

# Nonlinear black hole perturbation theory

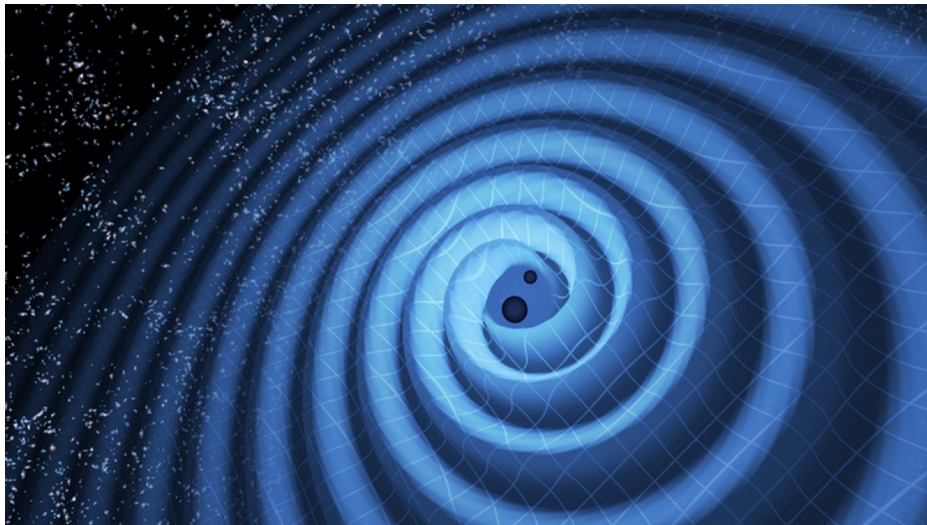
– Inspiral, merger, and ringdown of black hole binaries –

Adam Pound

New Directions in Theoretical Physics 5

Edinburgh, 6 January 2026

# Black hole binaries

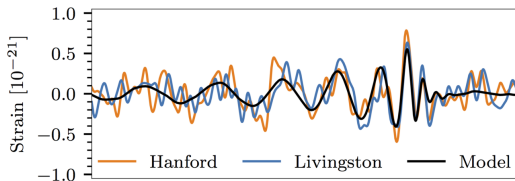
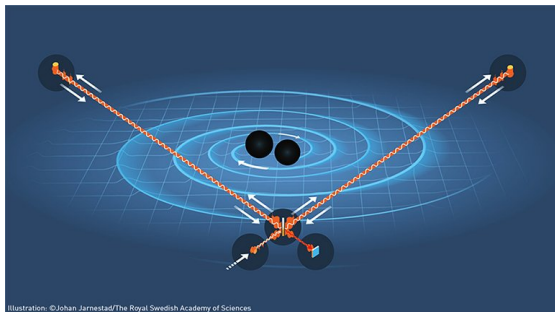


[Image credit: T. Pyle/LIGO]



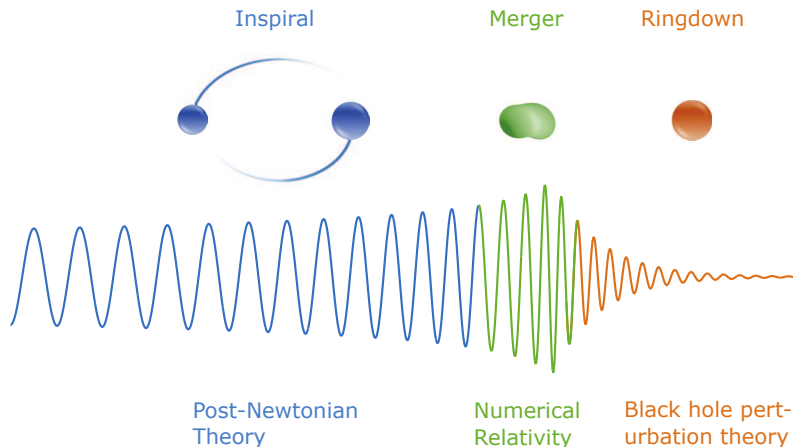
# Gravitational-wave astronomy

- hundreds of BH binaries detected since 2015
- first precise tests of strong gravity
- BHs of mass  $\sim 1M_{\odot}$  to  $> 100M_{\odot}$
- detection and interpretation *relies on accurate theoretical waveform models*



[Image credit: LIGO]

