TURBULENT CONVECTION IN STARS AND PLANETS: CHALLENGES AND PROGRESS

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New Directions in Theoretical Physics, Edinburgh

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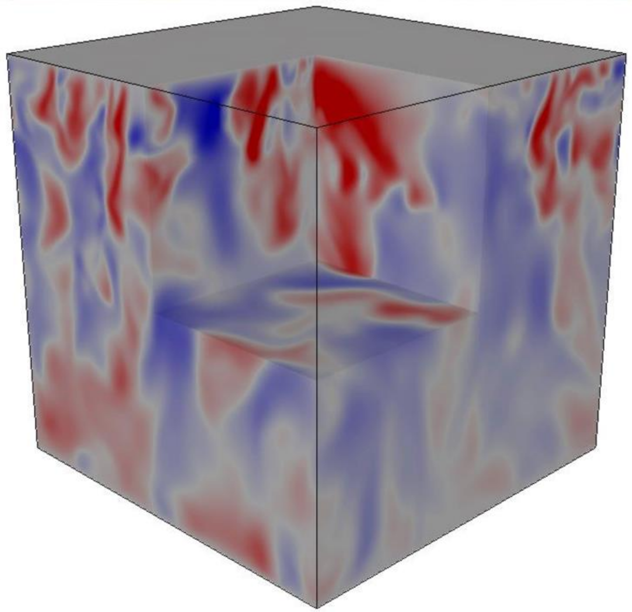

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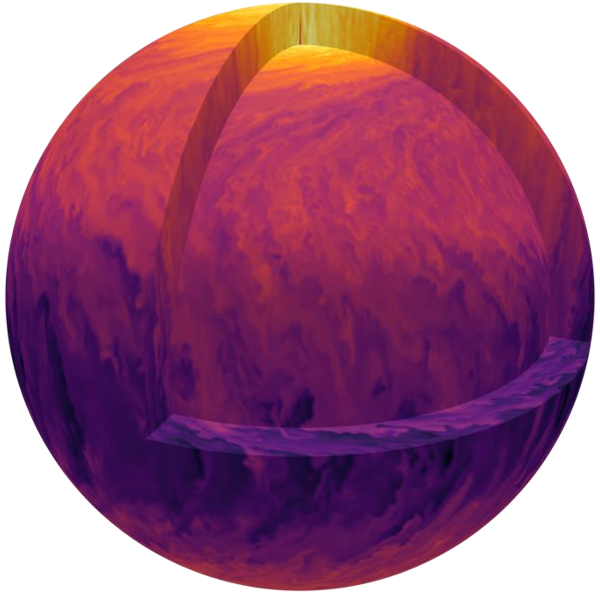


Convection in astrophysical systems

Convection is a ubiquitous process in a variety of geophysical and astrophysical contexts.

It is particularly important for understanding the dynamics of stellar and giant planet interiors.

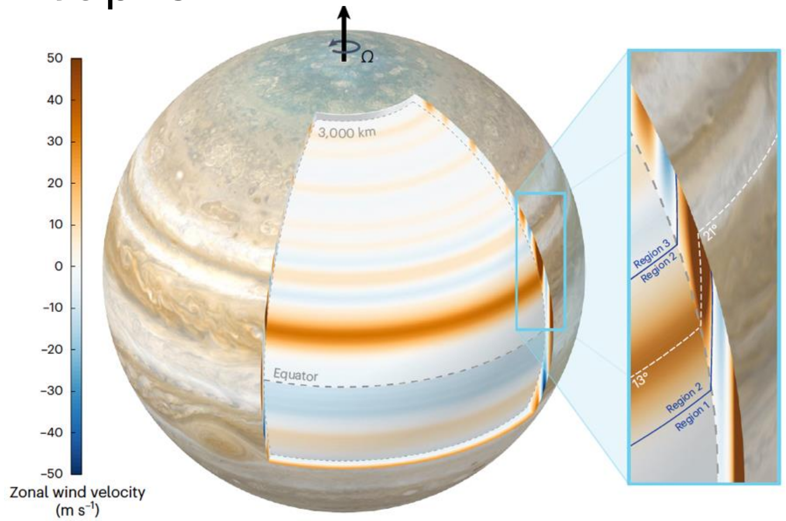
All stars transport at least some of their energy by convection.

Where this convection happens depends largely on the mass of the star.

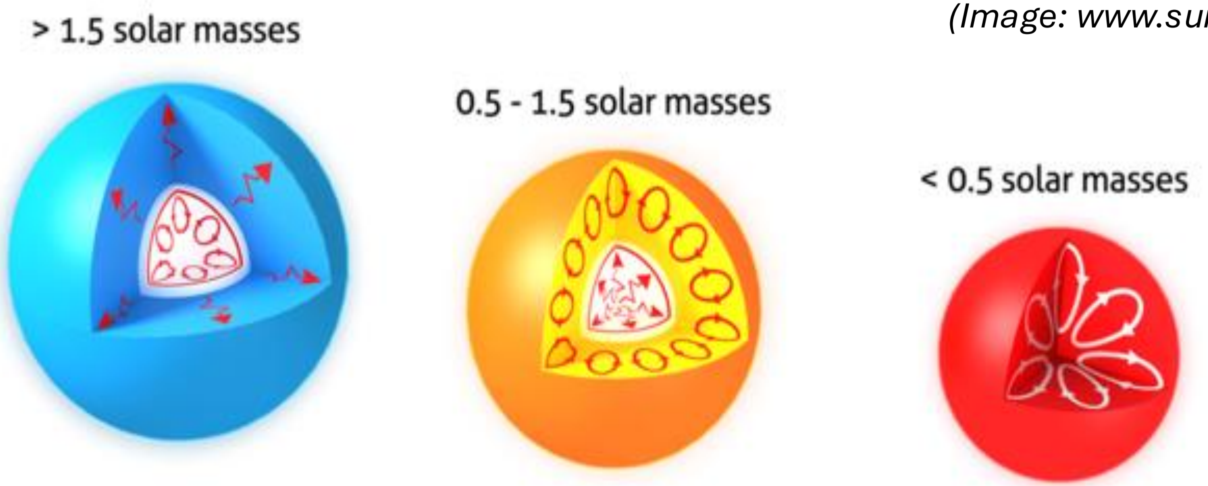
Convection also plays an important role in the gas giant planets.

Jupiter

(Kaspi+, Nat. Astronom., 2023)



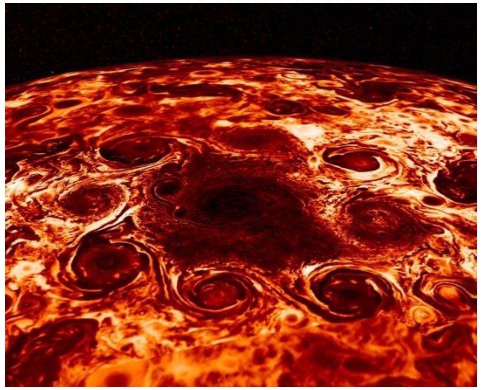
(Image: www.sun.org)



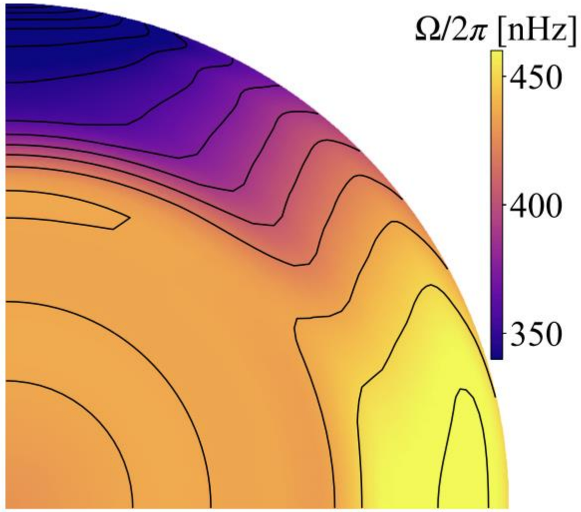
Physical characteristics: rotation and stratification

Rotation can be sufficiently rapid that convection becomes rotationally constrained.

