New Directions in Theoretical Physics V, Higgs Centre, U. of Edinburgh, 7.01.26

Violent Relaxation and Gentle Dissipation: Turbulent Lives of "Collisionless" Systems

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Toby Adkins, Robbie Ewart, Barry Ginat, David Hosking, Michael Nastac
(PPPL) (Princeton) (Oxford) (Cambridge) (Oxford)





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Oxford Physics

Stochastic Advection, Phase Mixing, and Landau Damping in a Collisionless Plasma

Alexander Schekochihin
(Oxford)

with

Anjor Kanekar, Bill Dorland (Maryland),
Greg Hammett (Princeton),
& Nuno Loureiro (IST Lisbon)

R.I.P.

Photo
by M. Koval,
2005

Nuno F. G. Loureiro (26.12.1977-16.12.2025)

https://www.cambridge.org/core/journals/journal-of-plasma-physics/announcements/in-memoriam/nuno-f-g-loureiro-1977-2025

Vlasov-Poisson Systems

$$\frac{\partial f}{\partial t} + \mathbf{v} \cdot \nabla f - (\nabla \varphi) \cdot \frac{\partial f}{\partial \mathbf{v}} = 0$$

$$\nabla^2 \varphi = 4\pi G m \int \mathrm{d}\mathbf{v} f$$

Liouville equation for phase-space density of particles of mass *m* in each other's gravitational field

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Collisional vs. Collisionless

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Liouville equation for phase-space density of particles of charge *-e* (electrons) in each other's electric field (+neutralizing ions)

Binary collisions (particle noise) push distribution towards a local Maxwellian equilibrium,

so "fluid" moments carry all info: $f(t,\mathbf{r},\mathbf{v}) = \frac{n(t,\mathbf{r})}{\left[2\pi T(t,\mathbf{r})/m\right]^{3/2}}\exp\left\{-\frac{m\left[\mathbf{v}-\mathbf{u}(t,\mathbf{r})\right]^2}{2T(t,\mathbf{r})}\right\}$

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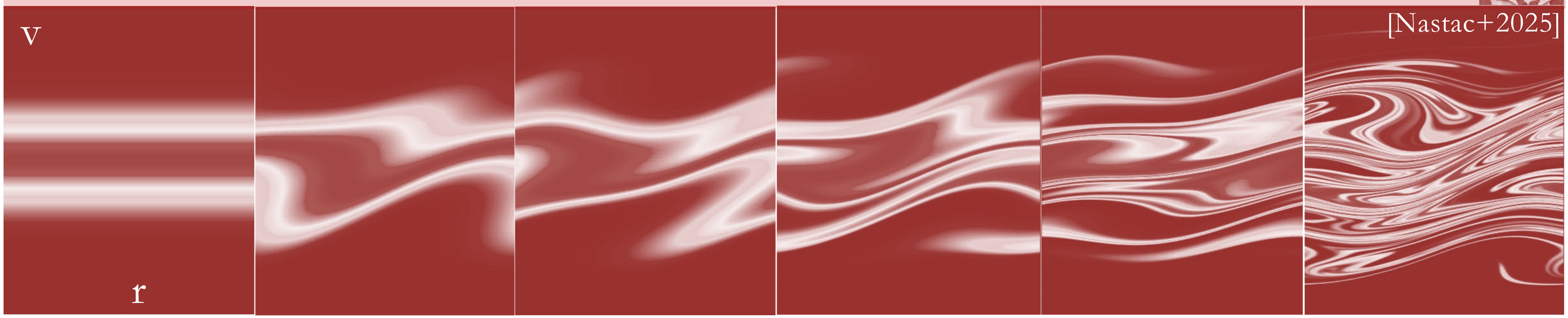
But dynamical times can be shorter than collision time: $\tau_{\rm dyn} \sim kv \sim \frac{ek\varphi}{mv_{\rm th}} \ll \tau_{\rm coll} \propto N$

Distribution gets mixed, linearly and nonlinearly, in (**r**,**v**) phase space:



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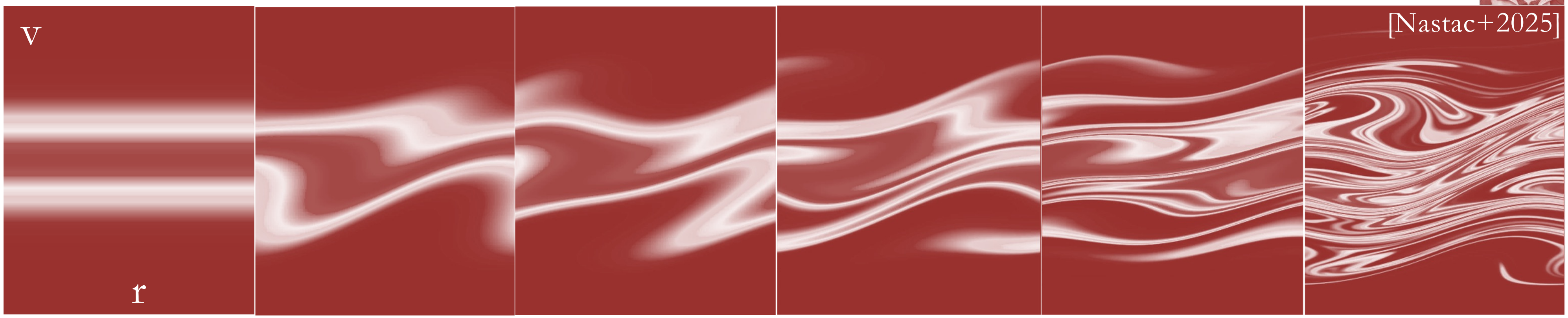

<u>Fundamental question</u>: *To what (universal?) state(s) does a collisionless system (want to) relax?*

Non-Maxwellian distributions are routinely measured (solar wind, cosmic rays...), typically flat-tops with tight tails or "fat" high-energy power-laws tails...

Collisionless Relaxation

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Liouville's equation conserves phase volume in (**r**,**v**) space,

so there is a continuum of constraints: $\rho(\eta) = \iint \mathrm{d}\mathbf{r}\mathrm{d}\mathbf{v}\,\delta\left(\eta - f(t,\mathbf{r},\mathbf{v})\right) = \mathrm{const}$

"waterbag content" (or Casimirs)

What do we know of relaxation constrained in this way?

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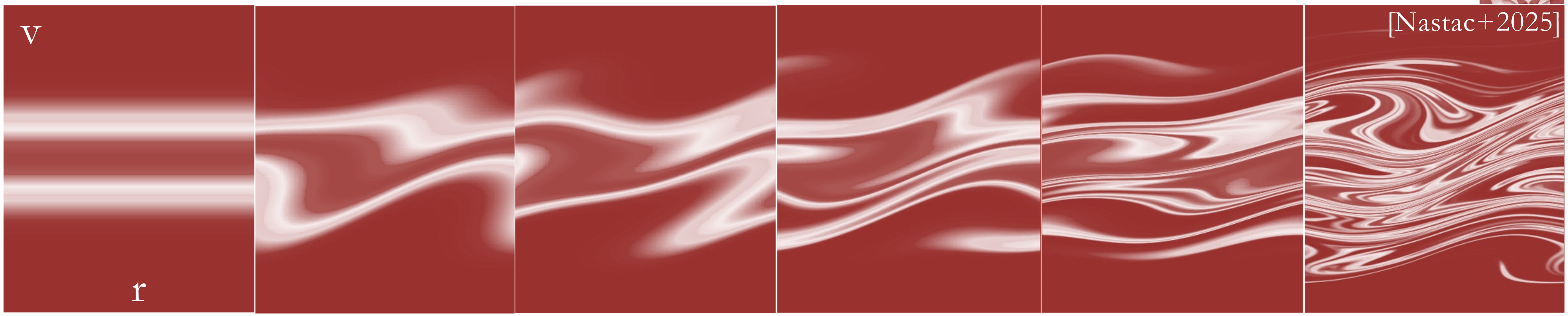

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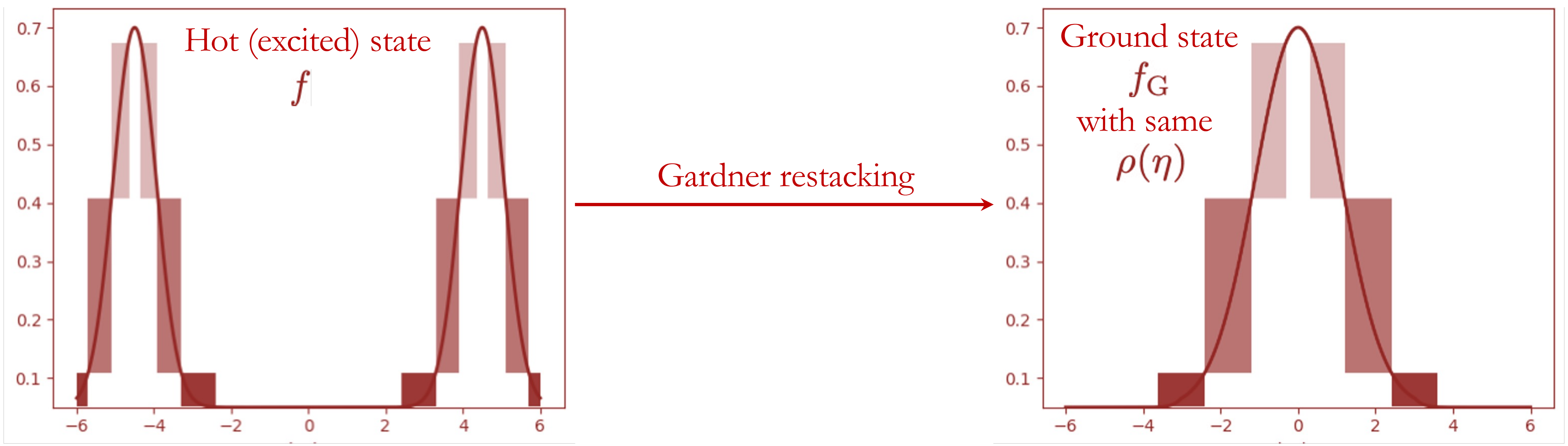
✓ <u>In an open system</u>, $f(\mathbf{r},\mathbf{v})$ will release energy until it's in "ground state" $f_{\mathrm{G}}(\mathbf{r},\mathbf{v})$

