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Role of conservation laws and fluxes in the self-organization of turbulence

Anna Frishman, Technion, Israel

New Directions in Theoretical Physics 5,
Jan 6th - Jan 9th 2026, Edinburgh

Outline

1. Turbulence why its everywhere and why its difficult
2. Simple picture for 3D turbulence, 2D turbulence (self-organization)

<u>3D turbulence + rotation</u>

4. What is the puzzle? (self-organization into 2D structures)
5. The role of waves (self-organization + wave turbulence)
6. Explaining the energy partition between 3D and 2D modes

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Waves drive the rise and fall of 2D flows in rotating turbulence

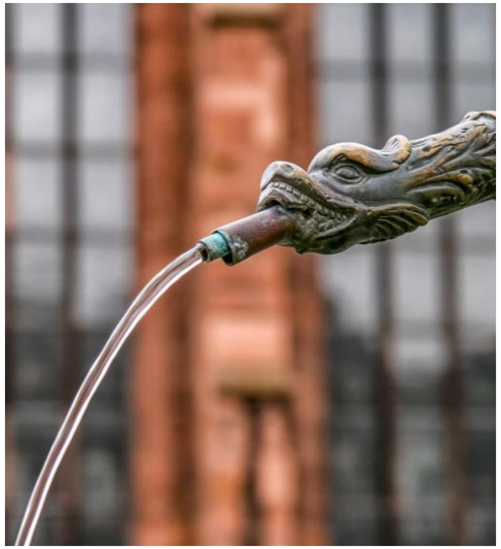
Anna Frishman, Technion, Israel

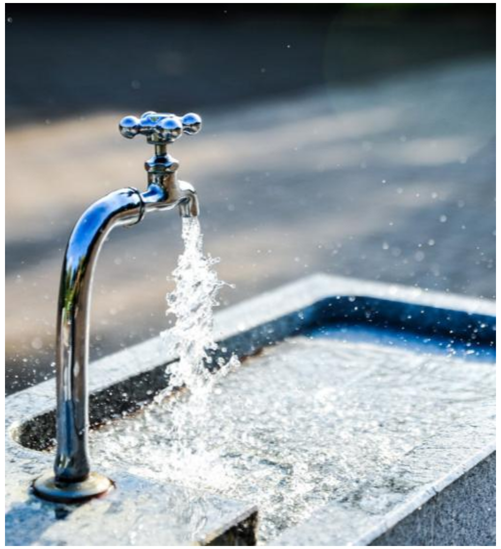
Sébastien Gomé

New Directions in Theoretical Physics 5,
Jan 6th - Jan 9th 2026, Edinburgh

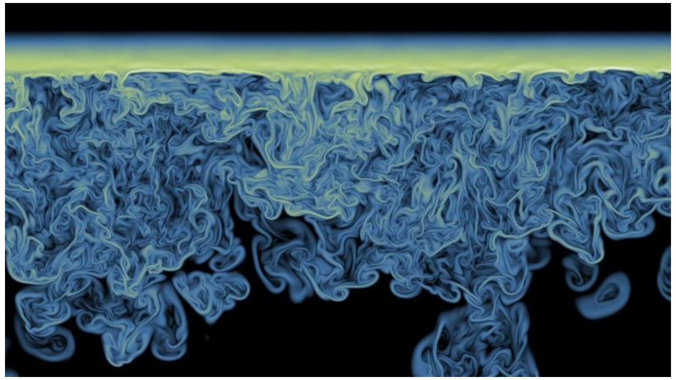
arXiv:2512.05253, arXiv:2509.18323

Turbulence: I know it when I see it





Visualization of turbulence
(temperature gradient in a cloud)
Mellado JFM (2010).



The control parameter: Reynolds number

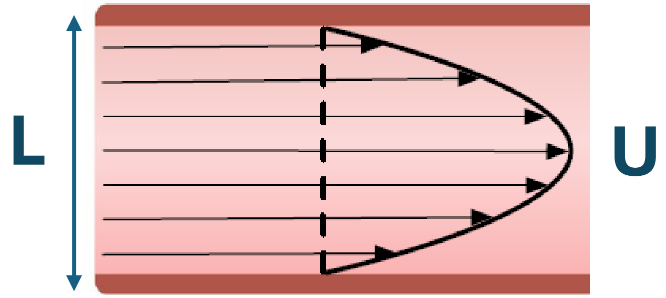
Inertia: Flow rate increases with increasing U, L

Friction: **Viscosity**, fluid property: $\nu\ [m^2]/[sec]$

$$Re = \frac{UL}{\nu}$$

Navier-Stokes

$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\nabla p + Re^{-1} \nabla^2 \boldsymbol{u},$$
$$\nabla \cdot \boldsymbol{u} = \boldsymbol{0}$$



The control parameter: Reynolds number

Inertia: Advection – **non-linear** effect

Friction: Loss of kinetic energy

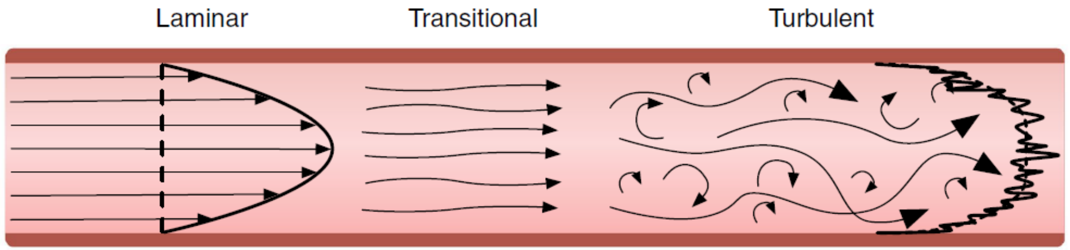
$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u} = -\boldsymbol{\nabla} p + Re^{-1} \nabla^2 \boldsymbol{u}, \qquad \boldsymbol{\nabla} \cdot \boldsymbol{u} = \boldsymbol{0}$$

$$Re = \frac{UL}{\nu}$$

The control parameter: Reynolds number

Inertia: Advection – **non-linear** effect

Friction: Loss of kinetic energy



$$Re = \frac{UL}{\nu}$$

Increasing non-linearity

$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \nabla \boldsymbol{u} = -\nabla p + Re^{-1} \nabla^2 \boldsymbol{u},$$
$$\nabla \cdot \boldsymbol{u} = 0$$

Typical Re is large!

Inertia: Advection – **non-linear** effect

Friction: Loss of kinetic energy

$$Re = \frac{UL}{\nu} \sim 10^4$$

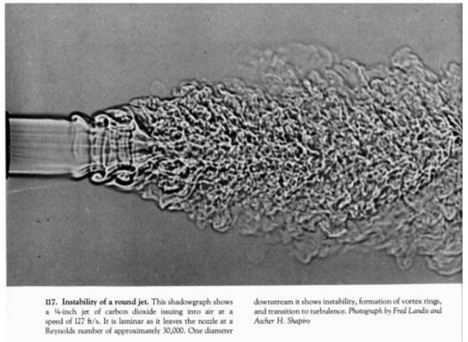
Water: $\nu = 10^{-6}\,\frac{\mathrm{m}^2}{\mathrm{s}}$

Turbulence across systems and scales

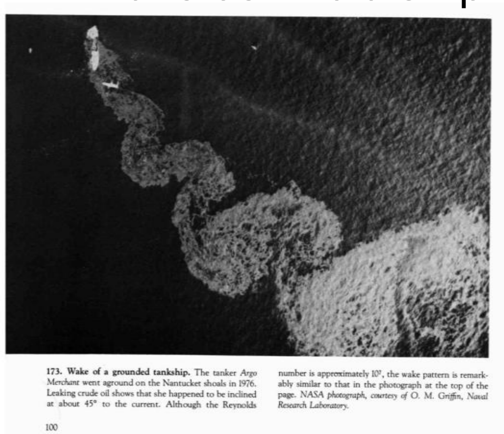
Turbulent mixing

Image from: Majchrzak et al, *Energies* (2023).

Turbulent Jet

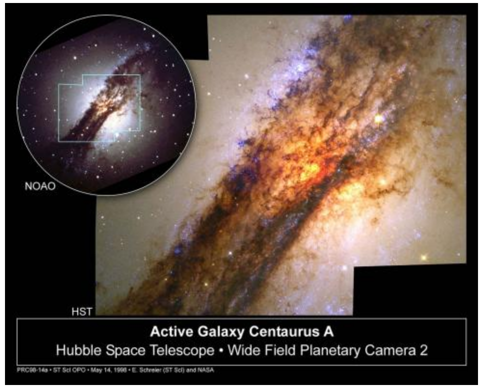


Wake behind a ship



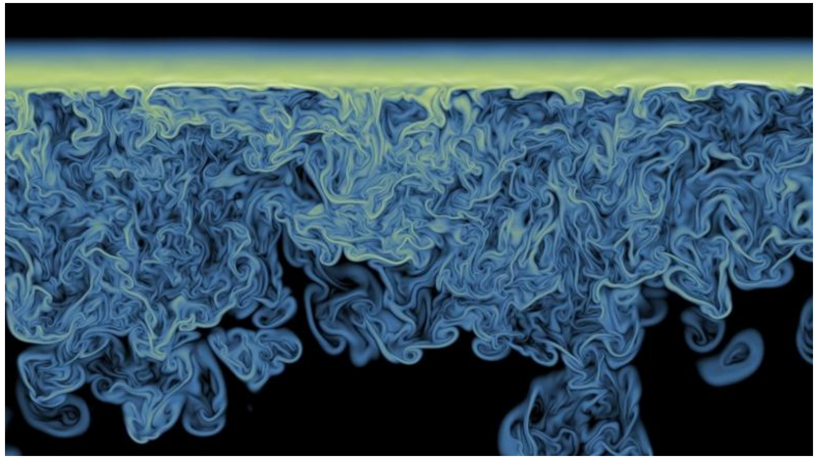
Turbulence in rivers and clouds

Turbulence in dust clouds

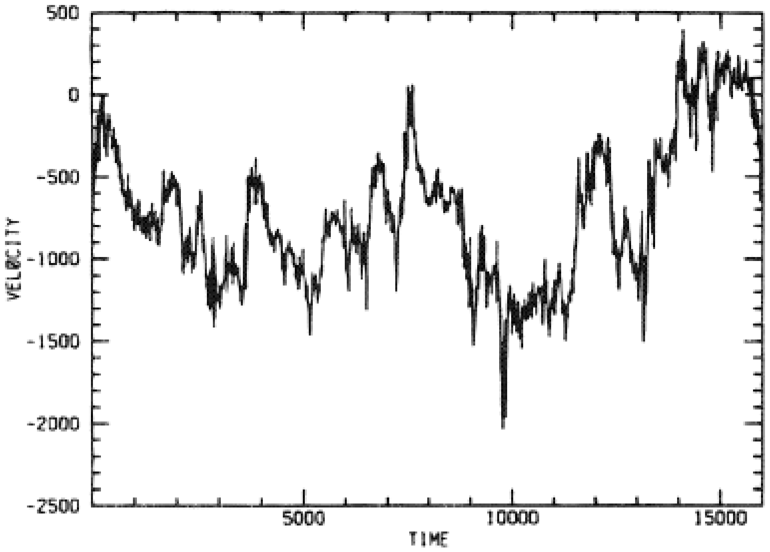


Fully developed turbulence = limit of $Re \to \infty$

Statistical mechanics of flow fields

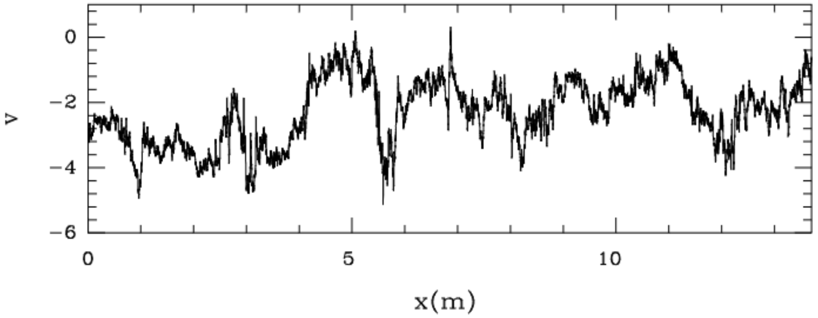


Velocity time series at a point



Sreenivasan (1991)

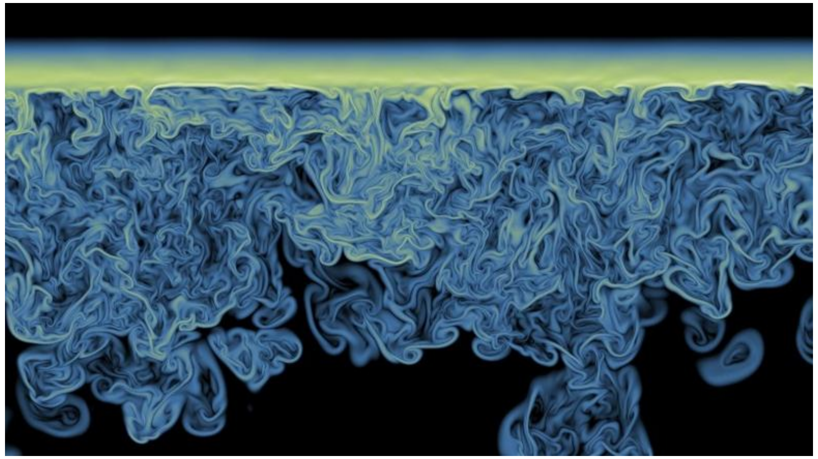
Velocity spatial variation at a given time



Arneodo et al (1997)

Statistical theory of Turbulence?

- Strongly interacting (highly non-linear, $Re \to \infty$)
- Driven far from equilibrium (fluxes play a key role)
- Long range interactions (sound waves equilibrate pressure)



$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u} = -\boldsymbol{\nabla} p + Re^{-1} \nabla^2 \boldsymbol{u} + \boldsymbol{f},$$
$$\boldsymbol{\nabla} \cdot \boldsymbol{u} = \boldsymbol{0}$$

Crash course on 3D turbulence

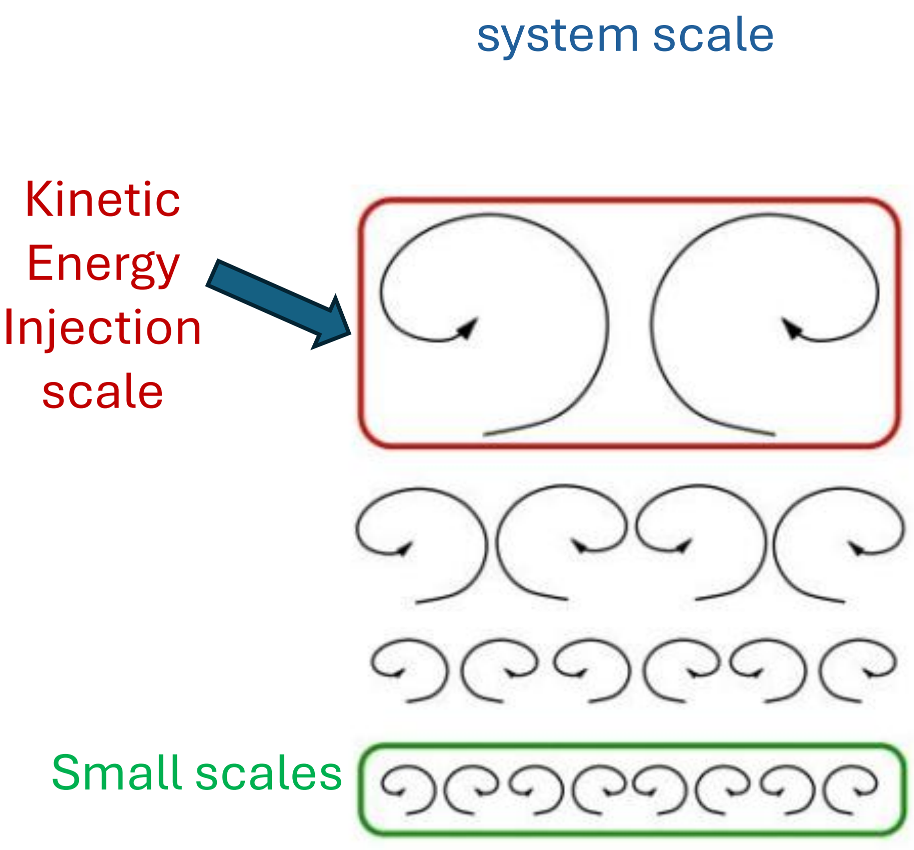
Importance of conservation laws:

Kinetic energy conserved without driving and viscosity

$$Re = \frac{UL}{\nu} \gg 1$$

Euler equation: $$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u} = -\boldsymbol{\nabla} p, \quad \boldsymbol{\nabla} \cdot \boldsymbol{u} = \boldsymbol{0}$$

Crash course on 3D turbulence

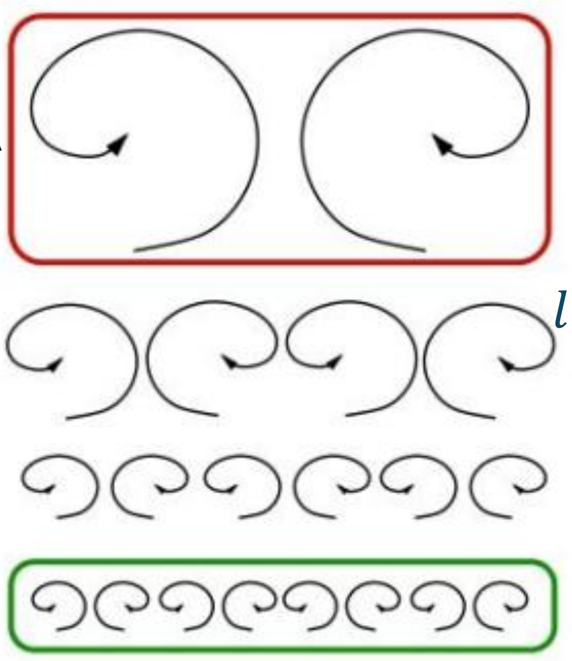


$$Re = \frac{UL}{\nu} \gg 1$$

Crash course on 3D turbulence

system scale

Kinetic Energy Injection scale



Small scales

$$Re = \frac{UL}{\nu} \gg 1$$

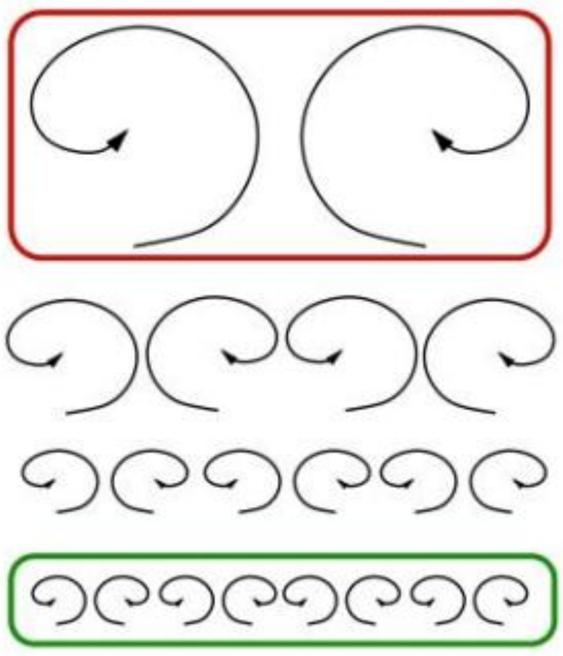
$$Re_l = \frac{U_l l}{\nu} \gg 1$$

Energy conserving interactions

Crash course on 3D turbulence

Domain scale

Kinetic Energy injection scale



$$Re = \frac{UL}{\nu} \gg 1$$

Flux of **Energy**

Π_k^E

$$Re_l = \frac{U_l l}{\nu} \gg 1$$

Energy conserving interactions

Small scales

Crash course on 3D turbulence

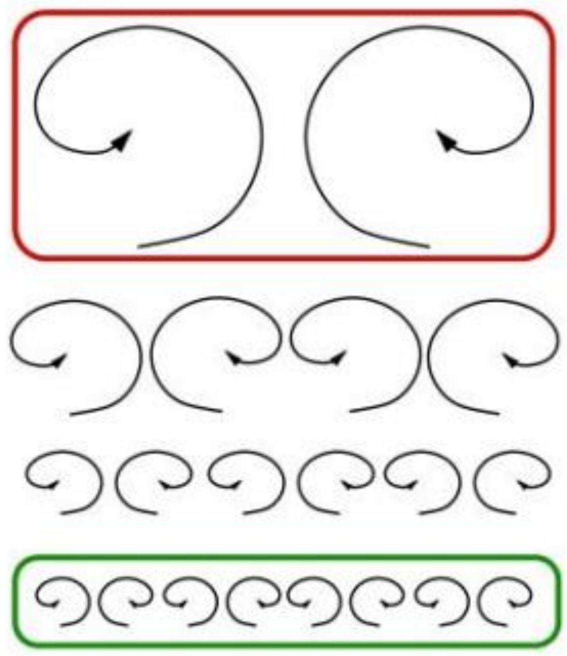
system scale

Energy injection

rate = ϵ

Flux of **Energy**

$$\Pi_k^E = \epsilon$$



Small scales

Energy dissipation

rate = ϵ

$$Re = \frac{UL}{\nu} \gg 1$$

Decreasing Reynolds number

$$Re_l = \frac{U_l l}{\nu}$$

$$Re_{l_\nu} = \frac{U_{l_\nu} l_\nu}{\nu} \approx 1$$

Finite viscous dissipation

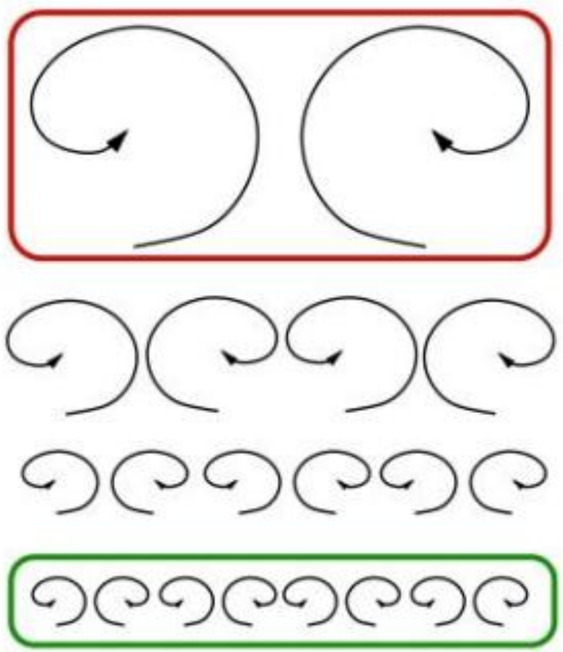
$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u} = -\boldsymbol{\nabla} p + \overbrace{Re^{-1} \nabla^2 \boldsymbol{u}}^{\text{Finite viscous dissipation}},$$

