good thermodynamics.

  There is a 'simple' map between the thermodynamics of the KK theory and SYM.

BLACK HOLES DUAL TO 1+1 SYM

- The GL instability indicates a **large** N 1st order phase transition between the localized and homogeneous phases for $r_L^2 \sim r_\beta$.
- The interpretation; a stringy winding mode in the original IIB theory has become light, and in this IIA can be seen as a supergravity d.o.f.
- Order parameter; the eigenvalue distribution of the spatial Wilson loop
- The large L_{YM} behaviour is governed by the homogeneous black string – this is T-duality invariant, and exists in both the IIA and IIB descriptions.
- However as L_{YM} gets smaller, then L gets bigger leading to the black holes to localization on the circle.
- This accounts for the transition to 0+1 SYM behaviour as $L_{YM} \to 0$.



r_β
$r_L^2 = c_{grav} r_\beta$
gravity localized phase
Deconfinement transition
gravity homogeneous phase
gravity limit
$P_L \neq 0$
?
GWW transition
Deconfinement transition
$P_L = 0$
$r_L^3 = c_{GWW} r_\beta$
BQM limit
r_L
$r_L^3 = c_{BQM} r_\beta$

RECENT DEVELOPMENTS - GRAVITY

- Some time ago Catterall, Joseph and I simulated 1+1 SYM for $N = 3, 4$.

  Interestingly at that time we knew from gravity the transition was 1st order but only partially had the gravitational solutions.

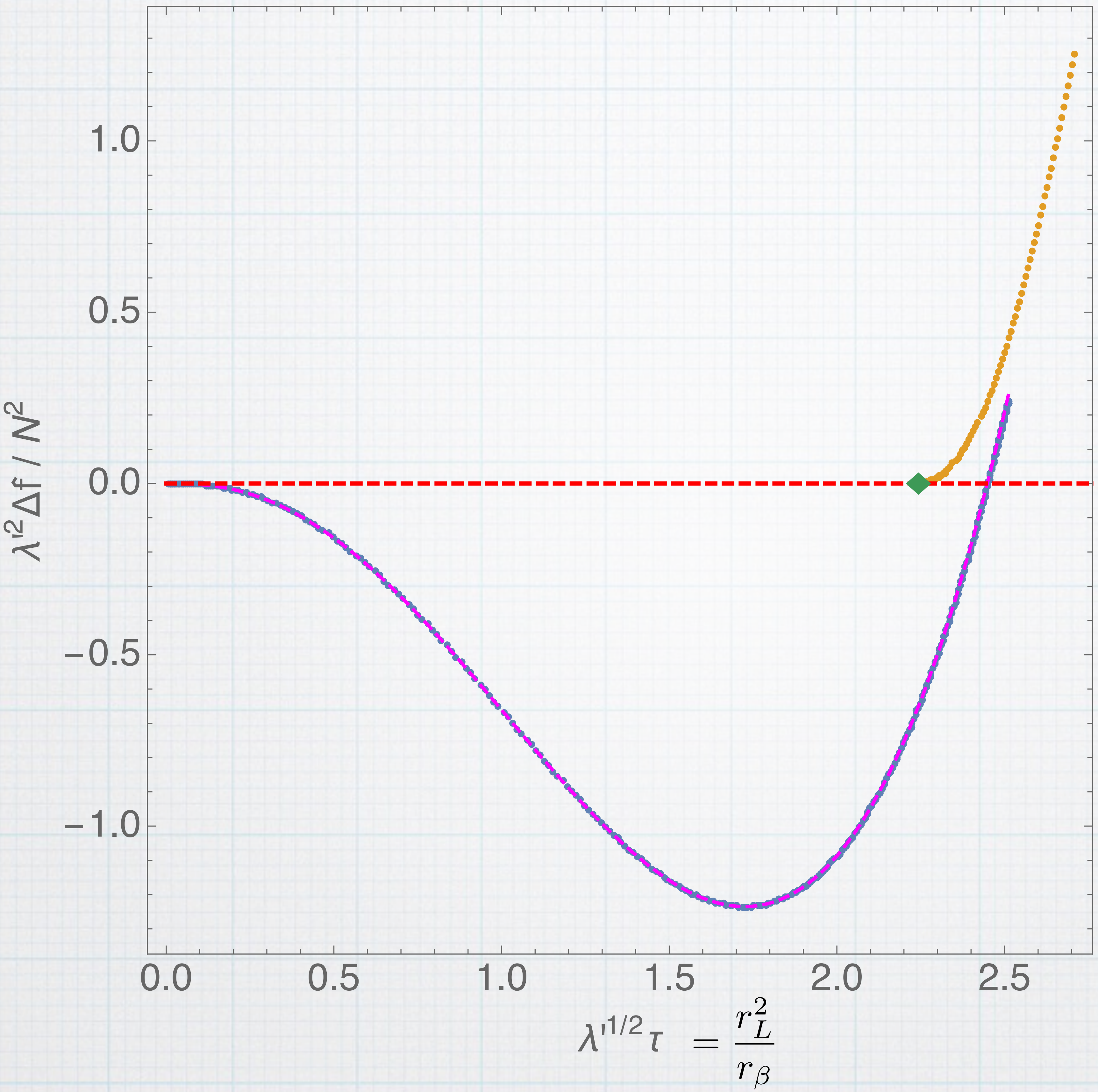
  The lattice simulations were too crude to really see the large N phase transition.

- The 9+1 KK inhomogeneous black strings and localized black holes were finally constructed in [Figueras, Murata, Reall '12 ; Dias, Santos, Way '17]

  Predicted transition; $r_L^2 = 2.45 r_\beta$ in gravity regime $r_\beta \gg 1$.

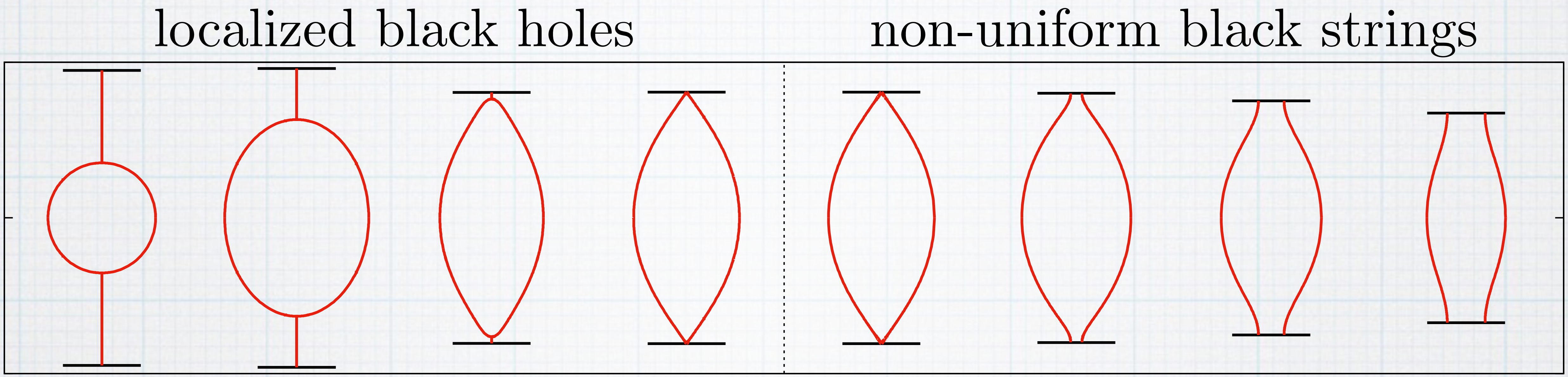
- Beautiful work by Kalisch et al confirmed a conjecture by Kol that there is critical behaviour near the topology transition; can this be seen on the lattice?

