

Lattice QCD comes of age



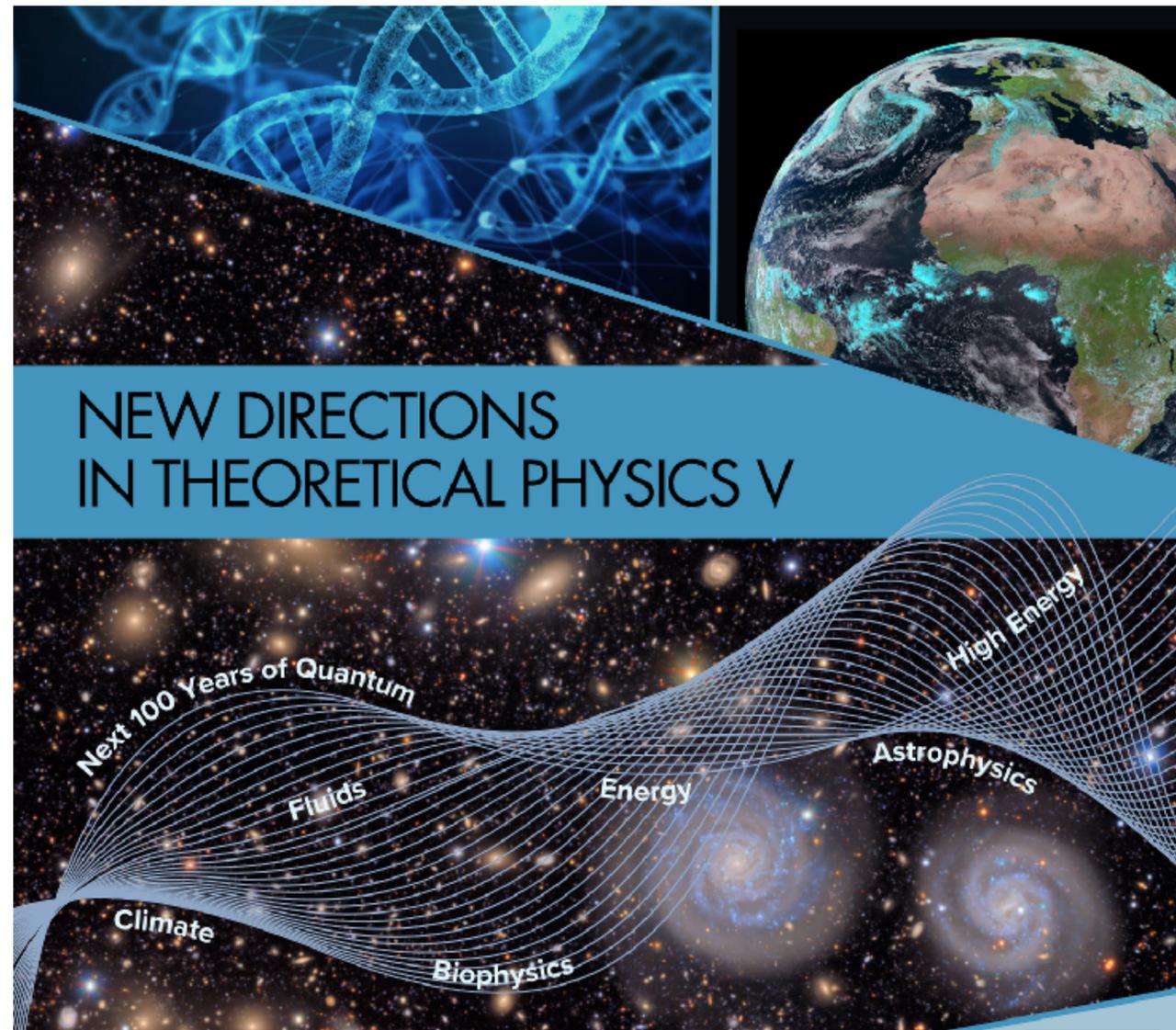
Aida X. El-Khadra

University of Illinois Urbana-Champaign

6 – 9 January 2026

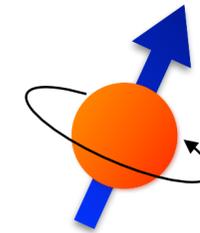


THE UNIVERSITY
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Outline

- The Standard Model of Particle Physics
 - open questions
- Introduction to QCD
- Introduction to Lattice QCD
- Two examples
 - rare B decay
 - muon $g-2$
- Conclusions and Outlook



Muon $g-2$ Theory Initiative

<https://muon-gm2-theory.illinois.edu/>

- “The anomalous magnetic moment of the muon in the SM: an update” [T. Aliberti et al, [arXiv:2505.21476](https://arxiv.org/abs/2505.21476), Phys. Repts. 1143 (2025) 0-157]
- “The anomalous magnetic moment of the muon in the SM”: [T. Aoyama et al, [arXiv:2006.04822](https://arxiv.org/abs/2006.04822), Phys. Repts. 887 (2020) 1-166.]



Flavour Lattice Averaging Group

<http://flag.itp.unibe.ch/2024/>

- “FLAG Review 2024” [S. Aoki et al, FLAG 2024 review, [arXiv:2411.04268](https://arxiv.org/abs/2411.04268)]

The Standard Model of Particle Physics

$$\mathcal{L}_{\text{SM}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\not{D}\psi + \bar{\psi}_i Y_{ij}\psi_j\phi + \text{h.c.} + |D_\mu\phi|^2 - V(\phi)$$

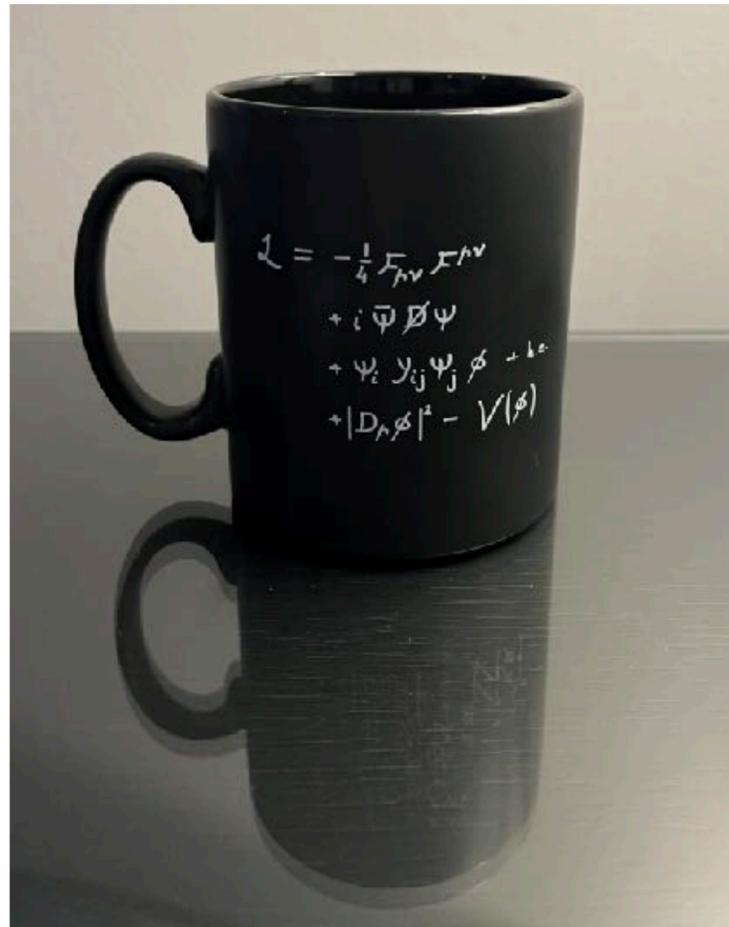
gauge field terms for
 $SU(3)_C \times SU(2)_L \times U(1)_Y$

fermion-gauge boson
interaction terms

Higgs-gauge boson
interactions

Higgs potential:
self interactions

Higgs Yukawa



The Standard Model of Particle Physics

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gauge field terms for $SU(3)_C \times SU(2)_L \times U(1)_Y$

fermion-gauge boson interaction terms

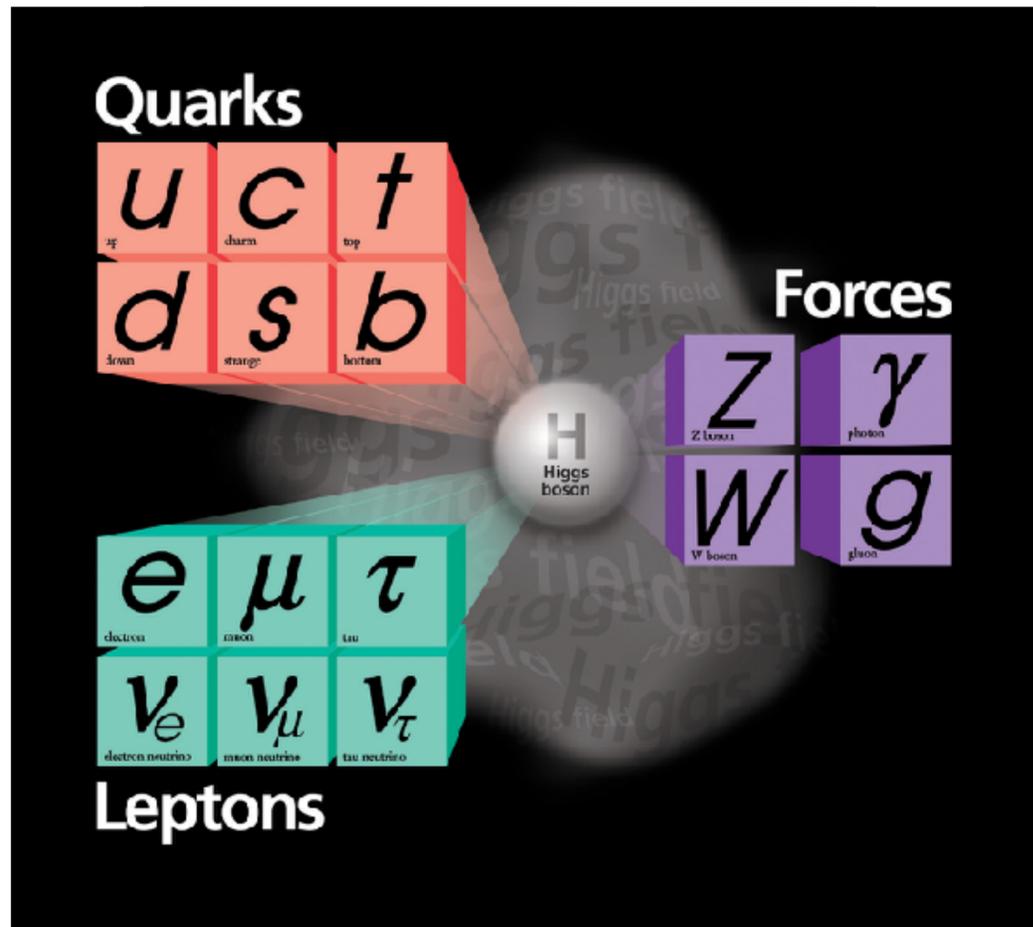
Higgs-gauge boson interactions

Higgs potential: self interactions

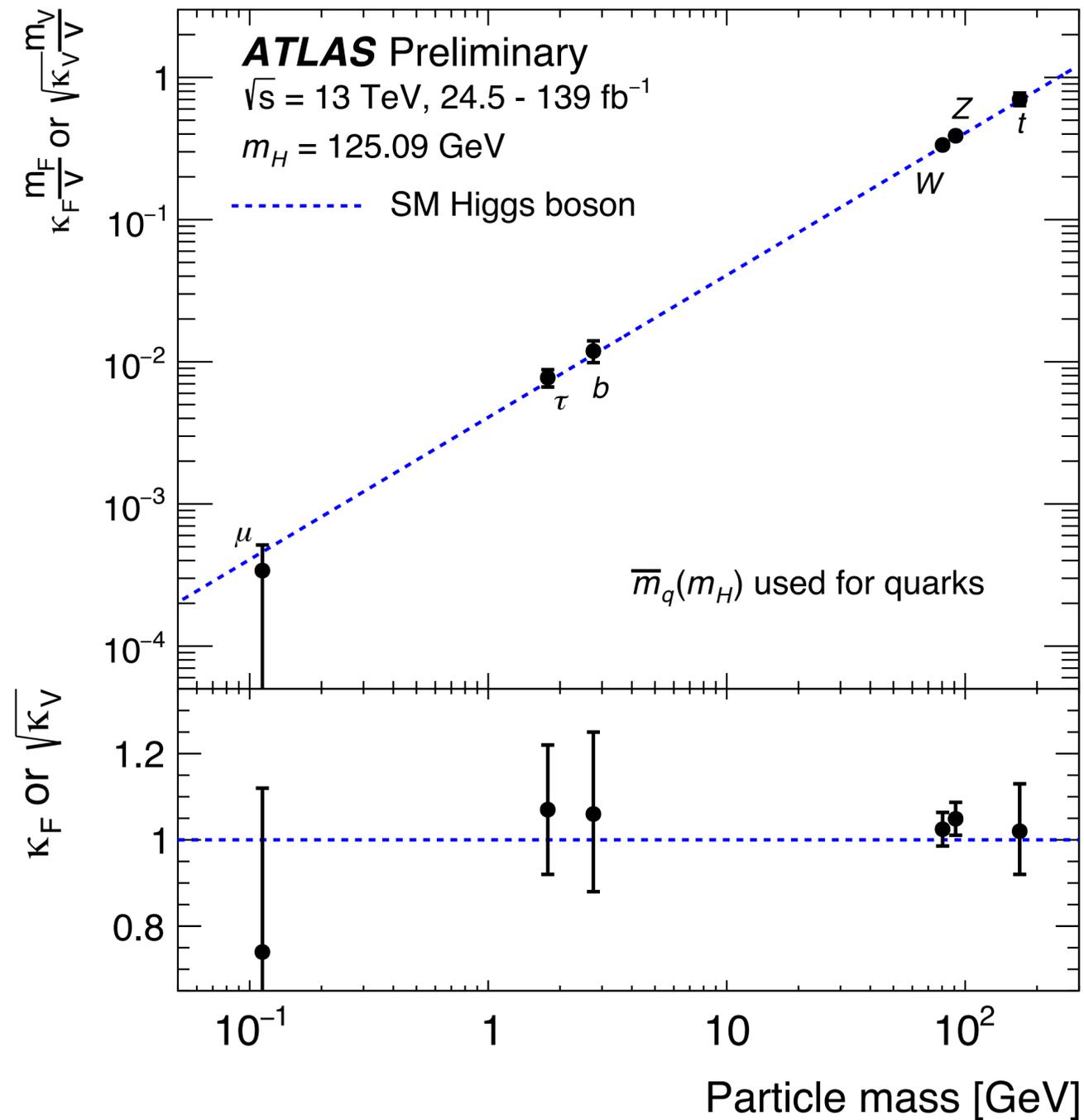
Higgs Yukawa

mass terms + fermion-Higgs interactions:

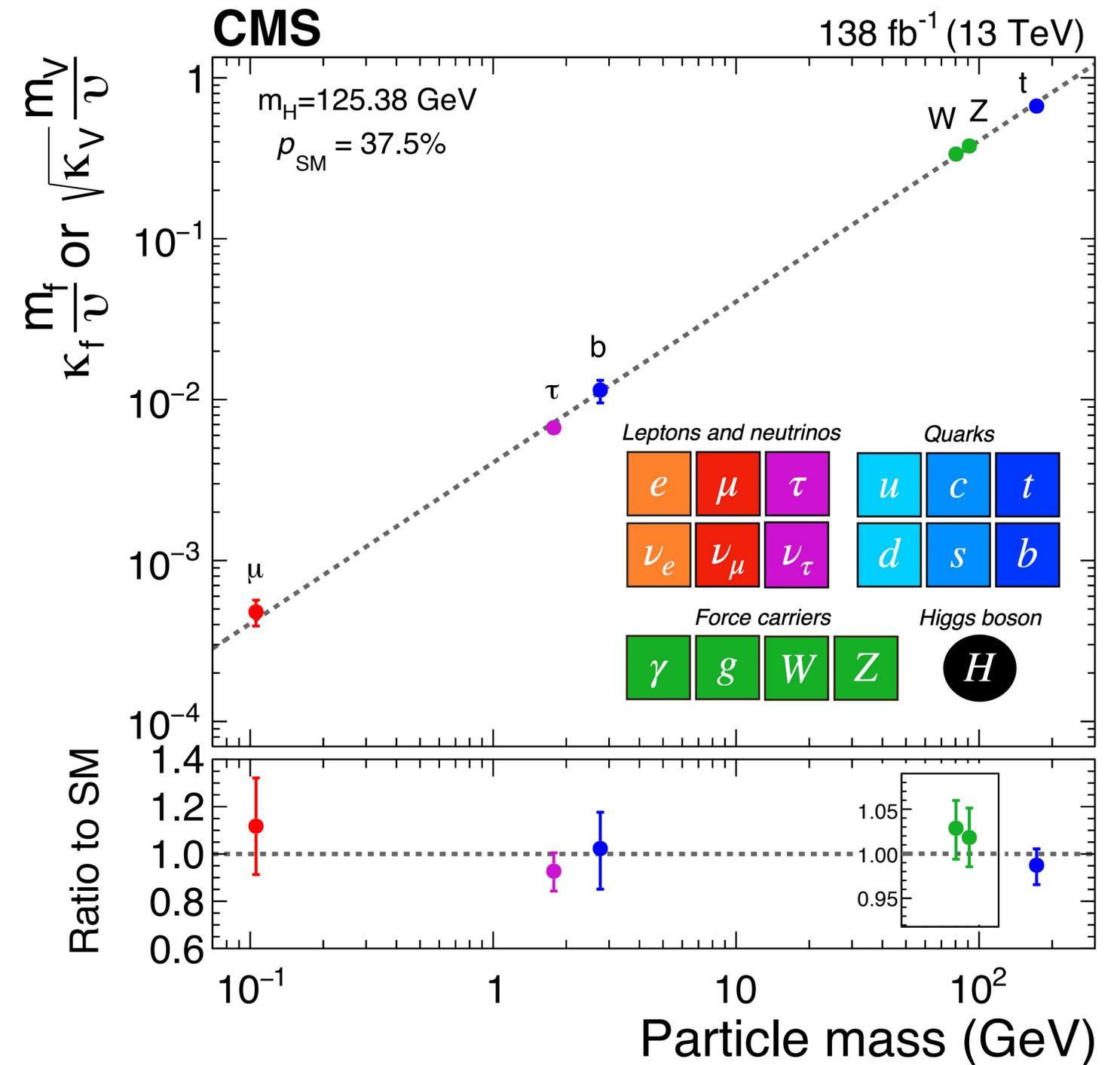
$$\sum_f m_f \bar{\psi}_f \psi_f [1 + H/v]$$



Higgs physics @ Large Hadron Collider (CERN)



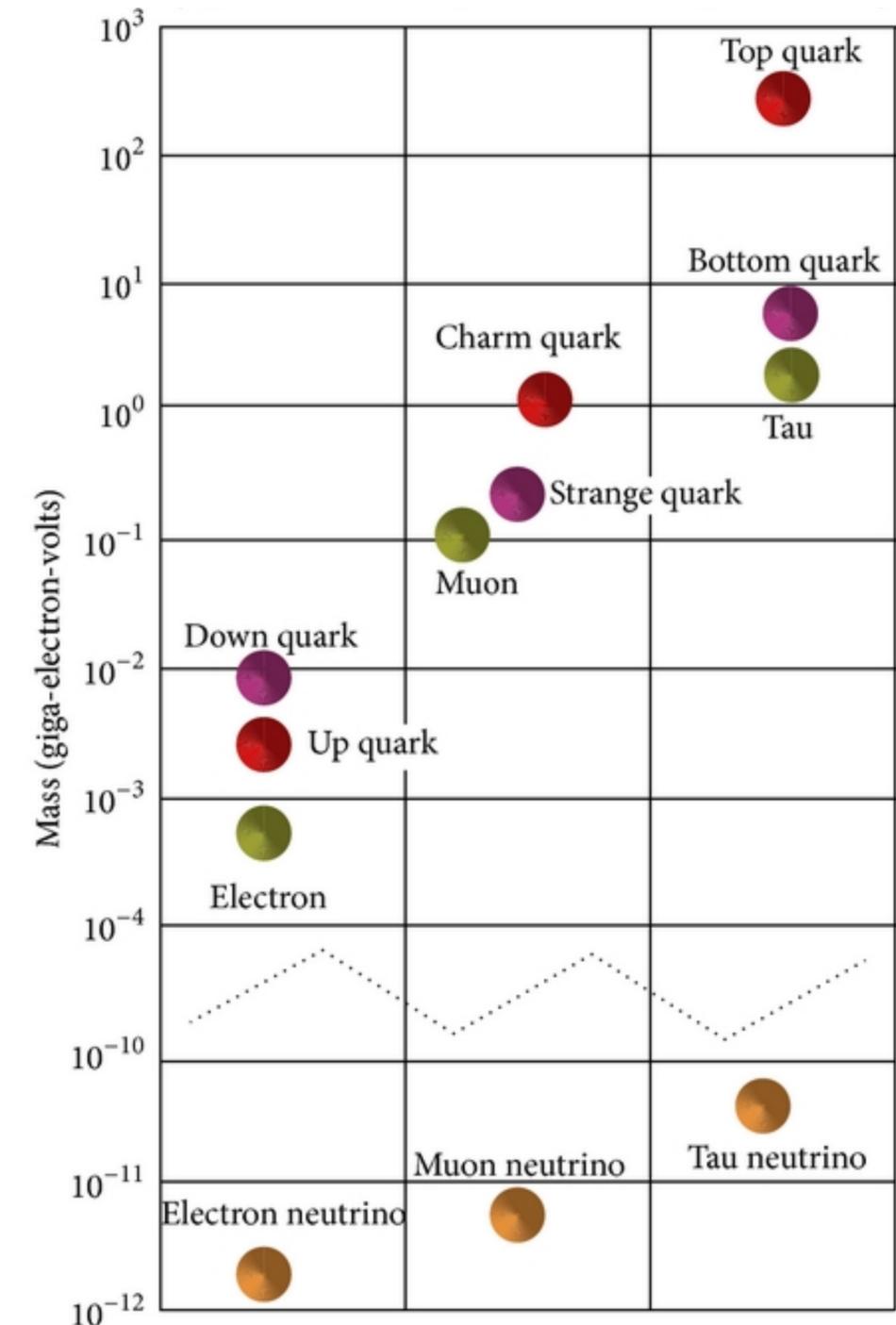
$$\frac{m_f}{v} \bar{\psi}_f \psi_f H$$



Open Questions

- ◆ Who ordered that? (Why three generations?)
- ◆ origins of fermion masses: Higgs-Yukawa
 - same for quarks, charged leptons, and neutrinos?
- ◆ origin of mass hierarchy
- ◆ structure of quark and neutrino mixing matrices
- ◆ matter — antimatter asymmetry
- ◆ Higgs mass $\simeq 125$ GeV
- ◆ nature of dark matter
- ◆ dark energy
- ⋮

Possible answers to these questions generally give rise to new particles and new interactions.



[G. Giacomelli et al, AHEP 2013 (2013)]

Open Questions in Flavor Physics

- quark flavor change caused in SM by weak interactions (mediated by charged W boson), and mixing described by CKM (Cabibbo, Kobayashi, Maskawa) matrix:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = V_{\text{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix} \quad V_{\text{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} \text{blue} & \text{purple} & \text{red} \\ \text{purple} & \text{blue} & \text{red} \\ \text{red} & \text{red} & \text{blue} \end{pmatrix}$$

- The CKM matrix is unitary in the SM \Rightarrow testable relations between CKM elements
- CP violation in quark sector, described by 1 complex phase in CKM matrix: unlikely enough to account for matter—anti-matter asymmetry
- Origin of hierarchy of neutrino mixing (aka PMNS) matrix?

neutrino mixing:

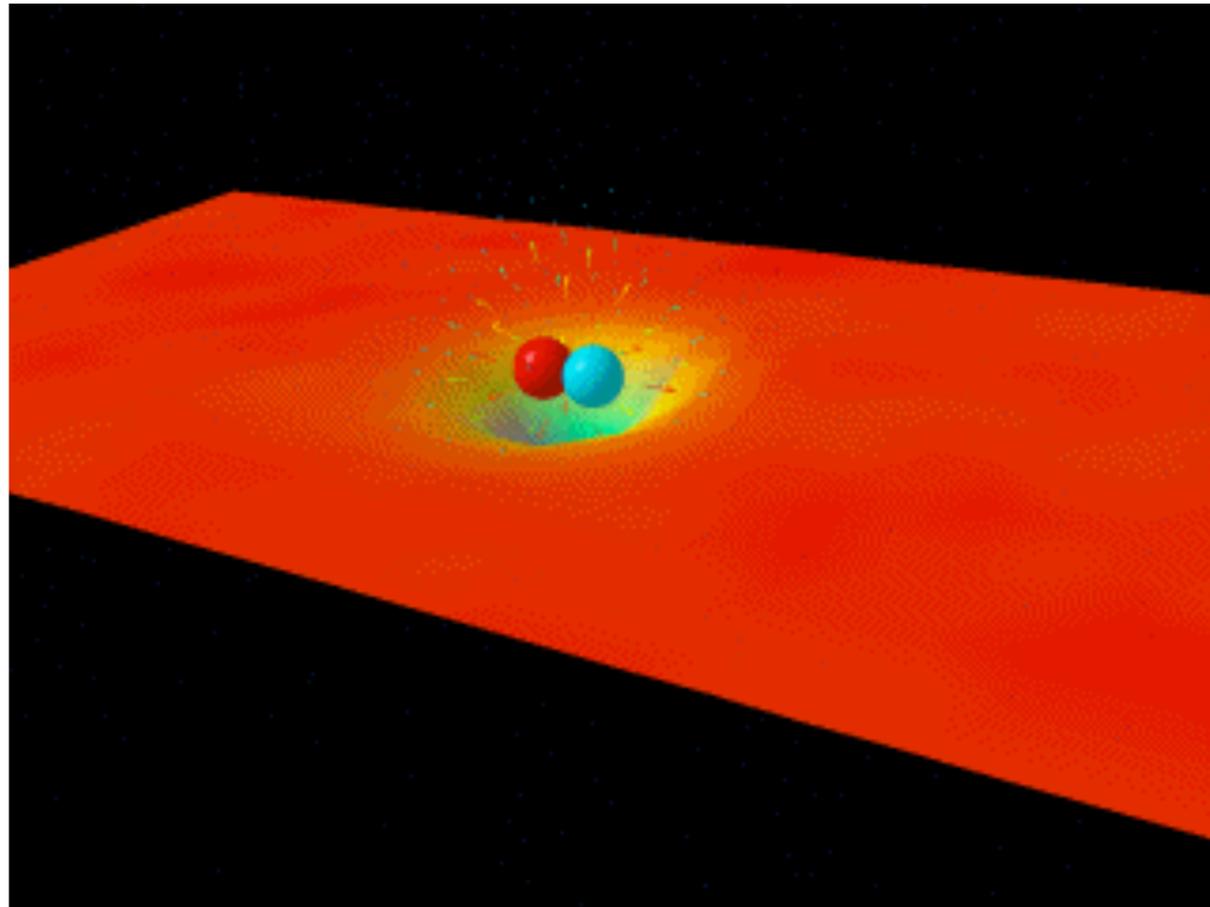
$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = U_{\text{PMNS}} \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} \quad U_{\text{PMNS}} = \begin{pmatrix} \text{blue} & \text{purple} & \text{red} \\ \text{purple} & \text{purple} & \text{blue} \\ \text{purple} & \text{purple} & \text{blue} \end{pmatrix}$$

- Origin of neutrino masses? CP violation in neutrino sector?
- no charged lepton flavor changing decays in SM \Rightarrow searches for processes such as $\mu \rightarrow e$
- lepton flavor universality, except for effects due to lepton mass differences $m_e \ll m_\mu \ll m_\tau$

QCD — SU(3) color

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4}F_{\mu\nu}^a F^{a\mu\nu} + \sum_f \bar{\psi}_f (i\not{D} - m_f)\psi_f$$

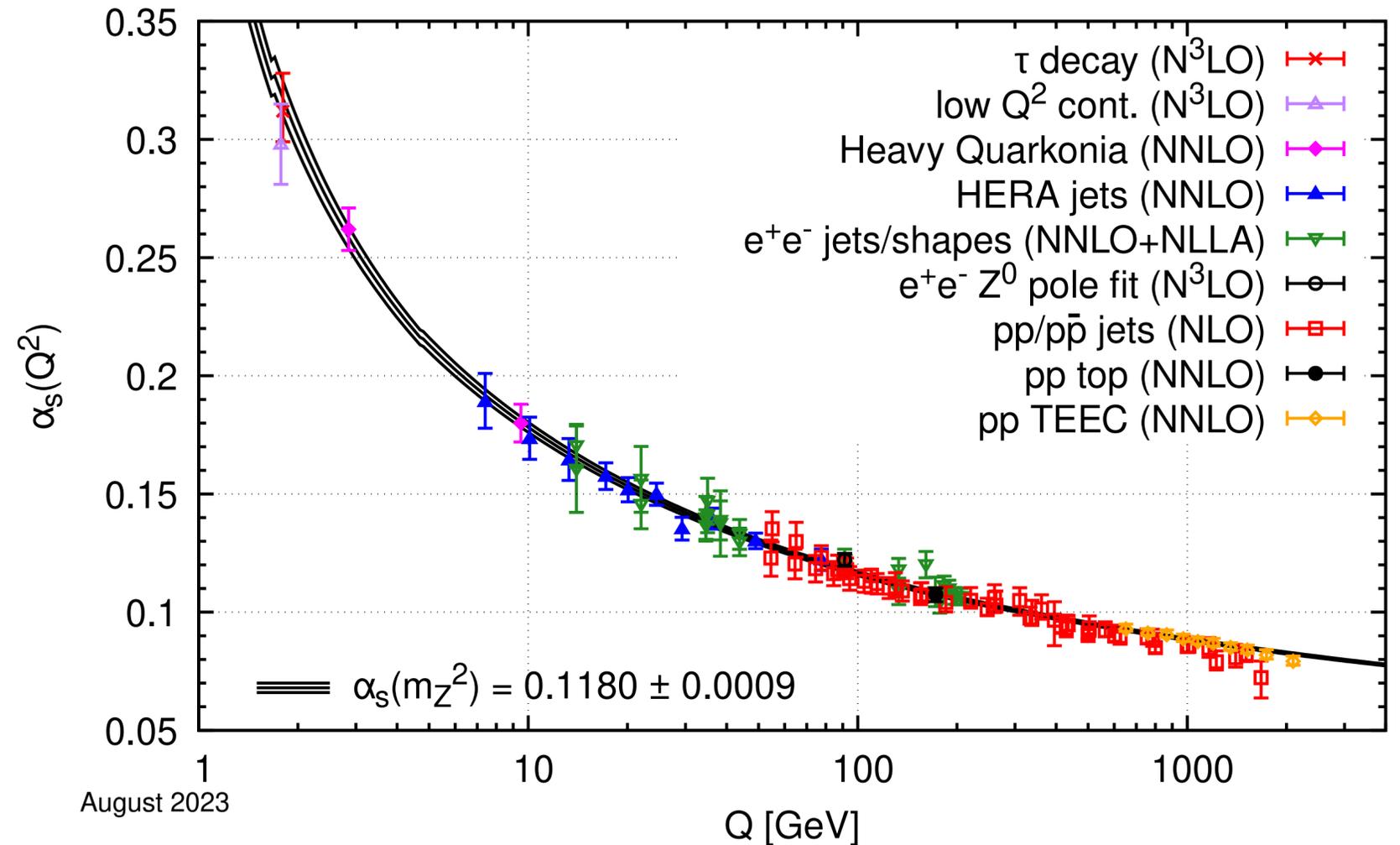
Quark confinement:
quarks are always in bound states
(a.k.a. hadrons)



Visualizations of Quantumchromodynamics

(<http://www.physics.adelaide.edu.au/theory/staff/leinweber/VisualQCD/Nobel/>)

Asymptotic freedom:
coupling strength decreases at higher
energies (shorter distances)



QCD:
$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \sum_f \bar{\psi}_f(i\not{D} - m_f)\psi_f$$

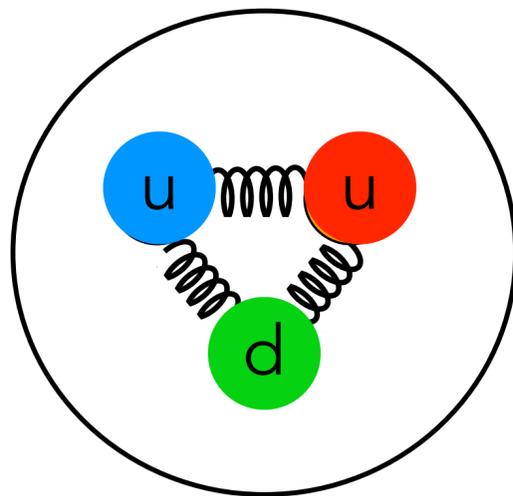
Asymptotic freedom \implies perturbation theory for high-energy (aka "hard scattering") processes.

