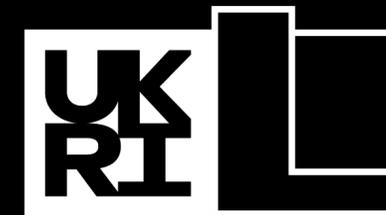


Topography, Turbulence, and Internal Waves: The Small-Scale Physics Driving the Global Ocean Circulation

Lois Baker
University of Edinburgh
6th January 2026
New Directions in Theoretical Physics

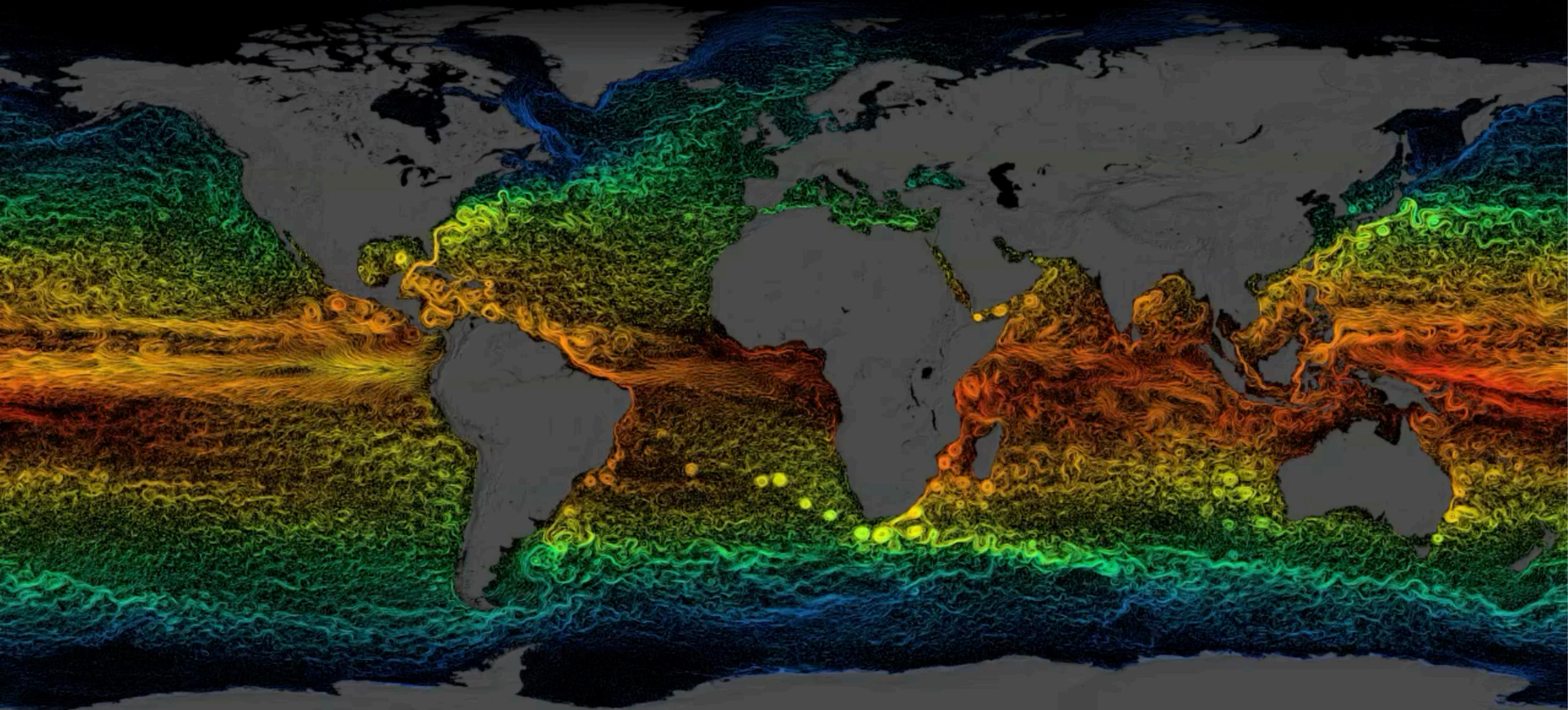


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Engineering and
Physical Sciences
Research Council

The Global Ocean: Nonlinear physics across scales

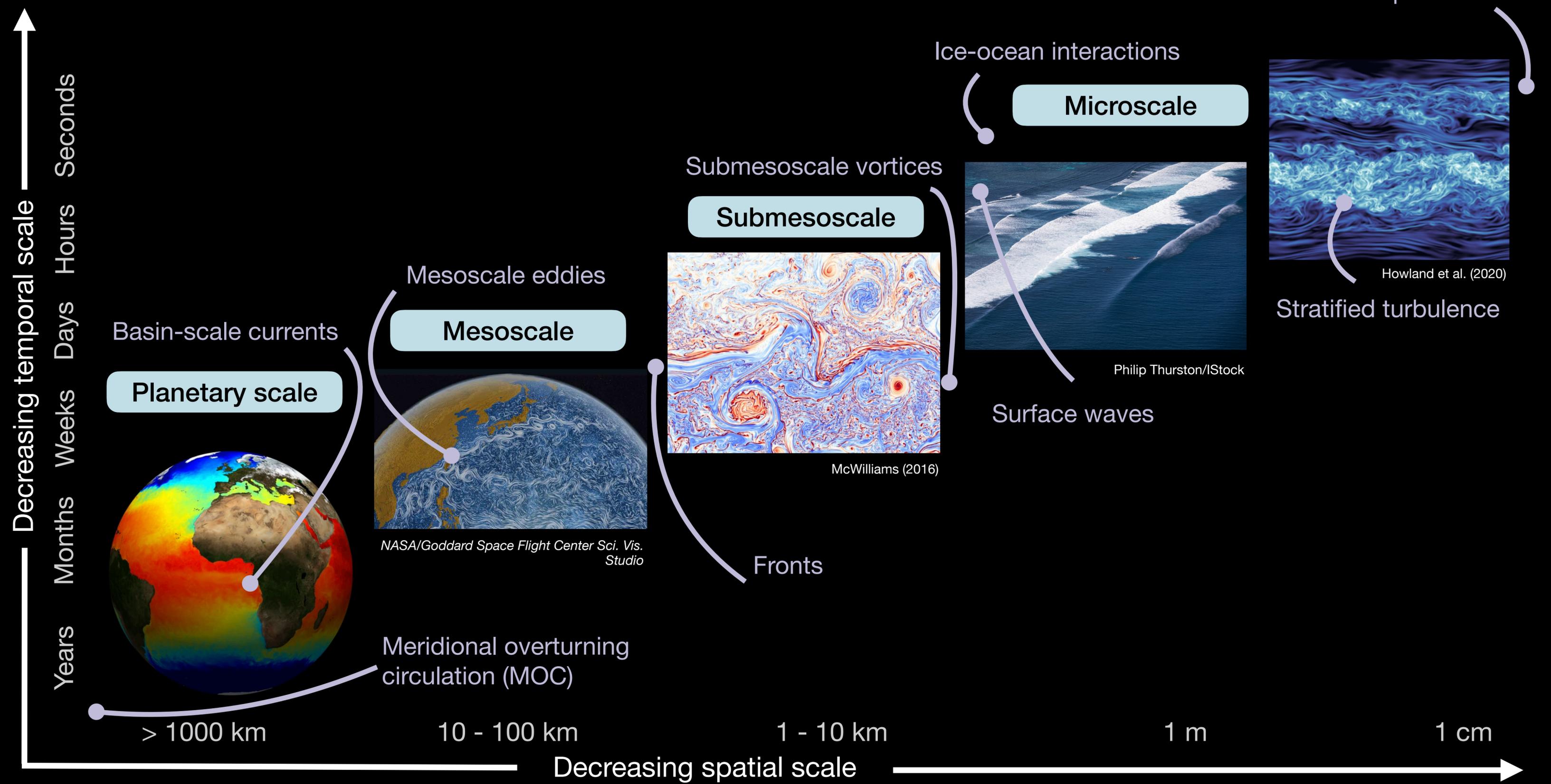


Sea Surface Temp (C)



Oceanic dynamics vary over ~ 8 orders of magnitude

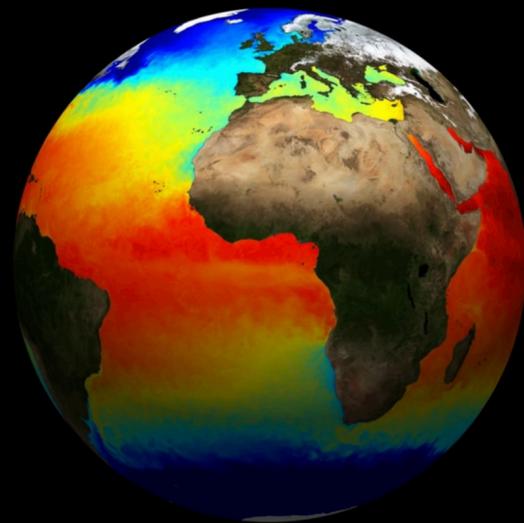
The scales of the ocean



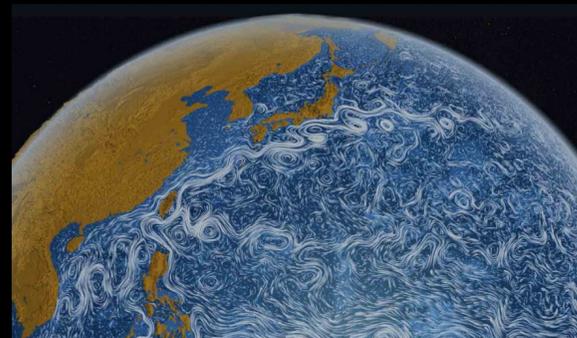
The dynamics of the ocean

Decreasing temporal scale

Planetary scale

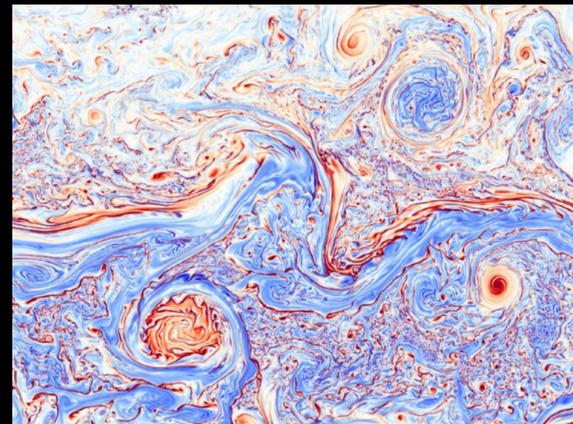


Mesoscale



NASA/Goddard Space Flight Center Sci. Vis. Studio

Submesoscale

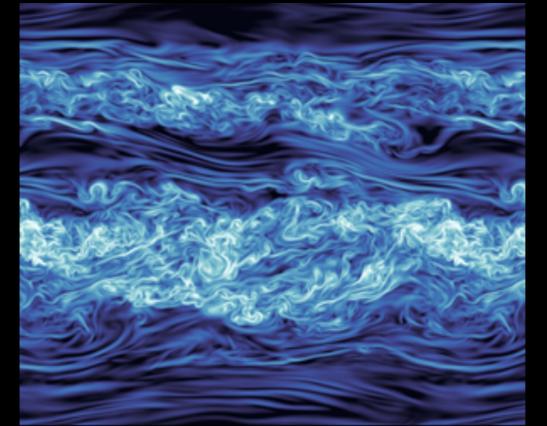


McWilliams (2016)

Microscale



Philip Thurston/IStock



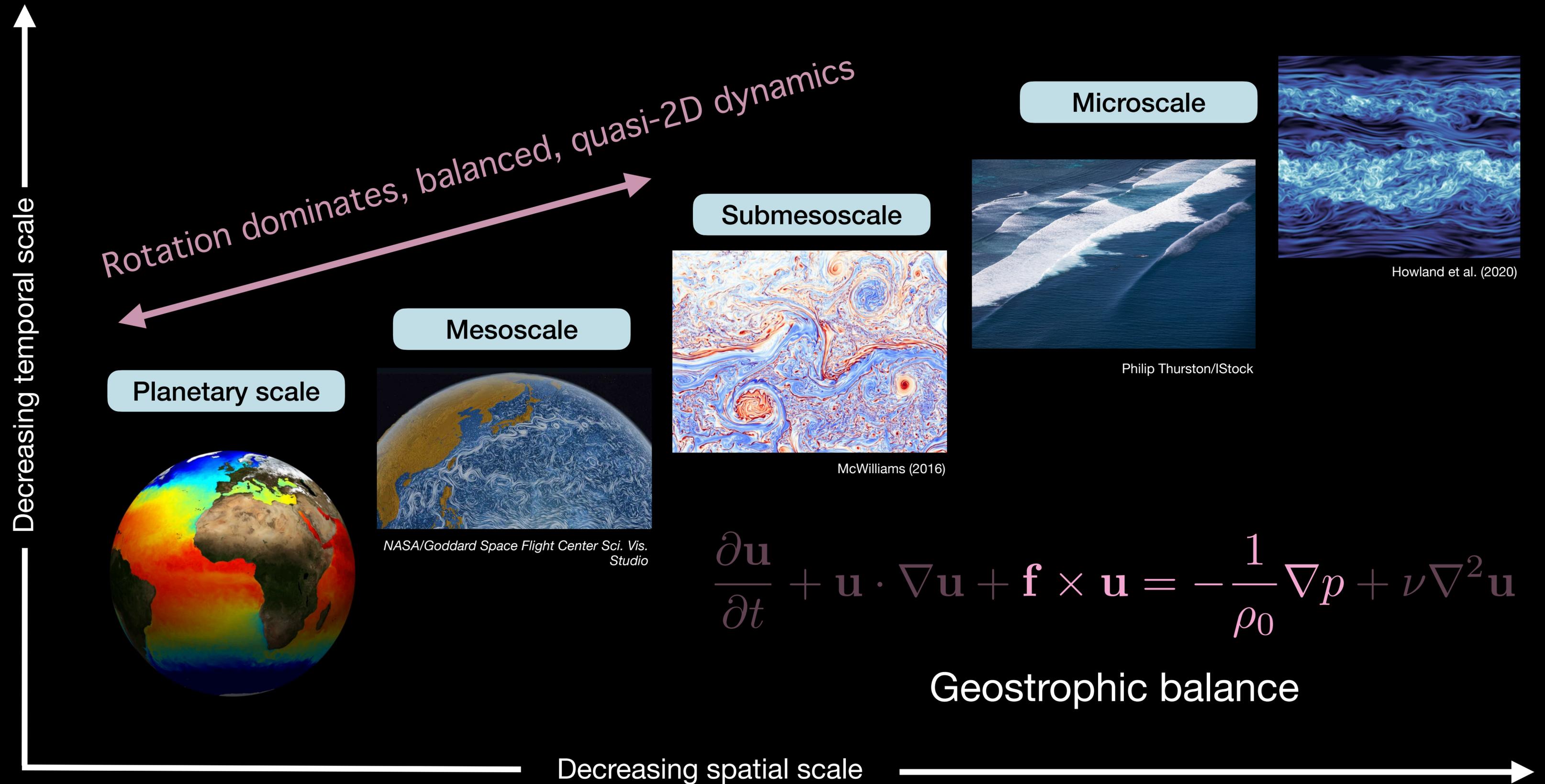
Howland et al. (2020)

$$\frac{\partial \mathbf{u}}{\partial t} + \mathbf{u} \cdot \nabla \mathbf{u} + \mathbf{f} \times \mathbf{u} = -\frac{1}{\rho_0} \nabla p + \nu \nabla^2 \mathbf{u}$$

Navier Stokes equation for horizontal momentum

Decreasing spatial scale

The dynamics of the ocean



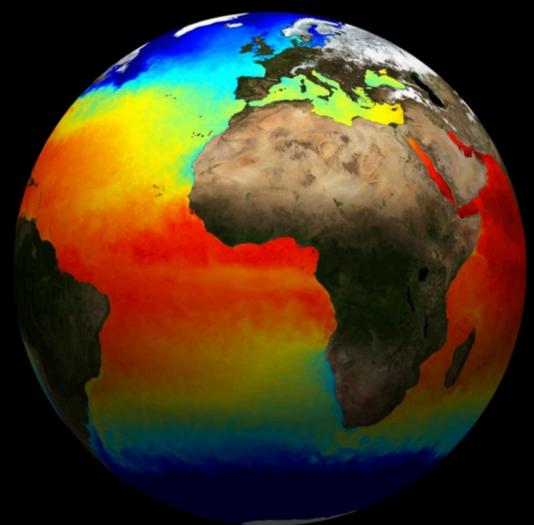
The dynamics of the ocean

Decreasing temporal scale

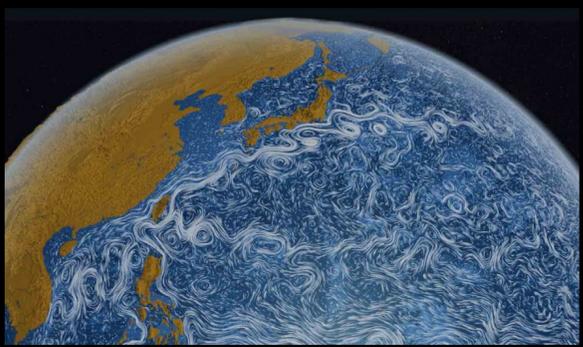
Rotation dominates, balanced, quasi-2D dynamics

Rotation negligible

Planetary scale

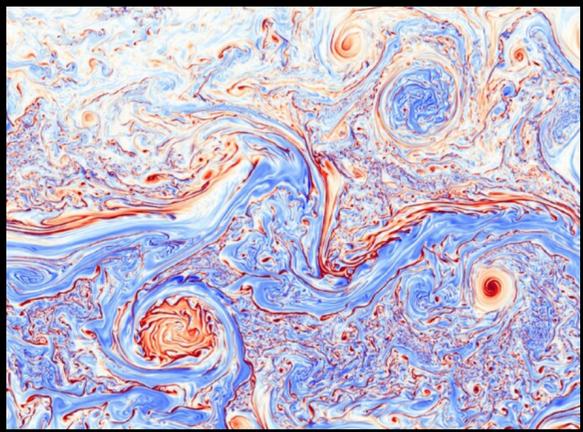


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Submesoscale

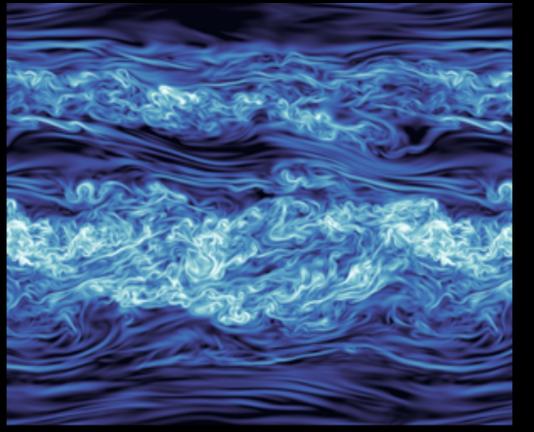


McWilliams (2016)