Gravity prediction

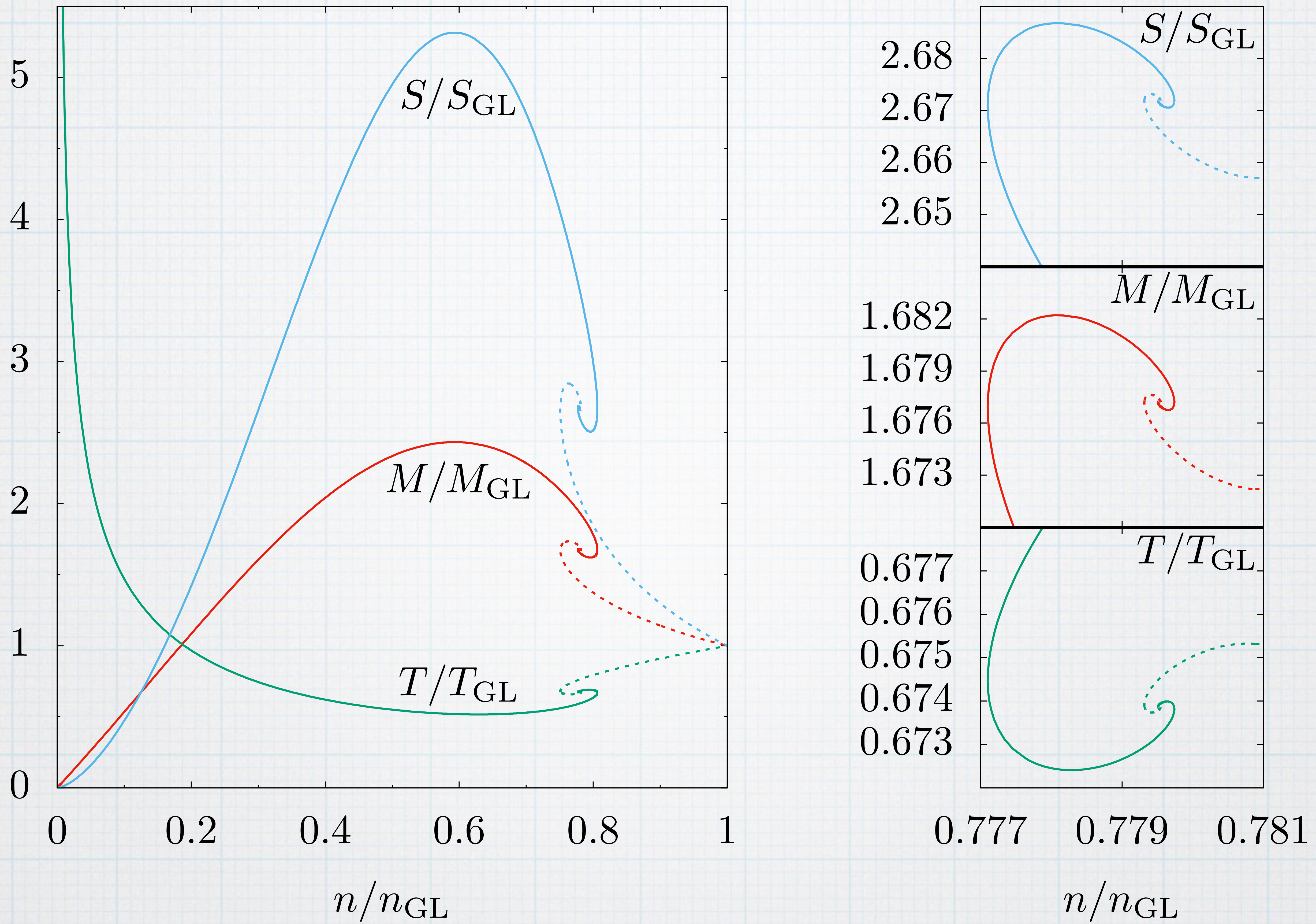


Dias, Santos, Way ’17

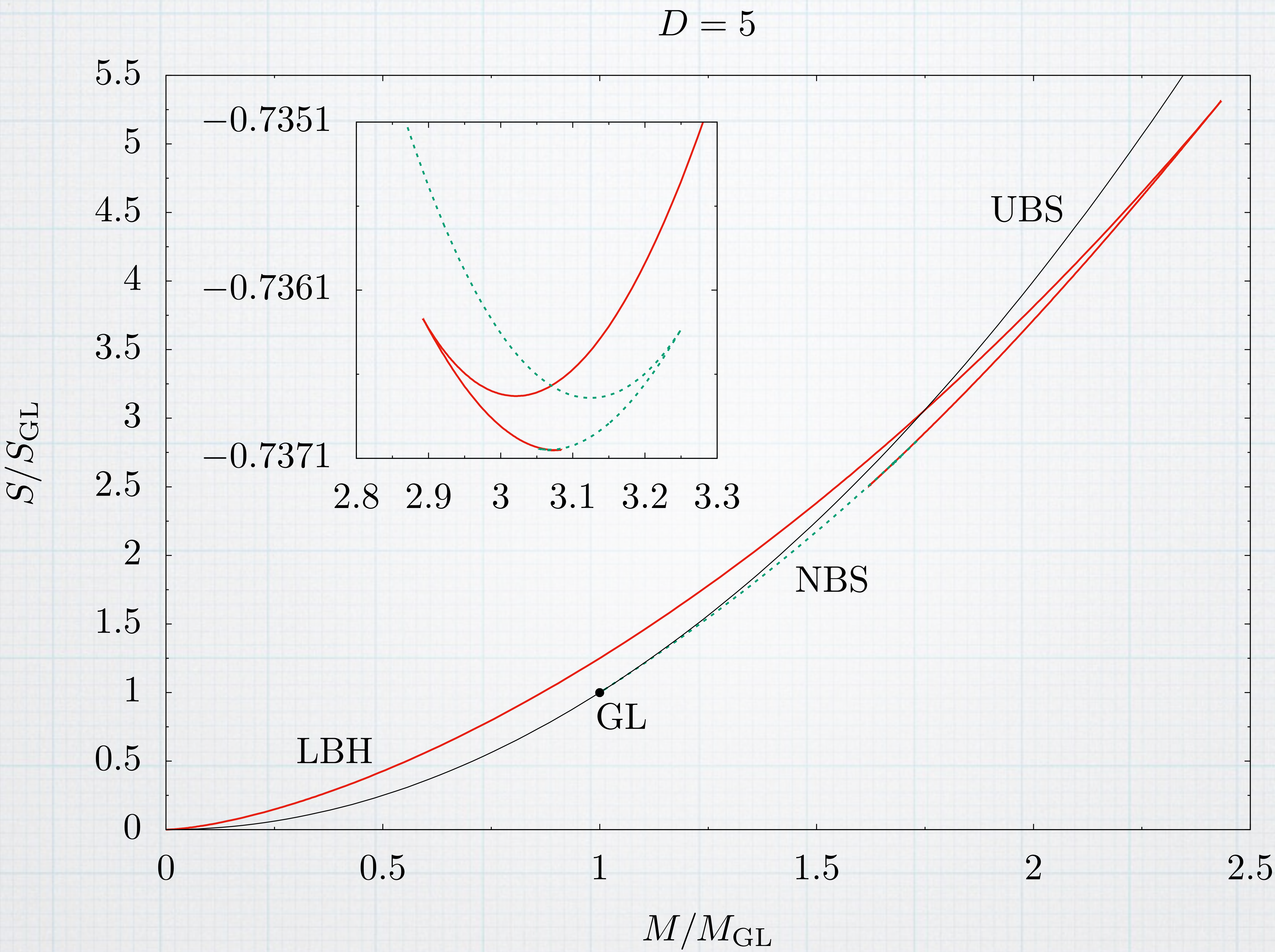
$$D = 5$$



Kalisch, Ansorg ’15, ’16; Kalisch, Mockel, Ammon ‘17



Kalisch, Ansorg '15, '16; Kalisch, Mockel, Ammon '17



Kalisch, Ansorg ’15, ’16; Kalisch, Mockel, Ammon ‘17

RECENT DEVELOPMENTS - 1+1 SYM

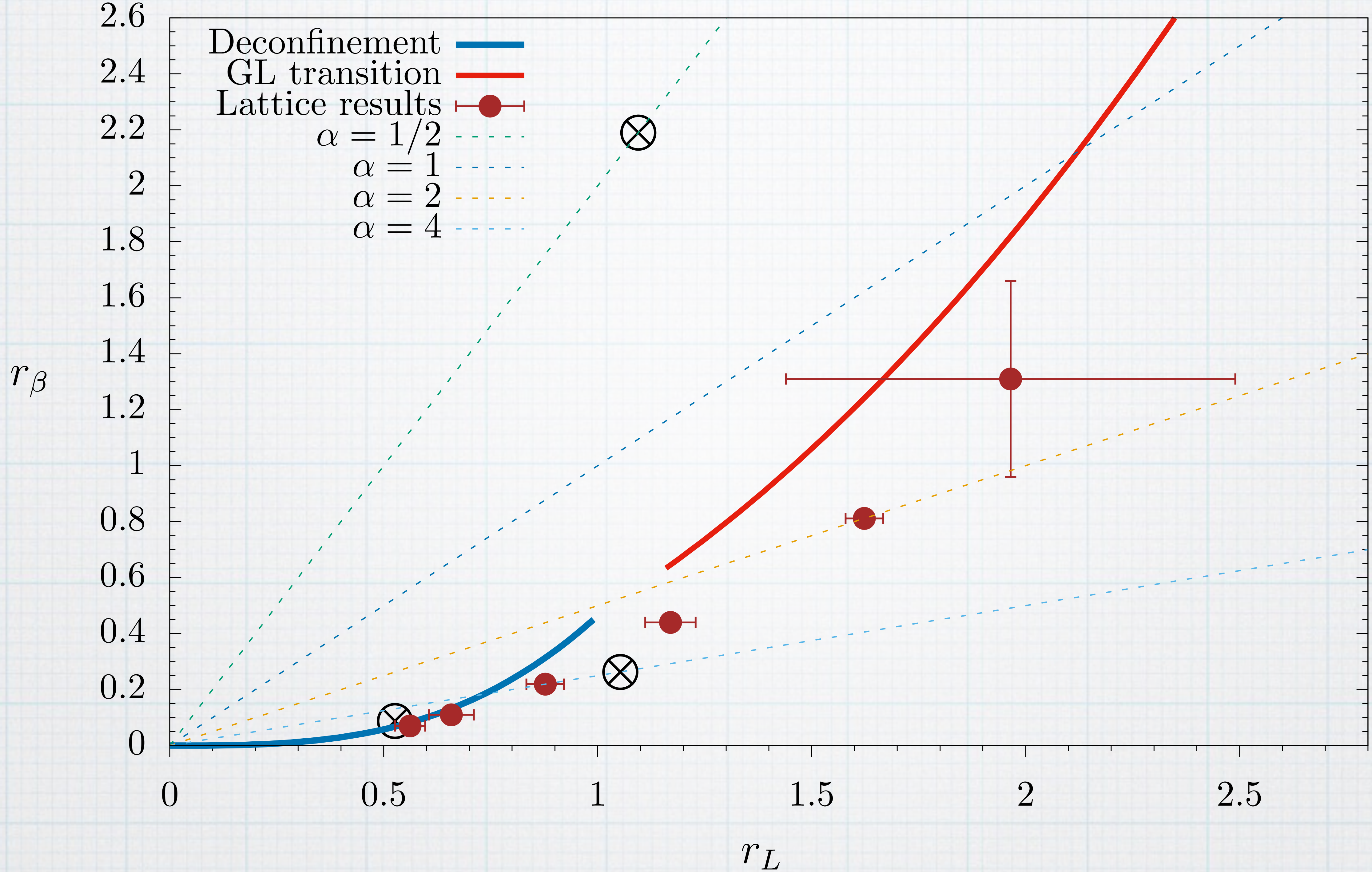
- Quenched study of 1+1 SYM, [Hanada, Romatschke '16]

