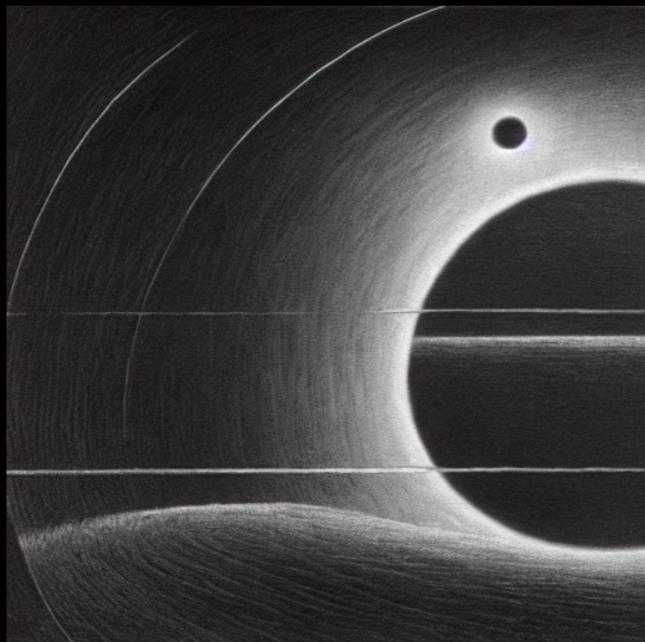# From Supercolliders to Black Holes

Clifford Cheung
California Institute of Technology

What defines a theory?

Consider how it works in particle mechanics.

$$S = \int dt \, \mathcal{L}$$

$$\mathcal{L} = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2$$

Consider how it works in particle mechanics.

$$S = \int dt\ \mathcal{L}$$

$$\mathcal{L} = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2$$

Consider how it works in particle mechanics.

locality in time

$$S = \int dt \, \mathcal{L}$$

parity symmetry

$$\mathcal{L} = \frac{1}{2}m\dot{x}^2 - \frac{1}{2}kx^2$$

parameters not fixed by symmetry are measured!

The same applies in quantum field theory (QFT).

$$S = \int d^4x \, \mathcal{L}$$

$$\mathcal{L} = \frac{1}{2} \partial_\mu \phi \partial^\mu \phi - \frac{1}{2} m^2 \phi^2 + \dots$$

The same applies in quantum field theory (QFT).

locality in spacetime

$$S = \int d^4x \, \mathcal{L}$$

Lorentz symmetry

$$\mathcal{L} = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi - \frac{1}{2}m^2\phi^2 + \dots$$

The same applies in quantum field theory (QFT).

locality in spacetime

$$S = \int d^4x \, \mathcal{L}$$

Lorentz symmetry

$$\mathcal{L} = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 + \ldots$$

parameters not fixed by
symmetry are measured!

physical principles

$\downarrow$

Lagrangian, Hamiltonian,
equations of motion, etc.
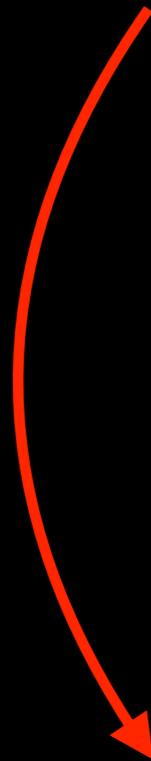
$\downarrow$

observables

physical principles

modern
scattering
amplitudes
program

~~Lagrangian, Hamiltonian, equations of motion, etc.~~
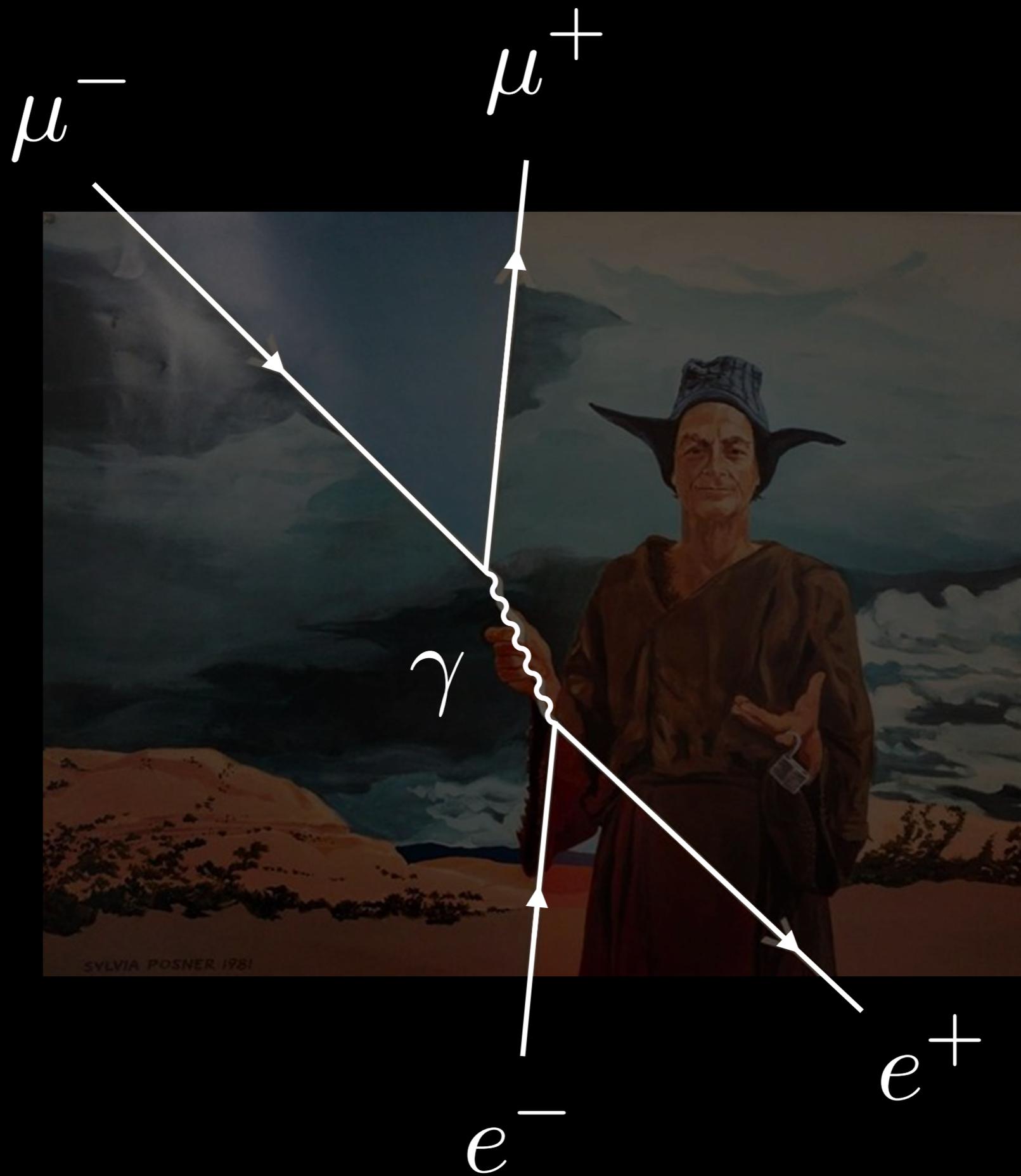
observables

