Holographic Aspects of Loop Quantum Gravity

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Numerical approaches to holography, quantum gravity and cosmology

Higgs Center, Edinburgh, May 21, 2018

Talk in a nutshell

- **Aim:**
  - Overview:

    Holography $\leftrightarrow$ Loop Quantum Gravity

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  - Complementary to standard techniques
  - Work by several groups on different aspects

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- **Message:**
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  - Complementary to standard techniques
  - Work by several groups on different aspects

- **Content:**
  - Holography in general spacetimes and 3+0 LQG
  - Ryu-Takayanagi from tensor networks and state counting
  - Boundary correlators and bulk singularity resolution

Outline

Gauge / gravity

[Maldacena '97; Gubser, Klebanov, Polyakov '98; Witten '98; ...]

Central conjecture: existence of theories, so that

$$\text{QG on } \mathrm{AdS}_{d+1} \times \mathcal{M} \quad \Leftrightarrow \quad \text{CFT on } \mathbb{R} \times \mathbb{S}^{d-1}$$

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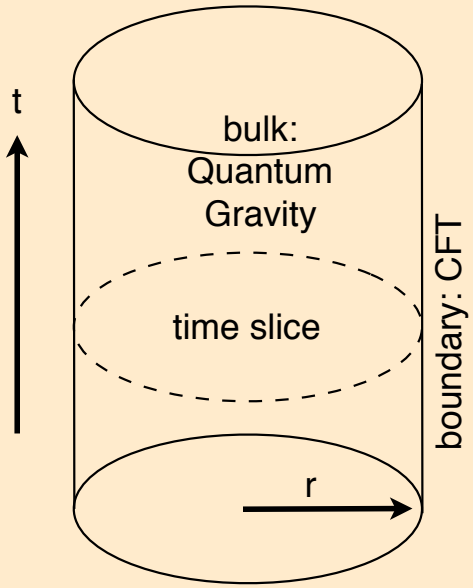

- Several examples
- Strong evidence in classical limit
- Some evidence beyond

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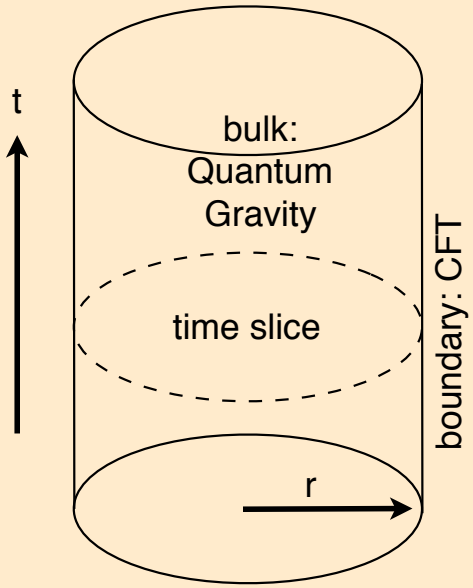

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$$Z_{\mathrm{QG}}\left[\phi_b^i\right] = \left\langle \exp\left(\int d_x^d \phi_b^i(x)\mathcal{O}_i(x)\right)\right\rangle_{\mathrm{CFT}}$$