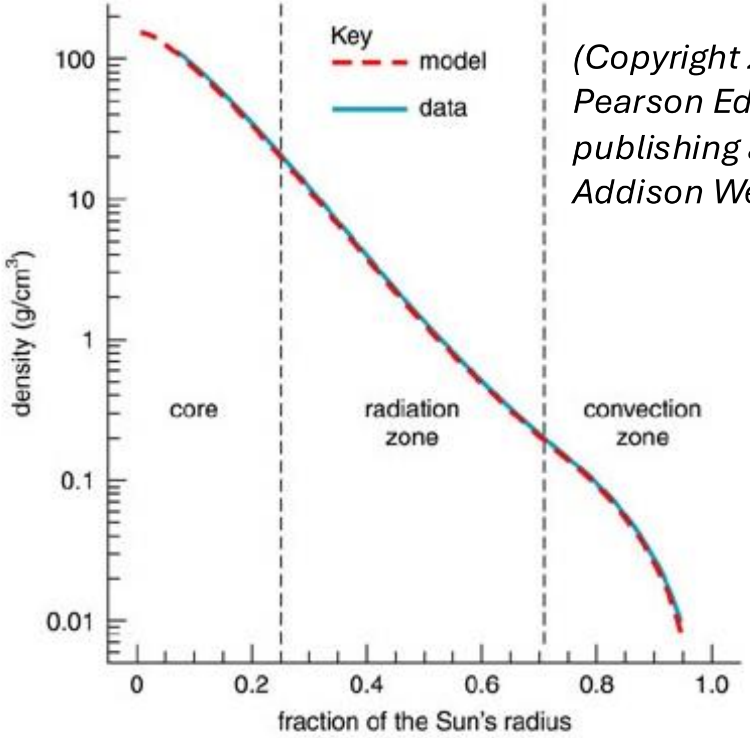
Can lead to the generation of large-scale flows such as the Sun's differential rotation and Jupiter's zonal winds.



(NASA/JPL-Caltech/SwRI/ ASI/INAF/JIRAM)



(Howe+, J. Phys.: Conf. Ser., 2011; Hotta+, ApJ, 2022)



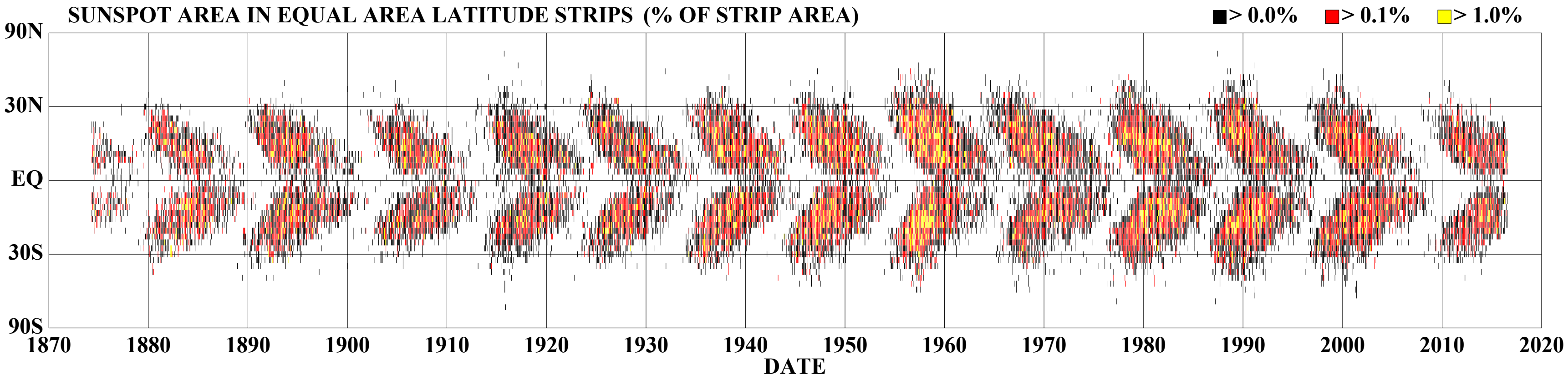
Stellar and giant planet interiors are also strongly stratified (density decreases rapidly with radius).

Stratification introduces a strong up-down asymmetry (broader, slower up-flows and narrower, faster down flows).

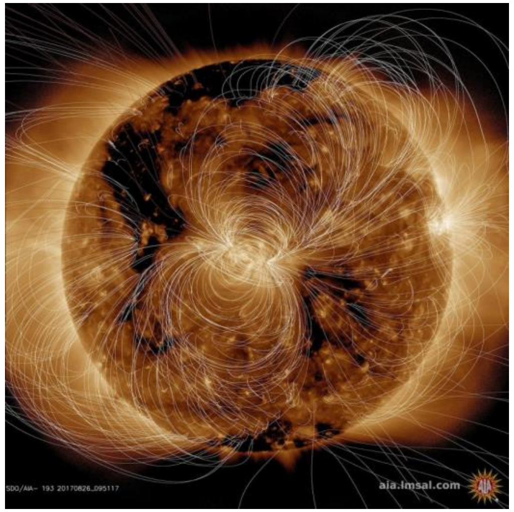
Some regions of the flow may be influenced by rotation more than others.

Physical characteristics: magnetism

Convection is thought to be important in the generation of magnetic fields through dynamo action.



(Credit: NASA Solar Physics)



(*Credit:* NASA / GSFC / SDO)

Summary

Need to account for the complex interactions between convection, rotation, stratification and magnetic fields.

Requires capturing behaviour across a wide range of scales, from small turbulent eddies to large-scale flows.

Complex nonlinear coupling between physical processes.

Mathematical Modelling

Dynamics governed by equations of Magnetohydrodynamics (MHD)

Rotation

$$\rho\left(\frac{\partial \mathbf{u}}{\partial t}+(\mathbf{u}\cdot\nabla)\mathbf{u}\right)=-\nabla P+\mathbf{j}\times\mathbf{B}-2\rho\mathbf{\Omega}\times\mathbf{u}+\rho'\mathbf{g}+\nabla\cdot(\mu\boldsymbol{\tau})$$

Magnetic field

$$c_v\rho\left(\frac{\partial T}{\partial t}+(\mathbf{u}\cdot\nabla)T\right)=-p\nabla\cdot\mathbf{u}+k\nabla^2T+\mu\frac{\partial u_i}{\partial x_j}\tau_{ij}+\eta\mu_0 j^2$$

Energy equation

$$\frac{\partial \rho}{\partial t}+\nabla\cdot(\rho\mathbf{u})=0$$

Mass continuity

Induction Equation

$$\frac{\partial \mathbf{B}}{\partial t}=\nabla\times(\mathbf{u}\times\mathbf{B})+\eta\nabla^2\mathbf{B} \qquad \nabla\cdot\mathbf{B}=0$$

Know the equations but difficulty lies in solving them over the wide range of spatial and temporal scales present in stellar and planetary interiors.

Dimensionless parameters

$\mathrm{Ra} \sim \frac{\text{buoyancy}}{\text{diffusion}}$ $\mathrm{Ta} \sim \frac{\text{Coriolis}}{\text{viscosity}}$

The molecular diffusivities in stellar and planetary interiors are tiny.

$\mathrm{Re} \sim \frac{UL}{\nu}$ $\mathrm{Rm} \sim \frac{UL}{\eta}$ $\mathrm{Pr} \sim \frac{\nu}{\kappa}$ $\mathrm{Pm} \sim \frac{\nu}{\eta}$

Parameter	Base of SCZ	Photosphere	Parameter	Base of SCZ	Photosphere
Ra	10^{20}	10^{16}	Ta	10^{27}	10^{19}
Re	10^{13}	10^{12}	Pr	10^{-7}	10^{-7}
Rm	10^{10}	10^{6}	Pm	10^{-3}	10^{-6}

(Ossendrijver, 2003)

Even if you had enough computational resources, the power you would require to simulate directly a Sun-like star would be 10^{22}W (equivalent to the power produced by a cool main sequence star).

(Credit: Petri Käpylä)

Modelling Approaches

Fully resolved simulations

Direct numerical simulation of the governing equations on massively-parallel computers.

In the wrong parameter regime but may be able to keep the correct relative force balances.

Scale-separated models

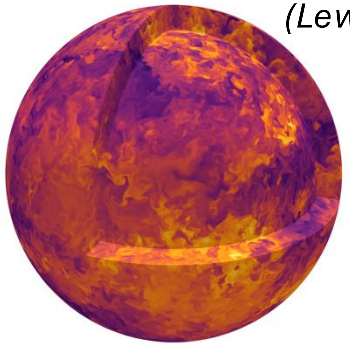
Direct numerical simulation of large-scale dynamics with sub-grid scale modelling to capture the effects of the unresolved scales (e.g., LES; quasilinear models).

Statistical/ensemble descriptions

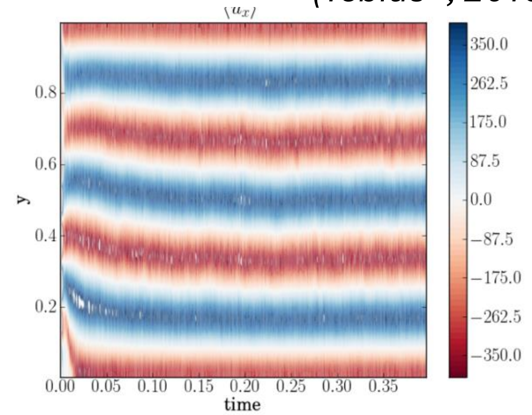
Evolution of correlation functions, spectra, or mean fields with closure approximations for unresolved statistical quantities.