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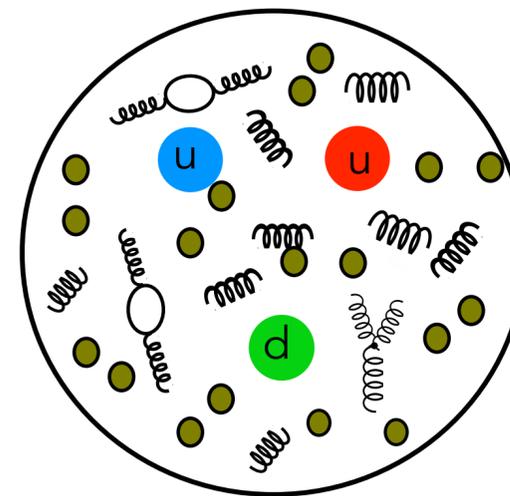
Asymptotic freedom \implies perturbation theory for high-energy (aka "hard scattering") processes.

Inside the proton...

Quark model view



QCD view



... a strongly interacting many-body bound state

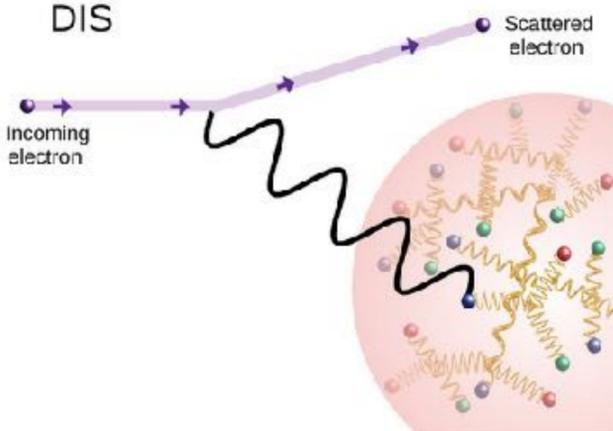
Need a nonperturbative method for QCD: Lattice Field Theory

QCD + QED

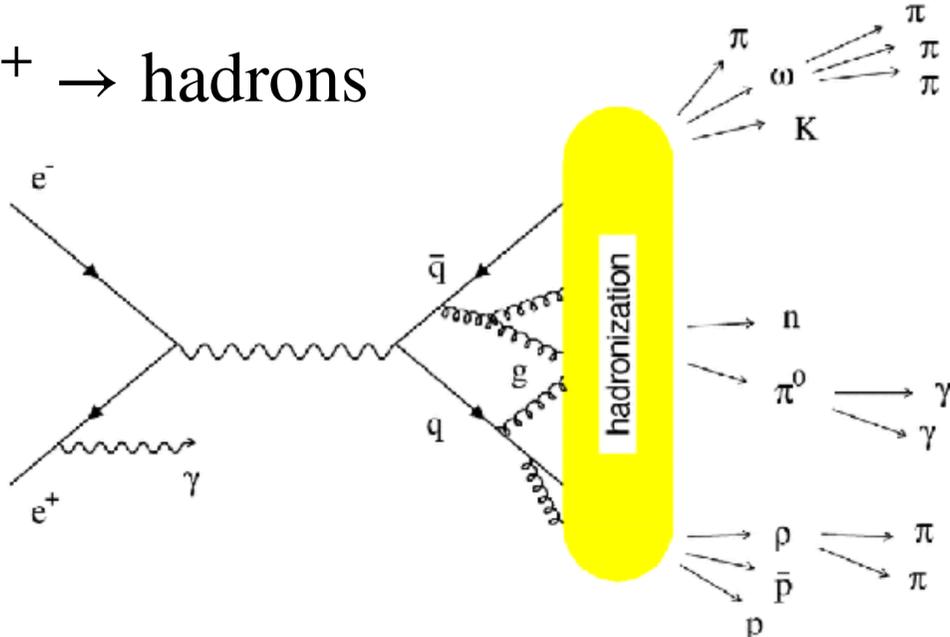
Quarks carry electric charge, so interact with photons:

Examples: Deep inelastic scattering

$$e + p \rightarrow X + e$$



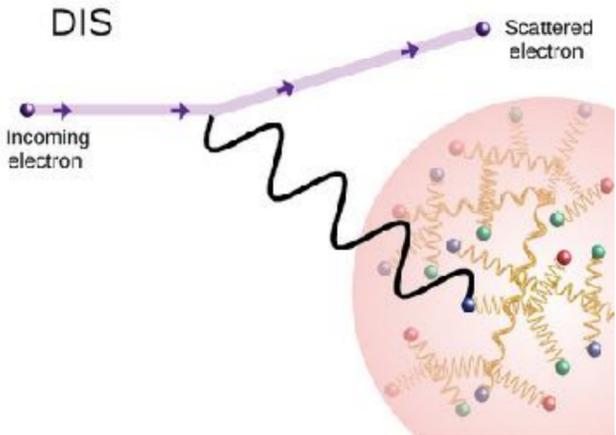
$$e^- + e^+ \rightarrow \text{hadrons}$$



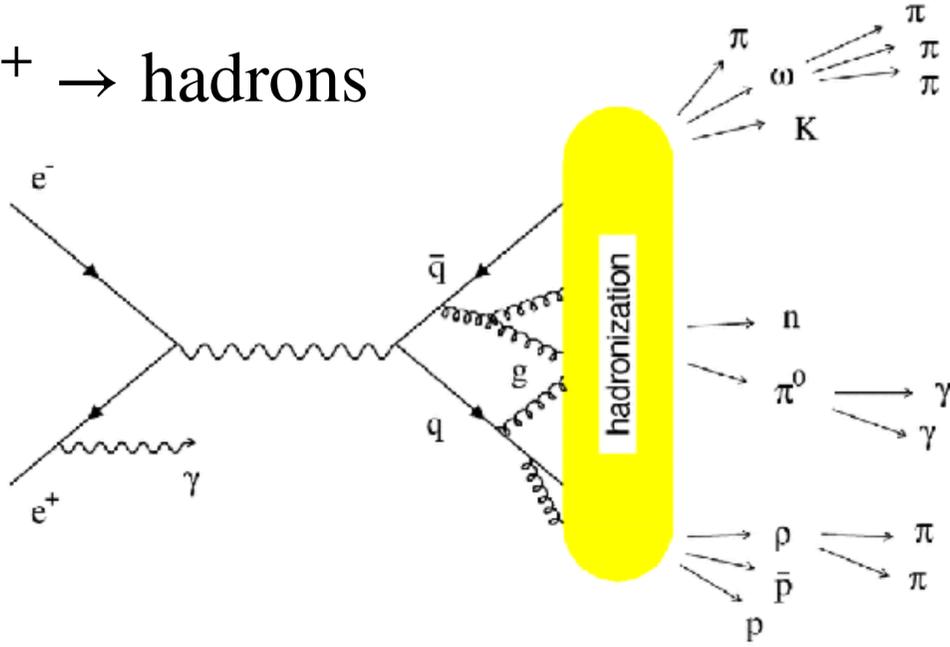
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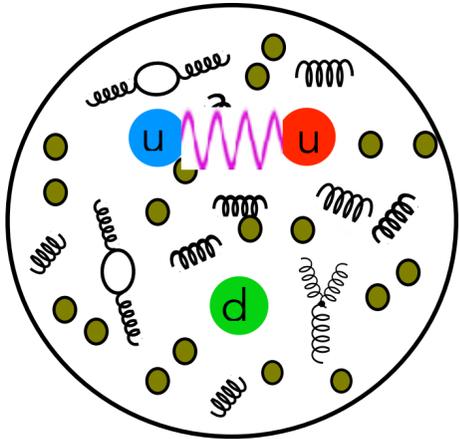
Examples: Deep inelastic scattering
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This means that the quarks inside a hadron can also exchange photons, etc...



⇒ QCD + QED: nonperturbative structure (distribution of electric charges)

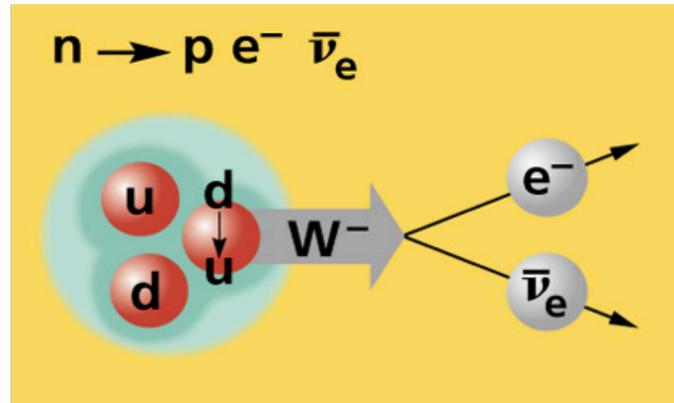
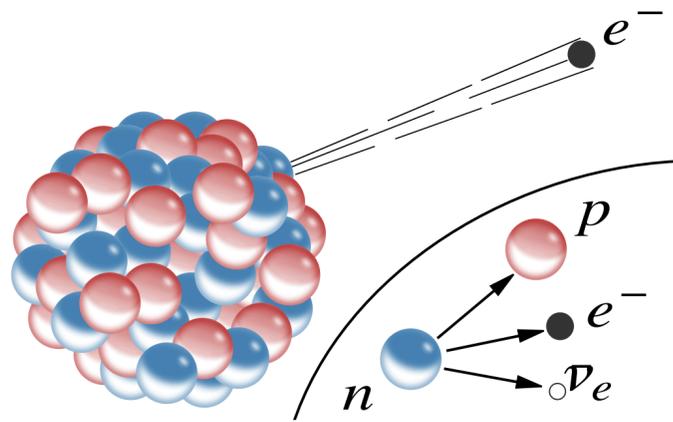
⇒ QED effects are small:
 ⇒ must be included for precision at $< 1\%$

$$\alpha = \frac{1}{137}$$

The Role of QCD in Flavor Physics

Quarks carry weak charge, so interact with W & Z bosons:

Example: Nuclear beta-decay

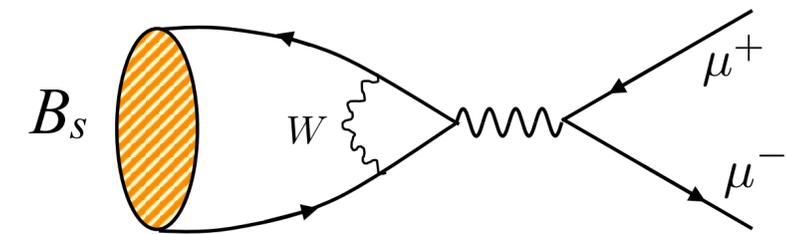
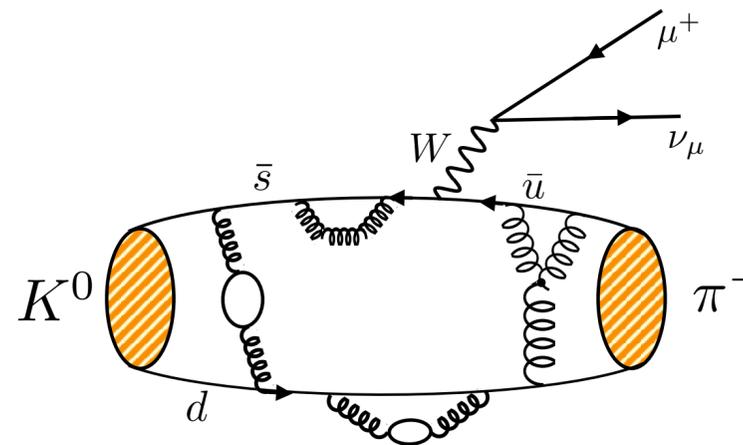


(CPEP particle physics chart)

Many other weak decay processes of hadrons, e.g.

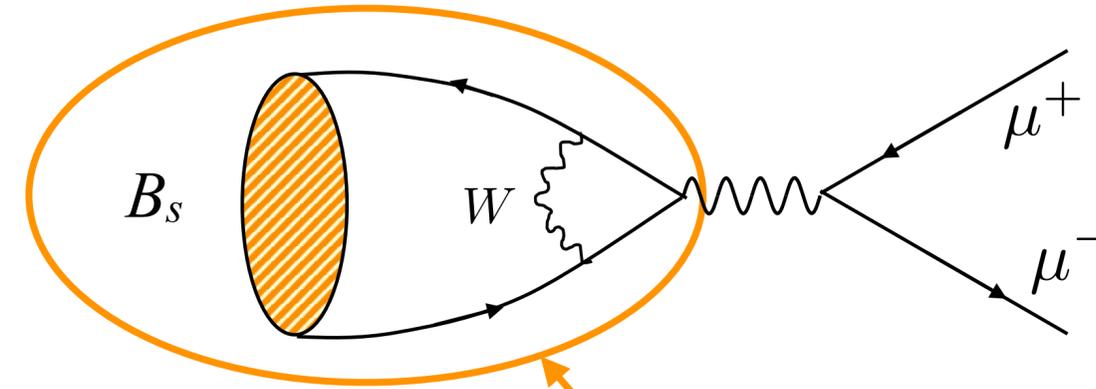
$$\begin{array}{llll}
 K^+ \rightarrow \ell^+ \nu_\ell(\gamma) & K^0 \rightarrow \pi^- \ell^+ \nu_\ell(\gamma) & \Delta M_K, \epsilon_K & \langle \pi\pi_{(I=2)} | \mathcal{H}^{\Delta S=1} | K^0 \rangle \\
 D^0 \rightarrow \pi^- \mu^+ \nu_\mu & D^0 \rightarrow K^- \mu^+ \nu_\mu & \Delta m_{d(s)} & \langle \pi\pi_{(I=0)} | \mathcal{H}^{\Delta S=1} | K^0 \rangle \\
 B_s \rightarrow \mu\mu & B^0 \rightarrow D^{*-} \mu^+ \nu_\mu & D_s \rightarrow \ell\nu\gamma & \Lambda_b \rightarrow (p, \Lambda_c, \Lambda) \ell\nu
 \end{array}$$

...



The role of (lattice) QCD in flavor physics

example: $B_s \rightarrow \mu^+ \mu^-$



Experiment vs. SM theory:

$$(\text{experiment}) = (\text{known}) \times (\text{CKM factors}) \times (\text{had. matrix element})$$

$$\Gamma(K^+ \rightarrow \ell^+ \nu_\ell (\gamma))$$

$$d\Gamma(B^0 \rightarrow D^{*-} \mu^+ \nu_\mu), \dots$$

$$B(B_s \rightarrow \mu\mu), \dots$$

$$\Delta m_{d(s)} \dots$$

Two main purposes:

- ◆ combine experimental measurements with LQCD results to determine SM parameters.
- ◆ confront experimental measurements with SM theory using LQCD inputs.

Lattice QCD

parameterize the MEs in terms of form factors, decay constants, bag parameters, ...

Outline

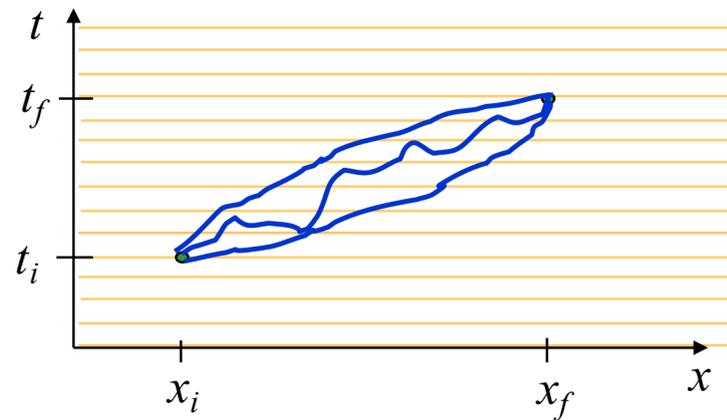
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Lattice Quantum Field Theory

Feynman's Path Integral in Quantum Mechanics

$$\langle x_f | e^{-iH(t_f-t_i)} | x_i \rangle = \int \mathcal{D}x(t) e^{iS}$$

$$x_k = x(t_k), \quad x_0 = x_i, \quad x_N = x_f \quad \int \mathcal{D}x(t) = \lim_{N \rightarrow \infty} \prod_{k=0}^N \int dx_k$$



For efficient numerical computation:
use Euclidean time $t \rightarrow it$ so that $e^{iS} \rightarrow e^{-S}$

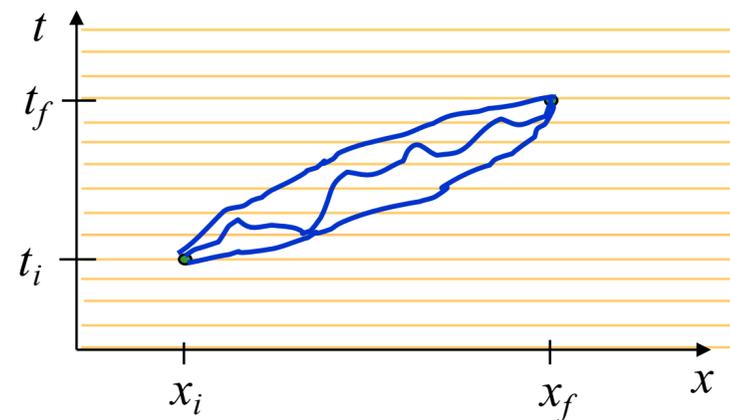
[see also Creutz & Freedman, [Annals Phys. 132 \(1981\) 427](#);
G.P. Lepage, [hep-lat/0506036](#)]

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Integrals are evaluated numerically using Monte Carlo methods.

For efficient numerical computation:
use Euclidean time $t \rightarrow it$ so that $e^{iS} \rightarrow e^{-S}$

[see also Creutz & Freedman, Annals Phys. 132 (1981) 427;
G.P. Lepage, hep-lat/0506036]

QM

$x(t)$

time lattice



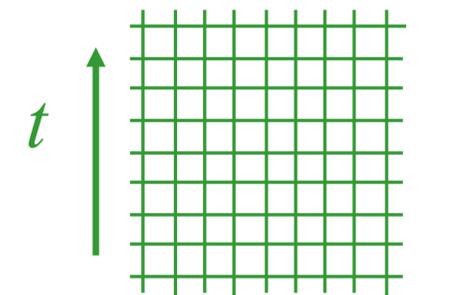
$$\int \mathcal{D}x(t)$$

N dimensions

QFT

$\phi(x)$

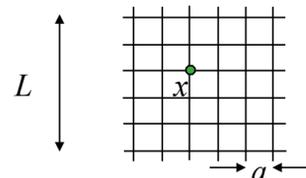
space-time lattice



$$\int \mathcal{D}\phi(x)$$

N^4 dimensions

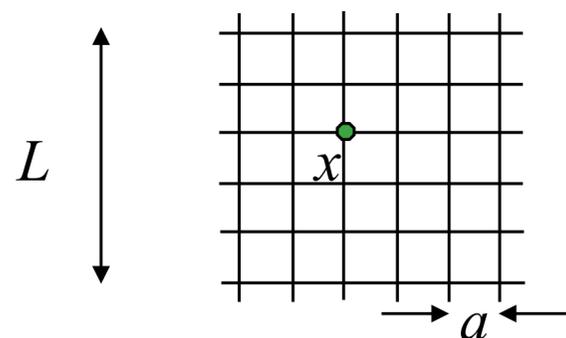
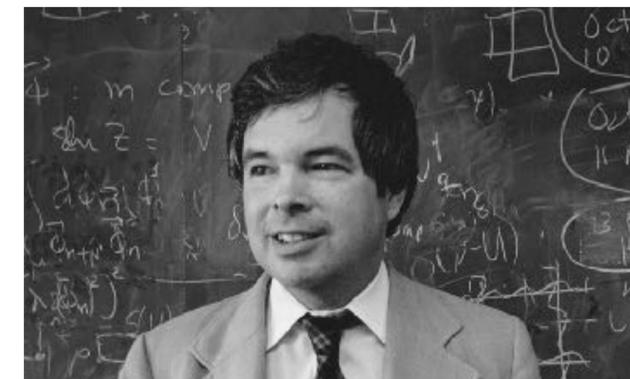
$$32^4 \rightarrow 144^4$$



Lattice QCD Introduction

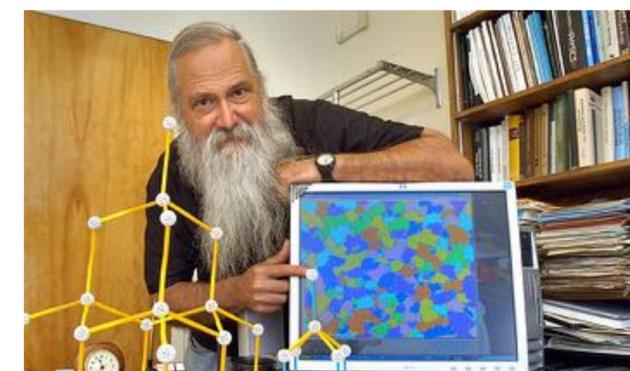
$$\mathcal{L}_{\text{QCD}} = \sum_f \bar{\psi}_f (\not{D} + m_f) \psi_f + \frac{1}{4} \text{tr} F_{\mu\nu} F^{\mu\nu}$$

Ken Wilson



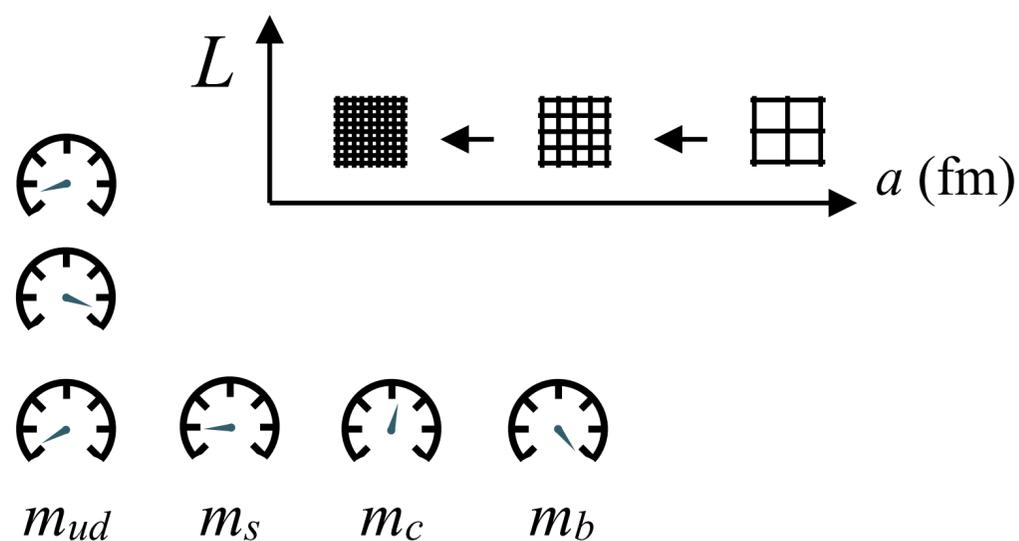
- ◆ discrete Euclidean space-time (spacing a)
derivatives \rightarrow difference operators, etc...
- ◆ finite spatial volume (L)
- ◆ finite time extent (T)

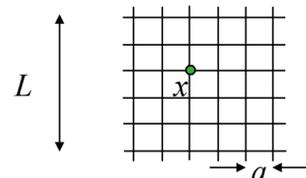
Mike Creutz



adjustable parameters

- ❖ lattice spacing: $a \rightarrow 0$
- ❖ finite volume, time: $L \rightarrow \infty, T > L$
- ❖ quark masses (m_f): $M_{H,\text{lat}} = M_{H,\text{exp}}$
tune using hadron masses
extrapolations/interpolations
 $m_f \rightarrow m_{f,\text{phys}}$



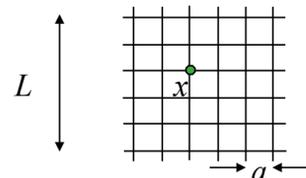


Lattice QCD Introduction

systematic error analysis

...of lattice spacing, chiral, heavy quark, and finite volume effects is based on [Effective Field Theory \(EFT\)](#) descriptions of QCD → *ab initio*

- finite a :
[Symanzik EFT](#) ← Asymptotic Freedom & Renormalizability
- light quark masses:
[Chiral Perturbation Theory](#) ← Chiral Symmetry & Spontaneous Symmetry Breaking
- heavy quarks:
[HQET](#) ← Heavy Quark symmetry
- finite L :
[finite volume EFT](#) ← Confinement & S-matrix



Lattice QCD Introduction

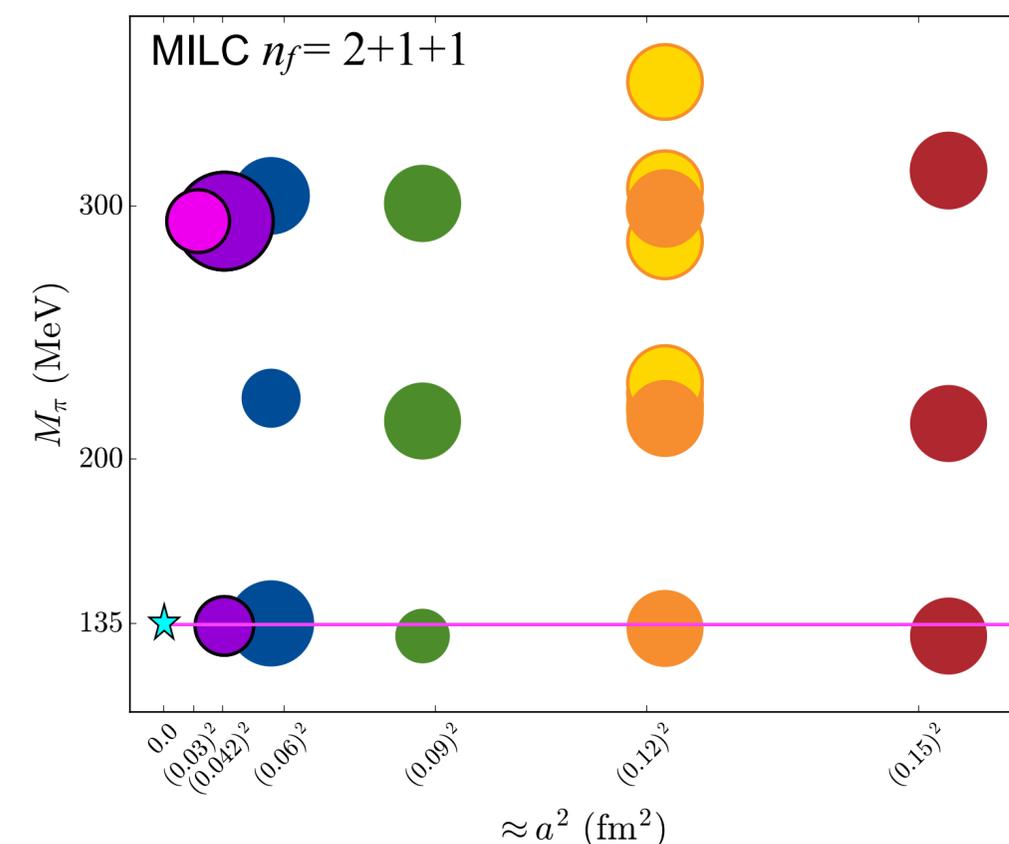
systematic error analysis

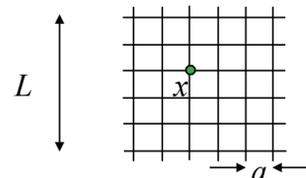
...of lattice spacing, chiral, heavy quark, and finite volume effects is based on **Effective Field Theory (EFT)** descriptions of QCD → **ab initio**

- finite a : **Symanzik EFT**
- light quark masses: **Chiral Perturbation Theory**
- heavy quarks: **HQET**
- finite L : **finite volume EFT**

In practice:

stability and control over systematic errors depends on the lattice action(s) employed, underlying simulation parameters (available computational resources), analysis choices, ...