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- **Dictionary**
  - How to guess / prove?
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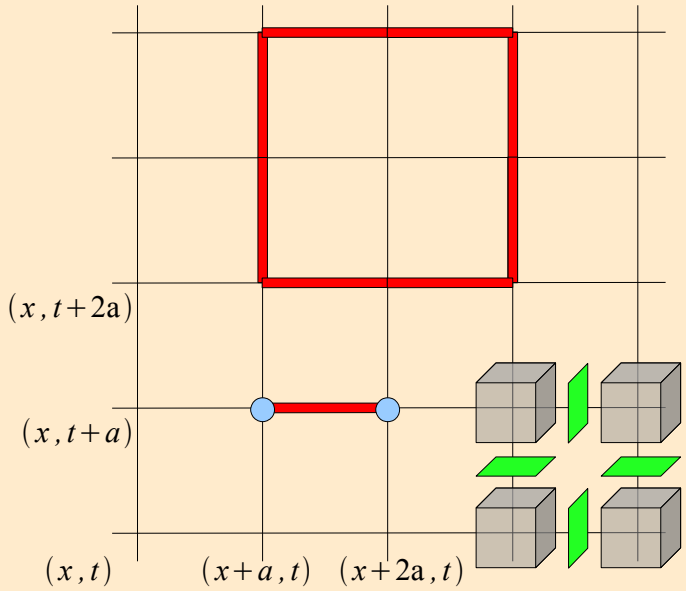
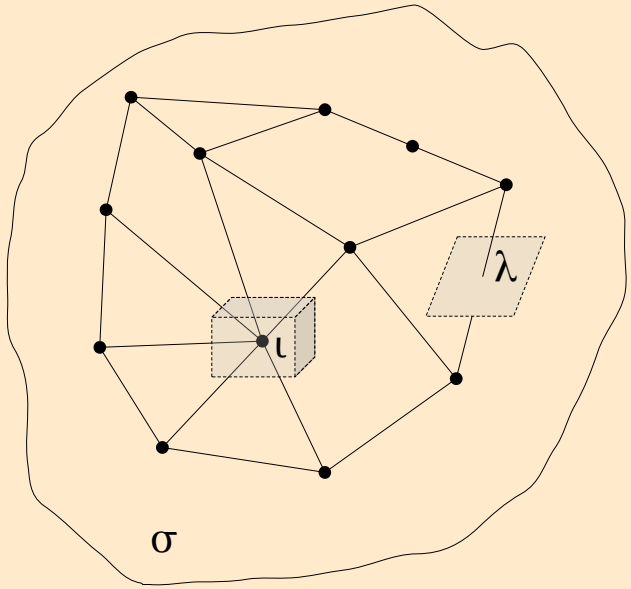
- **Asymptotically Anti-de Sitter** works best
  - Asymptotically flat, de Sitter?
  - Finite regions?

Loop quantum gravity

- Quantisation of classical gravity in connection variables
- Diffeomorphism-invariant extension of lattice gauge theory





[Ashtekar, Rovelli, Smolin, Lewandowski, Thiemann, . . .]

Loop quantum gravity

Status of the field

- Kinematic "quantum geometry" from rigorous Hilbert space techniques
- Dynamics largely unexplored / hard to track
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Main areas of progress

- 2+1 / 3+0 dimensions (topological)
  [Ponzano, Regge '68; Turaev, Viro '92; Rovelli '93; Freidel, Louapre '04; Barrett, Naish-Guzman '08; . . .]
- State counting / surface entropy
  [Krasnov '96; Rovelli '96; Ashtekar, Baez, Corichi, Kransov '97-; Engle, Noui, Perez '07-; . . .]
- Symmetry reduced quantisation $\rightarrow$ quantum cosmology
  [Bojowald '01-; Ashtekar, Bojowald, Lewandowski '03; Ashtekar, Pawlowski, Singh '06; . . .]

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General holography from QG

AdS/CFT relies on

- Asymptotic symmetry of AdS $\leftrightarrow$ global CFT symmetry
- Geometry of AdS near boundary $\leftrightarrow$ UV structure of CFT

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Derive dual theory directly from QG partition function!