Observables are computed with Feynman diagrams.

SYLVIA POSNER 1981

QFT makes astoundingly precise predictions that match beautifully with experimental results.
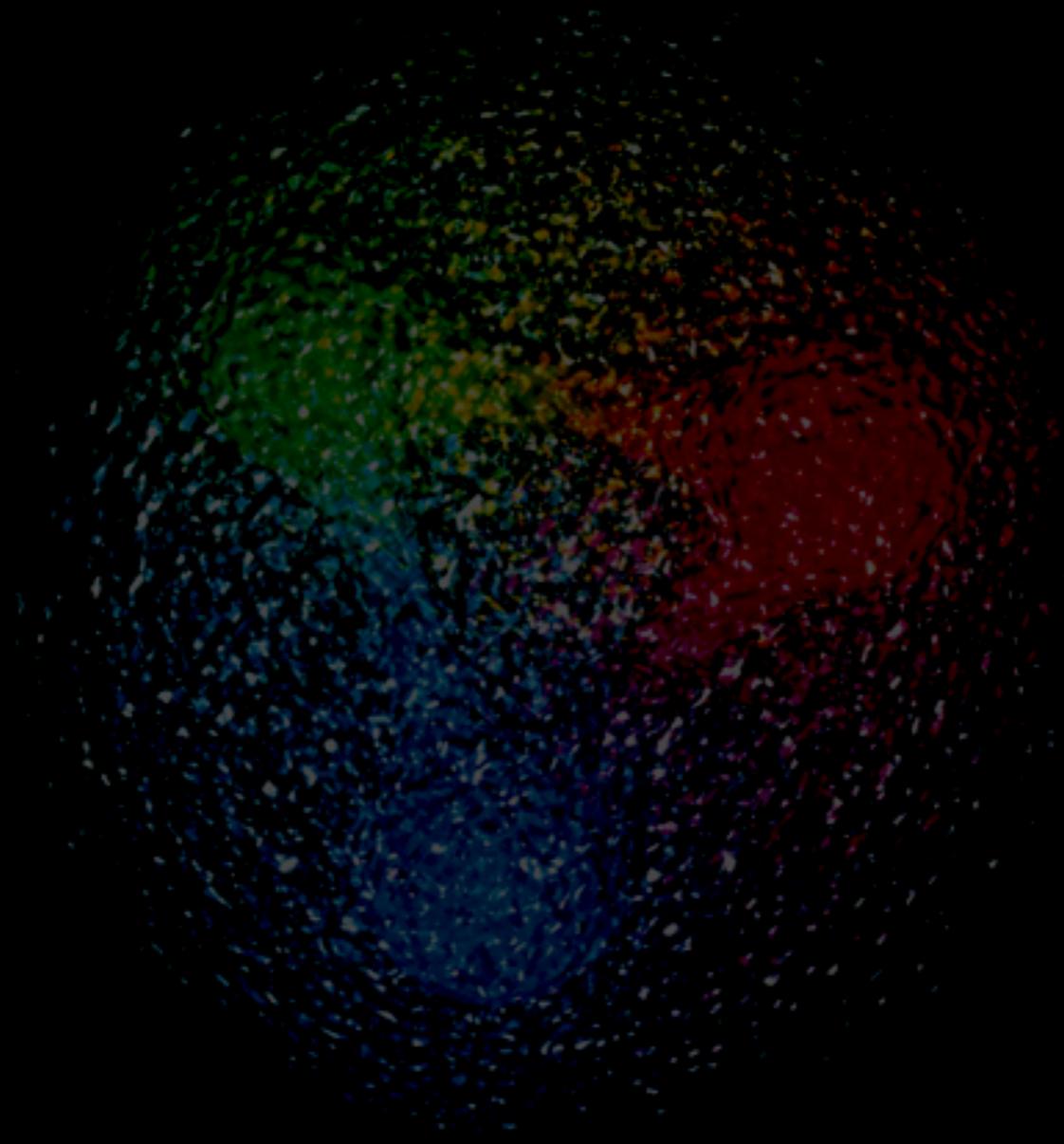
$$g/2 = 1.00115965219 \quad \text{(experiment)}$$

$$g/2 = 1.00115965217 \quad \text{(theory)}$$

But Feynman diagrams are *immensely* complicated.

