Lattice Quantum Gravity and Asymptotic Safety

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Asymptotic Safety

Weinberg proposed idea that gravity might be Asymptotically Safe in 1976 [Erice Subnucl. Phys. 1976:1]. This scenario would entail:

- Gravity is effectively renormalizable when formulated non-perturbatively. Problem lies with perturbation theory, not general relativity.
- Renormalization group flows of couplings have a non-trivial fixed point, with a finite dimensional ultraviolet critical surface of trajectories attracted to the fixed point at short distances.
- In a Euclidean lattice formulation the fixed point would show up as a second order critical point, the approach to which would define a continuum limit.

Lattice gravity

- Euclidean dynamical triangulations (EDT) is a lattice formulation that was introduced in the ’90’s. [Ambjorn, Carfora, and Marzuoli, The geometry of dynamical triangulations, Springer, Berlin, 1997] Lattice geometries are approximated by triangles with fixed edge lengths. The dynamics is contained in the connectivity of the triangles, which can be added or deleted.
- In lattice gravity, the lattice itself is a dynamical entity, which evolves in Monte Carlo time. The dimension of the building blocks can be fixed, but the effective fractal dimension must be calculated from simulations.
- EDT works perfectly in 2d, where it reproduces the results of non-critical bosonic string theory.
- The EDT formulation in 4d was shown to have two phases, a “collapsed" phase with infinite Hausdorff dimension and a branched polymer phase, with Hausdorff dimension 2. The critical point separating them was shown to be first order, so that new continuum physics is not expected. [Bialas et al, Nucl. Phys. B472, 293 (1996), hep-lat/9601024; de Bakker, Phys. Lett. B389, 238 (1996), hep-lat/9603024]

Causal Dynamical Triangulations

In the late 90's, Ambjorn and Loll introduced Causal Dynamical Triangulations (CDT) [NPB 536, 407 (1998), hep-th/9805108] . They introduced a causality condition, where only geometries that admit a time foliation are included in the path integral.

- Simulations from 2004-2005 show a good semi-classical limit, with (Euclidean) de Sitter space as a solution. [Ambjorn, et. al., PRD 78, 063544 (2008), arXiv:0807.4481.]
- Striking result is a running effective (spectral) dimension
- Effective (spectral) dimension runs from ~ 2 at short distances to ~ 4 at long distances. [Ambjorn, et. al., PRL 95, 171301 (2005), hep-th/0505113.]

Einstein Hilbert Action

Continuum Euclidean path-integral:

$$Z = \int \mathscr{D}g \; e^{-S[g]}, \tag{1}$$

$$S[g_{\mu\nu}] = -\frac{k}{2} \int d^d x \sqrt{\det g} (R - 2\Lambda), \tag{2}$$

where $k = 1/(8\pi G_N)$.

Discrete action

Discrete Euclidean (Regge) action is

$$S_E = k \sum 2 V_2 \delta - \lambda \sum V_4, \tag{3}$$

where $\delta = 2\pi - \sum \theta$ is the deficit angle around a triangular face, V_i is the volume of an i-simplex, and $\lambda = k\Lambda$. Can show that

$$S_E = -\frac{\sqrt{3}}{2}\pi k N_2 + N_4 \left(\frac{5\sqrt{3}}{2} k \arccos \frac{1}{4} + \frac{\sqrt{5}}{96} \lambda \right) \tag{4}$$

where N_i is the total number of i-simplices in the lattice. Conveniently written as

$$S_E = -\kappa_2 N_2 + \kappa_4 N_4. \tag{5}$$

Measure term

Continuum calculations suggest a form for the measure

$$Z = \int \mathscr{D}g \prod_x \sqrt{\det g}^{\beta} \; e^{-S[g]}, \tag{6}$$

Going to the discretized theory, we have

$$\prod_x \sqrt{\det g}^{\beta} \to \prod_{j=1}^{N_2} \mathscr{O}(t_j)^{\beta}, \tag{7}$$

where $\mathscr{O}(t_j)$ is the order of triangle t_j, i.e. the number of 4-simplices to which a triangle belongs. Can incorporate this term in the action by taking exponential of the log. β is a free parameter in simulations. Can interpret as an ultra-local measure term, since it looks like a product over local 4-volumes.