Philip Thurston/IStock

Microscale

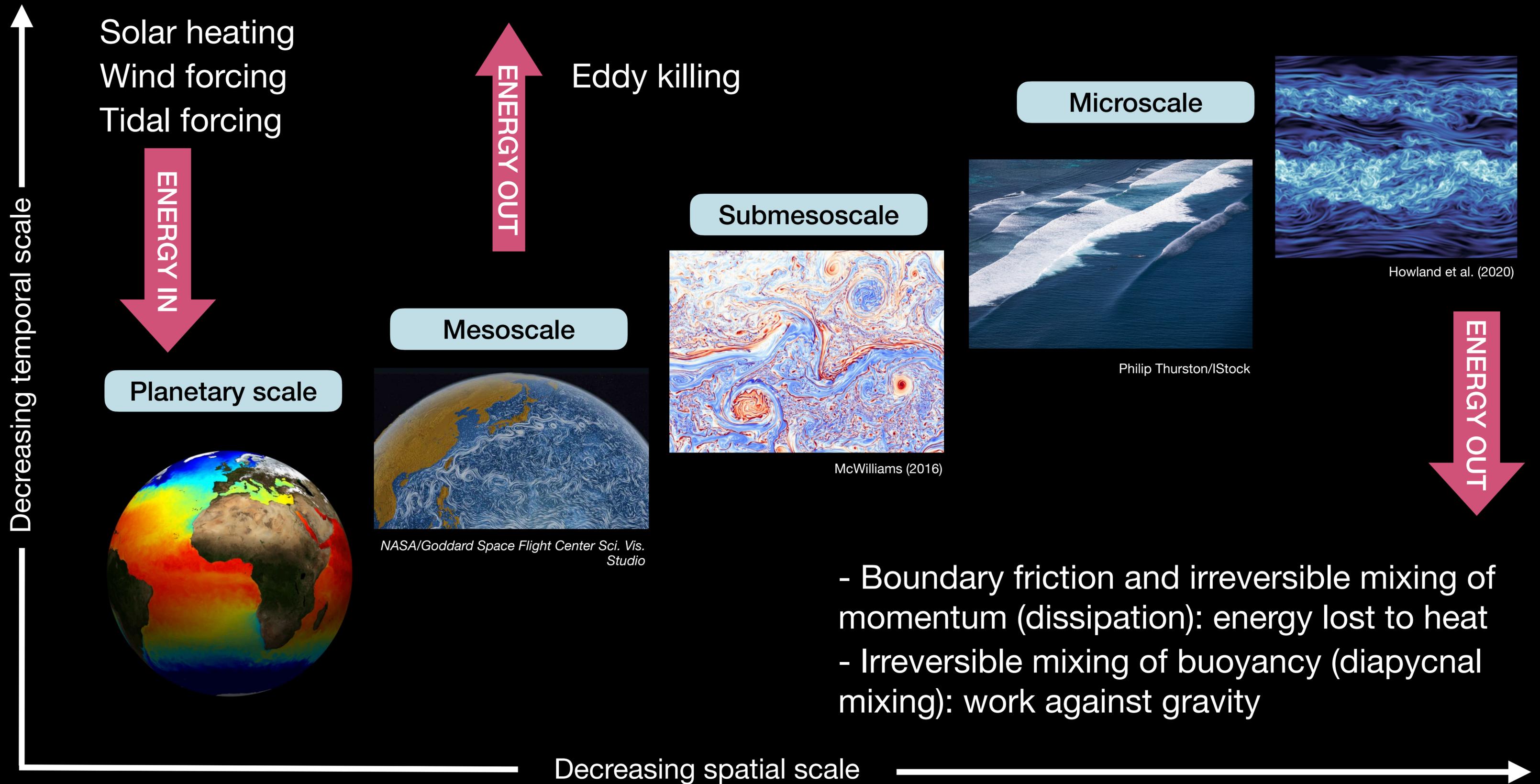


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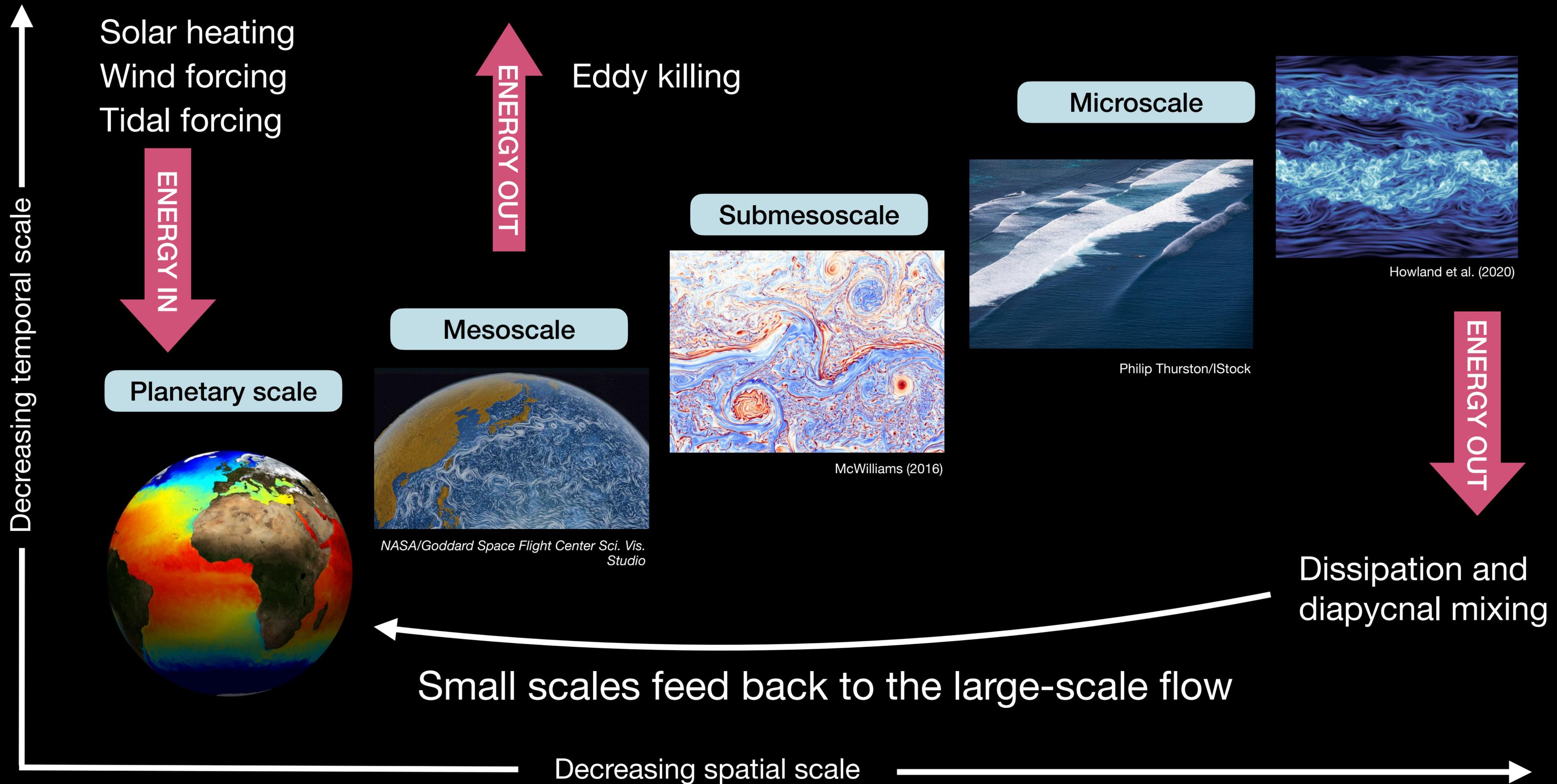
Decreasing spatial scale

Energy transfers across scales

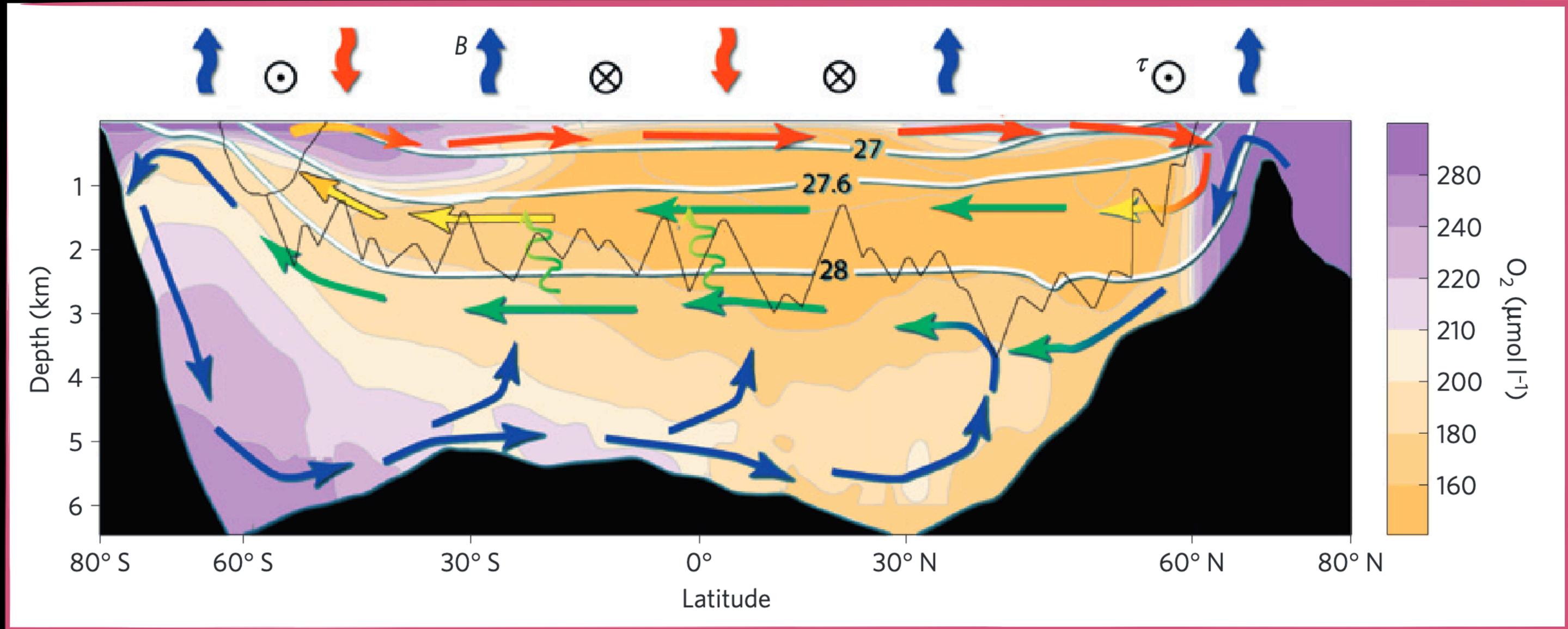


- Boundary friction and irreversible mixing of momentum (dissipation): energy lost to heat
- Irreversible mixing of buoyancy (diapycnal mixing): work against gravity

Energy transfers across scales



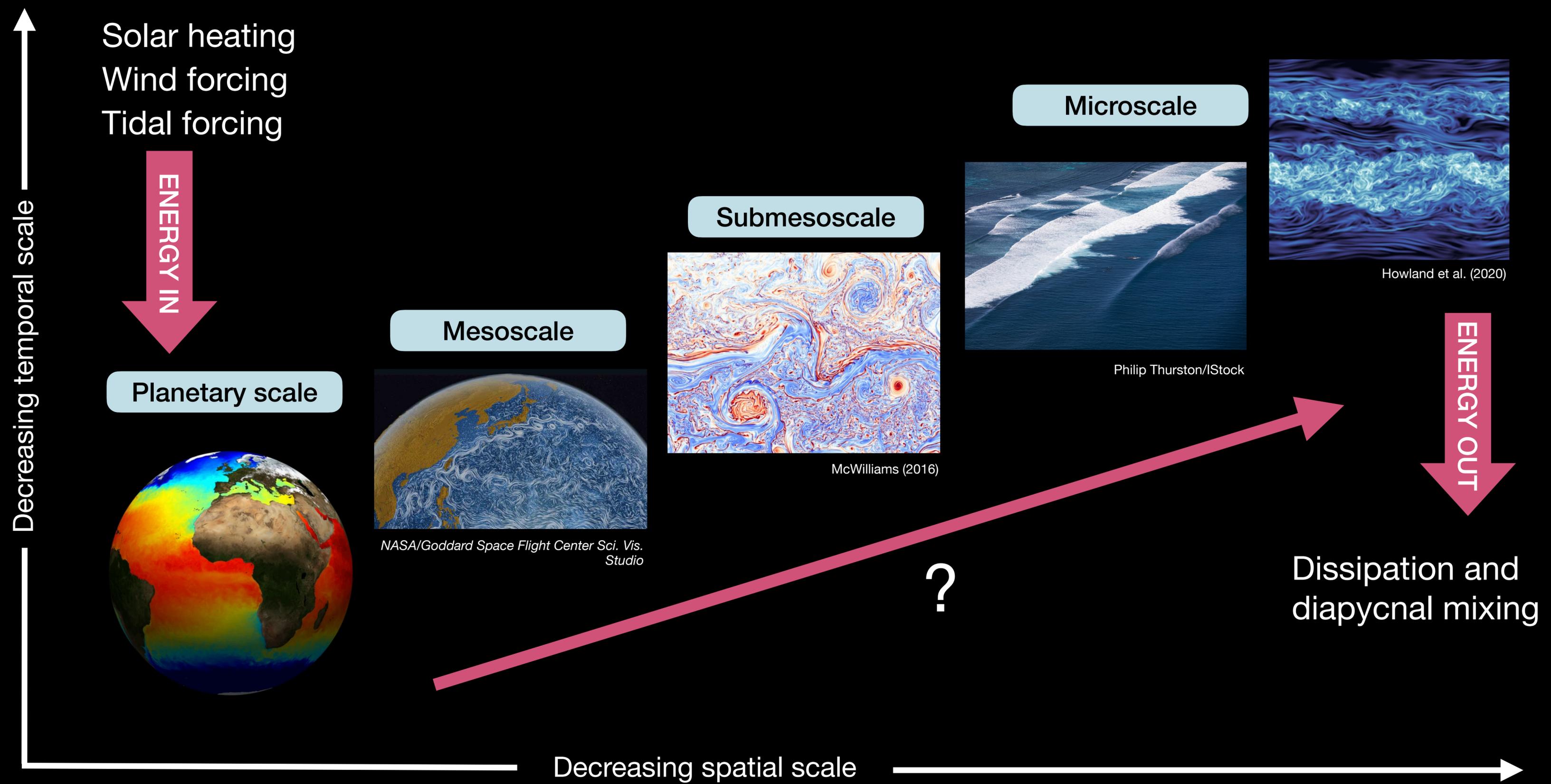
Why mixing matters: the buoyancy budget of the ocean



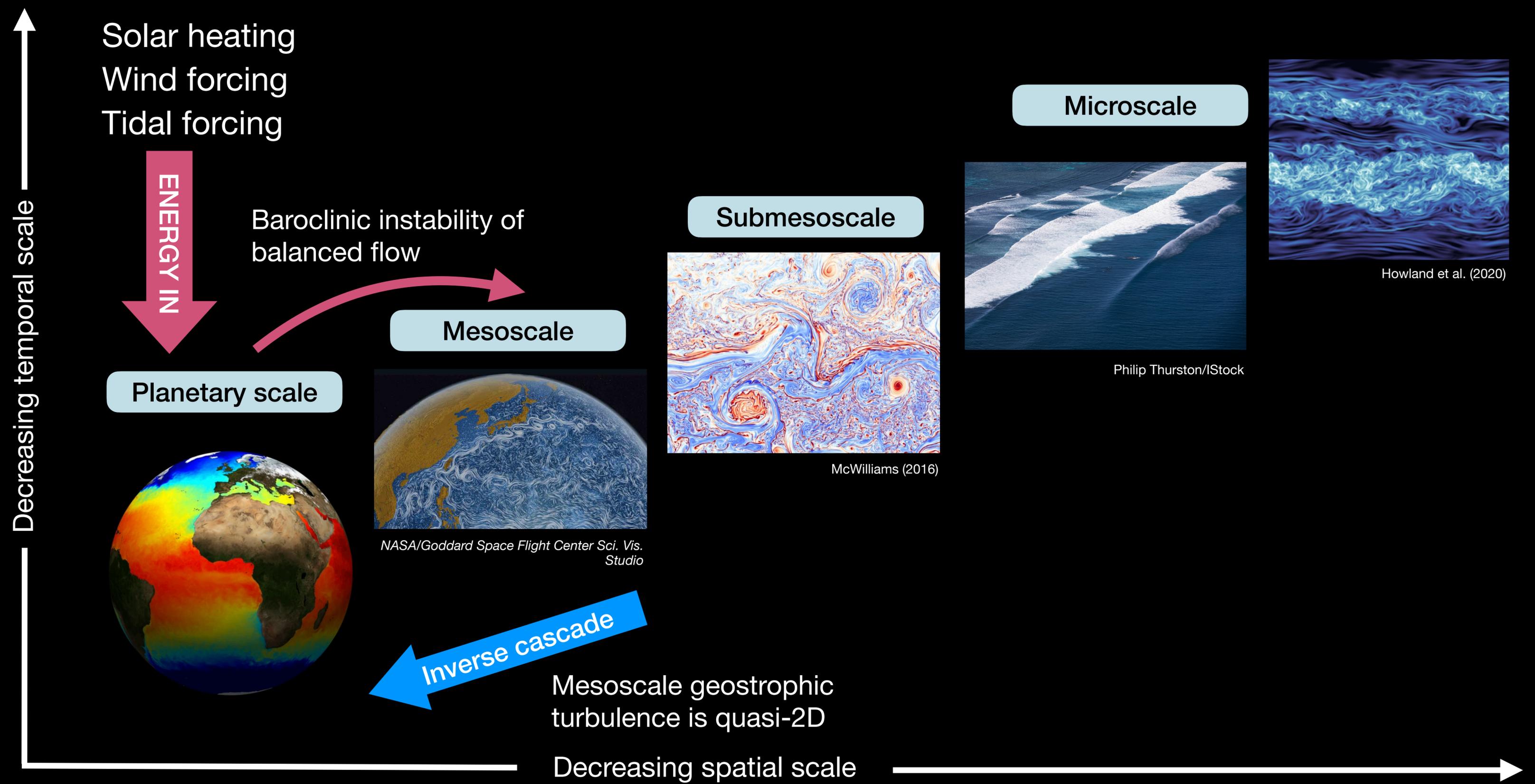
Marshall & Speer (2012)

Diapycnal mixing in the deep ocean is key to the thermohaline (overturning) circulation, which controls the ventilation of heat and carbon between the abyssal and surface ocean

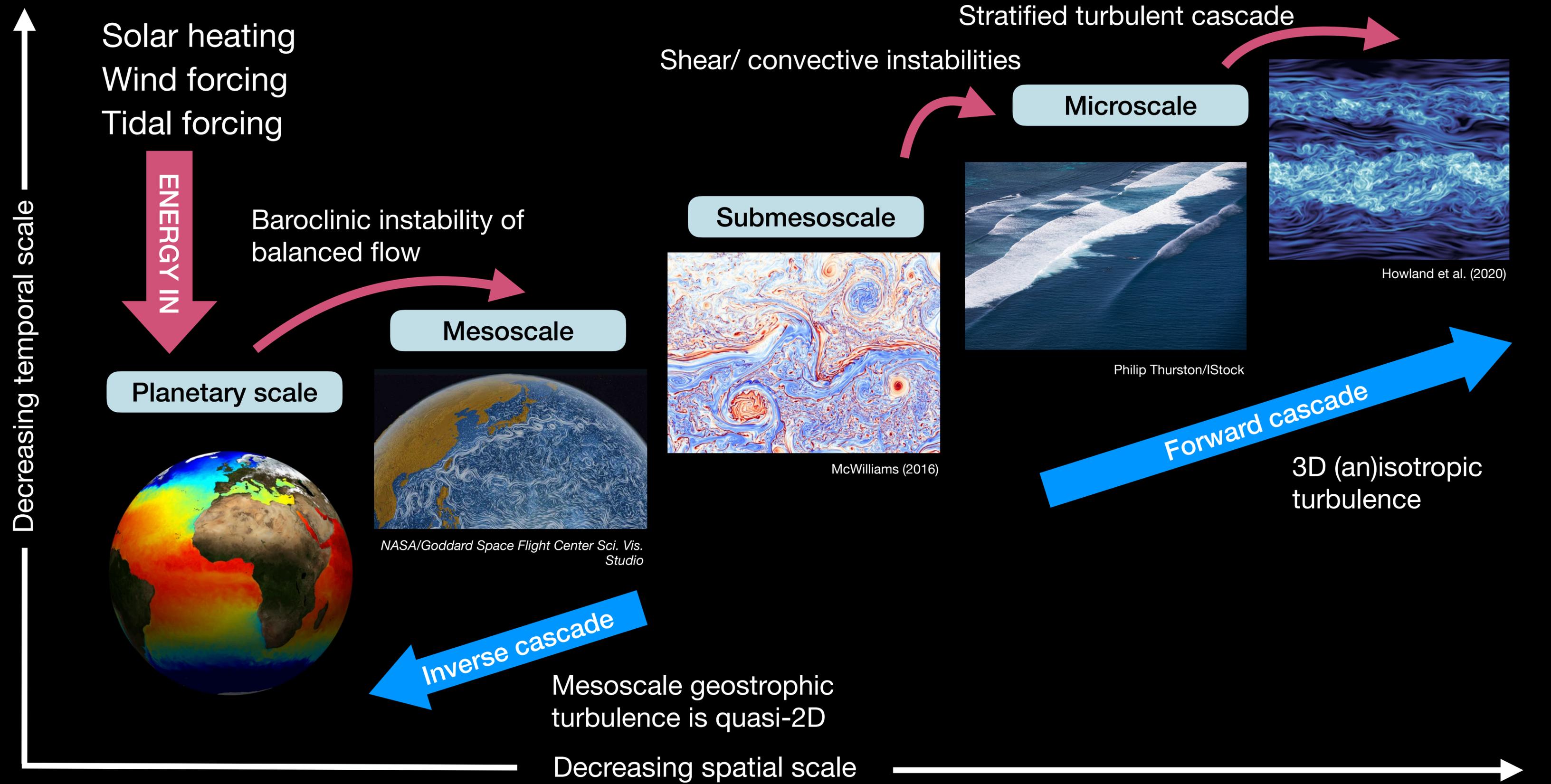
How does energy cascade from large to small scales?