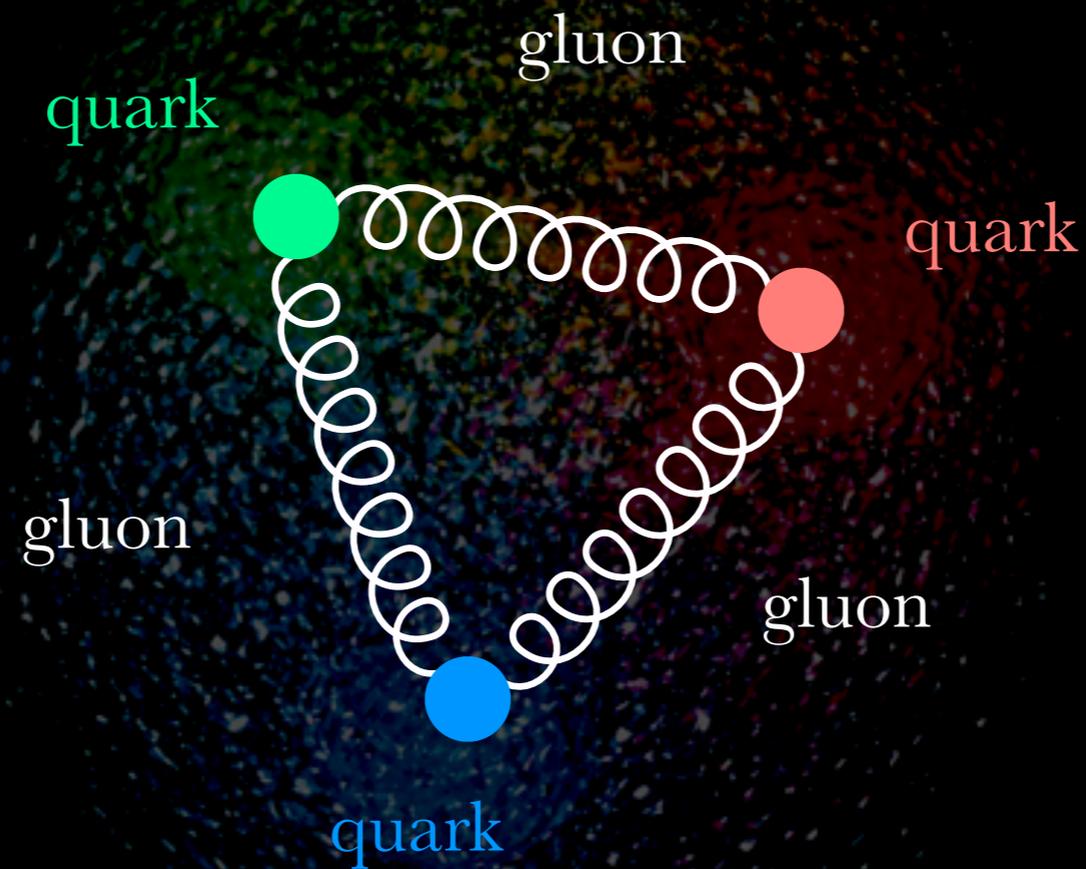But Feynman diagrams are *immensely* complicated.

proton

But Feynman diagrams are *immensely* complicated.

But Feynman diagrams are *immensely* complicated.

But Feynman diagrams are *immensely* complicated.