New Idea

Revisiting the EDT approach because other formulations (renormalization group and other lattice approaches) suggest that gravity is asymptotically safe.

New work done in collaboration with students (past and present) and postdoc: JL, S. Bassler, D. Coumbe, Daping Du, J. Neelakanta, (arXiv:1604.02745).

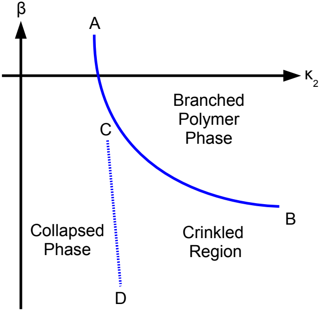
- Key new idea that inspired this study is that a fine-tuning of bare parameters in EDT is necessary to recover the correct continuum limit. This is in analogy to using Wilson fermions in lattice gauge theory to study quantum chromodynamics (QCD) with light or massless quarks. Striking similarities are seen. Coincidence?
- Previous work did not implement this fine-tuning, leading to negative results.

Simulations

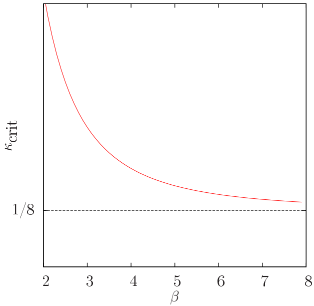
Methods for doing these simulations were introduced in the 90's. We wrote new code from scratch.

- The Metropolis Algorithm is implemented using a set of local update moves.
- We introduce a new algorithm for parallelizing the code, which we call parallel rejection. Exploits the low acceptance of the model, and partially compensates for it. Checked that it reproduces the scalar code configuration-by-configuration. Buys us a factor of ~ 10.

Phase diagram EDT vs. QCD with Wilson fermions



EDT

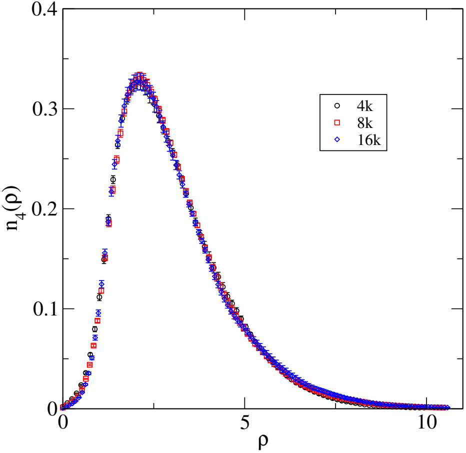


QCD

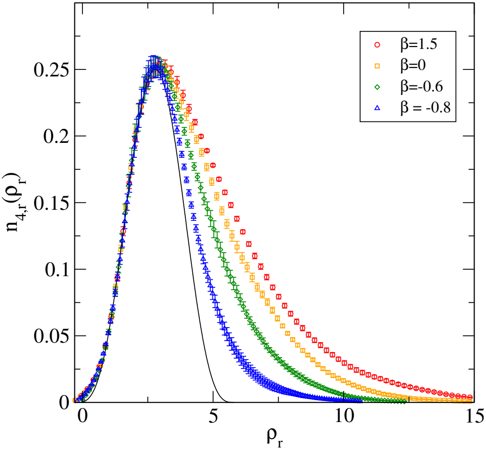
Main problems to overcome

- Must show recovery of semiclassical physics in 4 dimensions.
- Must show existence of continuum limit at continuous phase transition.