How does energy cascade from large to small scales?

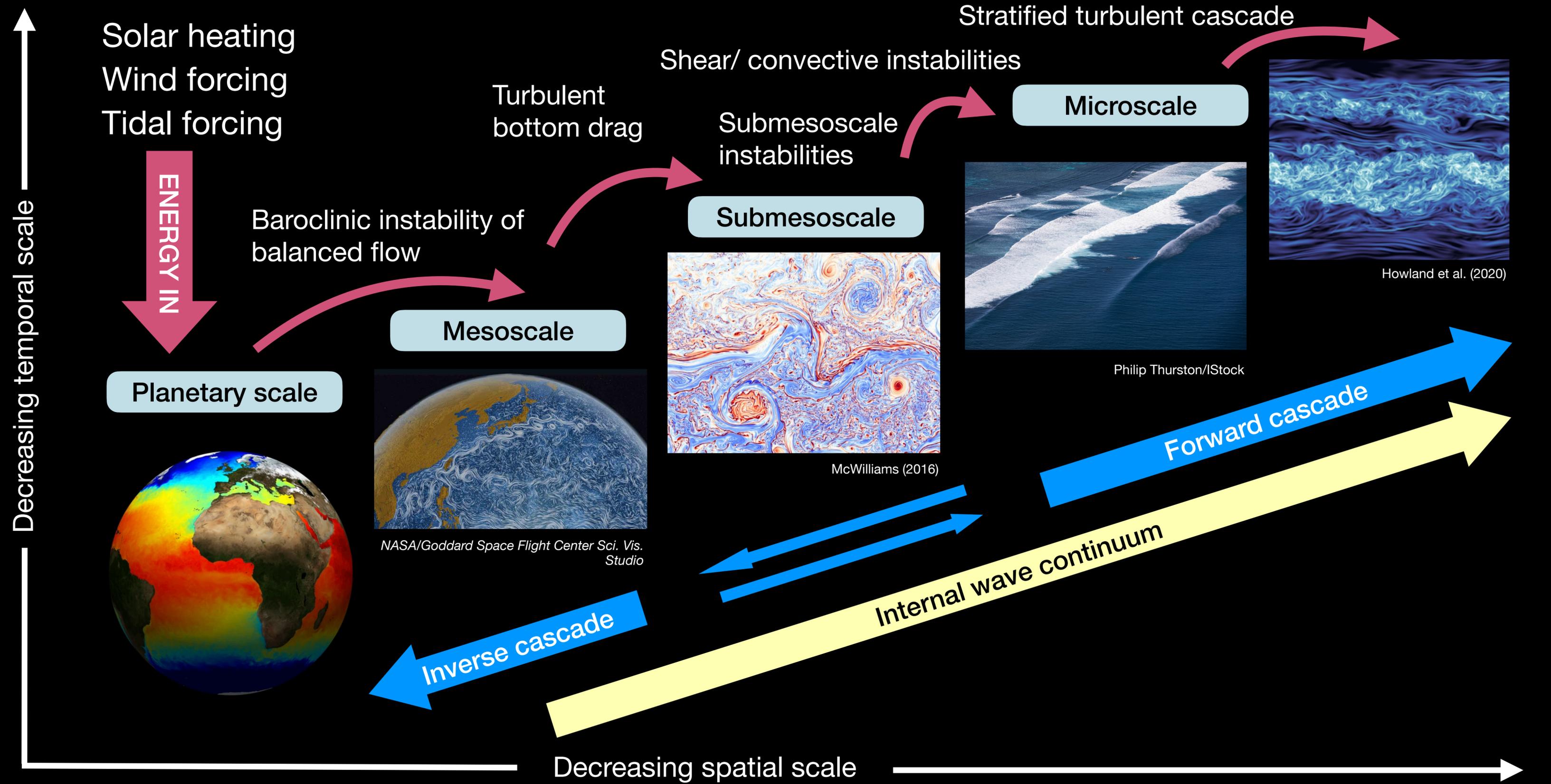


How does energy cascade from large to small scales?



How does energy cascade from large to small scales?

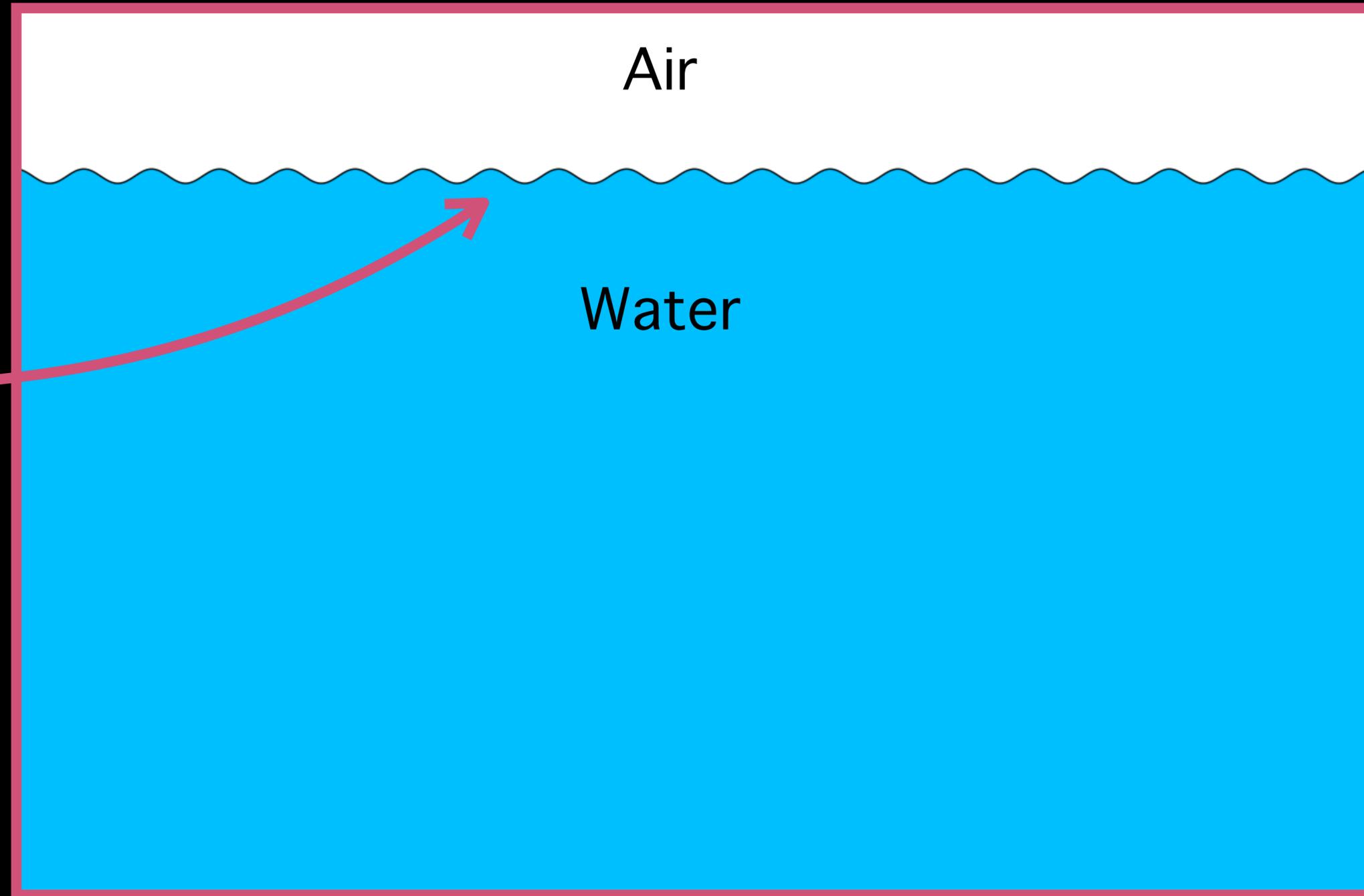
3D isotropic turbulent cascade



Internal gravity waves in the ocean



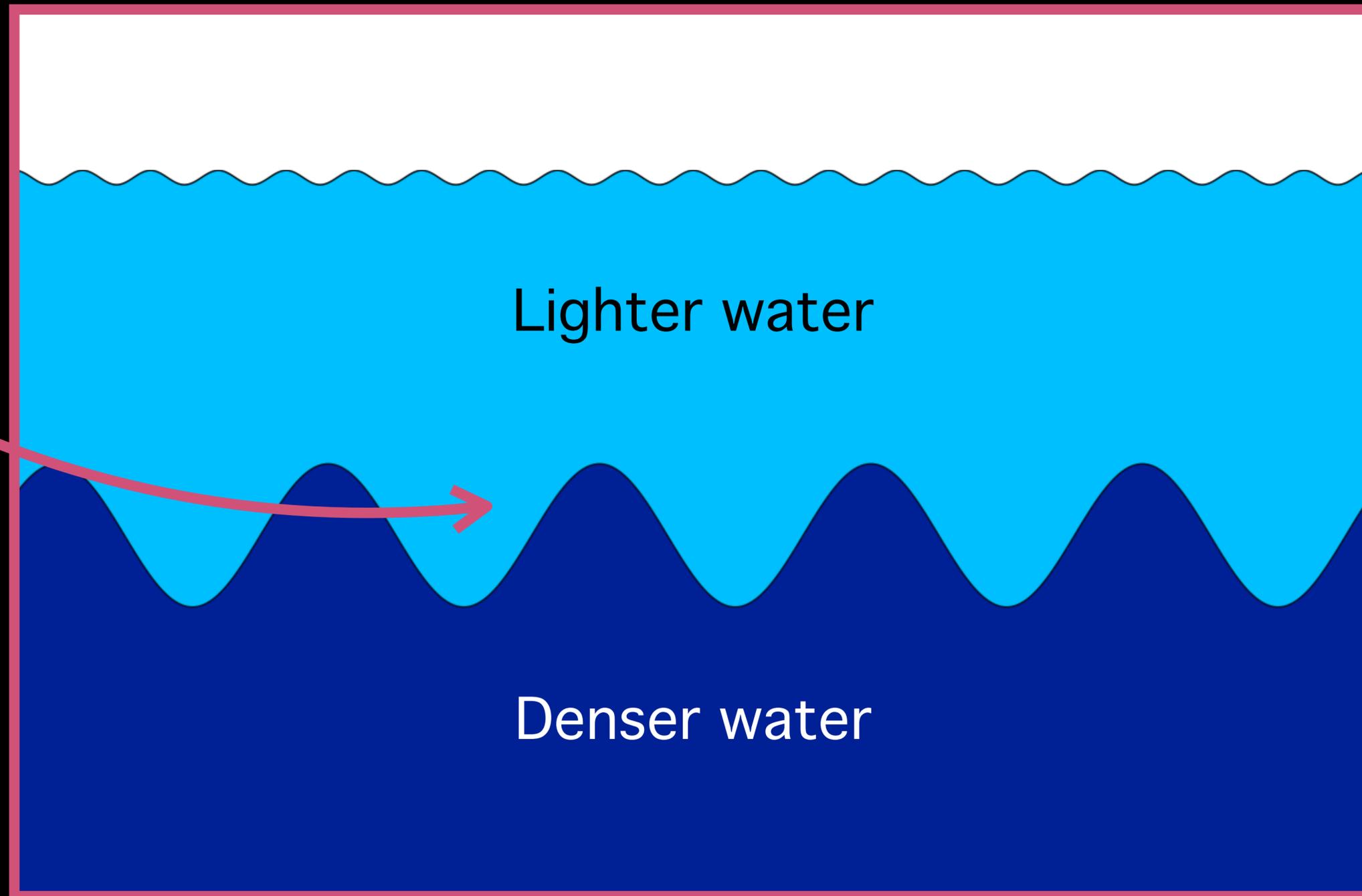
Surface
gravity waves



Internal gravity waves in the ocean



Internal
gravity waves



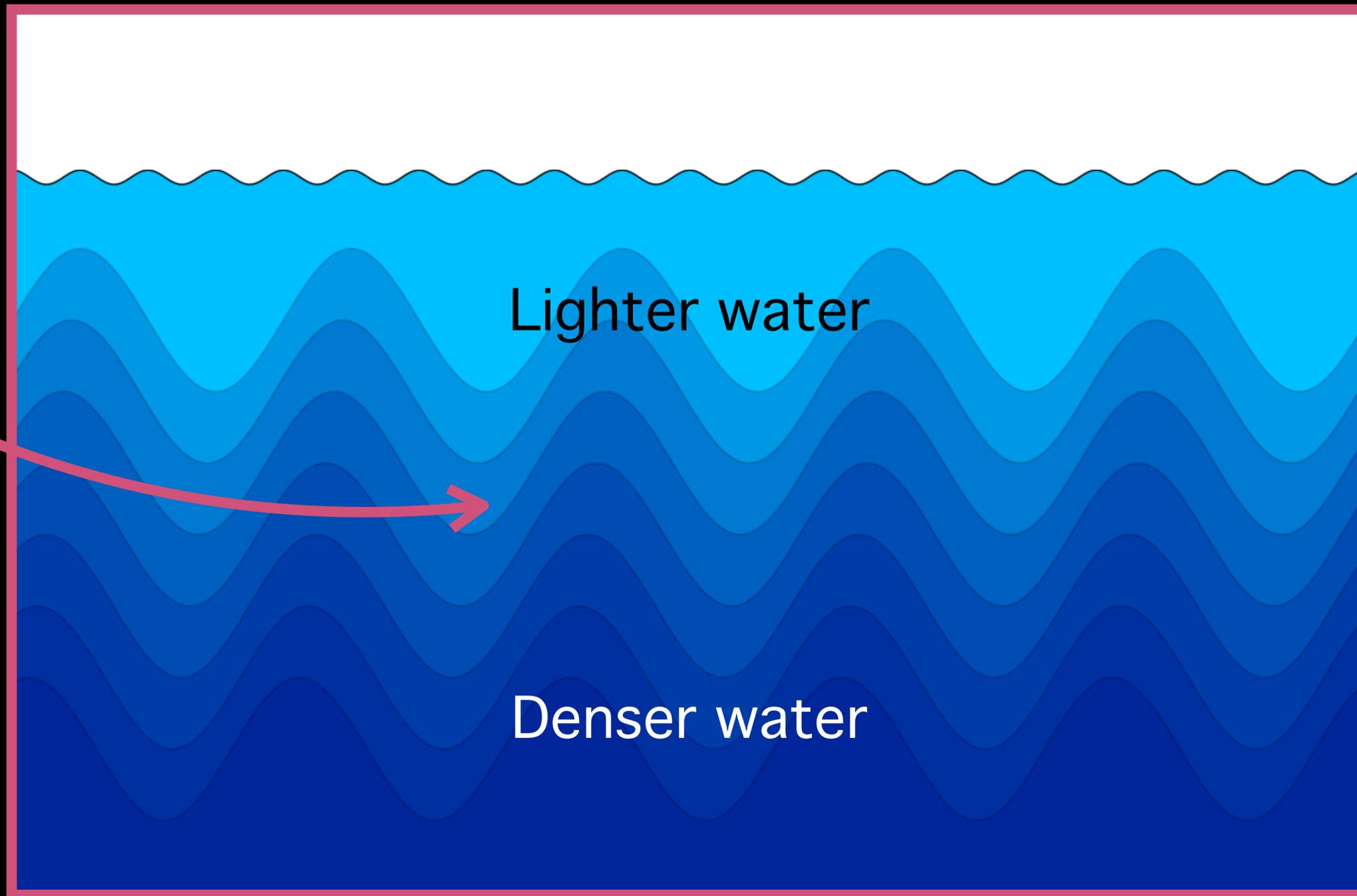
Lighter water

Denser water

Internal gravity waves in the ocean



Internal
gravity waves



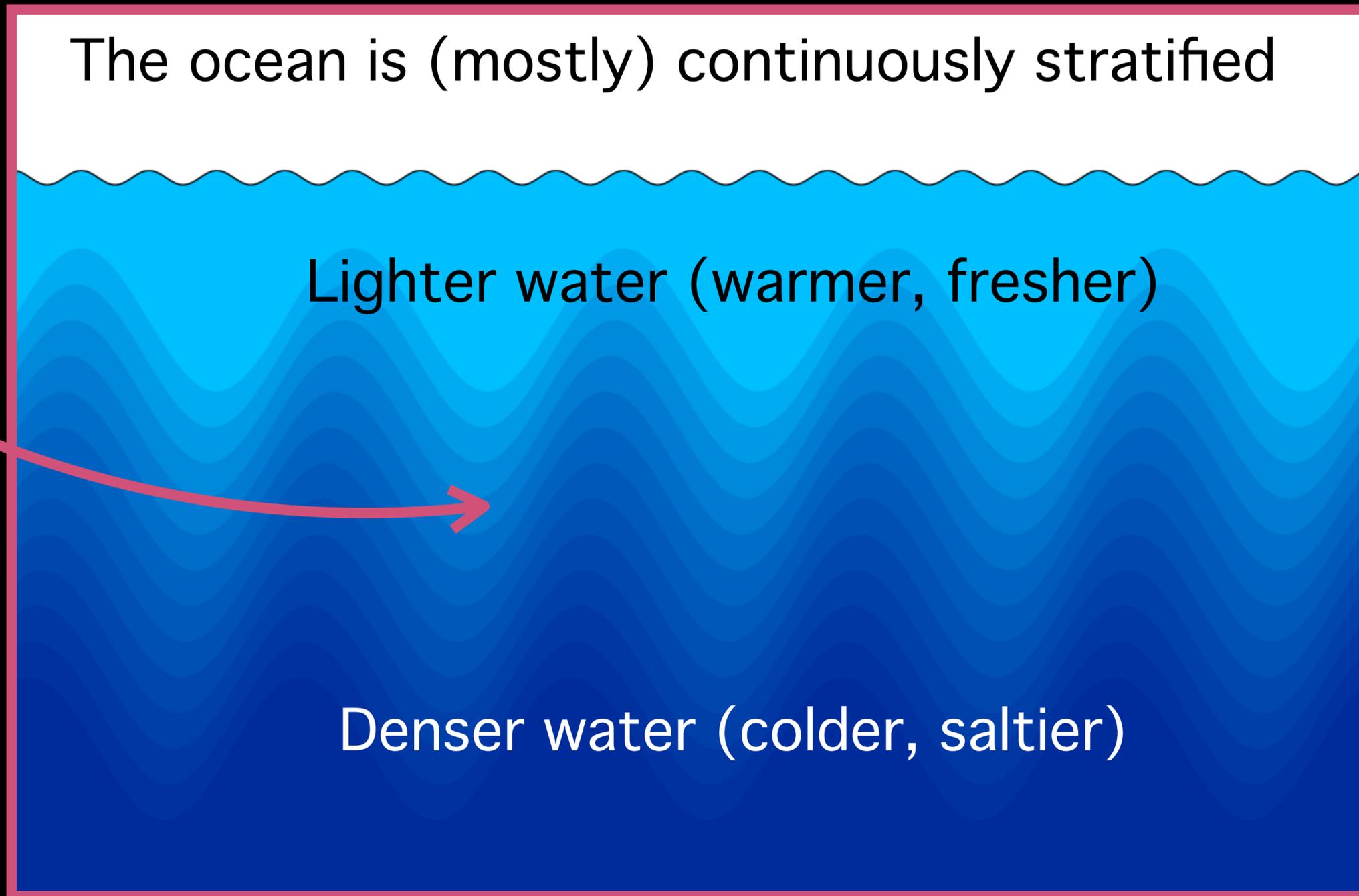
Lighter water

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Internal gravity waves in the ocean