[Gardner 1963: **"restacked" distribution** with the same waterbag content]



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✓ <u>In a closed system</u> that is well-mixed (=lots of fluctuations, so stirred or unstable), coarse-grained $\bar{f}(\mathbf{r},\mathbf{v})$ will relax to maximise entropy of the "hyper kinetic" pdf

$$S = \iiint \mathrm{d}\mathbf{r}\mathrm{d}\mathbf{v}\mathrm{d}\eta\, P\ln P, \quad P(\mathbf{r},\mathbf{v},\eta) = \overline{\delta\left(\eta - f(\mathbf{r},\mathbf{v})\right)}, \quad \bar{f} = \int \mathrm{d}\eta\eta P$$

subject to conservation of N, E, and $\rho(\eta)$:

$$\bar{f} = \frac{\int \mathrm{d}\eta\,\eta\, e^{-\beta\eta[\epsilon(\mathbf{r},\mathbf{v})-\mu(\eta)]}}{\int \mathrm{d}\eta\, e^{-\beta\eta[\epsilon(\mathbf{r},\mathbf{v})-\mu(\eta)]}}$$

[“**violent relaxation**”, Lynden-Bell 1967]

Lynden-Bell's Violent Relaxation

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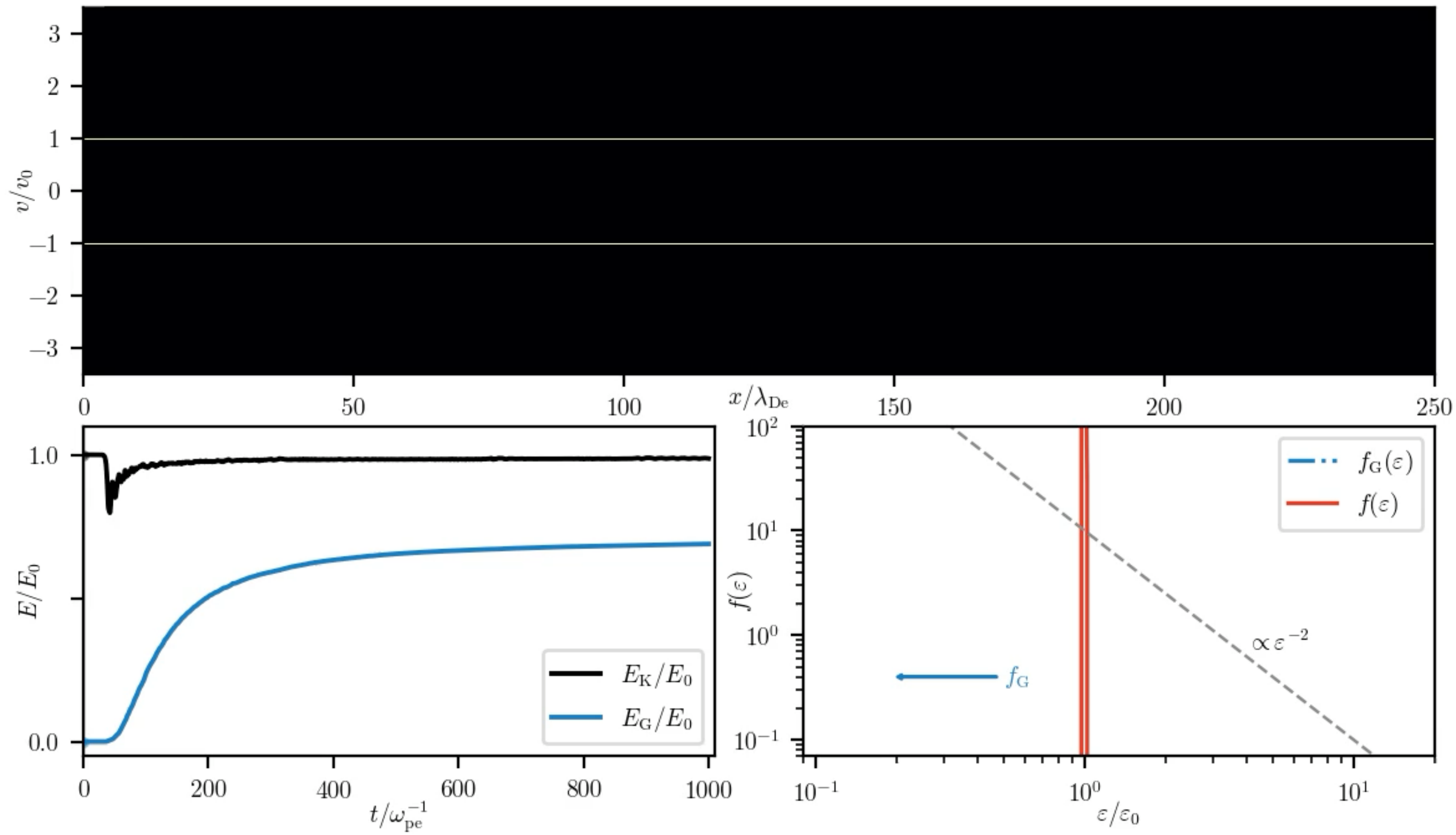

[**Two-stream instability** in 1D-1V, Ewart+2025]

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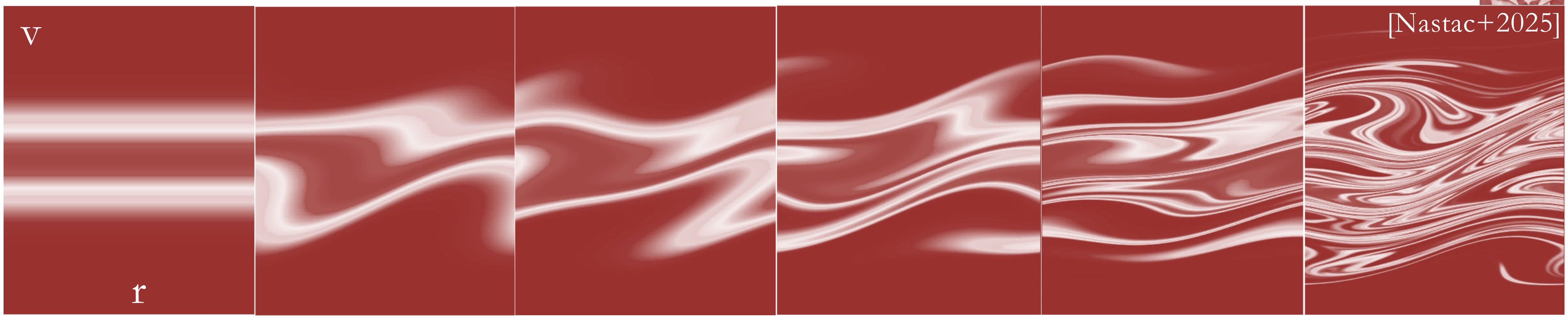

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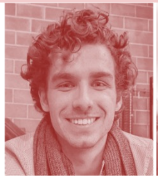
Effect of this: $f \to \bar{f}$ and $\rho(\eta)$ is renewed [Ewart, Hosking 2025]
...and the system remixes to Lynden-Bell maximum entropy with new constraints.
The process repeats "adiabatically" because $\bar{f}(\mathbf{r}, \mathbf{v})$ relaxes slightly faster ($\sim \tau_{\rm dyn}$) than $\rho(\eta)$ conservation is broken ($\sim \tau_{\rm dyn} \log N$) [Ewart+2025, Nastac+2025,2026]

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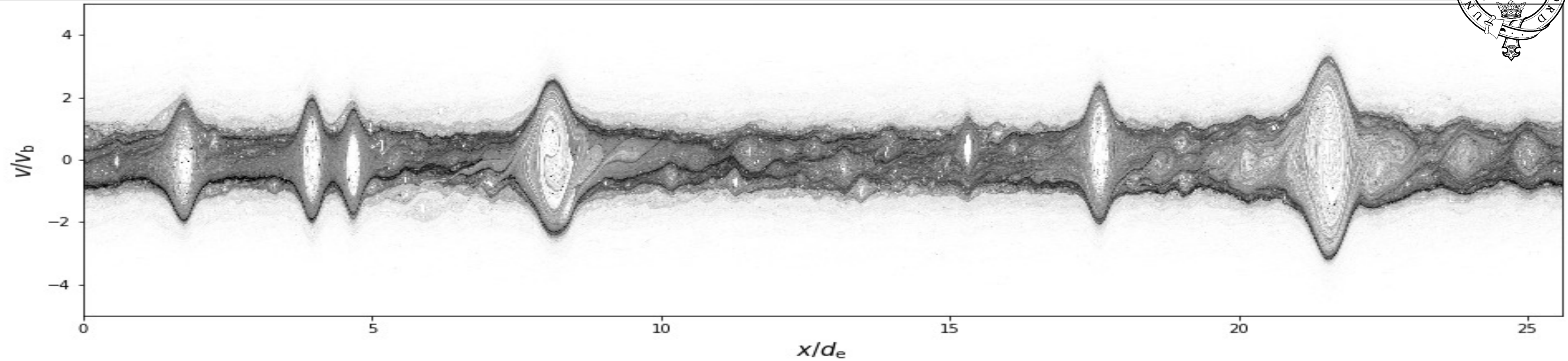