gluon

gluon

gluon

tree
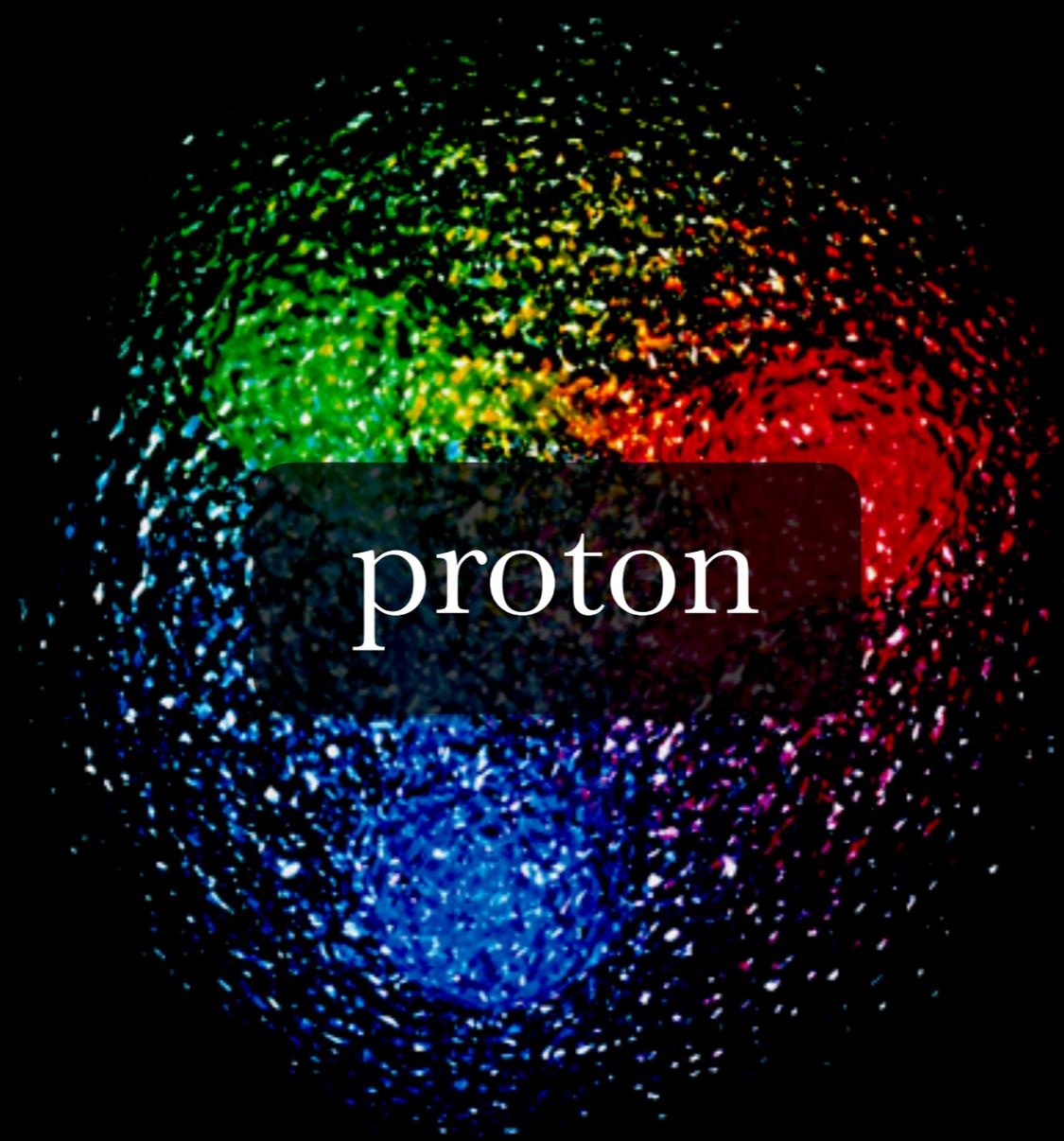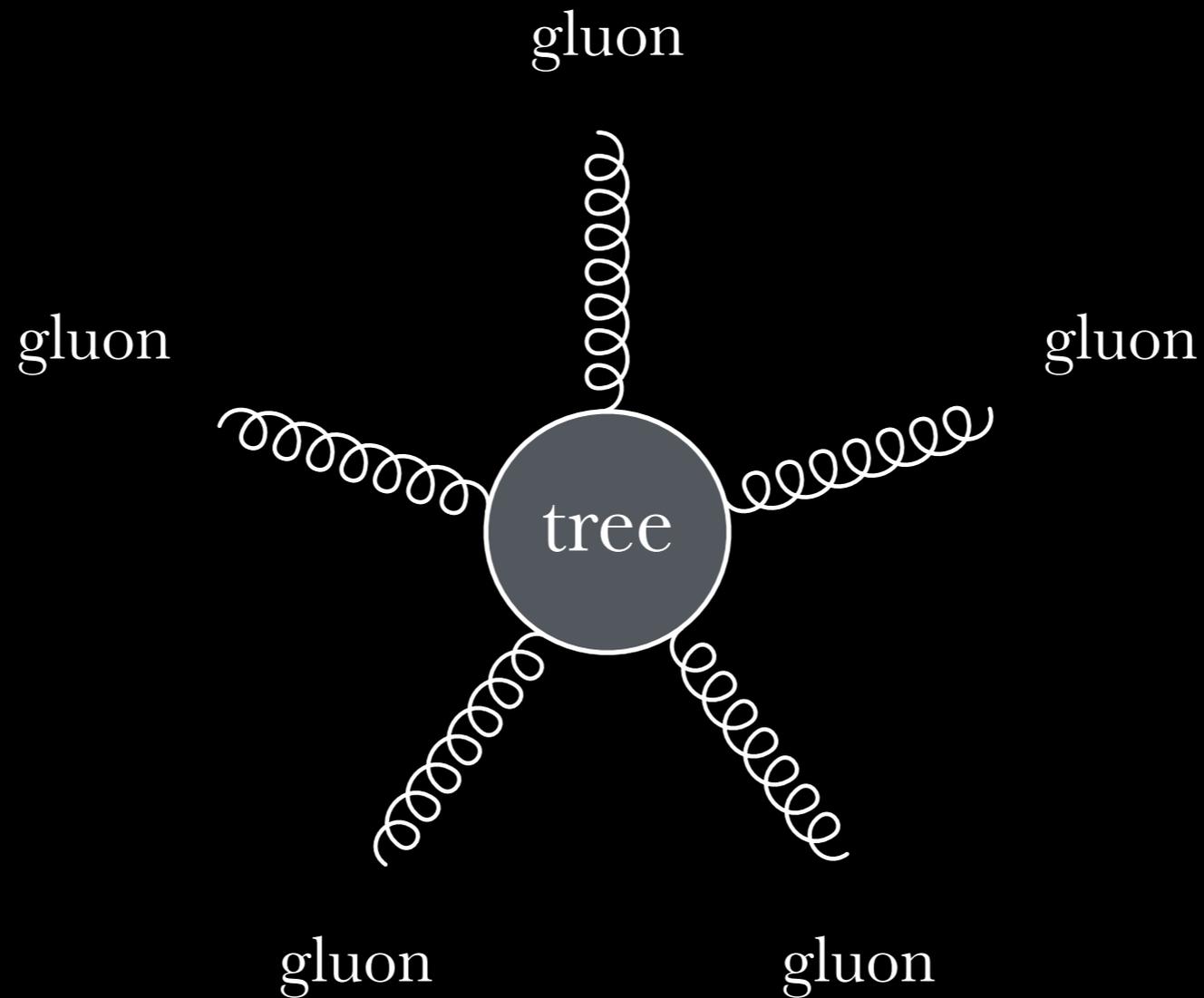
gluon