- Recently Catterall, Jha, Schaich and I had another go, [Catterall, Jha, Schaich, TW '17]

  This used a reduction of a 3+1 code, and a supersymmetric lattice construction due to [Kaplan, Unsal '05]

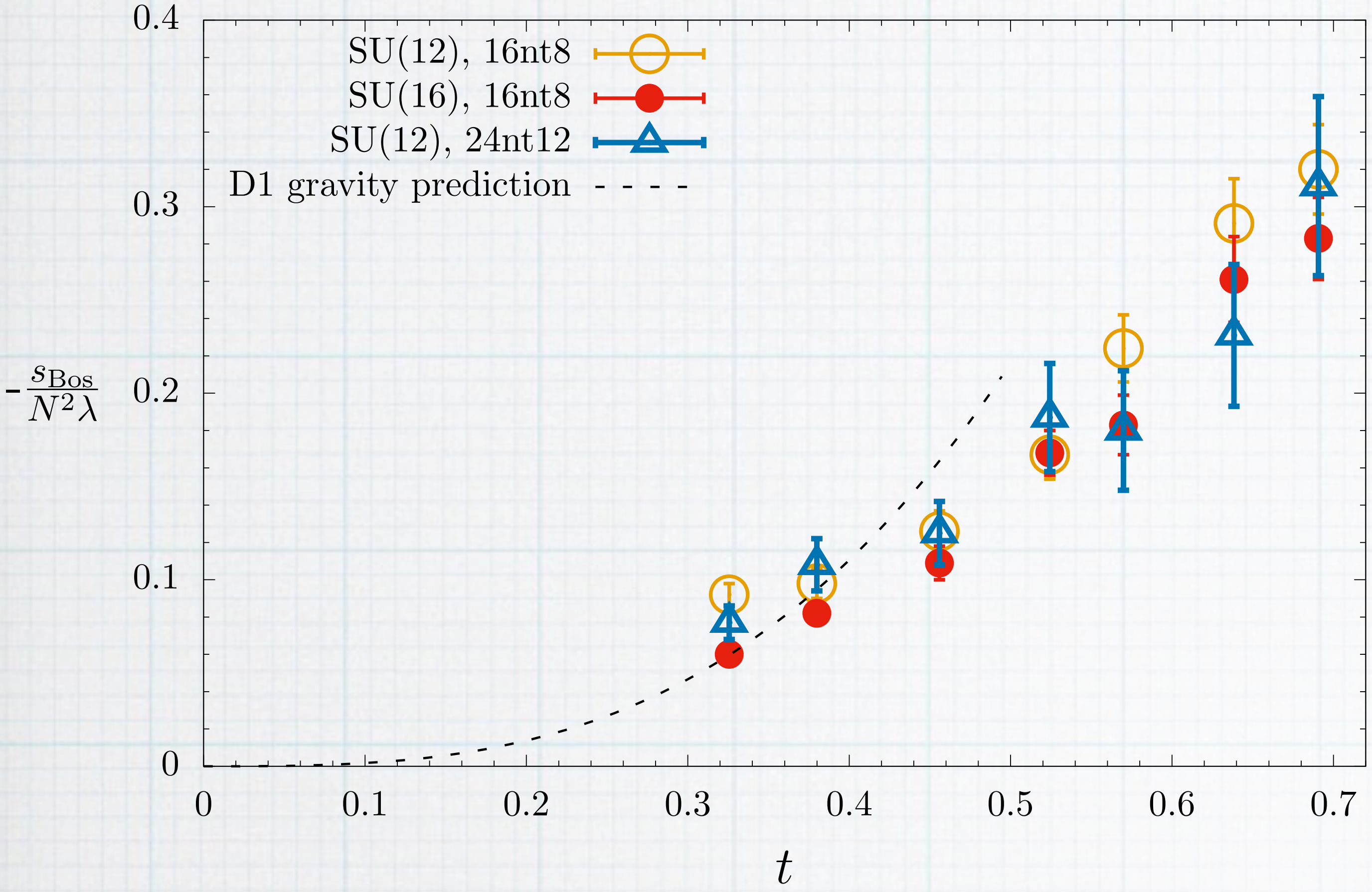
  In fact the Euclidean simulations are on a 'skewed' torus, but nonetheless the same phenomena occur, and in the gravity limit, it is again described by these KK black holes.