Crash course on 3D turbulence

system scale

Energy injection

rate = ϵ



$$Re = \frac{UL}{\nu} \gg 1$$

Flux of **Energy**

$$\Pi_k^E = \epsilon$$

Decreasing Reynolds number

$$Re_l = \frac{U_l l}{\nu}$$

Small scales

$$Re_{l_\nu} = \frac{U_{l_\nu} l_\nu}{\nu} \approx 1$$

"UV cutoff"

$Re \to \infty$: $k_\nu \to \infty$

Energy dissipation

rate = ϵ

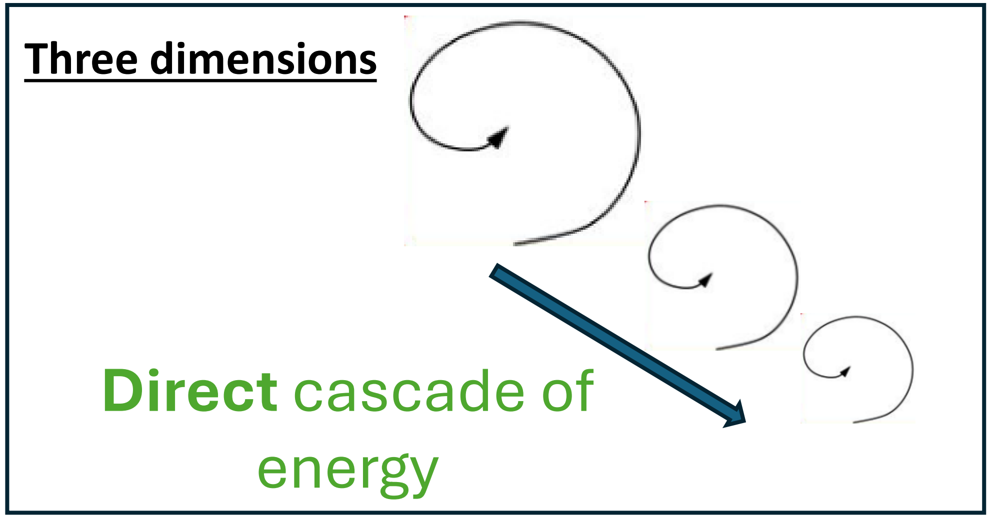
Finite viscous dissipation

$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u} = -\boldsymbol{\nabla} p + \overbrace{Re^{-1} \nabla^2 \boldsymbol{u}}^{\text{Finite viscous dissipation}},$$

Flavors of turbulence?

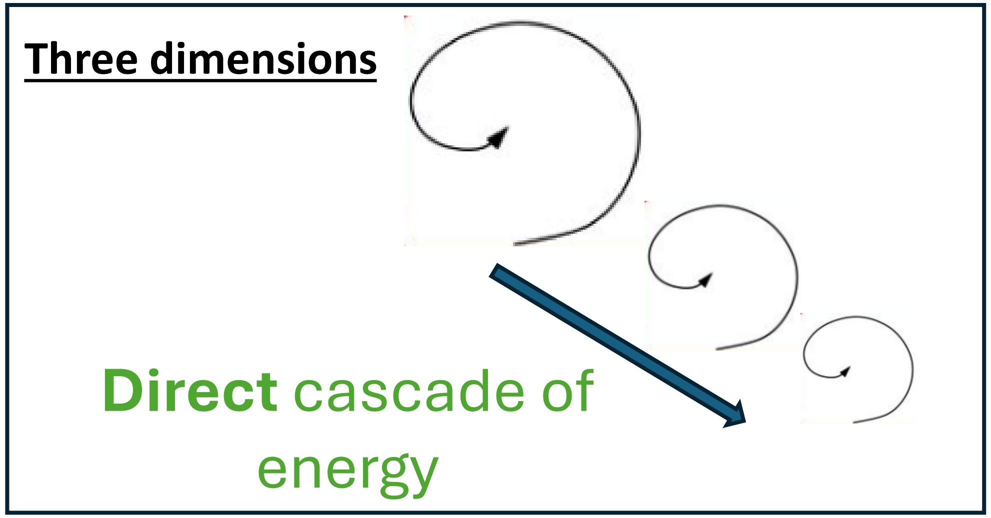
<u>3D turbulence</u>: small scales get generated from larger scales

"mixing"

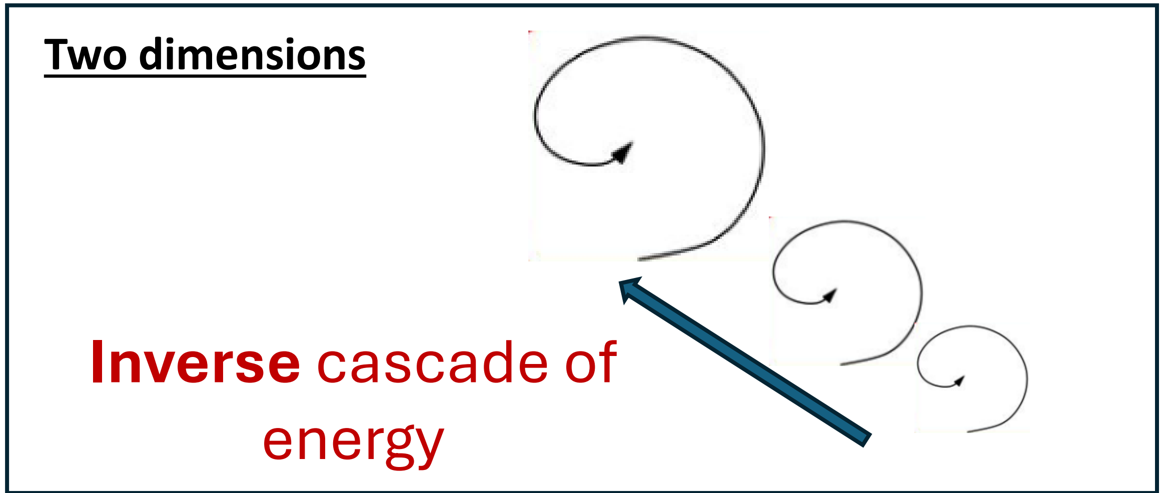


Flavors of turbulence?

<u>3D turbulence</u>: small scales get generated from larger scales
"mixing"

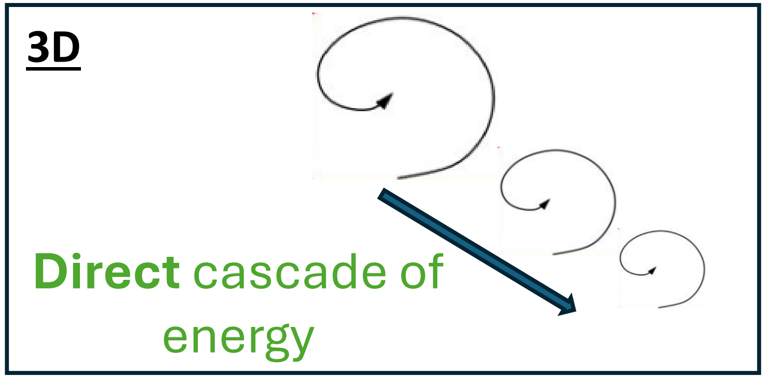


<u>2D turbulence:</u> large scale flows are generated from small scale turbulence
"self-organization"



Flavors of turbulence?

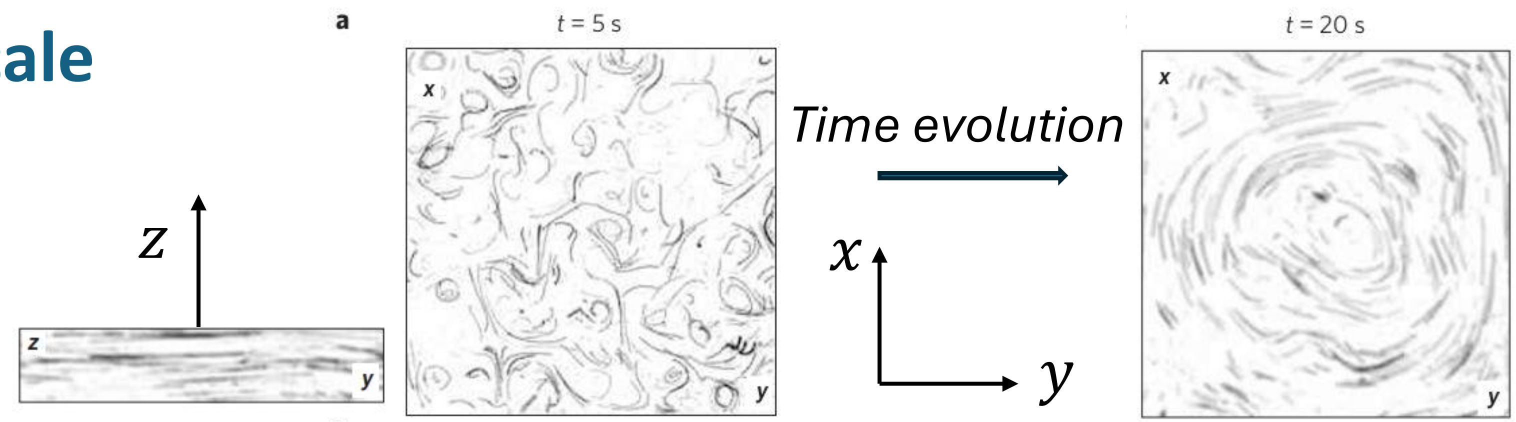
<u>3D turbulence</u>: small scales get generated from larger scales
"mixing"



<u>2D turbulence:</u> large scale flows are generated from small scale turbulence
"self-organization"

<u>Thin layer ~ 2D</u>

Thin fluid layer:



Xia *et al*, Nat. Phys. 2011

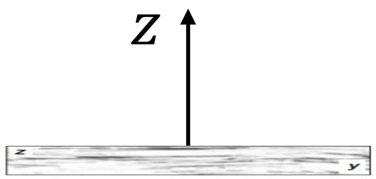
Crash course on 2D turbulence

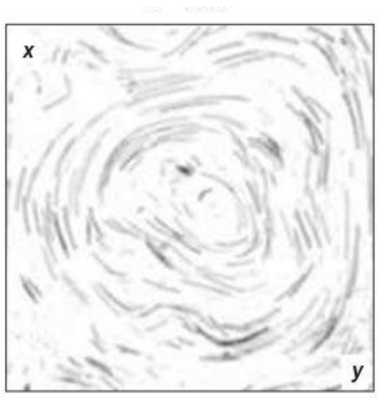
Two conserved quantities in absence of forcing and dissipation

Energy $$E = \frac{1}{2}\int (\vec{u} \cdot \vec{u})\, d^2x = \int E_k dk$$

Enstrophy $$Z = \frac{1}{2}\int (\nabla \times \vec{u})^2\, d^2x = \int k^2 E_k dk$$

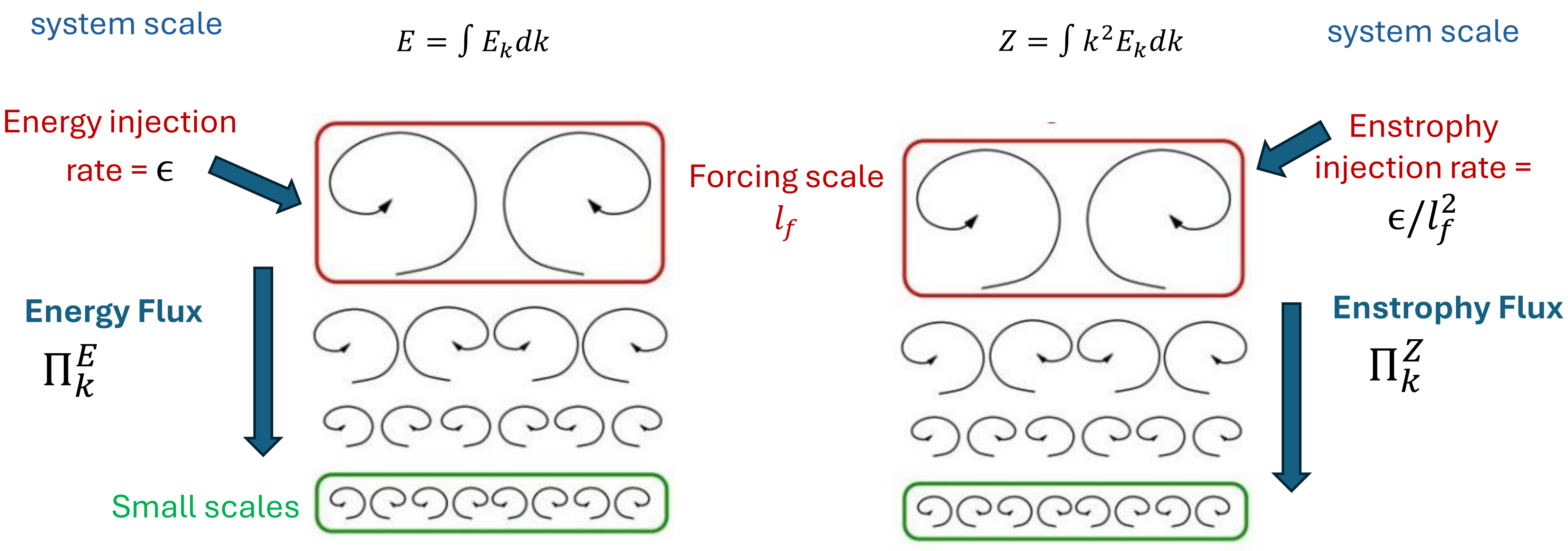
Thin layer
2D incompressible flow



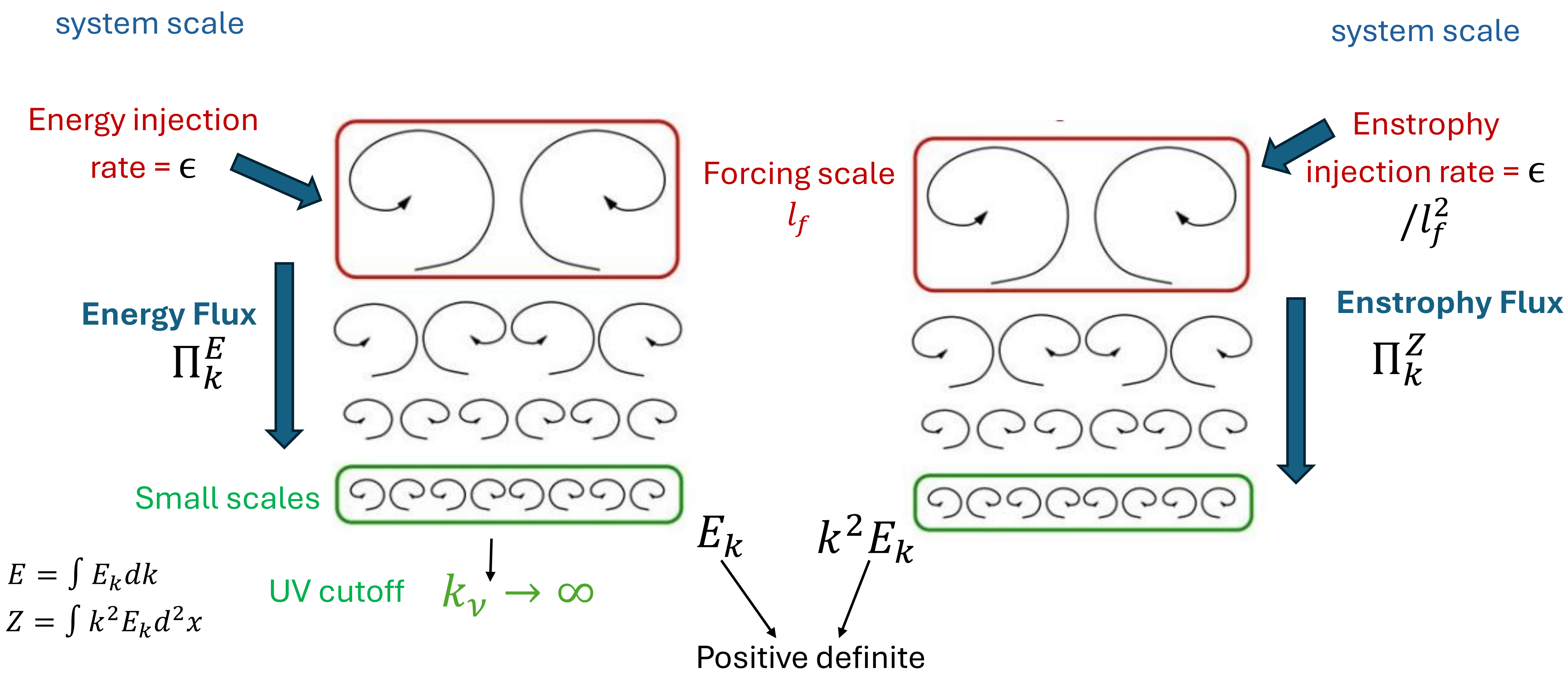


Conservation of **angular momentum** (per mass) in z direction for fluid elements