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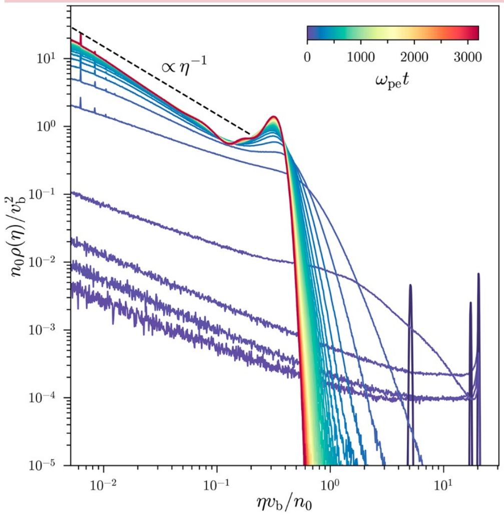
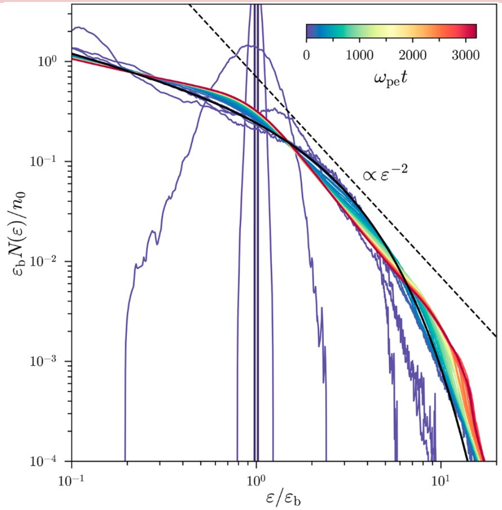
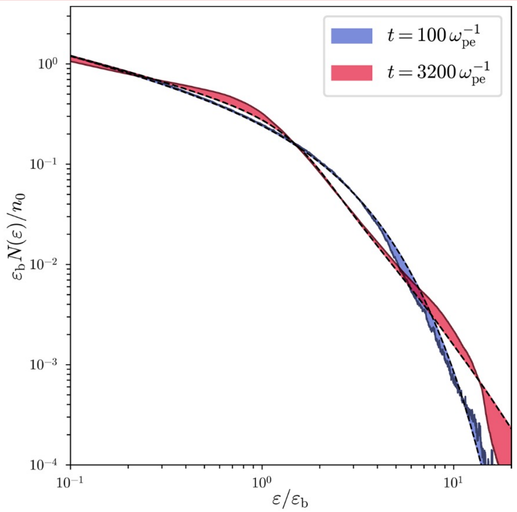
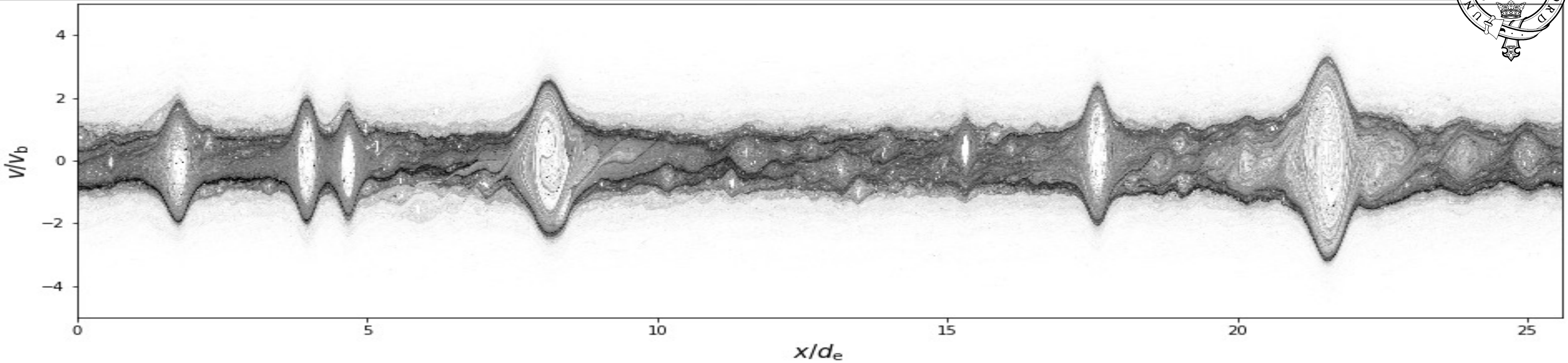






"Under the Hood": Phase-Space Cascade



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<u>The reason for this</u>: a Batchelor cascade of "*f-strophy*" $\int \mathrm{d}\mathbf{r}\mathrm{d}\mathbf{v}\, f^2 = \sum_{\mathbf{k},\mathbf{s}} |f_{\mathbf{k},\mathbf{s}}|^2$

(like milk in stirred coffee)

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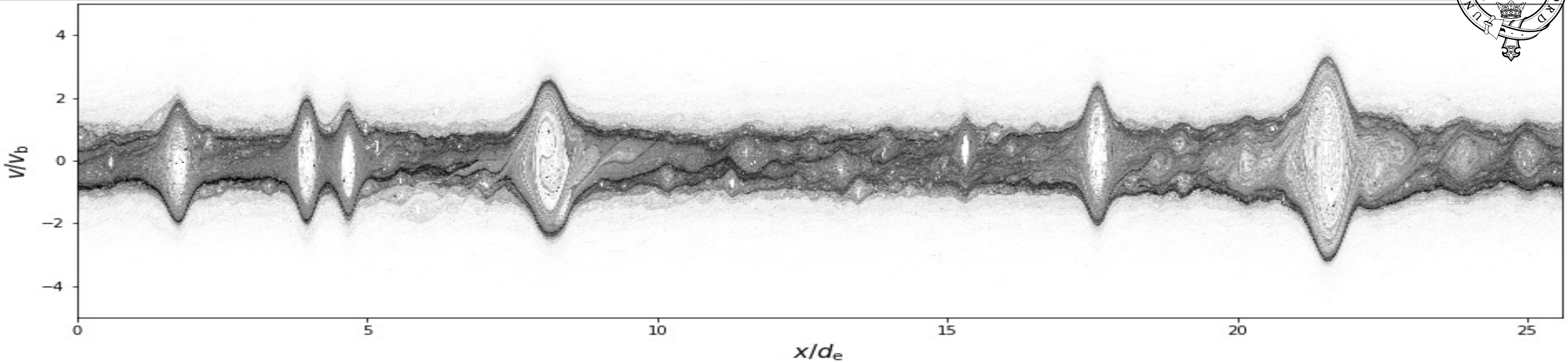

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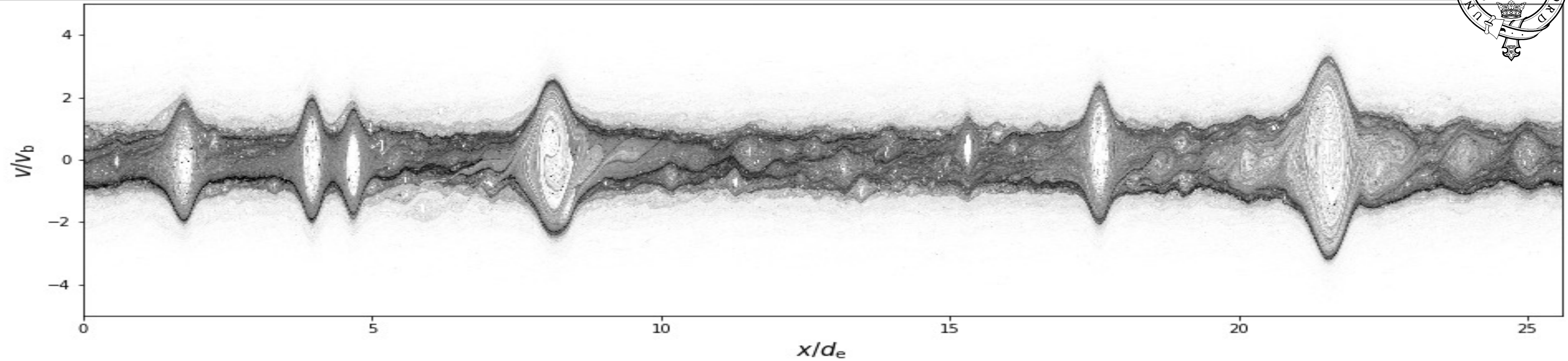

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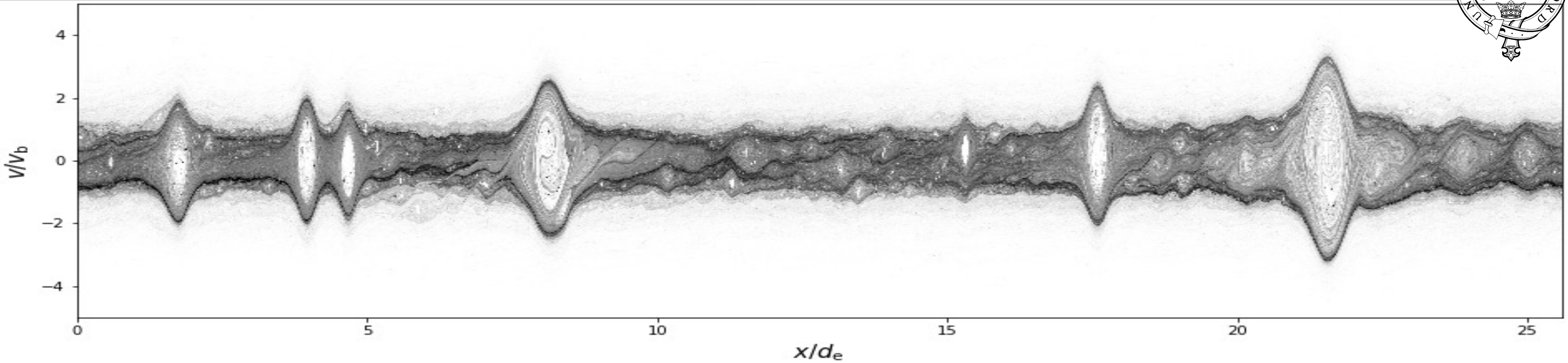