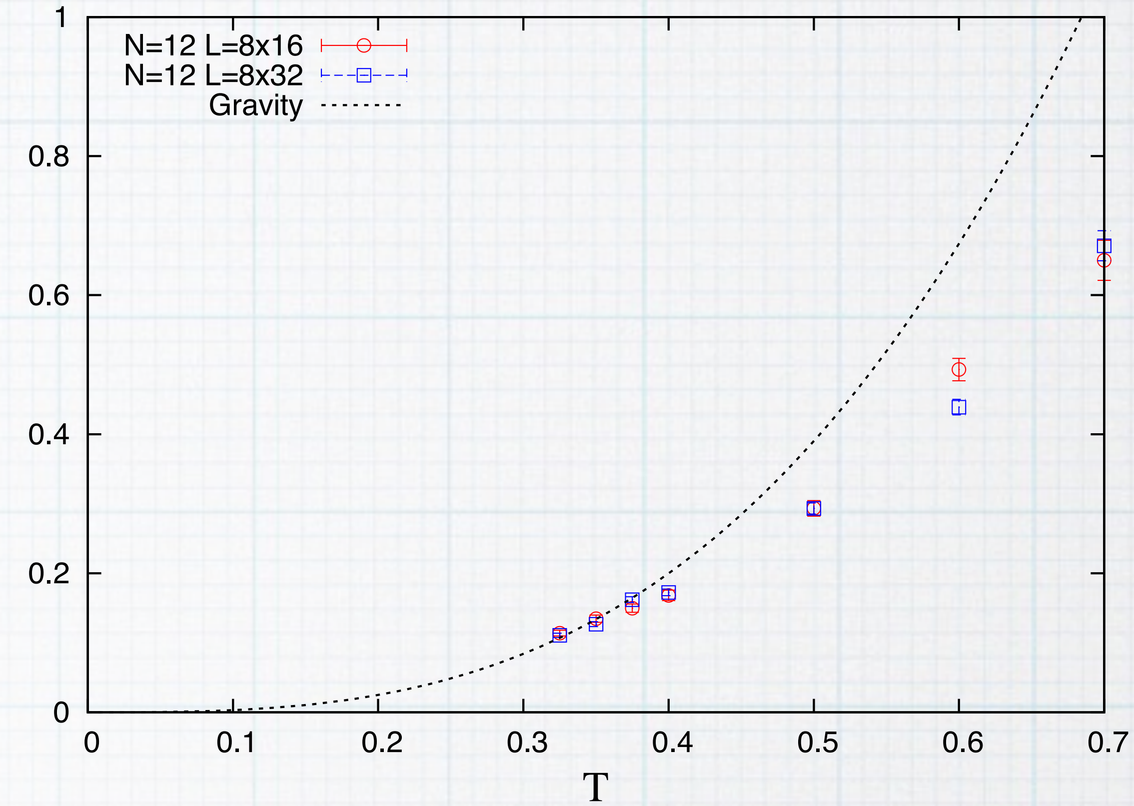
- Also [Kadoh, Giguere 17] reported results (not on a circle) in a conference proceeding.

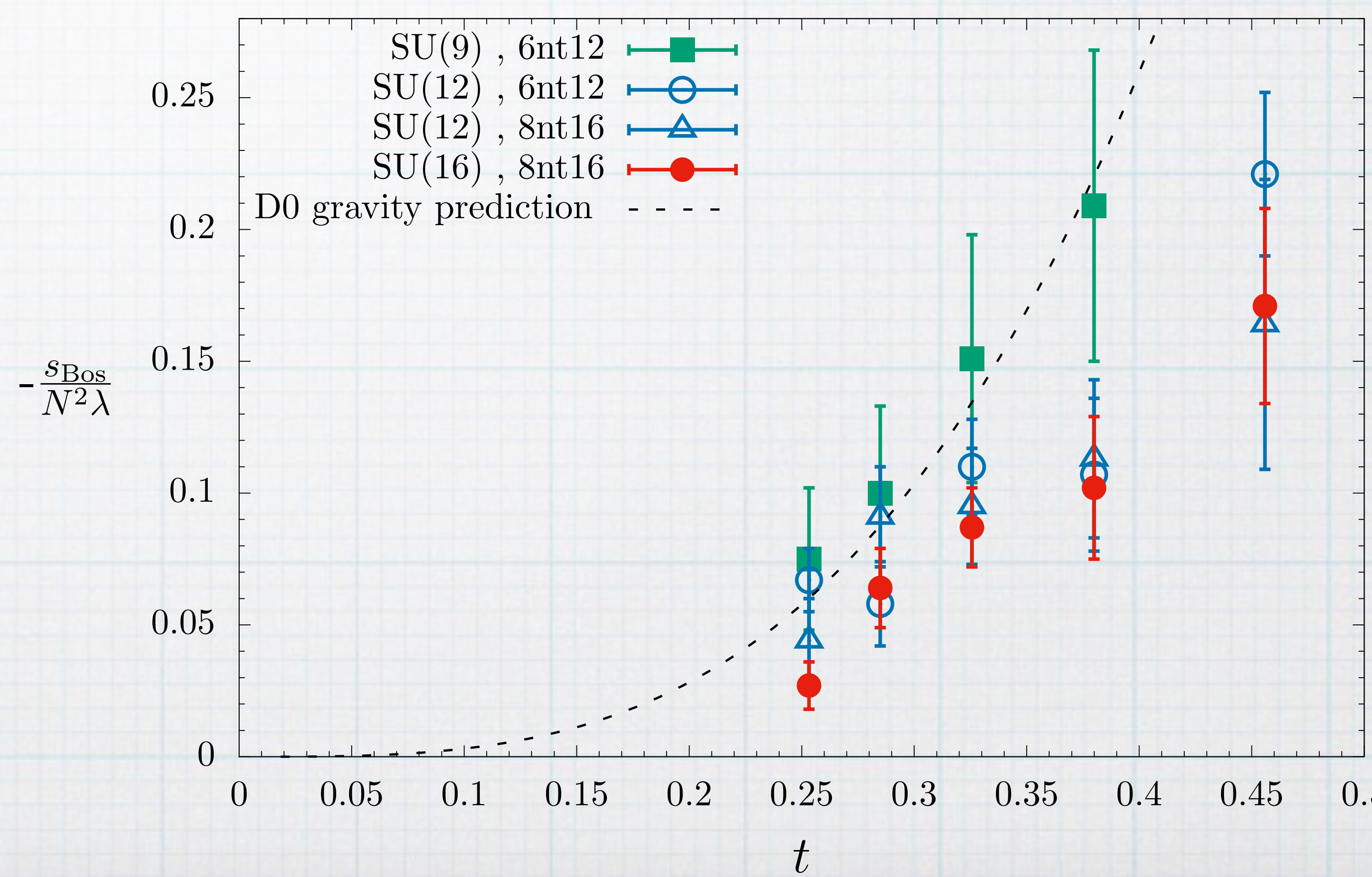


Catterall, Jha, Schaich, TW ’17

Kadoh + Giguere '17







Catterall, Jha, Schaich, TW '17

ADDING STRING CHARGE IN KALUZA-KLEIN THEORY

- Now consider again 5-d KK theory; but consider black holes carrying *string*-like charge, electrically charged under a 3 form field strength $F_{(3)}$.