gluon

5pt gluon amplitude

But Feynman diagrams are *immensely* complicated.
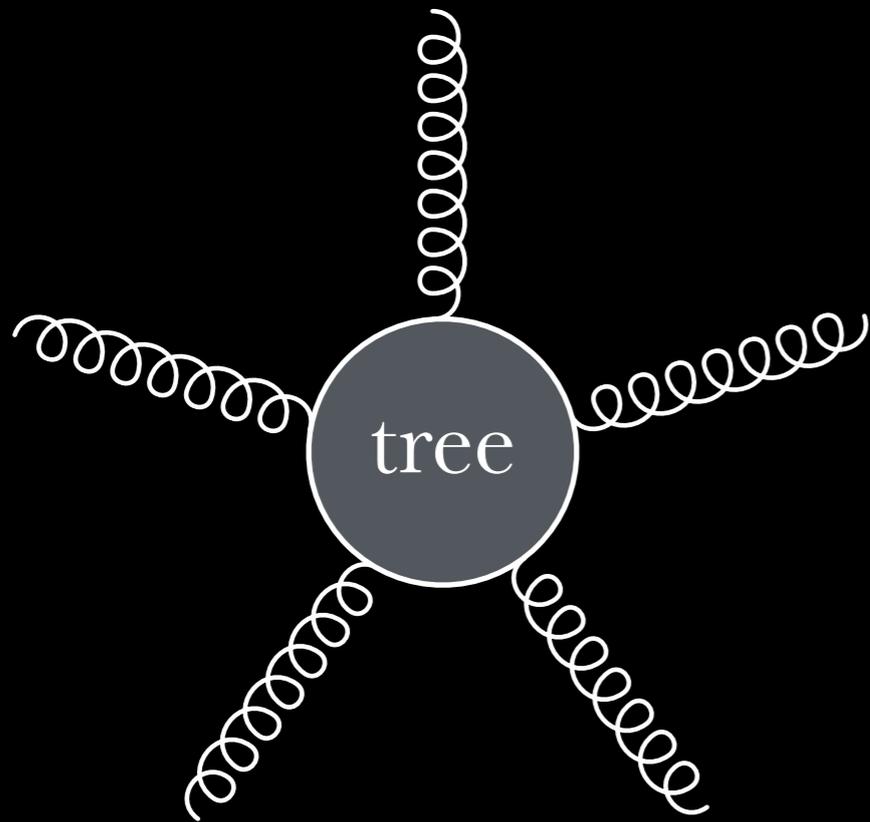
$$A(1^{h_1} 2^{h_2} 3^{h_3} 4^{h_4} 5^{h_5}) \quad =$$
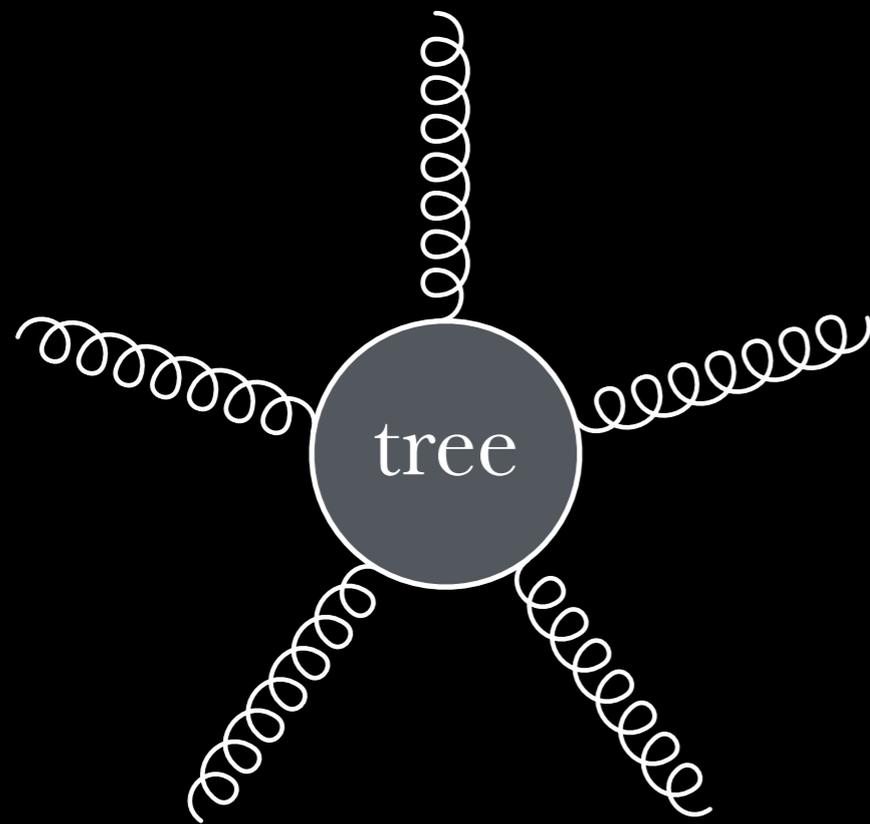
But Feynman diagrams are *immensely* complicated.

$$\sim \frac{(e_1 \cdot e_5)(p_1 \cdot e_2)(p_2 \cdot e_3)(p_3 \cdot e_4)}{(p_1 + p_2)^2 (p_4 + p_5)^2} + \ldots$$

tree

$$A(1^{h_1} 2^{h_2} 3^{h_3} 4^{h_4} 5^{h_5}) \quad =$$

But Feynman diagrams are *immensely* complicated.

$$A(1^+ \, 2^+ \, 3^+ \, 4^+ \, 5^+) \;=\; 0$$

But Feynman diagrams are *immensely* complicated.
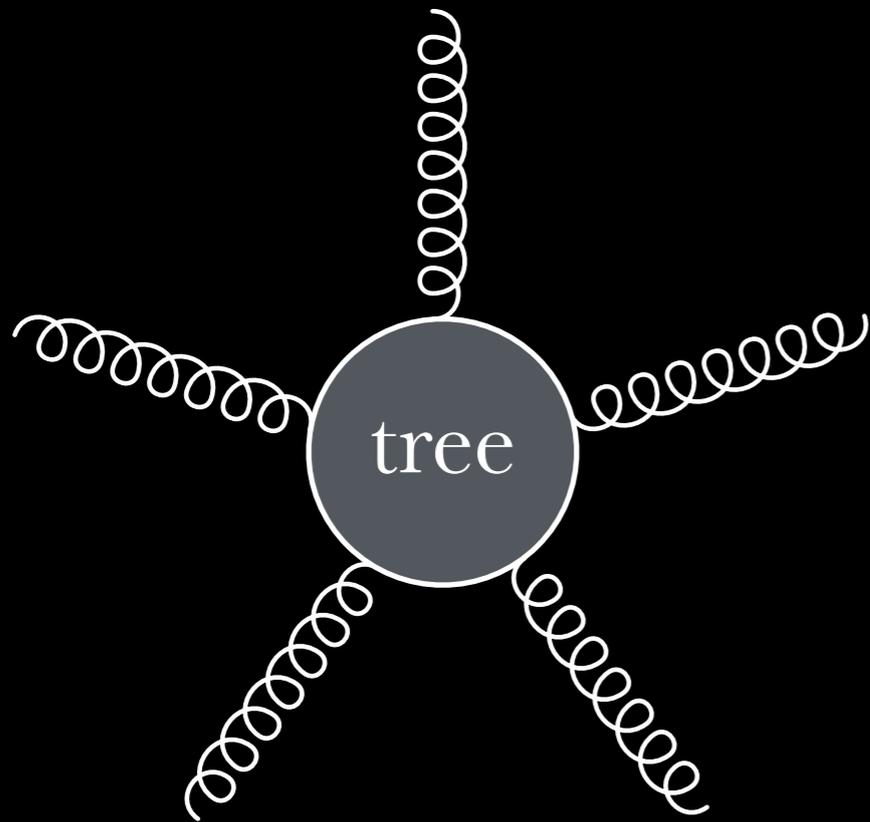
$$A(1^+ \, 2^+ \, 3^+ \, 4^+ \, 5^+) \;\; = \;\; 0$$

$$A(1^- \, 2^+ \, 3^+ \, 4^+ \, 5^+) \;\; = \;\; 0$$

But Feynman diagrams are *immensely* complicated.