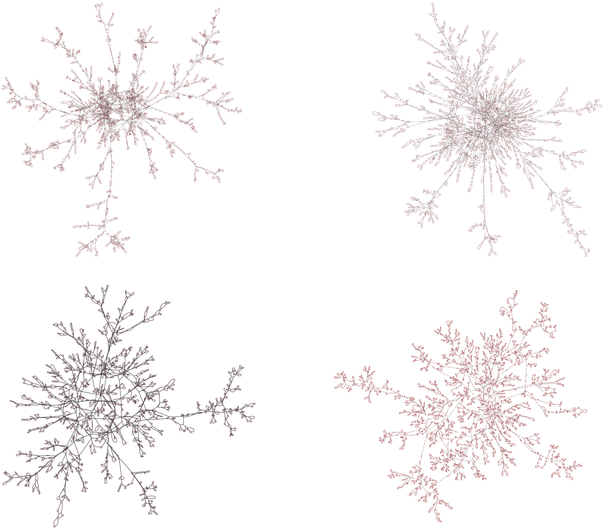
Three volume distribution



Three volume distribution



Visualization of geometries



Coarser to finer, left to right, top to bottom.

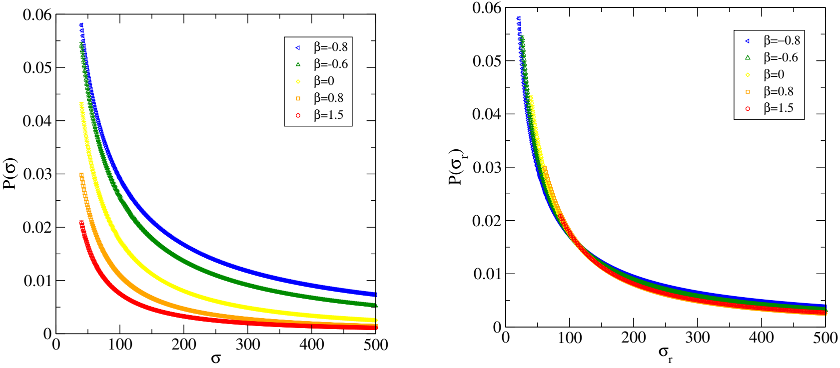
Diffusion process and the spectral dimension

Spectral dimension is defined by a diffusion process

$$D_S(\sigma) = -2\frac{d\log P(\sigma)}{d\log\sigma}, \tag{8}$$

where σ is the diffusion time step on the lattice, and $P(\sigma)$ is the return probability, i.e. the probability of being back where you started in a random walk after σ steps.

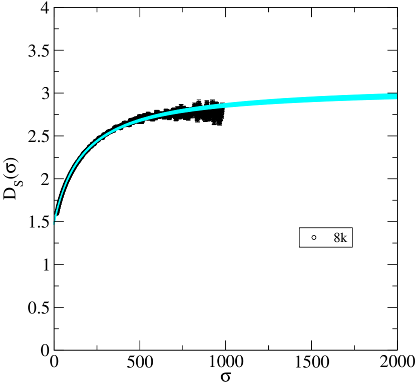
Relative lattice spacing



Return probability left and rescaled return probability right.

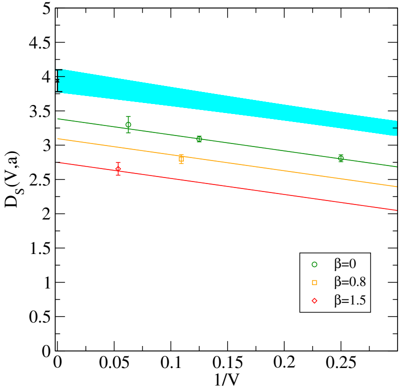
Spectral Dimension

χ^2/dof=1.25, p-value=17%
$D_S(\infty) = 3.090 \pm 0.041$, $D_S(0) = 1.484 \pm 0.021$