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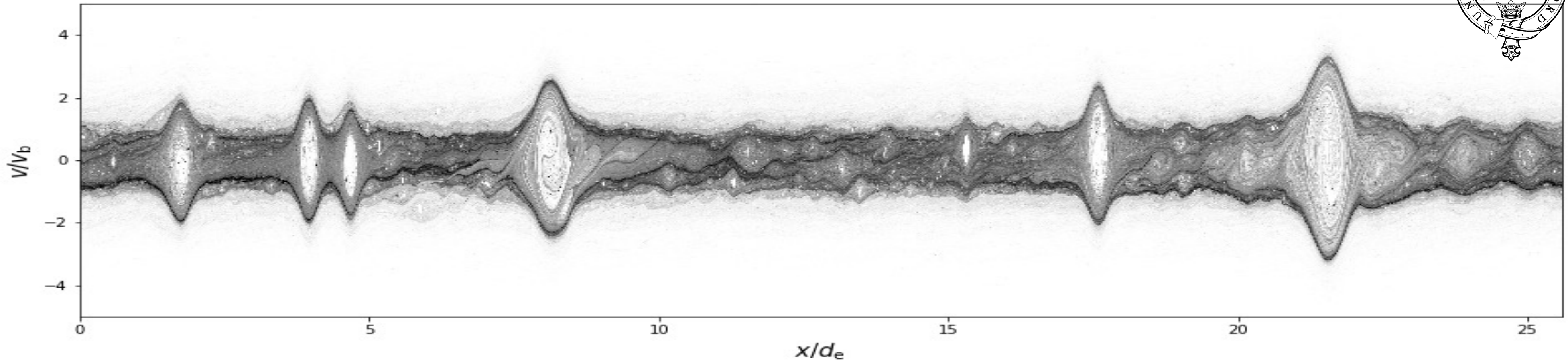
So k^{-1}, s^{-1} *f*-strophy spectra and cascade reaches cutoff in $\sim \tau_{\rm dyn} \log N$ time.

Universal Fluctuation Spectrum



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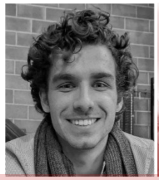


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Phase-space spectra $\quad |f_{\mathbf{k},\mathbf{s}}|^2 \propto \begin{cases} s^{-2d}, & k \ll \tau_{\rm dyn}^{-1} s \\ k^{-2d}, & k \gg \tau_{\rm dyn}^{-1} s \end{cases}$

...whence $\quad |E_{\mathbf{k}}|^2 \propto k^{-2(d+1)}, \quad |n_{\mathbf{k}}|^2 \propto k^{-2d}$

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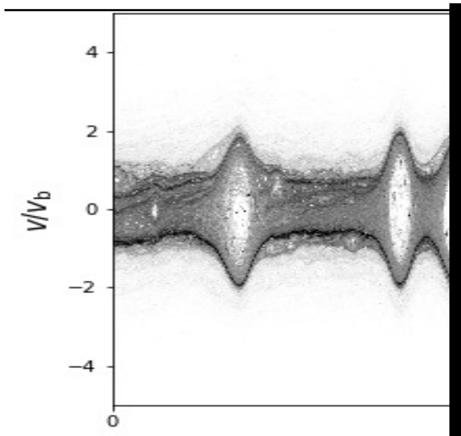

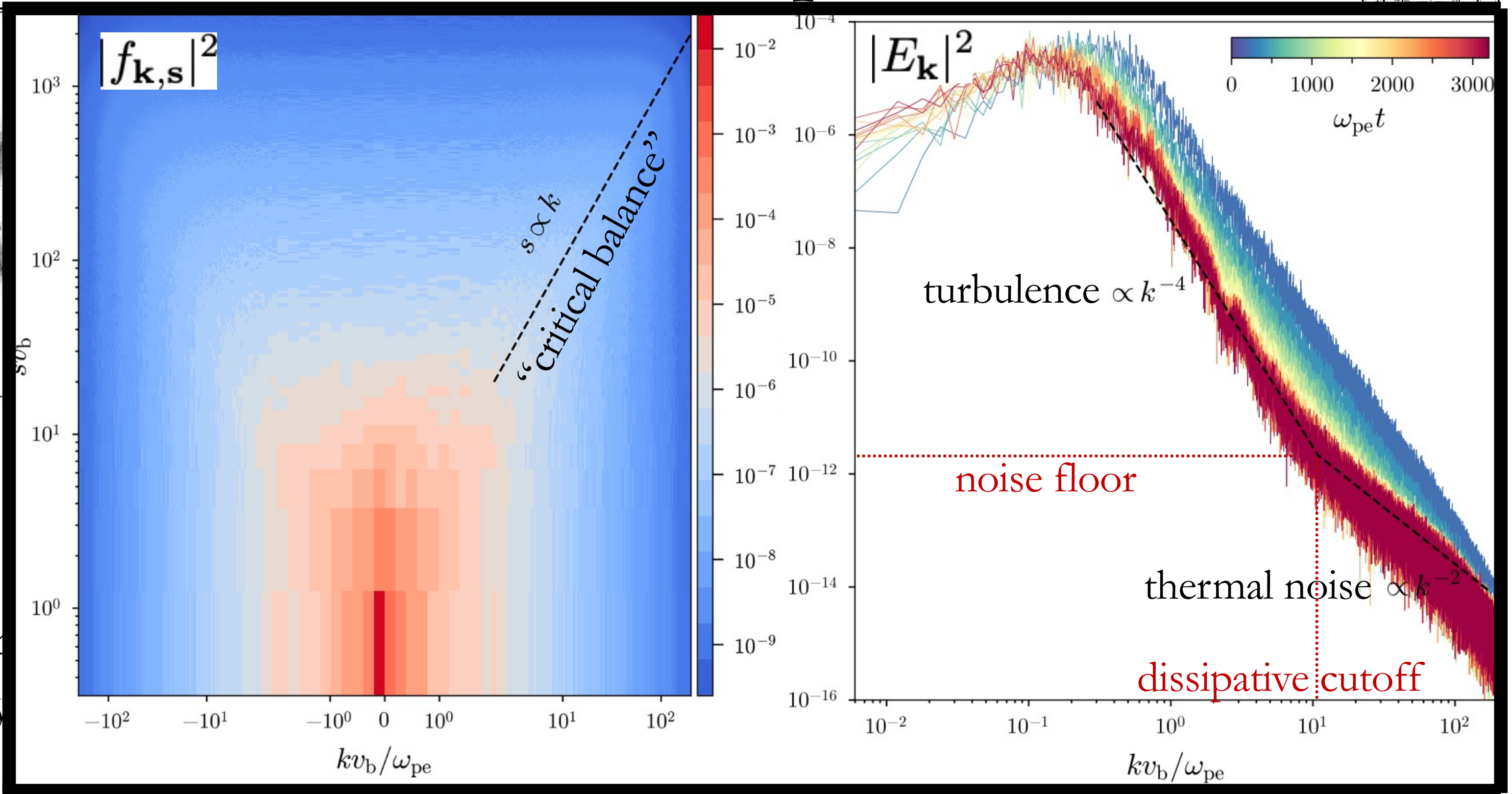

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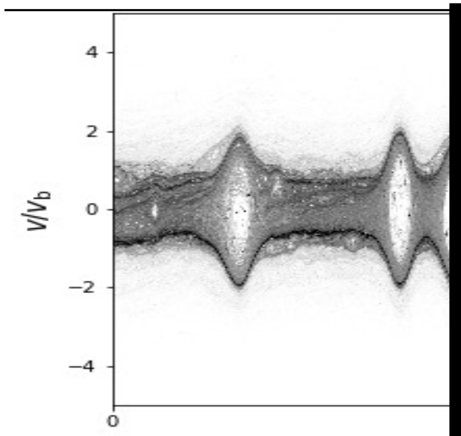
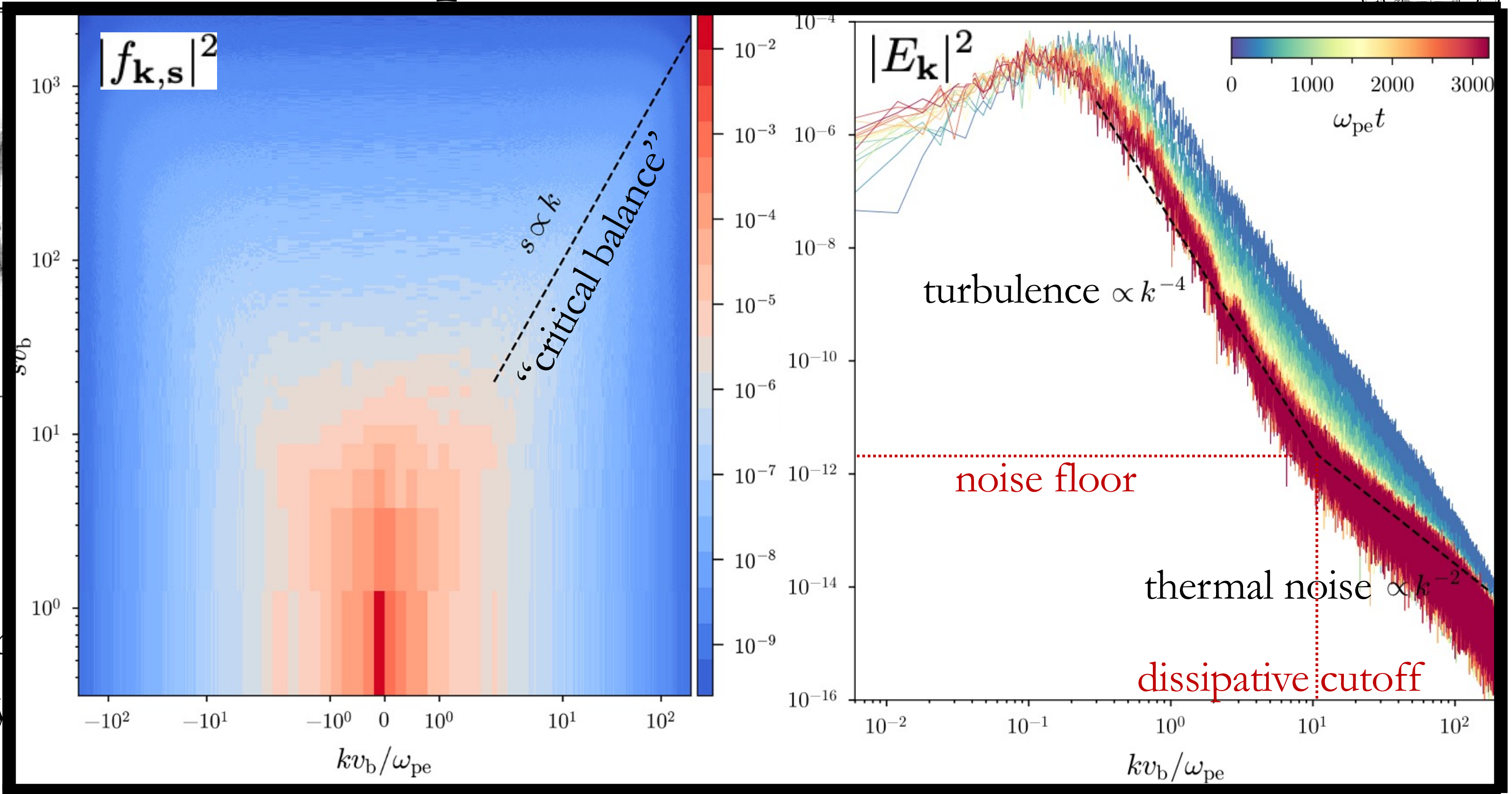
Dissipation Time