$$L = \int d^5x \sqrt{g} \left(R - F_{(3)} \wedge \star F_{(3)} \right)$$

- Consider entropy of fixed L, mass M and charge q ensemble.
- Homogeneous string;

$$ds^2 = -\left(1 - \frac{r_0}{r}\right) dt^2 + \frac{dr^2}{\left(1 - \frac{r_0}{r}\right)\left(1 - \frac{q^2}{3r_0 r}\right)} + r^2 d\Omega^2_{(2)} + \left(1 - \frac{q^2}{3r_0 r}\right) dz^2$$

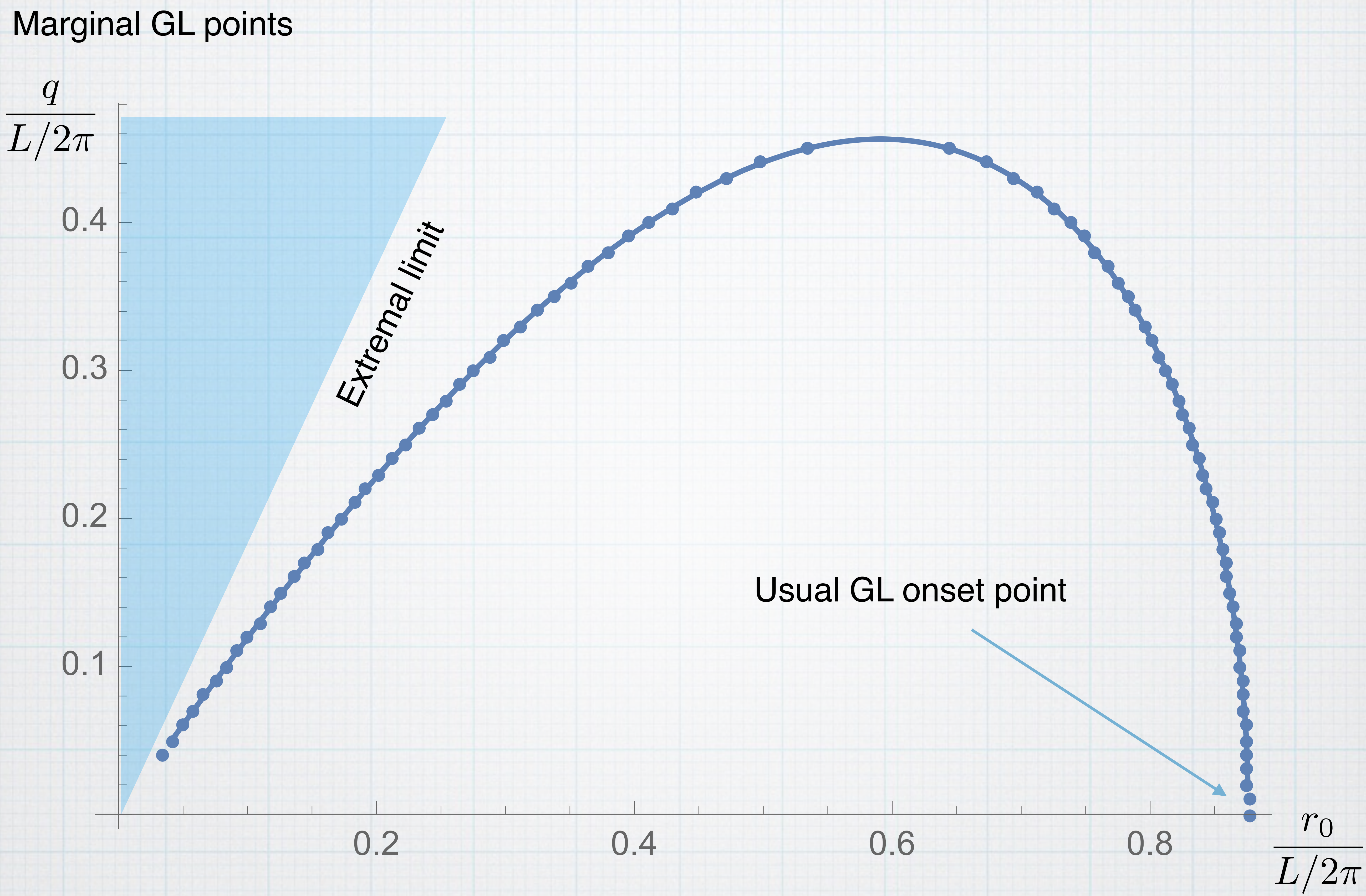
$$\star F_{(3)} = q\, \Omega_{(2)}$$

- For fixed L, q we have solutions for $r_0 \geq q/\sqrt{3}$

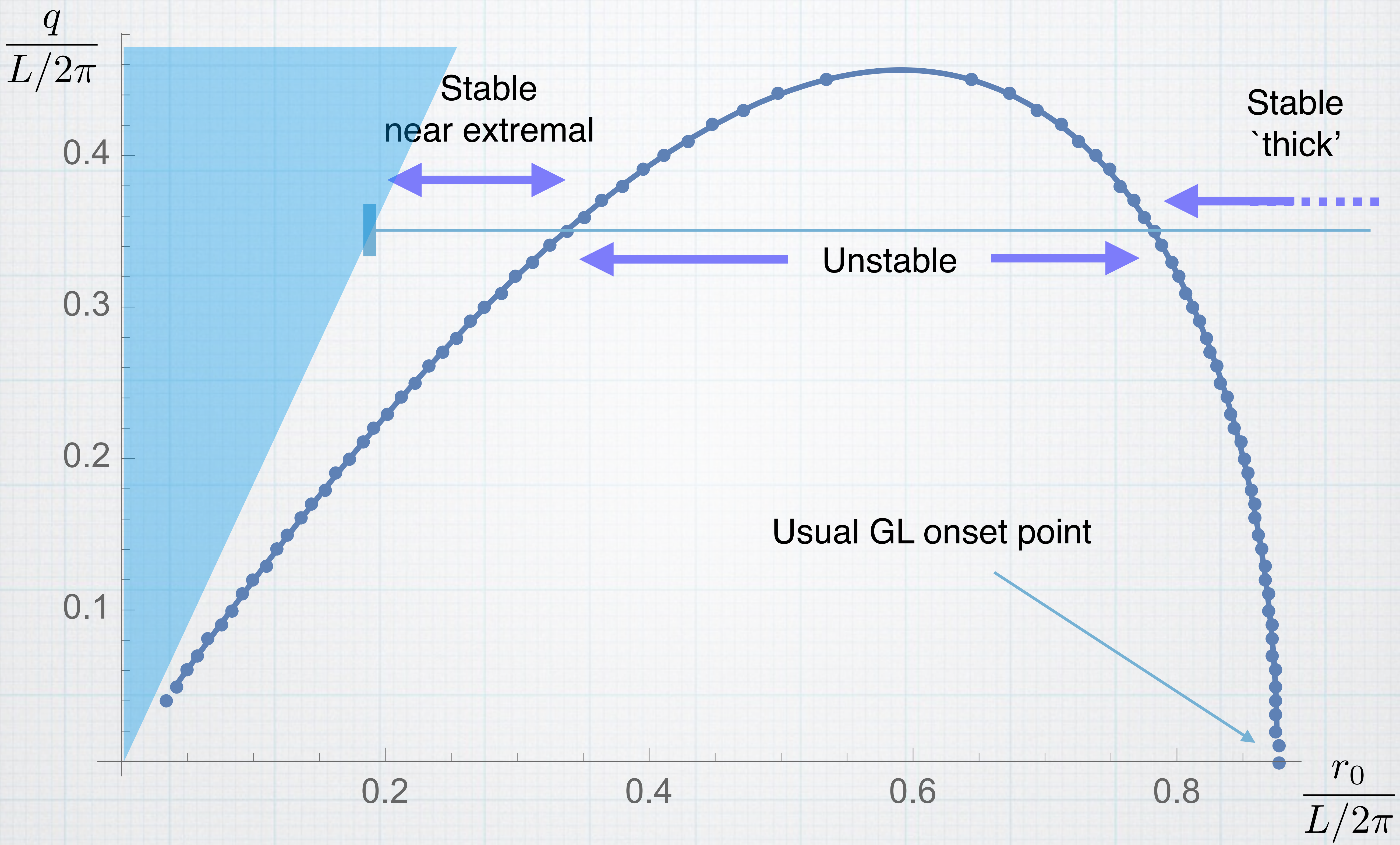
ADDING STRING CHARGE IN KALUZA-KLEIN THEORY

- BUT there cannot be localized solutions! They cannot carry the charge.
- What about the Gregory Laflamme instability? And where does it go?
- The strings are stabilized by the charge for $q > q_{max} = 0.457\frac{L}{2\pi}$ up to their extremal limit.
- But for $0 \leq q < q_{max}$ we find **two** marginal points with instability for $r_{GL_1} < r_0 < r_{GL_2}$.