Lattice QCD Introduction

Steps of a lattice QCD calculation:

$$\langle O \rangle \sim \int \mathcal{D}U \det(\mathcal{D} + m) O(U) e^{-S}$$

📍 generate gluon field configurations according to $\det(\mathcal{D} + m) e^{-S}$

➡ **statistical errors**

📍 Compute correlation functions (usually 2, 3, 4-pt functions),

e.g.

$$C_\pi(t) = \sum_{\mathbf{x}} \langle \pi(\mathbf{x}, t) \pi^\dagger(0, 0) \rangle \quad \pi(x) = \bar{\psi}(x) \gamma^5 \psi(x)$$

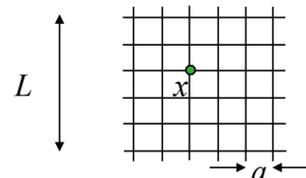
📍 Extract observables, such as hadron masses, energies, hadronic matrix elements, from correlation functions, e.g. via spectral decomposition

$$C_\pi(t) = \sum_n |\langle \pi | n \rangle|^2 \left(e^{-E_n t} + e^{-E_n (T-t)} \right) \quad \Rightarrow M_\pi, f_\pi, \dots$$

📍 Repeat for all gauge field ensembles, parameters (m_f, a, L, \dots)

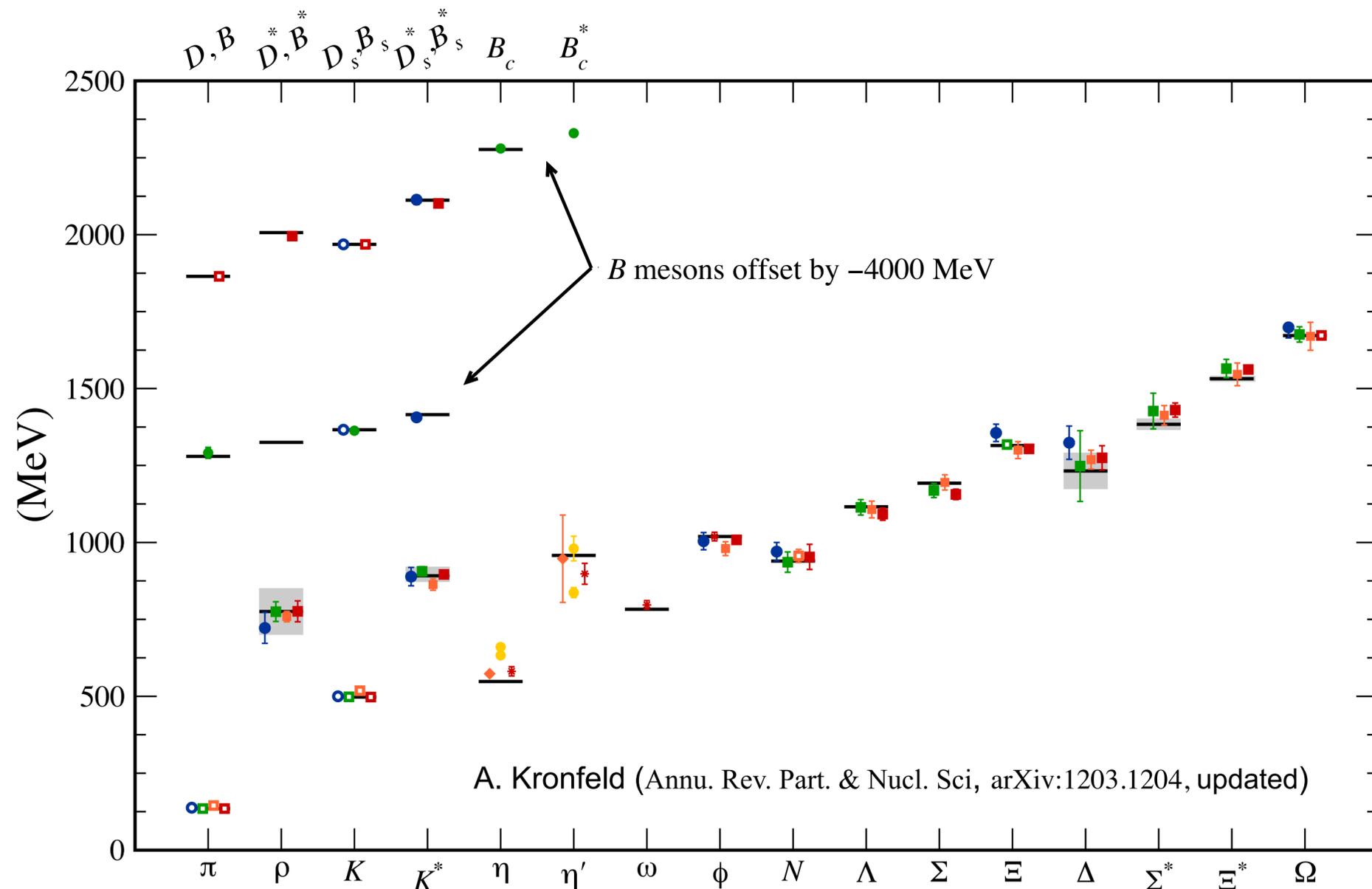
➡ extrapolate/interpolate to physical point in continuum and infinite volume

➡ **systematic errors**



(Lattice) QCD inputs

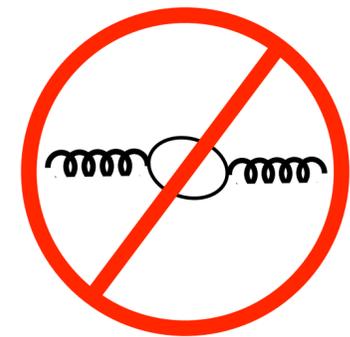
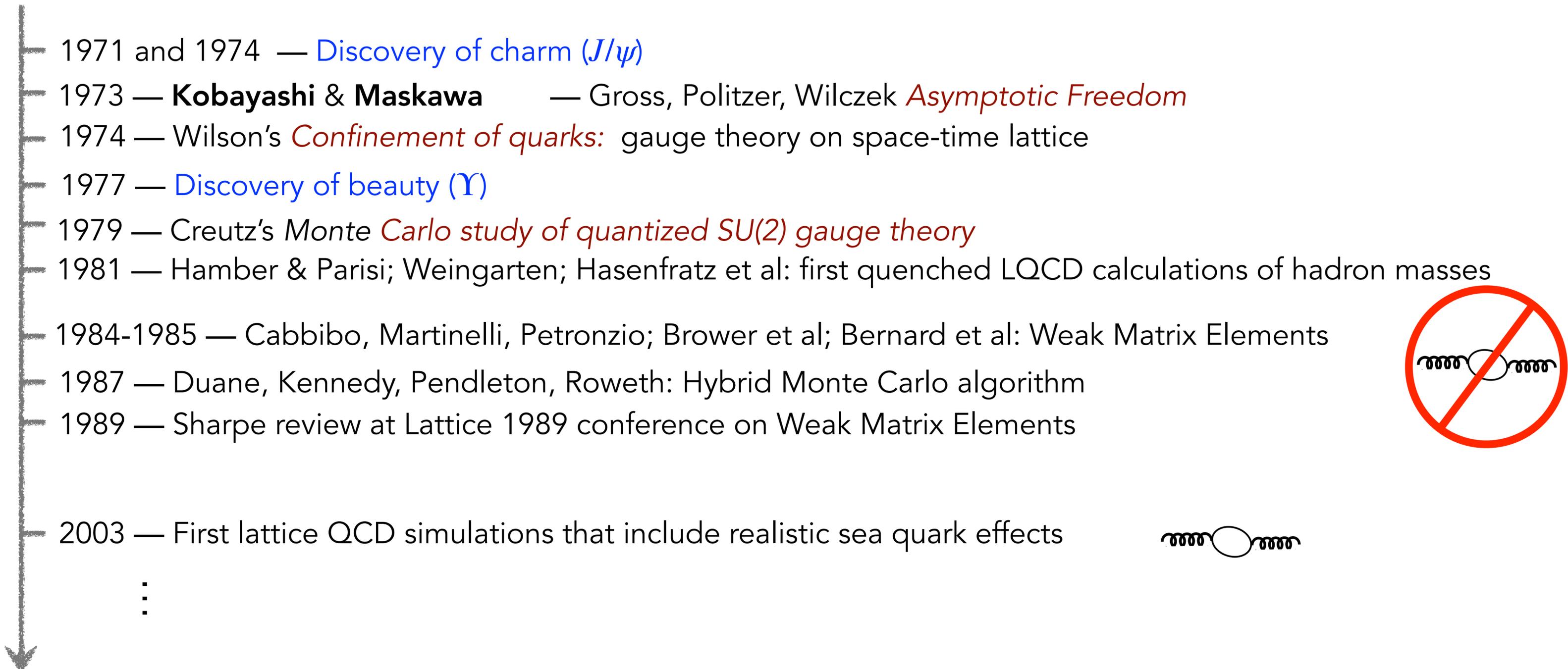
- free parameters in (lattice) QCD Lagrangian require experimental inputs
- **bare** quark masses, m_{ud}, m_s, m_c, m_b : fixed with exp. measured hadron masses, e.g., $M_\pi, M_K, M_{D_s}, M_{B_s}$
- lattice spacing in physical units (scale setting): f_π (or $f_K M_\Omega$ or ...) $\Rightarrow \alpha_s$



- all other quantities are pre/post dictions that can be compared to experiment.

- quark masses and couplings, m_f, α_s :
 \Rightarrow inputs to perturbative QCD calculations

a (selective) view of the history: LQCD for Flavor



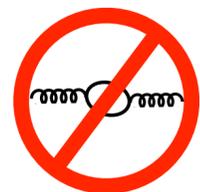
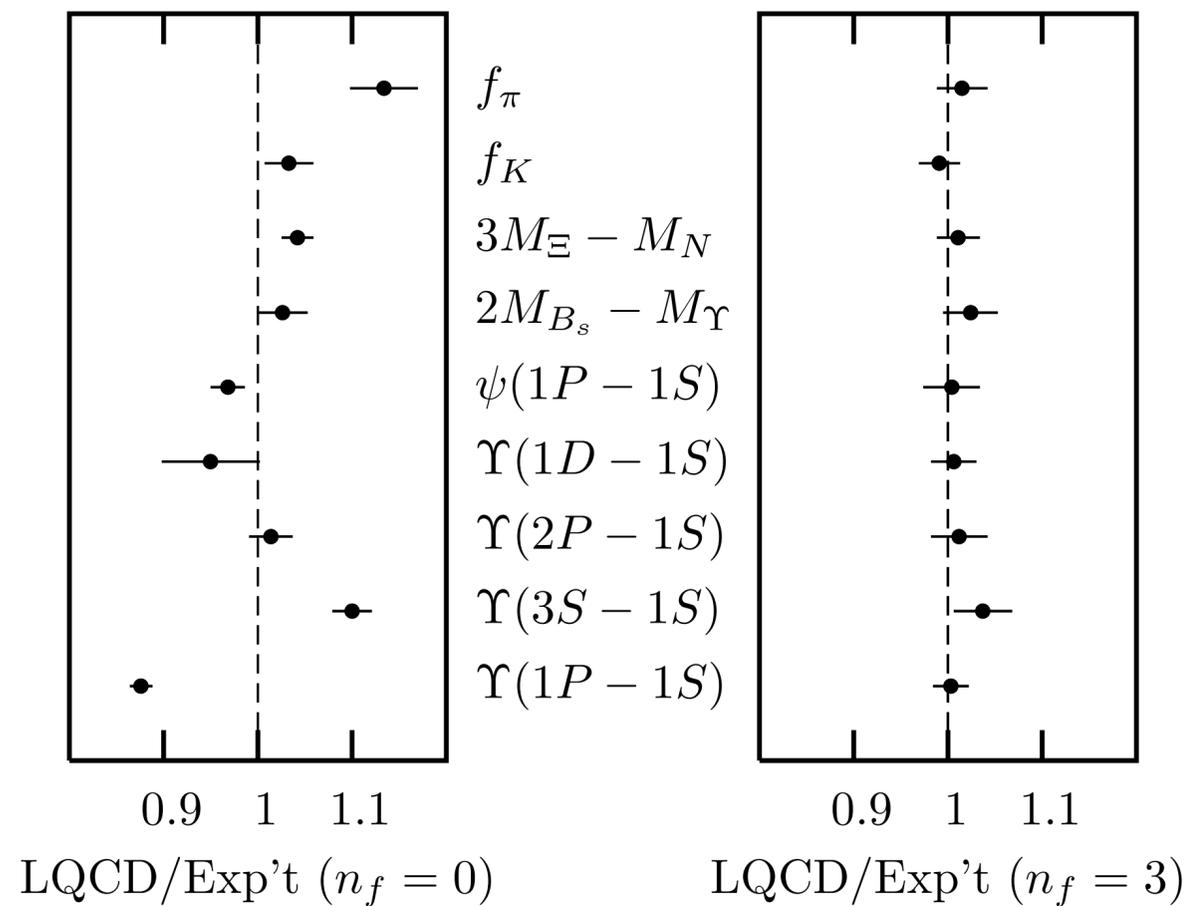
2003-2005: first "realistic" lattice QCD results

based on simulations with three flavors of sea quarks ($n_f = 2 + 1$):

C. Davies et al [HPQCD, MILC, Fermilab Lattice,
hep-lat/0304004, 2004 PRL]

Quenched QCD

full QCD



2003-2005: first "realistic" lattice QCD results

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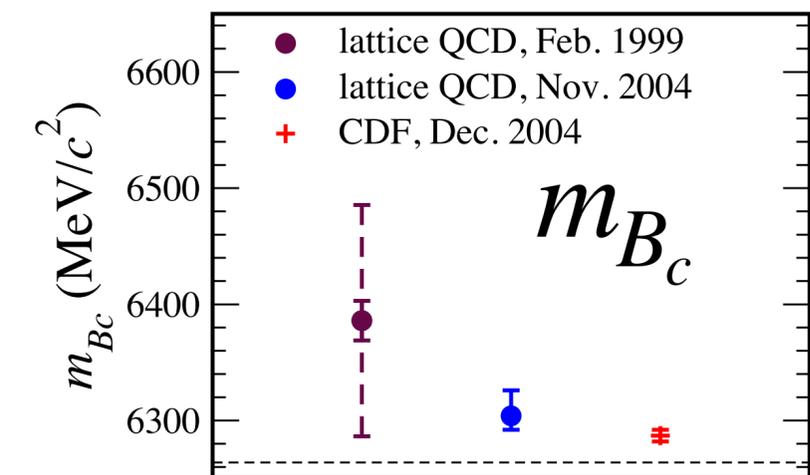
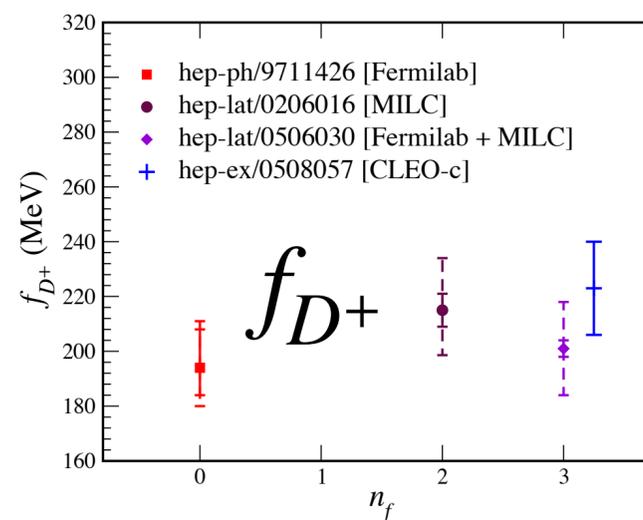
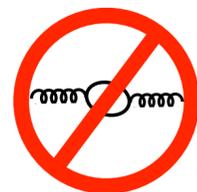
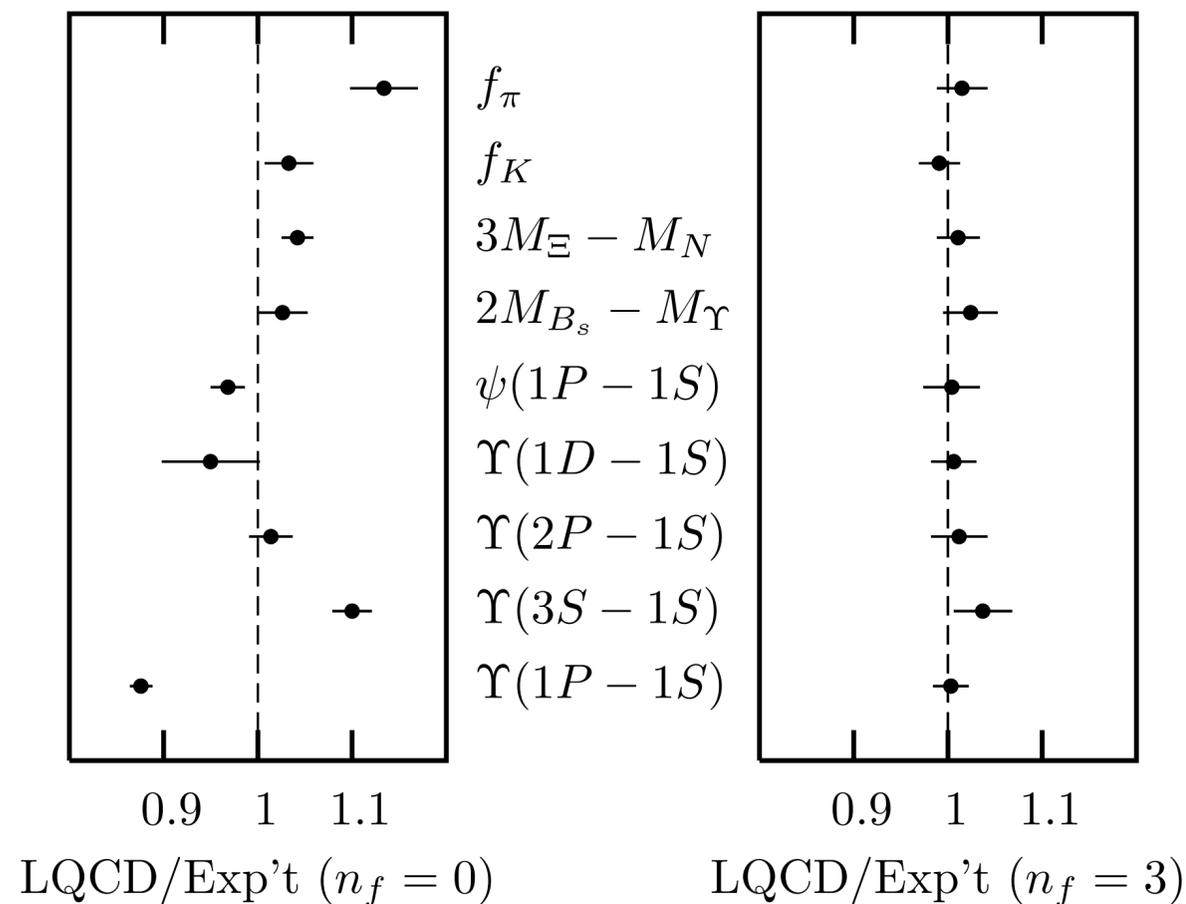
C. Davies et al [HPQCD, MILC, Fermilab Lattice, hep-lat/0304004, 2004 PRL]

A. Kronfeld et al [Fermilab Lattice, MILC, HPQCD, hep-lat/0509169, Int.J.Mod.Phys 2006]

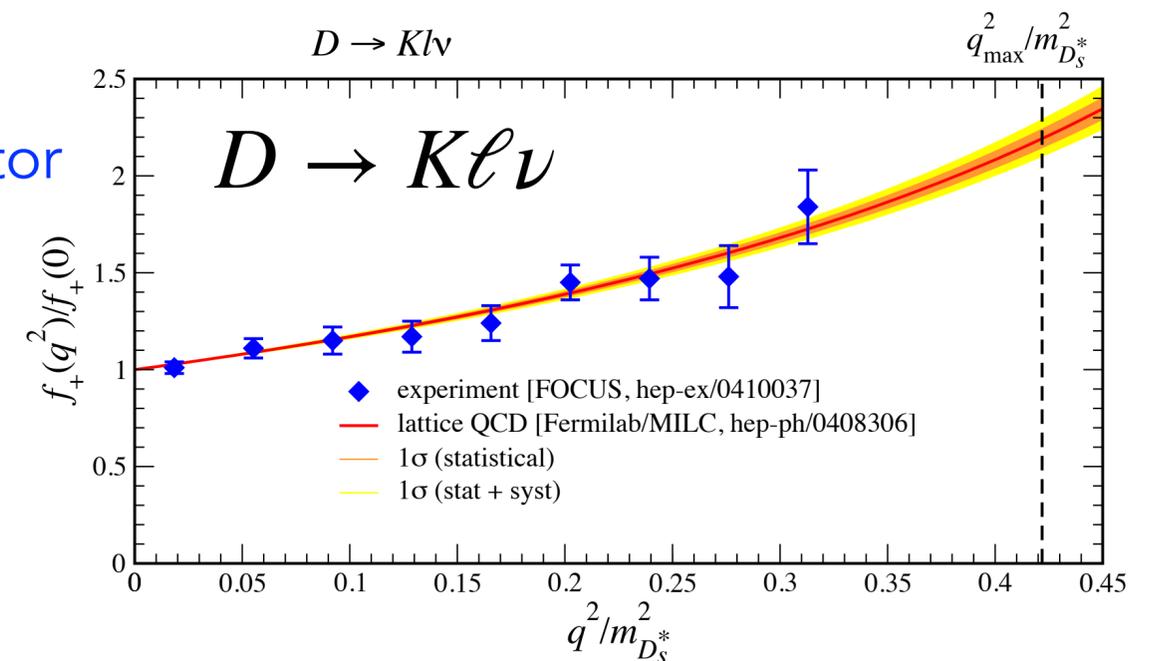
Quenched QCD

full QCD

First lattice QCD *predictions*, confirmed by experiment:



shape of form factor

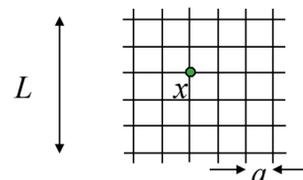


Snowmass 2013 present

<https://www.usqcd.org/documents/13flavor.pdf> and [J. Butler et al, arXiv:1311.1076]

Quantity	CKM element	2013 expt. error	2007 forecast lattice error	2013 lattice error	2018 forecast lattice error	2021 FLAG Average
f_K/f_π	$ V_{us} $	0.2%	0.5%	0.4%	0.15%	0.18 %
$f_+^{K\pi}(0)$	$ V_{us} $	0.2%	–	0.4%	0.2%	0.18 %
f_D	$ V_{cd} $	4.3%	5%	2%	< 1%	0.3 %
f_{D_s}	$ V_{cs} $	2.1%	5%	2%	< 1%	0.2 %
$D \rightarrow \pi \ell \nu$	$ V_{cd} $	2.6%	–	4.4%	2%	0.7 % [from 2212.12648]
$D \rightarrow K \ell \nu$	$ V_{cs} $	1.1%	–	2.5%	1%	0.6 %
$B \rightarrow D^* \ell \nu$	$ V_{cb} $	1.3%	–	1.8%	< 1%	~1.5 % [from 2105.14019, 2304.03137, 2306.05657]
$B \rightarrow \pi \ell \nu$	$ V_{ub} $	4.1%	–	8.7%	2%	~3 %
f_B	$ V_{ub} $	9%	–	2.5%	< 1%	0.7 % (0.6 % for f_{B_s})
ξ	$ V_{ts}/V_{td} $	0.4%	2–4%	4%	< 1%	1.3 %
Δm_s	$ V_{ts}V_{tb} ^2$	0.24%	7–12%	11%	5%	4.5 %
B_K	$\text{Im}(V_{td}^2)$	0.5%	3.5–6%	1.3%	< 1%	1.3 %

QED corrections dominant source of theory error



Lattice QCD Introduction

The State of the Art

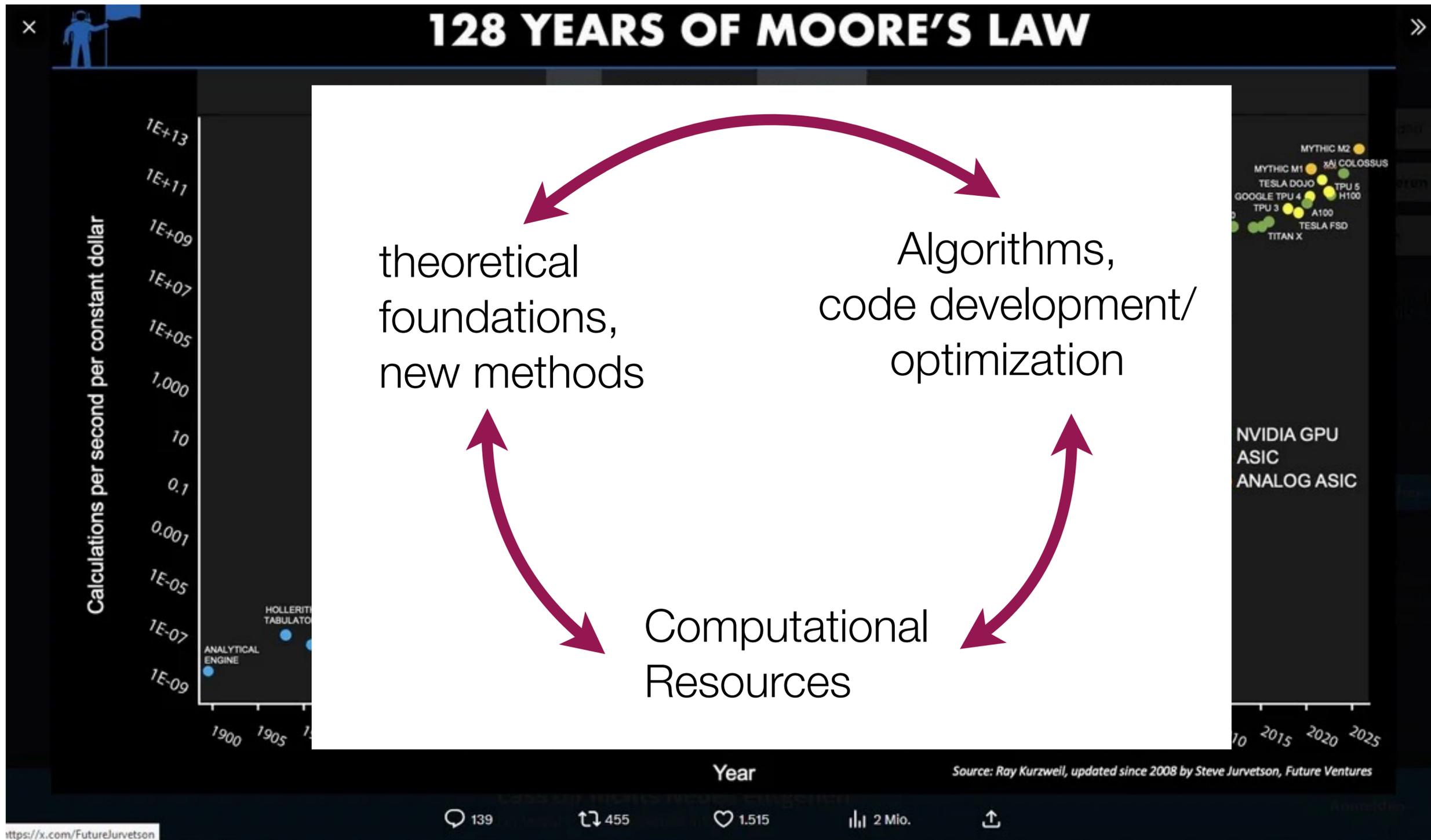
Lattice QCD calculations of simple quantities (with at most one stable meson in initial/final state) that **quantitatively account for all systematic effects** (discretization, finite volume, renormalization,...) in some cases with

- sub percent precision.
- total errors that are commensurate (or smaller) than corresponding experimental uncertainties.

Scope of LQCD calculations is increasing due to continual development of new methods:

- nucleon matrix elements
- nonleptonic kaon decays ($K \rightarrow \pi\pi, \epsilon', \dots$)
- resonances, scattering ($\pi\pi \rightarrow \rho, \dots$)
- long-distance effects ($\Delta M_K, \dots$)
- QED corrections
- radiative decay rates
- structure: PDFs, GPDs, TMDs, ...
- inclusive decay rates ($B \rightarrow X_c \ell \nu, \dots$)
- ...