- Finite region QG
- Boundary state / condition $\leftrightarrow$ dual theory

$$\langle \dots \rangle_{\text{Dual theory}(\phi^i_b)} := Z_{\text{QG}}\left[\phi^i_b\right]$$

$\rightarrow$ Euclidean 3d gravity best understood / solvable

[cf. neg. cos. constant: Castro, Gaberdiel, Hartman, Maloney, Volpato '11]

$3{+}0$ LQG, $\Lambda = 0$

3-dim. gravity is topological:

$$S = \int_M e_i \wedge F^i(A), \qquad \delta_{e_i} S = F^i(A) = 0$$

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Path integral:

$$Z(M) = \int \mathcal{D}e\, \mathcal{D}A\, e^{i \int_M e_i \wedge F^i(A)} \;\to\; \int \mathcal{D}A\; \delta\left(F^i(A)\right)$$

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Discretize on fixed simplicial decomposition:

$$Z_{\mathrm{PR}}(M) = \left(\prod_{\text{links } l} \int_{\mathrm{SU}(2)} dg_l \right) \prod_{\text{faces } f} \delta \left(\overleftarrow{\prod_{l \in f}} g_l^{\epsilon(l,f)} \right)$$

Needs regularization: Gauge fixing / quantum group

Holography from partition functions

Dual 2d Ising model

[Costantino '11; Dittrich, Hnybida '13; Bonzom, Costantino, Livine '15]

- Tri-valent boundary graph Γ on 2-sphere

$$\left(Z^{\text{Ising}}(\Gamma)\right)^2 Z^{\text{LQG}}(\Gamma) = \left(\prod_{\text{edges } e} \cosh(y_e)\right)^2 2^{2\#\text{vertices}}$$

- Ising couplings $y_e \leftrightarrow$ QG coherent state parameters

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Dual "twisted" 6-vertex model

[Dittrich, Goeller, Livine, Riello '17]

- Four-valent boundary graph Γ on twisted 2-torus
- Only spin 1/2 rep., "fuzzy parallelograms"
- Torus twist + monodromy integration in 6-vertex model:

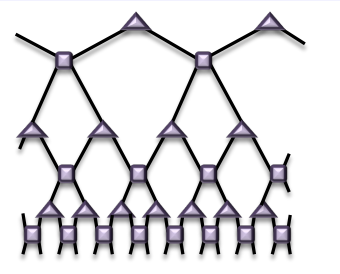
$$Z^{\text{LQG}}(\Gamma) = Z^{6\text{ vertex}}_{\text{twisted}}(\Gamma)$$

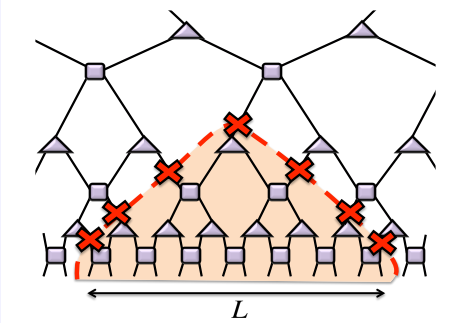
- Intertwiners $\leftrightarrow$ vertex parameters

Outline

Random tensor networks

- Approximate ground states of interacting many-body Hamiltonians
- Different types, here MERA (gapless systems) [figures from Orús, arXiv:1407.6552]





- $S_{\mathrm{EE}}(L) \sim$ min. # crossed legs

  [Swingle '09; . . . ; Hayden, Nezami, Qi, Thomas, Walter, Yang '16; . . .]

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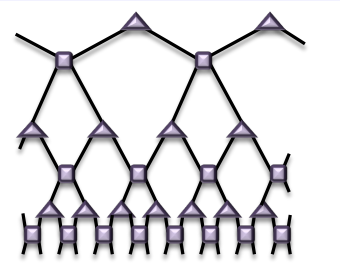

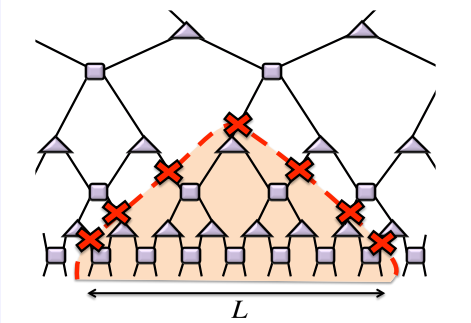

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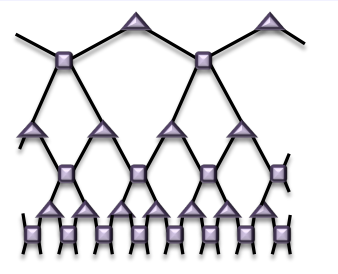