$$A(1^+ \ 2^+ \ 3^+ \ 4^+ \ 5^+ \ ) \quad = \quad 0$$

$$A(1^- \ 2^+ \ 3^+ \ 4^+ \ 5^+ \ ) \quad = \quad 0$$

$$A(1^- \ 2^- \ 3^+ \ 4^+ \ 5^+ \ ) \quad = \quad \frac{\langle 12 \rangle^3}{\langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 51 \rangle}$$

Park, Taylor (1986)
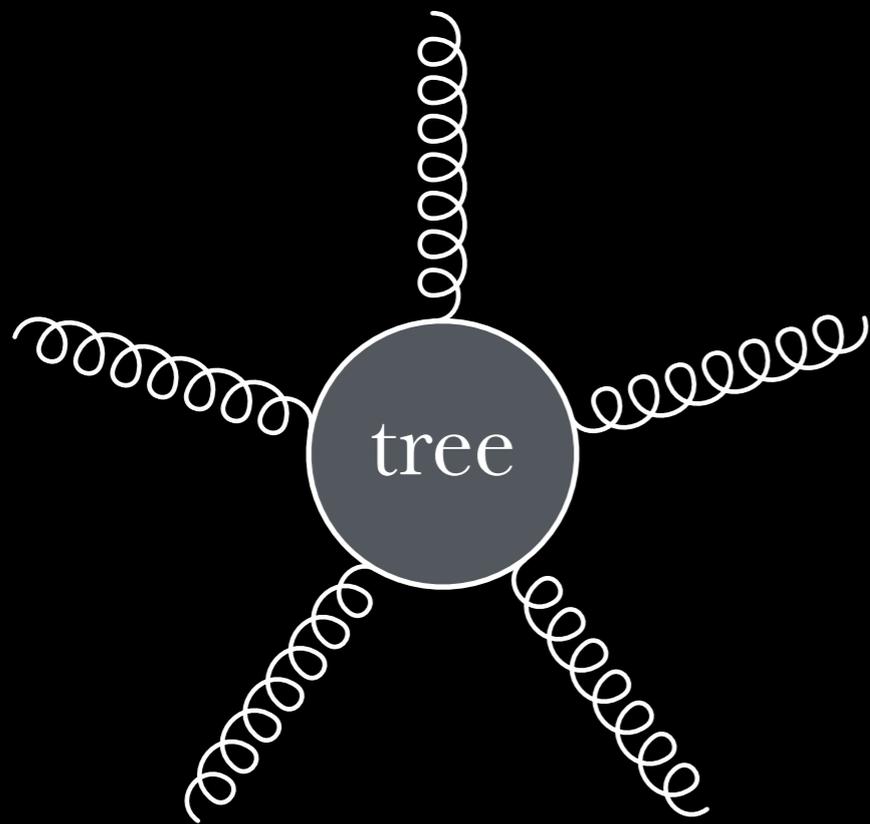
But Feynman diagrams are *immensely* complicated.
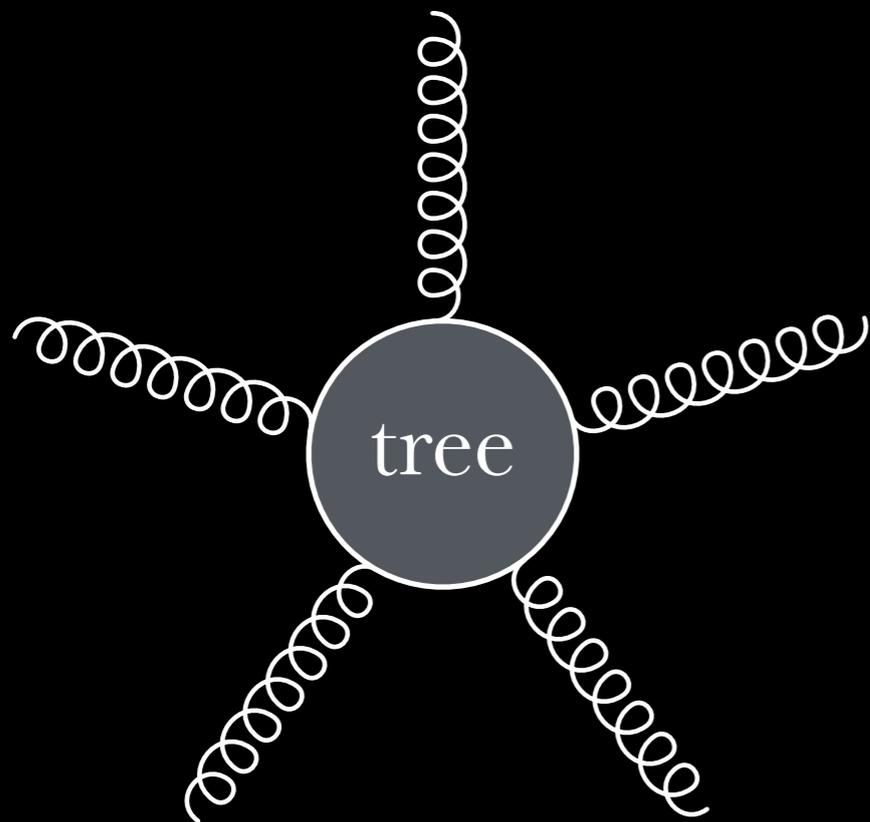
use the *right* variables

$$p_i^\mu = \sigma^\mu_{\alpha\dot\alpha} \lambda_i^\alpha \bar\lambda_i^{\dot\alpha}$$

$$\langle ij \rangle = \lambda_i^\alpha \lambda_j^\beta \epsilon_{\alpha\beta}$$

$$[ij] = \bar\lambda_i^{\dot\alpha} \bar\lambda_j^{\dot\beta} \epsilon_{\dot\alpha\dot\beta}$$

$$A(1^+\, 2^+\, 3^+\, 4^+\, 5^+\,) \quad = \quad 0$$

$$A(1^-\, 2^+\, 3^+\, 4^+\, 5^+\,) \quad = \quad 0$$

$$A(1^-\, 2^-\, 3^+\, 4^+\, 5^+\,) \quad = \quad \frac{\langle 12 \rangle^3}{\langle 23 \rangle \langle 34 \rangle \langle 45 \rangle \langle 51 \rangle}$$

Park, Taylor (1986)

A simplification also happens for gravitons.