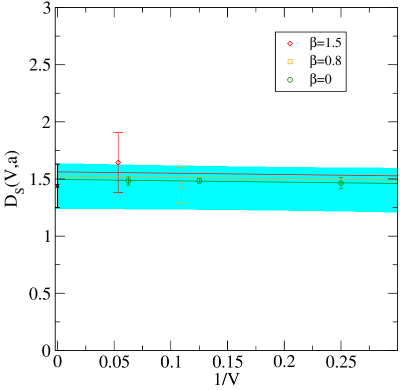
Infinite volume, continuum extrapolation

χ^2/dof=0.52, p-value=59%
$D_S(\infty) = 3.94 \pm 0.16$



Infinite volume, continuum extrapolation

χ^2/dof=0.17, p-value=84%
$D_S(0) = 1.44 \pm 0.19$



Consistent with holography?

Banks has a holography inspired argument against Asymptotic Safety that he gets from comparing the scaling of entropy with energy in a conformal field theory with that of a black hole, the idea being that a renormalizable theory should look like a CFT at high energies, whereas gravity should be dominated by black holes. Argument is semiclassical and could fall apart, but let's look at the lattice data:

$$S \sim E^{\frac{d-1}{d}}, \quad CFT \tag{9}$$

$$S \sim E^{\frac{d-2}{d-3}}, \quad GR \tag{10}$$

For these relations the relevant dimension is the spectral dimension if one lives on a fractal space.
The scaling agrees when $d = 3/2$. This is consistent with our result $D_S(0) = 1.44 \pm 0.19$. Amusing coincidence? In CDT one finds something similar (Coumb and Jurkiewicz, JHEP03 (2015) 151).

What does it mean?

Interesting results that suggest that the correct classical result might be restored in the continuum, large-volume limit. Analogy with Wilson fermions that inspired this study may tell us more.

We have to perform a fine-tuning, and long distance physics gets messed up by discretization effects. These things happen when the regulator breaks a symmetry of the quantum theory. In this case, natural to identify the symmetry as continuum diffeomorphism invariance.

If true, then β would not be a relevant parameter in a continuum formulation with diffeo invariance unbroken. (Would still need a measure term if the regulator broke scale invariance, but β would be fixed.)

Interesting because if true, number of relevant couplings in continuum theory could be less than 3.

The number of relevant parameters

Three adjustable parameters in the action: G, Λ, β.

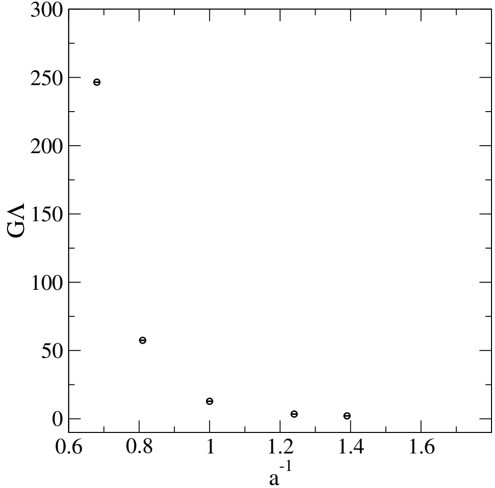
Nontrivial evidence that G and Λ are not separately relevant couplings. One of these is redundant, with $G\Lambda$ a relevant coupling. Only $G\Lambda$ approaches a constant near the fixed-point.

Further evidence that β is only relevant because the lattice regulator breaks the gauge symmetry. This symmetry should be an exact symmetry of the quantum theory, so β should not be a relevant parameter in the target continuum theory. Makes sense, since the local measure should not run.

In this case there would be only one relevant coupling! Maximally predictive theory with no adjustable parameters once the scale is set.