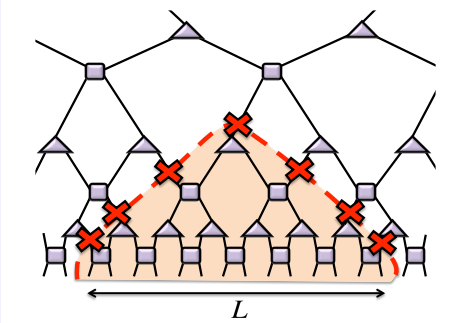

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$\rightarrow$ Model for discrete holography

- How to relate to continuum geometry / continuum RT-formula?

Deriving RT from random tensor networks

[Hayden, Nezami, Qi, Thomas, Walter, Yang '16]

- Average over random tensors $\leftrightarrow$ Ising model $\leftrightarrow$ RT-surface as domain wall
- Discrete RT formula for constant large bond dimension D:

$$S_{\mathrm{EE}}(L) = \log D \ \times \ \text{min. } \# \text{ crossed legs}$$

- Missing input: $\log D \ \leftrightarrow$ geometry

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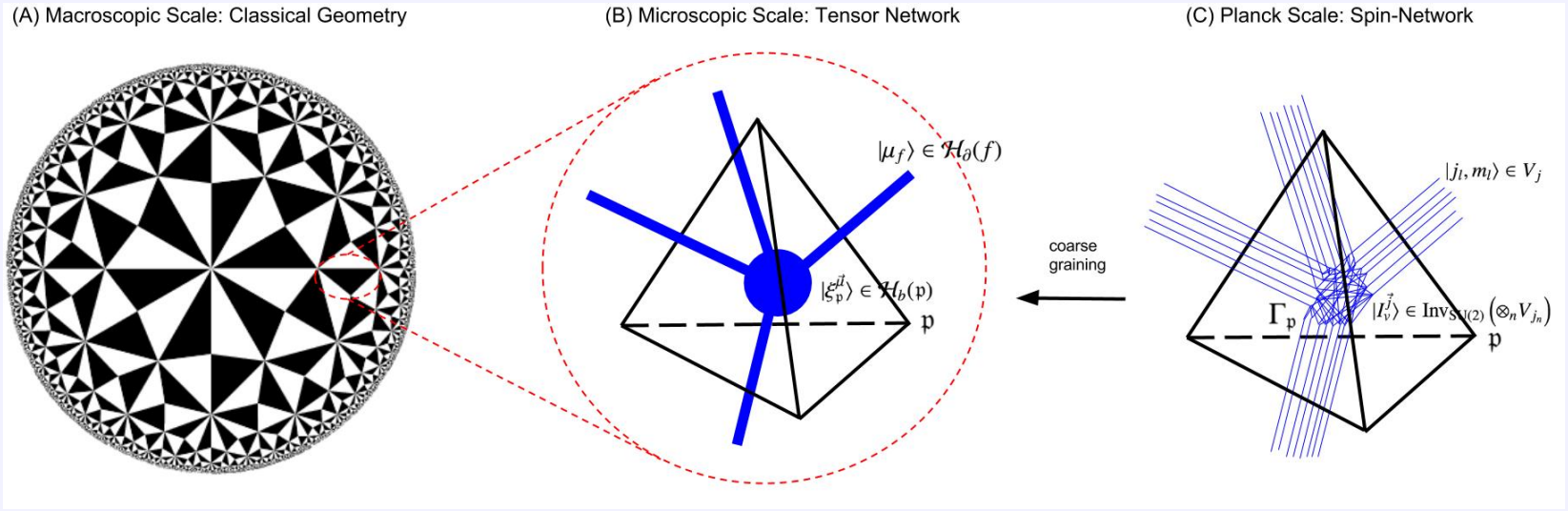
LQG interpretation [Han, Hung '16, figure from Han, Hung: arXiv:1610.02134]