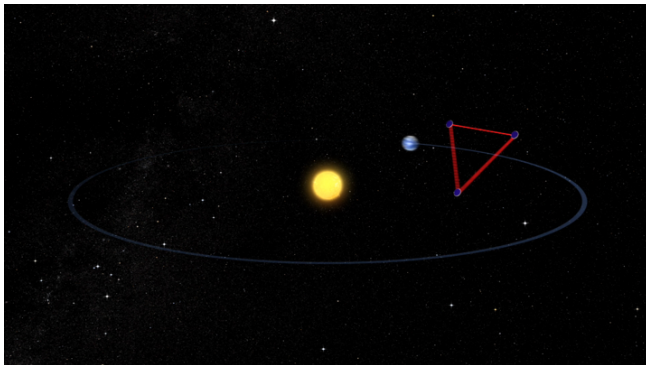
# Waveform modelling today



[Image adapted from Antelis and Moreno]

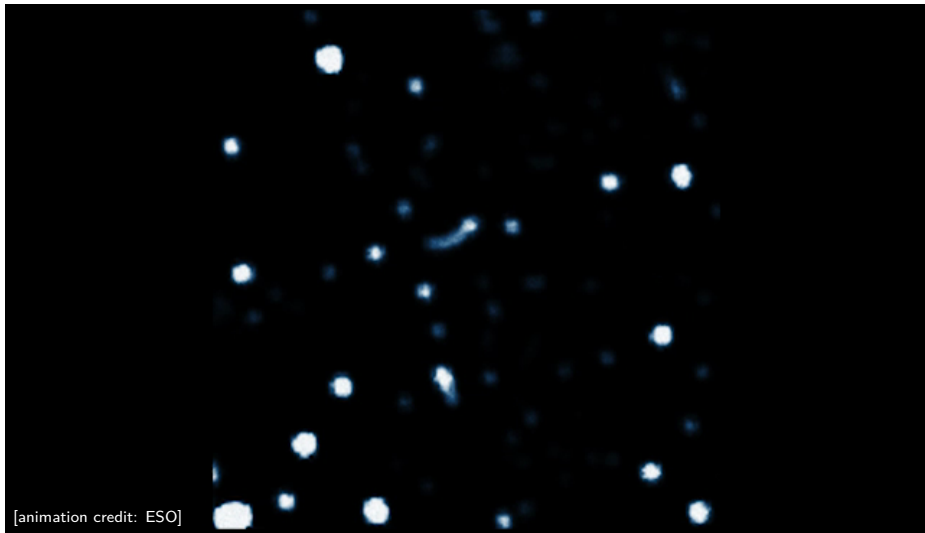
Fast waveform models combine these methods

# The Laser Interferometer Space Antenna (LISA)

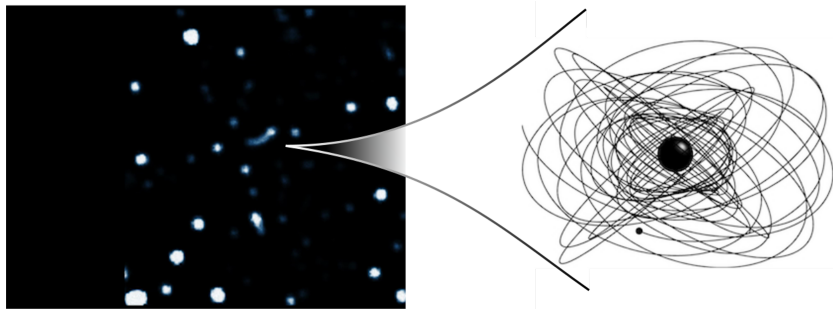


- £1.5B mission
- launch date July 2035
- no noise in space
  - ⇒ can detect much lower frequencies
  - ⇒ can detect much larger sources

# Massive black holes in galactic cores



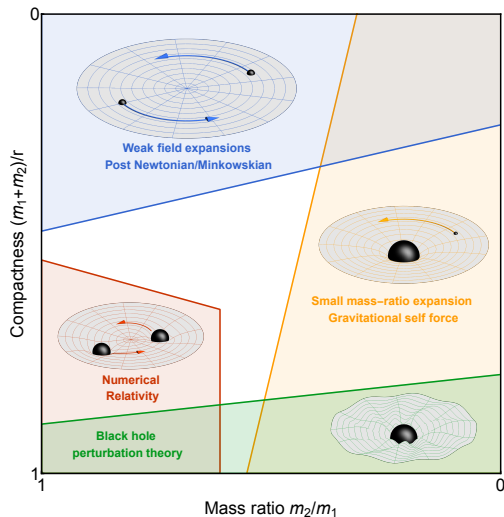
# Extreme-mass-ratio inspirals (EMRIs)