Moore's Law



Frontier 2022 (US)



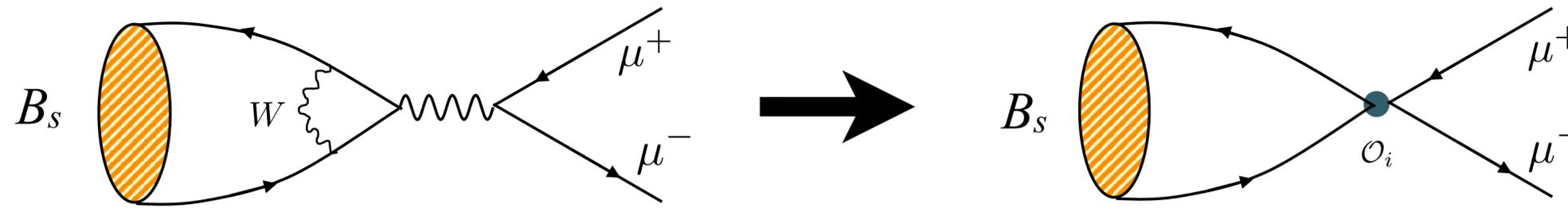
~37k AMD GPUs

Outline

- The Standard Model of Particle Physics
 - open questions
- Introduction to QCD
- Introduction to Lattice QCD
- Two examples
 - rare B decay
 - muon $g-2$
- Conclusions and Outlook



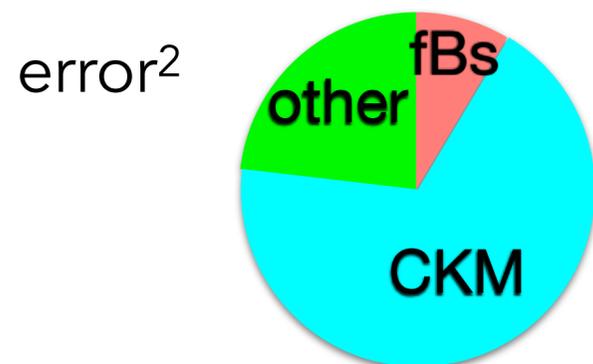
Rare leptonic decay $B_s \rightarrow \mu^+ \mu^-$



Standard Model predictions: Buras, et al [arXiv:1303.3820, JHEP 2013], Bobeth, et al [arXiv:1311.0903, PRL 2014; arXiv:2104.09521], Beneke et al [arXiv:1908.07011, JHEP 2019].

$$\bar{\mathcal{B}}(B_q \rightarrow \mu\bar{\mu}) = \frac{|\mathcal{N}_q|^2 M_{B_q}^3 f_{B_q}^2}{8\pi \Gamma_q^H} \beta_{q\mu} \left(\frac{m_\mu}{M_{B_q}}\right)^2 |C_{10}^{\text{eff}}|^2, \quad \beta_{q\mu} \equiv \sqrt{1 - \frac{4m_\mu^2}{M_{B_q}^2}}$$

$$\bar{\mathcal{B}}(B_s \rightarrow \mu\bar{\mu})_{\text{SM}} = (3.66 \pm 0.14) \cdot 10^{-9}$$

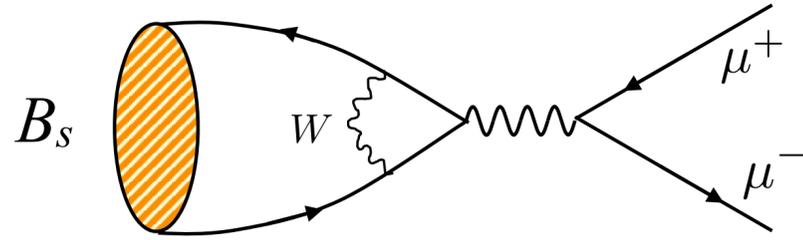


- includes structure-dependent QED corrections
- dominant uncertainty due to $|V_{cb}|$
- LQCD decay constant sub dominant source of uncertainty



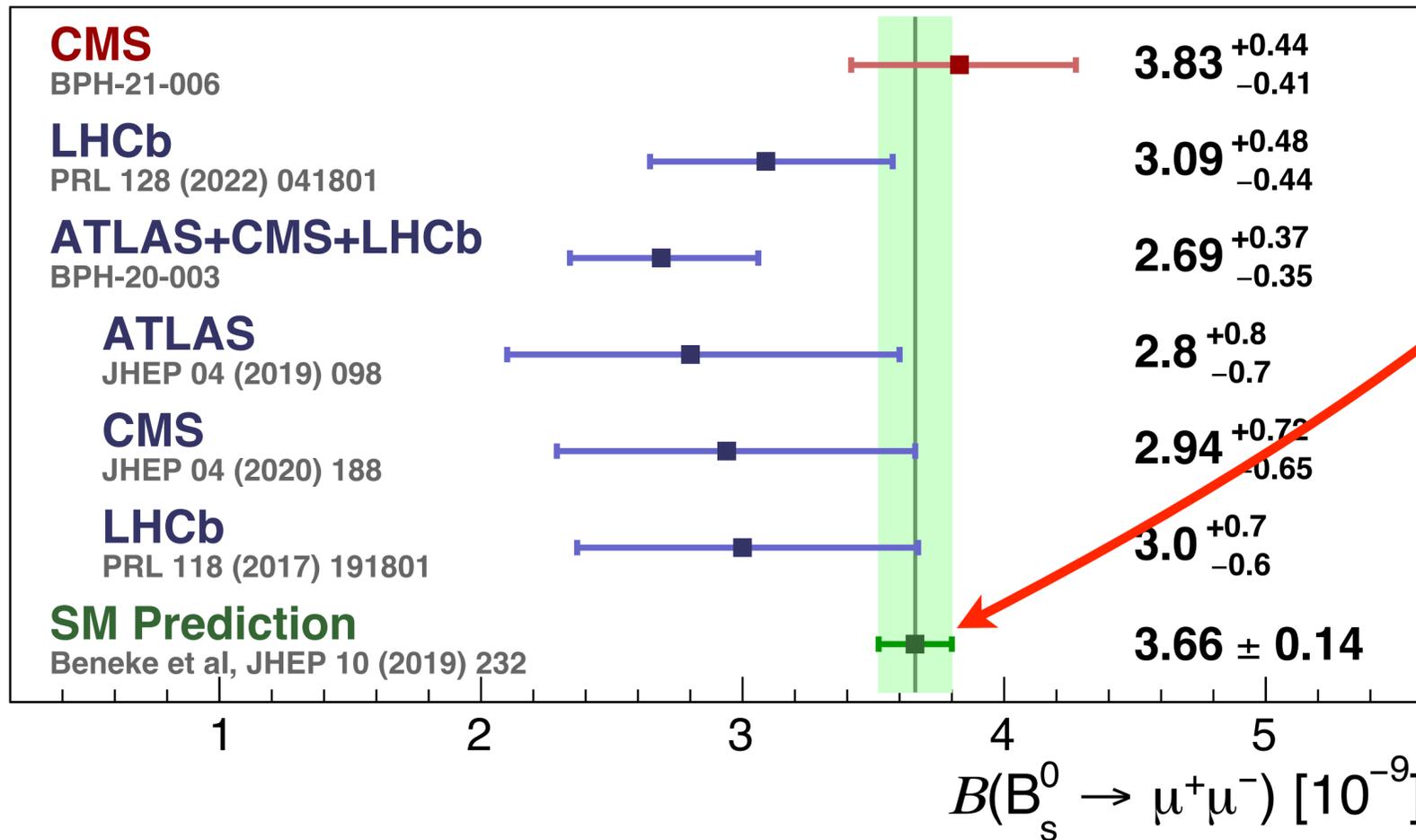
Rare leptonic decay

$$B_s \rightarrow \mu^+ \mu^-$$



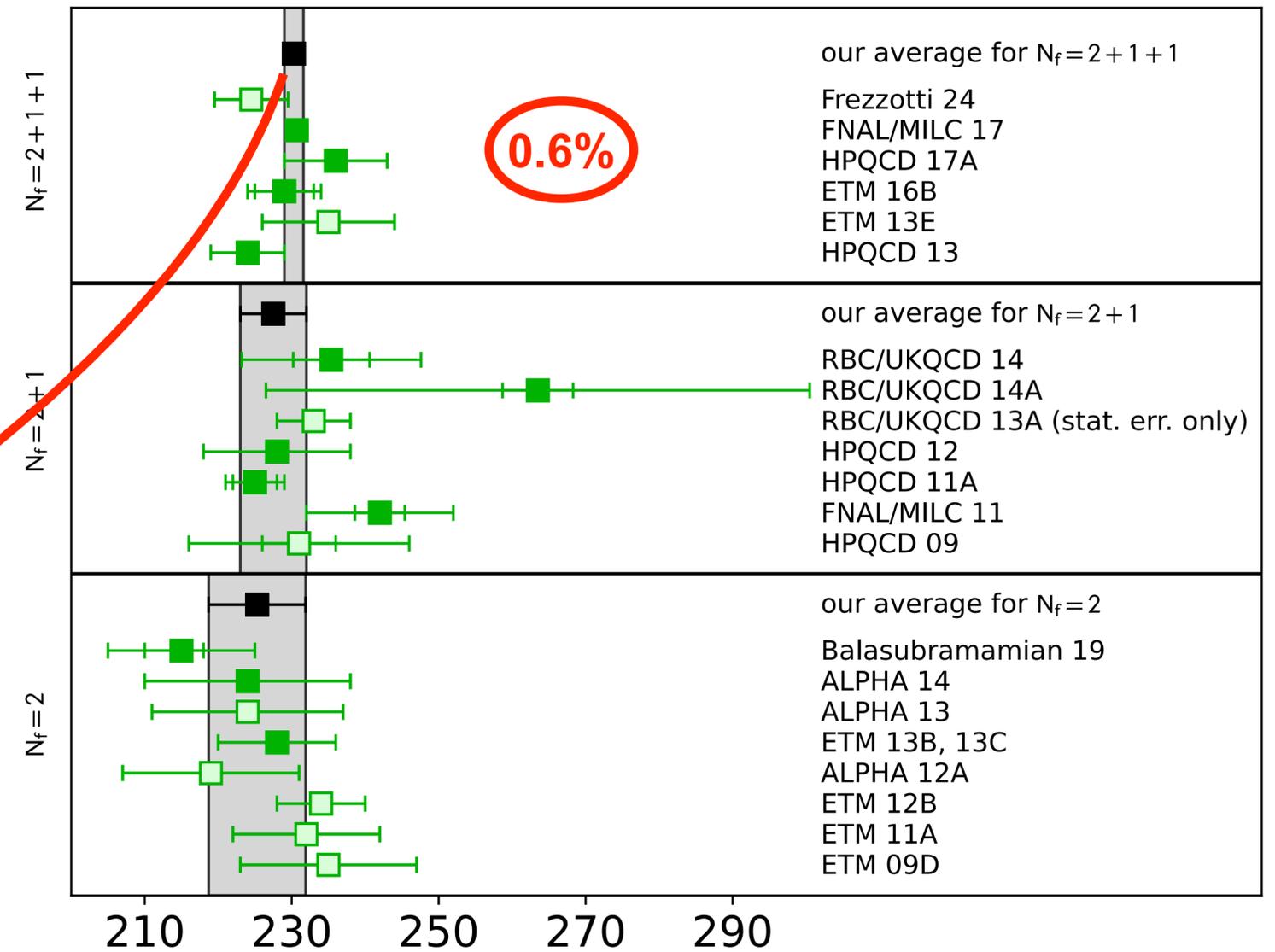
S. Aoki et al [FLAG 2024 review, arXiv:2411.04268]

[CMS, Phys Lett B 2023]



FLAG2024

f_{B_s} [MeV]



Outline

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lepton magnetic moment

The magnetic moment of charged leptons (e, μ, τ): $\vec{\mu} = g \frac{e}{2m} \vec{S}$

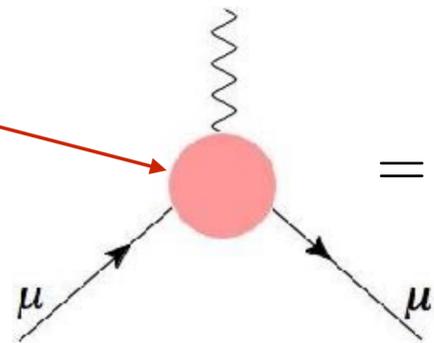
Dirac (leading order): $g = 2$

$$= (-ie) \bar{u}(p') \gamma^\mu u(p)$$

Quantum effects (loops):

$$\Rightarrow g = 2 \left(1 + \frac{\alpha}{2\pi} \right)$$

All SM particles contribute



$$= (-ie) \bar{u}(p') \left[\gamma^\mu F_1(q^2) + \frac{i\sigma^{\mu\nu} q_\nu}{2m} F_2(q^2) \right] u(p)$$

Note: $F_1(0) = 1$ and $g = 2 + 2 F_2(0)$

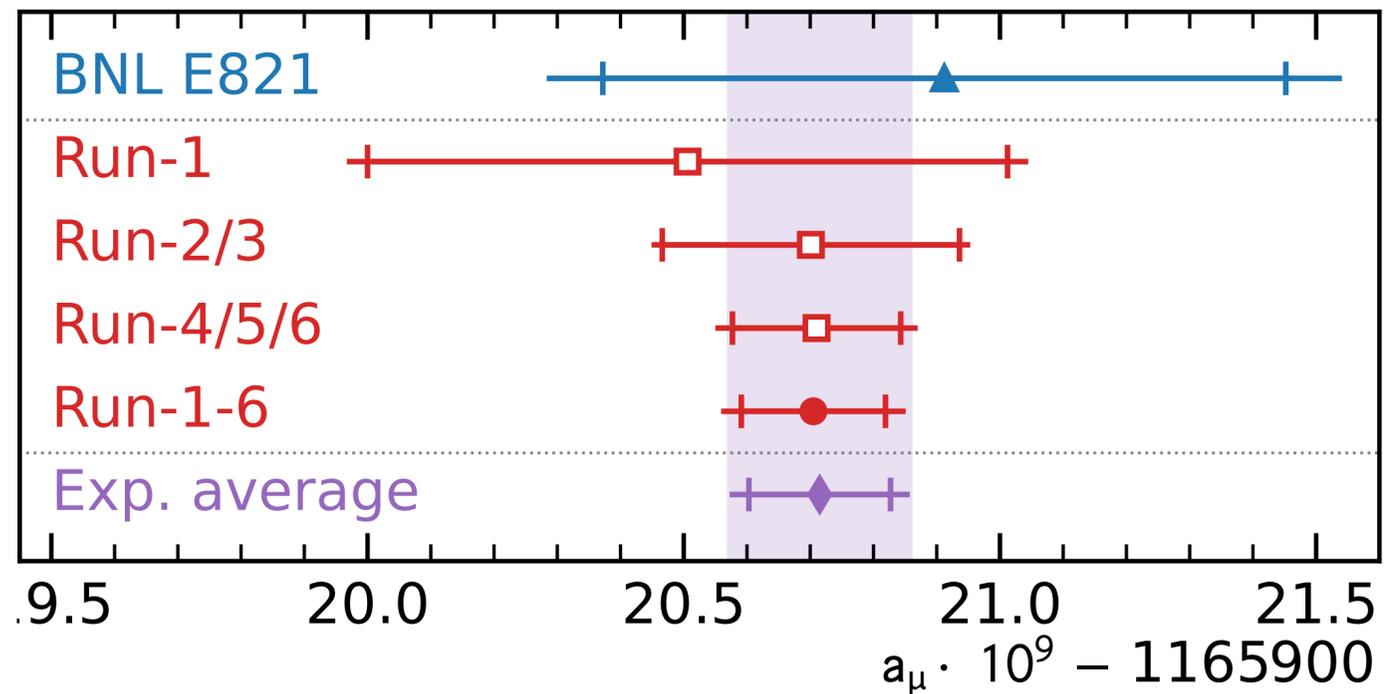
Anomalous magnetic moment:

$$a \equiv \frac{g - 2}{2} = F_2(0) = \frac{\alpha}{2\pi} + O(\alpha^2) + \dots = 0.00116\dots$$

Motivation: Fermilab muon g-2 experiment

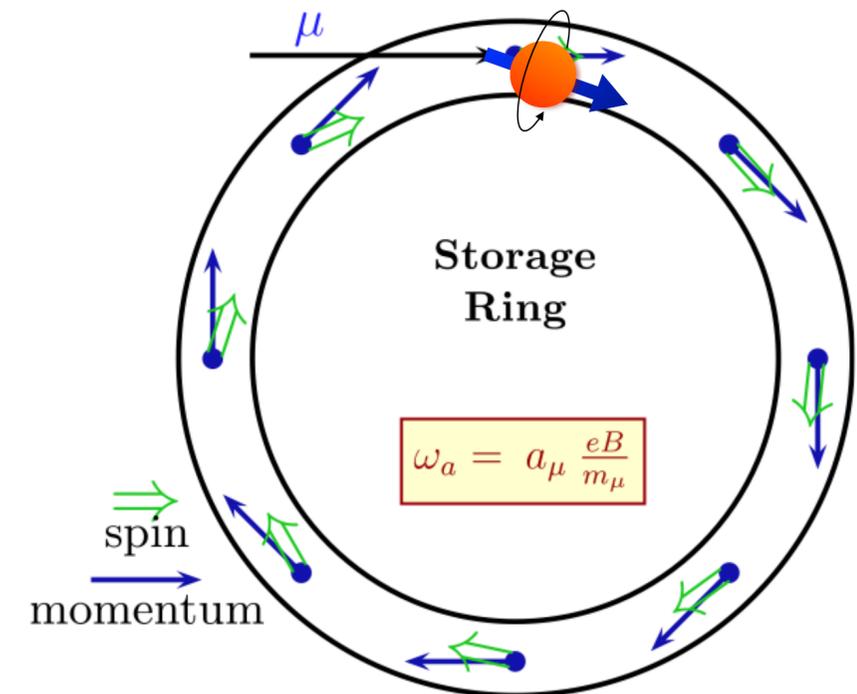
- The Fermilab experiment has released their final measurement results:
 - 07 Apr 2021: run 1 data [B. Abi et al, [Phys. Rev. Lett. 124, 141801 \(2021\)](#)]
 - 10 Aug 2023: run 2/3 data [D. Aguillard et al, [2308.06230](#)]
 - 03 Jun 2025: run 4/5/6 data [D. Aguillard et al, [2506.03069](#)]

From [Simon Corrodi @ Scientific Seminar, 03 Jun 2025](#)



$$a_\mu^{\text{exp}} = 1165920715 (145) \times 10^{-11}$$

124 parts per billion!



Muon g-2: SM contributions

$$a_\mu = a_\mu(\text{QED}) + a_\mu(\text{EW}) + a_\mu(\text{hadronic})$$

QED

$$a_\mu(\text{QED}) = A_1 + A_2 \left(\frac{m_\mu}{m_e}\right) + A_2 \left(\frac{m_\mu}{m_\tau}\right) + A_3 \left(\frac{m_\mu}{m_e}, \frac{m_\mu}{m_\tau}\right)$$

$$A_i = \sum_{n=0} \left(\frac{\alpha}{\pi}\right)^n A_i^{2n}$$

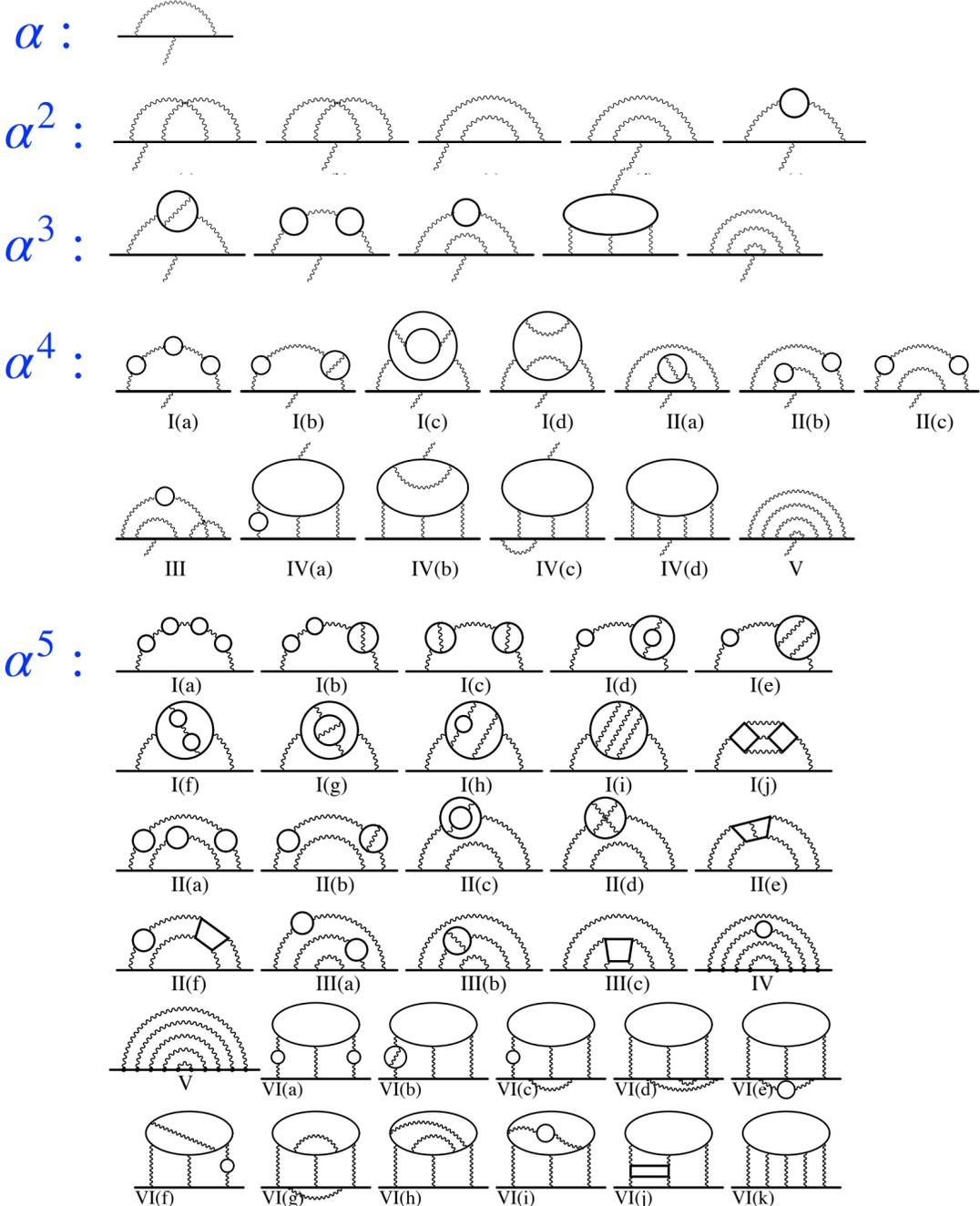
n	# of diagrams	Contribution x 10^{11}
1	1	116140973.32
2	7	413 217.63
3	71	30141.90
4	891	381.00
5	12672	5.08

$$a_\mu(\text{QED}) = 116\,584\,718.8(2) \times 10^{-11}$$

[T. Aoyama et al, arXiv:1205.5370, PRL;
T. Aoyama, T. Kinoshita, M. Nio, *Atoms* 7 (1) (2019) 28]



T. Kinoshita

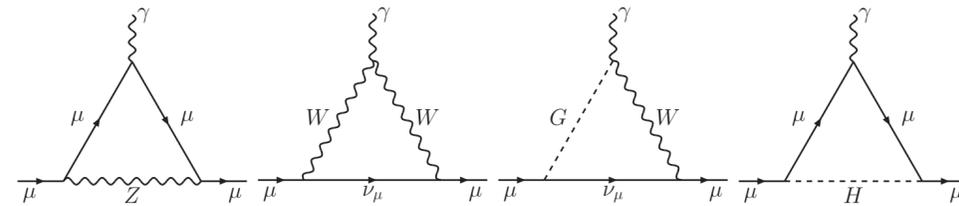


Muon g-2: SM contributions

$$a_\mu = a_\mu(\text{QED}) + a_\mu(\text{EW}) + a_\mu(\text{hadronic})$$

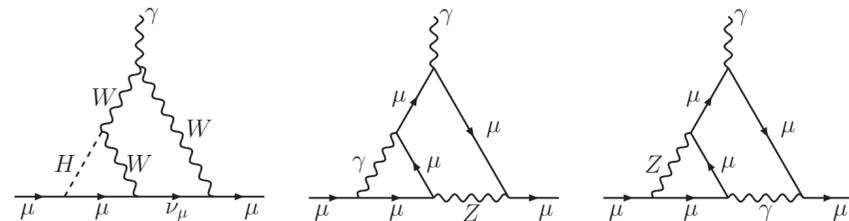
Electroweak
(contributions from W,Z,H bosons)

1-loop



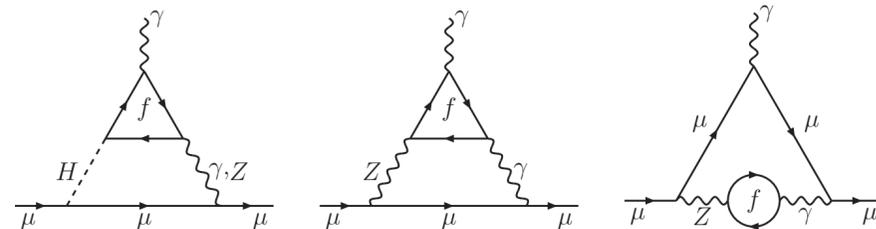
Compared to QED, suppressed by $\sim \frac{m_\mu^2}{M_W^2} \sim 10^{-6}$

2-loop



$$a_\mu(\text{EW}) = 154.4(4) \times 10^{-11}$$

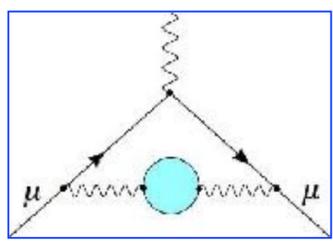
[A. Czarnecki et al, hep-ph/0212229, PRD;
C. Gnendinger et al, arXiv:1306.5546, PRD]

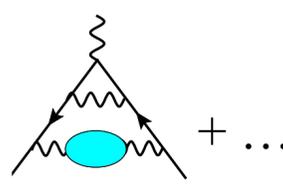


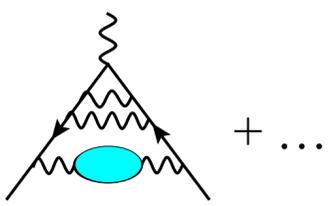
Muon g-2: SM contributions

$$a_\mu = a_\mu(\text{QED}) + a_\mu(\text{EW}) + a_\mu(\text{hadronic})$$

α^2 α^3 α^4



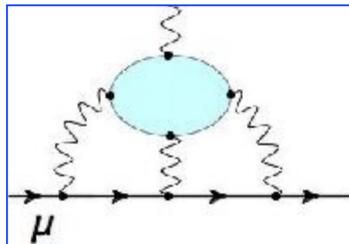


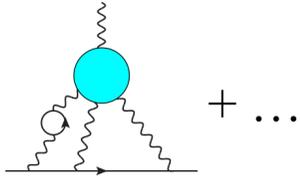


$$a_\mu(\text{hadronic}) = a_\mu^{\text{HVP, LO}} + a_\mu^{\text{HVP, NLO}} + a_\mu^{\text{HVP, NNLO}} + \dots$$

$$+ a_\mu^{\text{HLbL}} + a_\mu^{\text{HLbL, NLO}} + \dots$$

leading hadronic

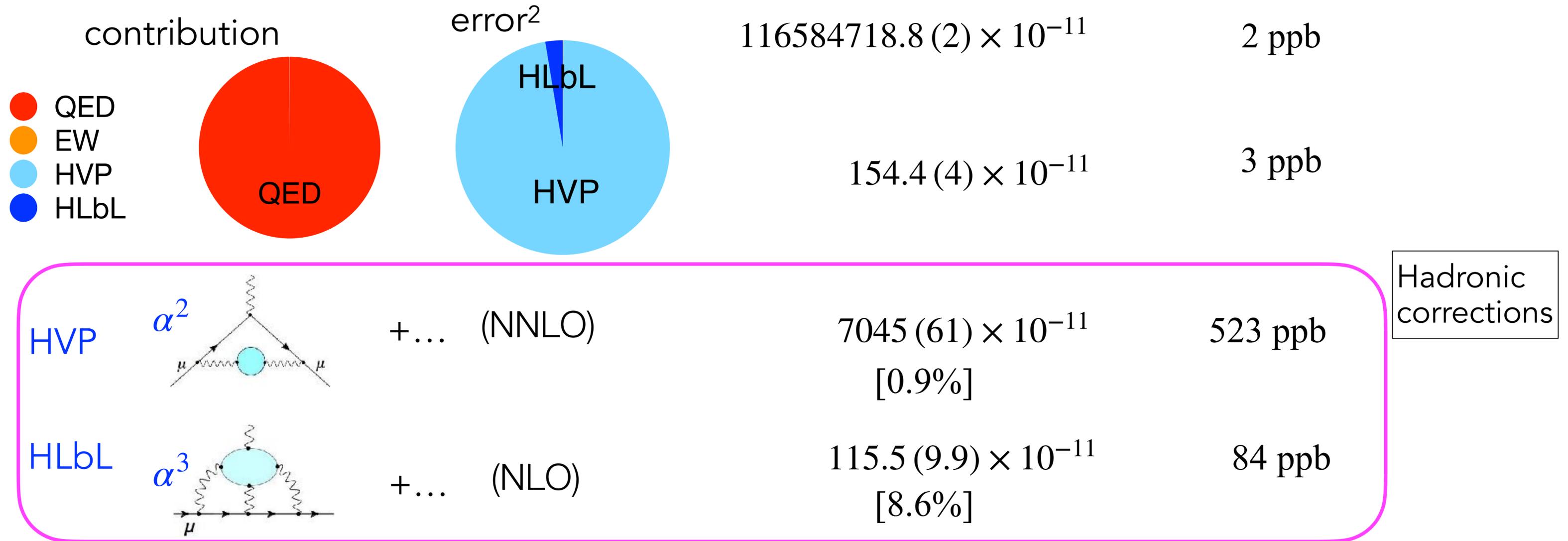




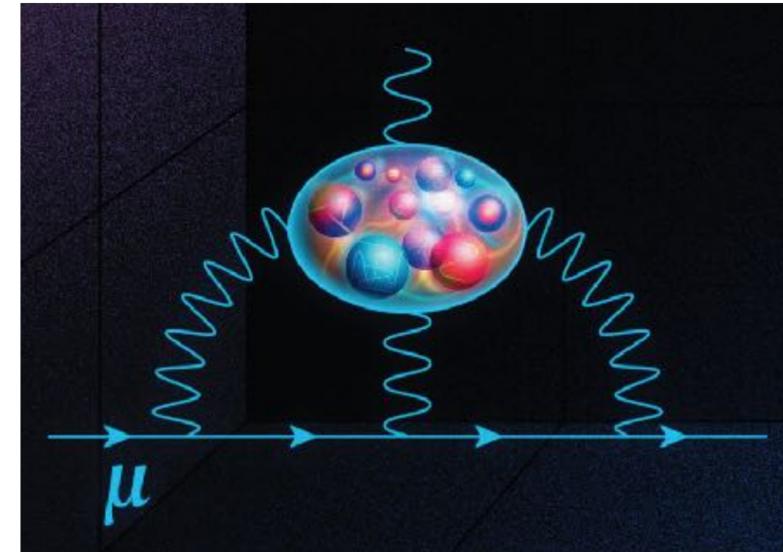
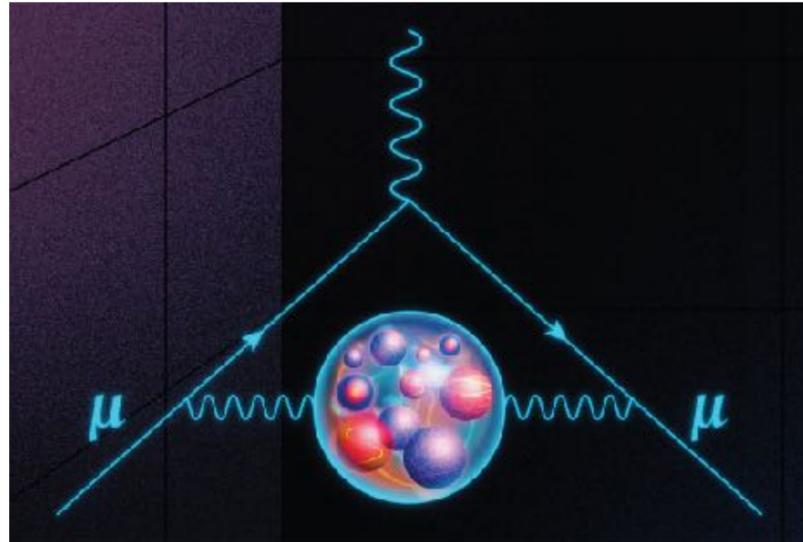
~ 10⁻⁷