Mean field & Two-point closure models (e.g., Reynolds stress, turbulent EMF modelling, CE2/DSS, EDQNM).

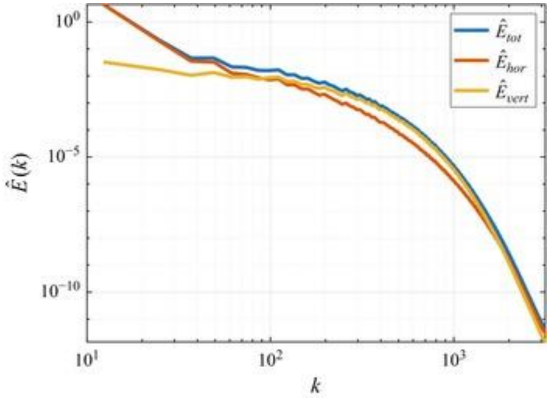
(Lewis+, ApJ, 2025)



(Tobias+, 2018)



(Lemm+, JFM, 2025)



Modelling Approaches

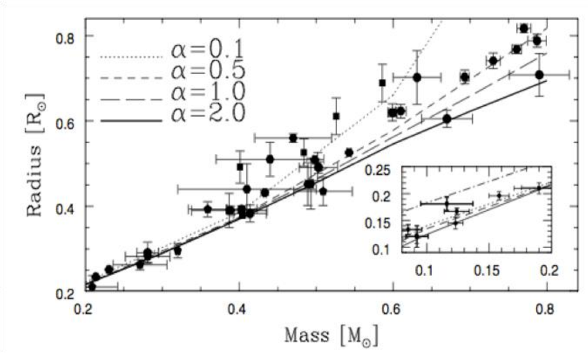
Reduced models

Simplification to low-dimensional systems capturing essential dynamics but that are easier to solve. (e.g., asymptotic regimes; Low order models/Galerkin expansions).

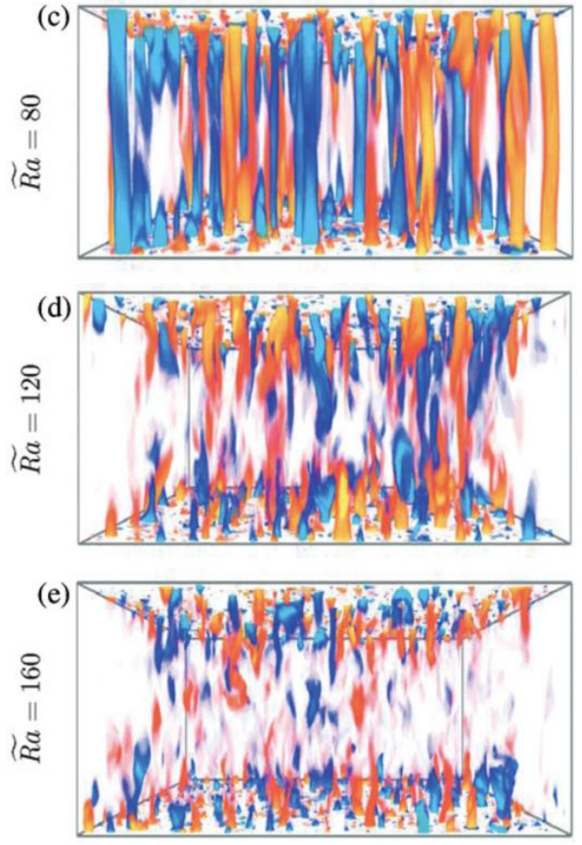
Theoretical frameworks (needed across all approaches)

Turbulent transport/closure theories

Mixing Length Theory (used to parametrise convective heat transport in 1D stellar evolution models).



(Chabrier+, A&A, 2007)



(Julien+, GAFD, 2012)

This talk

What do **fully resolved global simulations** tell us about turbulent convection and do the results agree with theoretical predictions of **Mixing Length Theory (MLT)**?

Fully resolved global simulations: Challenges

Convection in stellar and giant planet interiors is extremely turbulent and spans a vast range of scales making it impossible to simulate directly.

Molecular diffusivities in stars and planets are tiny and so it is generally assumed that the bulk properties of convection in real objects should not depend on these diffusivities.

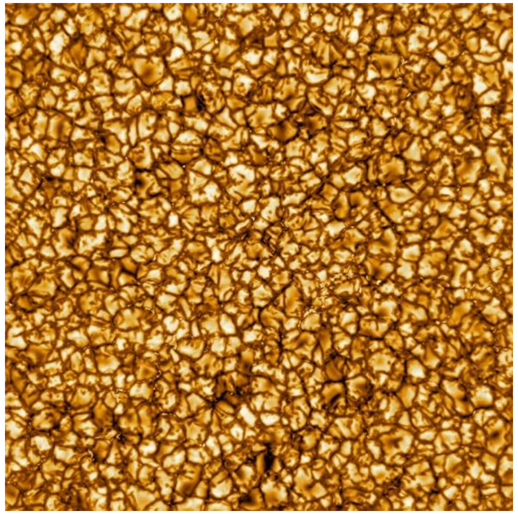
However, obtaining a "diffusion-free" regime in numerical simulations is challenging and results are often seen to depend on the diffusivities.

Challenging then to extrapolate results of simulations (conducted in parameter space far from reality) to real objects.

Also difficult to test diffusion-free theories of convection (e.g., MLT).