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and breaks

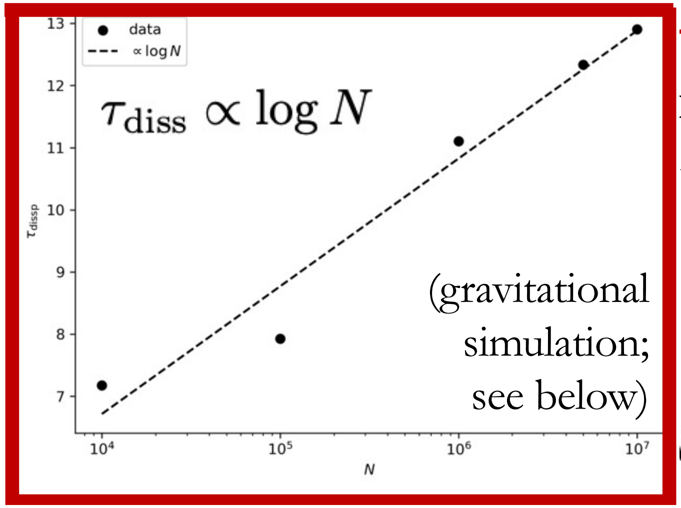
Effect of t

...and the sy

The proces

than $\rho(\eta)$ conservation is broken ($\sim \tau_{\rm dyn} \log N$)

[Ewart+2025, Nastac+2025,2026]



...or cascade of "*f-strophy*" $\int \mathbf{drdv} f^2 = \sum_{\mathbf{k},\mathbf{s}} |f_{\mathbf{k},\mathbf{s}}|^2$

...n stirred coffee)

Phase-space spectra $|f_{\mathbf{k},\mathbf{s}}|^2 \propto \begin{cases} s^{-2d}, & k \ll \tau_{\rm dyn}^{-1} s \\ k^{-2d}, & k \gg \tau_{\rm dyn}^{-1} s \end{cases}$

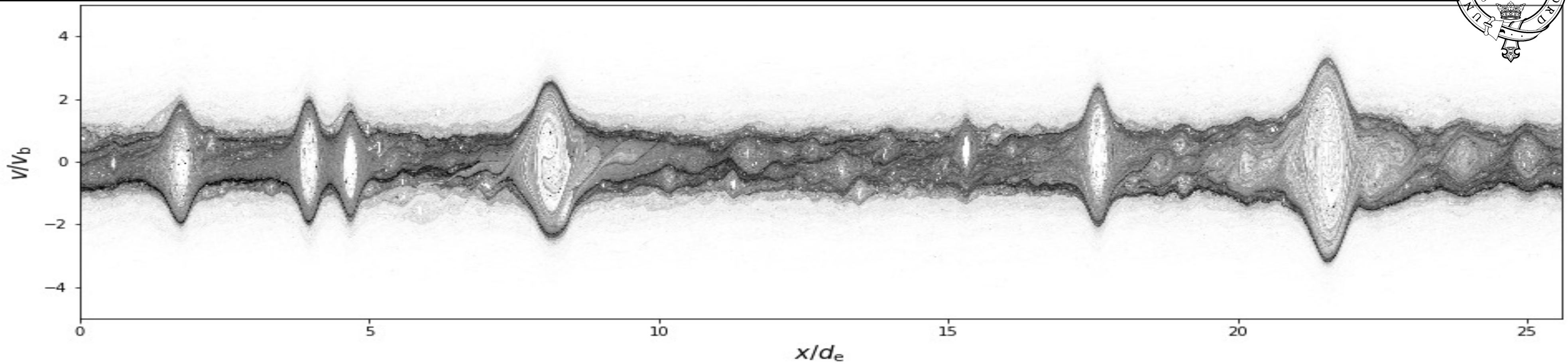
and cascade

...N time.

...whence $|E_{\mathbf{k}}|^2 \propto k^{-2(d+1)}$, $|n_{\mathbf{k}}|^2 \propto k^{-2d}$

supersedes thermal noise k^{-2}

Nonthermal Death



<u>In fact</u>, $\delta f = f - \bar{f}$ becomes highly fine-scaled, so eventually feels collisions and breaks phase-volume conservation! – *"Turbulent amnesia"*

Effect of this: $f \to \bar{f}$ and $\rho(\eta)$ is renewed [Ewart, Hosking 2025]

...and the system remixes to Lynden-Bell maximum entropy with new constraints.

The process repeats "adiabatically" because $\bar{f}(\mathbf{r}, \mathbf{v})$ relaxes slightly faster ($\sim \tau_{\rm dyn}$) than $\rho(\eta)$ conservation is broken ($\sim \tau_{\rm dyn} \log N$) [Ewart+2025, Nastac+2025,2026]

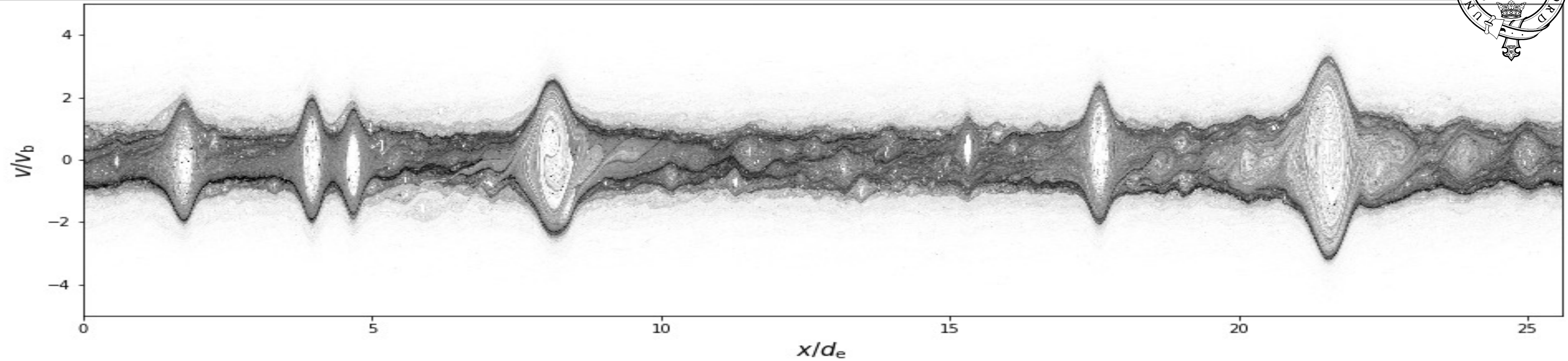
Where does this "cycle of violent relaxations" take the system?