Muon g-2: SM contributions

$$a_\mu = a_\mu(\text{QED}) + a_\mu(\text{EW}) + a_\mu(\text{hadronic})$$



Muon g-2: hadronic corrections



credit:
Samantha Koch, Fermilab

- ★ Hadronic contributions are obtained by integrating over all possible virtual photon momenta, integral is weighted towards low q^2 .
- ★ Cannot use perturbation theory to reliably compute the hadronic bubbles
- ★ Two-point & four-point functions:

$$\text{HVP: } \langle 0 | T \{ j_\mu j_\nu \} | 0 \rangle$$

$$\text{HLbL: } \langle 0 | T \{ j_\mu j_\nu j_\rho j_\sigma \} | 0 \rangle$$

Two independent approaches

1. Dispersive, data-driven
2. Lattice QCD



Muon $g-2$ Theory Initiative

Steering Committee

- Gilberto Colangelo (Bern)
- Achim Denig (Mainz)
- Aida El-Khadra (UIUC) **chair**
- Martin Hoferichter (Bern)
- Christoph Lehner (Regensburg University)
co-chair
- Laurent Lellouch (Marseille)
- Tsutomu Mibe (KEK)
J-PARC Muon $g-2$ /EDM experiment
- Lee Roberts (Boston)
Fermilab $g-2$ experiment
- Thomas Teubner (Liverpool)
- Hartmut Wittig (Mainz)

<https://muon-gm2-theory.illinois.edu>

- Maximize the impact of the Fermilab and J-PARC experiments
 - **quantify and reduce the theoretical uncertainties on the SM prediction**
- assess reliability of uncertainty estimates
- summarize the theory status: White Papers
- organize workshops to bring the different communities together:
 - [First plenary workshop near Fermilab: 3-6 June 2017](#)
 - ...
 - [Fifth plenary workshop hosted by Higgs Centre \(UK\): 5-9 Sep 2022](#)
 - ...
 - [Eight plenary workshop hosted by IJCLab \(Orsay, France\): 8-12 Sep 2025](#)
 - **Ninth plenary workshop to be hosted by UConn (US): 3-7 Aug 2026**

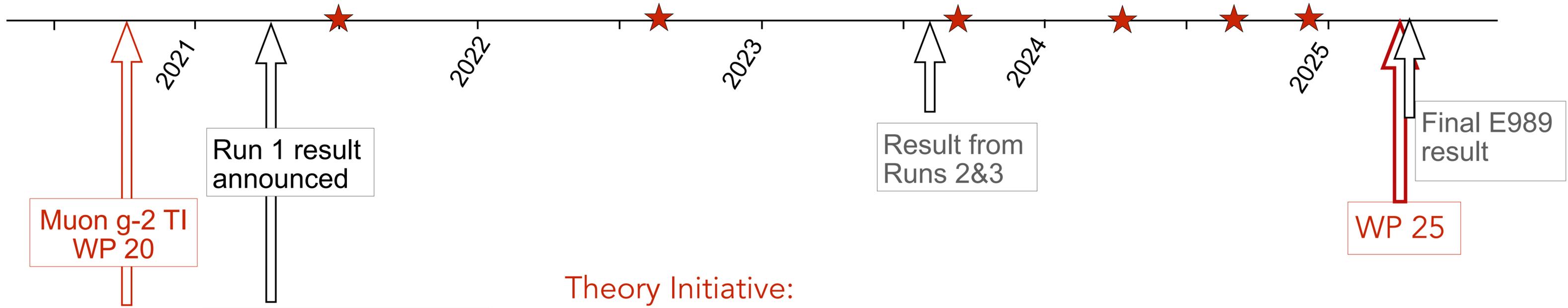
Timeline

FNAL E989

Run 4

Run 5

Run 6



Muon g-2 TI
WP 20

Run 1 result
announced

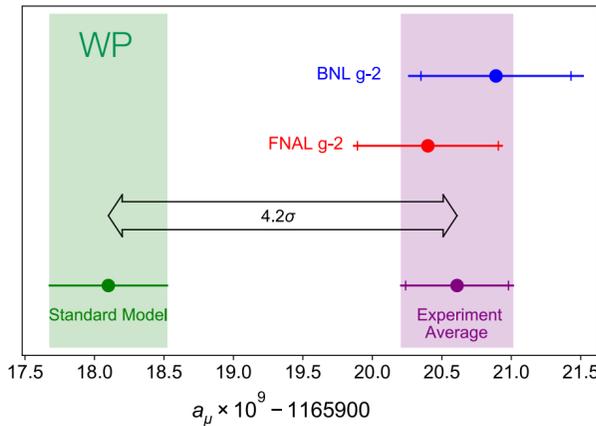
Result from
Runs 2&3

Final E989
result

WP 25

Theory Initiative:

- ☆ CMD-3 seminar (virtual): 27 March 2023 at 8:00am US CDT
- ☆ 2nd CMD-3 discussion meeting
- ☆ 8/9/2023: Status of Muon g-2 Theory in SM



- ★ TI workshops:
- Jun 2021 @ KEK (virtual)
- Sep 2022 @ Higgscentre

- Sep 2023 @ Bern
- Apr 2024 (virtual)

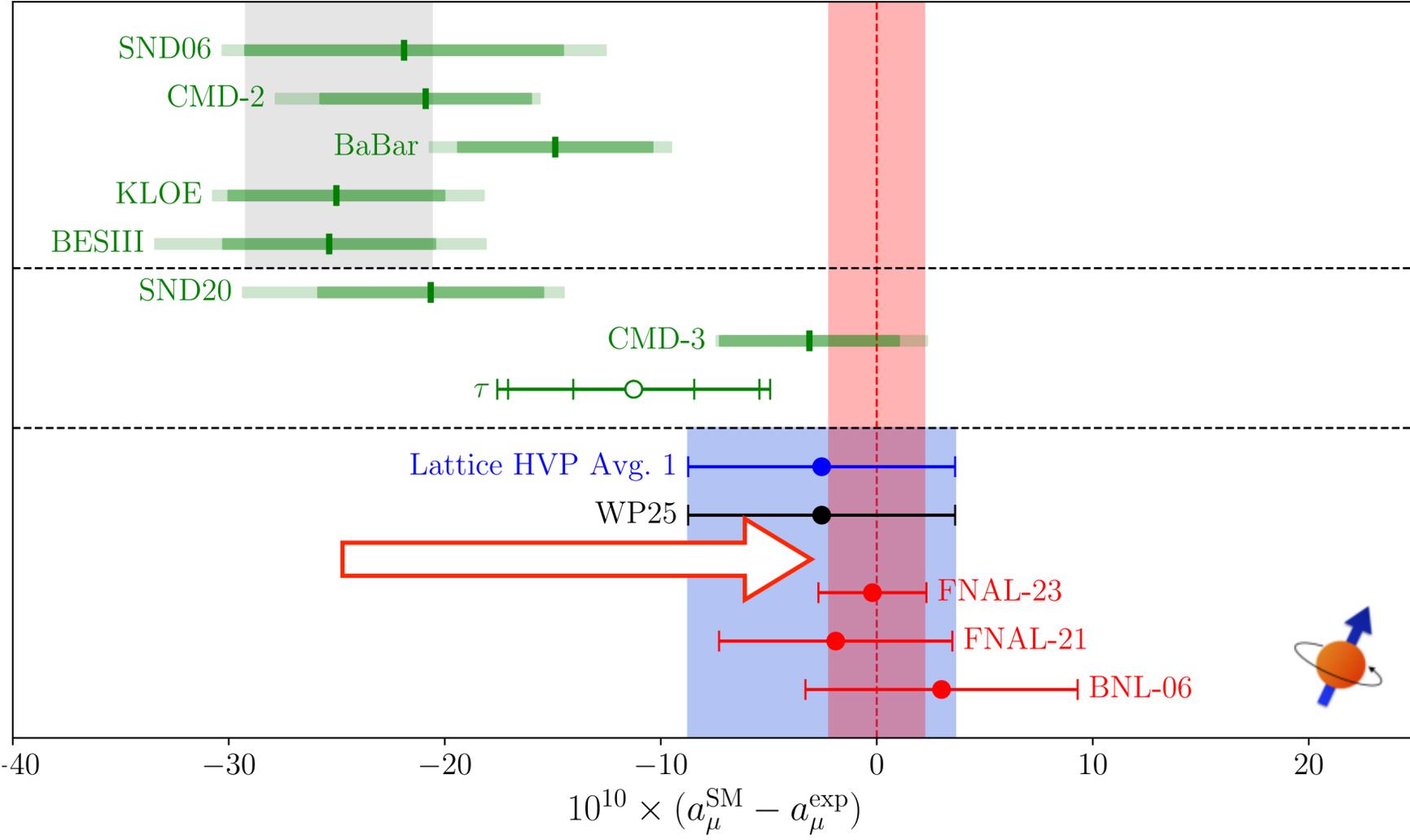
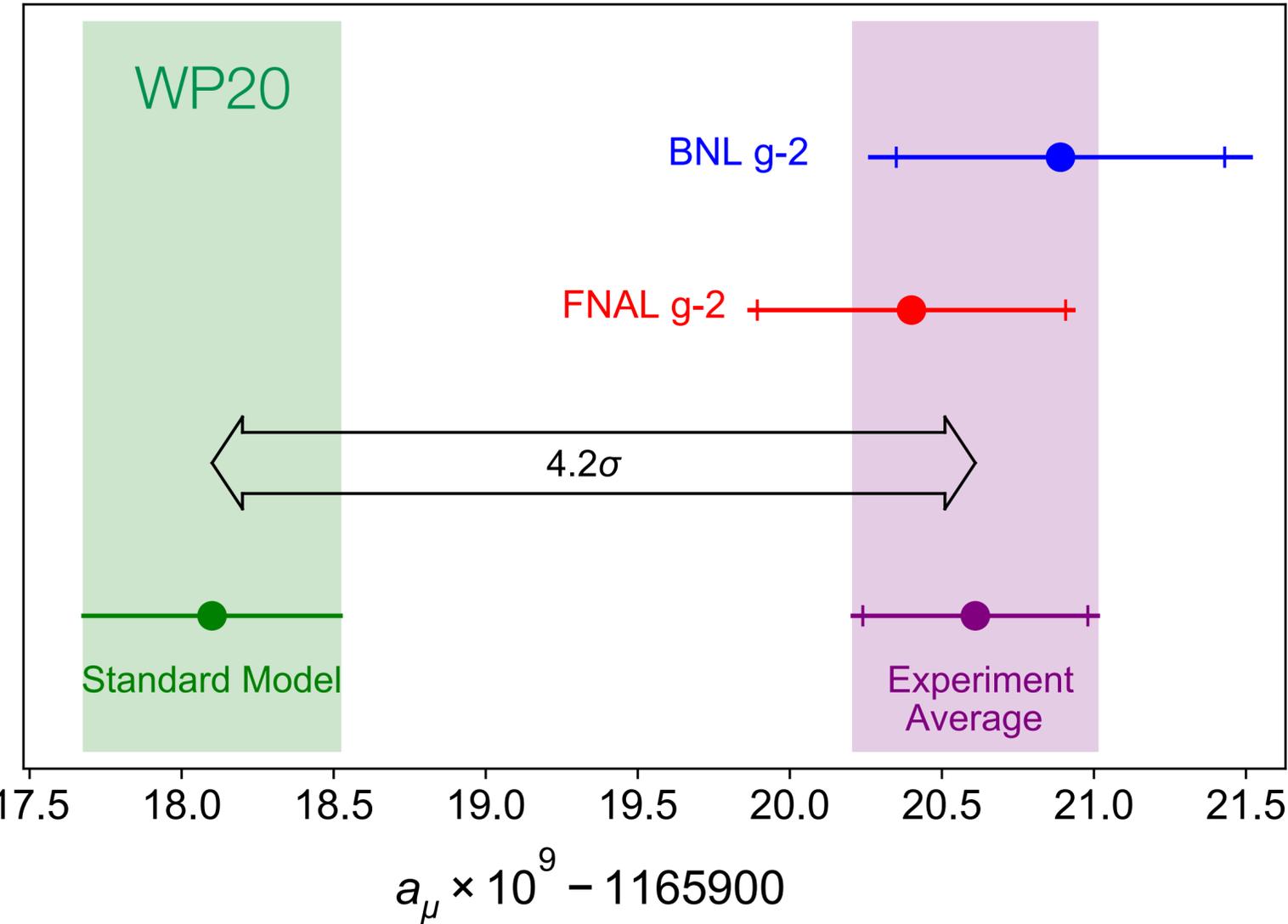
- Sep 2024 @ KEK & KMI
- Tau decays 2024 (virtual mini)
- Sep 2025 @ Orsay

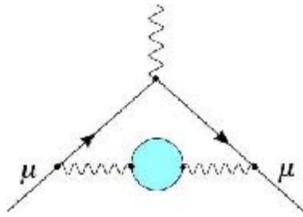
Experiment vs SM theory

$$a_\mu^{\text{SM}} = a_\mu^{\text{HVP}} + [a_\mu^{\text{QED}} + a_\mu^{\text{Weak}} + a_\mu^{\text{HLbL}}]$$

2021

2025





Hadronic Vacuum Polarization

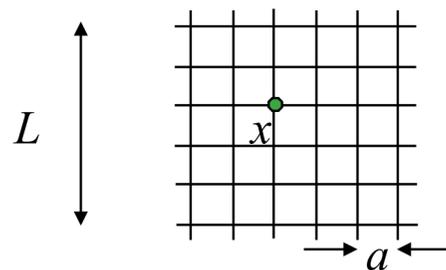
- Dispersive, data-driven:

HVP: integrate hadronic cross section over CM energy:

$$\text{Im}[\text{wavy line} \text{---} \text{blue circle} \text{---} \text{wavy line}] \sim |\text{wavy line} \text{---} \text{hadrons}|^2 \implies a_{\mu}^{\text{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int dq^2 \omega(q^2) \hat{\Pi}(q^2) = \frac{m_{\mu}^2}{12\pi^3} \int ds \frac{\hat{K}(s)}{s} \sigma_{\text{exp}}(s)$$

Many experiments (over 20+ years) have measured the e^+e^- cross sections for (almost) all channels over the needed energy range with increasing precision.

- Direct calculation using Euclidean Lattice QCD



Approximations:

discrete space-time (spacing a)

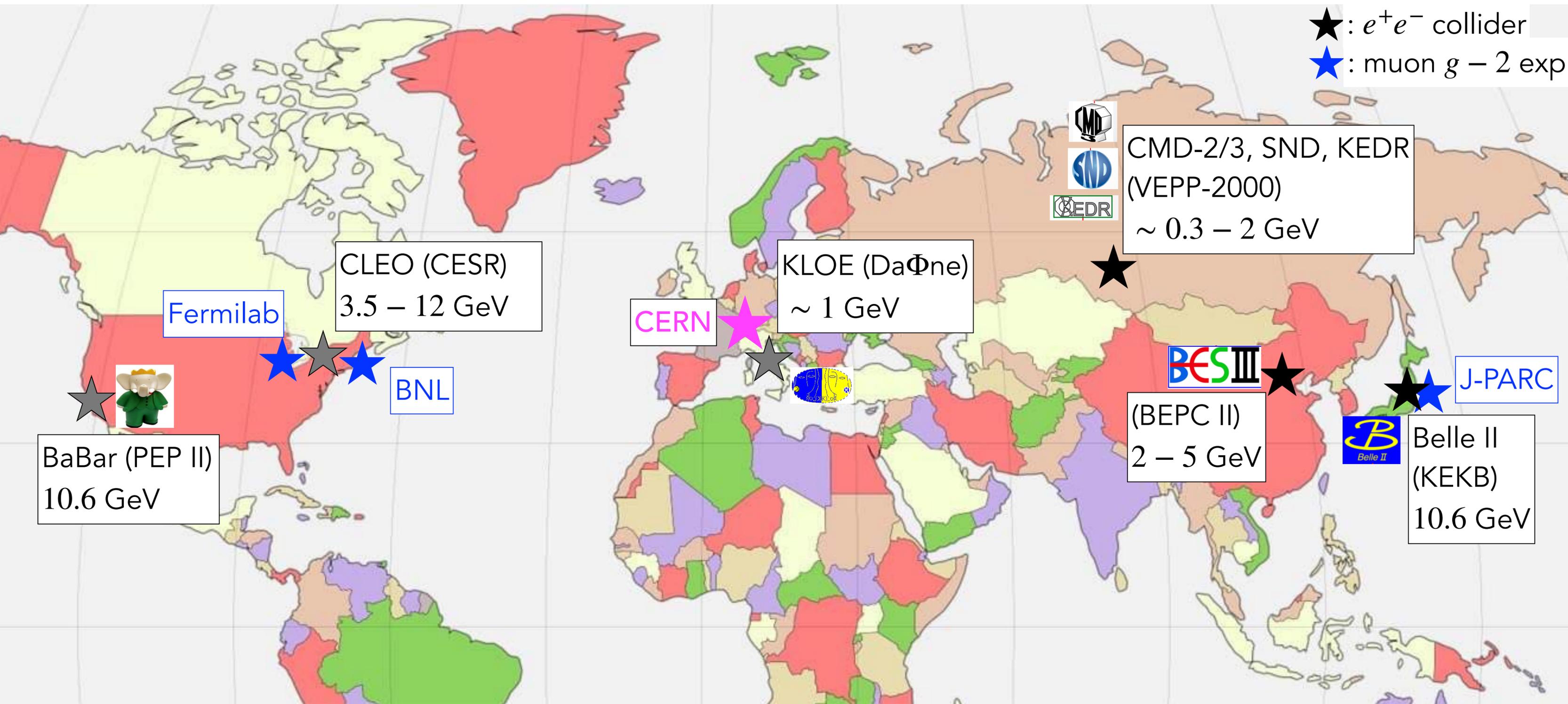
finite spatial volume (L), and time extent (T)

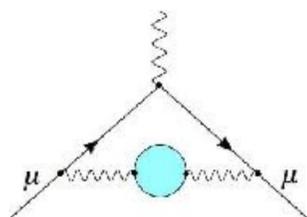
...

$$a_{\mu}^{\text{HVP,LO}} = 4\alpha^2 \int_0^{\infty} dt C(t) \tilde{w}(t)$$

- $ab-initio$ method to quantify QCD effects
- already used for simple hadronic quantities with high precision
- requires large-scale computational resources
- allows for entirely SM theory based evaluations

Overview of the experiments

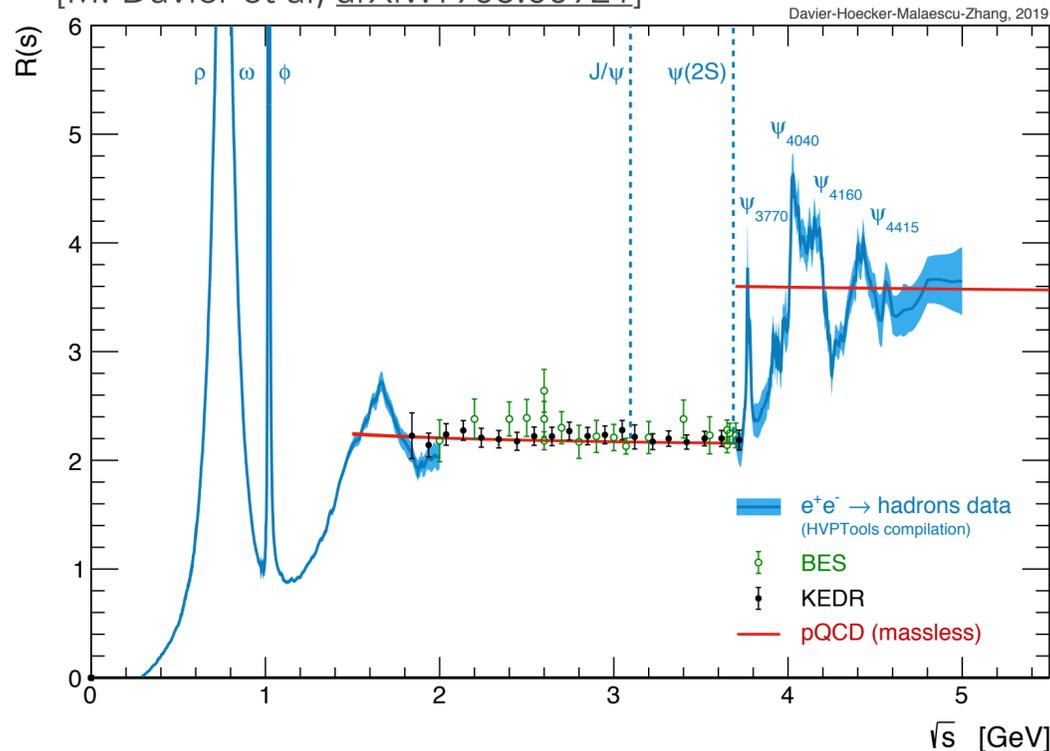




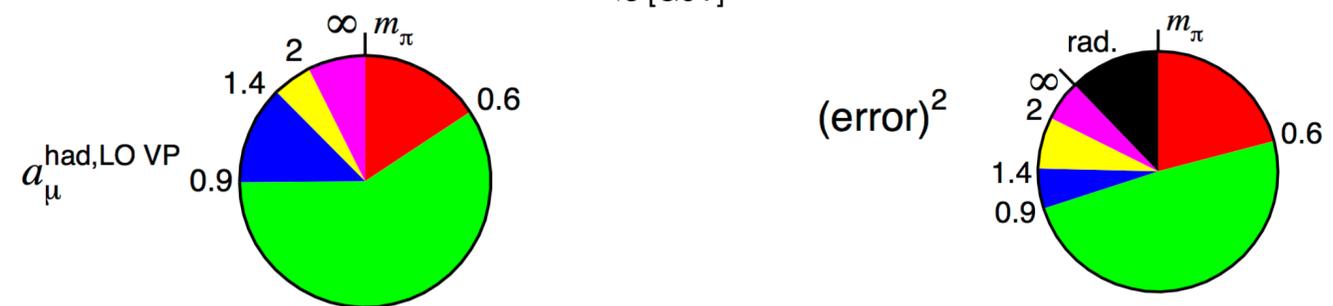
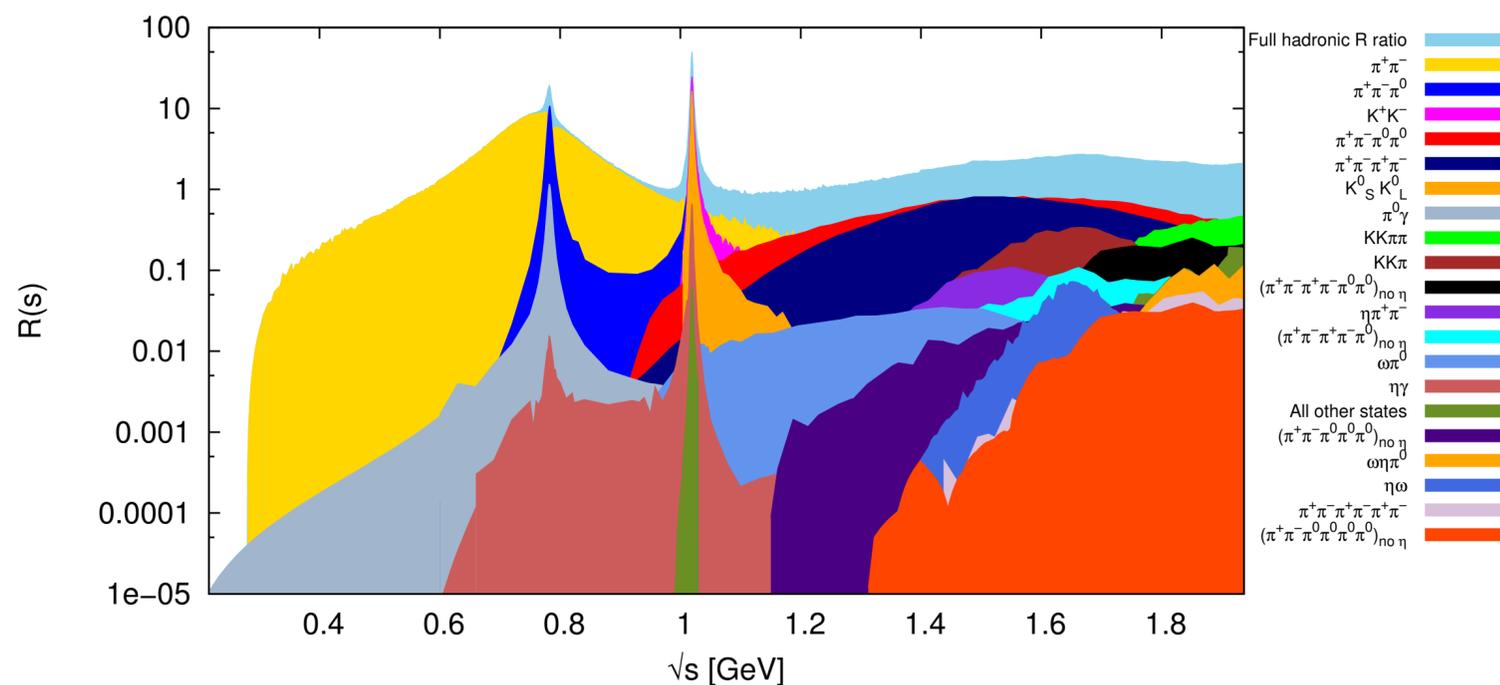
HVP: data-driven

$$a_{\mu}^{\text{HVP,LO}} = \frac{m_{\mu}^2}{12\pi^3} \int ds \frac{\hat{K}(s)}{s} \sigma_{\text{exp}}(s)$$

[M. Davier et al, arXiv:1908.00921]

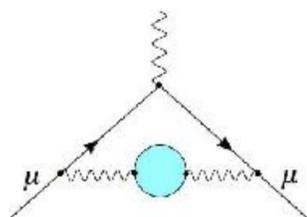


[A. Keshavarzi et al, arXiv:1802.02995]



- $\sigma_{\text{had}}(s)$ defined to include real & virtual photons
- direct integration method:** no modelling of $\sigma_{\text{had}}(s)$, summing up contributions from all hadronic channels
- total hadronic cross section $\sigma_{\text{had}}(s)$ from > 100 data sets in 35+ channels summed up to $\sqrt{s} \sim 2 \text{ GeV}$
- $\sqrt{s} > 2 \text{ GeV}$: inclusive data + pQCD + narrow resonances
- two independent compilations (DHMZ, KNT)

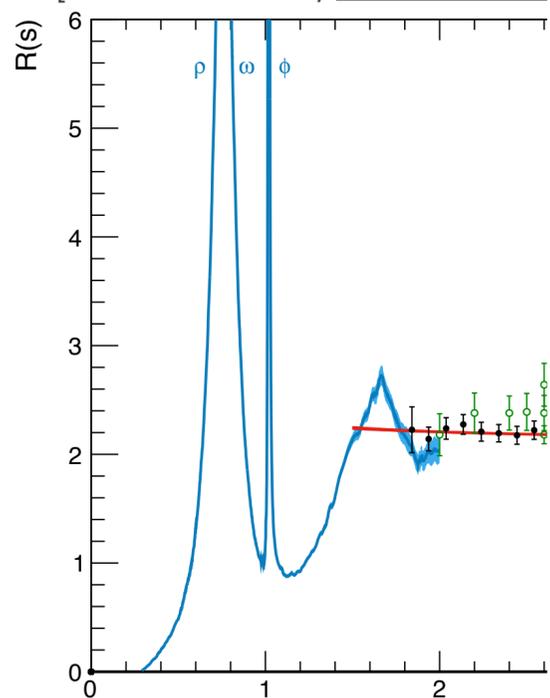
Tensions (of up to 3σ) between data sets:
 conservative procedure to include differences in error estimate



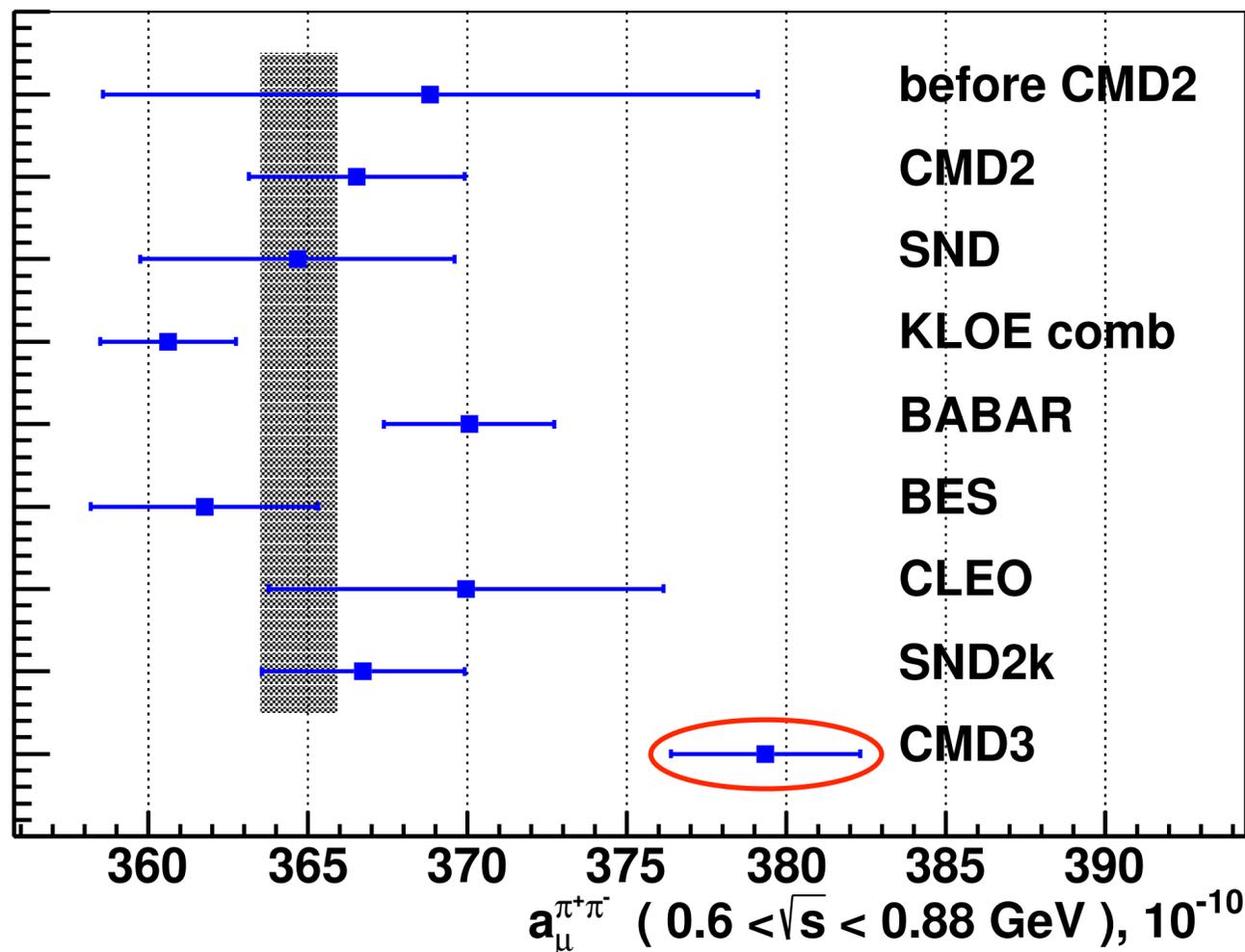
HVP: data-driven

$$a_{\mu}^{\text{HVP,LO}} = \frac{m_{\mu}^2}{12\pi^3} \int ds \frac{\hat{K}(s)}{s} \sigma_{\text{exp}}(s)$$