Jupiter

Credit: NASA/JPL-Caltech/SwRI/MSSS/ Gerald Eichstädt /Seán Doran

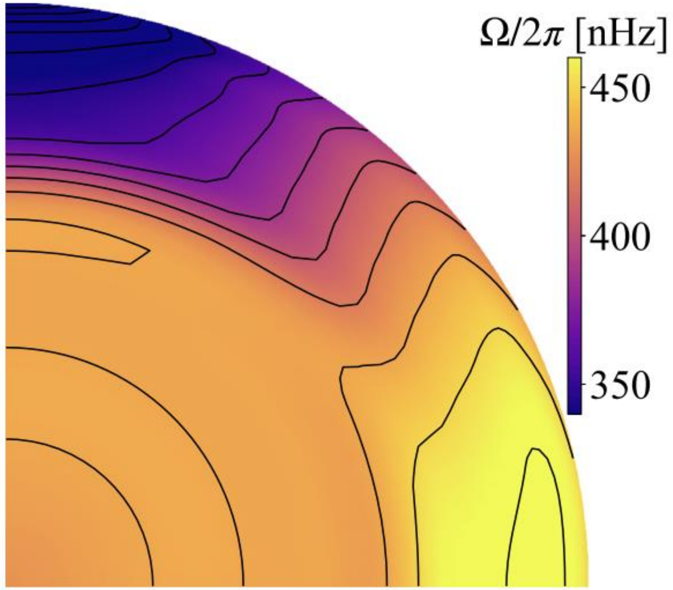


The Sun

Credit: NSO/NSF/AURA

Example: Solar differential rotation

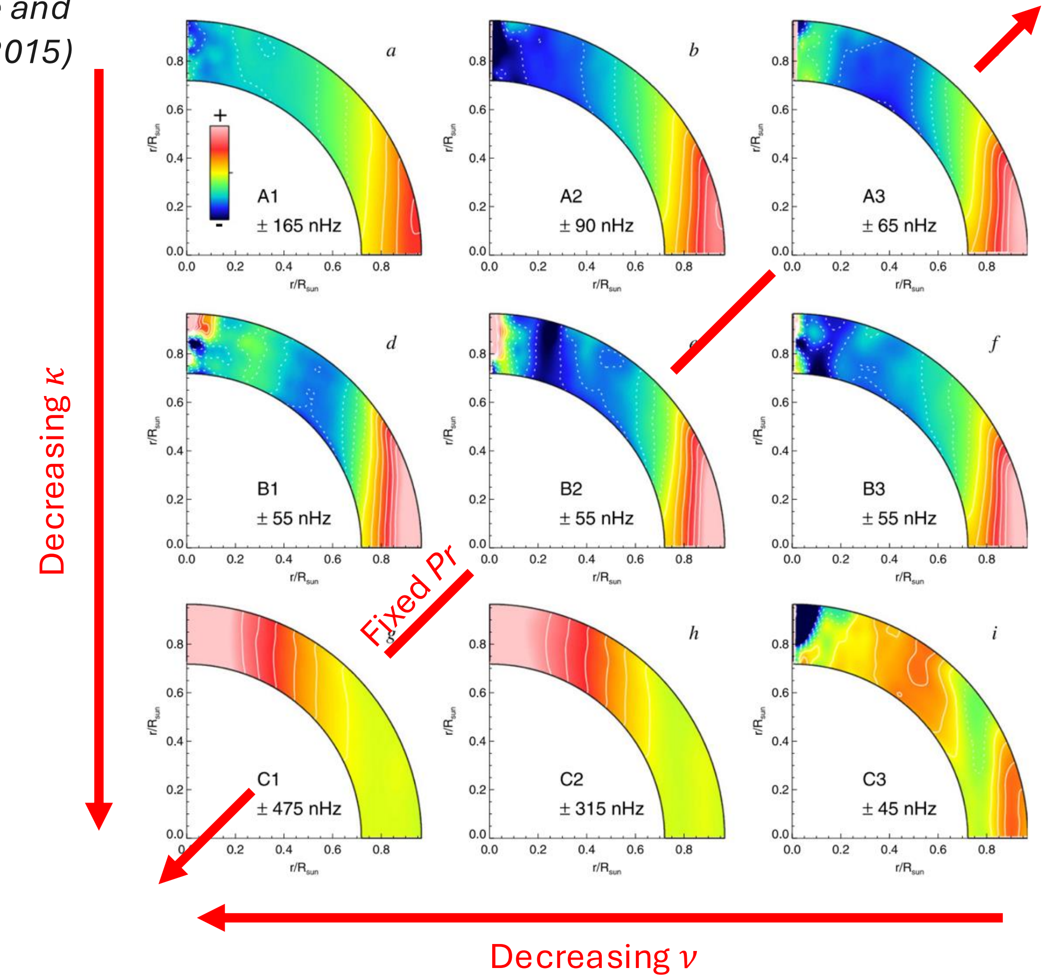
The Sun



(Howe+, J. Phys.: Conf. Ser., 2011; Hotta+, ApJ, 2022)

Numerical models

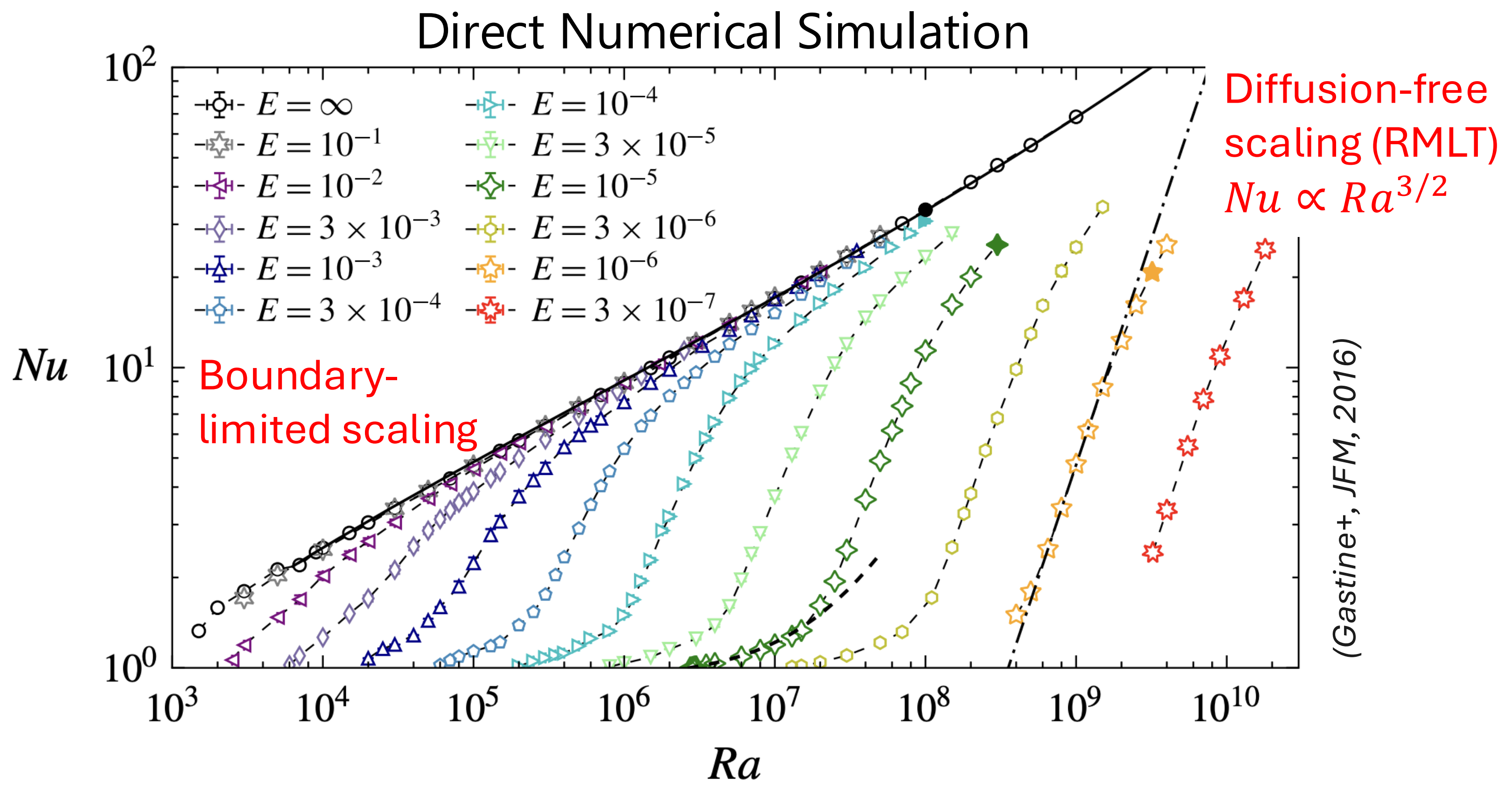
(Featherstone and Miesch, ApJ, 2015)



The Sun rotates faster at the equator than at the pole. This is called 'solar-like' differential rotation.

In models of the solar CZ, the differential rotation depends on the diffusivities.

Example: convective heat transport



In rotationally-influenced simulations, diffusion-free heat transport is obtained for rapid rotation.

As the rotational-constraint is lost, the convective heat transport exhibits 'boundary-limited' scaling (not diffusion-free).

Theories for convective heat transport are an important component of stellar evolution codes (e.g., MLT/RMLT).

We would like to be able to test these theories with numerical models.

Would ideally like a simulation setup that displays diffusion-free behaviour (for convective heat transport **and** other convective properties) across all rotation rates.

Convection driven by internal heating and cooling

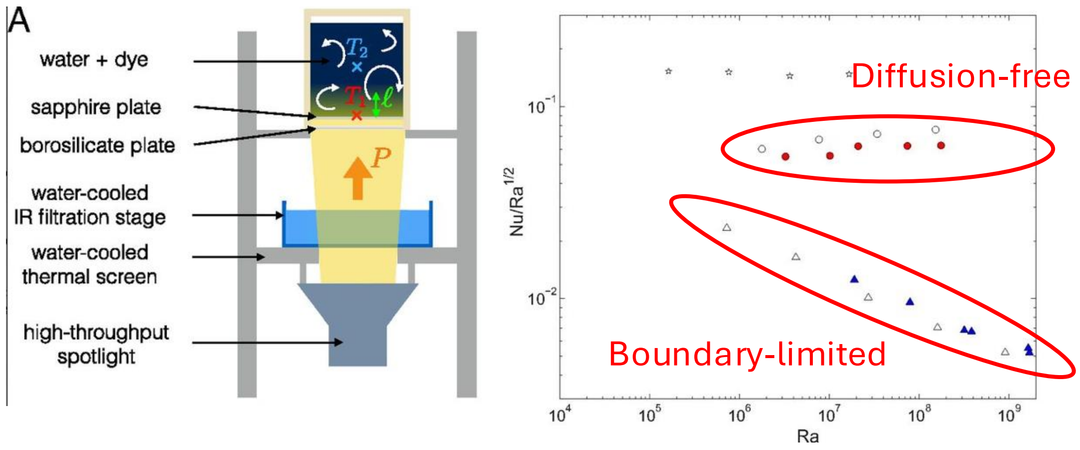
Alternative approach: drive convection by applying a prescribed distributed heating and cooling directly to the fluid (IH&C).

In 'boundary-driven' simulations, heat transport through the domain is throttled by the conductive flux in boundary layers.

Idealised simulations and laboratory experiments driven by IH&C have obtained diffusion-free dynamics for convective heat transport and other convective properties.