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$$\frac{\delta^3 S}{\delta\varphi_{\mu\nu}\delta\varphi_{\sigma'\tau'}\delta\varphi_{\rho''\lambda''}} \longrightarrow$$

$$\mathrm{Sym}[-\tfrac{1}{4}P_3(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\lambda})-\tfrac{1}{4}P_6(p^\sigma p^\tau\eta^{\mu\nu}\eta^{\rho\lambda})+\tfrac{1}{4}P_3(p\cdot p'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\lambda})+\tfrac{1}{2}P_6(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\rho}\eta^{\tau\lambda})+P_3(p^\sigma p^\lambda\eta^{\mu\nu}\eta^{\tau\rho})$$

$$-\tfrac{1}{2}P_3(p^\tau p'^\mu\eta^{\nu\sigma}\eta^{\rho\lambda})+\tfrac{1}{2}P_3(p^\rho p'^\lambda\eta^{\mu\sigma}\eta^{\nu\tau})+\tfrac{1}{2}P_6(p^\rho p^\lambda\eta^{\mu\sigma}\eta^{\nu\tau})+P_6(p^\sigma p'^\lambda\eta^{\tau\mu}\eta^{\nu\rho})+P_3(p^\sigma p'^\mu\eta^{\tau\rho}\eta^{\lambda\nu})$$

$$-P_3(p\cdot p'\eta^{\nu\sigma}\eta^{\tau\rho}\eta^{\lambda\mu})], \quad (2.6)$$

$$\frac{\delta^4 S}{\delta\varphi_{\mu\nu}\delta\varphi_{\sigma'\tau'}\delta\varphi_{\rho''\lambda''}\delta\varphi_{\iota'''\kappa'''}} \longrightarrow$$

$$\mathrm{Sym}[-\tfrac{1}{8}P_6(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\lambda}\eta^{\iota\kappa})-\tfrac{1}{8}P_{12}(p^\sigma p^\tau\eta^{\mu\nu}\eta^{\rho\lambda}\eta^{\iota\kappa})-\tfrac{1}{4}P_6(p^\sigma p'^\mu\eta^{\nu\tau}\eta^{\rho\lambda}\eta^{\iota\kappa})+\tfrac{1}{8}P_6(p\cdot p'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\lambda}\eta^{\iota\kappa})$$

$$+\tfrac{1}{4}P_6(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})+\tfrac{1}{4}P_{12}(p^\sigma p^\tau\eta^{\mu\nu}\eta^{\rho\iota}\eta^{\lambda\kappa})+\tfrac{1}{2}P_6(p^\sigma p'^\mu\eta^{\nu\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})-\tfrac{1}{4}P_6(p\cdot p'\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\rho\iota}\eta^{\lambda\kappa})$$

$$+\tfrac{1}{4}P_{24}(p\cdot p'\eta^{\mu\nu}\eta^{\sigma\rho}\eta^{\tau\lambda}\eta^{\iota\kappa})+\tfrac{1}{4}P_{24}(p^\sigma p^\tau\eta^{\mu\rho}\eta^{\nu\lambda}\eta^{\iota\kappa})+\tfrac{1}{4}P_{12}(p^\rho p'^\lambda\eta^{\mu\sigma}\eta^{\nu\tau}\eta^{\iota\kappa})+\tfrac{1}{2}P_{24}(p^\sigma p'^\rho\eta^{\tau\mu}\eta^{\nu\lambda}\eta^{\iota\kappa})$$

$$-\tfrac{1}{2}P_{12}(p\cdot p'\eta^{\nu\sigma}\eta^{\tau\rho}\eta^{\lambda\mu}\eta^{\iota\kappa})-\tfrac{1}{2}P_{12}(p^\sigma p'^\mu\eta^{\tau\rho}\eta^{\lambda\nu}\eta^{\iota\kappa})+\tfrac{1}{2}P_{12}(p^\sigma p^\rho\eta^{\tau\lambda}\eta^{\mu\nu}\eta^{\iota\kappa})-\tfrac{1}{2}P_{24}(p\cdot p'\eta^{\mu\nu}\eta^{\tau\rho}\eta^{\lambda\iota}\eta^{\kappa\sigma})$$

$$-P_{12}(p^\sigma p^\tau\eta^{\nu\rho}\eta^{\lambda\iota}\eta^{\kappa\mu})-P_{12}(p^\rho p'^\lambda\eta^{\nu\iota}\eta^{\kappa\sigma}\eta^{\tau\mu})-P_{24}(p_\sigma p'^\rho\eta^{\tau\iota}\eta^{\kappa\mu}\eta^{\nu\lambda})-P_{12}(p^\rho p'^\iota\eta^{\lambda\sigma}\eta^{\tau\mu}\eta^{\nu\kappa})$$

$$+P_6(p\cdot p'\eta^{\nu\rho}\eta^{\lambda\sigma}\eta^{\tau\iota}\eta^{\kappa\mu})-P_{12}(p^\sigma p^\rho\eta^{\mu\nu}\eta^{\tau\iota}\eta^{\kappa\lambda})-\tfrac{1}{2}P_{12}(p\cdot p'\eta^{\mu\rho}\eta^{\nu\lambda}\eta^{\sigma\iota}\eta^{\tau\kappa})-P_{12}(p^\sigma p^\rho\eta^{\tau\lambda}\eta^{\mu\iota}\eta^{\nu\kappa})$$

$$-P_6(p^\rho p'^\iota\eta^{\lambda\kappa}\eta^{\mu\sigma}\eta^{\nu\tau})-P_{24}(p^\sigma p'^\rho\eta^{\tau\mu}\eta^{\nu\iota}\eta^{\kappa\lambda})-P_{12}(p^\sigma p'^\mu\eta^{\tau\rho}\eta^{\lambda\iota}\eta^{\kappa\nu})+2P_6(p\cdot p'\eta^{\nu\sigma}\eta^{\tau\rho}\eta^{\lambda\iota}\eta^{\kappa\mu})]. \quad (2.7)$$