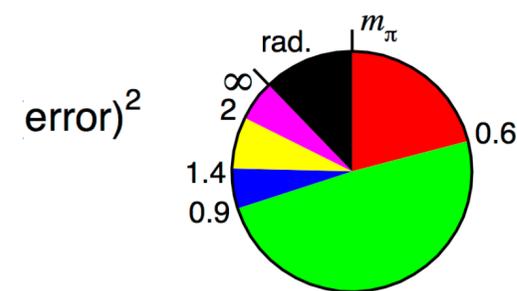
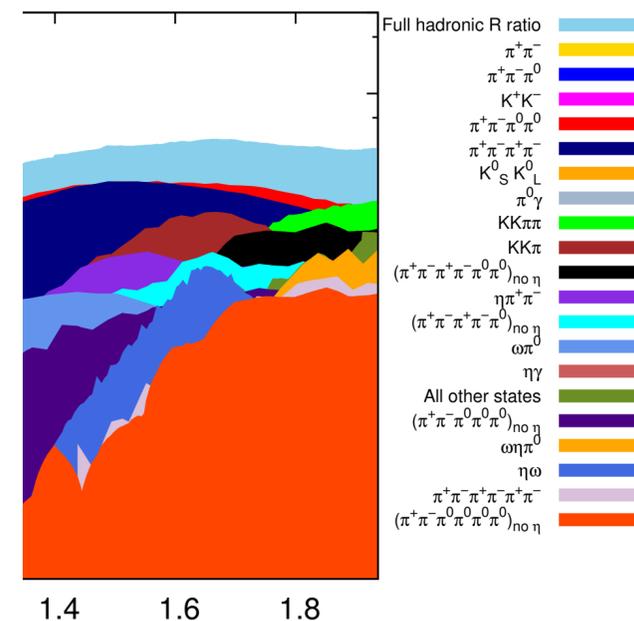
[M. Davier et al, arXiv:1908.00921]



Feb 2023: from CMD-3 [F. Ignatov et al, arXiv:2302.08834, PRD 2024]

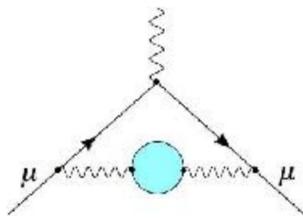


[arXiv:1802.02995]

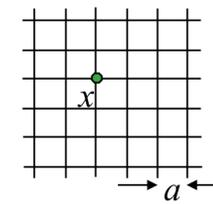


- $\sigma_{\text{had}}(s)$ defined to include direct integration
- summing up contributions from 35+ channels
- $\sqrt{s} > 2 \text{ GeV}$: inclusive data + pQCD + narrow resonances
- two independent compilations (DHMZ, KNT)

conservative procedure to include differences in error estimate



Lattice HVP: Introduction



Leading order HVP contribution:

$$a_{\mu}^{\text{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int dq^2 \omega(q^2) \hat{\Pi}(q^2)$$

[B. Lautrup, A. Peterman, E. de Rafael, Phys. Rep 1972;
E. de Rafael, Phys. Let. B 1994; T. Blum, PRL 2002]

- Calculate $a_{\mu}^{\text{HVP,LO}}$ in Lattice QCD

Start with correlation function of EM currents: $C(t) = \frac{1}{3} \sum_{i,x} \langle j_i^{\text{EM}}(x,t) j_i^{\text{EM}}(0,0) \rangle$ $j_{\mu}^{\text{EM}} = \sum_f q_f \bar{\psi}_f(x,t) \gamma_{\mu} \psi_f(x,t)$
 $f = u, d, s, c, \dots$

Fourier transform yields $\hat{\Pi}(Q^2) = 4\pi^2 \int_0^{\infty} dt C(t) \left[t^2 - \frac{4}{Q^2} \sin^2 \left(\frac{Qt}{2} \right) \right]$

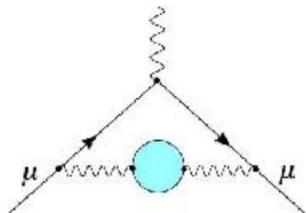
[D. Bernecker and H. Meyer,
arXiv:1107.4388, EPJA 2011]

so that $a_{\mu}^{\text{HVP,LO}}$ can be obtained as an integral over Euclidean time, aka time momentum representation (TMR):

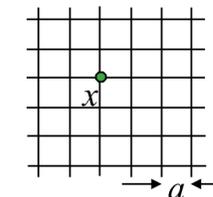
$$a_{\mu}^{\text{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^{\infty} dQ^2 w(Q^2) \hat{\Pi}(Q^2) = 4\alpha^2 \int_0^{\infty} dt C(t) \int_0^{\infty} dQ^2 w(Q^2) \left[t^2 - \frac{4}{Q^2} \sin^2 \left(\frac{Qt}{2} \right) \right]$$



$$a_{\mu}^{\text{HVP,LO}} = 4\alpha^2 \int_0^{\infty} dt C(t) \tilde{w}(t)$$



Lattice HVP: Introduction



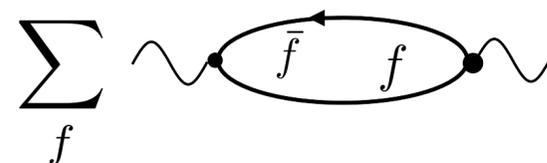
Calculate a_μ^{HVP} in Lattice QCD:

$$a_\mu^{\text{HVP,LO}} = \sum_f a_{\mu,f}^{\text{HVP,LO}} + a_{\mu,\text{disc}}^{\text{HVP,LO}}$$

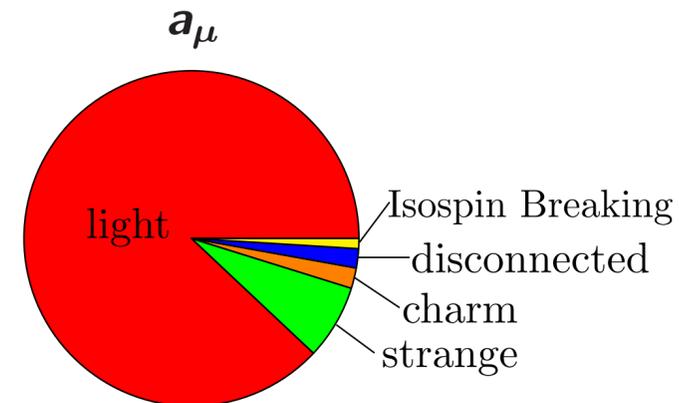
- Separate into connected for each quark flavor + disconnected contributions (gluon and sea-quark background not shown in diagrams)

Note: almost always $m_u = m_d$

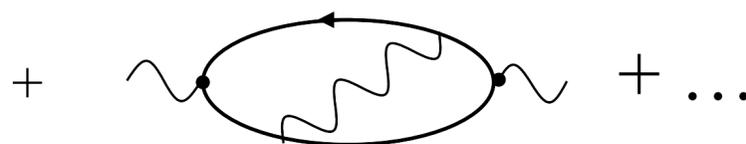
$f = ud, s, c, b$



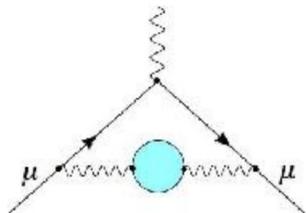
- light-quark connected contribution: $a_\mu^{\text{HVP,LO}}(ud) \sim 90\%$ of total
- s, c, b -quark contributions $a_\mu^{\text{HVP,LO}}(s, c, b) \sim 8\%, 2\%, 0.05\%$ of total
- disconnected contribution: $a_{\mu,\text{disc}}^{\text{HVP,LO}} \sim 2\%$ of total
- Isospinbreaking (QED + $m_u \neq m_d$) corrections: $\delta a_\mu^{\text{HVP,LO}} \sim 1\%$ of total



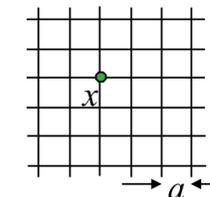
- need to add QED and strong isospin breaking ($\sim m_u - m_d$) corrections:



$$a_\mu^{\text{HVP,LO}} = a_\mu^{\text{HVP,LO}}(ud) + a_\mu^{\text{HVP,LO}}(s) + a_\mu^{\text{HVP,LO}}(c) + a_{\mu,\text{disc}}^{\text{HVP,LO}} + \delta a_\mu^{\text{HVP,LO}}$$



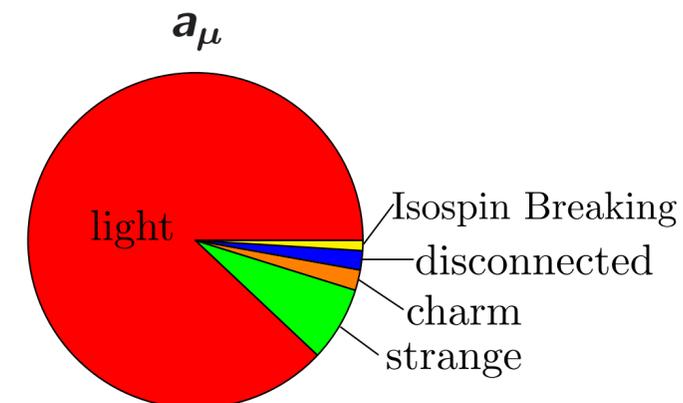
Lattice HVP: challenges



Calculate a_μ^{HVP} in Lattice QCD:

$$a_\mu^{\text{HVP,LO}} = \sum_f a_{\mu,f}^{\text{HVP,LO}} + a_{\mu,\text{disc}}^{\text{HVP,LO}}$$

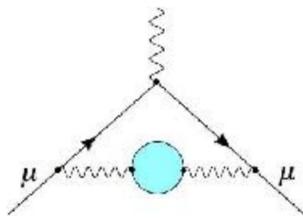
$$a_\mu^{\text{HVP,LO}} = a_\mu^{\text{HVP,LO}}(ud) + a_\mu^{\text{HVP,LO}}(s) + a_\mu^{\text{HVP,LO}}(c) + a_{\mu,\text{disc}}^{\text{HVP,LO}} + \delta a_\mu^{\text{HVP,LO}}$$



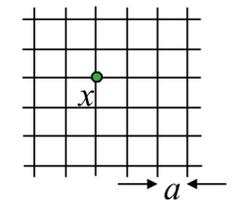
- $a_\mu^{\text{HVP,LO}}$ needed with $< 0.5\%$ precision
- subpercent statistical precision:
exponentially growing noise-to-signal in $C(t)$ as $t \rightarrow \infty$
affects light-quark contributions
- sizable finite volume effects
- sensitivity to scale setting uncertainty
- control discretization effects
- quark-disconnected diagrams: control noise
- include isospin-breaking effects

Separation of $a_\mu^{\text{HVP,LO}}$ into $a_\mu^{\text{HVP,LO}}(ud)$ and $\delta a_\mu^{\text{HVP,LO}}$ is scheme dependent.

- light-quark connected contribution:
 $a_\mu^{\text{HVP,LO}}(ud) \sim 90\%$ of total
- s, c, b -quark contributions
 $a_\mu^{\text{HVP,LO}}(s, c, b) \sim 8\%, 2\%, 0.05\%$ of total
- disconnected contribution:
 $a_{\mu,\text{disc}}^{\text{HVP,LO}} \sim 2\%$ of total
- Isospinbreaking (QED + $m_u \neq m_d$) corrections:
 $\delta a_\mu^{\text{HVP,LO}} \sim 1\%$ of total



Windows in Euclidean time



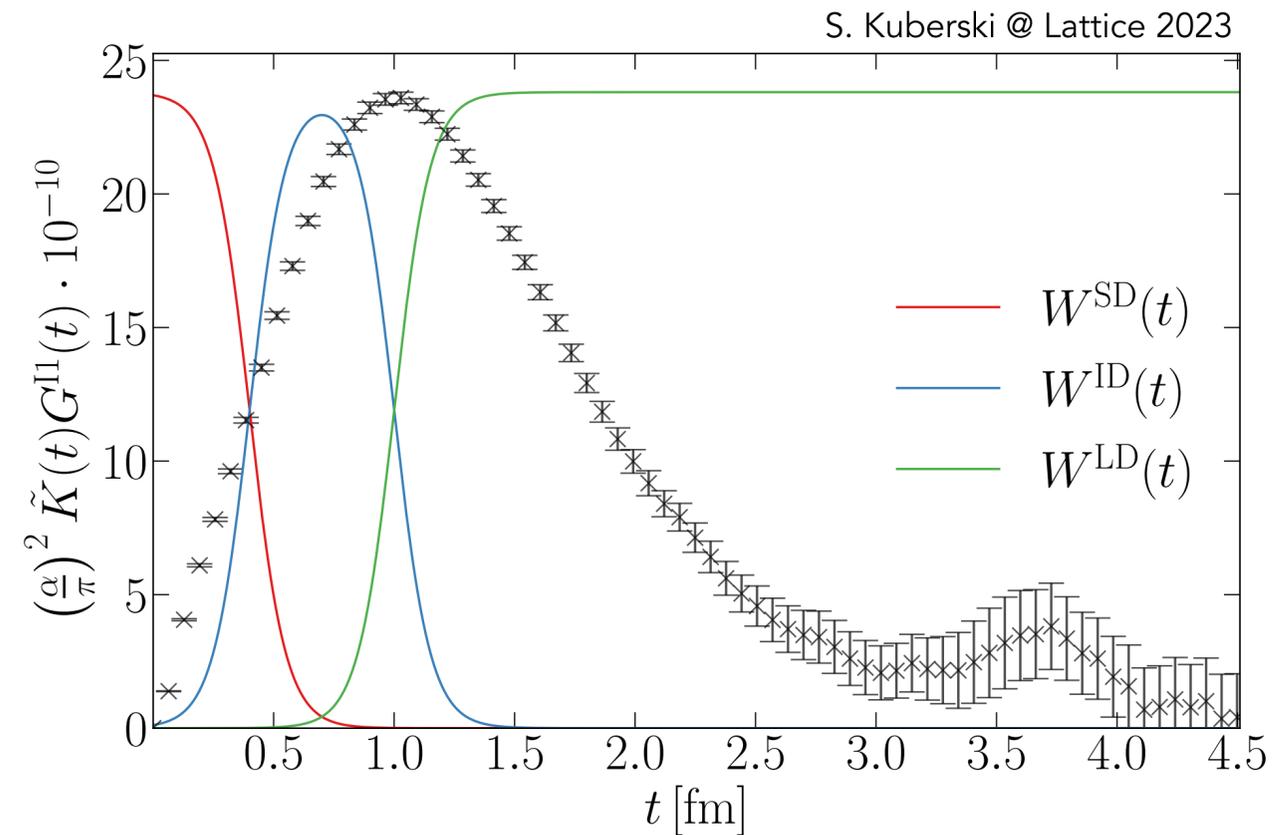
$$a_\mu^{\text{HVP,LO}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dt \tilde{w}(t) C(t)$$

- Use windows in Euclidean time to consider the different time regions separately

[T. Blum et al, arXiv:1801.07224, 2018 PRL]

$$t_0 = 0.4 \text{ fm}, t_1 = 1.0 \text{ fm}$$

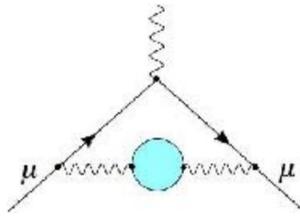
- Short Distance (SD) $t : 0 \rightarrow t_0$
- Intermediate (W) $t : t_0 \rightarrow t_1$
- Long Distance (LD) $t : t_1 \rightarrow \infty$



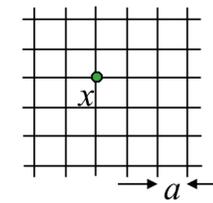
- disentangle systematics/statistics from long distance/FV and discretization effects
- intermediate window: easy to compute in lattice QCD; compare to disperse approach
- Internal cross check: compute each window separately (in continuum, infinite volume limits,...) and

combine:

$$a_\mu = a_\mu^{\text{SD}} + a_\mu^{\text{W}} + a_\mu^{\text{LD}}$$

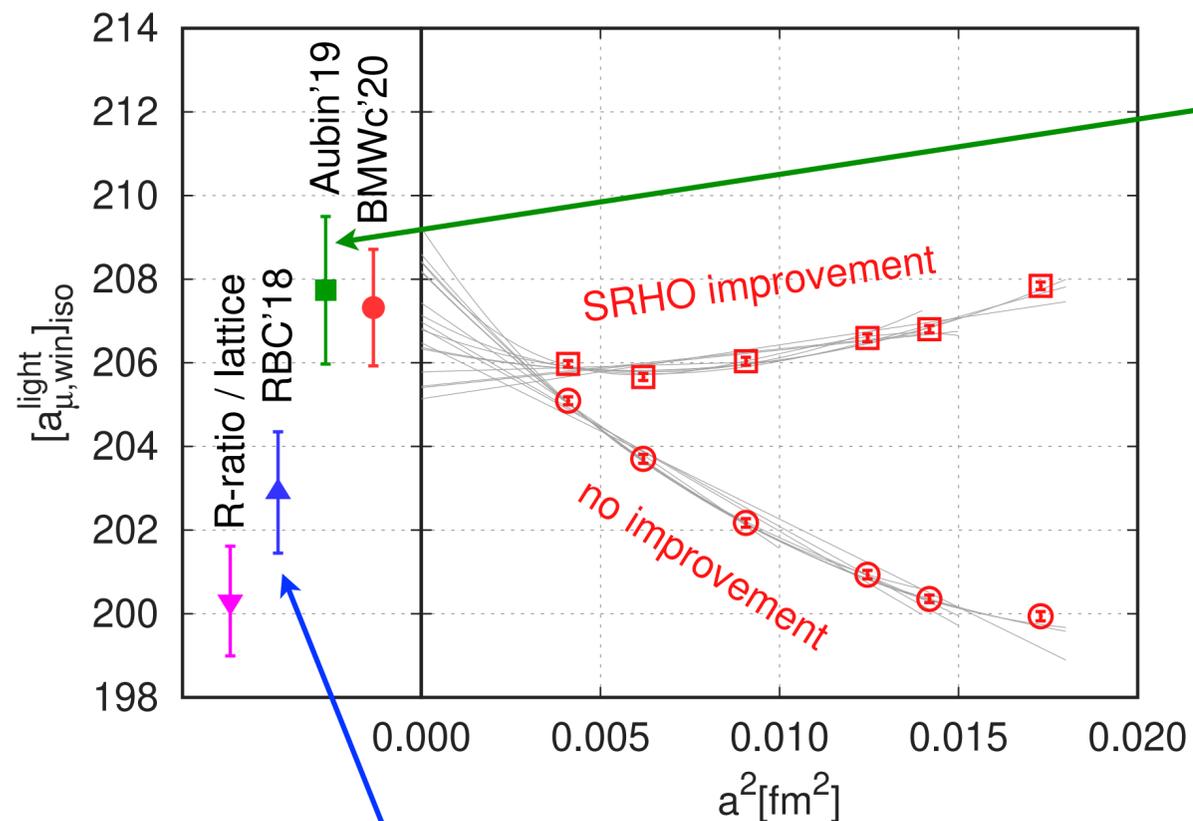


Lattice HVP: W (intermediate) window



BMW 20

[Sz. Borsanyi et al, arXiv:2002.12347, 2021 Nature]



Aubin et al -19

[C. Aubin et al, arXiv:1905.09307, PRD]

The hadronic vacuum polarization from lattice QCD at high precision

Nov 16 – 20, 2020
Europe/Zurich tmezone

Enter your search term



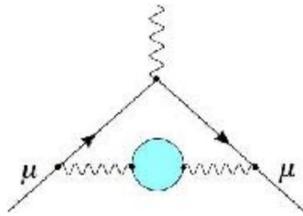
Virtual Muon g-2 Theory Initiative workshop (16-18 Nov 2020)

Agreement to:

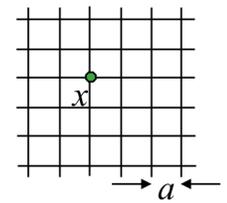
- compute the 3 window observables proposed by RBC/UKQCD
- first goal: intermediate window observable
- compute individual flavor contributions + IB-breaking effects
- ...

RBC/UKQCD-18

[T. Blum et al, arXiv:1801.07224, 2018 PRL]



Lattice HVP: W (intermediate) window



Mainz 2022

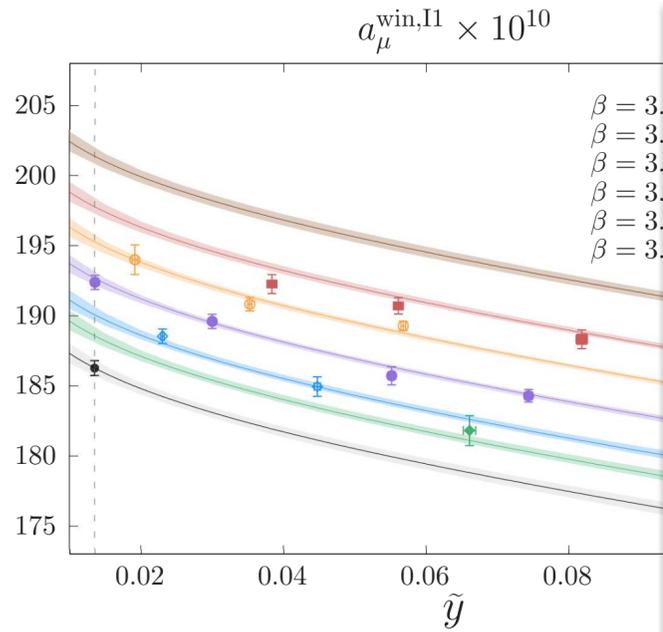
[M. Cè et al, arXiv:2206.06582, PRD]

ETM 2022

[C. Alexandrou et al, arXiv:2206.15084, PRD]

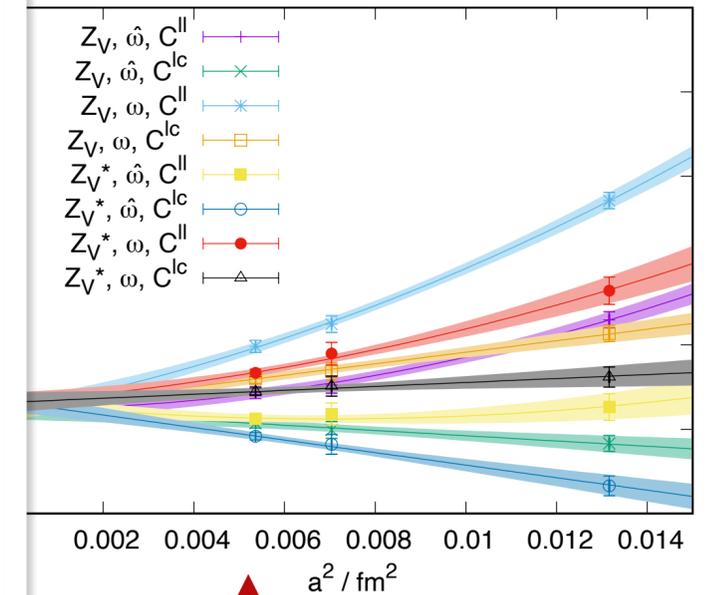
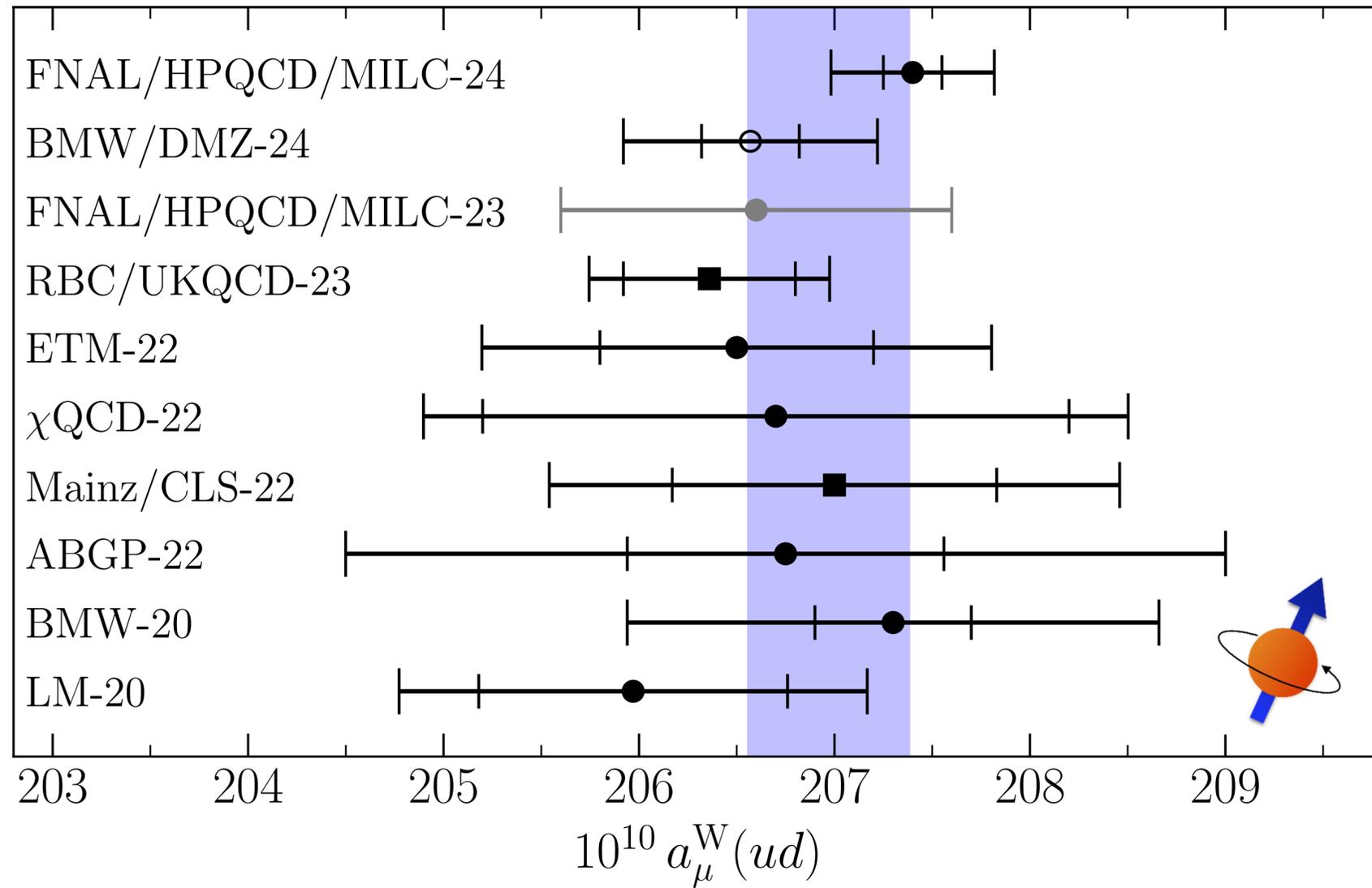
RBC/UKQCD 2023

[T. Blum et al, arXiv:2301.08696, PRD]

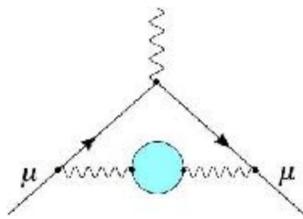


FNAL/HPQCD/MILC-24

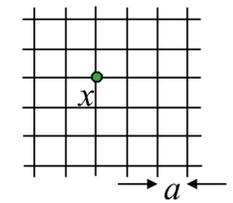
[A. Bazavov et al, arXiv:2206.06582, PRD]