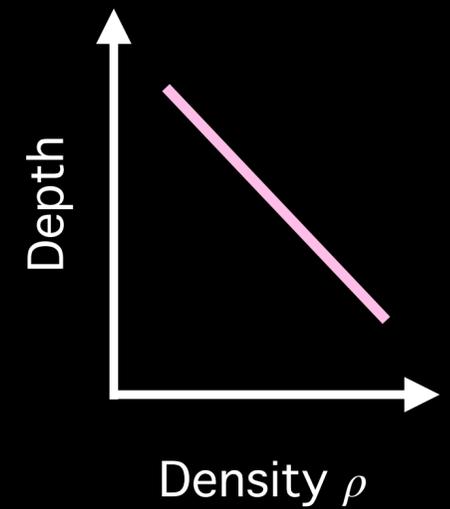


Internal
gravity waves

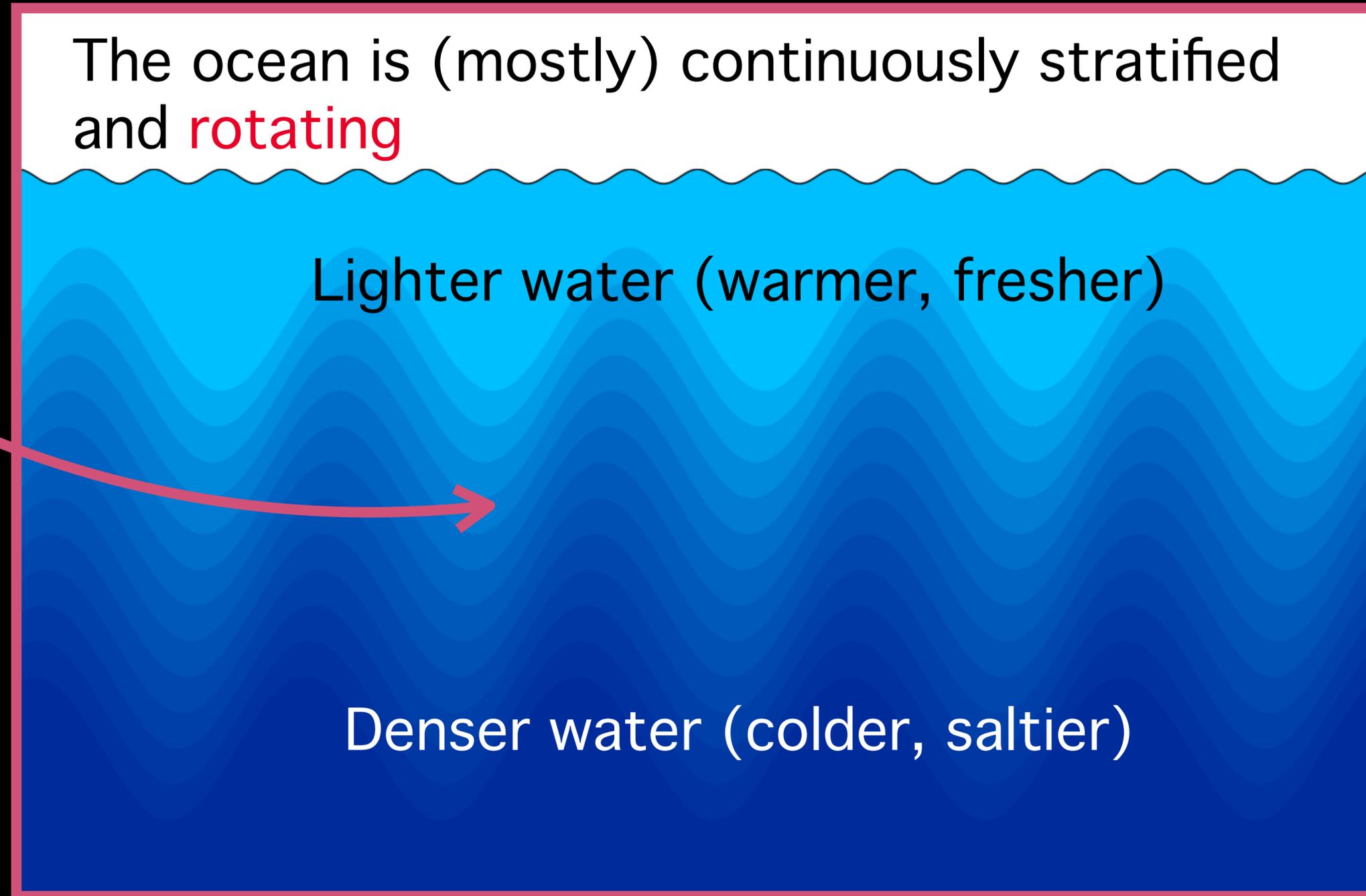


Buoyancy
frequency N

$$N^2 = -\frac{g}{\rho_0} \frac{\partial \rho}{\partial z} > 0$$



Internal gravity waves in the ocean



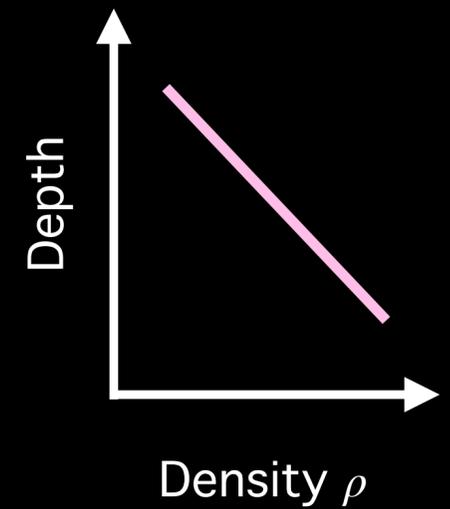
Internal inertia-gravity waves



Coriolis frequency
 f

Buoyancy frequency N

$$N^2 = -\frac{g}{\rho_0} \frac{\partial \rho}{\partial z} > 0$$



Internal gravity waves in the ocean



Internal inertia-gravity waves



Coriolis frequency
 f

The ocean is (mostly) continuously stratified and **rotating**

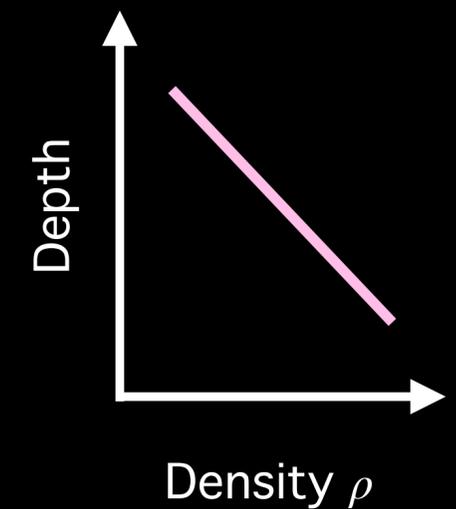
Lighter water (warmer, fresher)

$$f < \omega < N$$

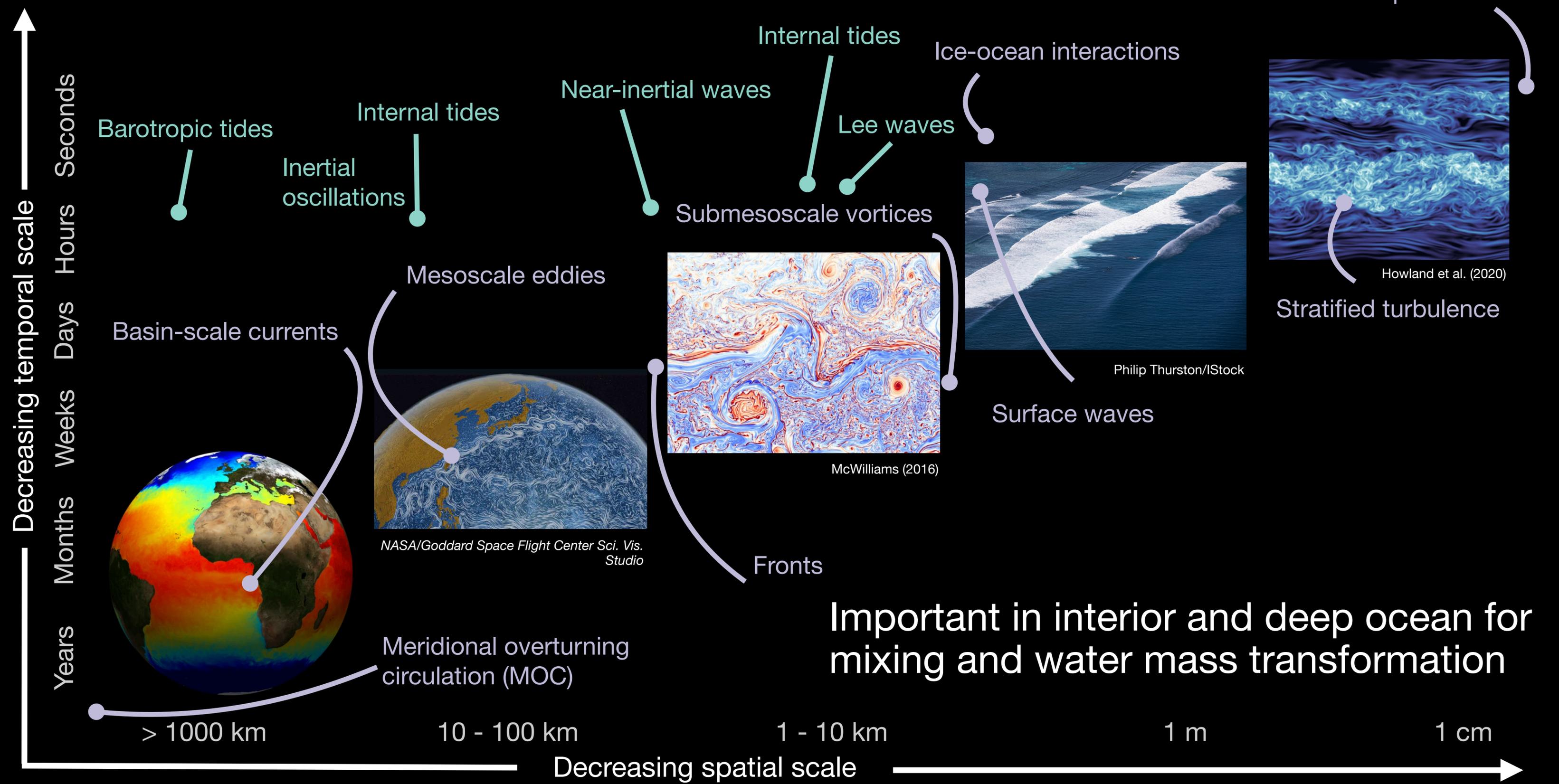
Denser water (colder, saltier)

Buoyancy frequency N

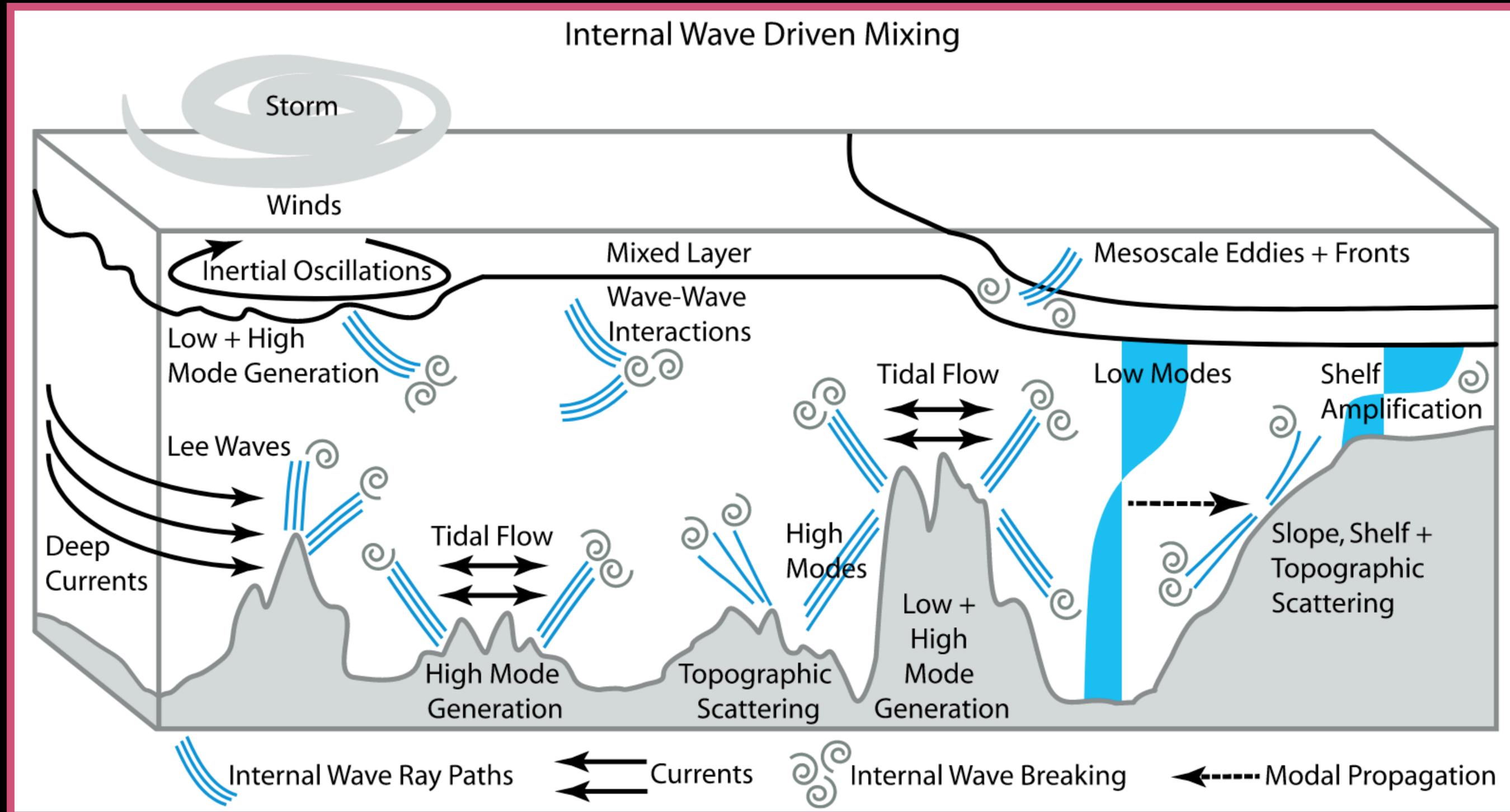
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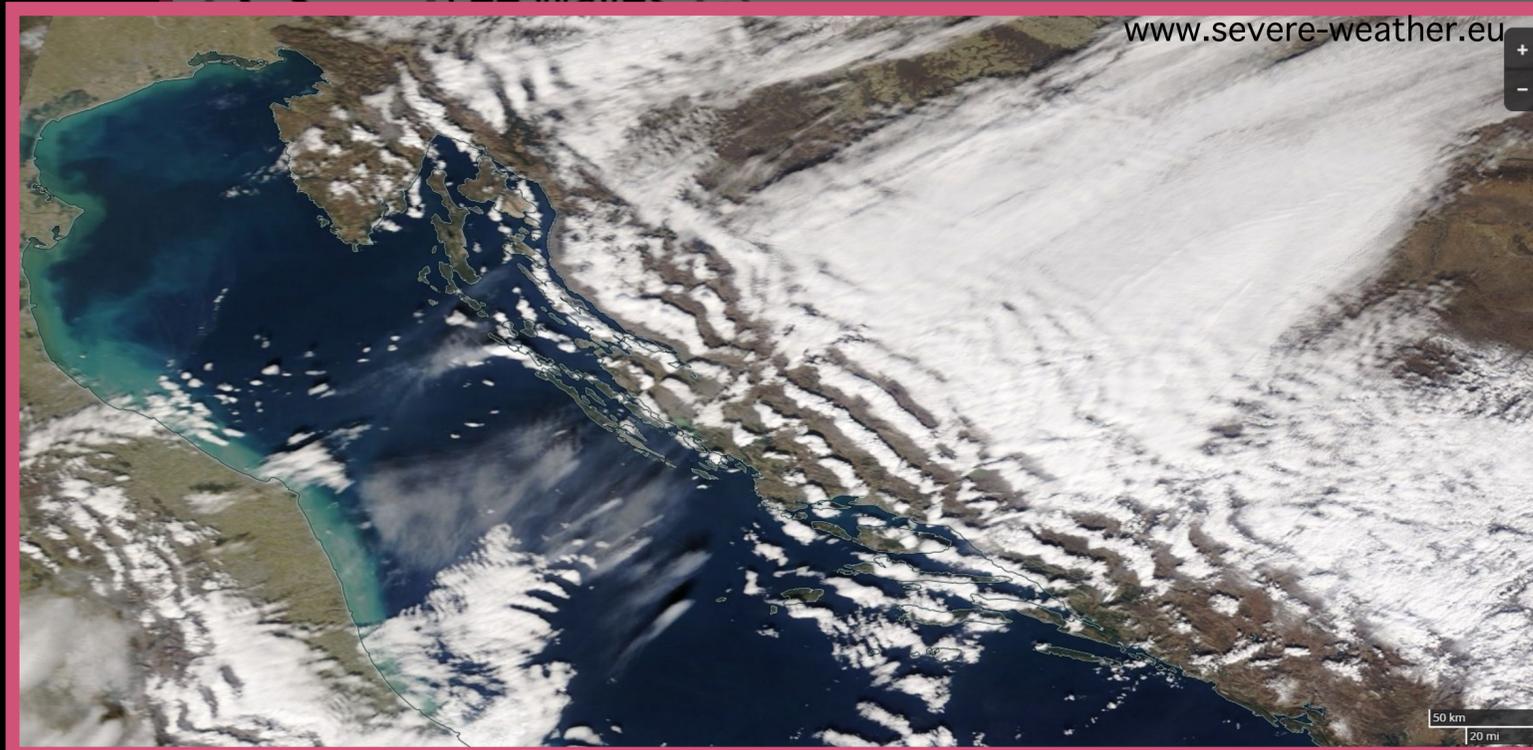
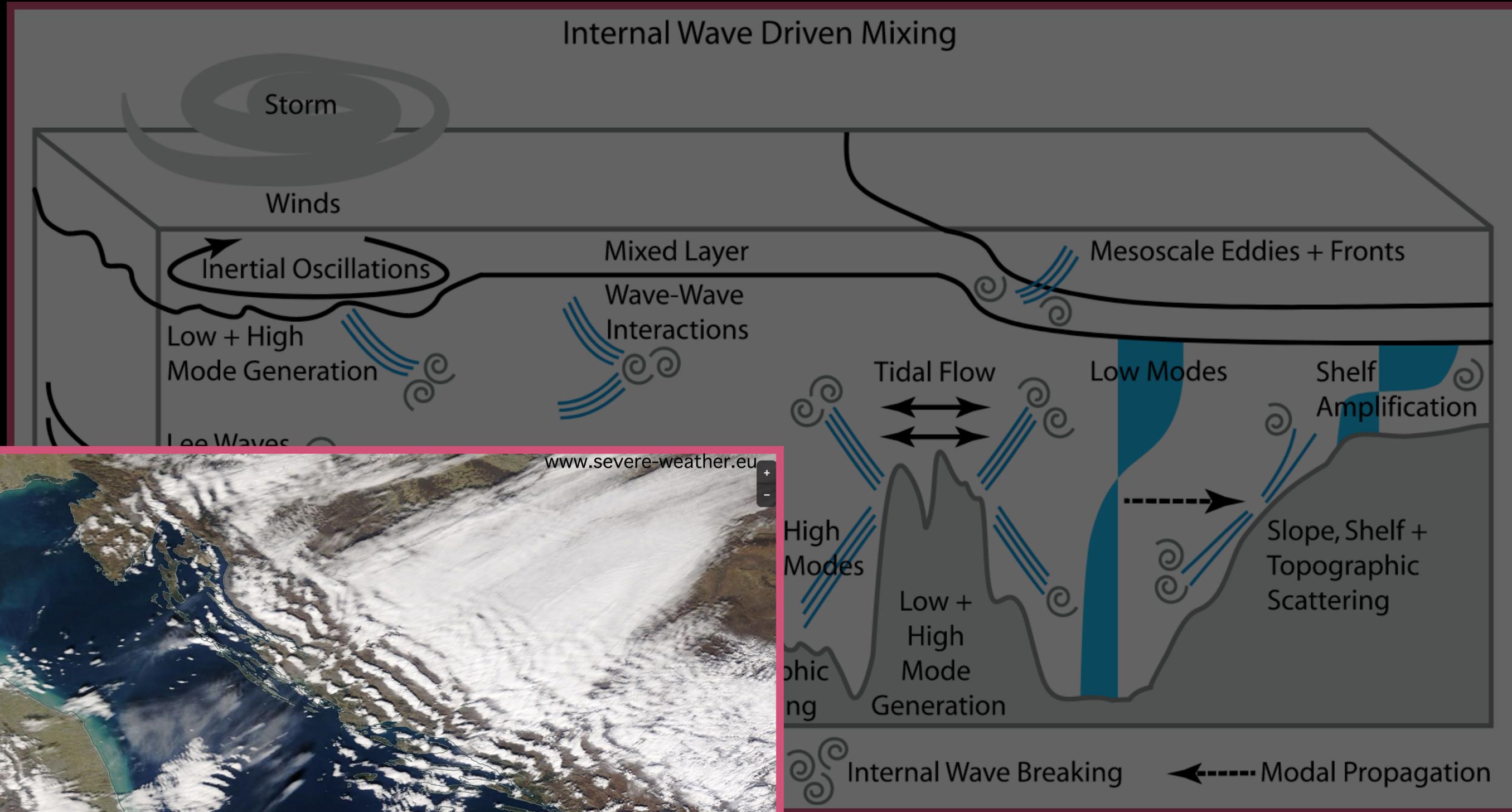
Internal waves fill the ocean