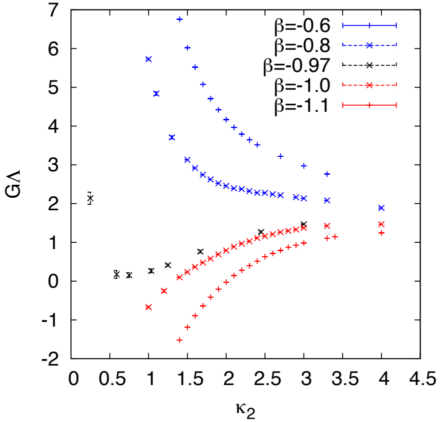
Performing the subtraction

Divergence of the form $\delta^4(0)$ in action should be cancelled by local measure term. Continuum calculations show that this fixes the value of β. A running β might introduce unphysical running of Λ. We see that the running of the bare $G\Lambda$ is not physical, if we are to interpret our geometries as semiclassical de Sitter space.



Performing the subtraction

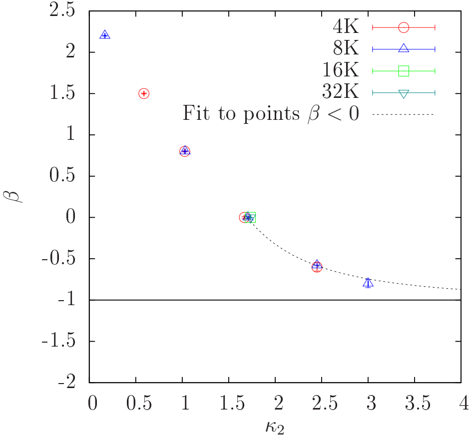
Assuming that we should find the running of Λ with β kept fixed, we study the running of $G\Lambda$ for different values of β. (κ_2 serves as a proxy for a^{-1}.)



The value of $\beta \approx -1$ is special in that it gives physical running, with the zero of $G\Lambda$ coinciding with its local minimum. This is expected for a semiclassical de Sitter solution.

Performing the subtraction

$\beta \approx -1$ is also compatible with the continuum value for β.



Field strength renormalization

For convenience in the following argument, we can rewrite the Lagrangian corresponding to the Einstein-Hilbert action as

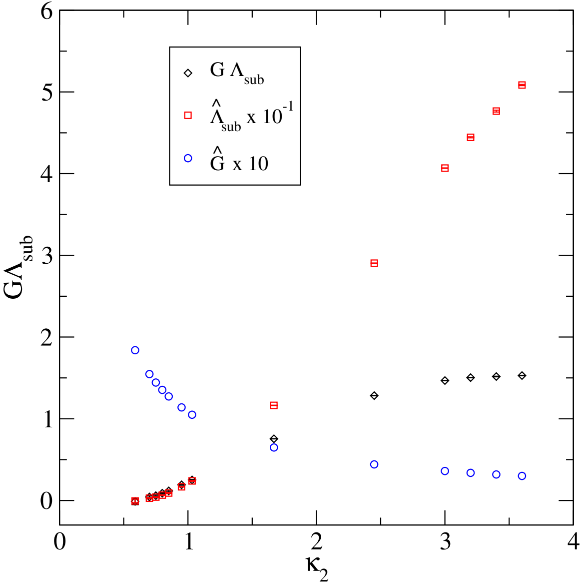
$$\mathscr{L} = \frac{\omega}{16\pi}\sqrt{g}(R - 2\omega\Lambda') \tag{11}$$

where ω and Λ' replace G and Λ. Then

$$\frac{\partial\mathscr{L}}{\partial\omega} = \frac{1}{16\pi}\sqrt{g}(R - 4\omega\Lambda'). \tag{12}$$

which vanishes by the equations of motion, suggesting that ω is a redundant parameter.

Running of $G\Lambda$



Conclusions

Important to test the picture presented here against other approaches, renormalization group and other lattice formulations.

If this holds, lattice provides a nonperturbative definition of a renormalizable quantum field theory of general relativity.

Back-up Slides