- while in LISA band, stellar-mass object executes  $\sim 10^5$  tight, intricate orbits around massive black hole
- precise probe of BH spacetime and properties of galactic cores

# Waveform modelling for tomorrow

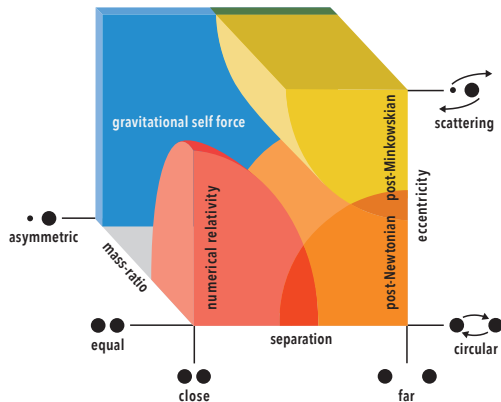
- NR: “exact”
- PN: low speeds, weak fields
- PM: weak-field scattering
- SF: small mass ratio
- ringdown: near-Kerr



[Credit: LISA Waveforms White Paper]

# Waveform modelling for tomorrow

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[Credit: Carvalho & GWSky]

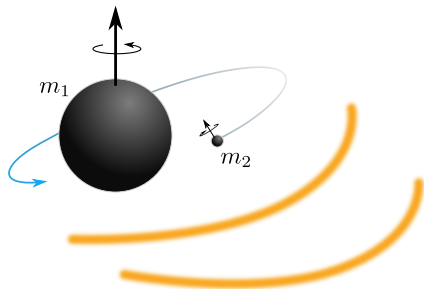
## Gravitational self-force theory



# The EMRI challenge

- strong gravitational fields, highly relativistic speeds  
⇒ PN fails
- system is in band for  $\sim 10^5$  intricate cycles  
⇒ NR would need  $\sim 10^9$  years per waveform
- we need to generate theoretical waveforms in  $\ll 1\text{s}$

# Gravitational self-force theory [Mino &co., Wald &co, Detweiler & Whiting, Poisson, Barack, Harte, AP, van de Meent, Wardell, ...]



- $\epsilon = 1/q = m_2/m_1 \ll 1$
- small body perturbs spacetime:

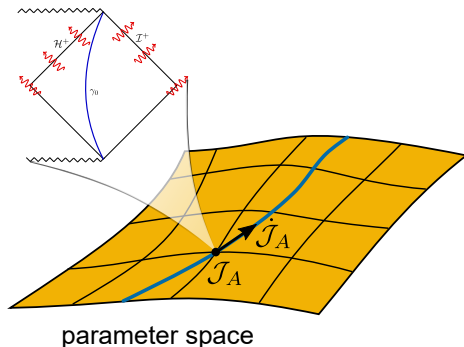
$$g_{\alpha\beta} = g_{\alpha\beta}^{\text{Kerr}} + \epsilon h_{\alpha\beta}^{(1)} + \epsilon^2 h_{\alpha\beta}^{(2)} + \dots$$

- perturbation affects  $m_2$ 's motion:

$$\frac{D^2 z^\alpha}{d\tau^2} = \epsilon f_{(1)}^\alpha + \epsilon^2 f_{(2)}^\alpha + \dots$$

# Multiscale expansion [Hinderer and Flanagan; AP &co]

## & fast waveforms [Katz, Chua, Hughes, Warburton; & Chapman-Bird et al.]



**Offline step**  $\gg 10^6$  CPU hrs

- formulate field equations on orbital phase space  $(\varphi^A, \mathcal{J}_A)$

$$\Rightarrow h = \sum_n \epsilon^n h_{\ell m}^{(n)}(\varphi^A, \mathcal{J}_A) {}_{-2}Y_{\ell m}$$

- store  $h_{\ell m}^{(n)}$  and  $\dot{\mathcal{J}}_A$  on  $\mathcal{J}_A$  grid

**Online step**  $\lesssim 100\text{ms}$

- solve ODEs for  $\varphi^A(t)$  and  $\mathcal{J}_A(t)$  and sum up modes

# Post-adiabatic ( $n$ PA) counting and nonlinearities

- in the long, slow inspiral...