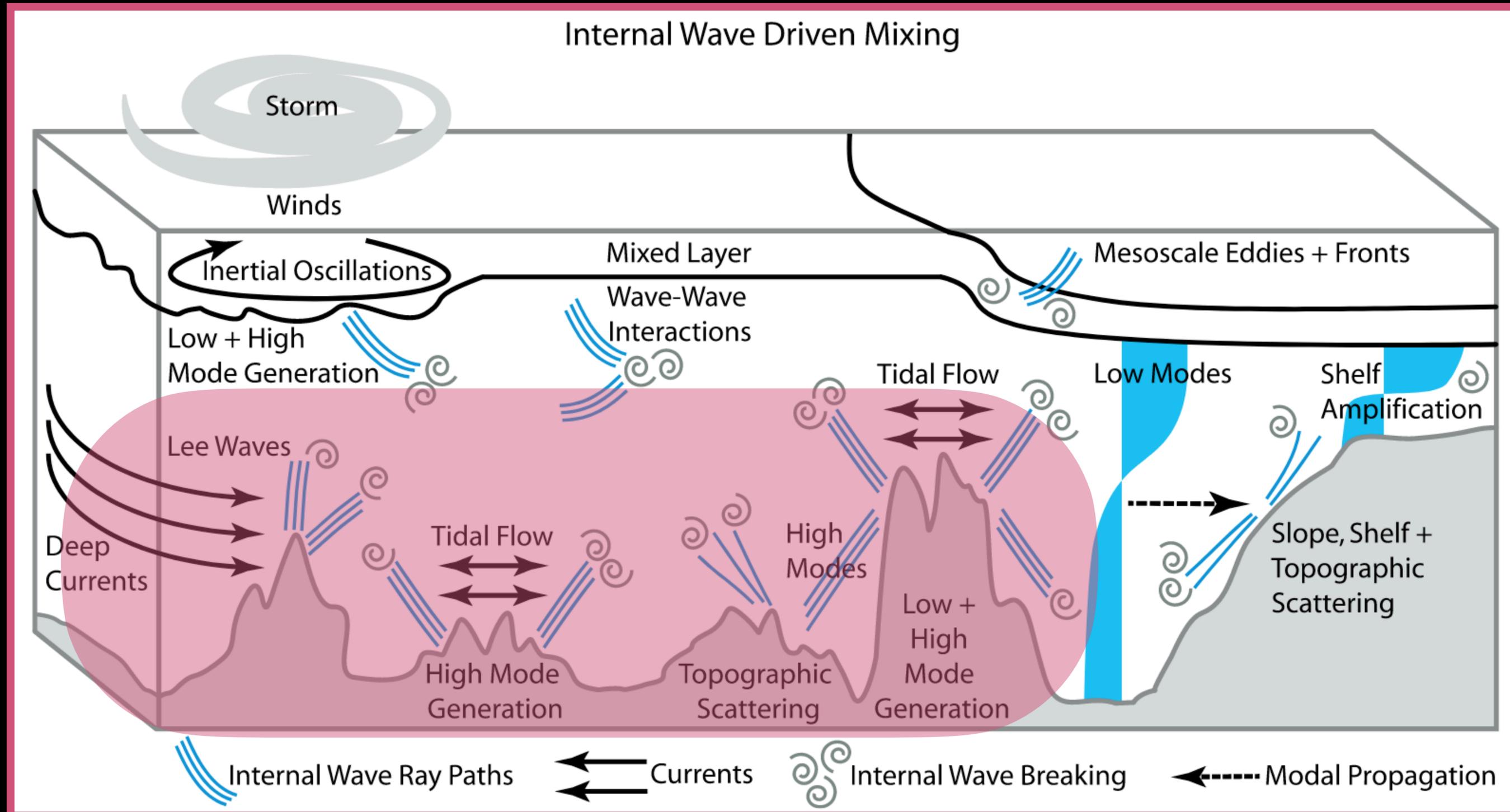
Internal waves in the ocean



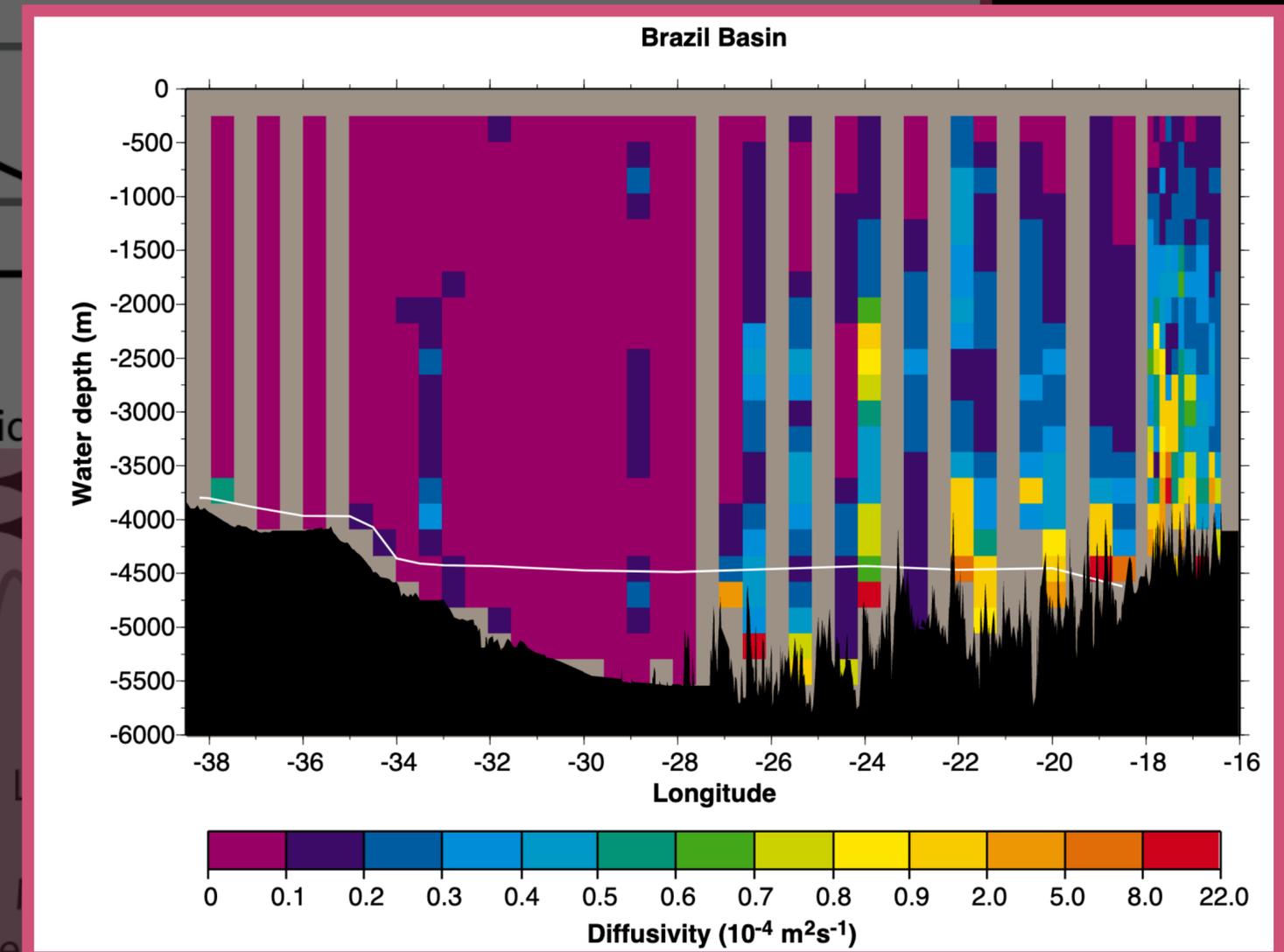
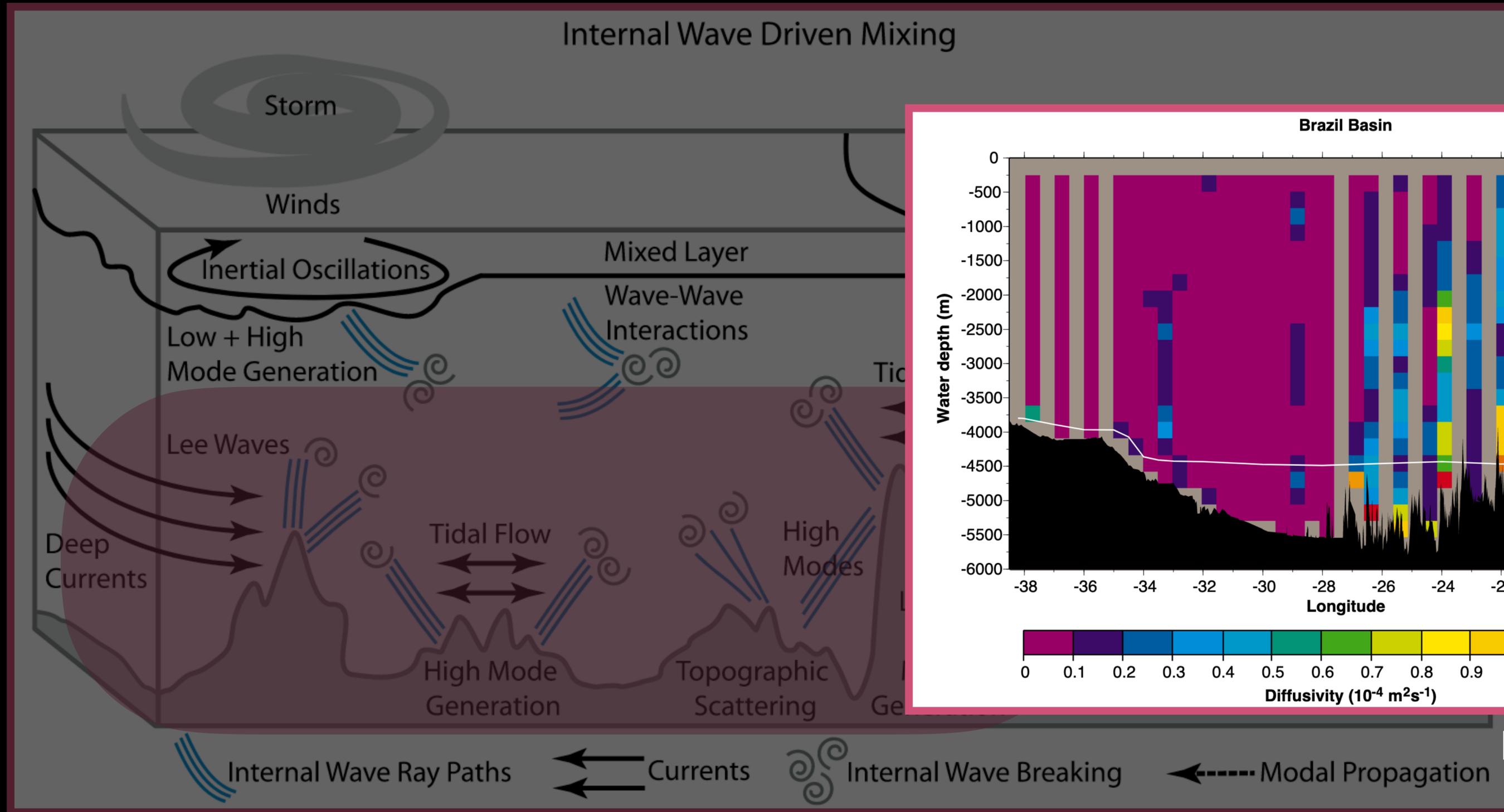
Internal waves in the atmosphere



Internal waves in the ocean



Internal waves in the ocean

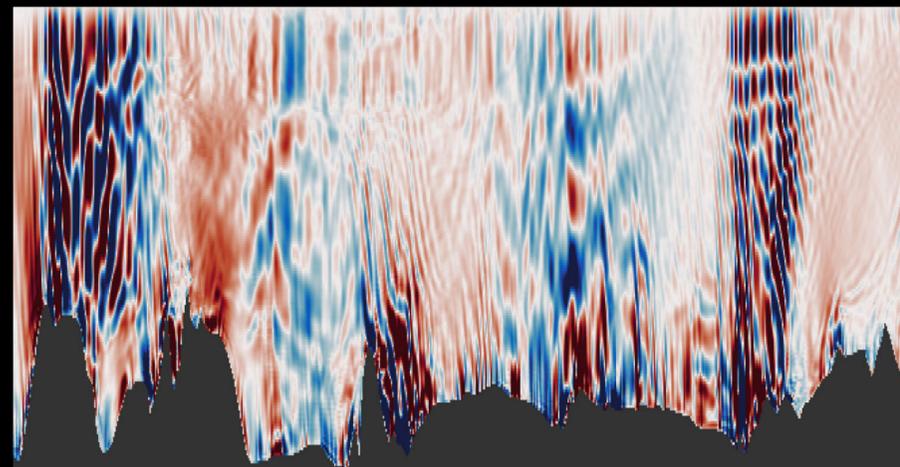


Polzin 1997

Representing internal waves in climate models



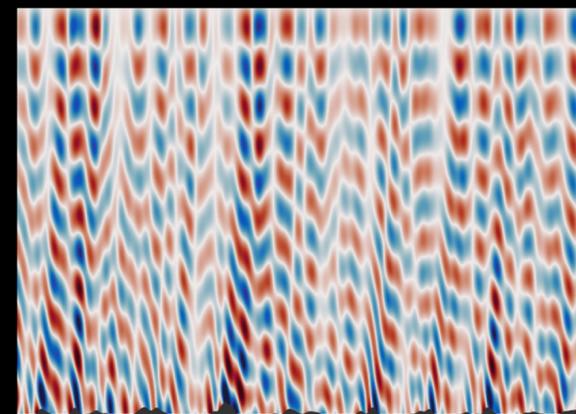
- > Most internal waves are not resolved in global ocean models, and certainly not in coupled climate models
- > Representing their effect on the energy and buoyancy budgets of the ocean through mixing and dissipation is key to getting climate models right
- > To do this, we use physics-informed parameterisations of internal waves derived from theory or wave-resolving numerical models



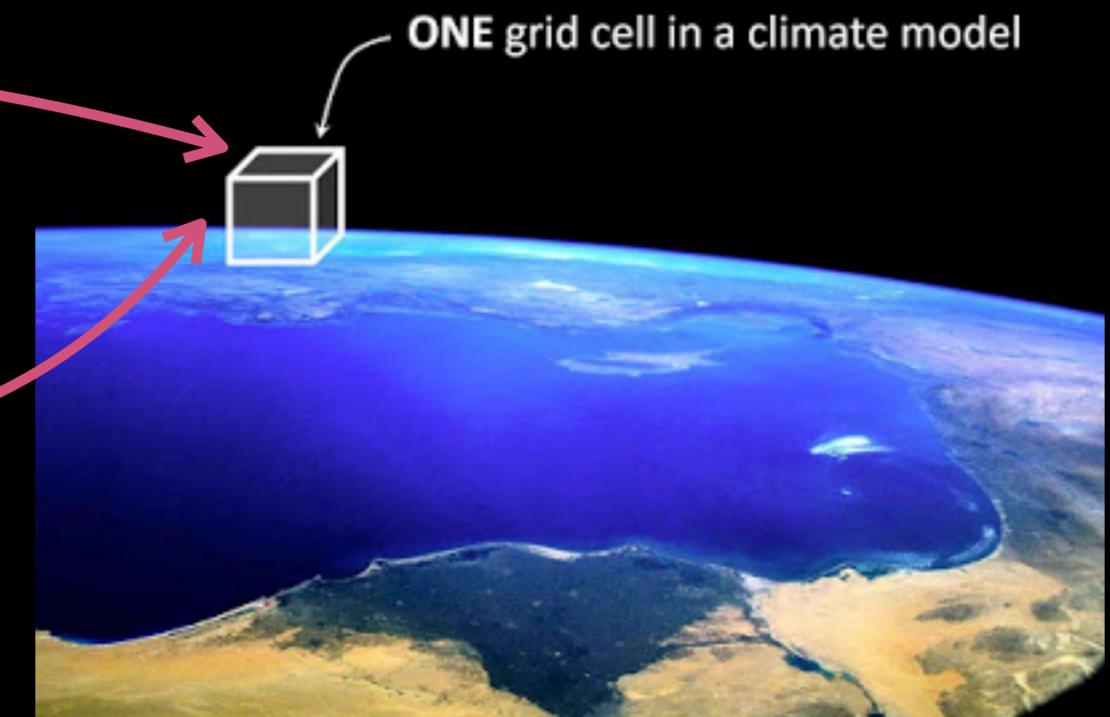
Baker & Mashayek (2022)

Numerical models

Theoretical models



Baker & Mashayek (2021)

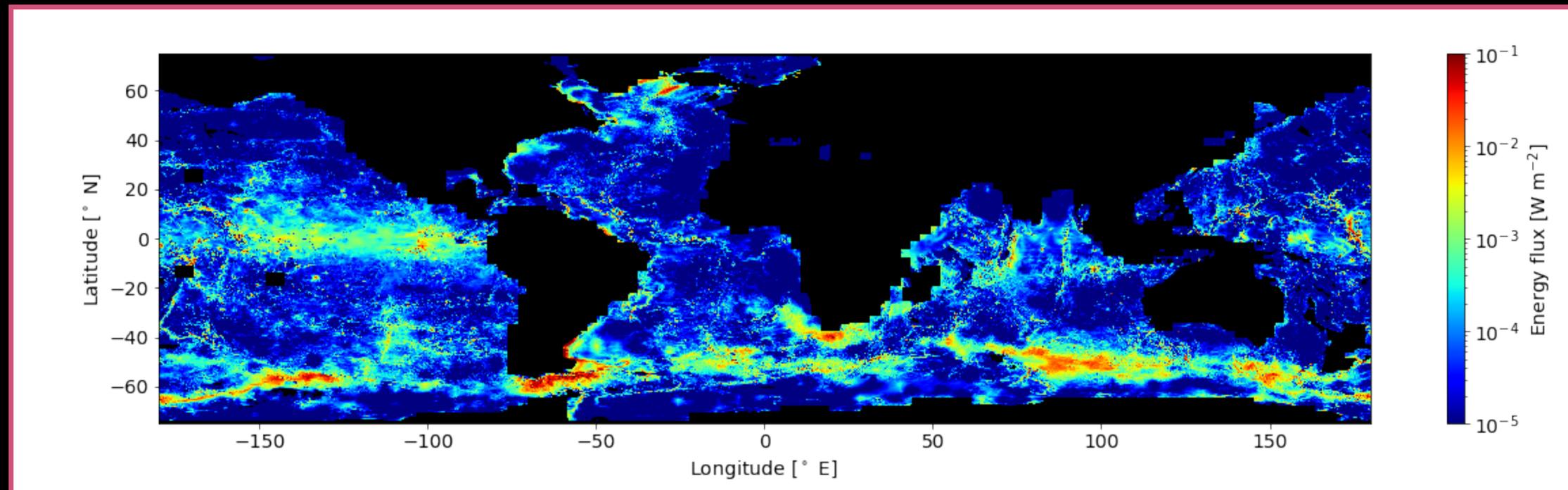
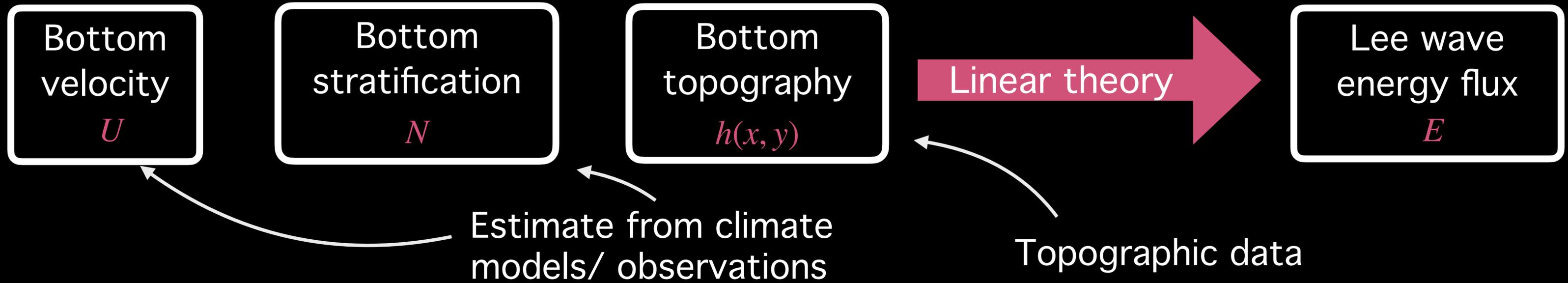