Geometric RT from LQG

- Codim. 2 area from bond dimension $\leftrightarrow$ surface (black hole) entropy
- State counting: [Krasnov '96; Rovelli '96; Ashtekar, Baez, Corichi, Kransov '97-; ...]

$$D \sim \exp(A)$$

- Generic codim. 2 surfaces and dimensions [Husain '98; NB '13,'14]

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Geometric RT from LQG

- Repeat computation for generic large bond dimensions $D \sim \exp(A)$

  $\rightarrow$ discrete Nambu-Goto path integral

  $\rightarrow$ minimal surface [Han, Hung '16]

- Correct entanglement spectrum from Wheeler-de Witt wave function in 3d [Han, Huang '17]

Outline

Classical limit and singularities

Classical supergravity $\leftrightarrow$ Strongly coupled large N Super-Yang Mills

$\rightarrow$ Field theory signatures of gravitational singularities?

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Beyond classical gravity: usual strategy

- Non-perturbative string theory defined via AdS/CFT
- Quantum gravity from field theory

  [Hertog, Horowitz '04, '05; Das, Michelson, Narayan, Trivedi '06; Turok, Craps, Hertog '07; Barbón, Rabinovici '11; Smolkin, Turok '12;]

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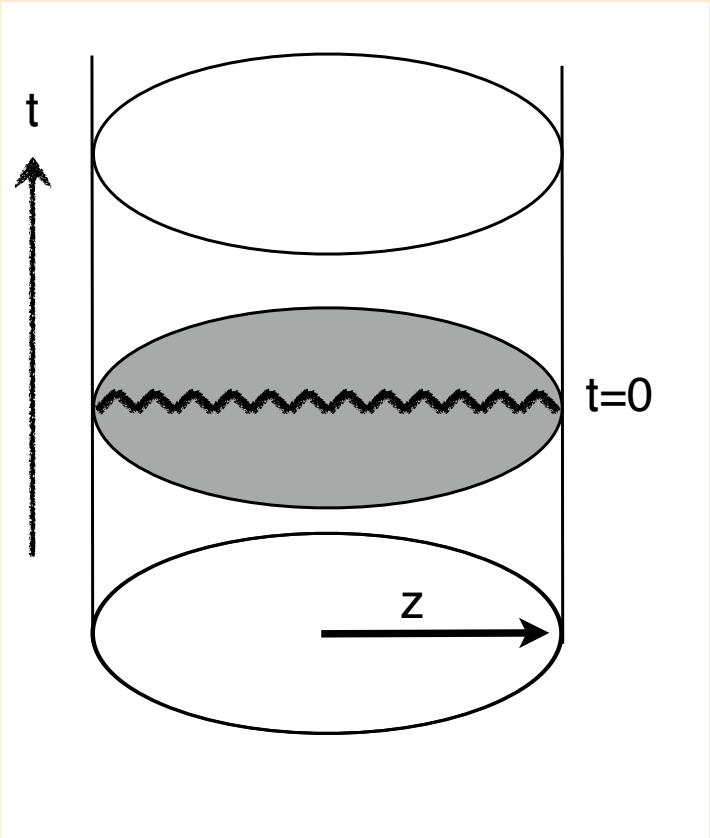
Here: work on gravity side

$\rightarrow$ Quantum gravity: signatures of resolved singularities?