In the long run, everything is dead ("cold", degenerate):

$$\delta f \to 0, \quad f\ ,\ \bar{f} \to f_{\rm G} \leftrightarrow \rho(\eta), \quad E \to E_{\rm G}$$

(we don't know if this state is universal, but we know that $\rho(\eta) \to \eta^{-1}$ as $\eta \to 0$)

Maximum Entropy is Minimum Energy



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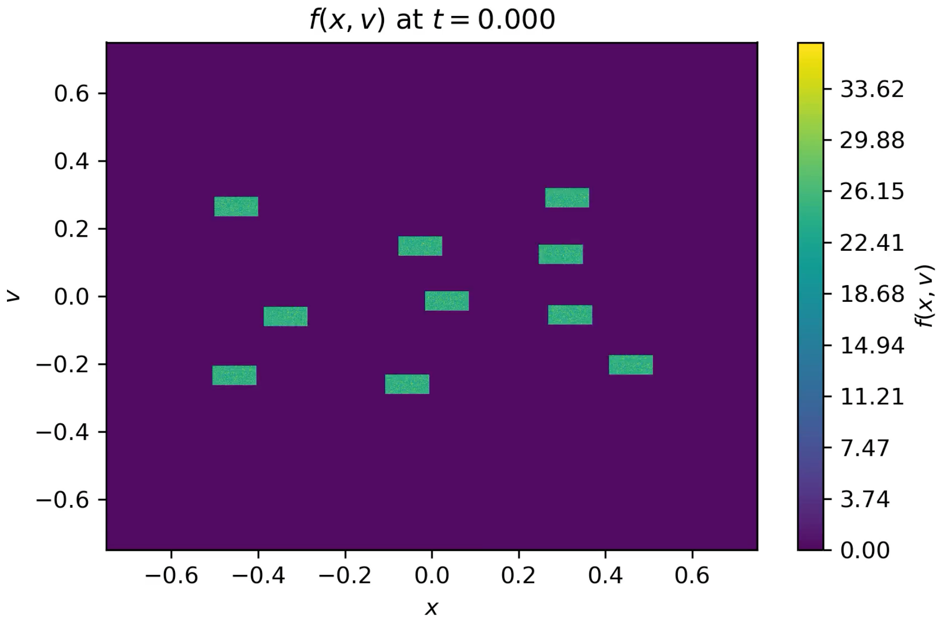

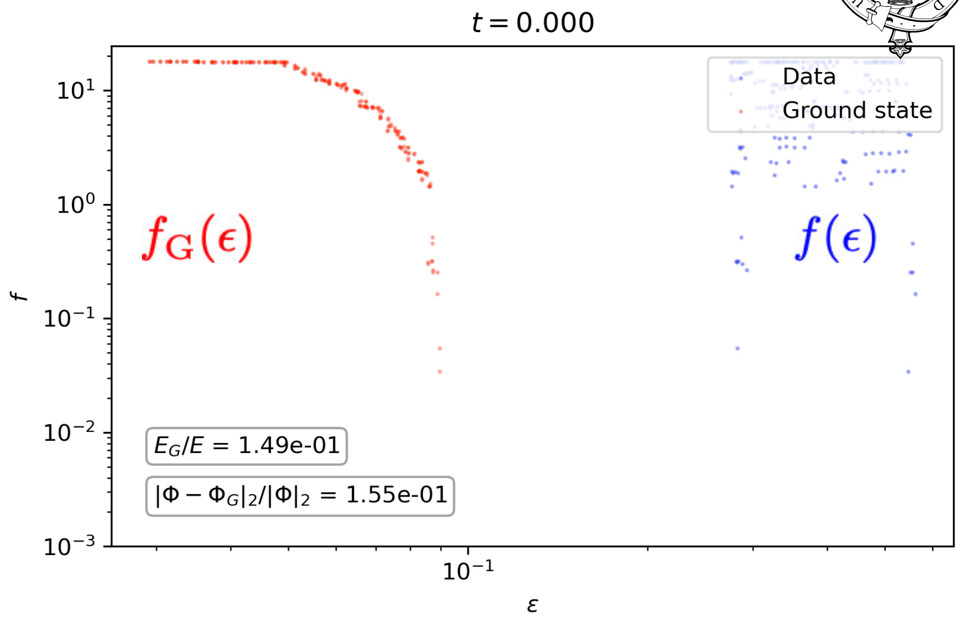

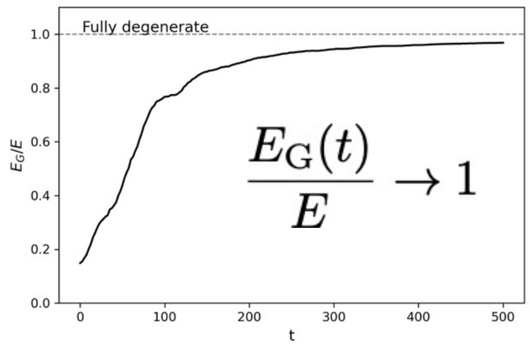

[Nastac+2026]

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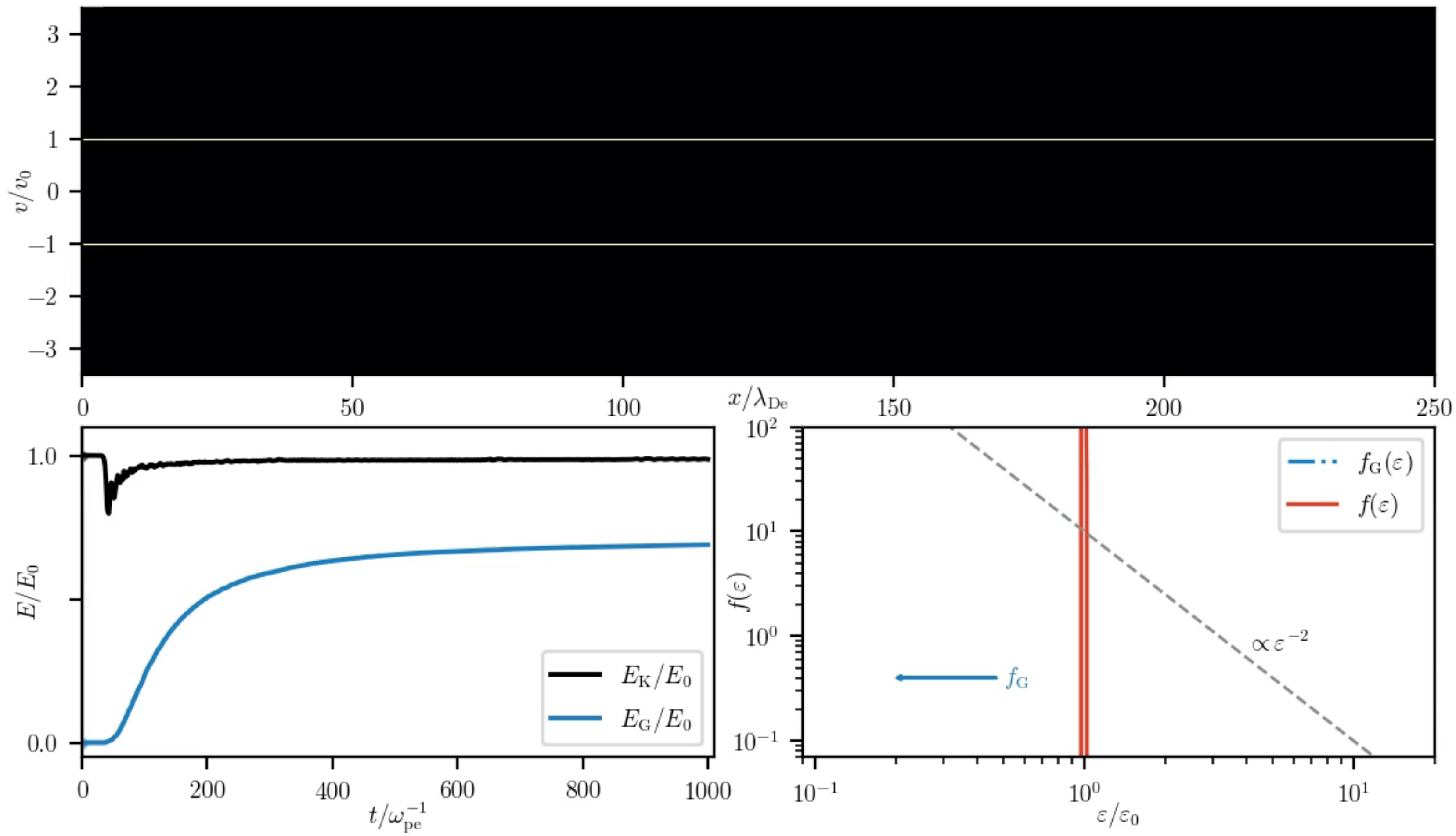
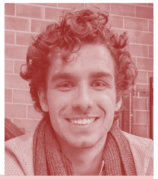


[Ewart+2025]