ONE grid cell in a climate model

Fig: Sciencebuffs.org, Kris Karnauskas

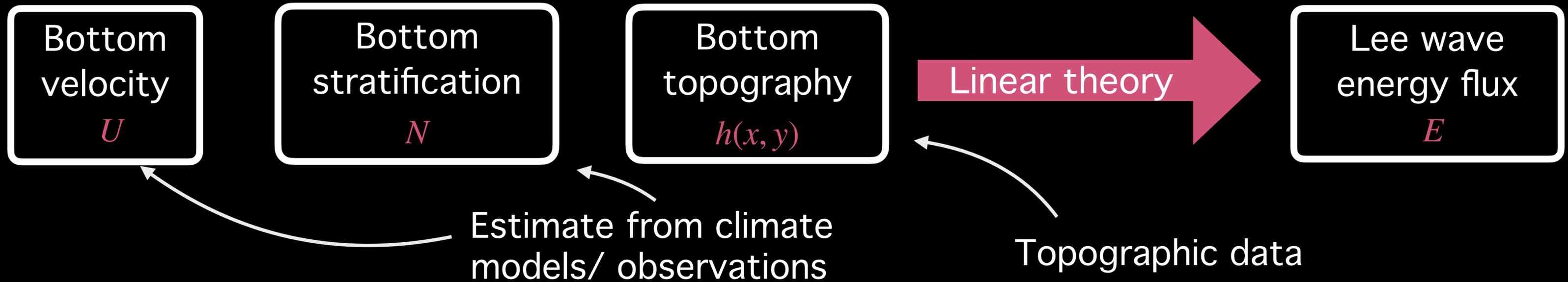
Example: Lee wave parameterisation

1. Estimate generation rates using linear theory

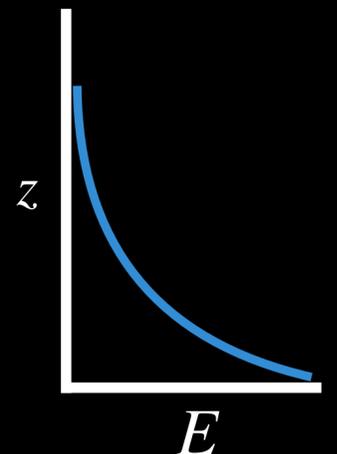


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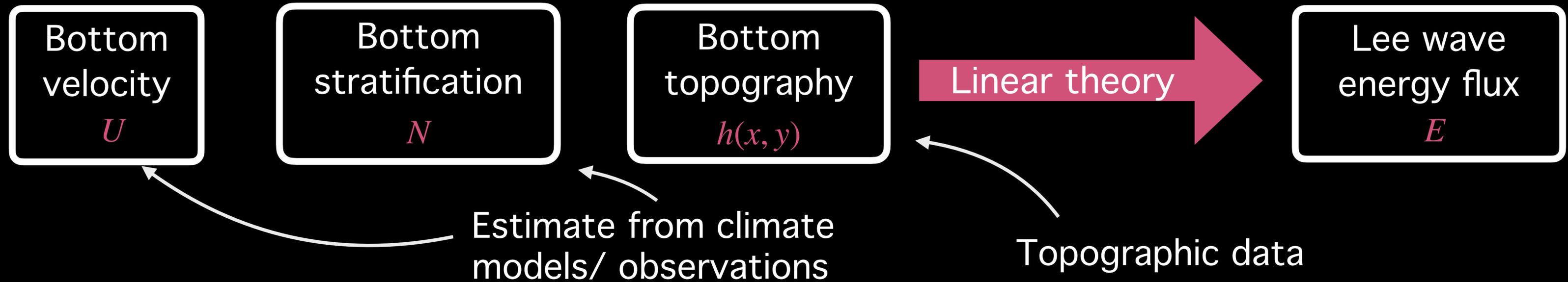


2. Typically assume that lee wave energy decays exponentially from the bottom, with some empirical decay scale $\sim 300 - 900\text{m}$ and dissipates locally



Example: Lee wave parameterisation

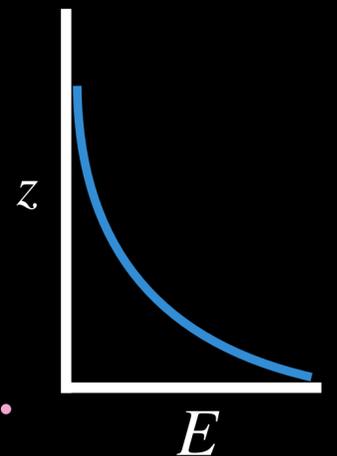
1. Estimate generation rates using linear theory



2. Typically assume that lee wave energy decays exponentially from the bottom, with some empirical decay scale $\sim 300 - 900\text{m}$ and dissipates locally

3. Convert energy flux to energy dissipation rate, infer corresponding diffusivity (Osborn, 1980), and plug into numerical model

$$\frac{\partial b}{\partial t} + \mathbf{u} \cdot \nabla b = \kappa \frac{\partial^2 b}{\partial z^2} + \dots$$

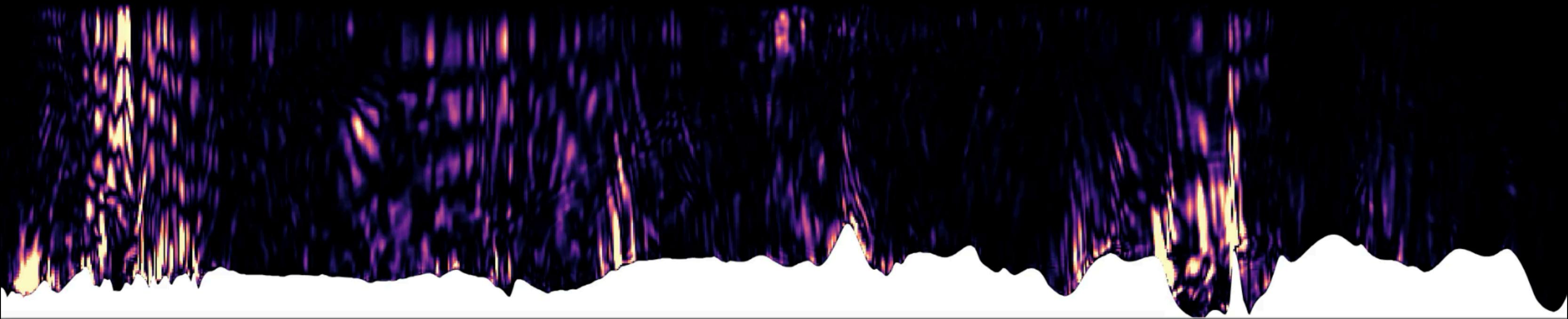


Challenges (and how we might address them)



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- > Observations are sparse
- > Very difficult to simulate scales of both internal waves and turbulent mixing



Challenges (and how we might address them)

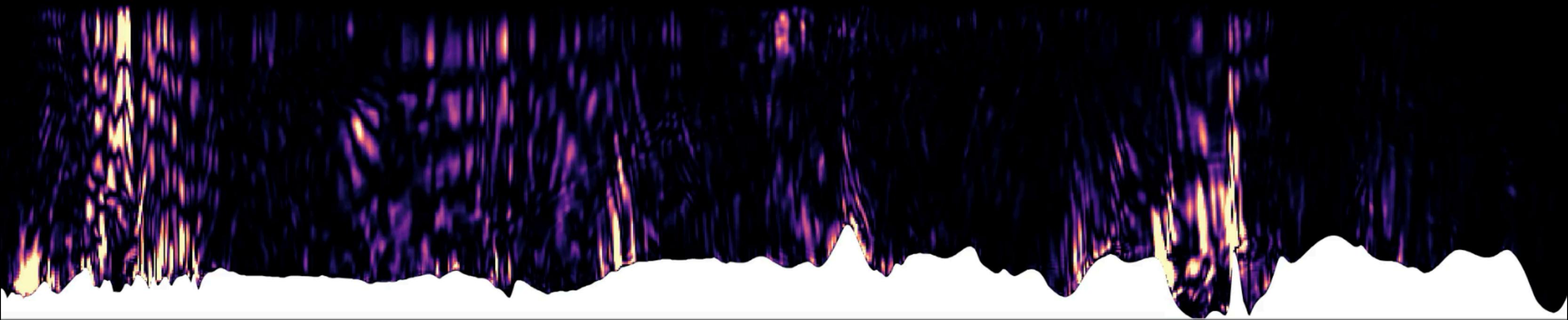


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Data limited regime - unclear if there is a role for machine learning

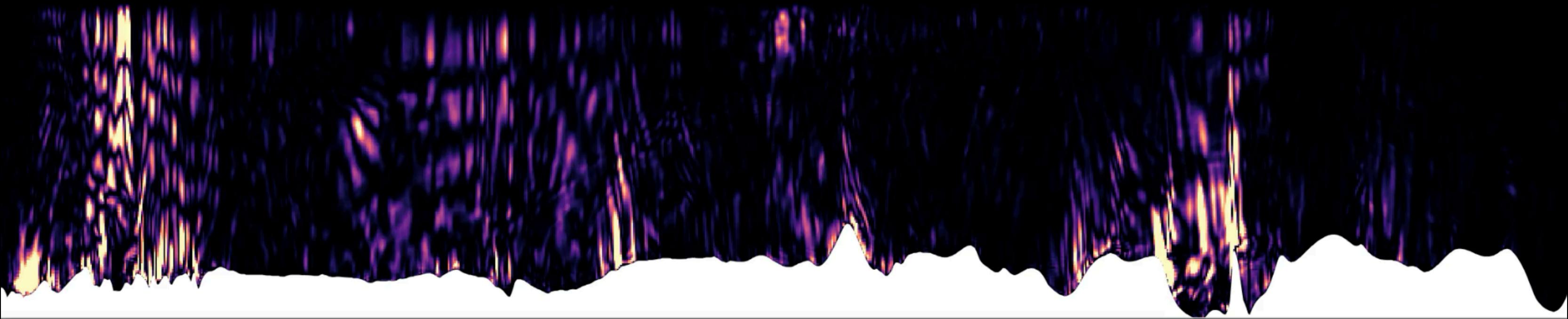


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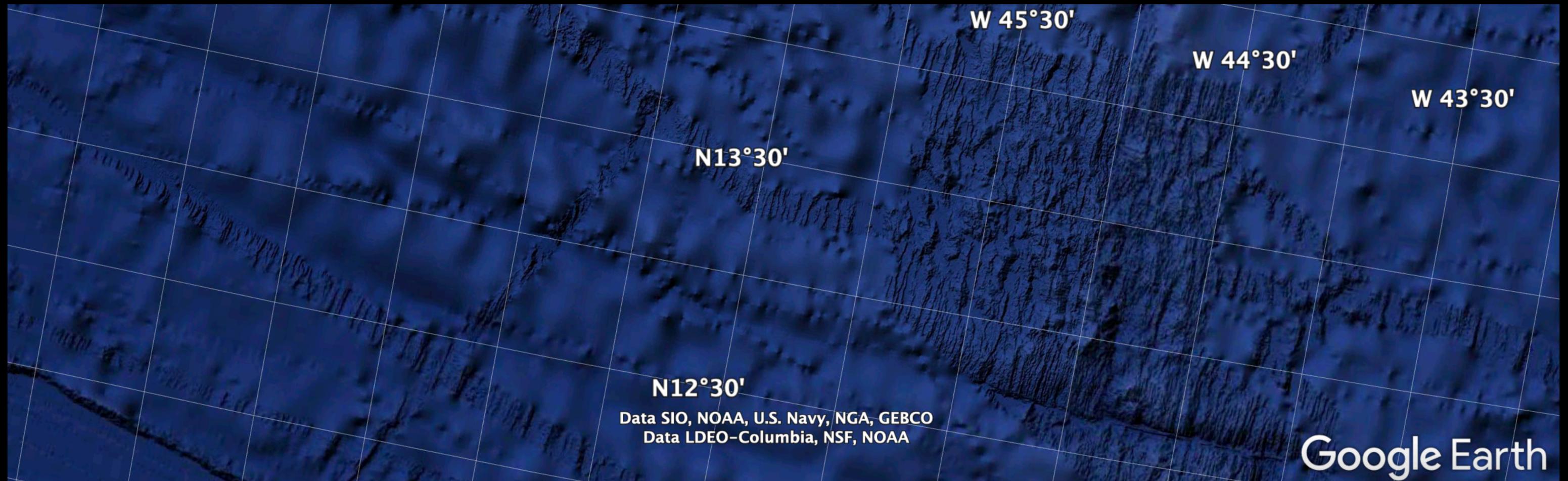
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of EDINBURGH

- > Observations are sparse
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- > Availability and representation of topography



Challenge: availability of topographic data

- > Ocean bathymetry isn't mapped at sufficient resolution for wave generation (~ 1 km). High-resolution data only available where a ship has surveyed.
- > Current workaround - assume a spectral form for abyssal hill topography at unresolved scales (Goff & Jordan 1988)



Challenge: availability of topographic data

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- > Current workaround - assume a spectral form for abyssal hill topography at unresolved scales (Goff & Jordan 1988)
- > Seabed 2030 mapping initiative to collate and collect new data

This table shows the minimum resolutions we expect to achieve at each depth range by Seabed 2030.

DEPTH RANGE	GRID CELL SIZE	% OF WORLD OCEAN FLOOR
0–1500 m	100 × 100 m	13.7
1500–3000 m	200 × 200 m	11
3000–5750 m	400 × 400 m	72.6
5750–11,000 m	800 × 800 m	2.7

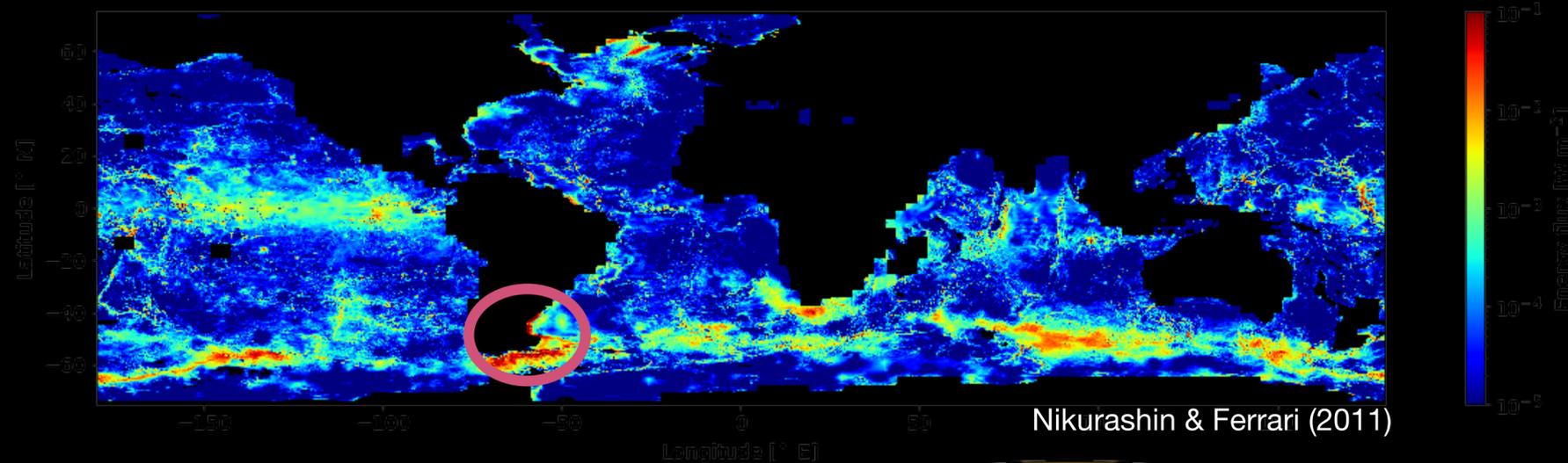
THE NIPPON FOUNDATION-GEBCO
SEABED
2030

<https://seabed2030.org/>

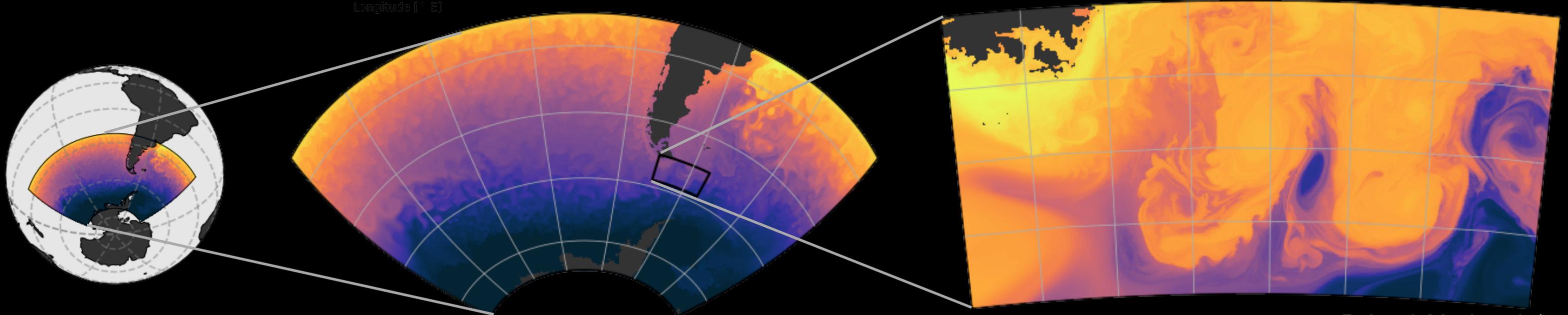


Challenge: nonlinearities at steep topography

- > How does linear theory do on steep, nonlinear, topography?
- > Case study: Drake Passage in the Southern Ocean - hot-spot for lee wave generation