Two-point correlators in Kasner

[Engelhardt, Horowitz '14; Engelhardt, Horowitz, Hertog '15]



Boundary: $ds_4^2(t) = -dt^2 + \sum_{i=1}^{3} t^{2p_i} dx_i^2, \quad p_i \in \mathbb{R}$

Bulk: $ds_5^2 = \frac{1}{z^2}\left(dz^2 + ds_4^2(t)\right)$

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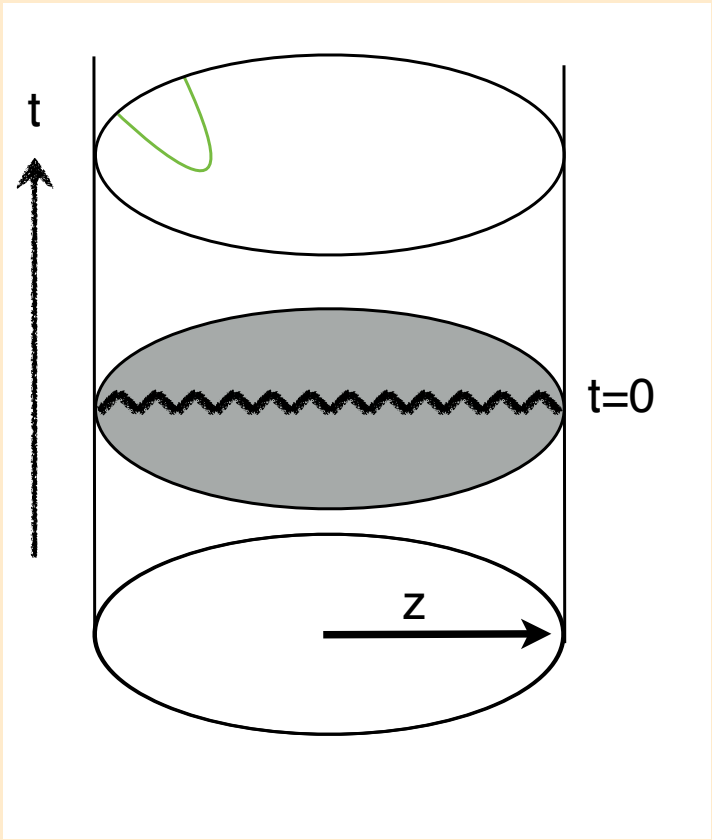

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Geodesic approximation: (heavy scalar operators)

$$\langle \mathcal{O}(x)\mathcal{O}(-x)\rangle \sim \exp(-\Delta L_{\text{ren}})$$

Δ: conformal weight of $\mathcal{O}$
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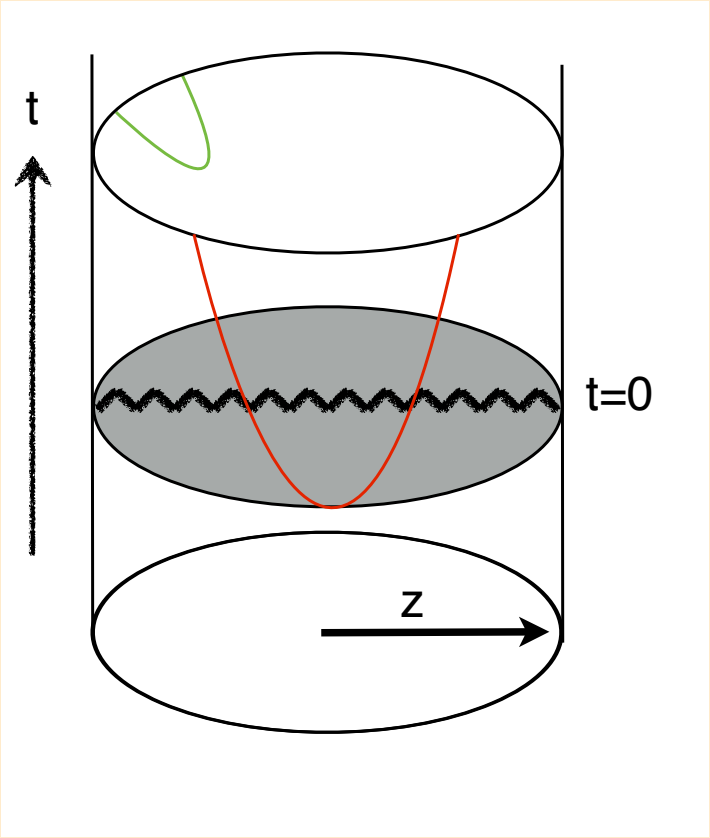