Crash course on 2D turbulence



Crash course on 2D turbulence

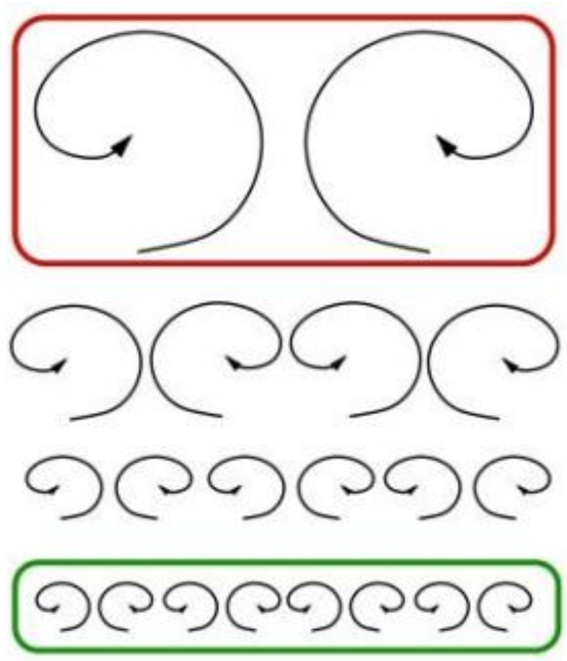


Crash course on 2D turbulence

system scale

Energy injection rate = ϵ

Energy Flux Π_k^E



Small scales

Forcing scale l_f

E_k

$E = \int E_k dk$

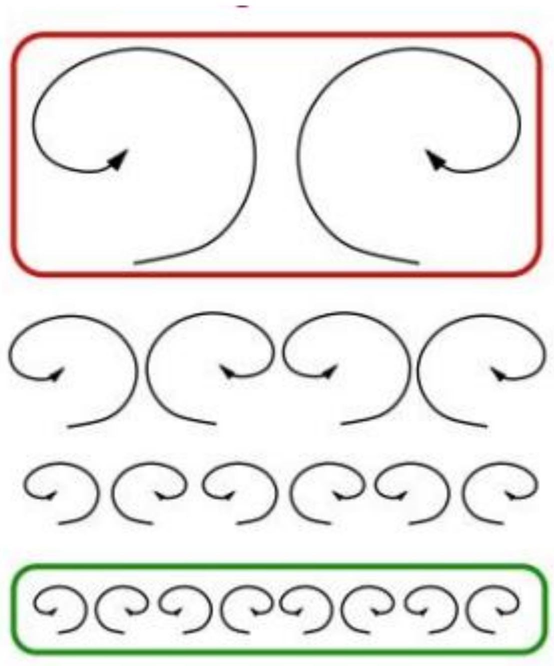
$Z = \int k^2 E_k dk$

UV cutoff $k_\nu \to \infty$

system scale

Enstrophy injection rate = ϵ / l_f^2

Enstrophy Flux Π_k^Z



$k^2 E_k$

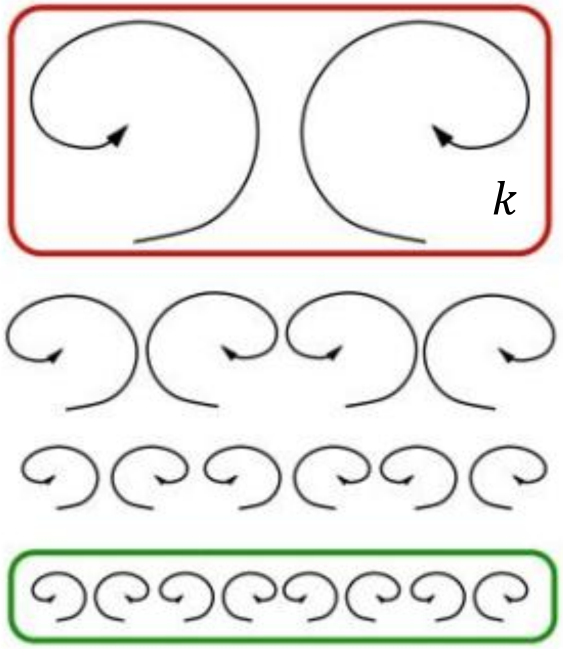
Large k: $\Pi_k^Z \sim k^2 \Pi_k^E$

Crash course on 2D turbulence

system scale

Energy injection rate = ϵ

Energy Flux Π_k^E



Small scales

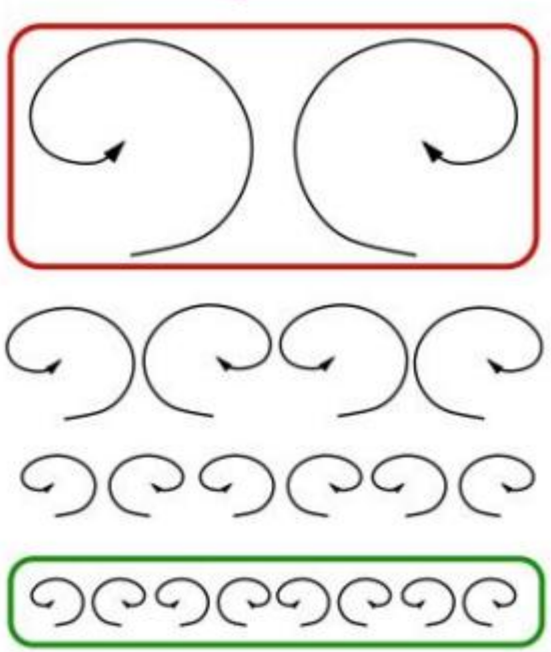
Forcing scale l_f

E_k $k^2 E_k$

system scale

Enstrophy injection rate = ϵ / l_f^2

Enstrophy Flux Π_k^Z

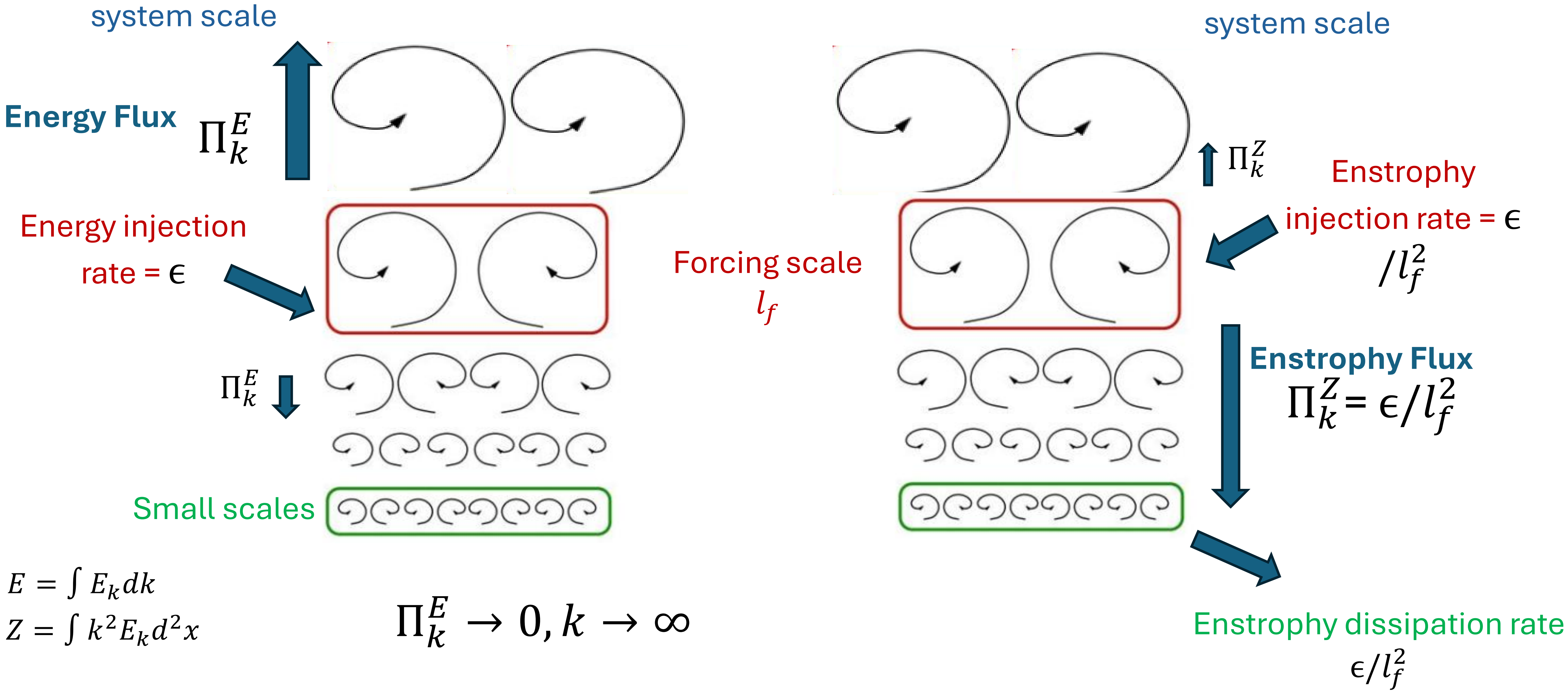


$E = \int E_k dk$

$Z = \int k^2 E_k d^2 x$

$\Pi_k^E \to 0, k \to \infty$ ← Large k: $\Pi_k^Z \approx k^2 \Pi_k^E$

Crash course on 2D turbulence

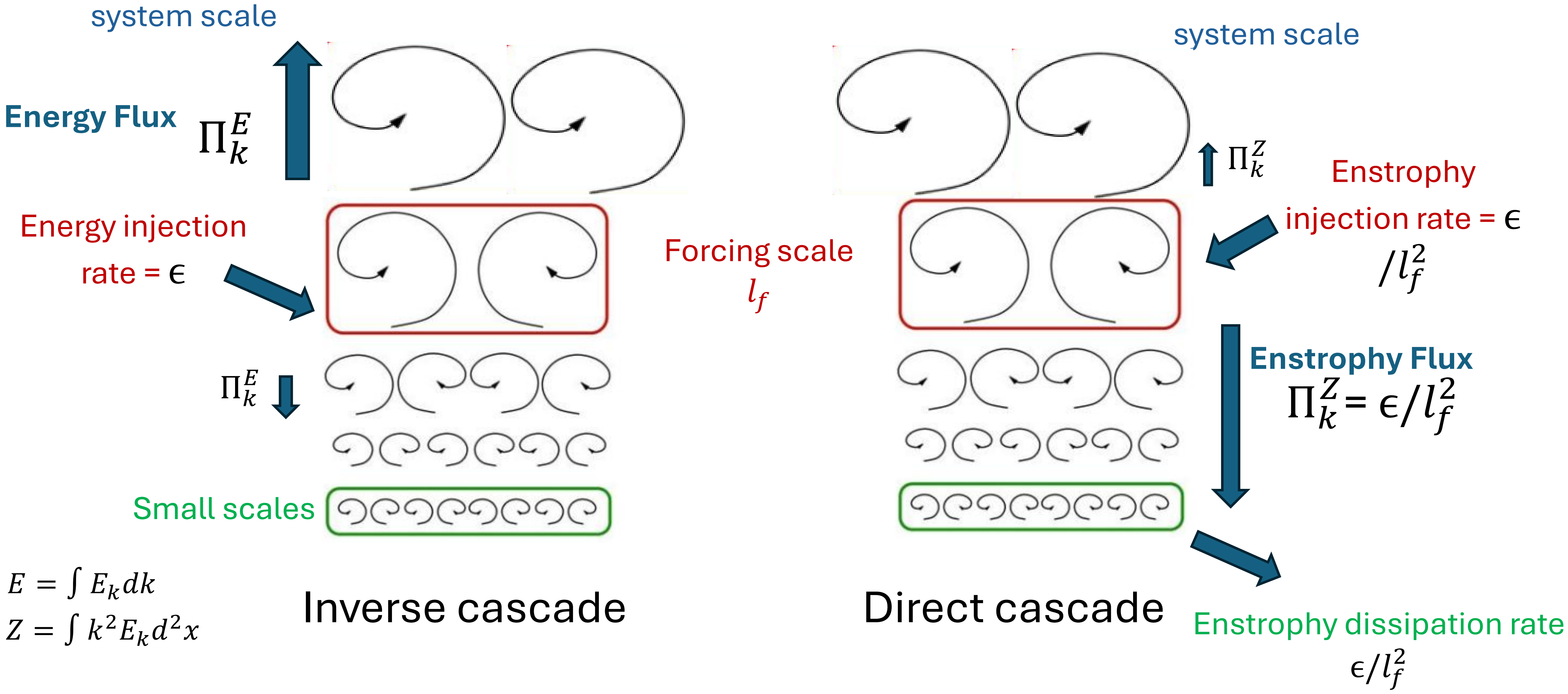


$E = \int E_k dk$

$Z = \int k^2 E_k d^2x$

$$\Pi_k^E \to 0, k \to \infty$$

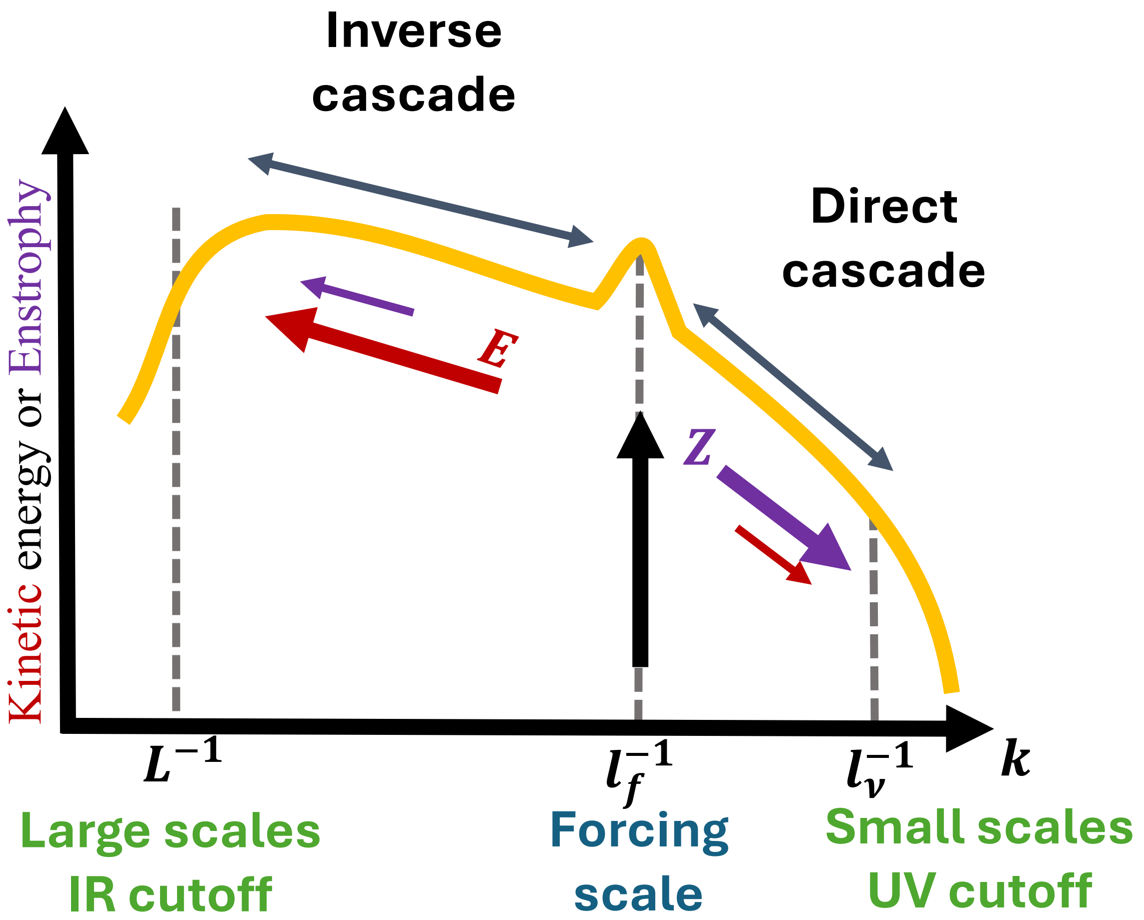
Double cascade in 2D turbulence



$E = \int E_k dk$

$Z = \int k^2 E_k d^2 x$

Double cascade in 2D turbulence



$E = \int E_k dk$

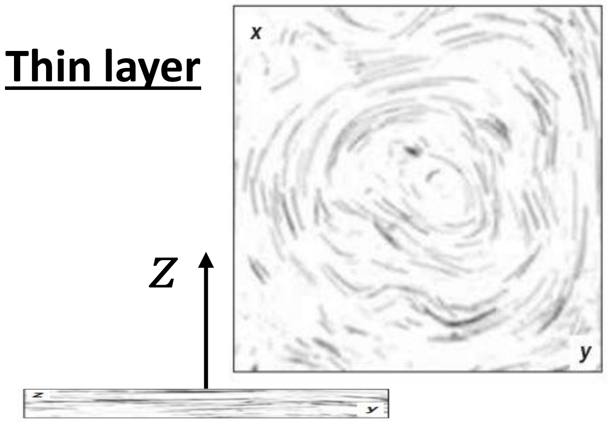
$Z = \int k^2 E_k d^2x$

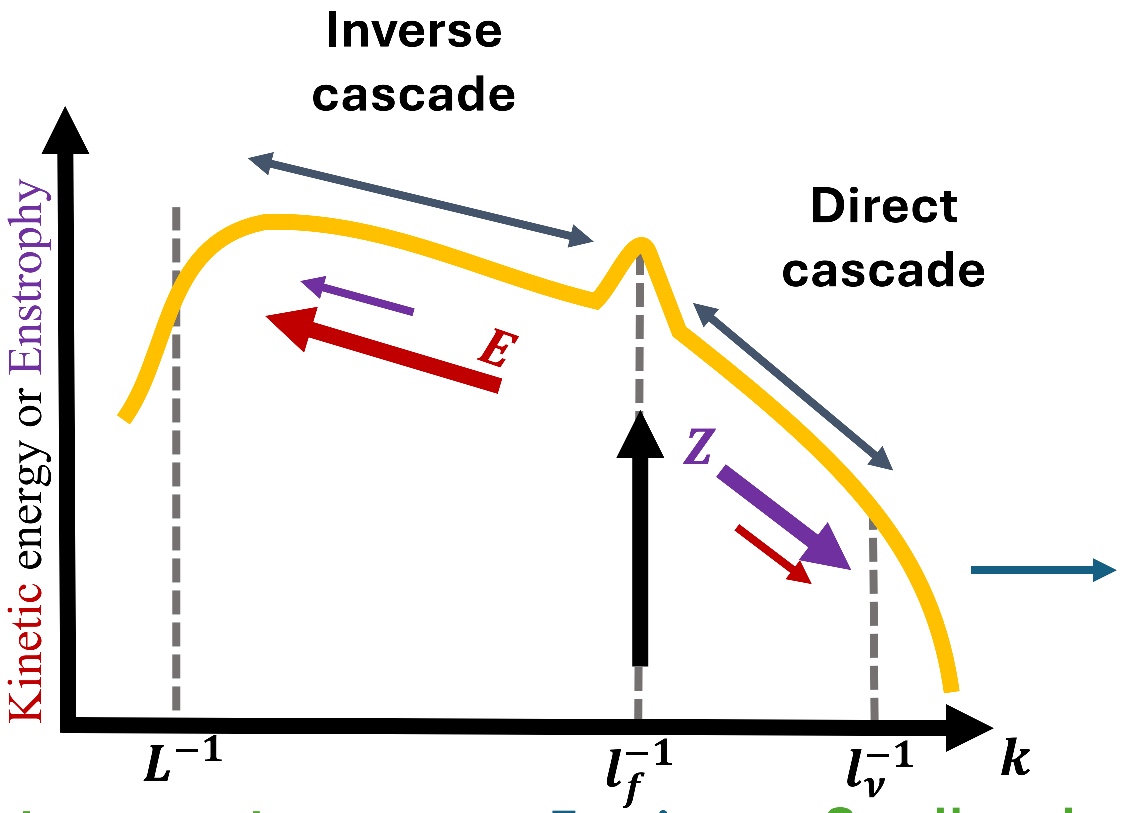
Double cascade in 2D turbulence

Slow dissipation at large scale

Energy gets accumulated at k~1/L

Kraichnan 1967





Large scales IR cutoff

Forcing scale

Small scales UV cutoff

Energy injection rate

ϵ

Small scale dissipation rate:

$$D^Z = \Pi^Z = \epsilon / l_f^2$$

$$D^E = l_\nu^2 / l_f^2 \epsilon \to 0$$

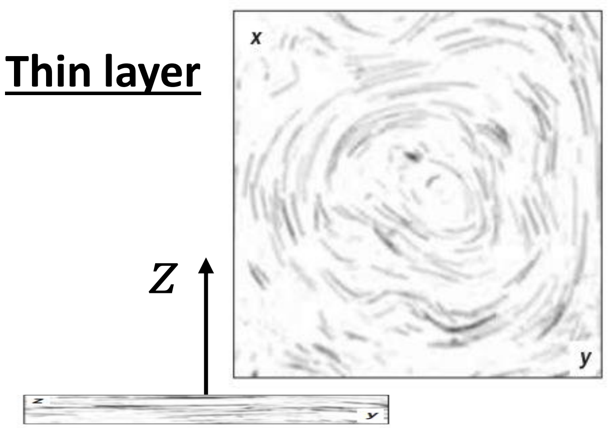
$$E = \int E_k dk$$

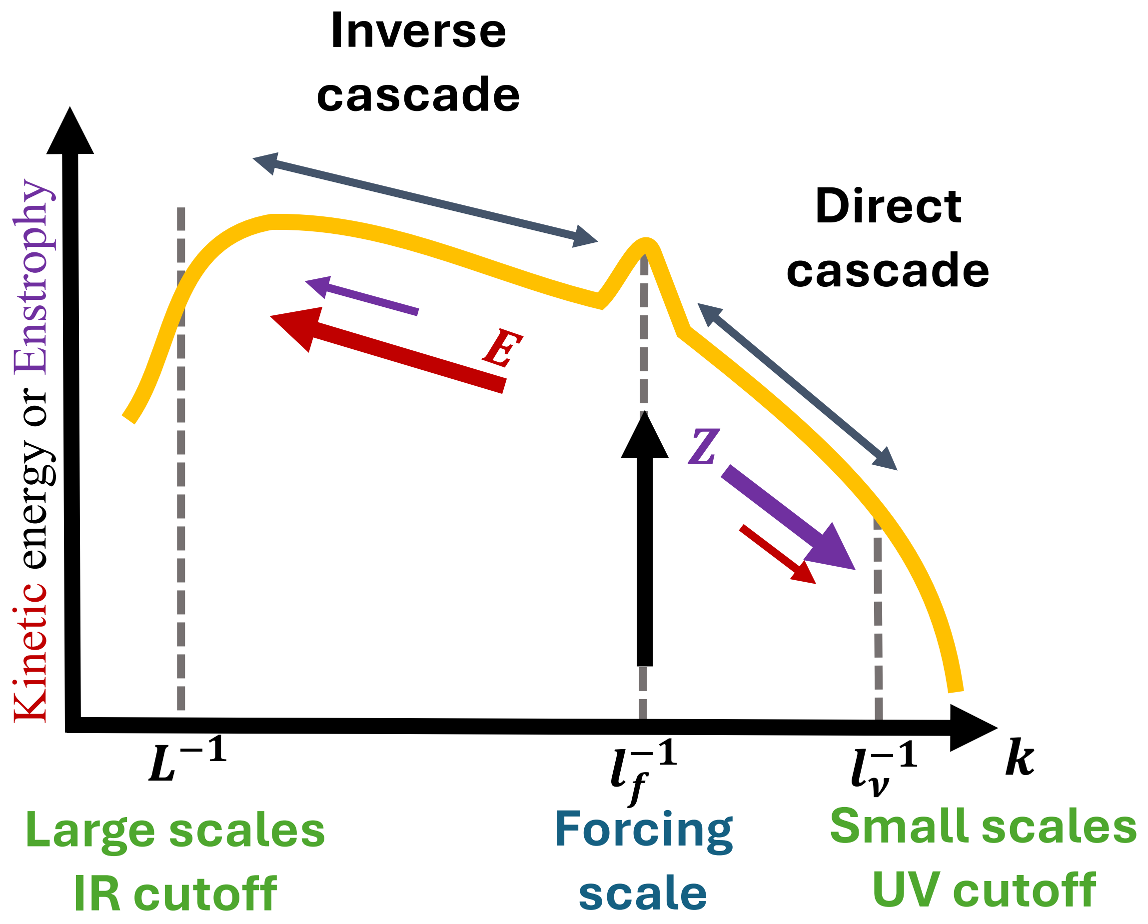
$$Z = \int k^2 E_k d^2 x$$

Self-organization of 2D turbulence

Slow dissipation at large scale

“Condensate” = large amplitude flow





$E = \int E_k dk$

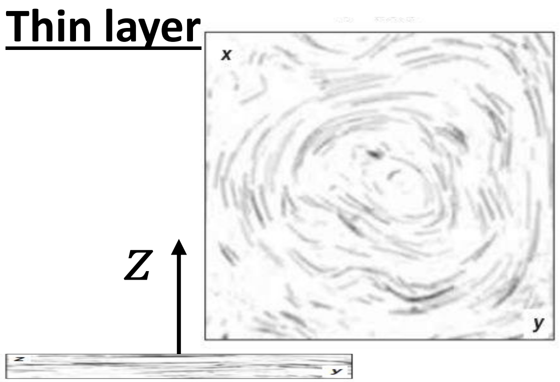
$Z = \int k^2 E_k d^2x$

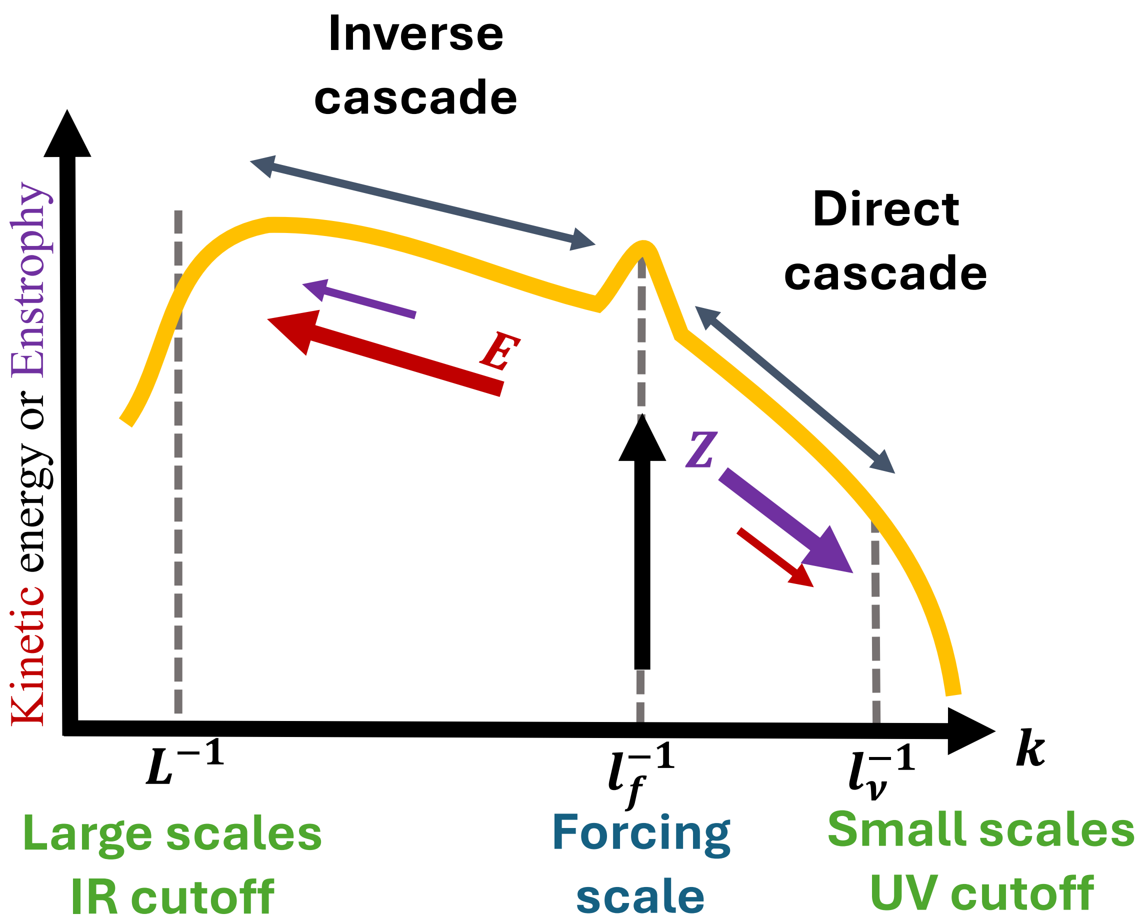
Self-organization of 2D turbulence

“Condensate” = large amplitude flow

Inverse energy transfer: from turbulence to condensate

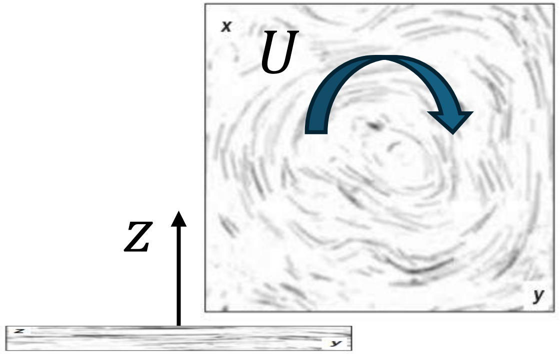
Energy dissipation: via condensate (in this talk quantified by Re)





Self-organization of 2D turbulence

Statistical description?



"condensate"

Perturbative self-consistent approach

(strong mean flow, weak fluctuations)

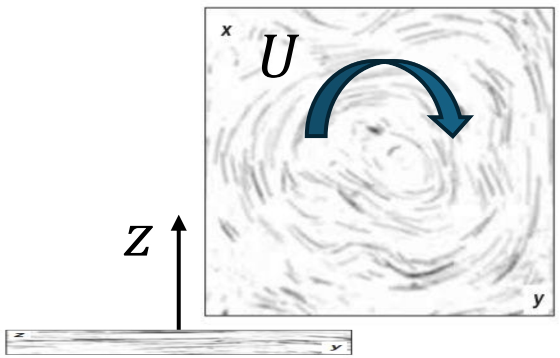
"quasi-linear" treatment for steady state

Review on of such approaches: Marston, J. B., and S. M. Tobias. Ann. Rev. Fluid Mechanics (2023)

Connections with

Self-organization of 2D turbulence

Statistical description?



"condensate"

Perturbative self-consistent approach
(strong mean flow, weak fluctuations)

"quasi-linear" treatment for steady state

Can use hierarchical approach for mean flow operator:
"small" viscosity, scale –separation, PT symmetry

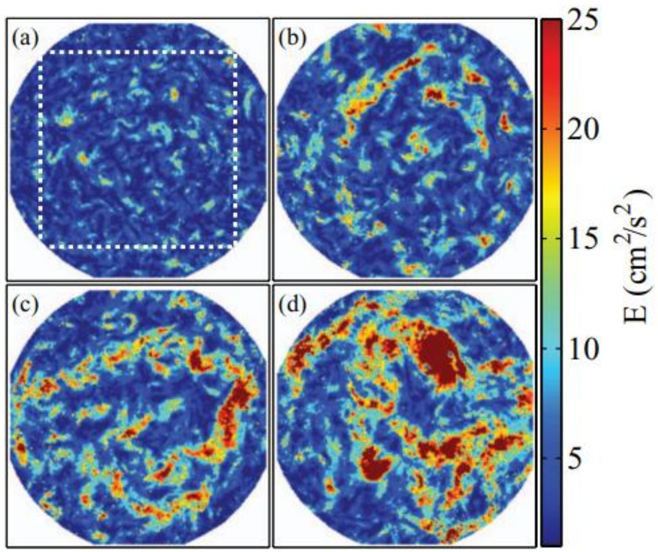
Laurie, et al, PRL 2014
Kolokolov, Lebedev PRE 2016
Woillez, Bouchet. *EPL* 2017
AF Phys. Fluids, 2017
AF, Herbert, PRL 2018
Svirsky, Herbert, AF, PRL 2023, Svirsky, AF, PRL 2025

Rotation: self-organization of 3d flow (deep layer)?

Rotating 3D flows : $\Omega \parallel \hat{z}$

Energy piles up in **2D large** scales

(2D = z invariant)



RESEARCH ARTICLE | AUGUST 13 2013

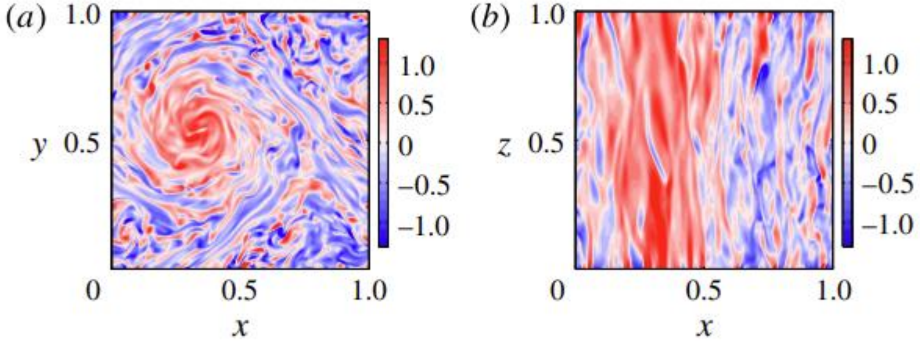
Experimental quantification of inverse energy cascade in deep rotating turbulence

Ehud Yarom; Yuval Vardi; Eran Sharon

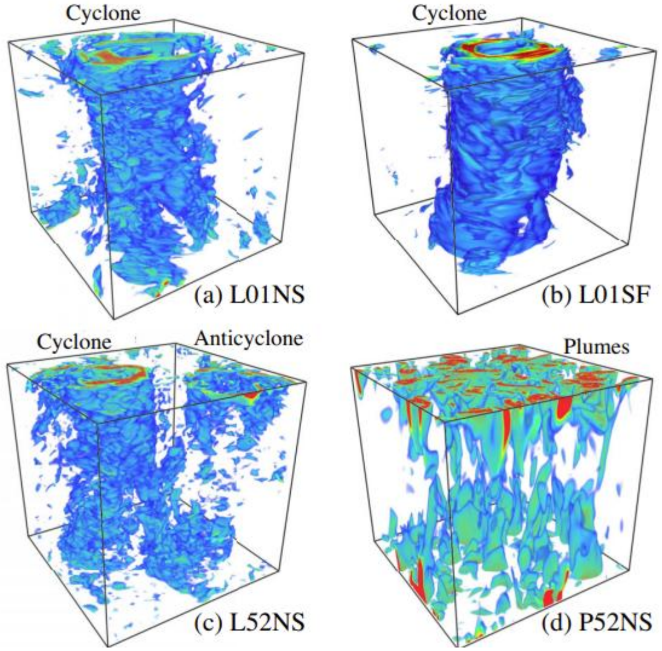
J. Fluid Mech. (2014), vol. 758, pp. 407–435.
doi:10.1017/jfm.2014.542

Large-scale vortices in rapidly rotating Rayleigh–Bénard convection

Céline Guervilly[1,†], David W. Hughes[1] and Chris A. Jones[1]



Condensates in 3d rotating flow



PHYSICAL REVIEW LETTERS 125, 214501 (2020)

Competition between Ekman Plumes and Vortex Condensates in Rapidly Rotating Thermal Convection

Andrés J. Aguirre Guzmán,[1] Matteo Madonia,[1] Jonathan S. Cheng,[1,*] Rodolfo Ostilla-Mónico,[2] Herman J. H. Clercx,[1] and Rudie P. J. Kunnen[1,†]

$\Omega \parallel \hat{z}$

What does rotation do?

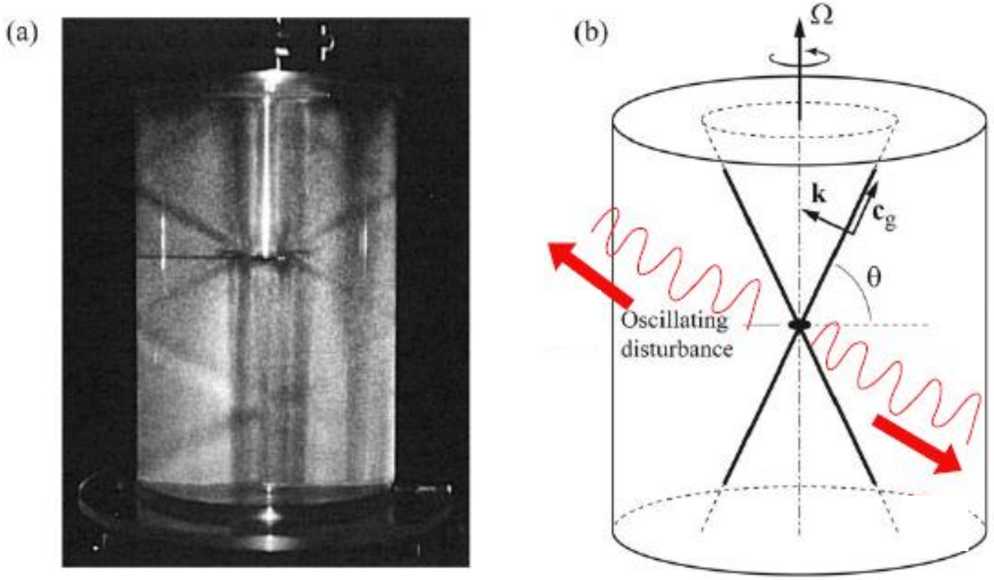
Coriolis force: Waves

Inertial Waves

$$\boldsymbol{h}_k^s e^{i\omega_k^s t + i\boldsymbol{k}\cdot\boldsymbol{x}}$$

$$\omega_k^s = 2\Omega s \frac{k_z}{k}$$

$$s = \pm 1$$

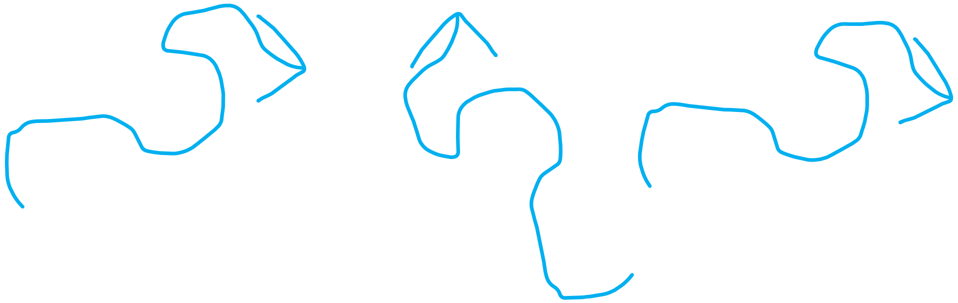


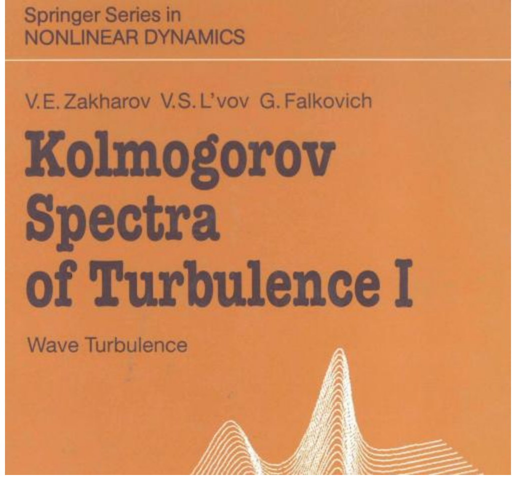
Görtler (1969)
Yarom & Sharon (2014)

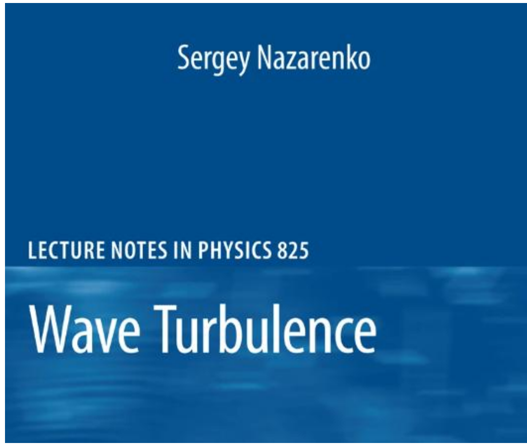
$$\Omega \parallel \hat{z}$$

Wave turbulence

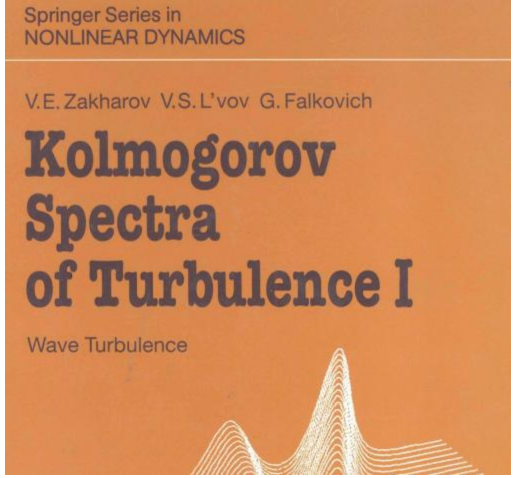
Yet another class of turbulence....