$$\begin{aligned}\frac{d\varphi^A}{dt} &= \Omega^A(\mathcal{J}_B) \\ \frac{d\mathcal{J}_A}{dt} &= \epsilon \underbrace{F_A^{(0)}(\mathcal{J}_B)}_{0\text{PA}} + \epsilon^2 \underbrace{F_A^{(1)}(\mathcal{J}_B)}_{1\text{PA}} + \dots\end{aligned}$$

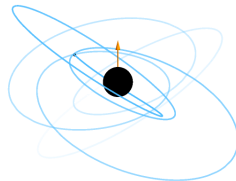
- 1PA needed for parameter estimation [Burke et al., Mathews et al.,]
- even 0PA evolution is a nonlinear effect:

$$F_A^{(0)} \propto \partial_t h^{(1)} \partial_t h^{(1)}$$

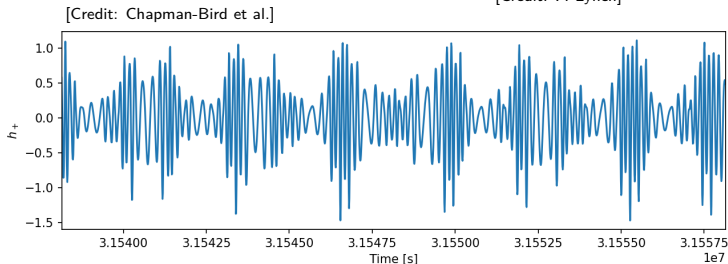
# State of the art at 0PA

[Isoyama et al., Hughes et al., Chapman-Bird et al., Warburton, Nasipak, ...]

- years-long waveforms for complex orbits
- currently used in LISA pipeline development

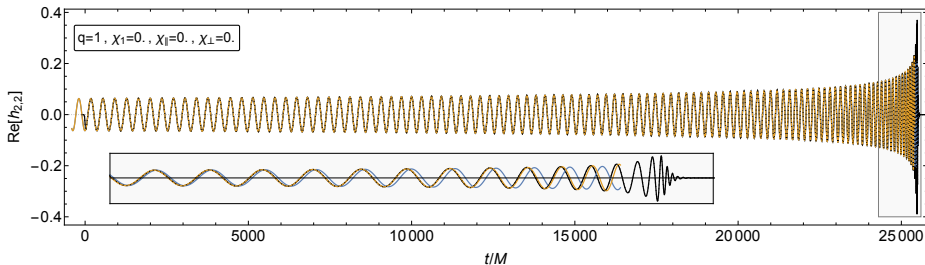


[Credit: P. Lynch]



# State of the art in accuracy: complete 1PA waveforms [AP, Warburton, Wardell, &co]

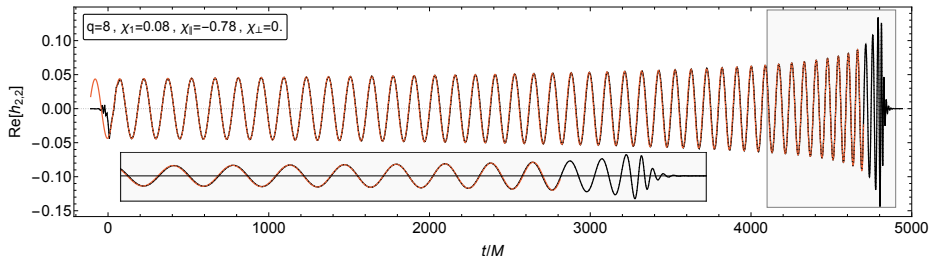
- quasicircular, generic secondary spin, linear primary spin
- comparison with NR:



[Mathews, AP, Warburton, Wardell]

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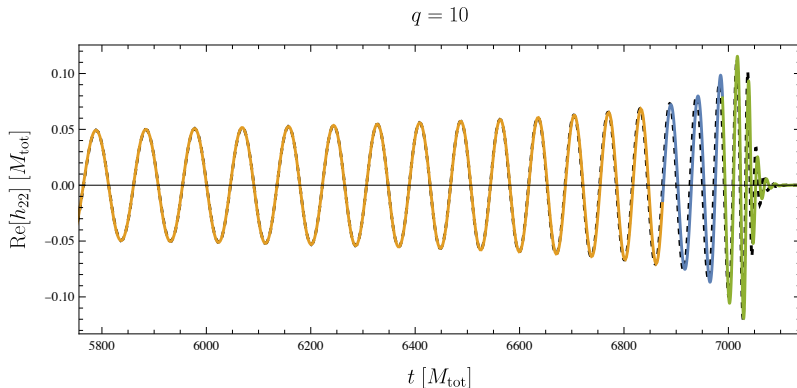