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Main result

Geodesic passing singularity $\leftrightarrow$ finite distance pole in 2-point correlator

Loop quantum cosmology

- Minisuperspace quantization, e.g. volume v + mean curvature b
- Quantize Weyl elements $e^{i\mu v + i\nu b}$ with GNS functional $\omega(e^{i\mu v + i\nu b}) = \delta_{\nu,0}$
- Quantize b as $\widehat{\sin(b\lambda)}/\lambda$

[Bojowald '01-; Ashtekar, Bojowald, Lewandowski '03; Ashtekar, Pawlowski, Singh '06; ...]

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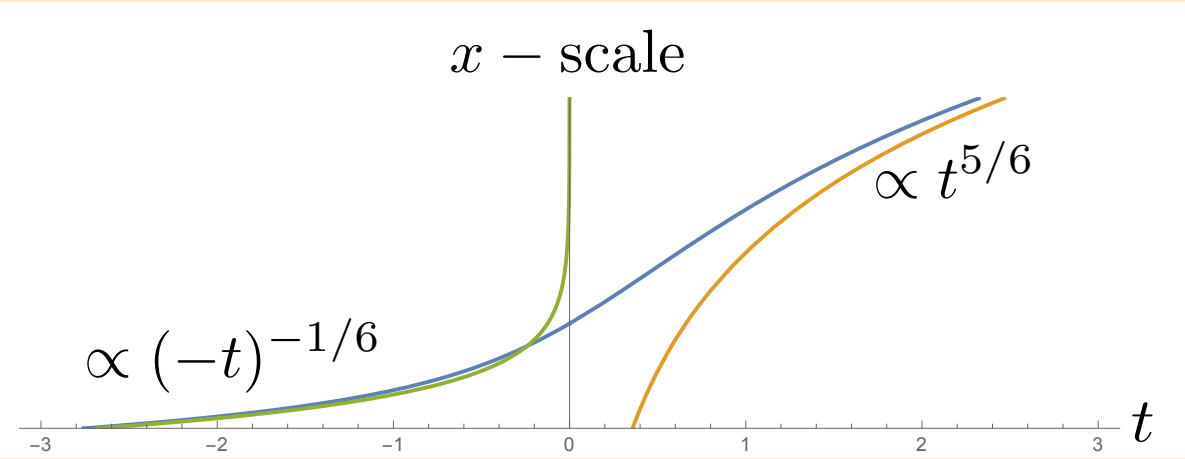

- Quantum bounce interpolates between classical solutions
- Transitions between different Kasner solutions [Gupt, Singh '12]

Improved 2-point correlators

Strategy

- Goal: quantum corrected Kasner-AdS
- Here: 4d geometry quantized, z-direction classical
- Future work: 5d quantum geometry