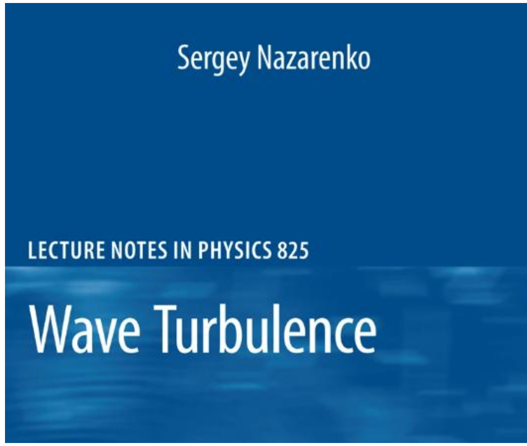






Wave turbulence



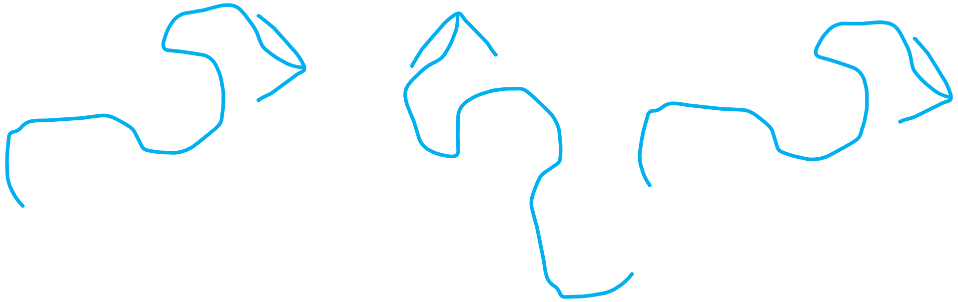


Yet another class of turbulence....

Main idea we will need: <u>in wave dominated regime</u>

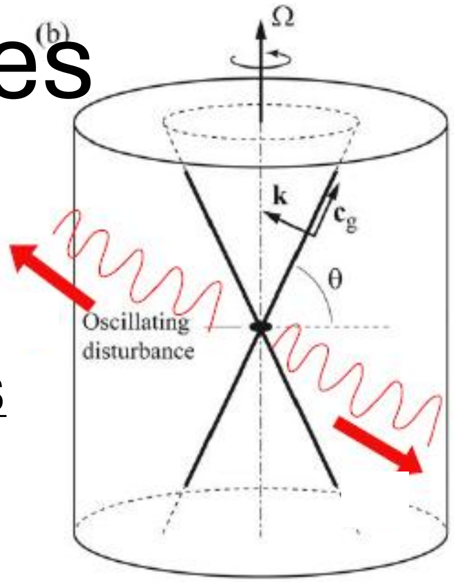
Interactions are restricted to resonant ones:

$$\omega_k + \omega_p + \omega_q = 0$$



(Oscillating factor in front of interaction averages to zero otherwise)

Rotating turbulence waves + 2d modes



3D waves **cannot** transfer energy to/from 2D modes through exact resonances

2D modes $k_z = 0$ have zero frequency:

$$\omega_k^s = 2\Omega s \frac{k_z}{k} \quad s = \pm 1$$

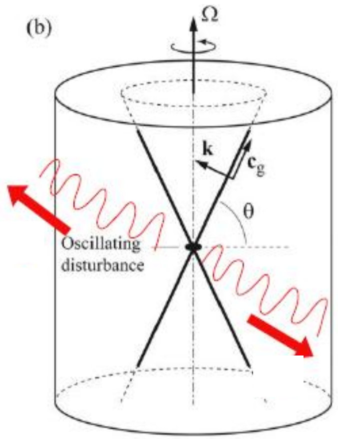
Greenspan 1969, Smith & Waleffe 1999
Shavit et. al 2024

Inverse cascade in Rotating turbulence

3D waves **cannot** transfer energy to/from 2D modes through exact resonances

3d and 2d decouple....

Answer: inverse cascade due to energy injected into 2D



Babin (1999)
Gallet (2015)
Seshasayanan & Alexakis (2018)

PHYSICS OF FLUIDS VOLUME 11, NUMBER 6 JUNE 1999

Transfer of energy to two-dimensional large scales in forced, rotating three-dimensional turbulence

Leslie M. Smith
Departments of Mathematics & Mechanical Engineering, University of Wisconsin–Madison, Madison, Wisconsin 53706
Fabian Waleffe
Departments of Mathematics & Engineering Physics, University of Wisconsin–Madison, Madison, Wisconsin 53706

PHYSICAL REVIEW LETTERS **132,** 224001 (2024)

Direct Measurement of Energy Transfer in Strongly Driven Rotating Turbulence

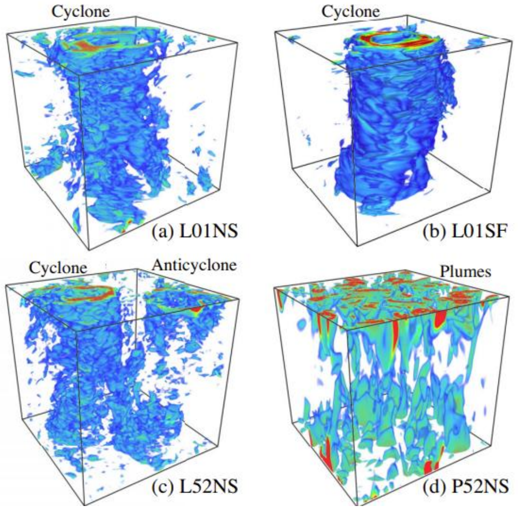
Omri Shaltiel, Alon Salhov, Omri Gat, and Eran Sharon

PRL **102,** 014503 (2009) PHYSICAL REVIEW LETTERS week ending 9 JANUARY 2009

Energy Transfer by Inertial Waves during the Buildup of Turbulence in a Rotating System

Itamar Kolvin, Kobi Cohen, Yuval Vardi, and Eran Sharon*

Energy injected in 3d reaches 2d!



PHYSICAL REVIEW LETTERS **125,** 214501 (2020)

Competition between Ekman Plumes and Vortex Condensates in Rapidly Rotating Thermal Convection

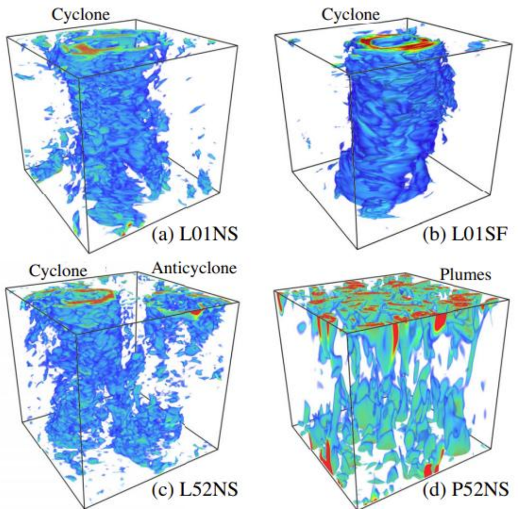
Andrés J. Aguirre Guzmán,[1] Matteo Madonia,[1] Jonathan S. Cheng,[1,*] Rodolfo Ostilla-Mónico,[2] Herman J. H. Clercx,[1] and Rudie P. J. Kunnen[1,†]

$\Omega \parallel \hat{z}$

Puzzles:

- Why is energy transfer directionally from 3d to 2d?

- How can waves transfer energy to 2d modes?



Sébastien Gomé

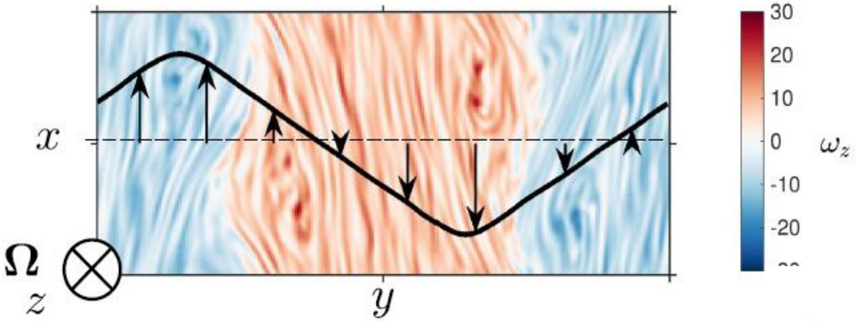
Our focus:

Use 3D small-scale stochastic forcing, scale l_f

Consider *steady state* with condensate

Numerical simulations:

$L_y = 2L_x = 2L_z \quad L_y/l_f = 10$



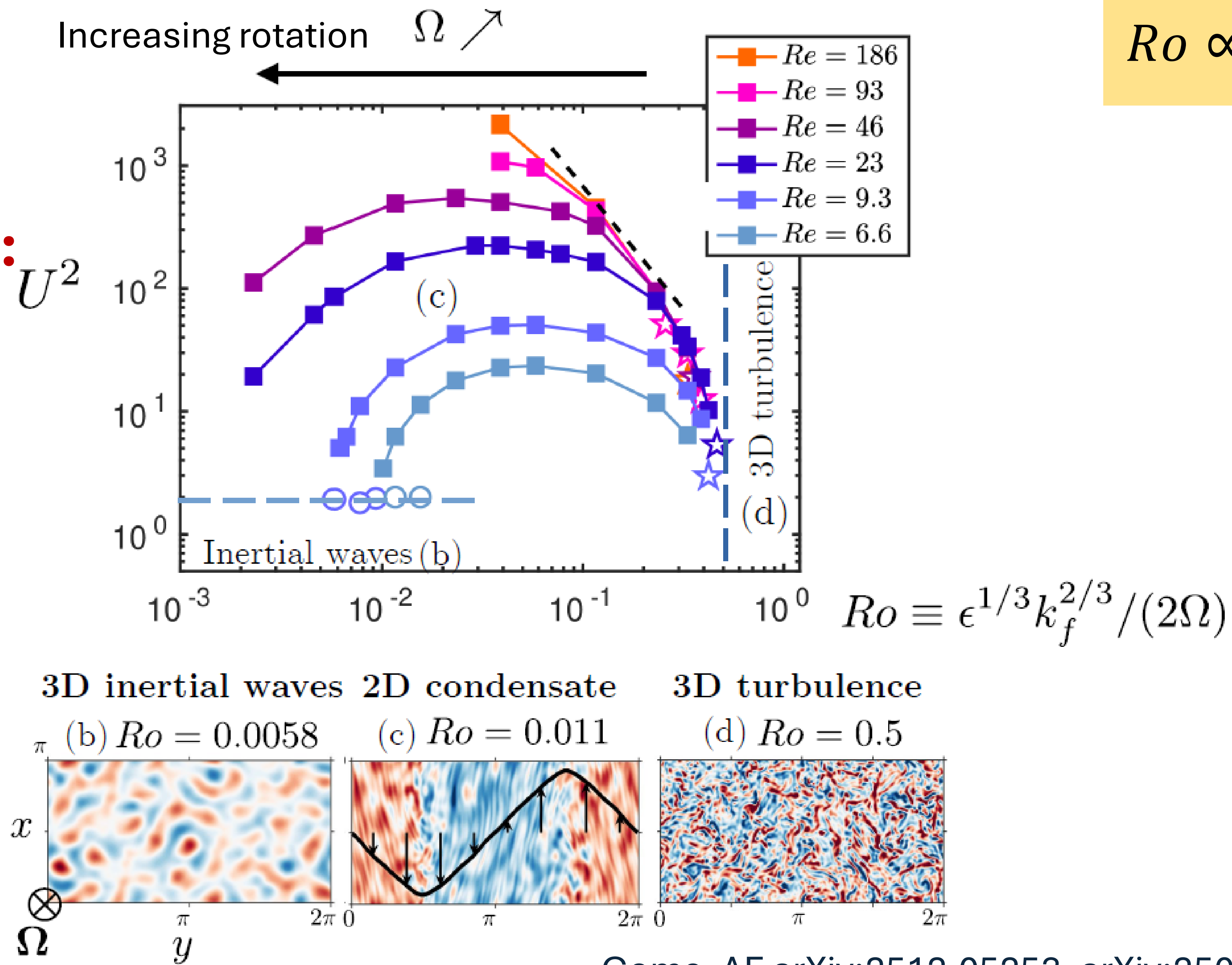
Using hyper-viscosity, $256 \times 512 \times 256$

Varying parameters:

$$Ro \propto \frac{1}{\Omega}, Re, \frac{L_y}{l_f}, \frac{L_z}{l_f}, \frac{L_x}{l_f}$$

Energy in 2d:

$$Ro \propto \frac{1}{\Omega}, Re$$



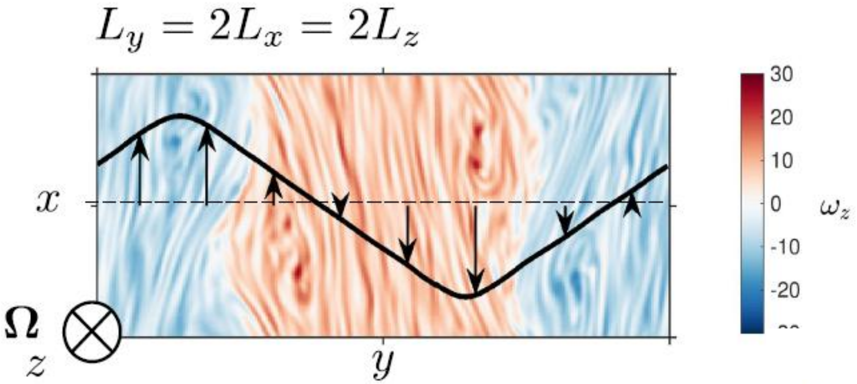
Gome, AF arXiv:2512.05253, arXiv:2509.18323

Puzzles:

- Why is energy transfer directional from 3d to 2d?

- How can waves transfer energy to 2d modes?

- Quantitative: how much energy is transferred to 2D with varying parameters?



$$Ro \propto \frac{1}{\Omega}, Re \ , \frac{L_y}{l_f}, \frac{L_z}{l_f}, \frac{L_x}{l_f}$$

Sébastien Gomé

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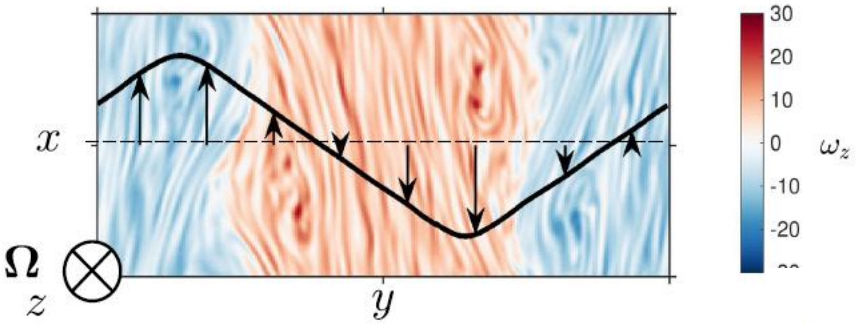


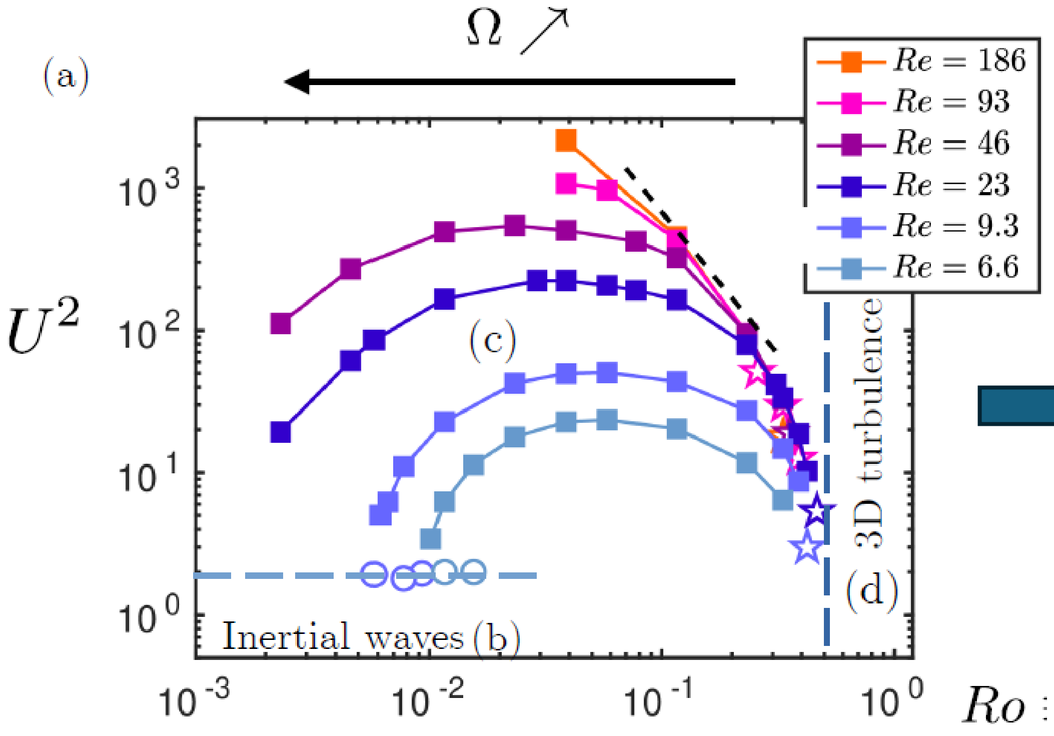
Using hyper-viscosity, $256 \times 512 \times 256$