  Thus ‘thick’ and near extremal black strings are stable.


Marginal GL points
$\frac{q}{L/2\pi}$
0.4
0.3
0.2
0.1
Extremal limit
Usual GL onset point
0.2
0.4
0.6
0.8
$\frac{r_0}{L/2\pi}$

Marginal GL points



ADDING STRING CHARGE IN KALUZA-KLEIN THEORY

- Inhomogeneous black strings emerge from the marginal points; [Kudoh, Miyamoto '06] (for $D \geq 6$)

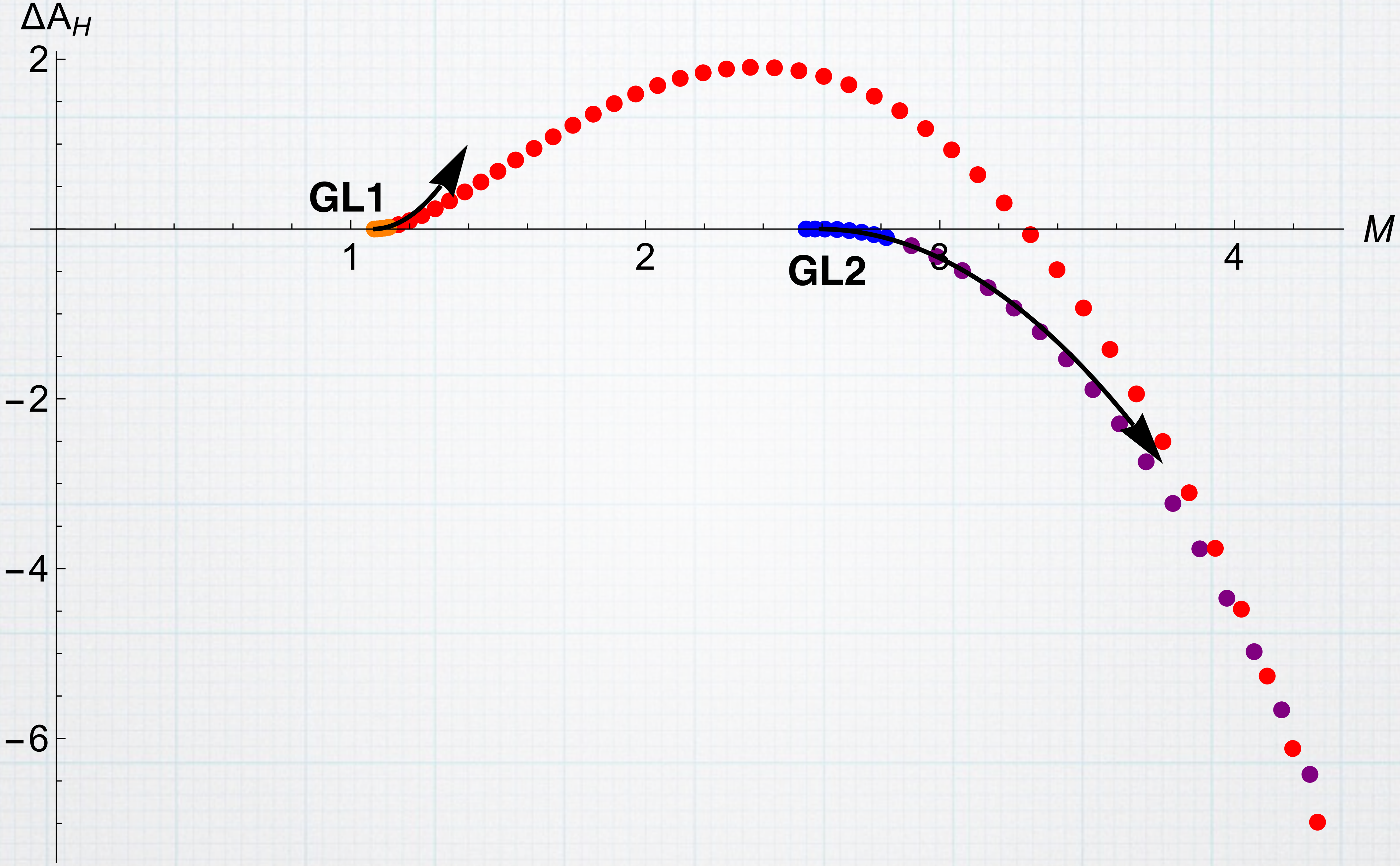
  Their motivation was to show the charge could stabilize inhomogeneous black strings.

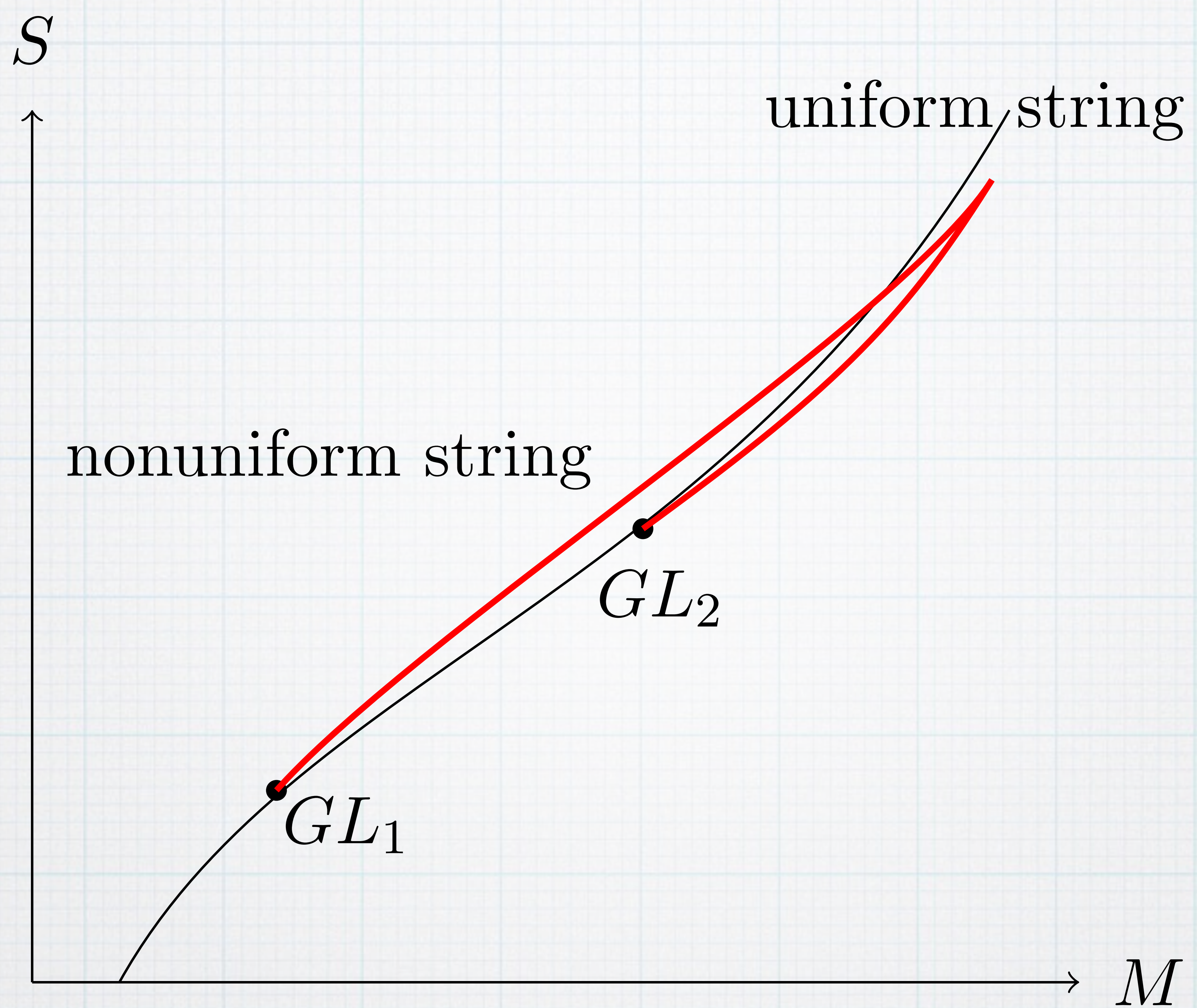
- The picture we find (in $D = 5$) is that these inhomogeneous extend from r_{GL_1} to r_{GL_2}.