E.g. Barker + (2014), Lepot+ (2018), Currie+ (2020), Bouillaut+ (2021), Kazemi+ (2022), Joshi-Hartley+ (2025)

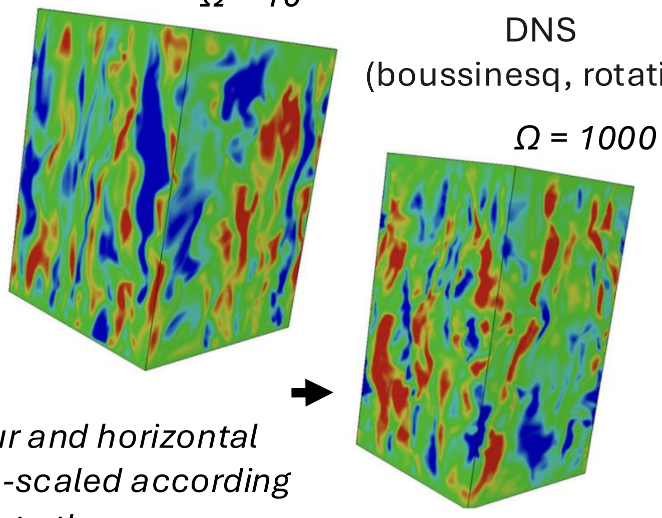
Lab experiment *(Lepot+, PNAS, 2018)*



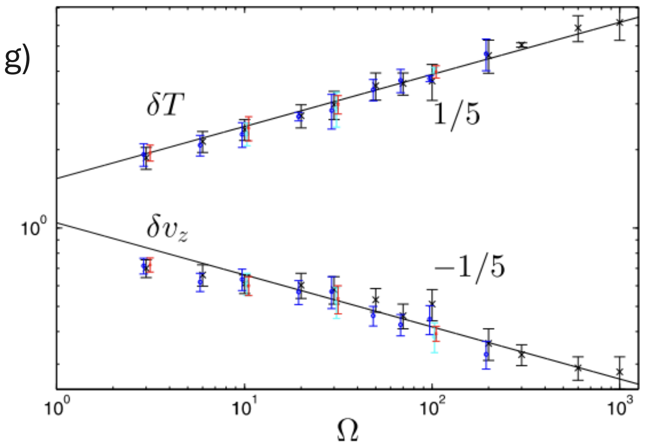
Ω = 10

DNS (boussinesq, rotating)

Ω = 1000



Colour and horizontal scale re-scaled according to theory



(Barker+, ApJ, 2014)

Do the same ideas hold in a spherical geometry?

Aims

Study the dynamics of internally heated and cooled, rotating spherical shell convection.

1. Is it possible to obtain ‘diffusion-free’ scaling behavior for the convective heat transport, for both slow and rapid rotation?

2. Are other aspects of the flow diffusion free?
   (e.g., convective velocities, zonal flows, flow morphology)

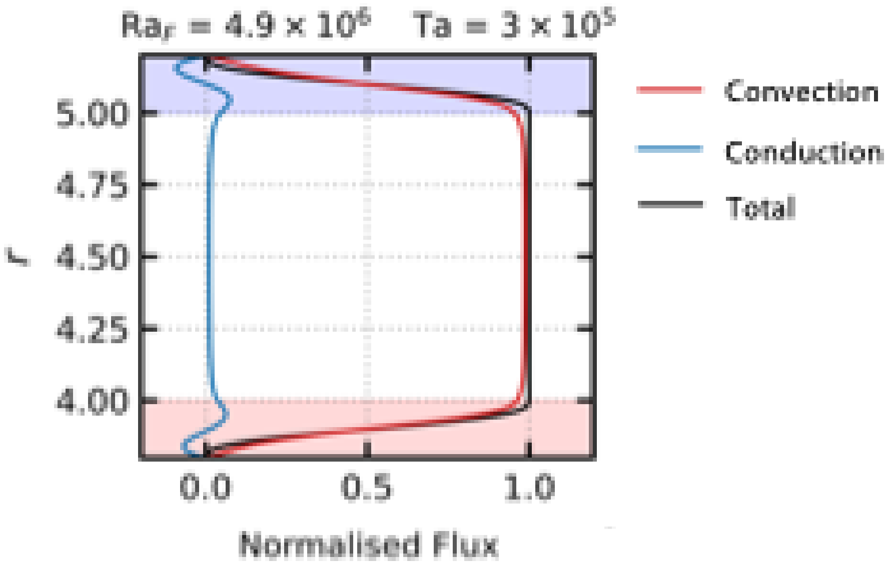
3. How do the results compare with predictions from MLT/RMLT?

Spherical shell simulations

Work with Neil Lewis, Tom Joshi-Hartley, Steve Tobias & Matt Browning.

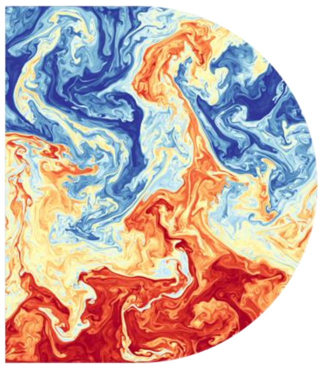
We conduct simulations of Boussinesq, rotating spherical shell convection driven by IH&C.

$$\nabla \cdot \boldsymbol{u} = 0$$
$$\frac{\mathrm{D}\boldsymbol{u}}{\mathrm{D}t} + 2\boldsymbol{\Omega} \times \boldsymbol{u} = -\nabla p + \tilde{g}(r) T \mathbf{e}_r + \nu \boldsymbol{\nabla}^2 \boldsymbol{u}$$
$$\frac{\mathrm{D}T}{\mathrm{D}t} = q(r) + \kappa \nabla^2 T$$



Heating is applied below $r = r_i$ and cooling is applied above $r = r_o$, each over a depth δ.

Boundary conditions are insulating, stress-free, and impenetrable.



We analyse the dynamics within the convection zone (CZ) between r_i and r_o.

Equations are solved using the pseudospectral code *Dedalus*.

(Burns+, Phys. Rev. Res., 2020)

Parameter survey

The dynamics of the system are determined by:

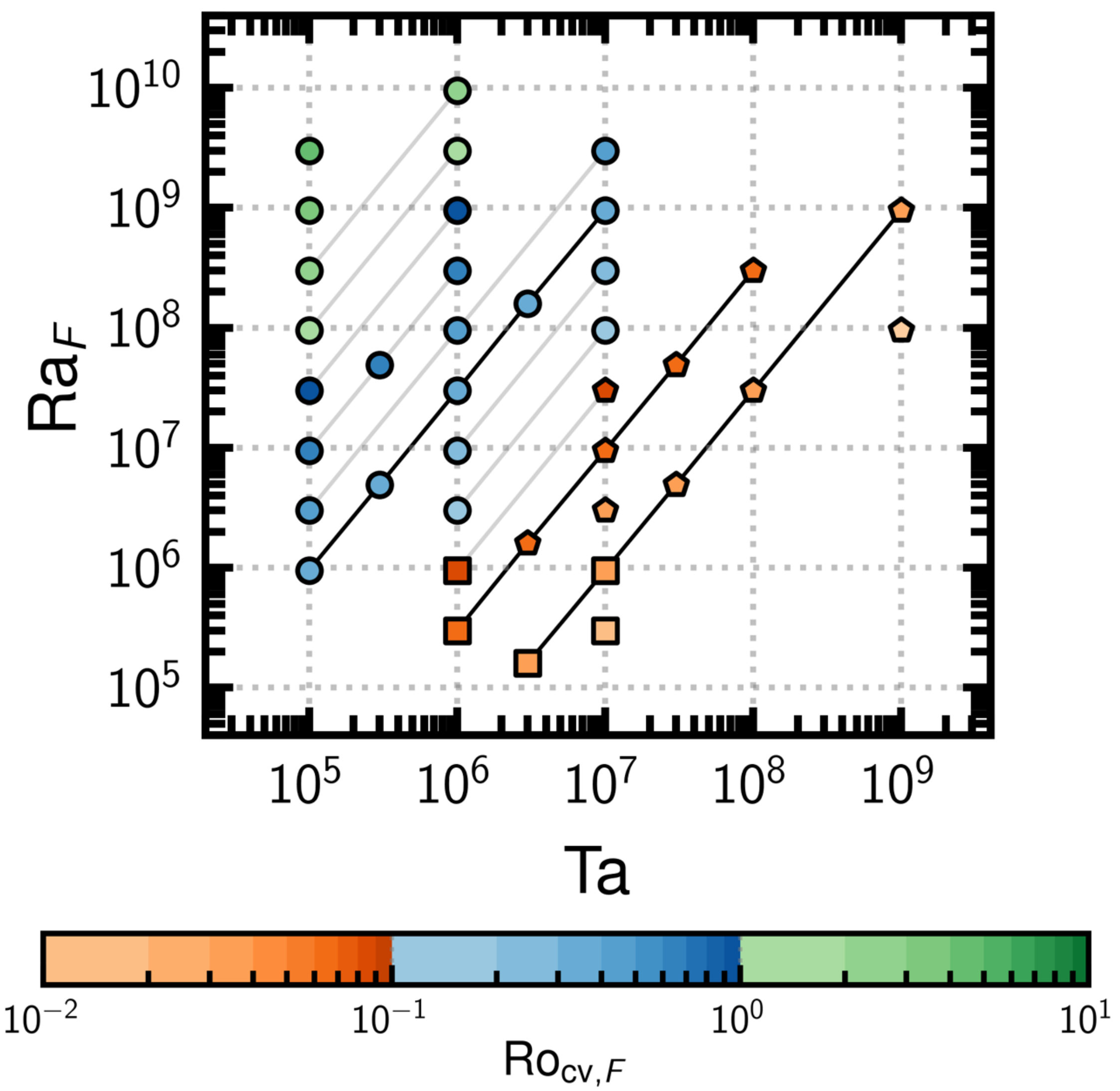
$$Ra_F \equiv \frac{d^4 F}{\nu\kappa^2}, \quad Ta \equiv \frac{4\Omega^2 d^4}{\nu^2}, \quad Pr \equiv \frac{\nu}{\kappa}, \quad \eta \equiv \frac{r_i}{r_o}, \text{ and } \quad \tilde{\delta} \equiv \frac{\delta}{d}.$$