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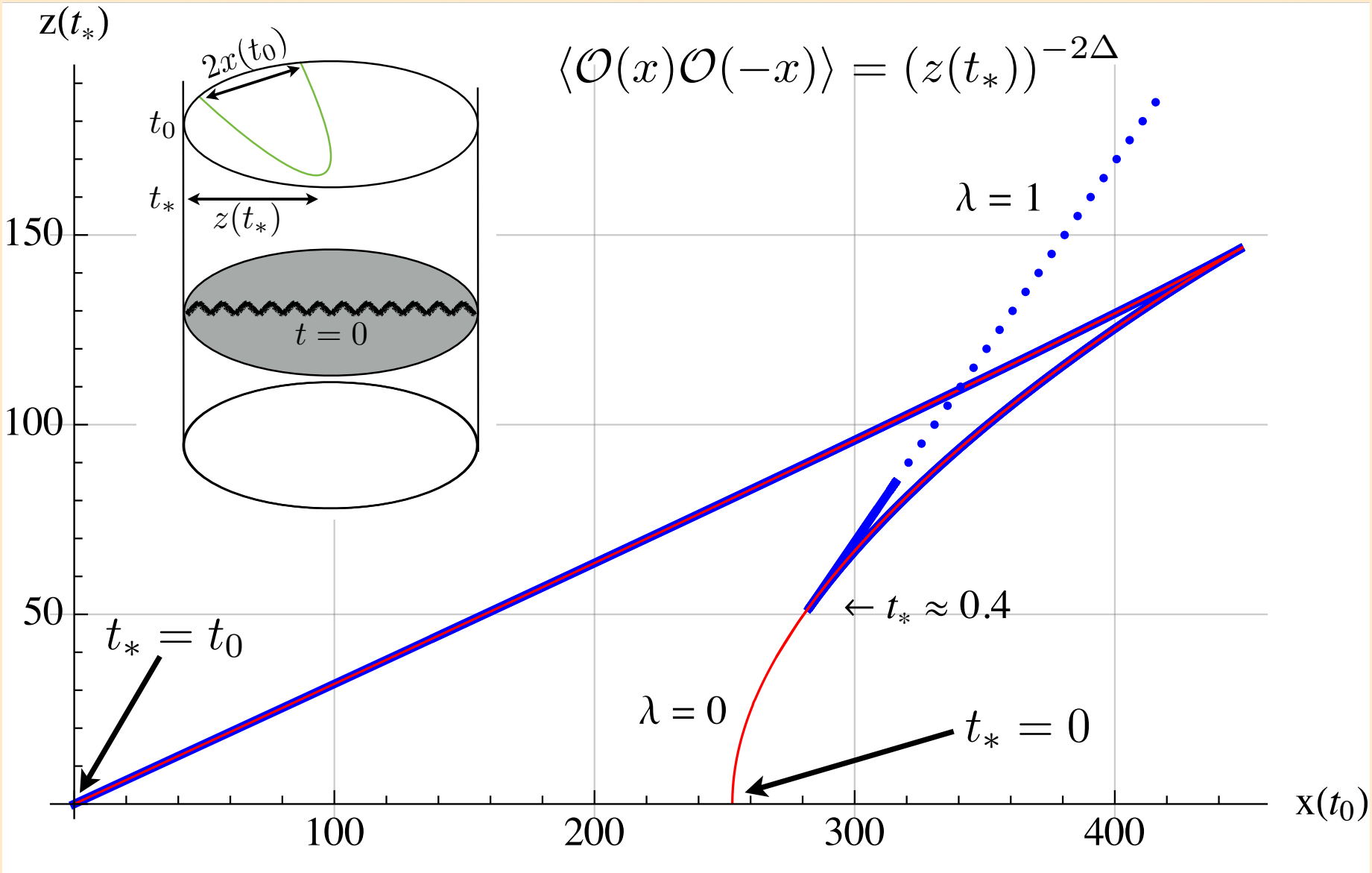
Results

[NB, Schäfer, Schliemann '16; NB, Mele, Münch '18]

- Finite distance pole in 2-point correlator resolved
- Result non-trivial with proper bulk 5d Planck scale
- Kasner transitions problematic in 4d / z - split



Analytical result (above): no Kasner transition, 4d Planck scale [NB, Schäfer, Schliemann '16]

Numerics: 5d Planck scale + Kasner transitions qualitatively similar [NB, Mele, Münch '18]

Signatures of the resolved singularity

Dual of the resolved singularity

- Finite distance bump instead of pole
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Discussion

- So far: prototype calculation
- Goal: find system where independent field theory computation possible

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  - Singularity resolution

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