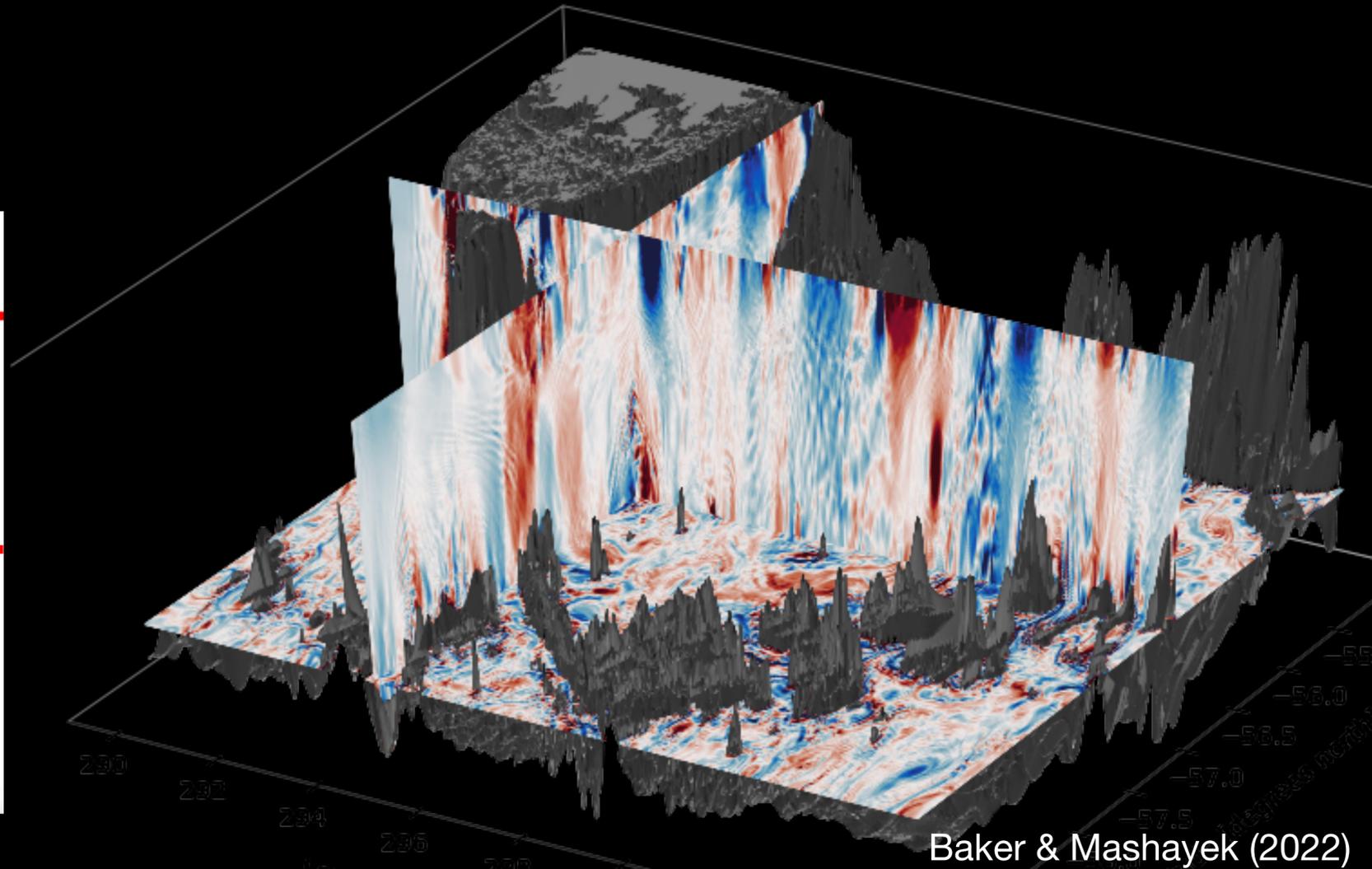
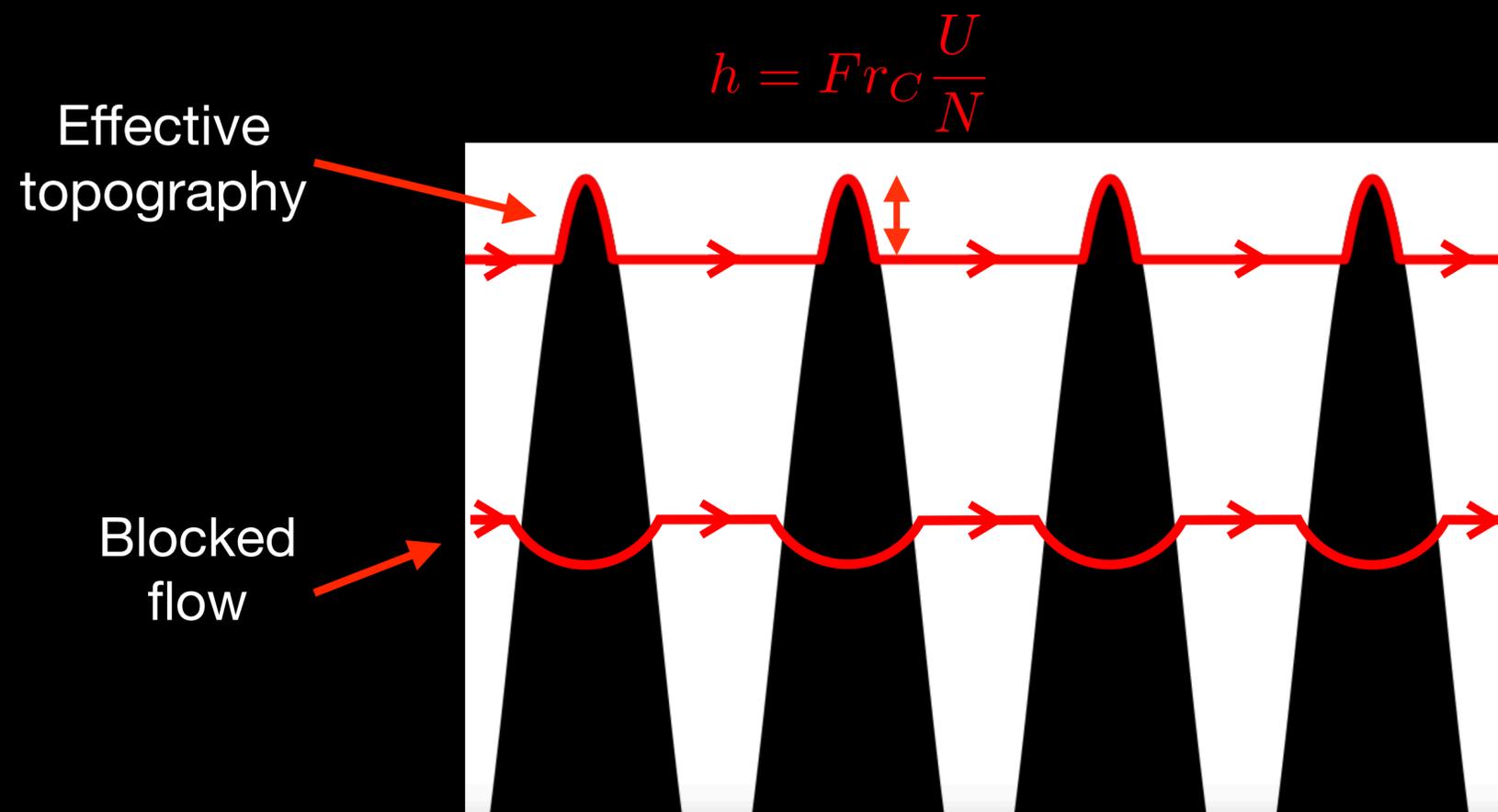
Realistically forced, nested ~ 600 m resolution nonlinear numerical simulation



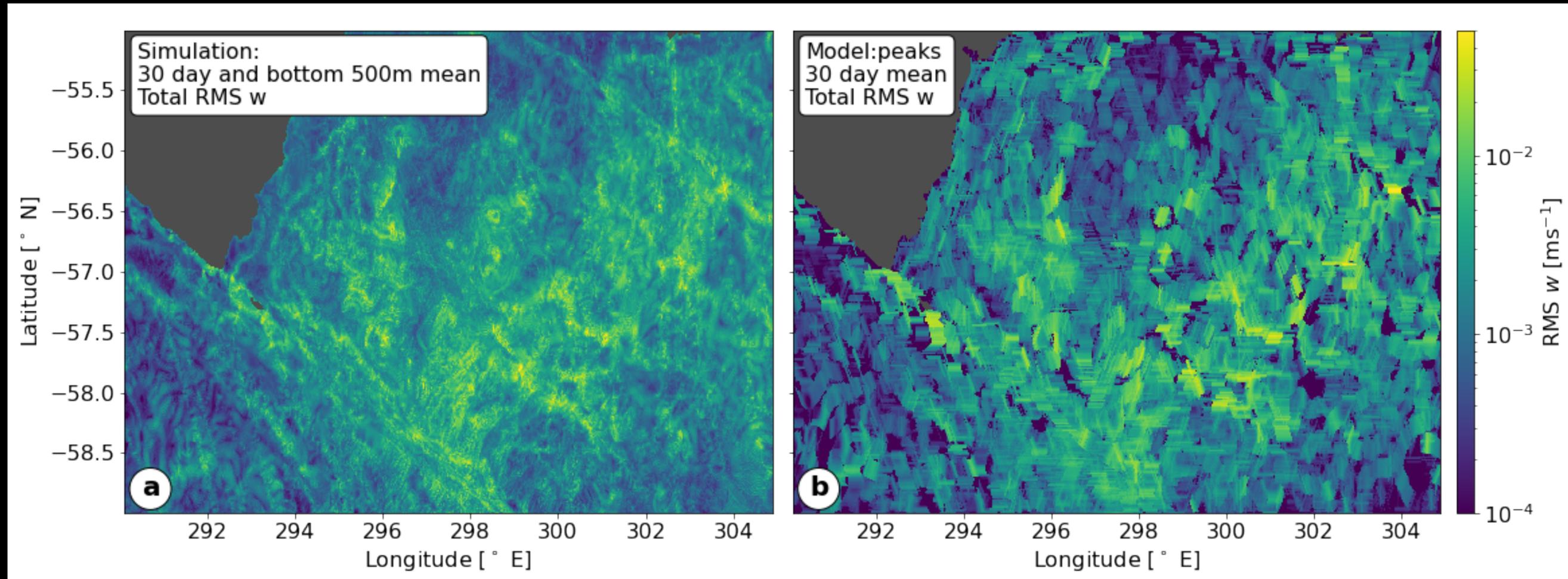
Challenge: nonlinearities at steep topography



- > Idea: compare linear theory with nonlinear simulation
- > For linear theory, need $Fr = NH/U \ll 1$. Here, $Fr \sim 100$ at high ridges
- > Develop parameterisation to represent 'effective topography'



Challenge: nonlinearities at steep topography



Baker & Mashayek (2022)

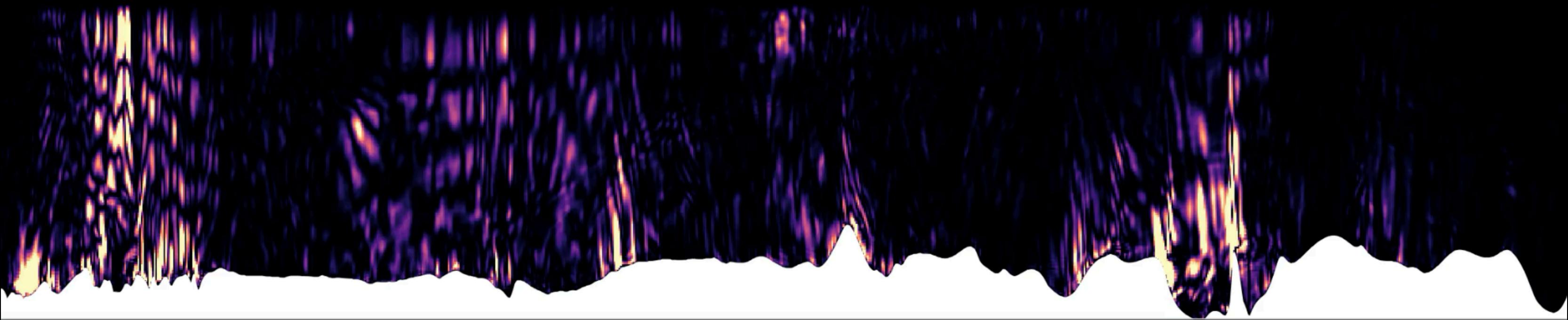
- > Flux calculated from simulated wave field matches new parameterisation well
- > Current estimates could be over-predicting lee wave energy flux (as seen in observations)

Challenges (and how we might address them)



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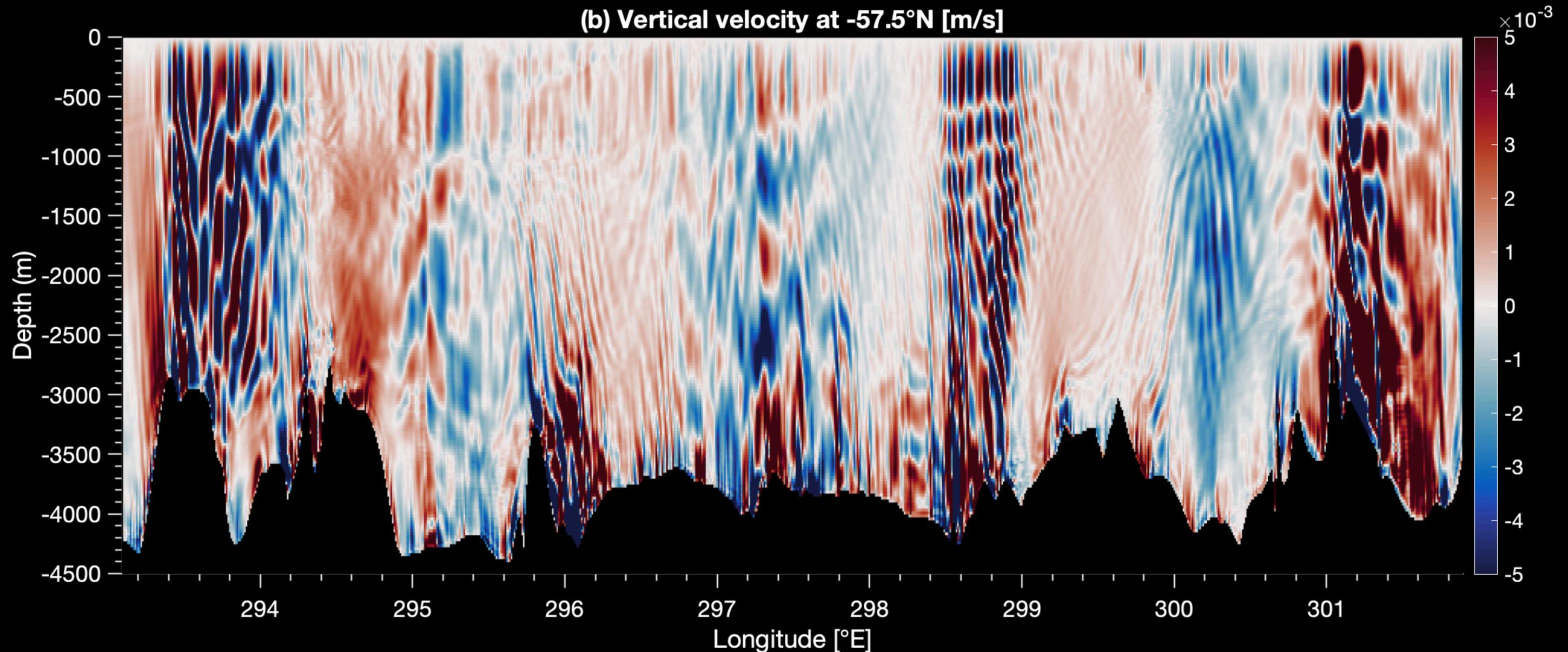
- > Observations are sparse
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- > Availability and representation of topography
- > Wave interactions with other dynamics



Challenge: identifying waves in numerical simulations



- > Generally identify waves as high-frequency motions $f < \omega < N$
- > Internal waves are Doppler-shifted by background flows - only frequency in frame of flow (intrinsic frequency) satisfies frequency criterion

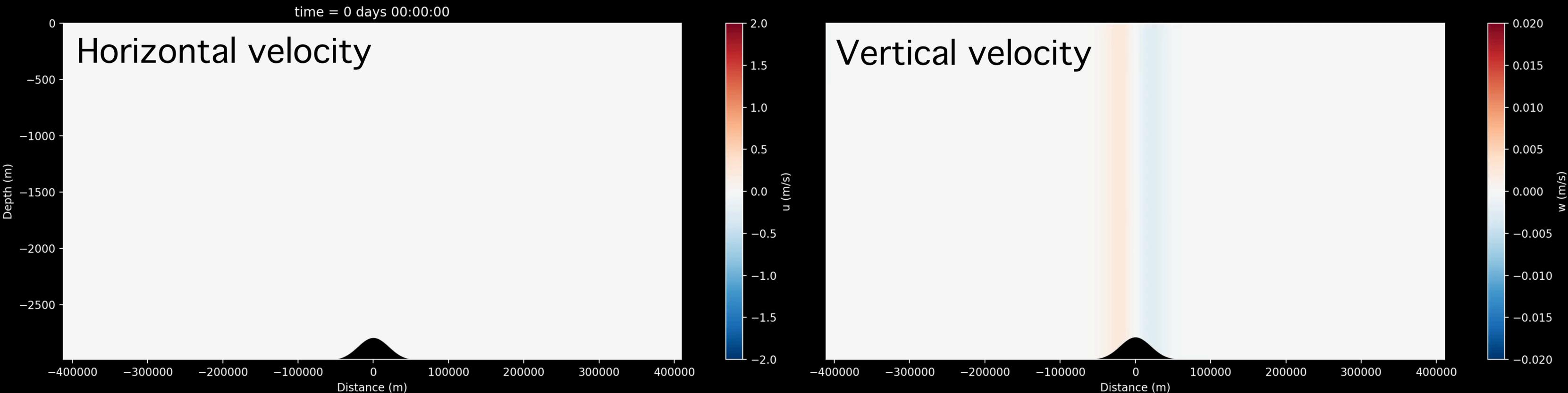


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- > Numerical methods: particle tracking (Shakespeare et al. 2021), and new PDE-based methods (Kafiabad & Vanneste 2023, Baker et al. 2025, Minz et al. 2025)

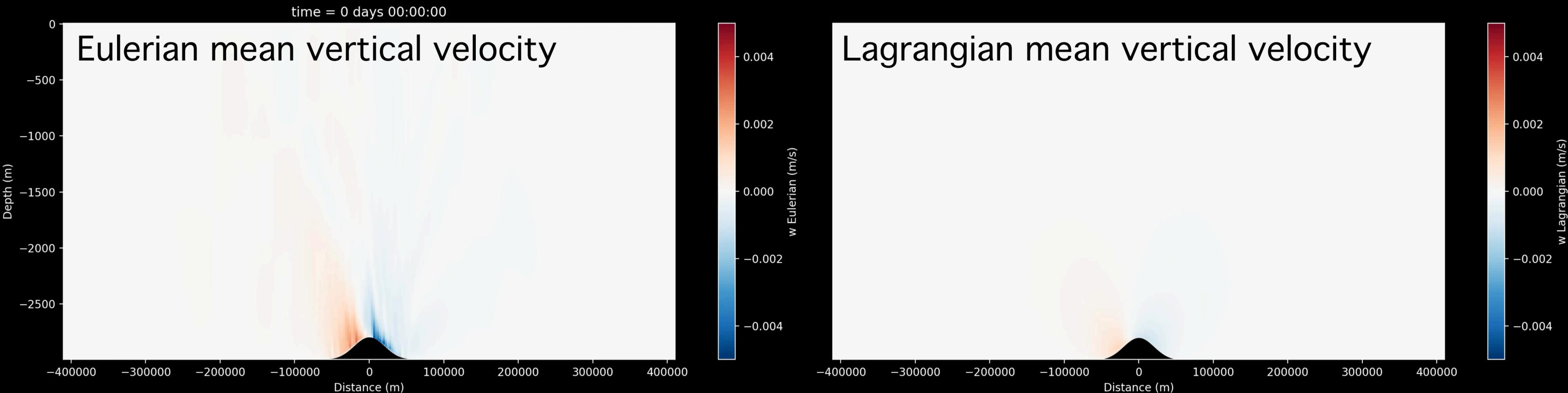
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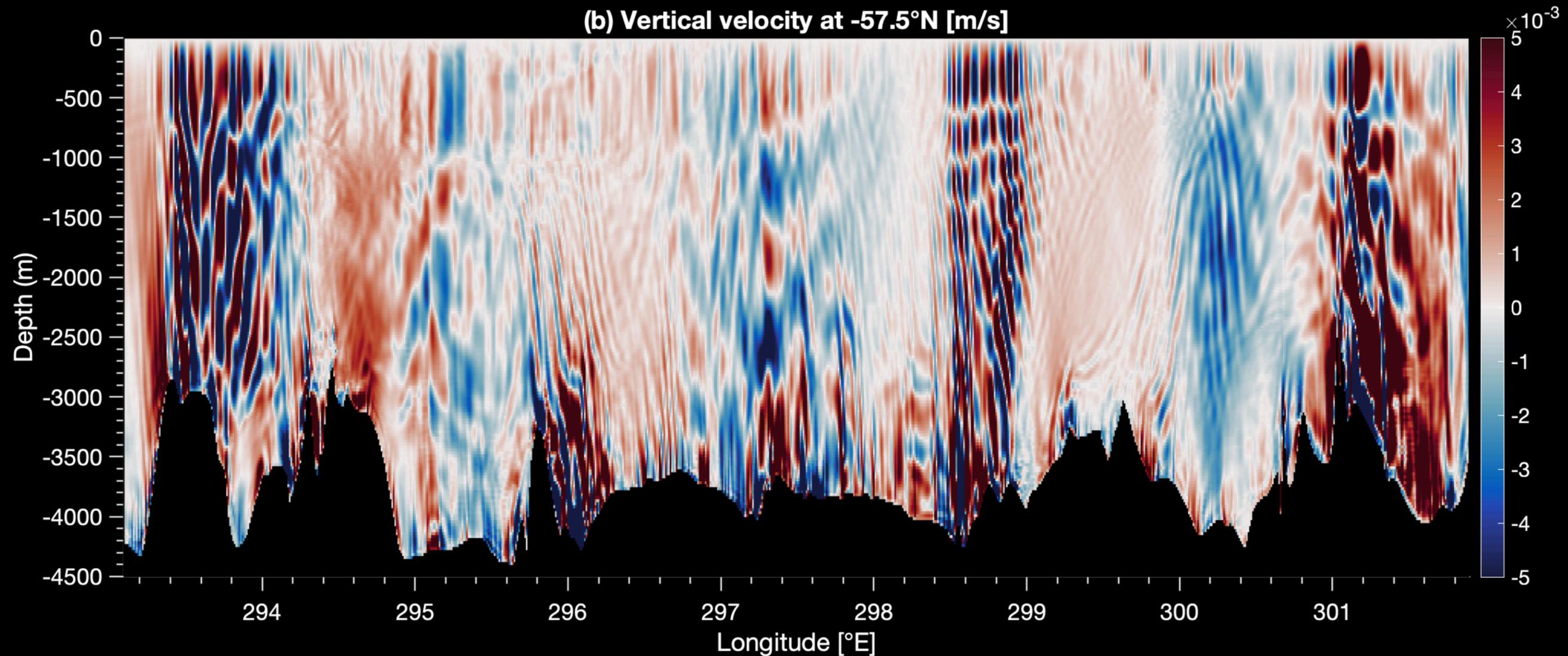
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Challenge: Wave interactions with background flow



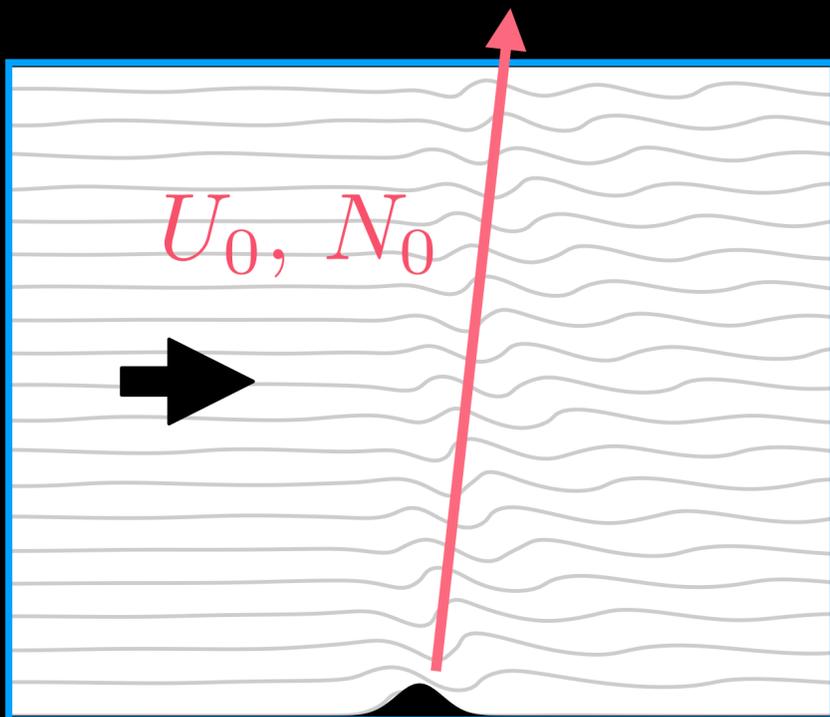
- > Linear theory assumes that internal waves propagate in an inviscid, infinite-depth, uniform background flow
- > What happens when background flow and stratification vary with height above bottom, and ocean has finite depth and viscosity?