In all simulations, $Pr = 1, \eta = 0.8$ and $\tilde{\delta} = 0.2$.

We are interested in comparing simulations with fixed

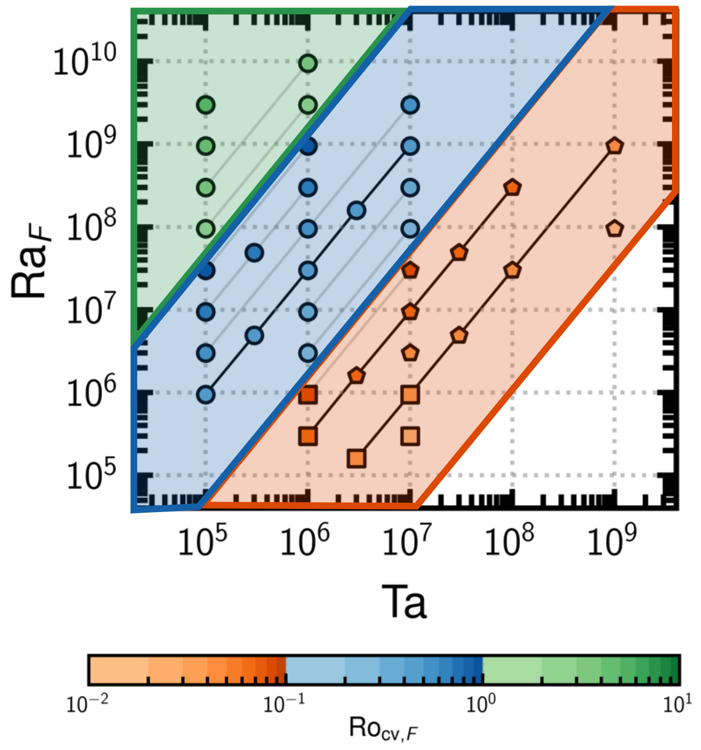
$$Ro_{cv,F} = \left(\frac{Ra_F}{Ta^{\frac{3}{2}} Pr^2}\right)^{1/3} = (dF)^{1/3} \frac{1}{2\Omega d}.$$

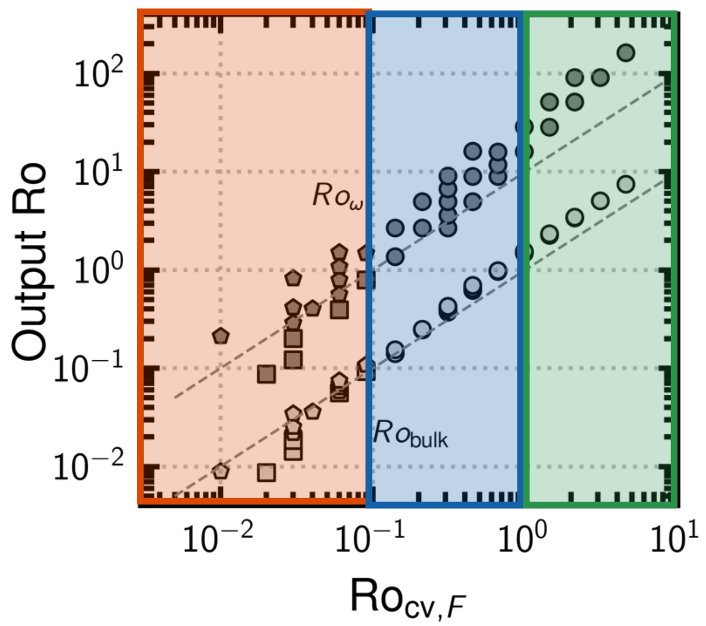
For a given $Ro_{cv,F}$, different combinations of Ra and Ta correspond to different values for the diffusivities.



Regimes

Our results will show that it is useful to categorise our simulations using three 'dynamical regimes'





$$Ro_{\text{bulk}} = \frac{U}{\Omega d}$$

$$Ro_\omega = \frac{\omega}{\Omega}$$

(i) rotationally-constrained

(ii) rotationally-influenced

(iii) rotationally-uninfluenced

$Ro_{cv,F} < 0.1$ ($Ro_\omega < 1$; $Ro_{bulk} << 1$)

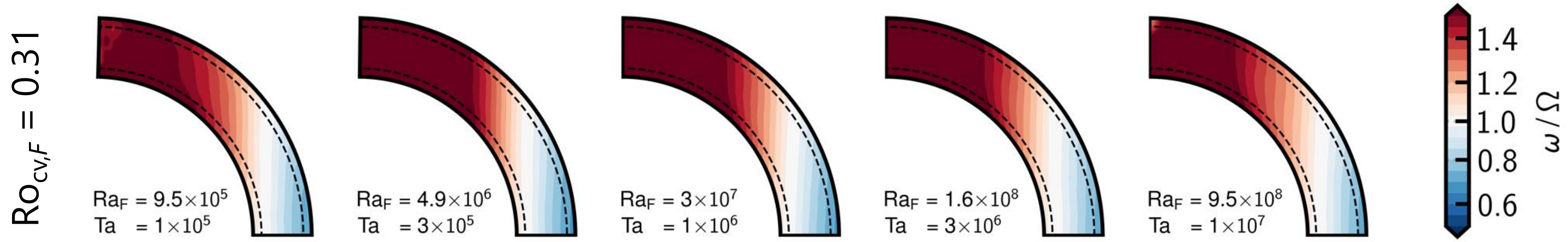
$0.1 < Ro_{cv,F} < 1$ ($Ro_\omega > 1$; $Ro_{bulk} < 1$)

$Ro_{cv,F} > 1$ ($Ro_\omega >> 1$; $Ro_{bulk} > 1$)

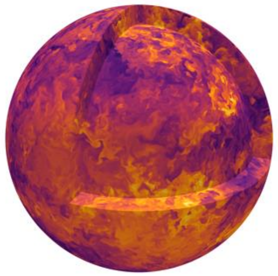
Differential rotation

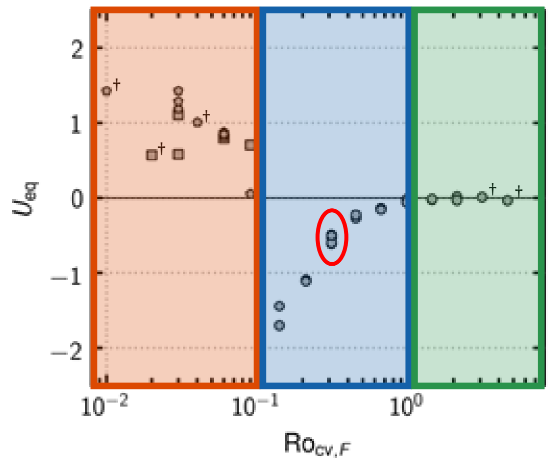
Azimuthally-averaged angular velocity normalised by the rotation rate.

(ii) rotationally-influenced



Decreasing diffusivity





Differential rotation

Azimuthally-averaged angular velocity normalised by the rotation rate.