- Various behaviours are possible – we have computed these using Gubser's perturbative approach and also using a non-linear pde numerical approach.

[work with Krai Cheamsawat]

$$\frac{q}{L/2\pi} = 0.31$$

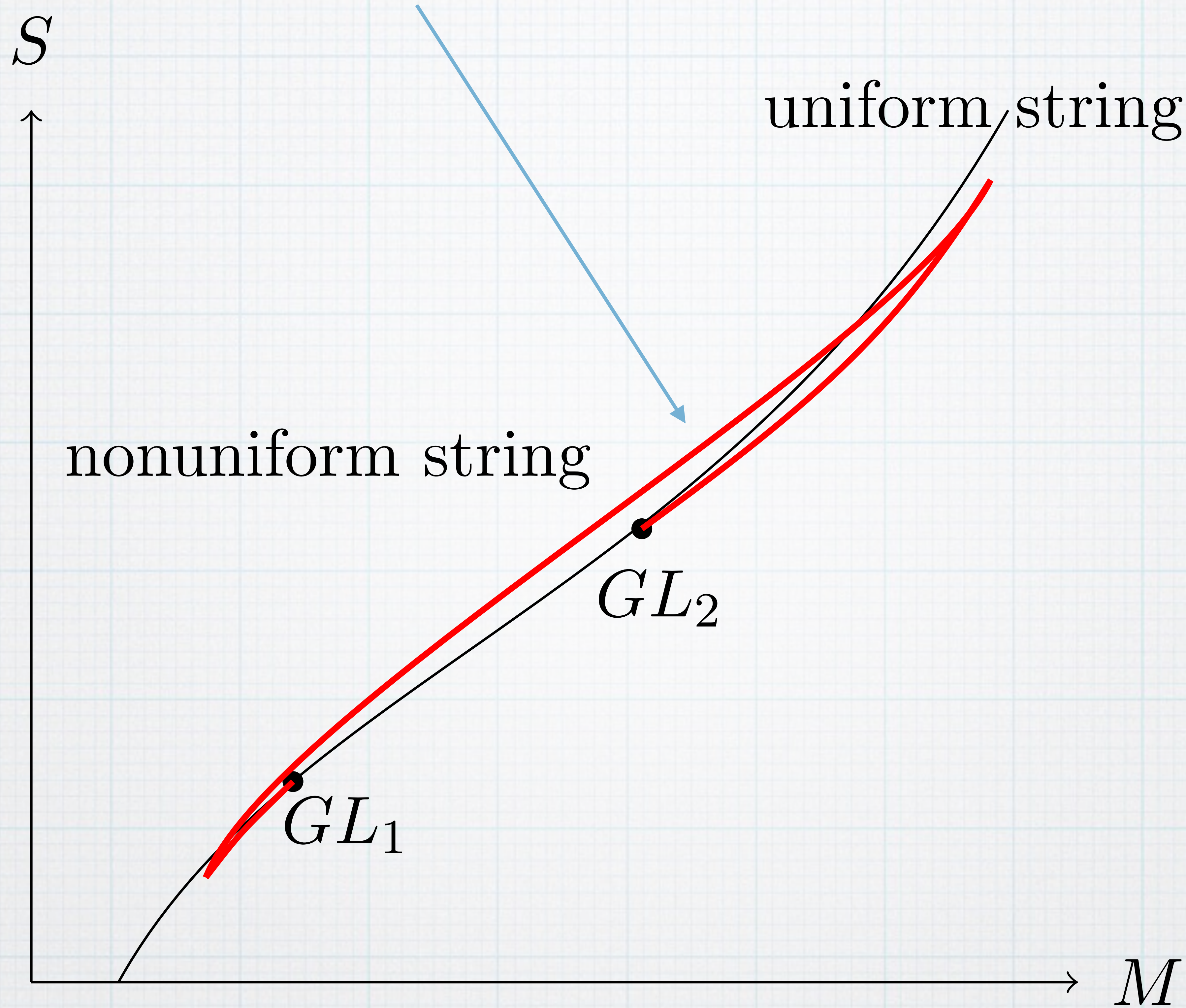




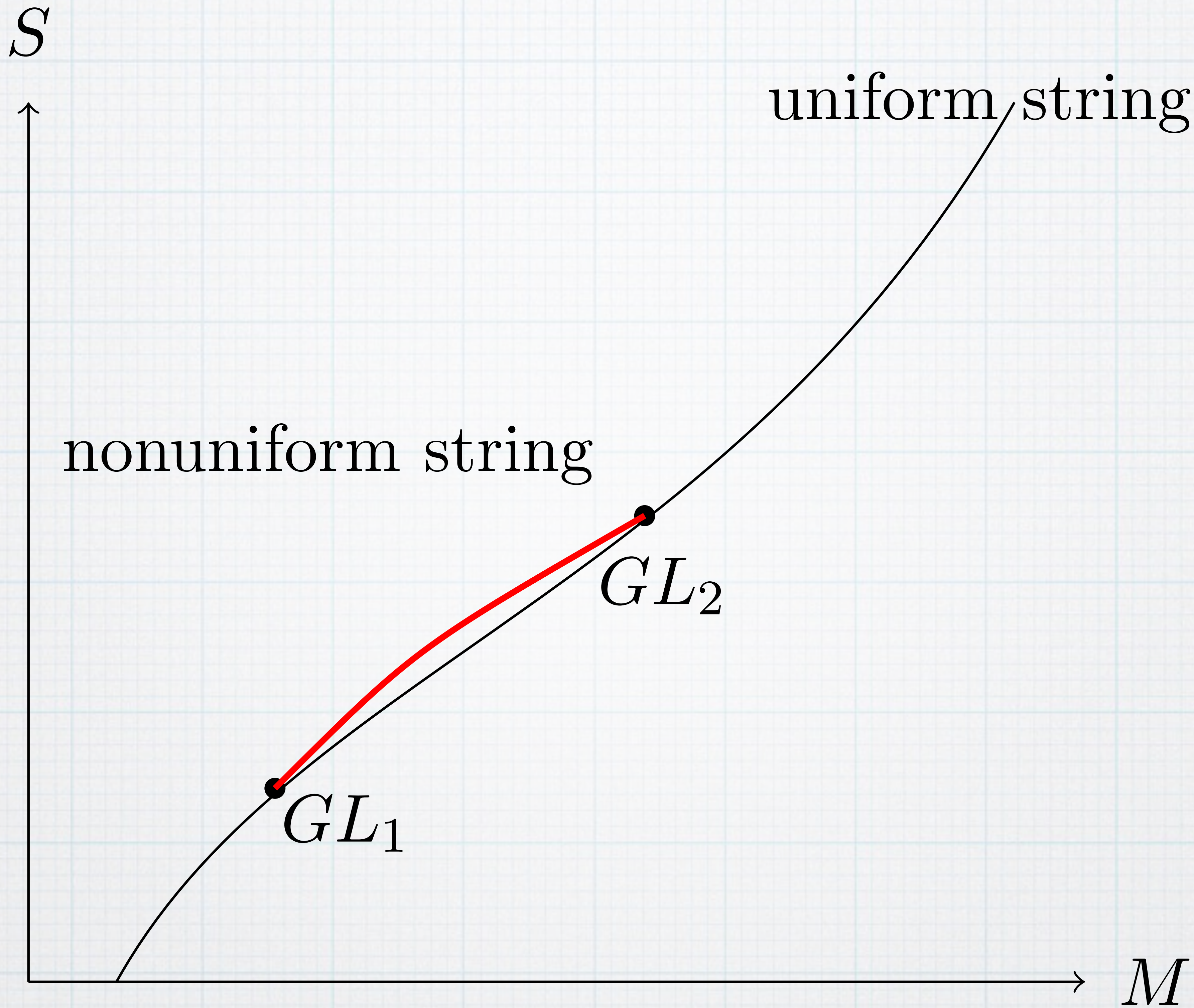
S
uniform string
nonuniform string
GL_2
GL_1
M