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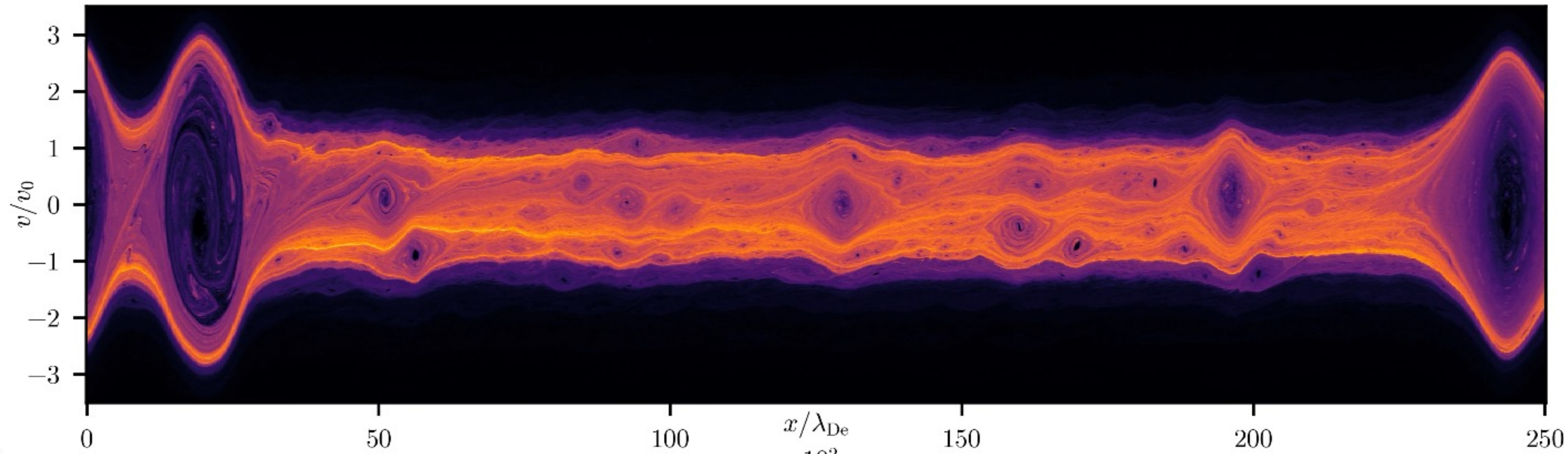
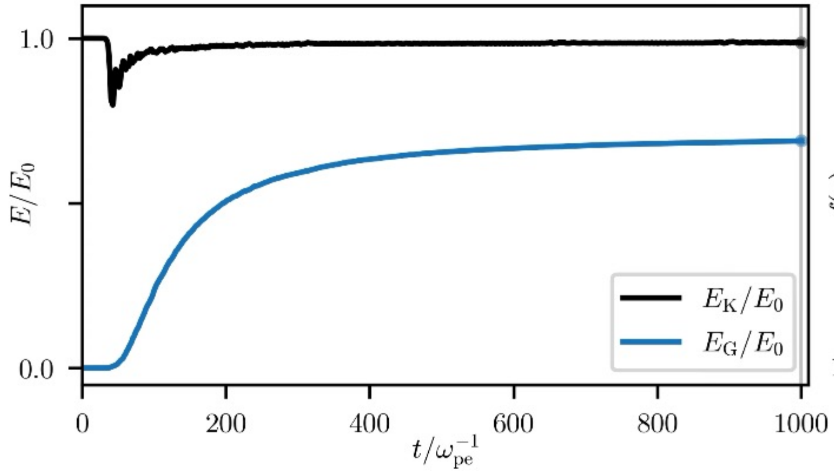
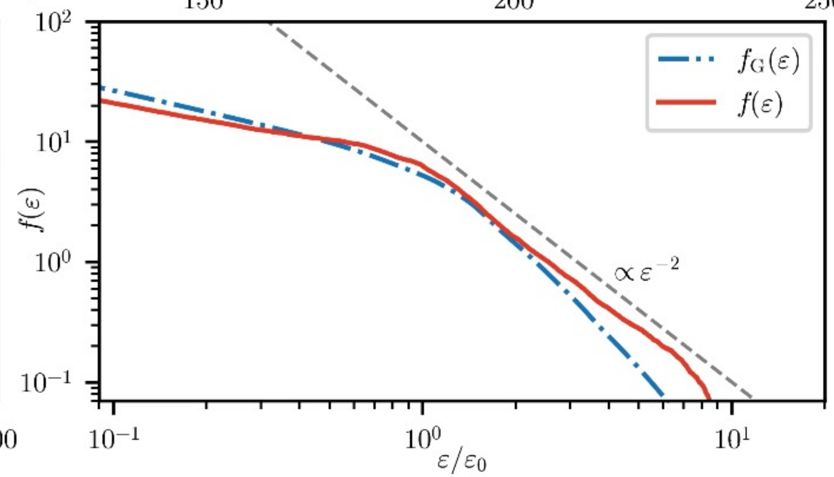
Intermediate Universality







[Ewart+2025]

There are long-lived "non-degenerate" transient states with $E \gg E_{\rm G}$.

In this limit, particle-energy distribution develops a universal high-energy tail:

$$\bar{f}_\epsilon(\epsilon) \propto \epsilon^{-2}$$

(which follows from universal $\rho(\eta)$; Ewart+2023)

(interesting because of observed power-law tails in solar wind, cosmic rays)

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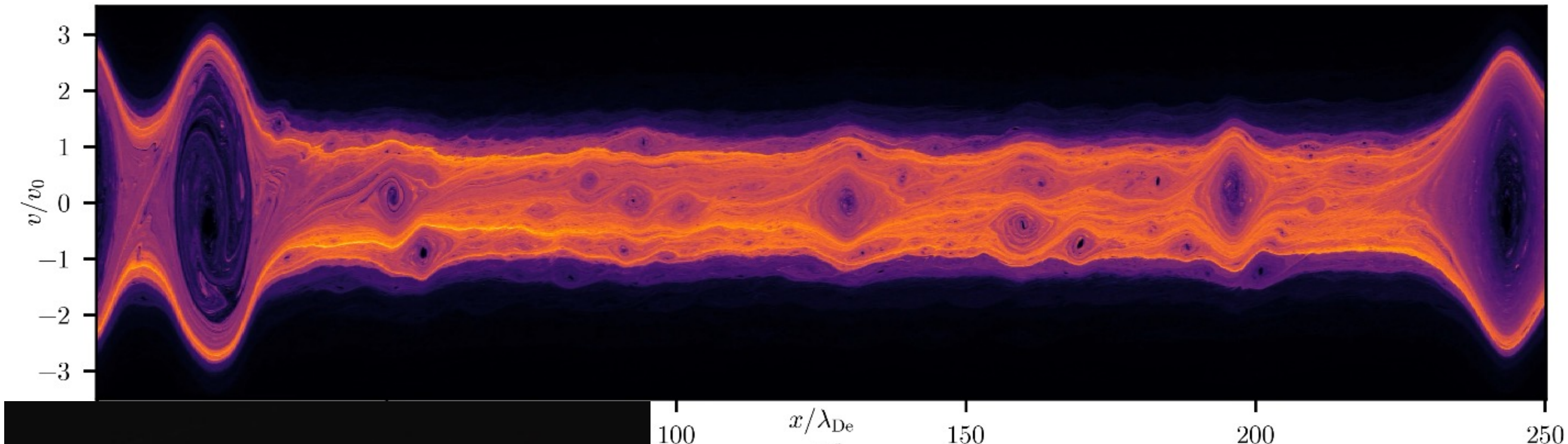
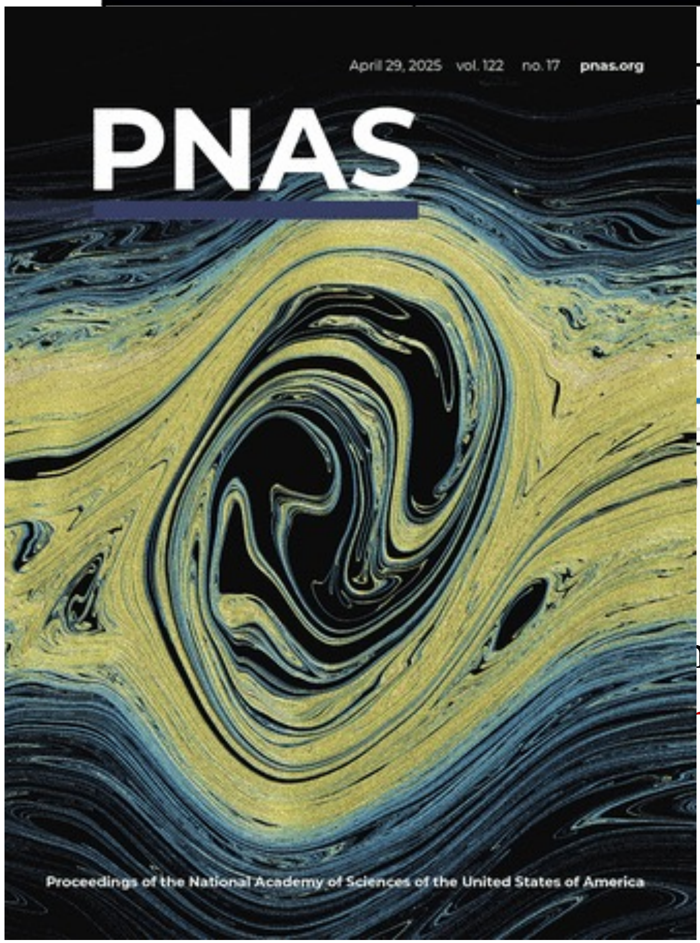
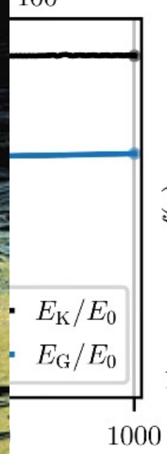
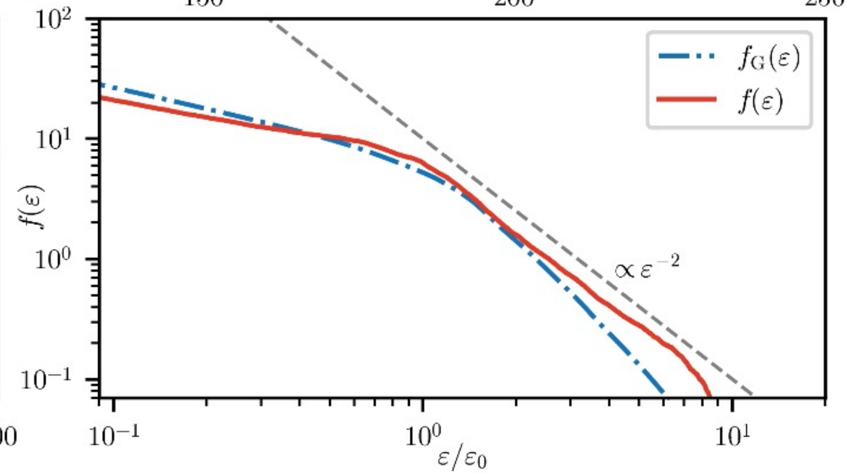
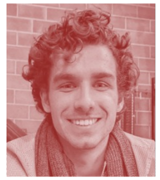








[Ewart+2025]