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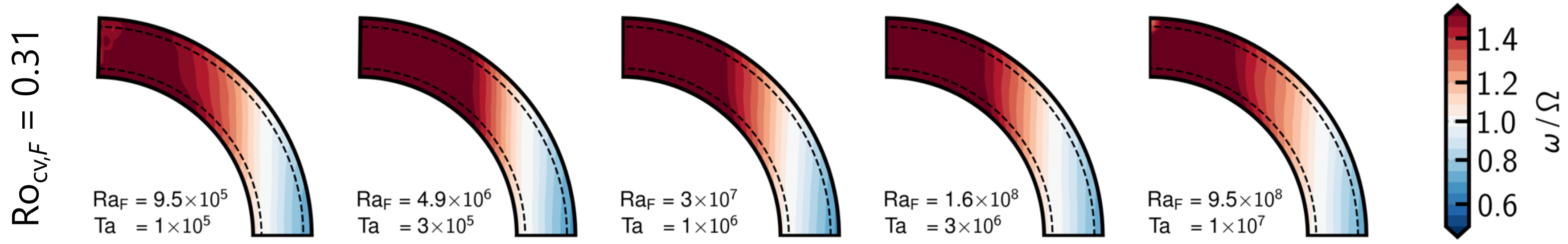
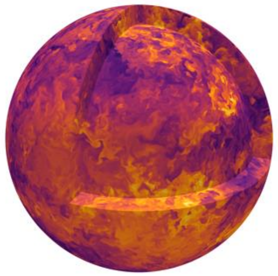
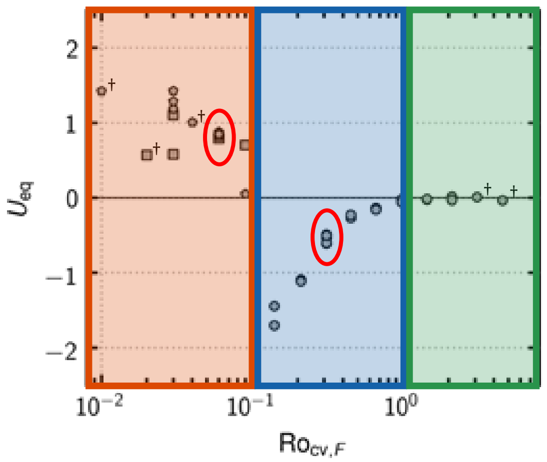
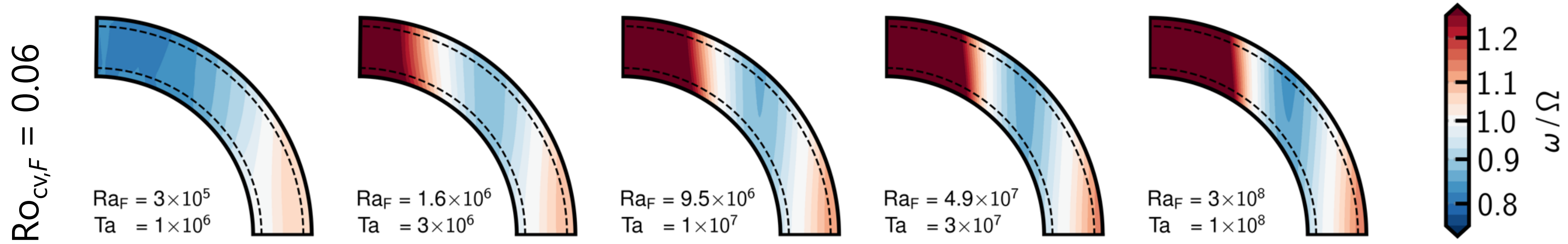
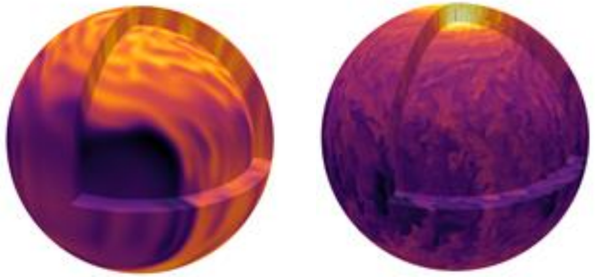






(i) rotationally-constrained



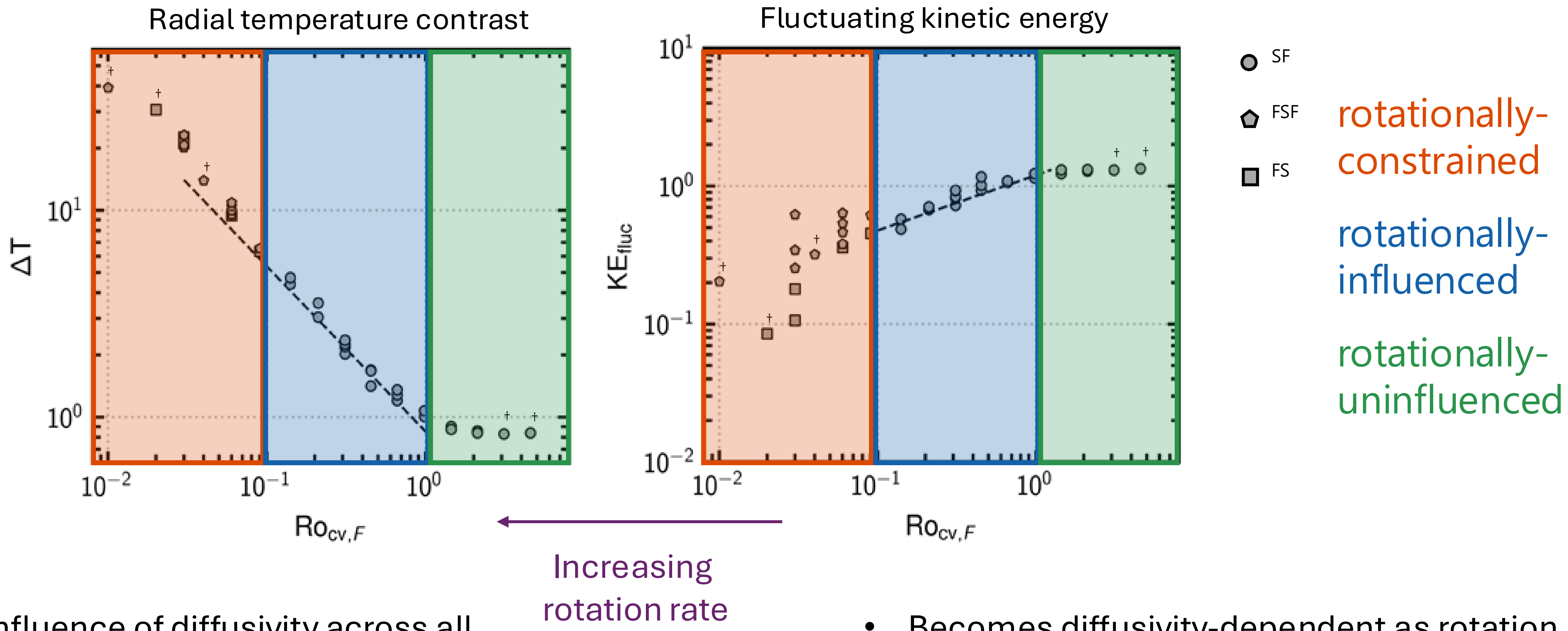


Decreasing diffusivity →

Mean temperature gradient and convective velocities

Diagnostics are pseudo-dimensional and plotted against $Ro_{cv,F}$. As $Ro_{cv,F}$ is independent of the viscosity, data will collapse onto one curve if diffusion-free.

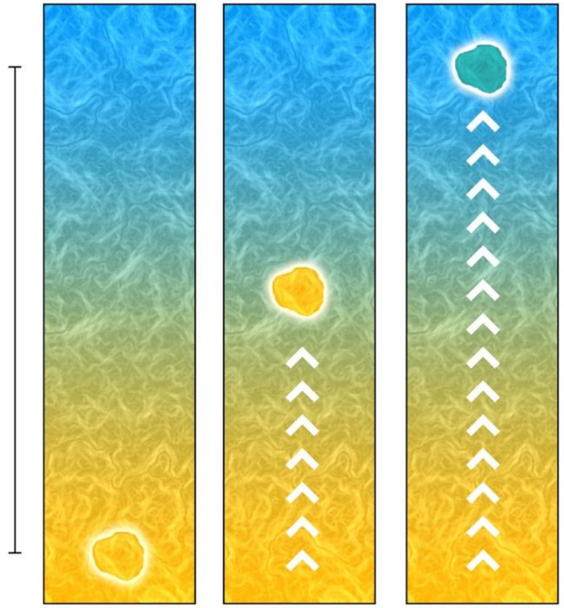
Independent of $Ro_{cv,F}$ when $Ro_{cv,F} > 1$.



- Minimal influence of diffusivity across all simulations.
- Becomes diffusivity-dependent as rotation rate is increased (small *Ro*).

(Rotating) Mixing Length Theory

(Wikipedia)



Basic idea: - Seek growing mode solutions to linearised governing equations.
- Assume that heat transport is dominated by largest modes.
- Assume linear growth is balanced by the nonlinear cascade rate.
- Assume temperature is mixed over a 'mixing length'.
- Assume temperature and velocity are correlated.