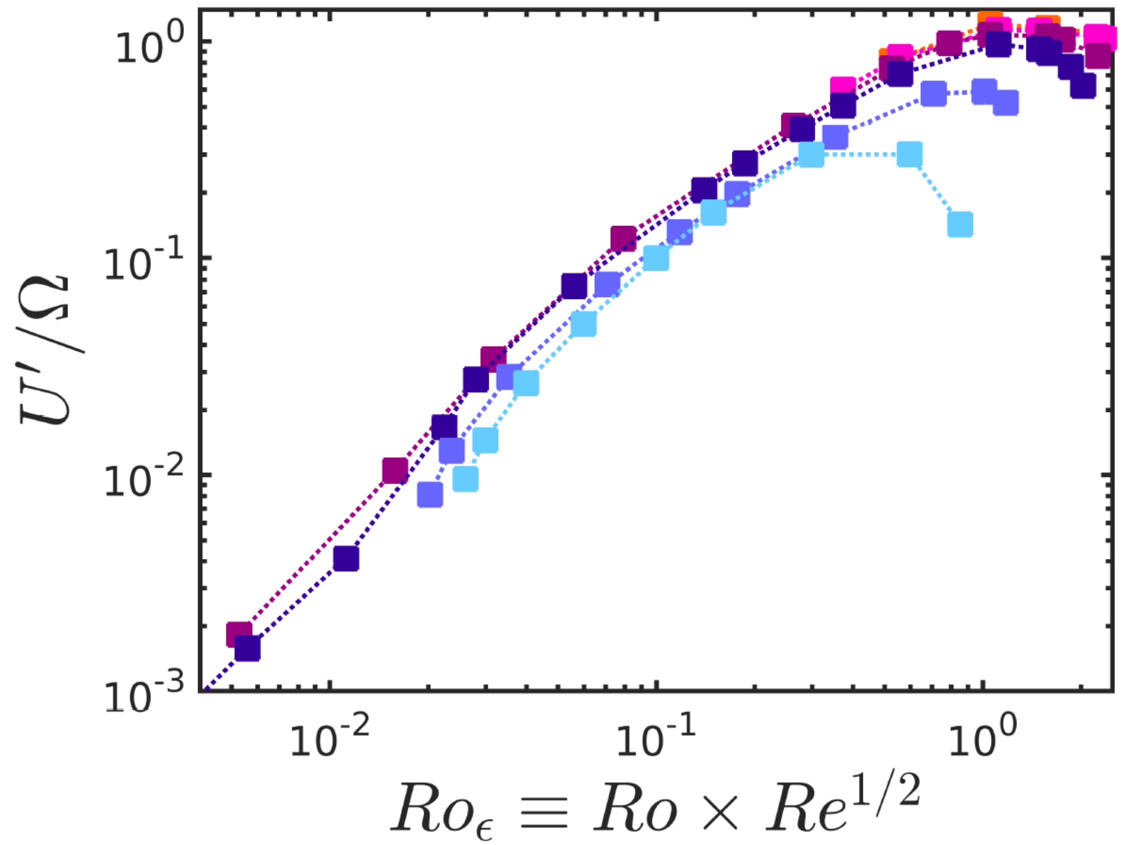
Analytical theory:

- Observables: condensate amplitude, 3D-2D energy transfer
- **Quasi-linear** treatment (perturbative) + **scale separation** $l_f/L \ll 1$
- Wave turbulence: restricted by **resonances**

Gome, AF arXiv:2512.05253, arXiv:2509.18323

Quantitative theory for amplitude





$$(U')^2 \equiv \int \left(\partial_y U'\right)^2 dy/L_y$$

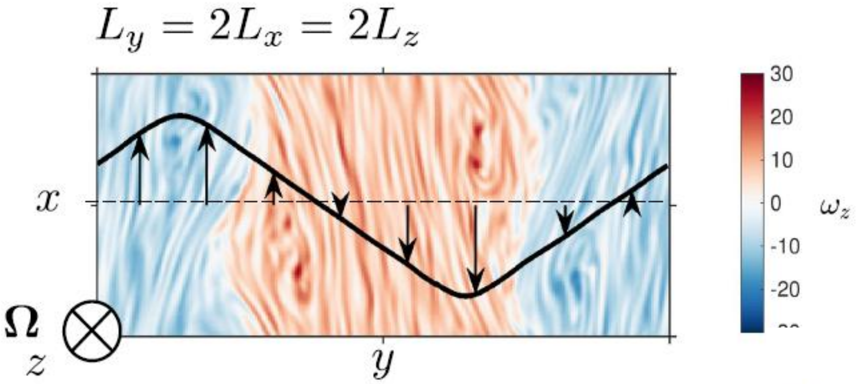
$$Ro \propto \frac{1}{\Omega}, Re$$

Puzzles:

- Why is energy transfer directional from 3d to 2d?

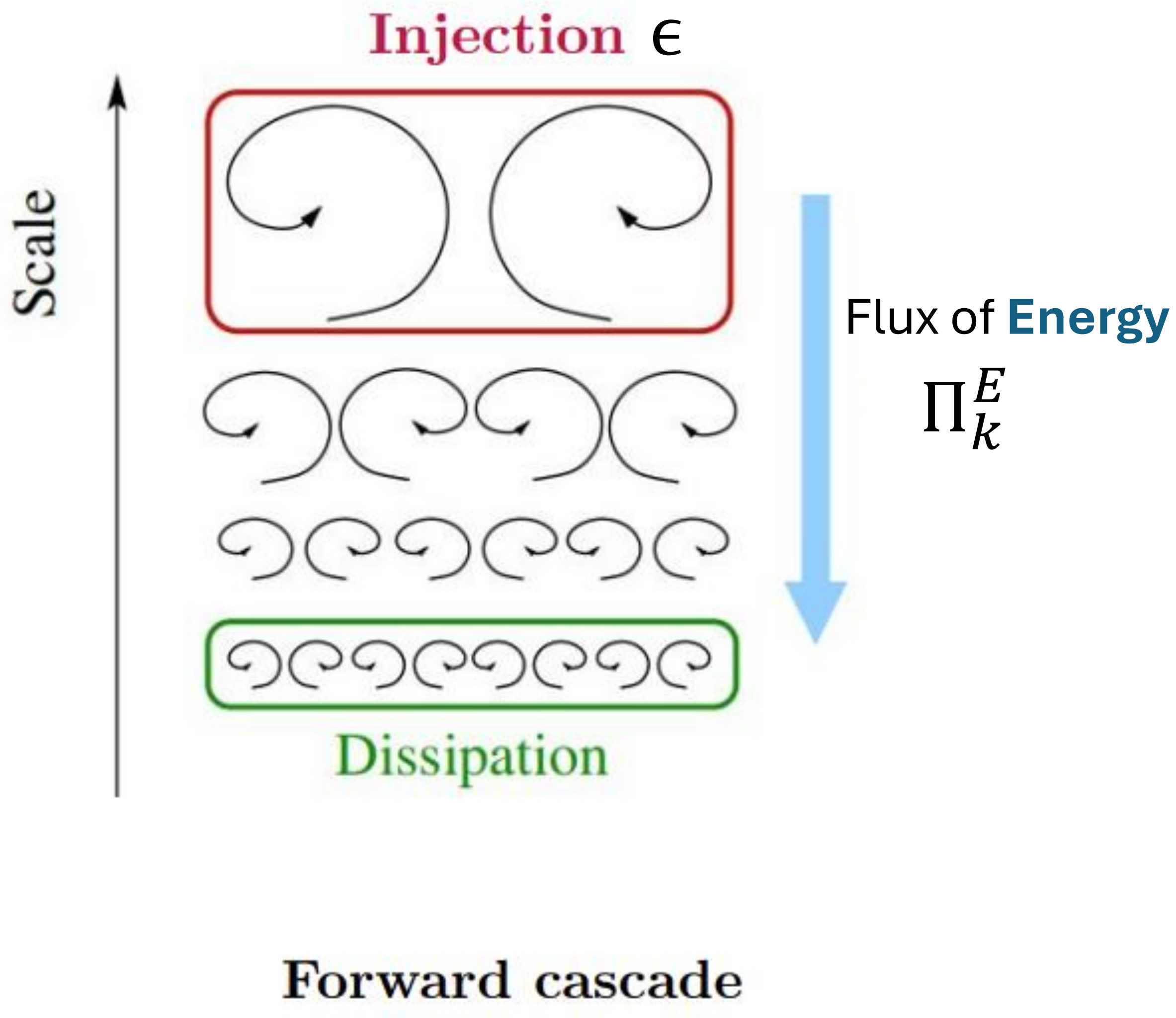
- How can waves transfer energy to 2d modes?

- Quantitative: how much energy is transferred to 2D with varying parameters?



$$Ro \propto \frac{1}{\Omega}, Re\ , \frac{L_y}{l_f}, \frac{L_z}{l_f}, \frac{L_x}{l_f}$$

Conservation laws in 3d (+ rotation)?



Helicity is conserved:

$$H = \int \vec{u} \cdot (\nabla \times \vec{u}) \mathrm{d}^2\mathrm{x}$$

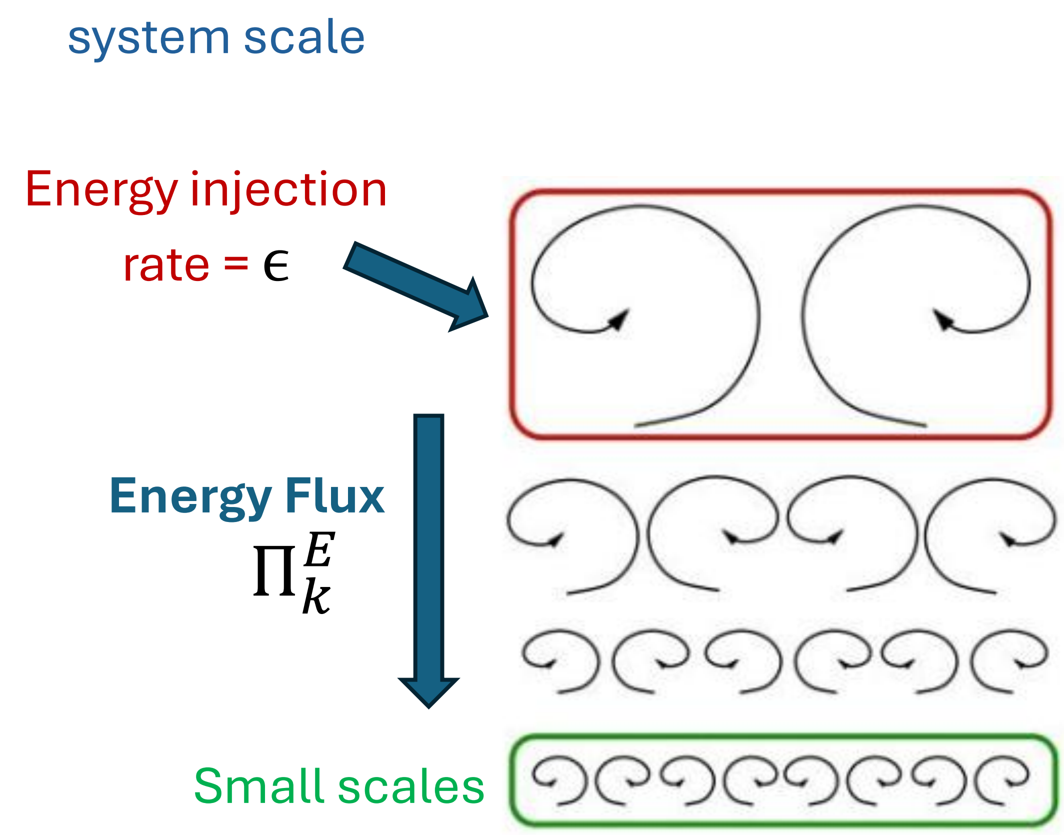
Fourier:

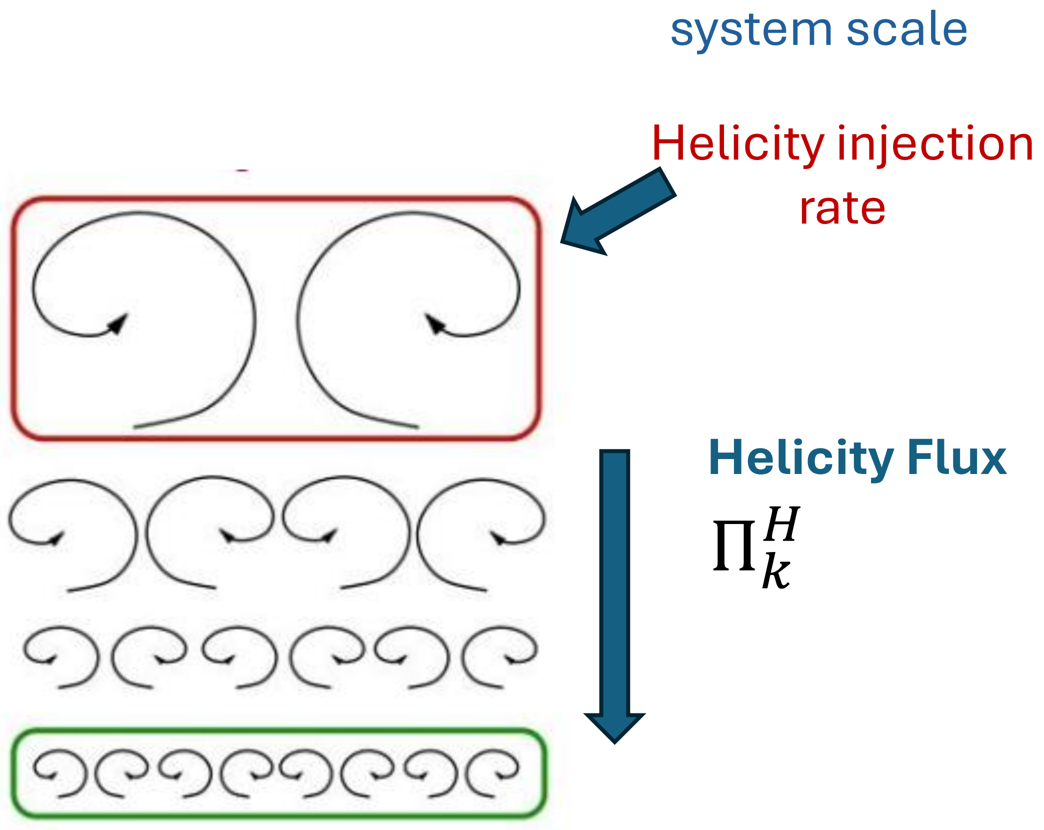
$$H_k = \mathrm{k}\, \mathrm{E}_\mathrm{k}^+ - \mathrm{k}\mathrm{E}_\mathrm{k}^-$$

Positive helicity modes

Negative helicity modes

Helicity conservation does not generically lead to self-organization





$$E = \int E_k d^3k = \int (E_k^+ + E_k^-) d^3k$$

$$H = \int k(E_k^+ - E_k^-) d^3k$$

Can have: $\Pi_k^H \approx k(\Pi_k^{E^+} - \Pi_k^{E^-})$

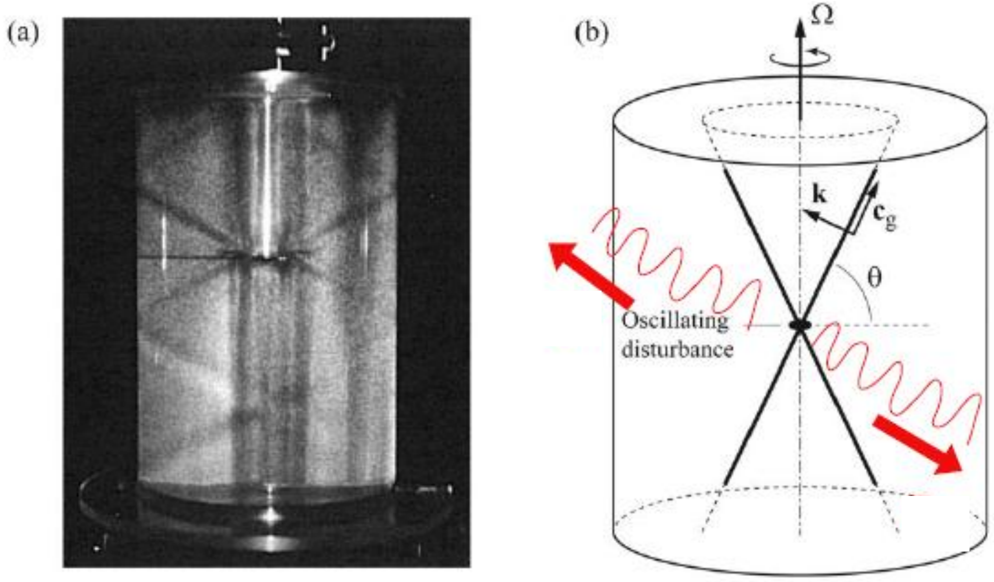
Chen, Chen, and Eyink. *POF* (2003).

Rotation: Waves carry sign-definite helicity

Inertial Waves $\boldsymbol{h}_k^s e^{i\omega_k^s t + i\boldsymbol{k}\cdot\boldsymbol{x}}$ Carry sign definite helicity $s = \pm 1$

$$\omega_k^s = 2\Omega s \frac{k_z}{k}$$

$s = \pm 1$



Görtler (1969)
Yarom & Sharon (2014)

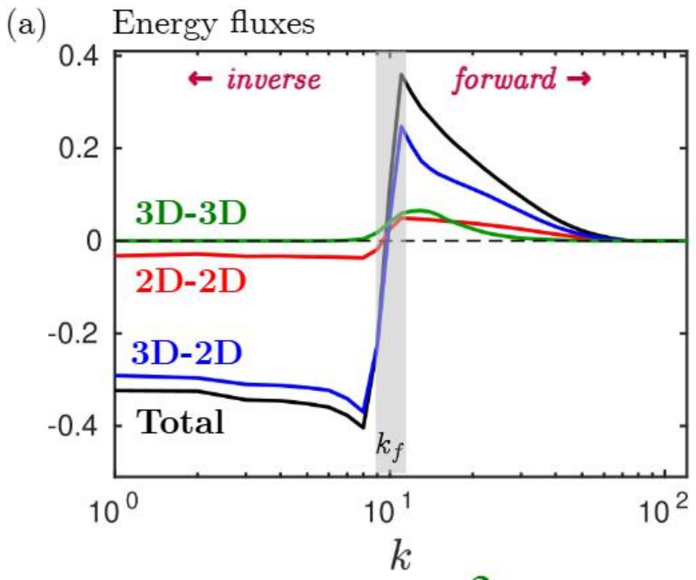
Helicity plays an important role:
Buzzicotti, *PRF* (2018).
Clark Di Leoni. PRF (2020)

$\Omega \parallel \hat{z}$

2d Condensate - 3d waves interactions

Fast rotation, interactions restricted **close to** resonant:

Waves: $\omega_p^s + \omega_q^{s'} \approx 0$ $\omega_k^s = 2\Omega s \frac{k_z}{k}$ $s = \pm 1$



$\omega_k + \omega_p + \omega_q = 0$

2D: $\omega_k = 0$

3D

2d Condensate - 3d waves interactions

Fast rotation, interactions restricted **close to** resonant:

Waves: $\omega_p^s + \omega_q^s \approx 0$ $\qquad$ $\omega_k^s = 2\Omega s \frac{k_z}{k}$ $\qquad$ $s = \pm 1$

1. Only same helicity waves interact! $s = s'$

2d Condensate - 3d waves interactions

Fast rotation, interactions restricted **close to** resonant:

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1. Only same helicity waves interact! $s = s'$

2. 2d Condensate is large scale → Waves helicity is (approx.) **conserved**

2d Condensate - 3d waves interactions

Fast rotation, interactions restricted **close to** resonant:

Waves: $\omega_p^s + \omega_q^s \approx 0$ $\quad s = \pm 1 \quad$ $\omega_k^s = 2\Omega s \dfrac{k_z}{k}$

Only same helicity waves interact

+

Waves helicity is (approx.) **conserved**

H^+, H^- of the waves conserved separately

2d Condensate - 3d waves interactions

Only same helicity waves interact

+

Waves helicity is (approx.) **conserved**

$H^{+} = \int k\, E_k^{+} dk \,, H^{-} = -\int k E_k^{-} dk$ conserved for waves

Finite amount of wave energy cannot go to very small scales + advection: (positive but vanishing energy flux to small scales)

Transfer **from** 3d **to** 2d condensate

2d Condensate - 3d waves interactions

Only same helicity waves interact

+

Waves helicity is (approx.) **conserved**

$H_k^+ = \mathrm{k}\,\mathrm{E}_\mathrm{k}^+,\ \mathrm{H}_\mathrm{k}^- = \mathrm{k}\mathrm{E}_\mathrm{k}^-$ conserved for waves

Finite amount of wave energy cannot go to very small scales + advection:

Transfer **from** 3d **to** 2d condensate

Argument for general large scale 2D mode

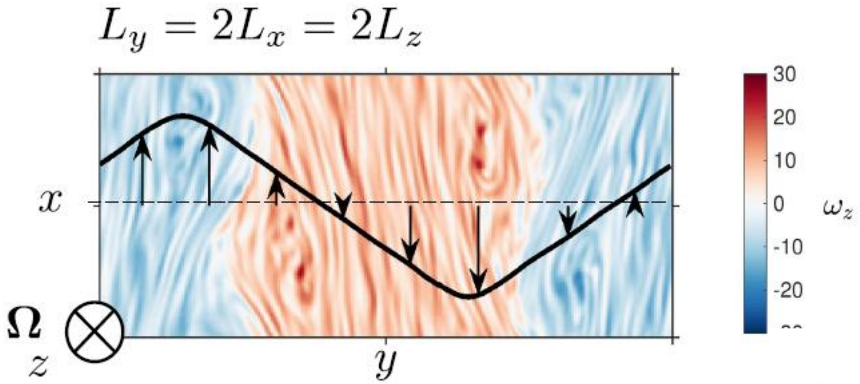
Puzzles:

Emergent conservation law

- Why is energy transfer directional from 3d to 2d?