Blind analyses



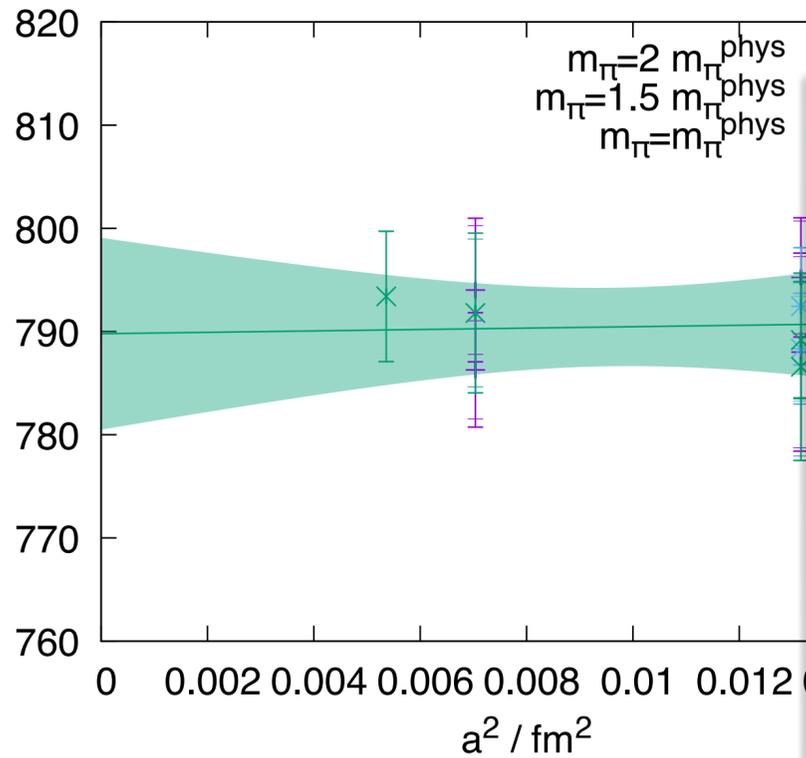
Lattice HVP: LD window



Fall 2024: RBC/UKQCD, Mainz, FNAL/HPQCD/MILC — all from blind analyses

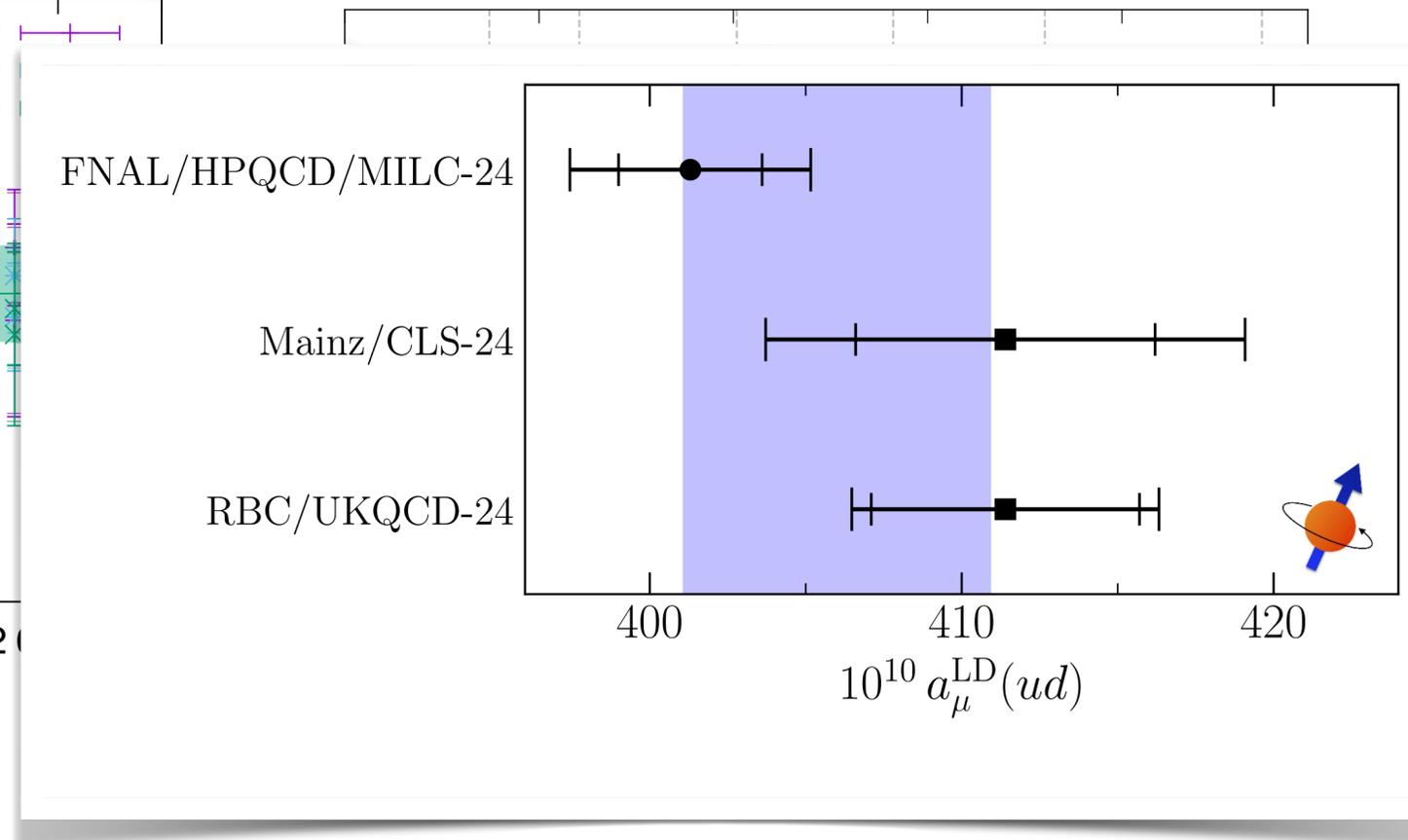
RBC/UKQCD

[T. Blum et al, arXiv:2410.20590, PRL]



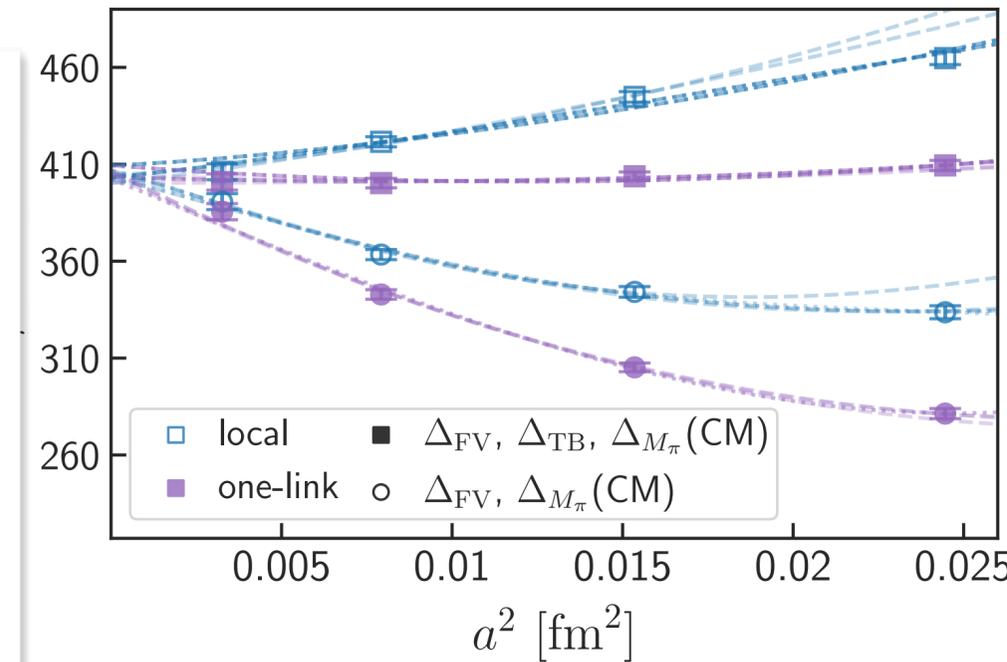
Mainz/CLS

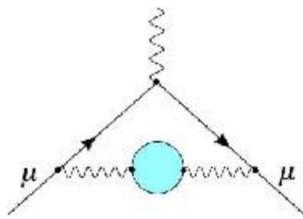
[D. Djukanovic., arXiv:2411.07969, JHEP]



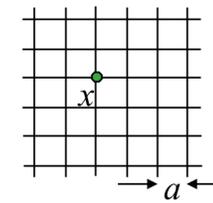
FNAL/HPQCD/MILC

[A. Bazavov et al, arXiv:2412.18491, PRL]

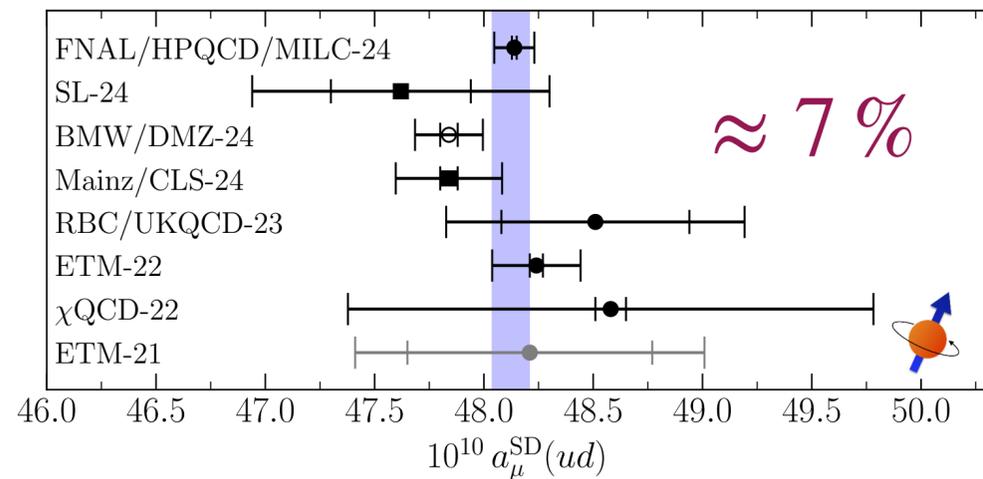




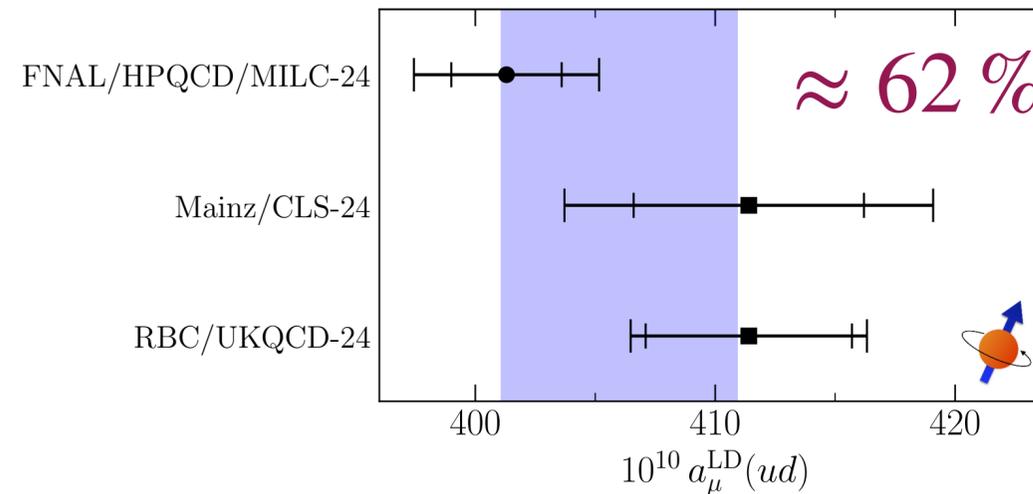
Light-quark connected (ud)



Short Distance (SD)



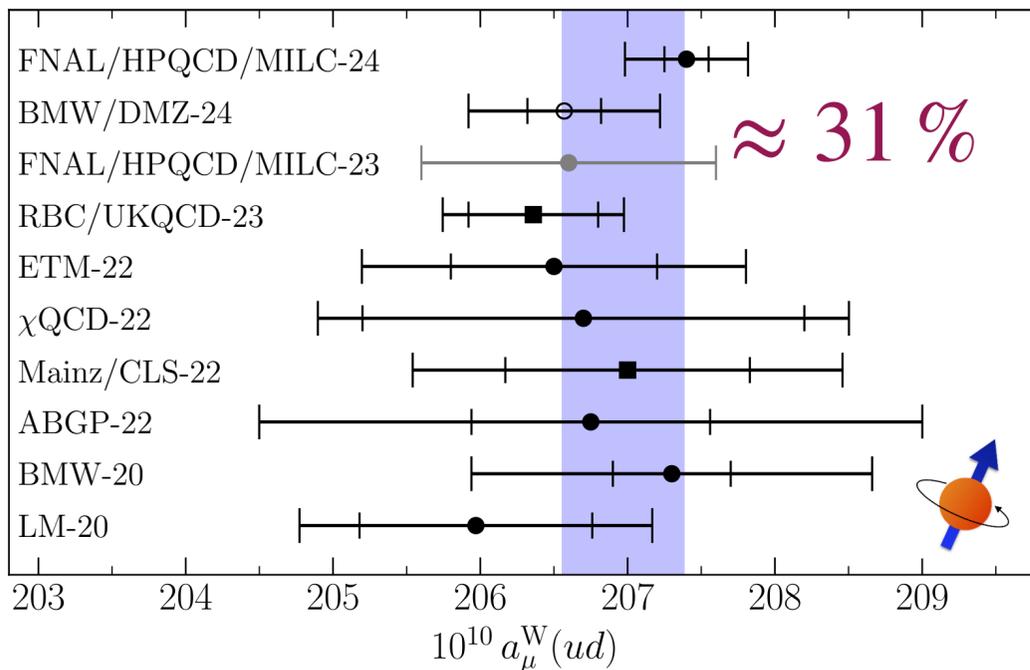
Long Distance (LD)



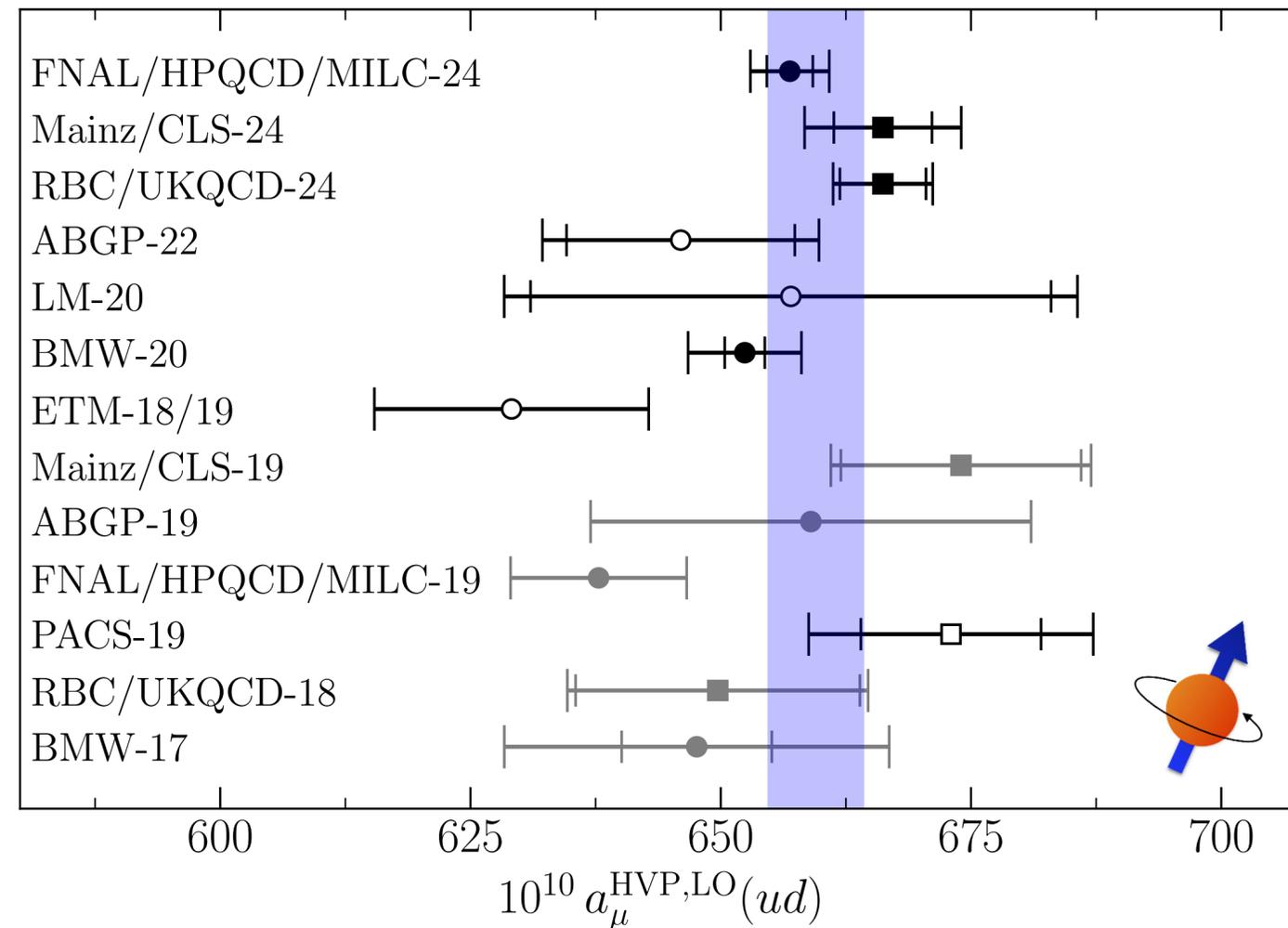
Fall 2024:

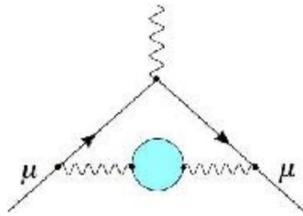
- RBC/UKQCD, Mainz, FNAL/HPQCD/MILC
- all from blind analyses

Intermediate (W)



Full integrand
(full)





Lattice HVP: consolidated averages



light

strange

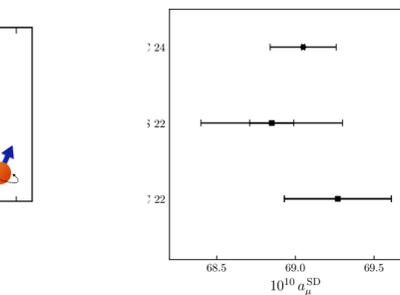
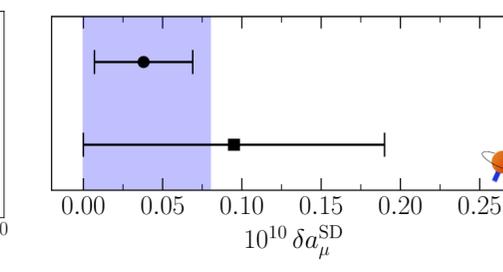
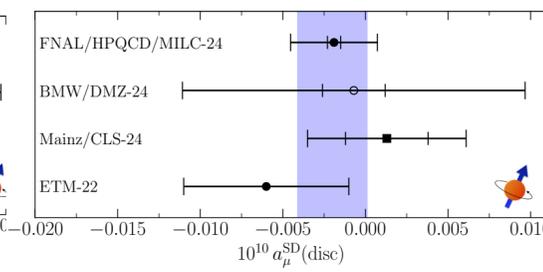
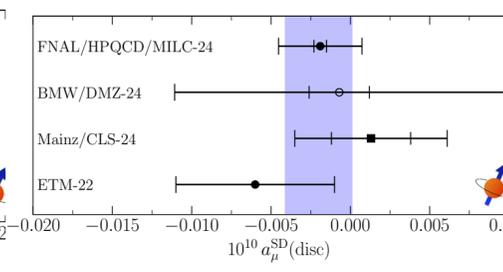
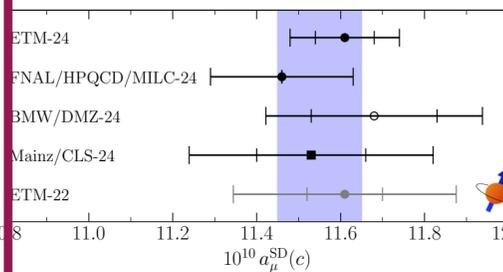
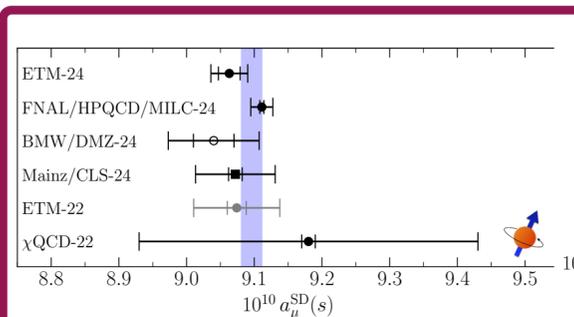
charm

disc

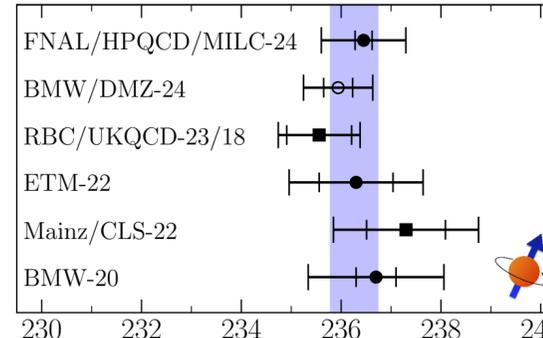
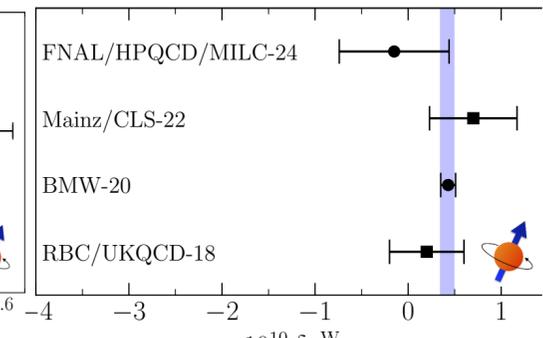
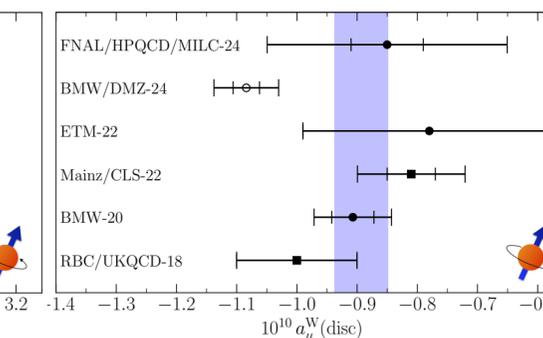
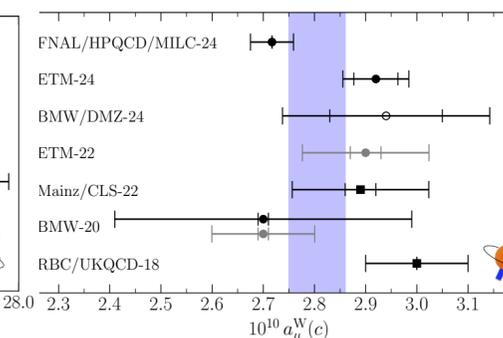
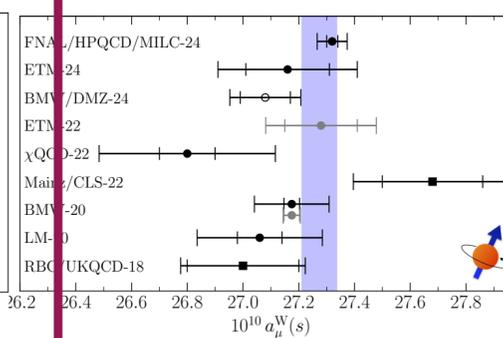
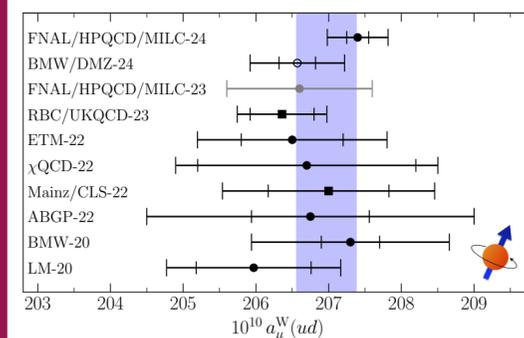
IB

Total

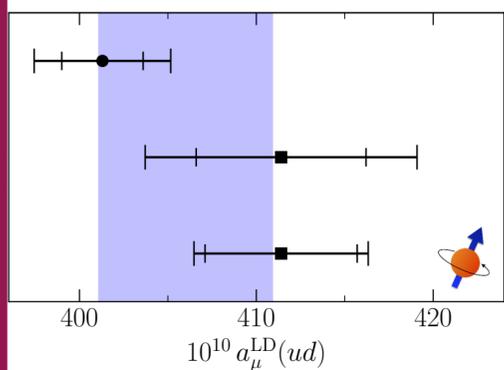
SD



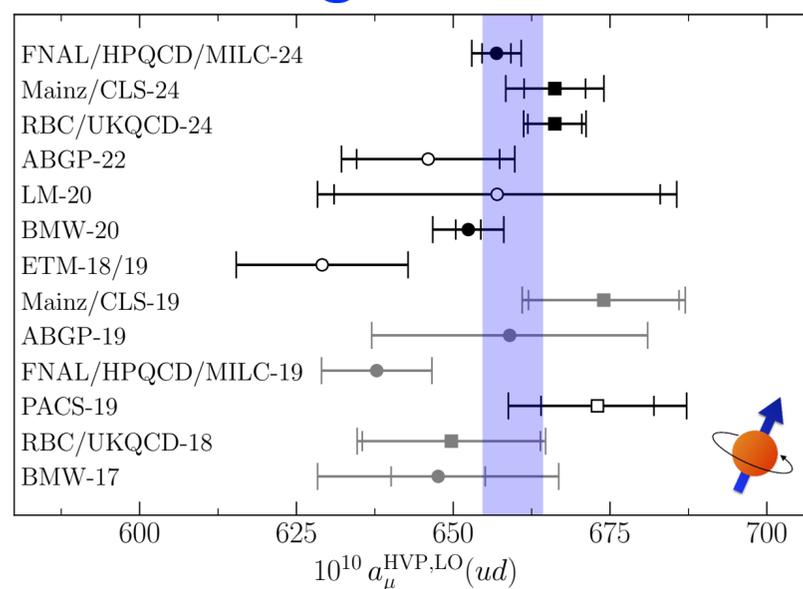
W



LD



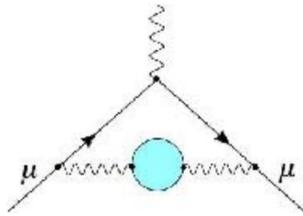
Full, light



+ strange, charm, disc, IB



Lattice HVP



Lattice HVP: consolidated averages



light

strange

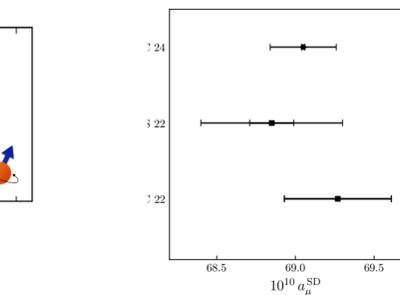
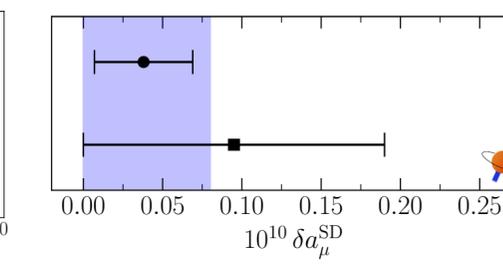
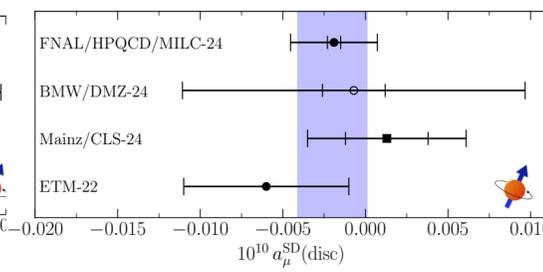
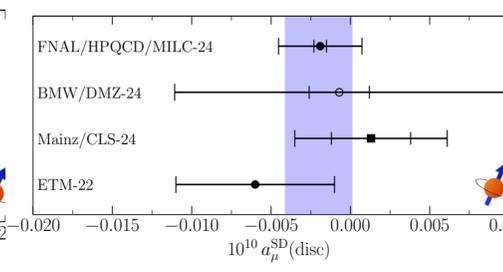
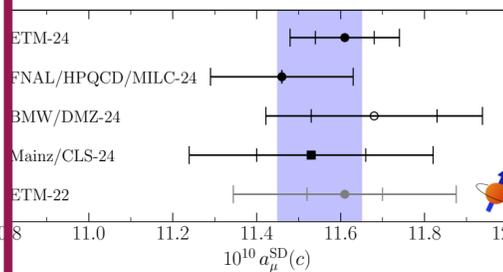
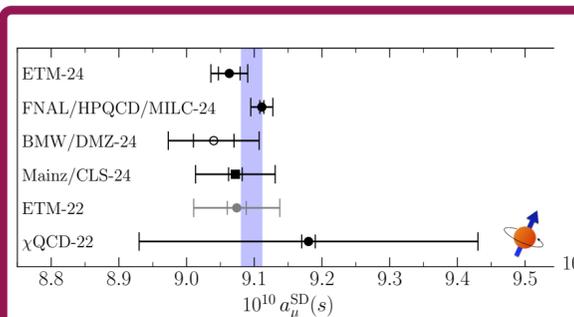
charm

disc

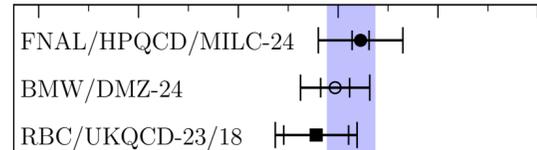
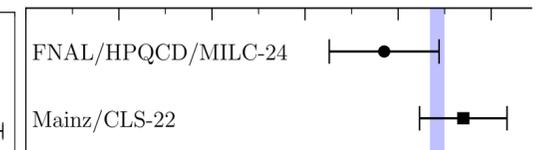
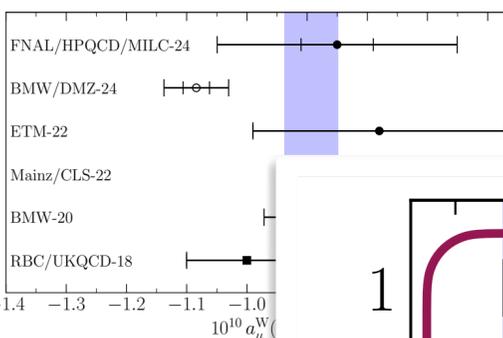
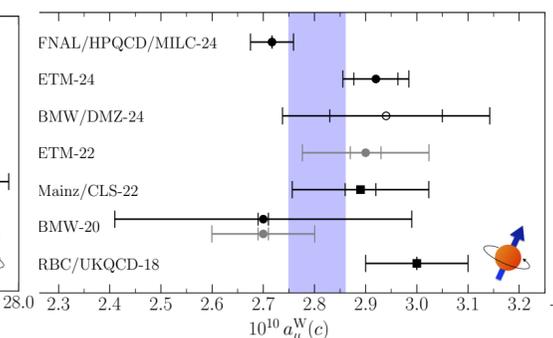
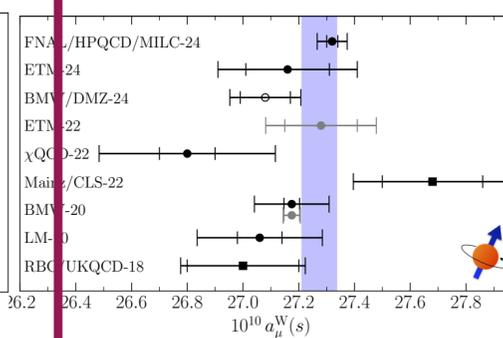
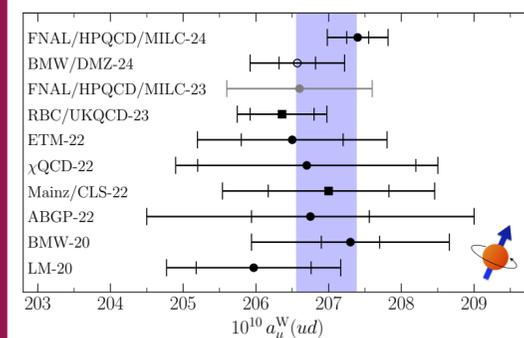
IB