Mixing Length Theory:

$$u \sim F^{\frac{1}{3}} H^{\frac{1}{3}} \qquad -\frac{\mathrm{d}T_0}{\mathrm{d}z} = N_*^2 \sim \frac{F^{\frac{2}{3}}}{H^{\frac{4}{3}}} \qquad T \sim \frac{F^{\frac{2}{3}}}{H^{\frac{1}{3}}}$$

Rotating Mixing Length Theory (in the limit of **rapid enough rotation**):

$$u \sim \frac{H^{\frac{1}{5}} F^{\frac{2}{5}}}{\Omega^{\frac{1}{5}}} \qquad -\frac{\mathrm{d}T_0}{\mathrm{d}z} = N_*^2 \sim \frac{F^{\frac{2}{5}} \Omega^{\frac{4}{5}}}{H^{\frac{4}{5}}} \qquad T \sim \frac{F^{\frac{3}{5}} \Omega^{\frac{1}{5}}}{H^{\frac{1}{5}}} \qquad k_h \sim \frac{\Omega^{\frac{3}{5}}}{H^{\frac{3}{5}} F^{\frac{1}{5}}}$$

Convection is inhibited by rotation. As Ω increases, k_h and N_* increase, while u and T decrease.

Good agreement with these scalings has been found in Cartesian simulations as long as the zonal flows remain weak.

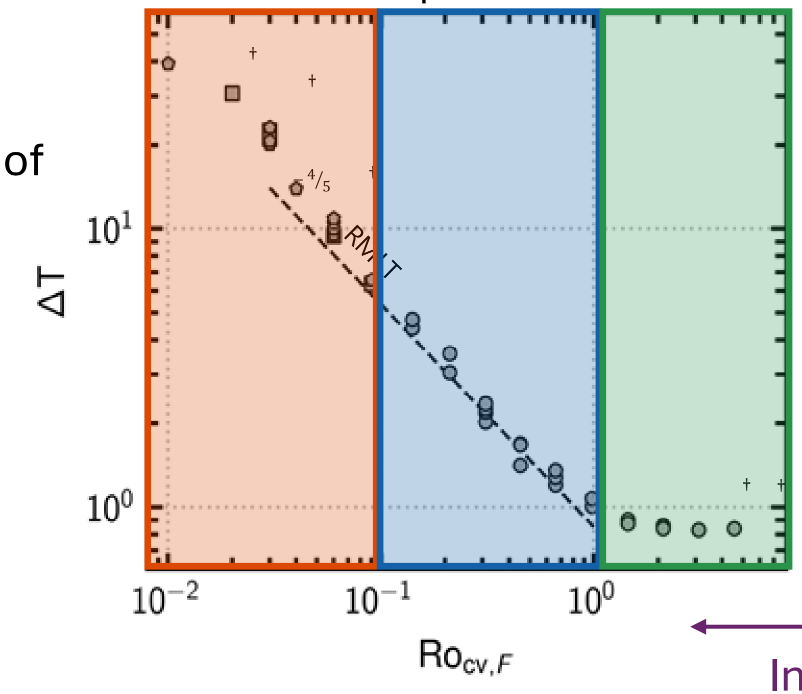
(e.g., Barker+, ApJ, 2014; Stevenson, GAFD, 1979; Böhm-Vitense, Zeitschrift für Astrophysik, 1958)

Comparison with RMLT

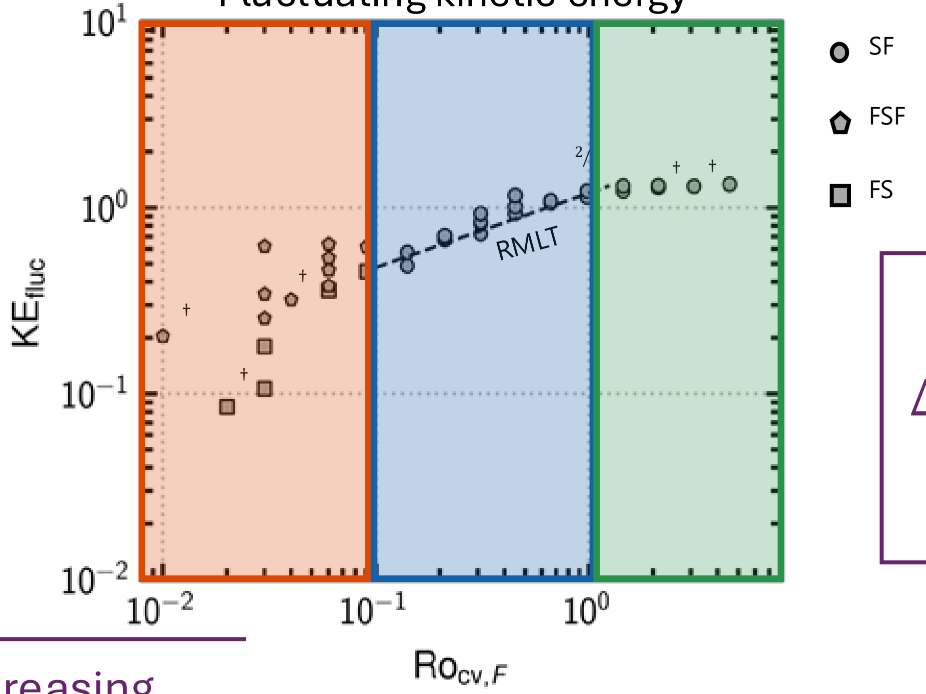
Diagnostics are pseudo-dimensional and plotted against $Ro_{cv,F}$. As $Ro_{cv,F}$ is independent of the viscosity, data will collapse onto one curve if diffusion-free.

Independent of $Ro_{cv,F}$ when $Ro_{cv,F} > 1$.

Radial temperature contrast



Fluctuating kinetic energy



RMLT

$\Delta T \sim Ro_{cv,F}{}^{-4/5}$

KE $\sim Ro_{cv,F}{}^{2/5}$

Increasing rotation rate

- Minimal influence of diffusivity.
- Consistent with RMLT for intermediate rotation, something else going on for rapid rotation (but still diffusion-free).

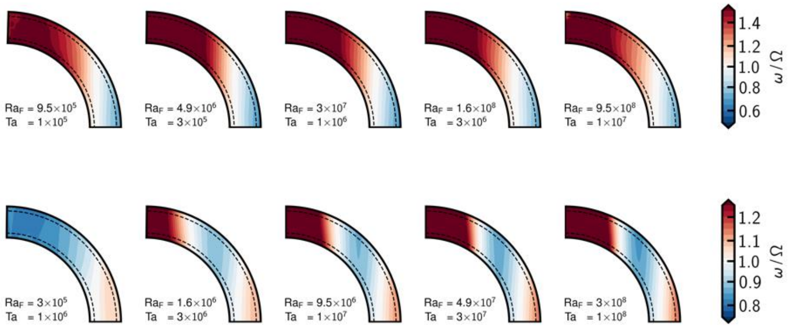
- Intermediate $Ro_{cv,F}$ scaling for KE_{fluc} consistent with RMLT.
- Diffusivity-dependent as rotation rate is increased (small *Ro*).

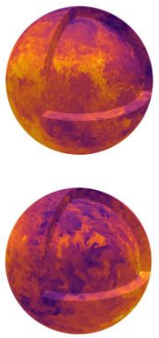
Summary of spherical shell simulations

Is it possible to obtain 'diffusion-free' scaling behaviour for the convective heat transport (and other properties), for both slow and rapid rotation?

Does MLT/RMLT describe the simulation behaviour accurately?

	Ro			Nu / ΔT		Re / $\mathrm{KE_{fluc}}$	
Name	$Ro_{\mathrm{cv,F}}$	Ro_ω	Ro_{bulk}	Diff.-free?	Scaling	Diff.-free?	Scaling
Rot.-constrained	$Ro < 0.1$	$Ro \lesssim 1$	$Ro \ll 1$	✓	Offset RMLT	✗	VAC
Rot.-influenced	$0.1 \leq Ro < 1$	$Ro \gtrsim 1$	$Ro < 1$	✓	RMLT	✓	CIA
Rot.-uninfluenced	$Ro \geq 1$	$Ro \gg 1$	$Ro \geq 1$	✓	MLT	✓	Ultimate





Possible to obtain zonally-averaged zonal flow statistics and flow 'morphologies' that are diffusion-free (for sufficient supercriticality).

(Lewis+ ApJ, in press)

Future directions

Extend simulations further into rotationally-constrained regime. E.g., lower $Ro_{cv,F}$ and greater Ra_c .

Explore latitudinal variation.

Generalise model to include compressibility and magnetism.