- How can waves transfer energy to 2d modes?

- Quantitative: how much energy is transferred to 2D with varying parameters?



$$Ro \propto \frac{1}{\Omega}, Re\ , \frac{L_y}{l_f}, \frac{L_z}{l_f}, \frac{L_x}{l_f}$$

Puzzles:

- Why is energy transfer directional from 3d to 2d?

- How can waves transfer energy to 2d modes?

- Quantitative: how much energy is transferred to 2D with varying parameters?

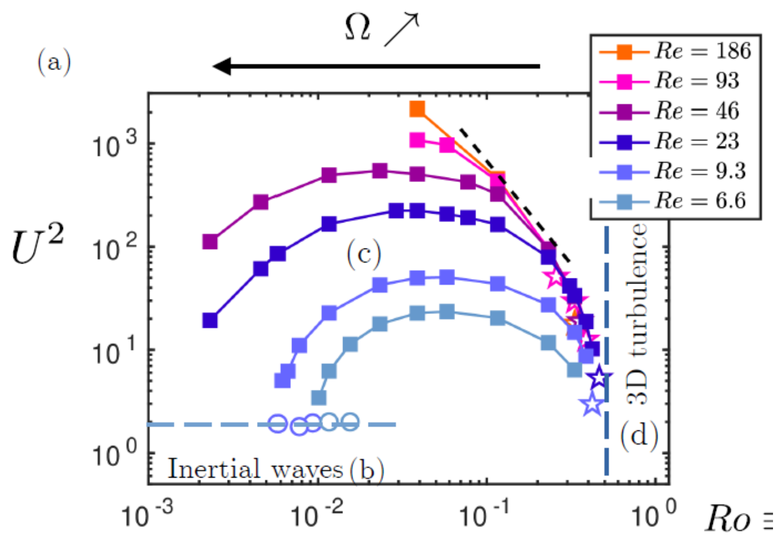
All the energy goes to the condensate?

Puzzles:

- How can waves transfer energy to 2d modes?

- Quantitative: how much energy is transferred to 2D with varying parameters?

All the energy goes to the condensate?



2d Condensate - 3d waves interactions

Fast rotation, interactions restricted **close to** resonant:

Waves: $\omega_p^s + \omega_q^{s'} < U' = \frac{1}{\tau_U}$ $\omega_k^s = 2\Omega s \frac{k_z}{k}$ $s = \pm 1$

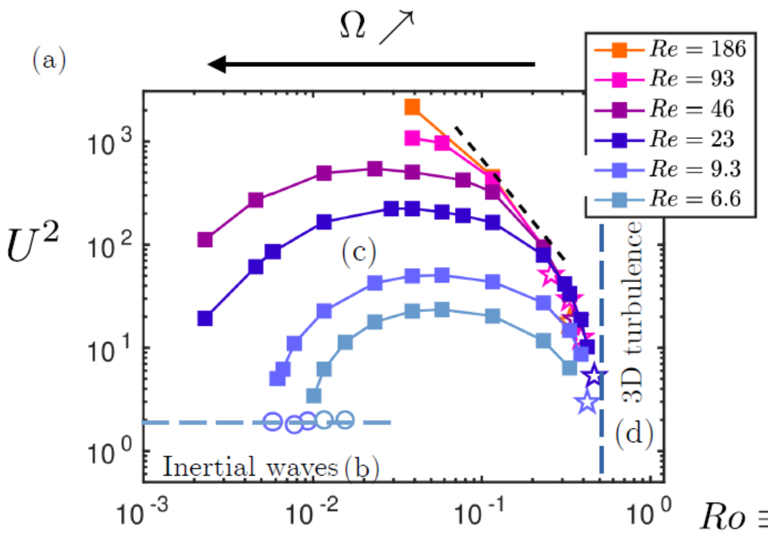
Puzzles:

- How can waves transfer energy to 2d modes?

- Quantitative: how much energy is transferred to 2D with varying parameters?

2D-3D Interactions are not exactly resonant!

$$\omega_p^s + \omega_q^{s'} < U' = \frac{1}{\tau_U}$$

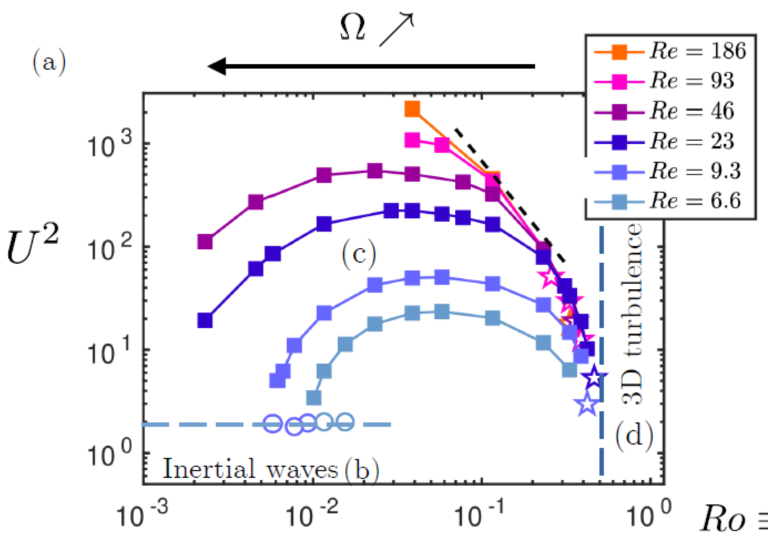


Puzzles:

- How can waves transfer energy to 2d modes?

- Quantitative: how much energy is transferred to 2D with varying parameters?

$$\omega_p^s + \omega_q^{s'} < U' = \frac{1}{\tau_U}$$

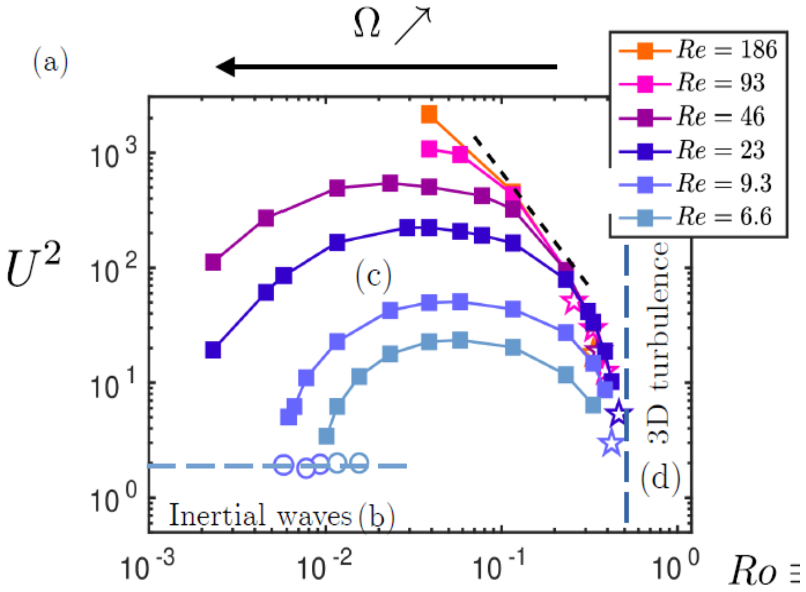


2d Condensate - 3d waves interactions

Fast rotation, interactions restricted **close to** resonant:

Waves: $$\omega_p^s + \omega_q^{s'} < U' = \frac{1}{\tau_U}$$

3D-2D interactions are selected if they are within this tolerance



2d Condensate - 3d waves interactions

Fast rotation, interactions restricted **close to** resonant:

Waves: $$\omega_p^s + \omega_q^{s'} < U' = \frac{1}{\tau_U}$$

$$s = \pm 1 \quad \omega_k^s = 2\Omega s \frac{k_z}{k}$$

Energy balance for condensate:

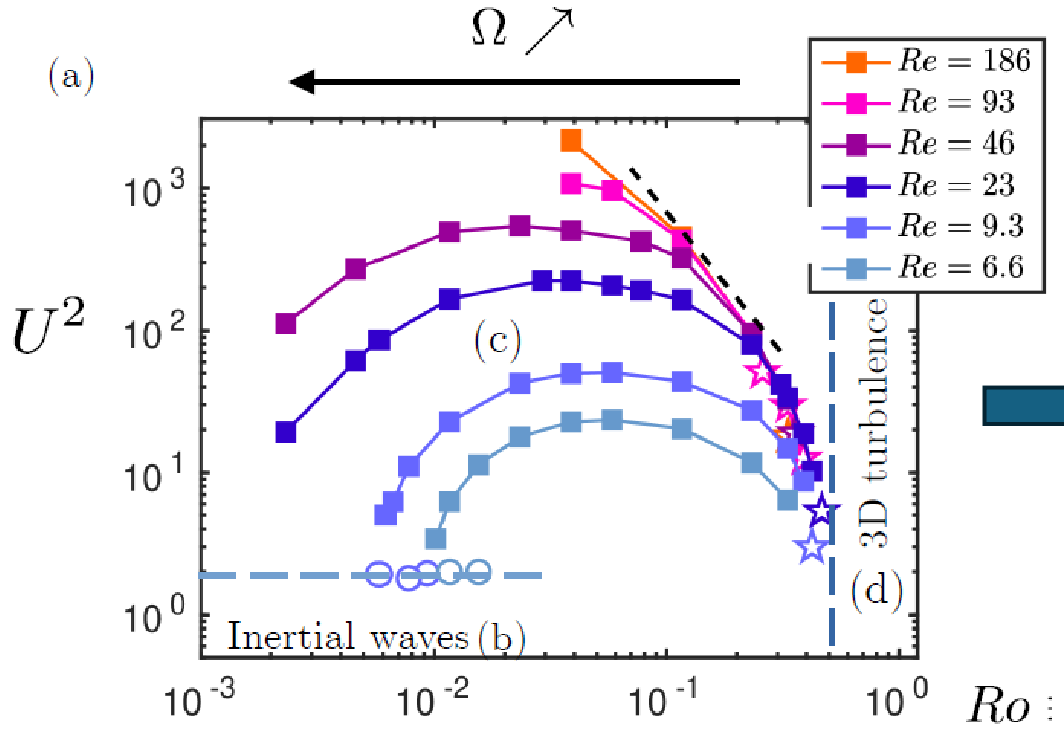
$$\nu U'^2 = \epsilon \, T_{3D-2D} \left(\frac{U'}{\Omega}\right)$$

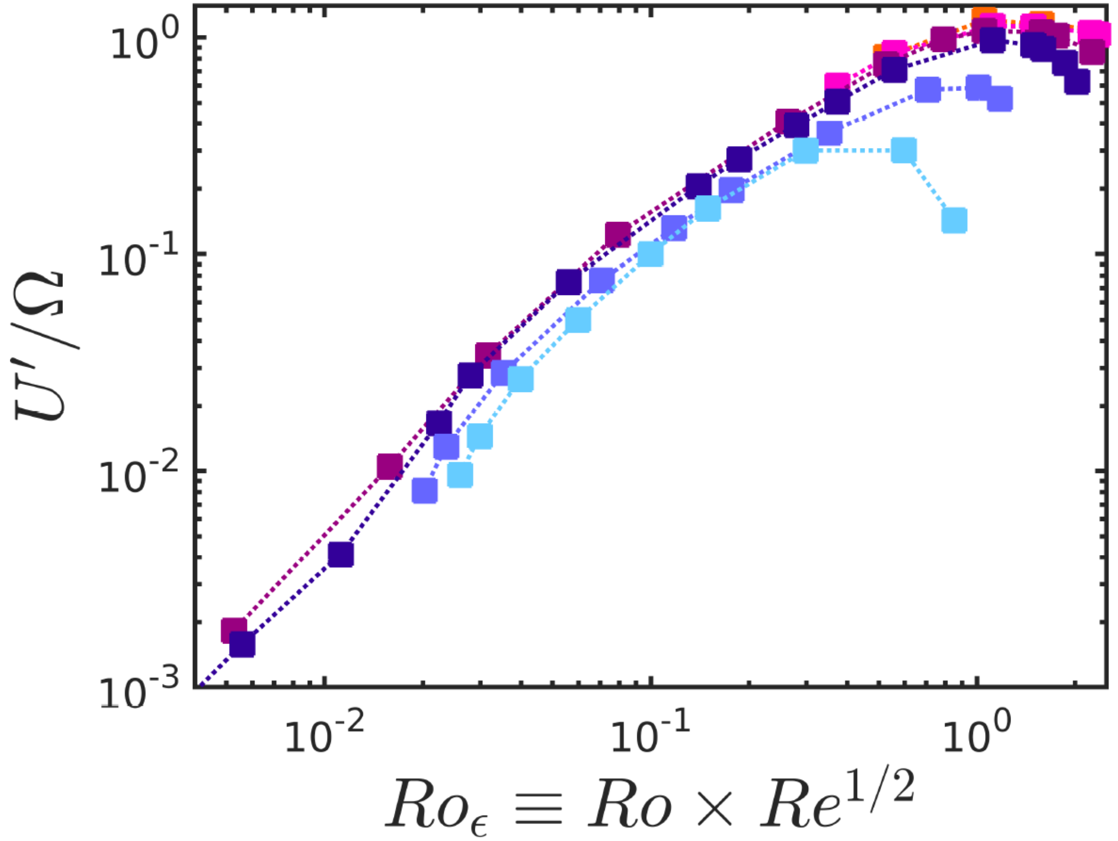
Energy dissipation rate by condensate

Energy transfer rate from 3D to condensate

Total energy injection rate

Relevant observable and parameter:





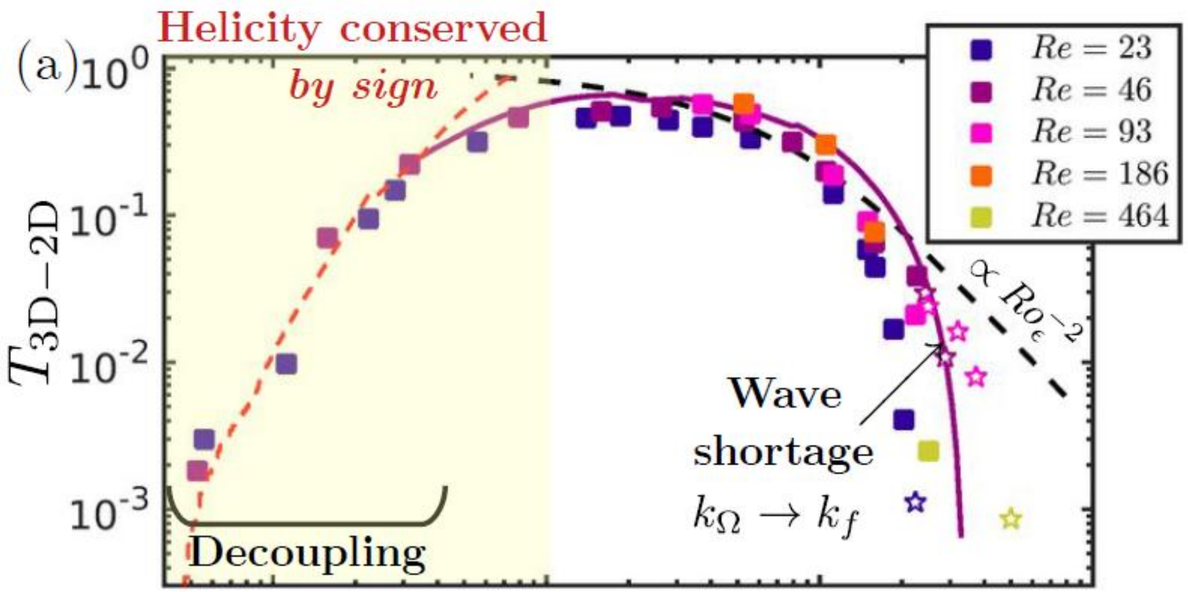
$$Ro \propto \frac{1}{\Omega}, Re$$

Inspired by: Kolokolov, et. al. PRF 2020, Parfenyev, et al. POF (2021).

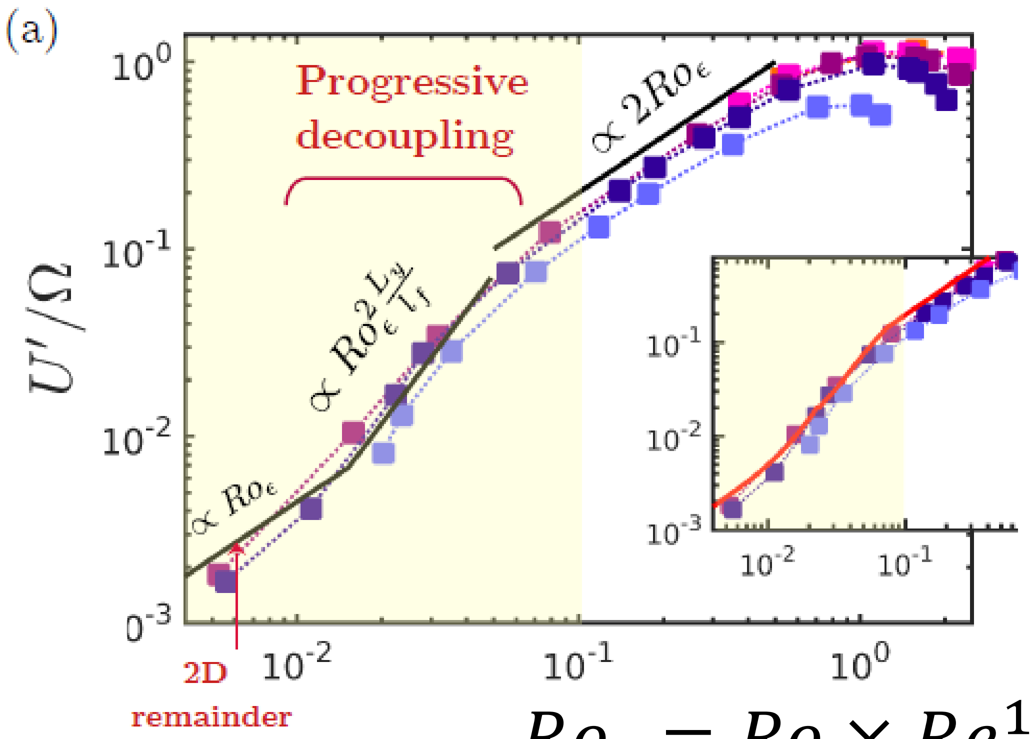
Most energy reaches condensate:

$$Ro \propto \frac{1}{\Omega}, Re$$

$$\frac{l_f}{2L_y} < Ro_\epsilon \ll \frac{8l_f}{L_x}: \quad U' \sim \sqrt{\frac{\epsilon}{\nu}}$$



$$Ro_\epsilon = Ro \times Re^{1/2}$$



$$Ro_\epsilon = Ro \times Re^{1/2}$$

2d Condensate - 3d waves interactions

Fast rotation, interactions restricted **to same helicity**:

Waves: $\omega_p^s + \omega_q^s \ll U'$ $s = \pm 1$ $\omega_p^s \propto s\,\Omega$

Becomes too restrictive as $\frac{U'}{\Omega}$ **decreases**

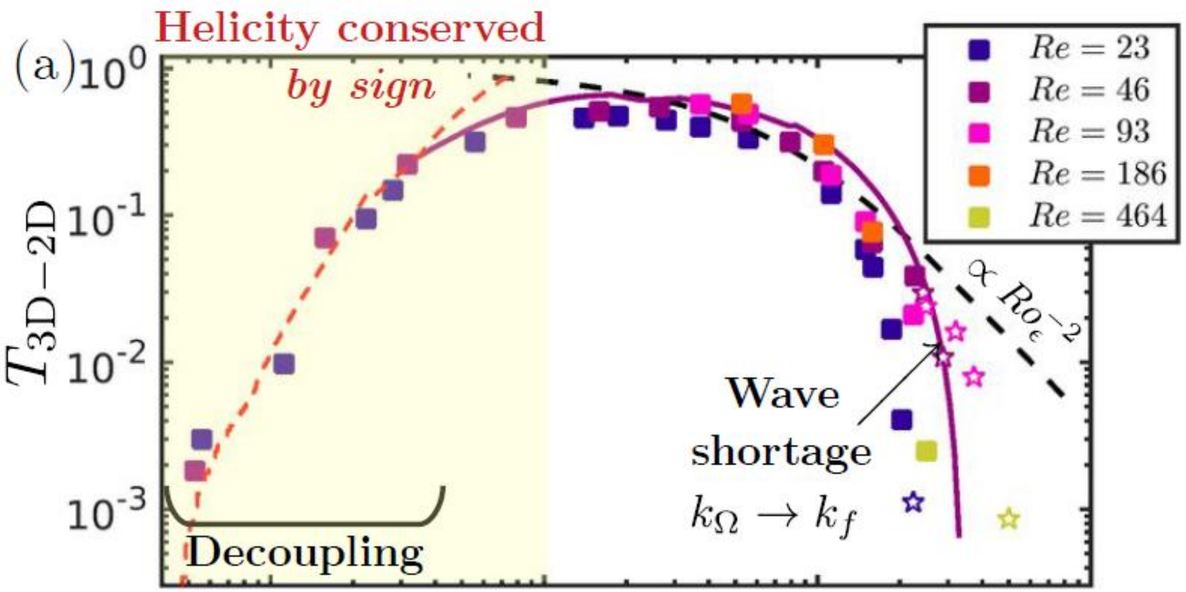
(finite detuning, if the condensate time scale is slow, interaction averages out to zero)

$$\frac{1}{\tau_U} \sim U' \sim \sqrt{\frac{\epsilon}{\nu}} \ll \frac{2\pi}{L_y} \partial_{p_y} \omega_p^s$$

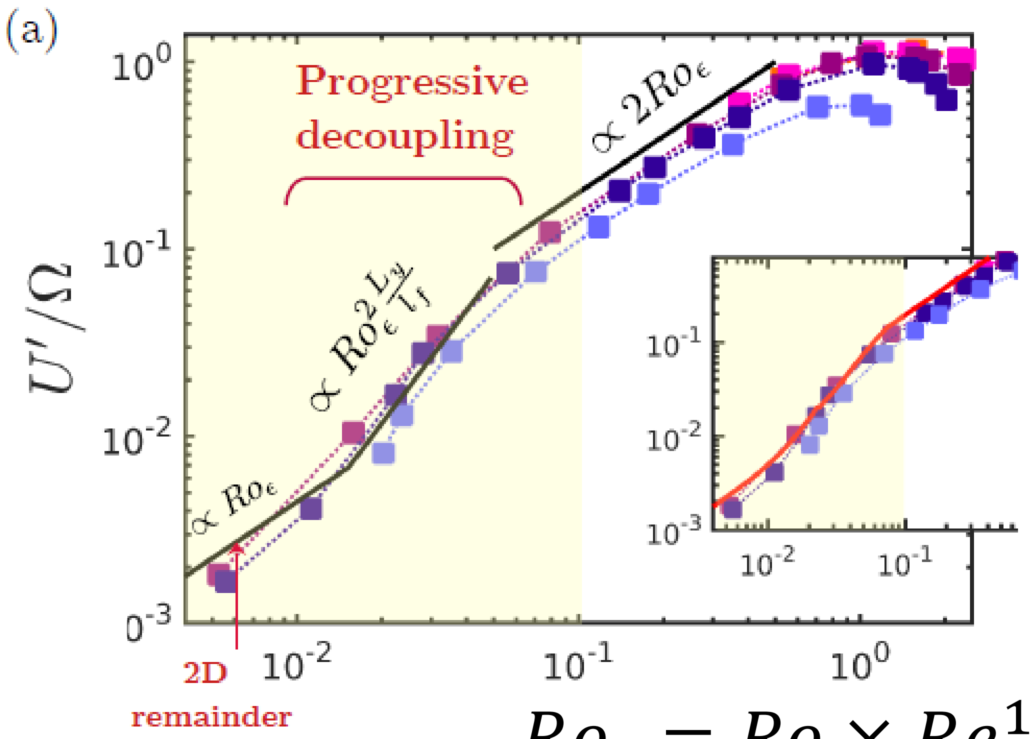
$$\nu U'^2 = T_{3D-2D}\left(\frac{U'}{\Omega}\right)$$

Gradual 3D-2D decoupling for high rotation:

For more and more modes: $\frac{1}{\tau_U} \sim U' \ll \omega_p^s + \omega_q^s,\qquad \mathrm{Ro}_\epsilon < \frac{\mathrm{l_f}}{2\mathrm{L_y}}$



$Ro_\epsilon = Ro \times Re^{1/2}$



$Ro_\epsilon = Ro \times Re^{1/2}$

$Ro \propto \frac{1}{\Omega}, Re$

2d Condensate - 3d waves interactions

Interactions restricted **close to** resonant:

Waves: $$\omega_p^s + \omega_q^{s'} < U' = \frac{1}{\tau_U}$$

$$\omega_k^s = 2\Omega s \frac{k_z}{k}$$

$$s = \pm 1$$

If the frequencies are sufficiently small this condition is not restrictive even for $s \neq s'$...

$$\omega_p^s + \omega_q^{-s} \approx 2\omega_p^s \ll 1/U'$$

2d Condensate - 3d waves interactions

Interactions restricted **close to** "resonant":

Waves:

$$\omega_p^s + \omega_q^{s'} < U' = \frac{1}{\tau_U}$$

$$\omega_k^s = 2\Omega s \frac{k_z}{k}$$

$$s = \pm 1$$

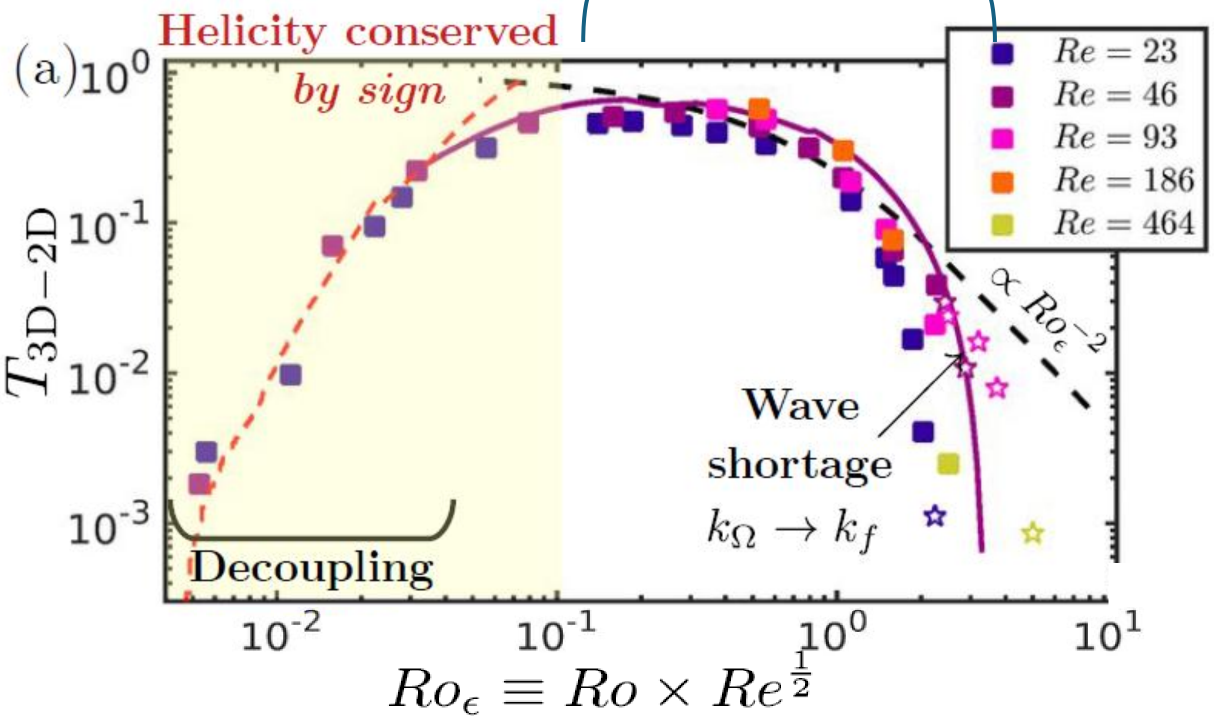
Wave energy is transferred to smaller scales, frequencies decrease, this condition becomes less restrictive…

A sector of waves with interactions **mixing positive and negative helicity** waves appears, **which extracts energy from the condensate**.

$$\omega_p^s + \omega_q^{-s} \approx 2\omega_p^s \ll 1/U'$$

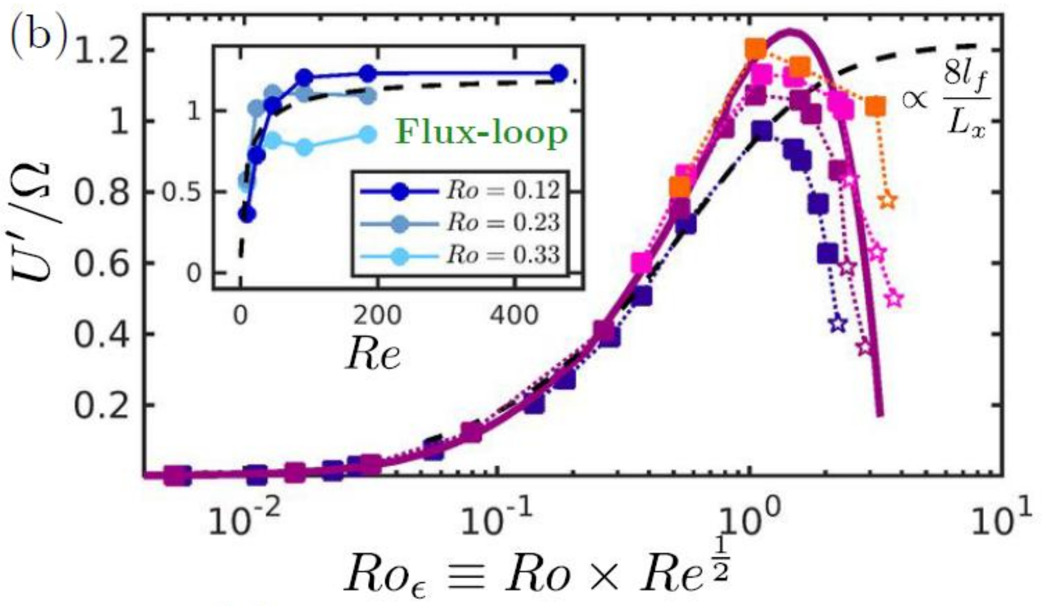
Inverse and direct transfers co-exist

Wave-helicity mixing
interactions are present



$Ro_\epsilon >\sim \frac{8l_f}{L_x}$:

Wave-helicity mixing
interactions are present

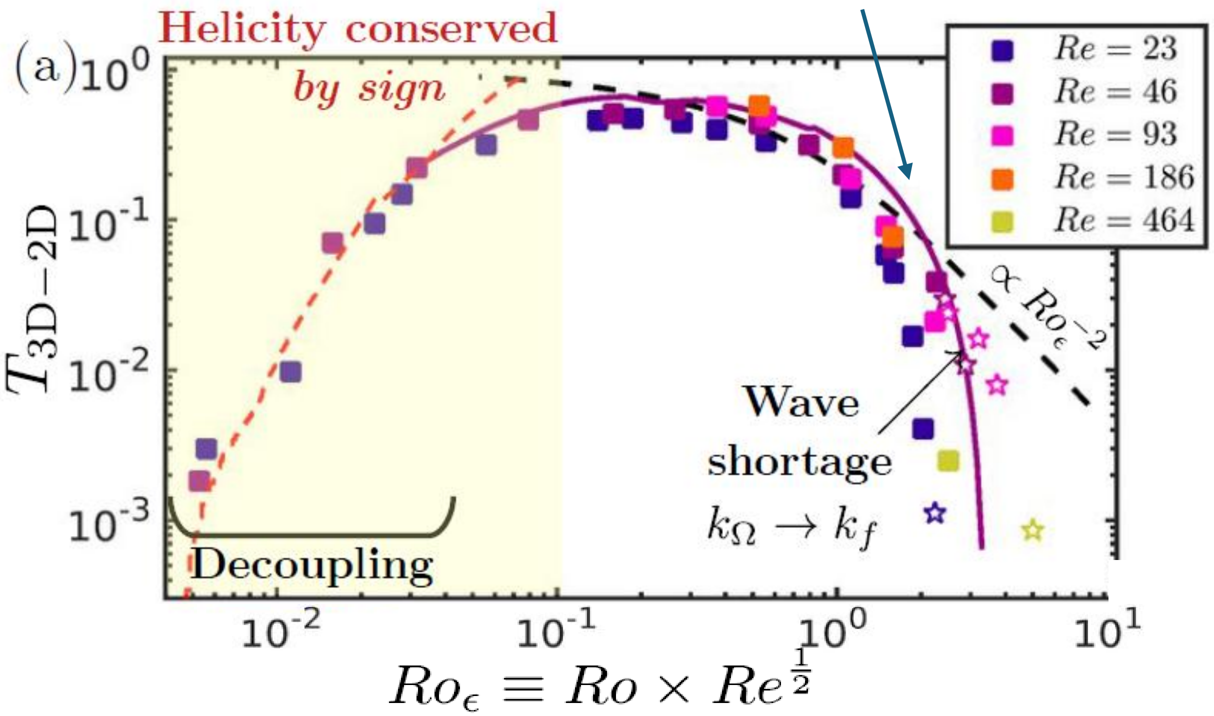


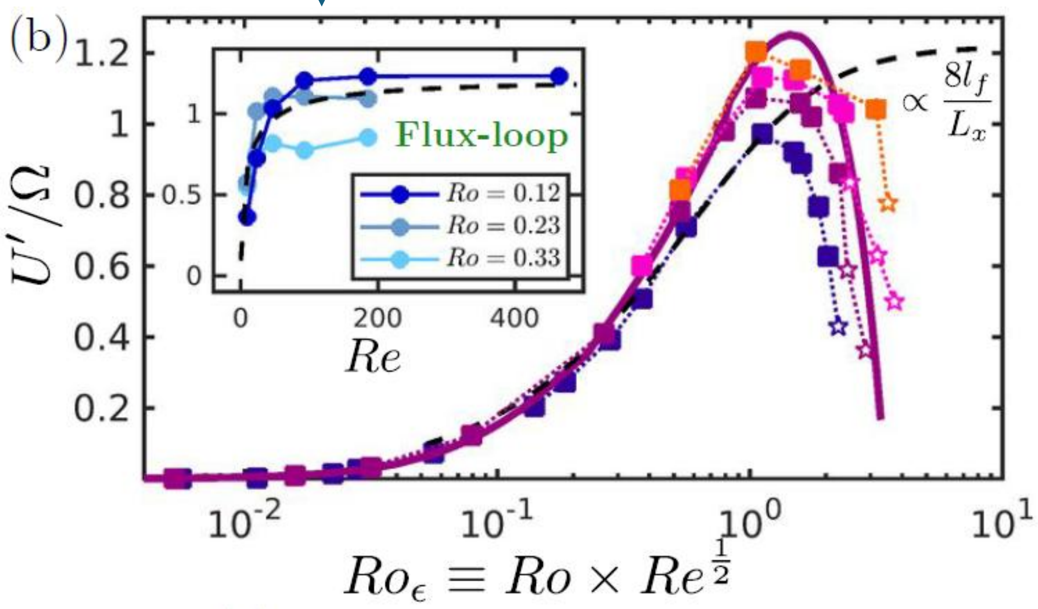
The condensate shear rate tunes the size of the conserving/non conserving sectors, such that energy balance is satisfied.

Increasing Reynolds number? Flux loop

$$\nu U'^2 = \epsilon T_{3D-2D}\left(\frac{U'}{\Omega}\right) \to 0\,, U' \sim \Omega$$

$T_{3D-2D} \sim Ro_\epsilon^{-2}$



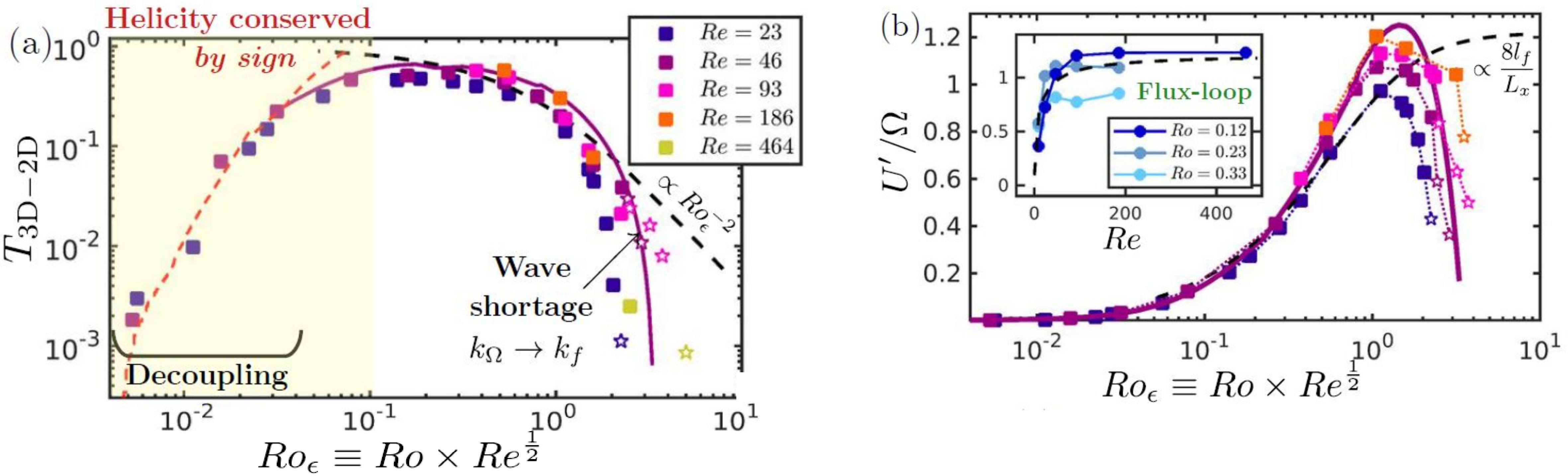


Seshasayanan 2018

$U' \sim \Omega$

Asymptotic state: transfer to/from waves almost balanced

Decreasing rotation: lose the helicity conserving sector



The scaling with Ro_ϵ is eventually lost, as modes become unrestricted by rotation

Summary

- 2d large scale flow gets energy from 3d waves due to (approximate) separate **conservation** of each sign of the helicity for the waves
- For **fast enough rotation** the condensate decorrelates from the waves (gradually)
- For **slow enough rotation**, mixed helicity sign interactions are allowed for some waves, and those extract energy from the condensate

Thank you!

Take home messages

- **Role of conservation laws…**

- **Waves alter interactions → can generate new conservation laws**

- **Finite time-scale separation gives rise to a rich transitional region**