Total

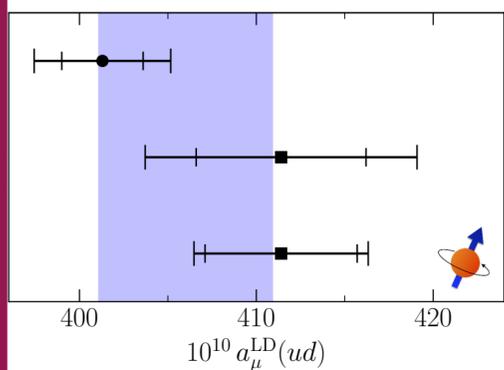
SD



W



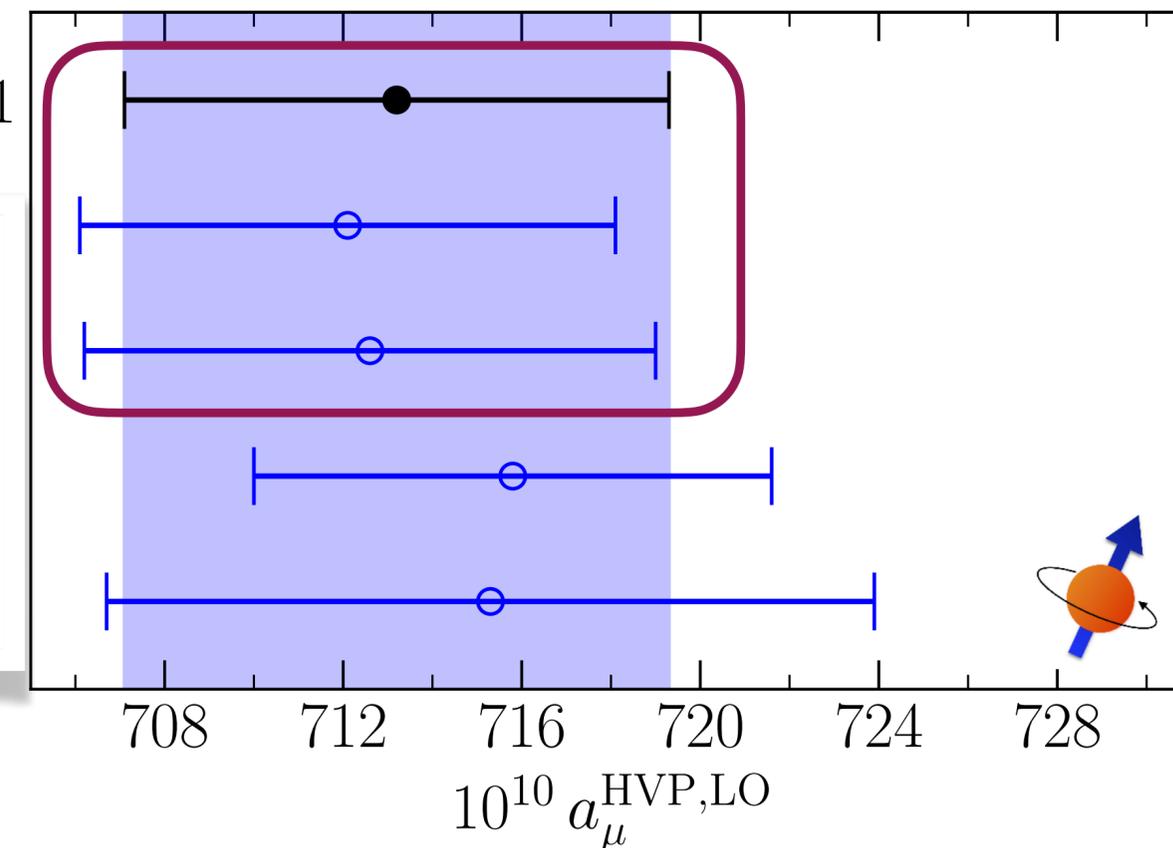
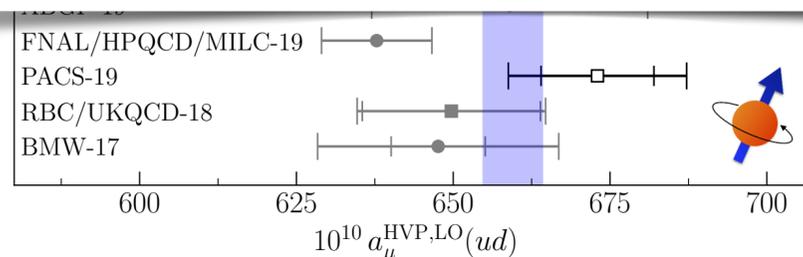
LD

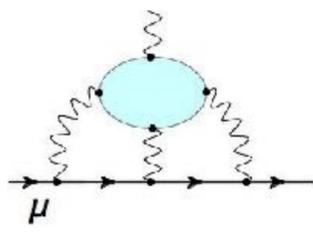


Averages maximize the number of independent lattice inputs:

17 papers

8 independent lattice collaborations



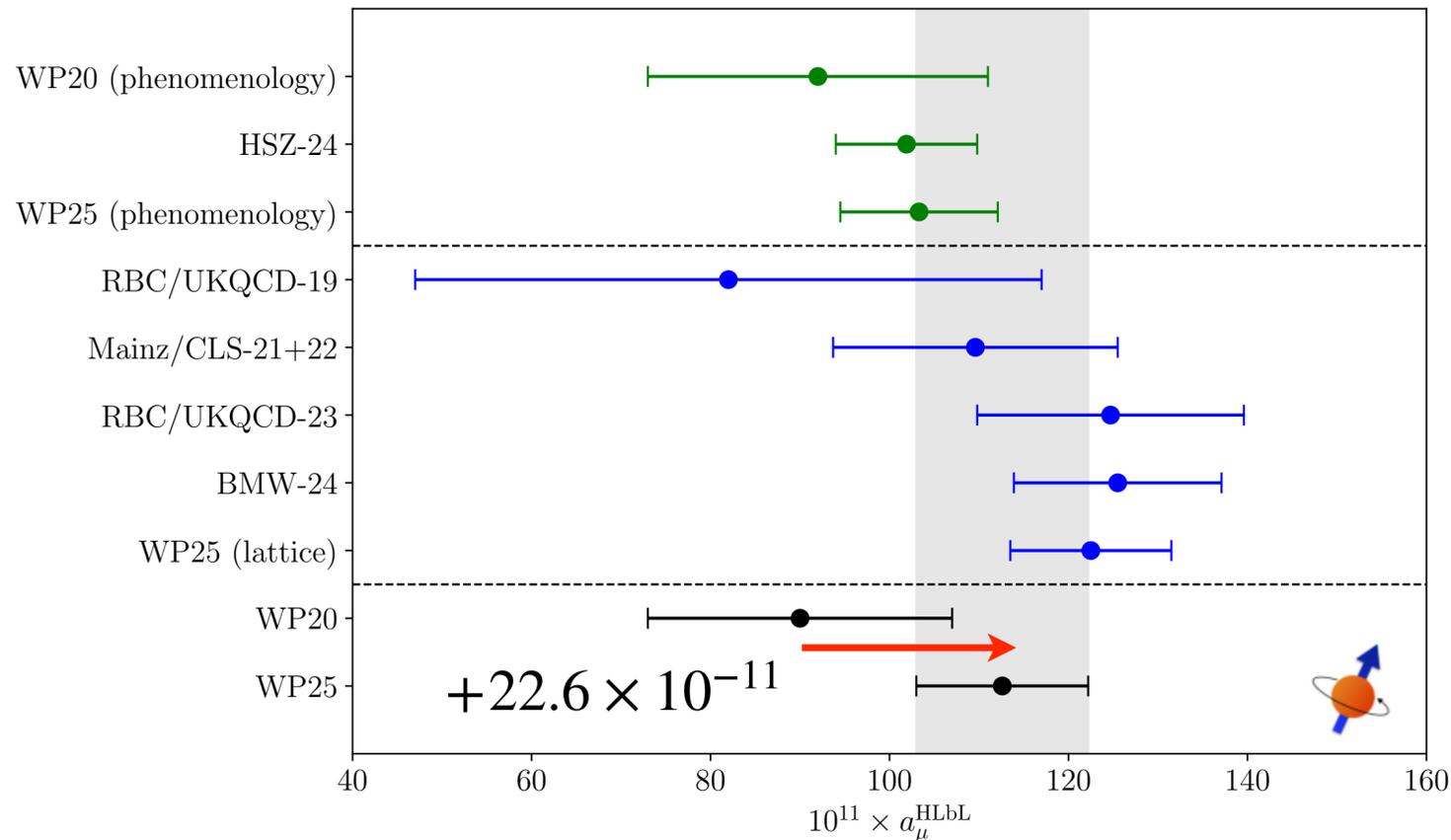


Hadronic Light-by-Light: Summary

$$a_{\mu}^{\text{HLbL}}$$

$$a_{\mu}^{\text{SM}}$$

$$a_{\mu}^{\text{HVP}} + [a_{\mu}^{\text{QED}} + a_{\mu}^{\text{Weak}} + a_{\mu}^{\text{HLbL}}]$$



Dispersive approach:

[Colangelo et al, 2014; Pauk & Vanderhaegen 2014; ...; Hoferichter et al, 2024]

- ◆ model independent
- ◆ significantly more complicated than for HVP
- ◆ provides a framework for data-driven evaluations
- ◆ can also use lattice results as inputs
- ◆ ongoing work on tensors
- ◆ now 10% uncertainty

Lattice QCD+QED:

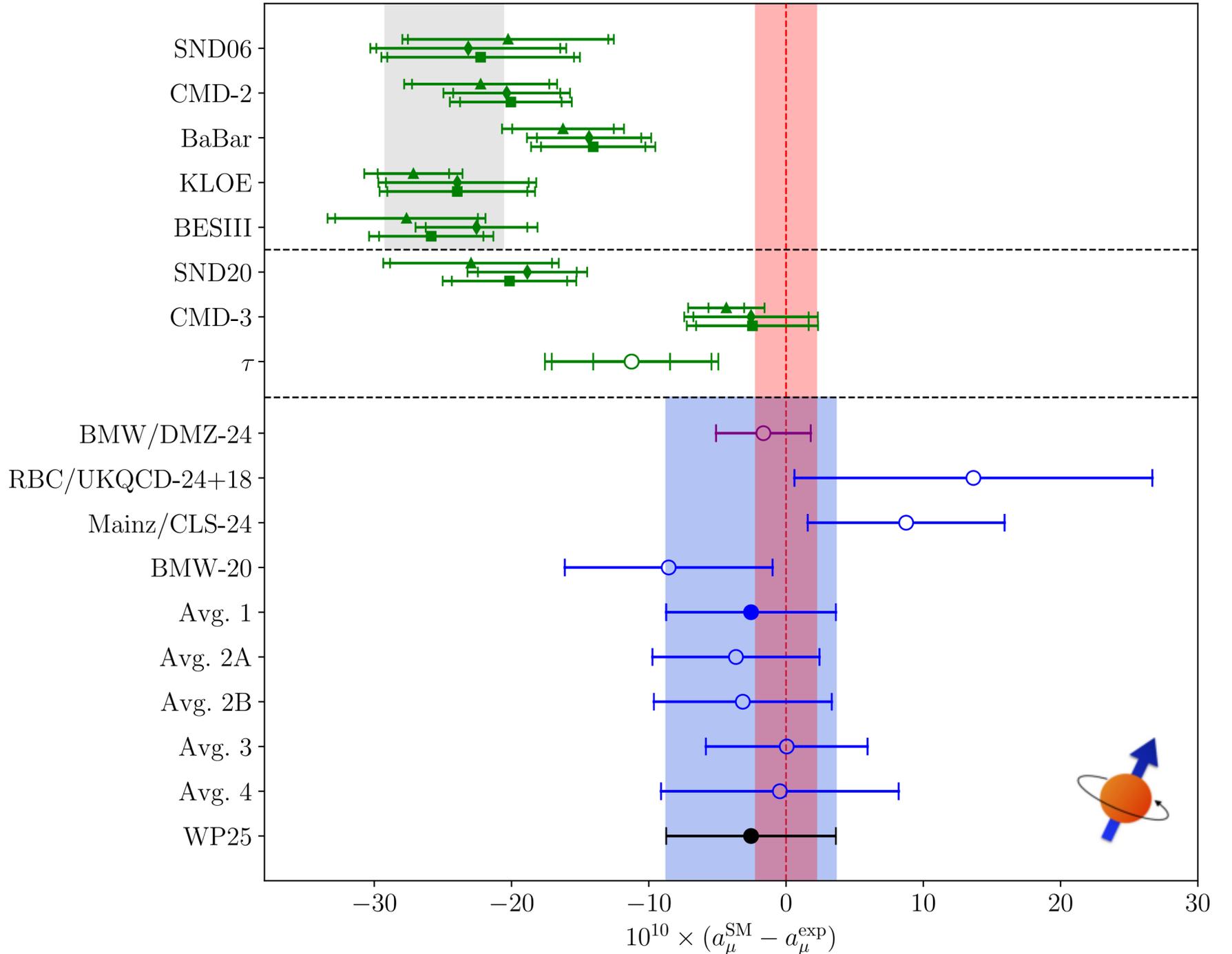
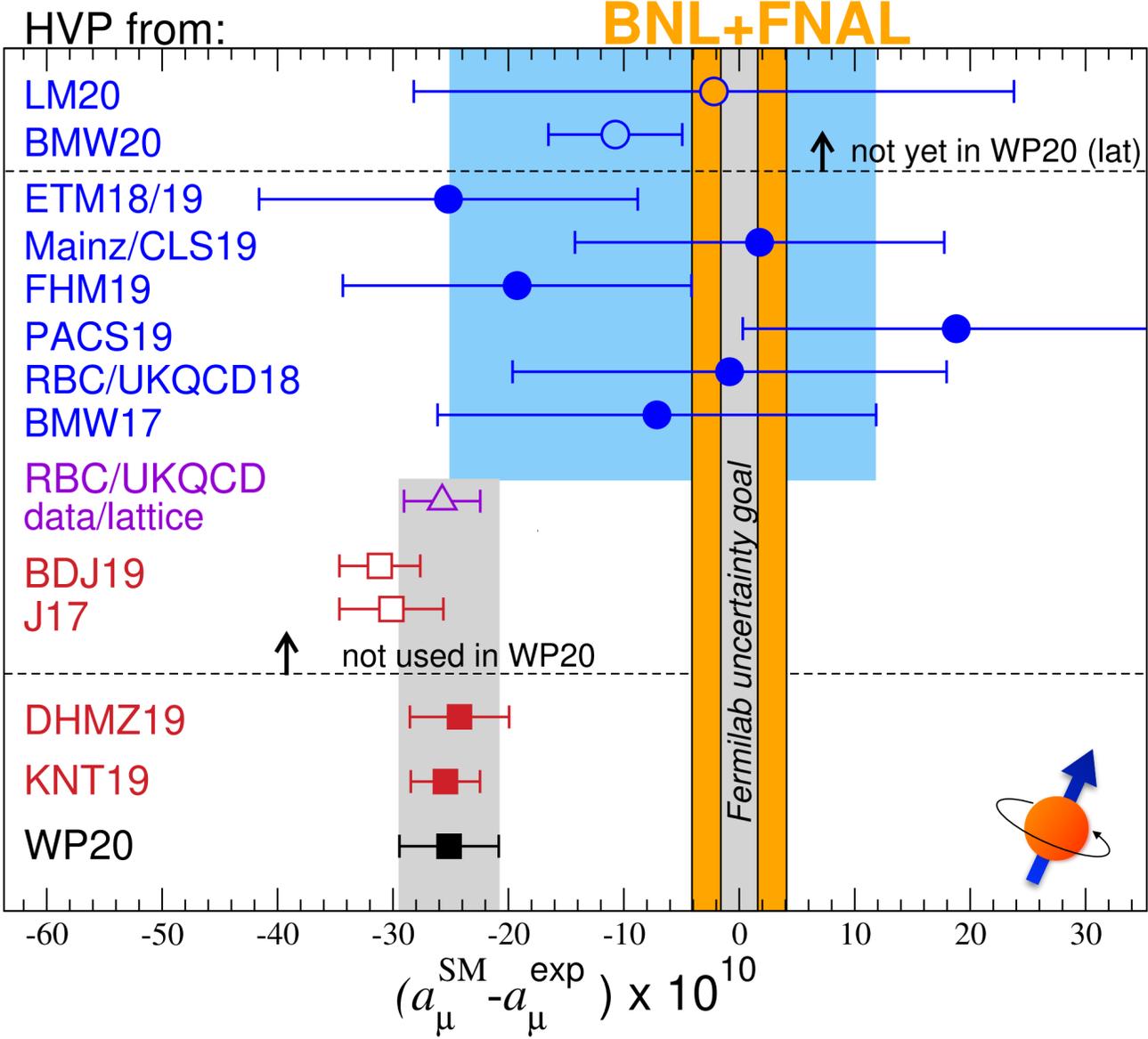
- ◆ Independent calculations by three groups (RBC/UKQCD, Mainz, BMW)
- ◆ consistent with each other and with previous calculations
- ◆ ongoing LQCD calculations of π, η, η' transition form factors to determine pseudo scalar pole contributions [Mainz, ETMC, BMW]

Experiment vs SM theory

$$a_\mu^{\text{SM}} = a_\mu^{\text{HVP}} + [a_\mu^{\text{QED}} + a_\mu^{\text{Weak}} + a_\mu^{\text{HLbL}}]$$

2025

2021



Science Magazine: 2025 breakthrough of the year runner-up

<https://www.science.org/content/article/breakthrough-2025>

2025 BREAKTHROUGH OF THE YEAR

PHOTO ESSAY

RUNNERS-UP

BREAKDOWNS

MULTIMEDIA

RUNNER-UP

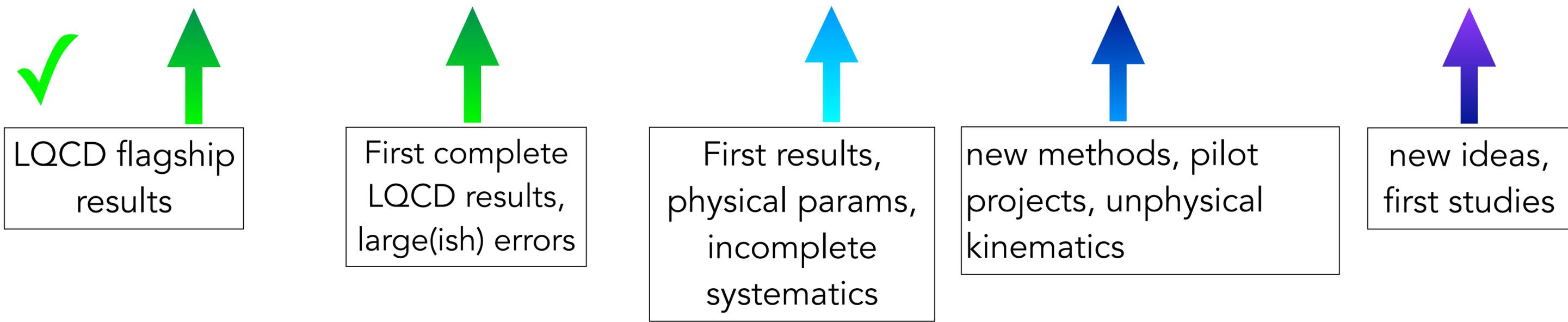
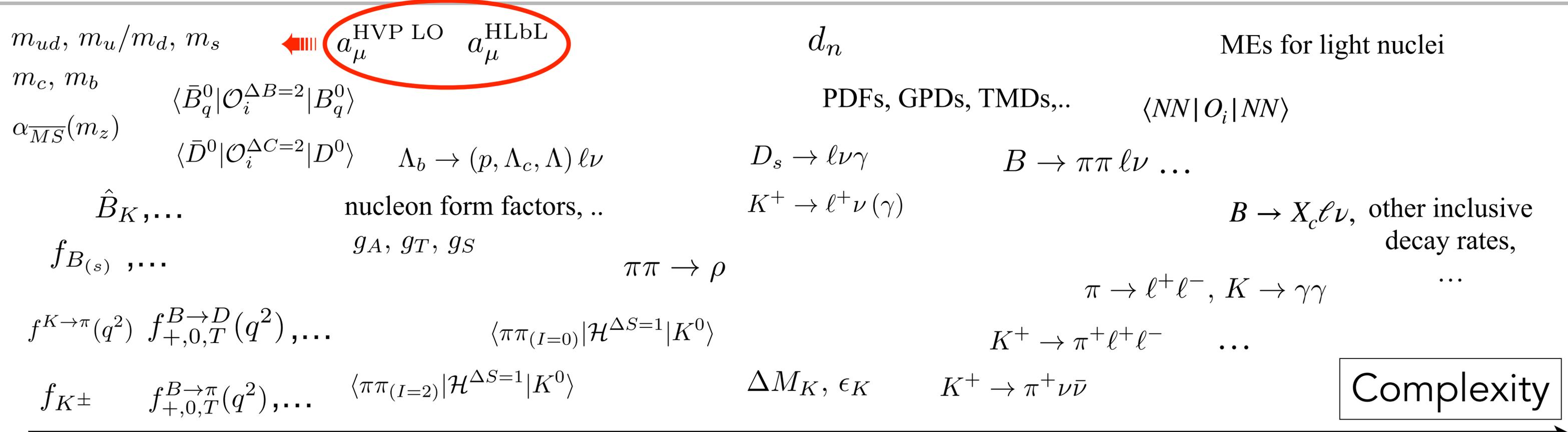
Triumph of calculation helps resolve particle mystery

BY ADRIAN CHO

For decades, particle physicists have longed for something—anything—their prevailing theory, the standard model, cannot explain. In June, perhaps the most tantalizing sign of a new mystery vanished when [a long-running experiment](#) reported that, contrary to its earlier claims, a particle called [the muon was not more magnetic than the standard model predicts](#). Behind the disappointment lurks a triumph: Theorists were finally able to calculate the muon's magnetism precisely from scratch using a technique called lattice gauge theory.

The muon is a heavier, unstable cousin of the electron. Its magnetism gets a small boost, denoted $g-2$, from particles that, because of quantum uncertainty, flit in and out of the vacuum surrounding the muon. If those “virtual” particles include ones not found in the standard model, the muon's magnetism could differ from the theory's predictions. Beginning in 2001, a U.S. experiment called Muon $g-2$ [suggested the muon's](#)

Summary and Outlook



Outlook

Connections and Extensions

- Generalized Symmetries:
 - ↔ LFTs with Symmetric Mass Generation
- S-matrix bootstrap
 - ↔ resonances, multi hadrons in LQCD
- pQCD ↔ LQCD
- precision physics ↔ model building
- LFTs for DM models, BSM
- vacuum decay in Euclidean LFT
- ...

Old problems and New Paradigms

- sign problem:
 - finite density, QCD phase diagram
- real-time dynamics
- Chiral gauge theories

- AI
- Quantum Computers and quantum simulators

Schematic of a pp scattering event

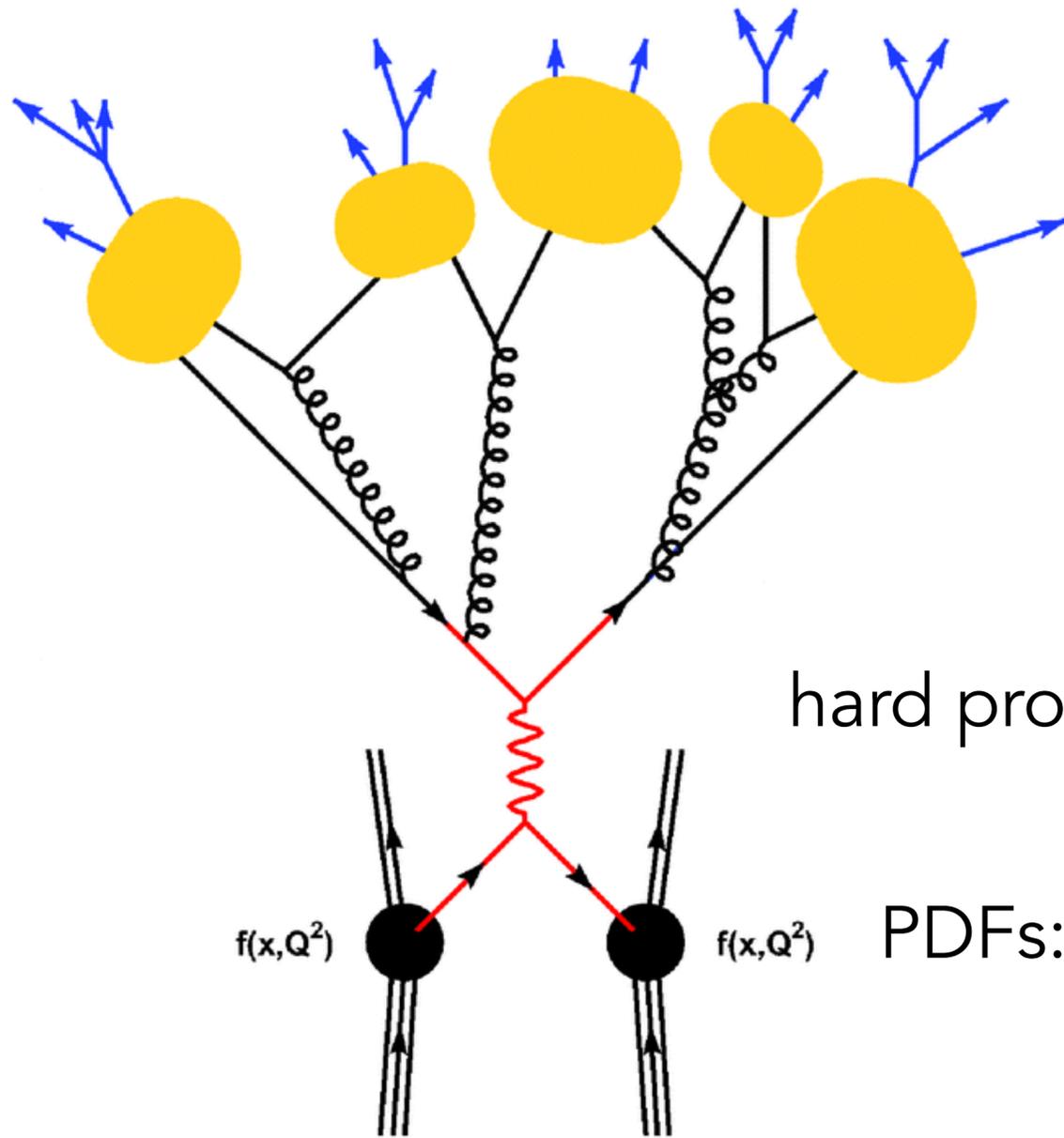
The dream: a completely *ab-initio* theoretical description of the scattering process from beginning to end \Rightarrow quantum computers

parton showers + hadronization:
nonperturbative — use models, approximations

hard process: perturbative QCD — Lattice QCD inputs for m_f, α_s

PDFs: nonperturbative — determine from other experiments

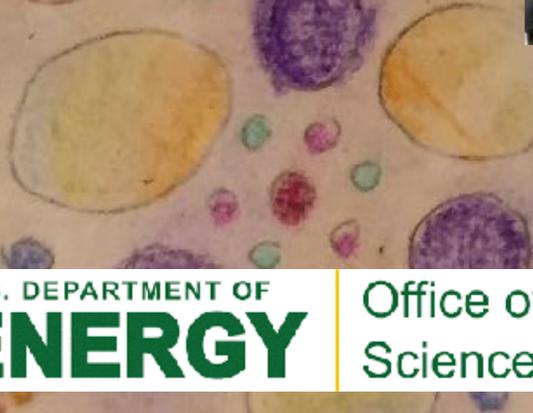
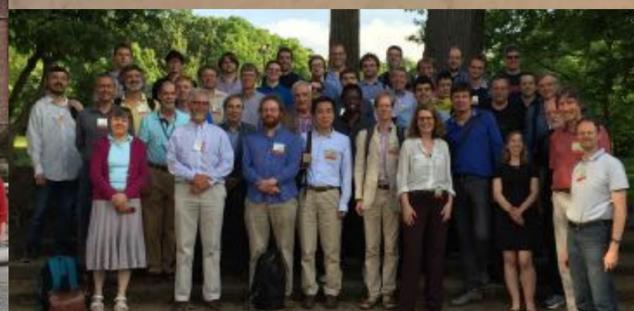
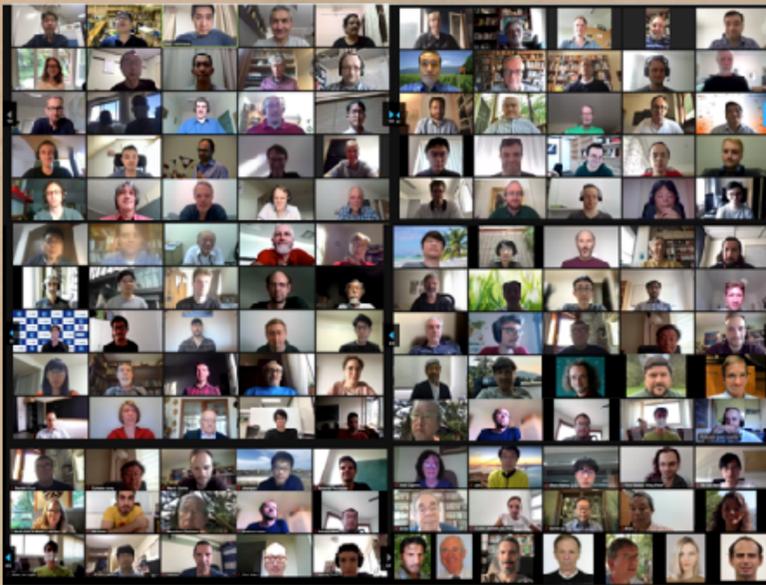
\Rightarrow future Lattice QCD inputs



Fermilab Lattice and MILC Collaboration

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- ❖ Claude Bernard (Wash U)
- ❖ **Nick Cassar** (CSU)
- ❖ **Akhil Chauhan** (UIUC)
- ❖ David Clarke (KFZ Julich)
- ❖ **Mingwei Dai** (UIUC)
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- ❖ **Dean Valois** (Granada)
- ❖ Ruth Van de Water (FNAL)





Thank you!