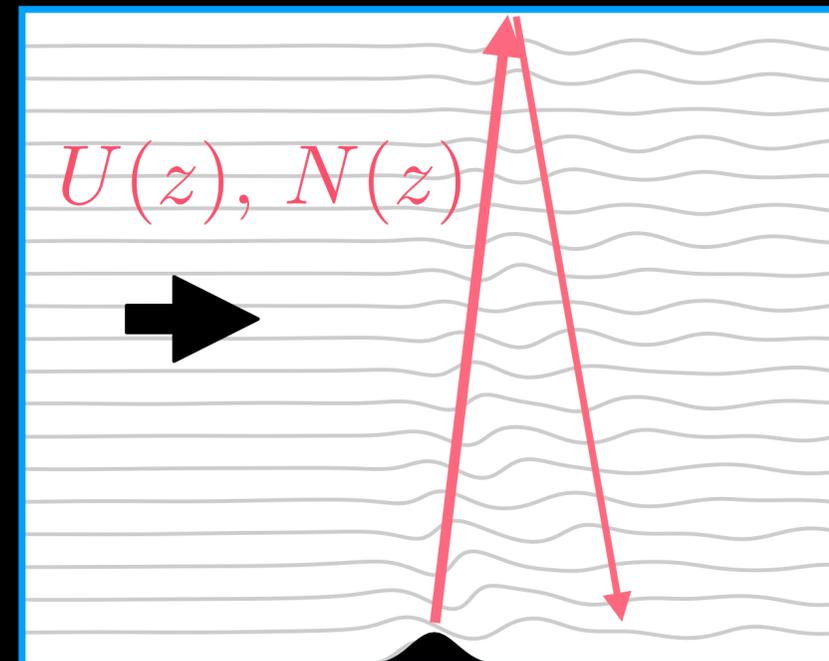
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Classical model (Bell 1975)

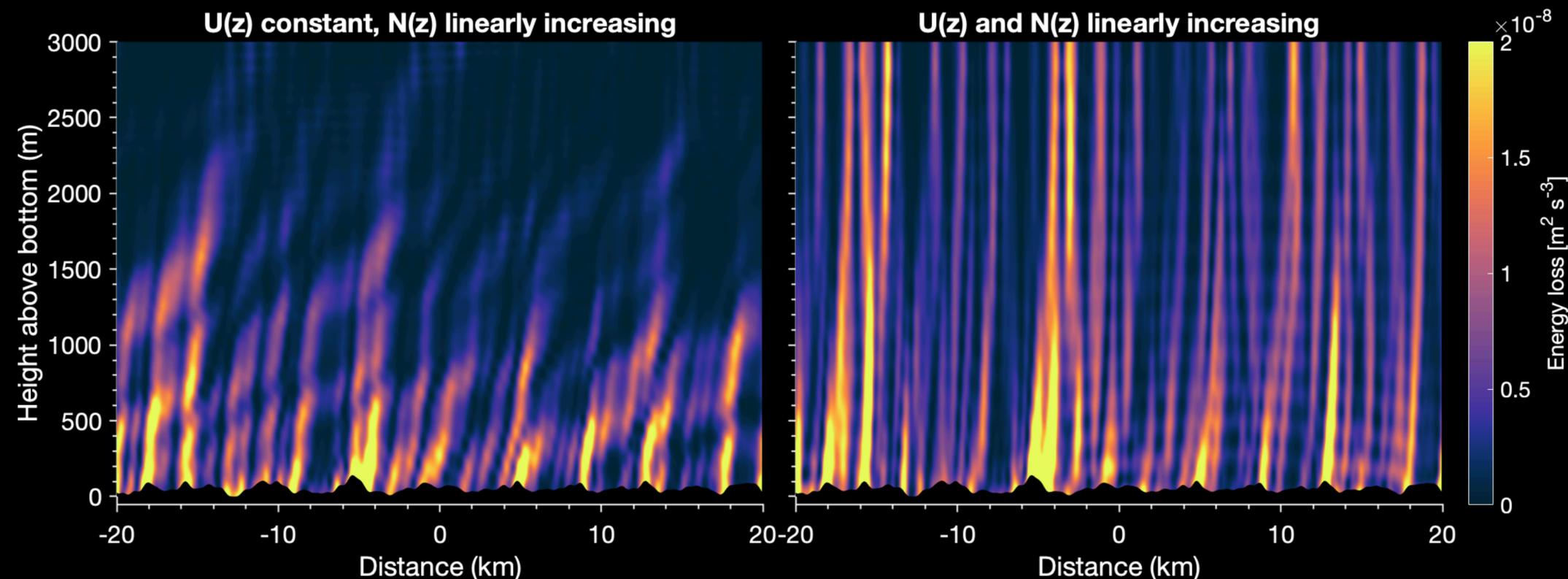


Modified model



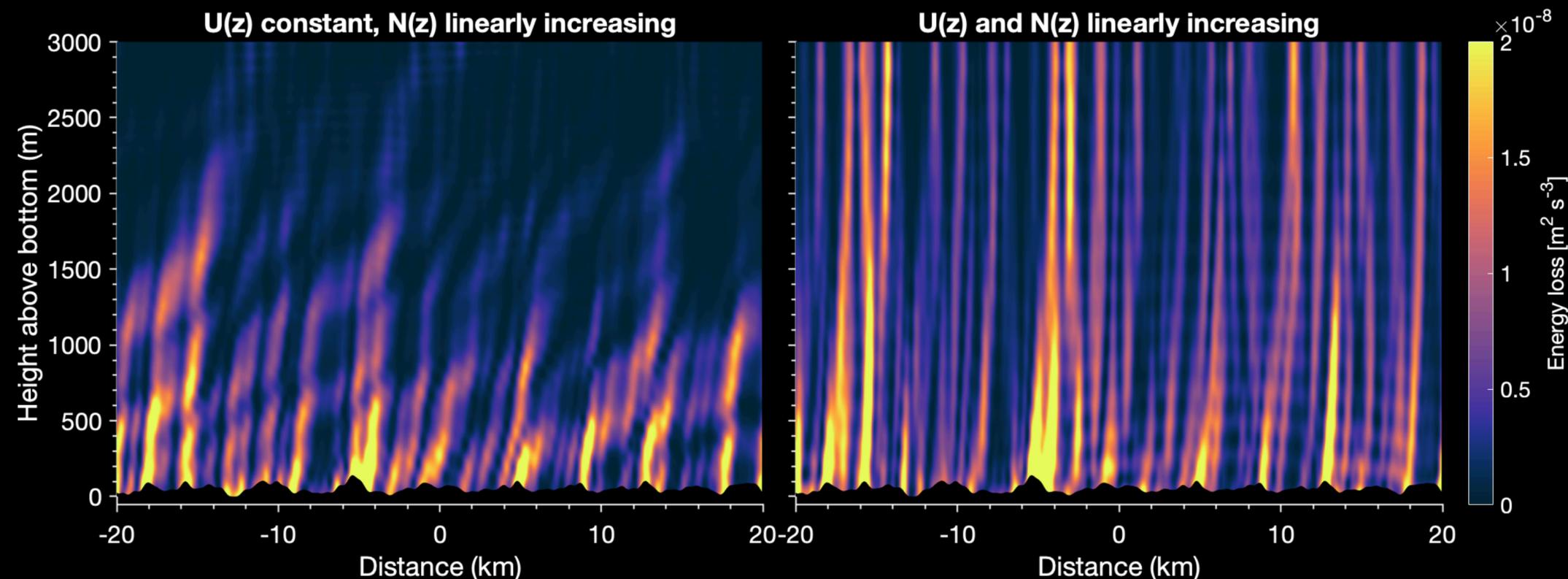
Challenge: Wave interactions with background flow

- > Linear theory assumes that internal waves propagate in an inviscid, infinite-depth, uniform background flow
- > What happens when background flow and stratification vary with height above bottom, and ocean has finite depth and viscosity?
- > Energy flux modified by interferences with reflected waves, and interaction with mean flow (including critical levels - need Green's function solutions (Maitland-Davies & Buhler 2025)).



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- > Still an important role for simple, linear physics in improving ocean parameterisations

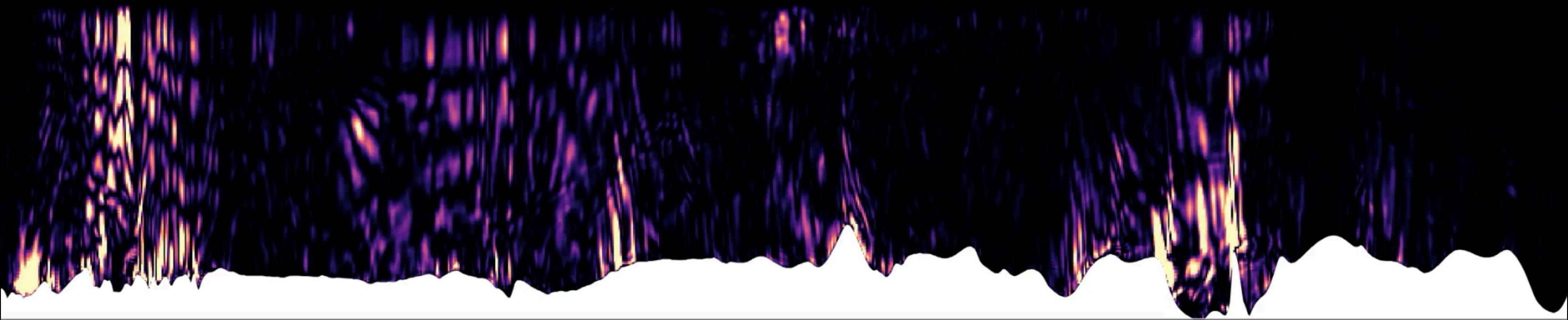


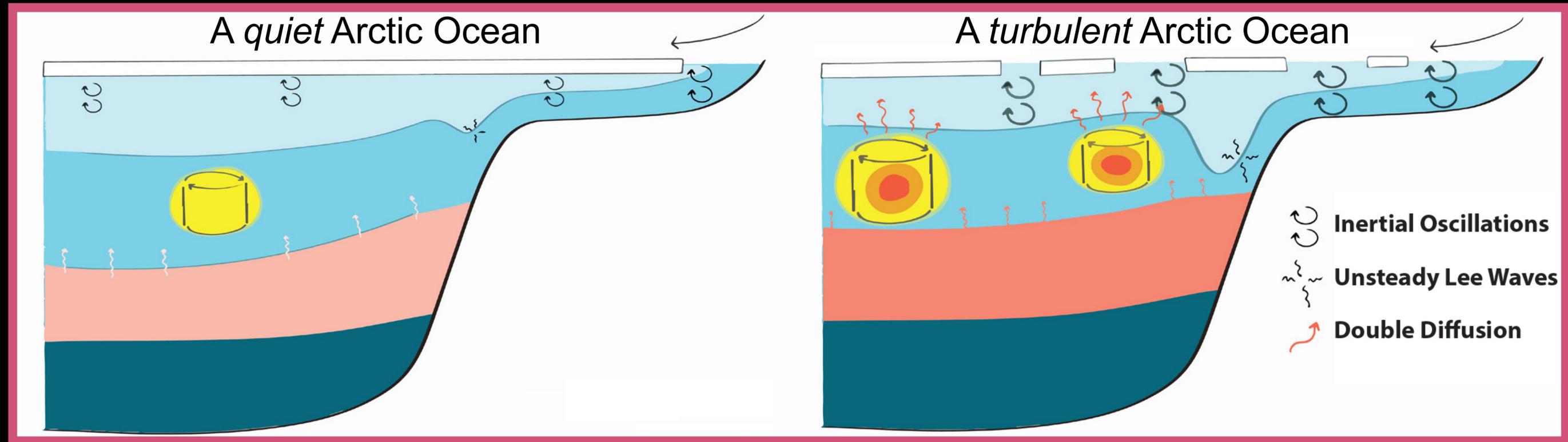
Challenges (and how we might address them)



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- > Observations are sparse
- > Very difficult to simulate scales of both internal waves and turbulent mixing
- > Availability and representation of topography
- > Wave interactions with other dynamics
- > Mixing feedbacks in a changing climate



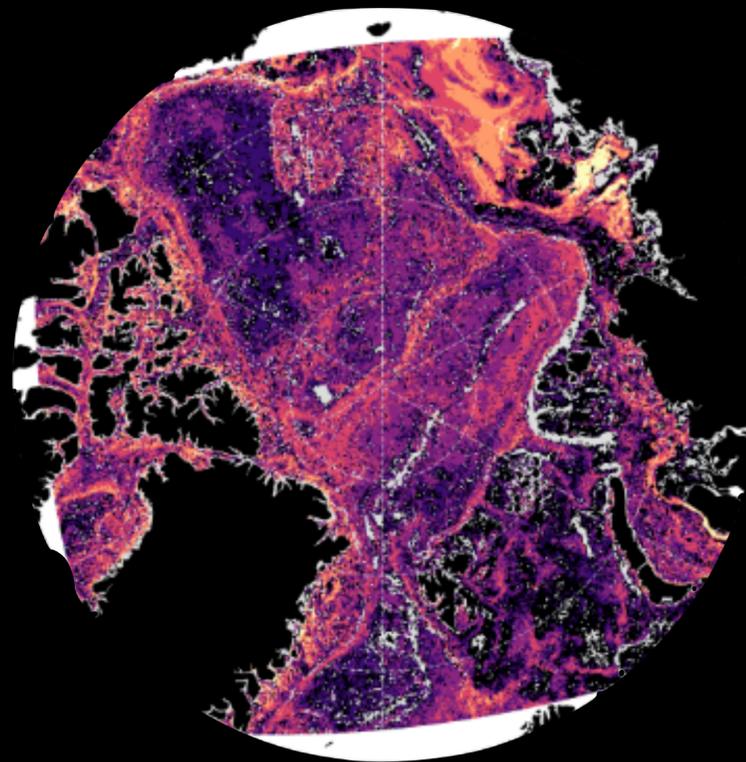


Rippeth & Fine 2022

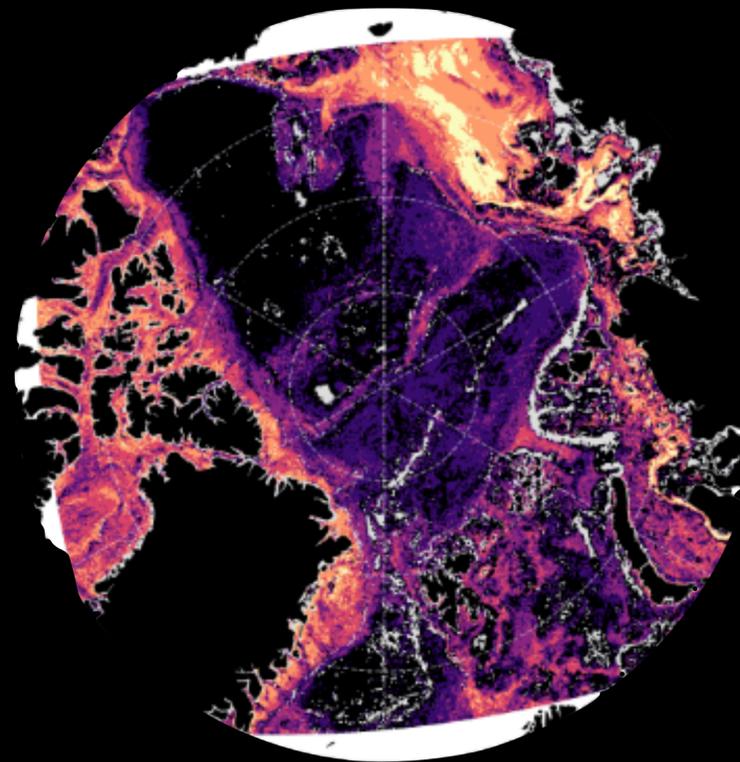
- > The Arctic is generally considered very 'quiet' with low turbulent mixing - no primary frequency internal tides since $\omega < f$, and weak mean flows.
- > As sea ice cover reduces, more ocean-atmosphere coupling and Arctic expected to spin-up (Li et al 2024) - more internal waves and turbulent mixing.
- > In the Arctic, warm subsurface waters sit underneath a cold surface layer. Could turbulence mix the warm waters into the surface layer and melt sea ice, causing a positive feedback?

- > We don't currently have wave generation estimates in the Arctic - mixing is poorly constrained
- > Extended linear theory with interdependent internal tides and lee waves (Shakespeare 2020) can be used to investigate wave generation pathways in the Arctic

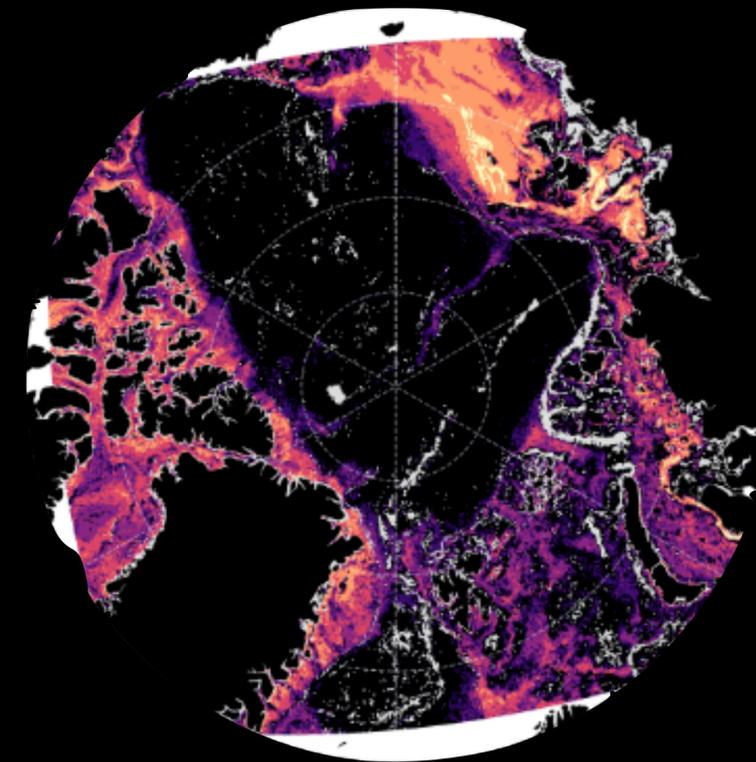
Lee waves



Doppler-shifted
internal tides



Higher harmonic
internal tides



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- > And many more...

Thank you!