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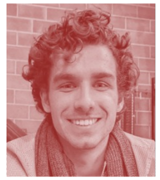
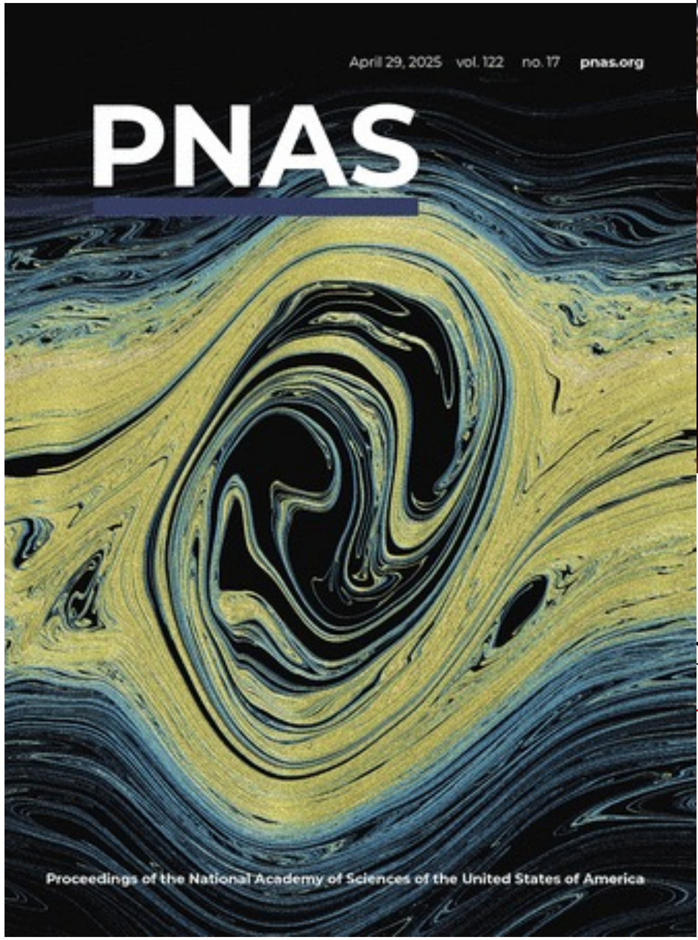
*How and why does the system stay in this **"hot"** state?*

[Ewart+2025]





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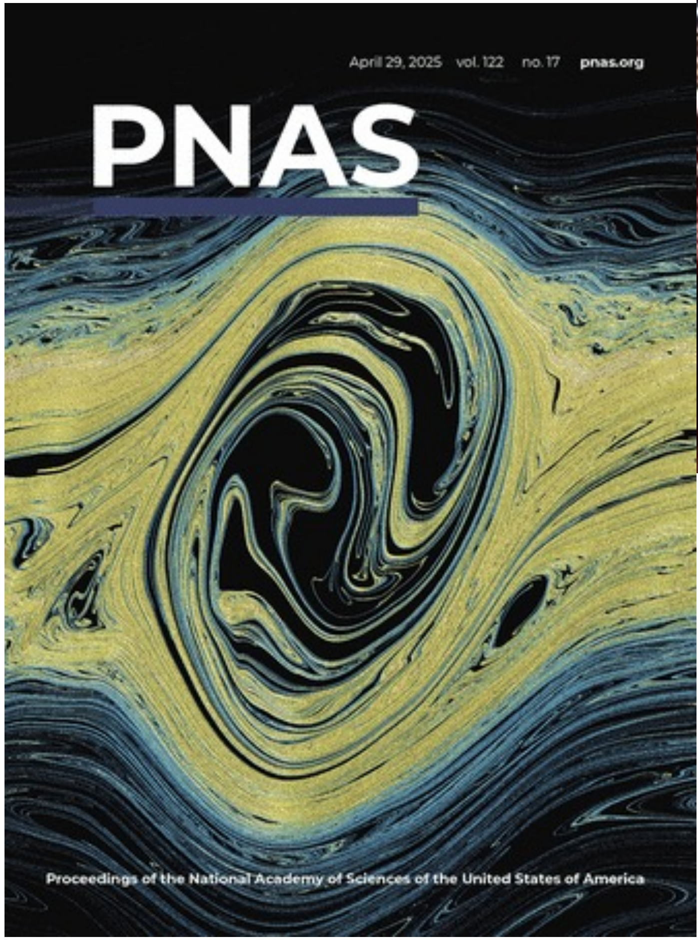


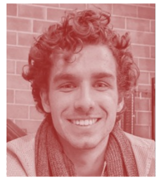
How and why does the system stay in this ***"hot"*** *state?*

It stays hot by storing some of its energy in long-lived structures, phase-space **holes** in plasmas, collapsed **halos** in gravity.

$$E \gg E_{\mathrm{G}}$$

They have their own dynamics, and stop the system from rearranging itself into a minimum-energy state.

[Ewart+2025]



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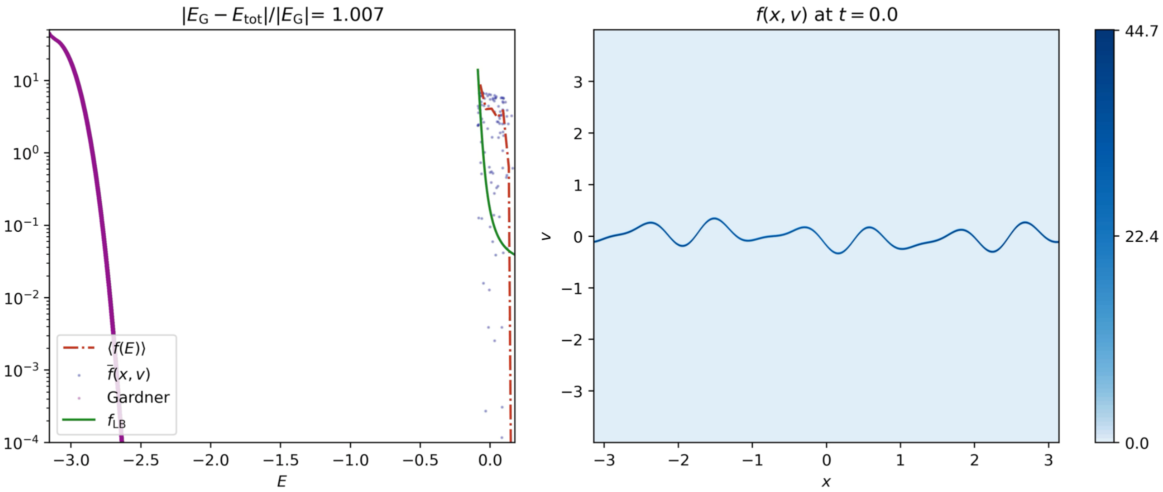

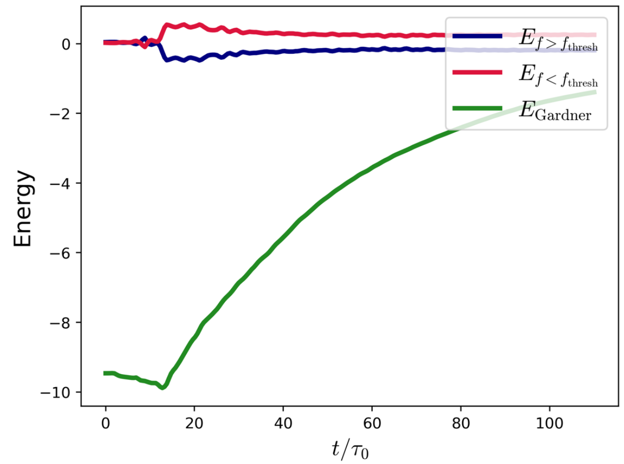

[Ginat+2025, 2026]

[Ewart+2025]

This is what clumping cold dark matter does *(“cosmological turbulence”)*

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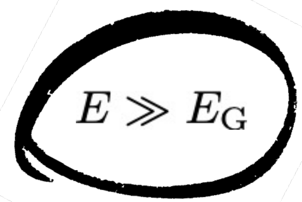

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✓ **Lynden-Bell's violent relaxation** to maximum-entropy state subject to phase-space volume conservation ("Casimirs") appears to work on dynamical times.

✓ Chaotic dynamics lead to a **turbulent cascade** of *f*-strophy towards small scales in phase space (constant-flux, Batchelor-like). This leads to

– **universal fluctuation spectrum** $|E_{\mathbf{k}}|^2 \propto k^{-2(d+1)}$, $|n_{\mathbf{k}}|^2 \propto k^{-2d}$,
superseding the thermal-noise spectrum in turbulent systems;

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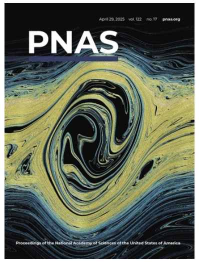

PNAS RESEARCH ARTICLE | PHYSICS 2025 Vol. 122 No. 17 e2417813122

Relaxation to universal non-Maxwellian equilibria in a collisionless plasma

Robert J. Ewart[a,b,1], Michael L. Nastac[a,c], Pablo J. Bilbao[d], Thales Silva[d], Luís O. Silva[d], and Alexander A. Schekochihin[a,e]

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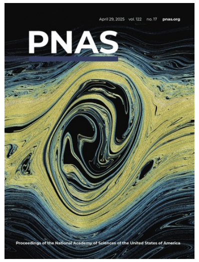

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Application of these ideas to fluid systems (stratified atmospheres, fusion plasmas) → D. Hosking

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Metastability of stratified magnetohydrostatic equilibria and their relaxation

D.N. Hosking[1,2,†], D. Wasserman[3] and S.C. Cowley[4]

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R. Ewart


M. Nastac


B. Ginat

✓ In the long run, Casimirs change so as to bring the **minimum-energy** (Gardner-restacked) state in alignment with the **maximum-entropy** state, making the latter degenerate (statistically "**cold**").


D. Hosking

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➢ *Is there a* ***universal shape*** *of this frozen equilibrium?*

➢ *How to calculate the "collisionless* ***collision integral****" (EFT) for time evolution of the distribution? (hence how to treat* ***non-equilibrium systems*** *with drive and transport)*

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➢ *How to predict* ***hole/halo dynamics****, survival time, and* $E/E_{\rm G}$ *?*