[Mathews, AP, Warburton, Wardell]

# Inspiral-merger-ringdown in self-force theory

[Kuchler, Compere, AP]

$\lesssim 100\text{ms}$  for complete, first-principles waveforms:





## Nonlinear ringdown

# Nonlinear effects in ringdown

- at linear order, exponentially damped “ringing”:

$$h_{\ell m}^{\text{ringdown}} = \sum_{n \geq 0} h_{\ell mn} e^{-i\omega_{\ell mn}^{\text{QNM}} t} + \dots$$

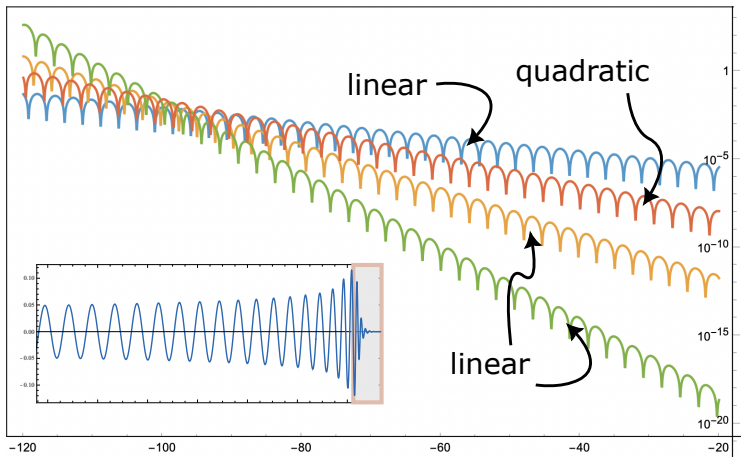
- at second order, “quadratic QNMs” (QQNMs) arise:

$$\omega_{\ell mn}^{\text{QQNM}} = \omega_{\ell_1 m_1 n_1}^{\text{QNM}} + \omega_{\ell_2 m_2 n_2}^{\text{QNM}}$$

- NR ringdown suggests strong QQNMs [London et al., Cheung et al., Mitman et al.,]

# Second-order ringdown calculations

[Ripley et al., Ma et al., Bourg et al., Bucciotti et al., ...]



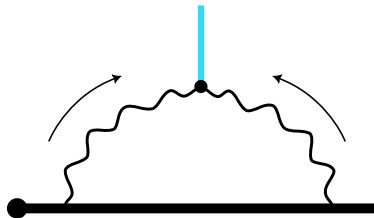
[Bourg, Kuchler, Leather, AP, et al.]

## Gravitational memory

# Gravitational memory

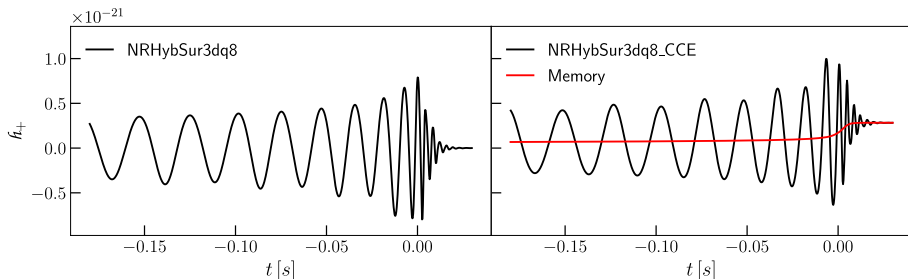
memory is...

- permanent displacement after passage of GW
- GWs interacting to source a “soft” GW
- evolution of conserved charges associated with asymptotic symmetries

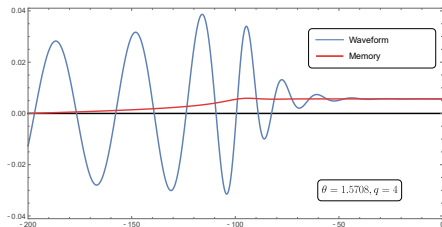


# Prominent memory in the merger

## Memory in NR [Mitman et al.]



## Memory in self-force theory [Kuchler, Cunningham, AP, et al.]



# Conclusion

- future of GW astronomy depends on new theoretical tools
- nonlinear self-force & BH perturbation theory:
  - essential for modelling EMRIs
  - powerful tool for studying black hole binaries with any mass ratio
  - can now precisely probe nonlinear (quadratic, cubic, secular) effects

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Thank you for listening