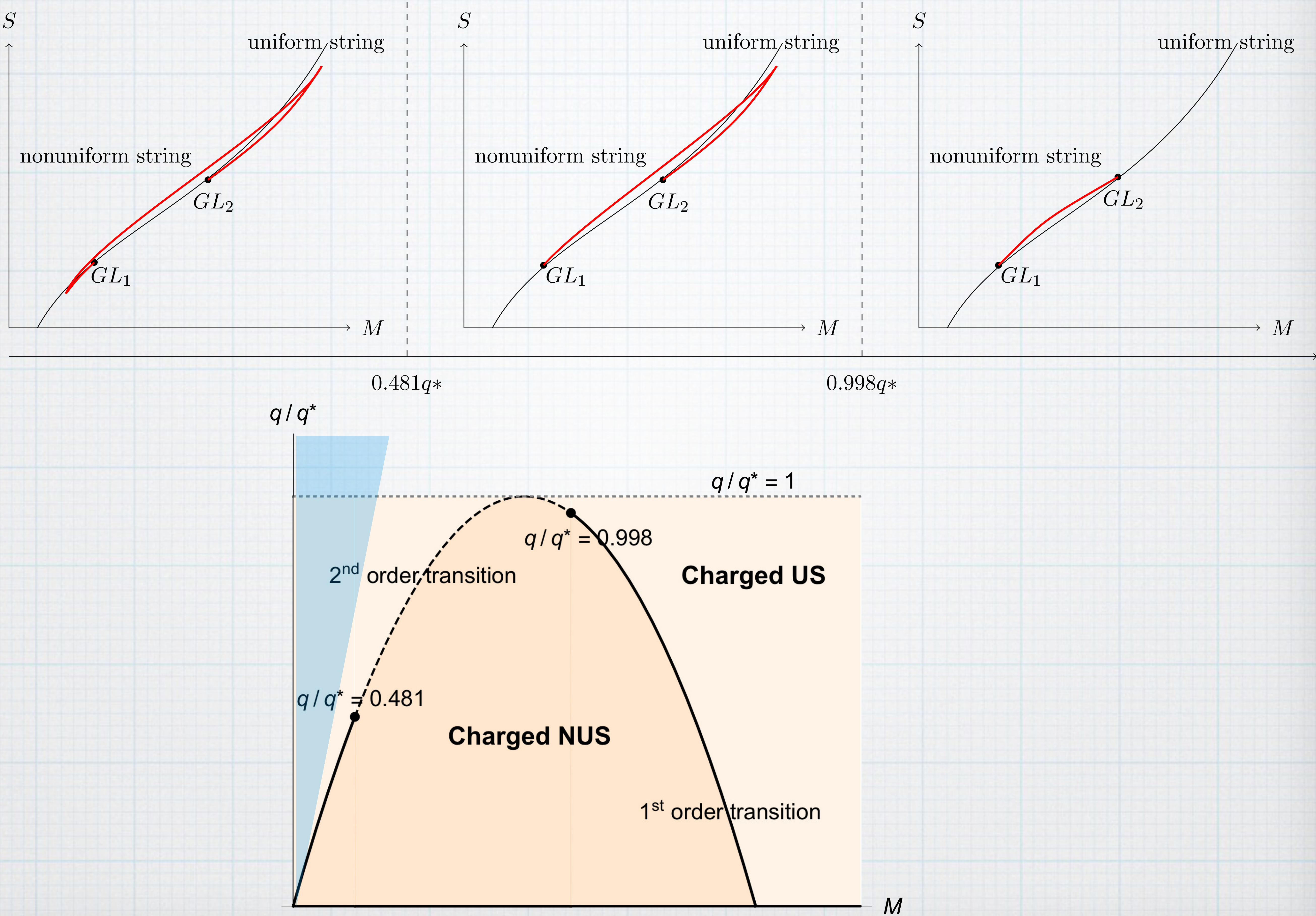
For small q

Presumably become similar to uncharged localised solutions



For large q





S
uniform string
nonuniform string
GL_2
GL_1
M
S
uniform string
nonuniform string
GL_2
GL_1
M
S
uniform string
nonuniform string
GL_2
GL_1
M
$0.481q*$
$0.998q*$
q / q*
q / q* = 1
q / q* = 0.998
2nd order transition
Charged US
q / q* = 0.481
Charged NUS
1st order transition
M

ADDING STRING CHARGE IN KALUZA-KLEIN THEORY

- We infer that stable inhomogeneous solutions are found for any $q > 0$ and presumably form the end point of the GL instability.
- Presumably for $q \to 0$ the inhomogeneous charged strings ‘limit’ to the localized solutions with flux tubes connecting them.
- Thus *any* non-zero string charge appears to change the picture of cosmic censorship violation – is this violation really what we would call *generic*?

NEW PREDICTIONS FOR 1+1 SYM

- Considering 11-d black membranes, with electric membrane charge, wrapped on a circle size L is analogous – we see the same behaviour, ie. homogeneous and inhomogeneous solutions related by GL instability.

- Using a trick related to the solution generating technique earlier we can relate this to 1+1 SYM.

- Boosting on the 11-circle and then reducing on it gives IIA black strings carrying D0 *and* NS 3-form electric field strength.

- Now T-dualizing to IIB theory gives a 10-d sugra solution, with D1 charge wrapping a circle size $L_{YM} \sim \frac{1}{L}$ but now also carrying *momentum* on this circle.

NEW PREDICTIONS FOR 1+1 SYM

- This then provides the holographic description of 1+1 SYM on a circle in the usual way;

  Hence in the Lorentzian setting adding momentum to a finite energy plasma on a circle may result in a rich phase structure.

  Presumably there is no longer a compactly supported eigenvalue distribution?

- Can we see this in SYM? For example at high temperature limit could dynamical simulations perhaps see similar phase structure?

SUMMARY

- The structure of black holes with one direction compactified to a circle is a fascinating system.
- We are still learning about its structure, and its implications for cosmic censorship and holography.
- There is now a fascinating interplay between two numerical and exotic fields – black hole solutions and lattice SYM.