3pt graviton Feynman vertex

4pt graviton Feynman vertex

DeWitt (1967)

A simplification also happens for gravitons.

$$\frac{\delta^3 S}{\delta \varphi_{\mu\nu} \delta \varphi_{\sigma'\tau'} \delta \varphi_{\rho''\lambda''}} \longrightarrow$$

$$\mathrm{Sym}[-\tfrac{1}{4}P_3(p \cdot p' \eta^{\mu\nu} \eta^{\sigma\tau} \eta^{\rho\lambda}) - \tfrac{1}{4}P_6(p^\sigma p^\tau \eta^{\mu\nu} \eta^{\rho\lambda}) + \tfrac{1}{4}P_3(p \cdot p' \eta^{\mu\sigma} \eta^{\nu\tau} \eta^{\rho\lambda}) + \tfrac{1}{2}P_6(p \cdot p' \eta^{\mu\nu} \eta^{\sigma\rho} \eta^{\tau\lambda}) + P_3(p^\sigma p^\lambda \eta^{\mu\nu} \eta^{\tau\rho})$$

$$-\tfrac{1}{2}P_3(p^\tau p'^\mu \eta^{\nu\sigma} \eta^{\rho\lambda}) + \tfrac{1}{2}P_3(p^\rho p'^\lambda \eta^{\mu\sigma} \eta^{\nu\tau}) + \tfrac{1}{2}P_6(p^\rho p^\lambda \eta^{\mu\sigma} \eta^{\nu\tau}) + P_6(p^\sigma p'^\lambda \eta^{\tau\mu} \eta^{\nu\rho}) + P_3(p^\sigma p'^\mu \eta^{\tau\rho} \eta^{\lambda\nu})$$

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$$+\tfrac{1}{4}P_6(p \cdot p' \eta^{\mu\nu} \eta^{\sigma\tau} \eta^{\rho\iota} \eta^{\lambda\kappa}) + \tfrac{1}{4}P_{12}(p^\sigma p^\tau \eta^{\mu\nu} \eta^{\rho\iota} \eta^{\lambda\kappa}) + \tfrac{1}{2}P_6(p^\sigma p'^\mu \eta^{\nu\tau} \eta^{\rho\iota} \eta^{\lambda\kappa}) - \tfrac{1}{4}P_6(p \cdot p' \eta^{\mu\sigma} \eta^{\nu\tau} \eta^{\rho\iota} \eta^{\lambda\kappa})$$

$$+\tfrac{1}{4}P_{24}(p \cdot p' \eta^{\mu\nu} \eta^{\sigma\rho} \eta^{\tau\lambda} \eta^{\iota\kappa}) + \tfrac{1}{4}P_{24}(p^\sigma p^\tau \eta^{\mu\rho} \eta^{\nu\lambda} \eta^{\iota\kappa}) + \tfrac{1}{4}P_{12}(p^\rho p'^\lambda \eta^{\mu\sigma} \eta^{\nu\tau} \eta^{\iota\kappa}) + \tfrac{1}{2}P_{24}(p^\sigma p'^\rho \eta^{\tau\mu} \eta^{\nu\lambda} \eta^{\iota\kappa})$$

$$-\tfrac{1}{2}P_{12}(p \cdot p' \eta^{\nu\sigma} \eta^{\tau\rho} \eta^{\lambda\mu} \eta^{\iota\kappa}) - \tfrac{1}{2}P_{12}(p^\sigma p'^\mu \eta^{\tau\rho} \eta^{\lambda\nu} \eta^{\iota\kappa}) + \tfrac{1}{2}P_{12}(p^\sigma p^\rho \eta^{\tau\lambda} \eta^{\mu\nu} \eta^{\iota\kappa}) - \tfrac{1}{2}P_{24}(p \cdot p' \eta^{\mu\nu} \eta^{\tau\rho} \eta^{\lambda\iota} \eta^{\kappa\sigma})$$

$$-P_{12}(p^\sigma p^\tau \eta^{\nu\rho} \eta^{\lambda\iota} \eta^{\kappa\mu}) - P_{12}(p^\rho p'^\lambda \eta^{\nu\iota} \eta^{\kappa\sigma} \eta^{\tau\mu}) - P_{24}(p_\sigma p'^\rho \eta^{\tau\iota} \eta^{\kappa\mu} \eta^{\nu\lambda}) - P_{12}(p^\rho p'^\iota \eta^{\lambda\sigma} \eta^{\tau\mu} \eta^{\nu\kappa})$$

$$+P_6(p \cdot p' \eta^{\nu\rho} \eta^{\lambda\sigma} \eta^{\tau\iota} \eta^{\kappa\mu}) - P_{12}(p^\sigma p^\rho \eta^{\mu\nu} \eta^{\tau\iota} \eta^{\kappa\lambda}) - \tfrac{1}{2}P_{12}(p \cdot p' \eta^{\mu\rho} \eta^{\nu\lambda} \eta^{\sigma\iota} \eta^{\tau\kappa}) - P_{12}(p^\sigma p^\rho \eta^{\tau\lambda} \eta^{\mu\iota} \eta^{\nu\kappa})$$

$$-P_6(p^\rho p'^\iota \eta^{\lambda\kappa} \eta^{\mu\sigma} \eta^{\nu\tau}) - P_{24}(p^\sigma p'^\rho \eta^{\tau\mu} \eta^{\nu\iota} \eta^{\kappa\lambda}) - P_{12}(p^\sigma p'^\mu \eta^{\tau\rho} \eta^{\lambda\iota} \eta^{\kappa\nu}) + 2P_6(p \cdot p' \eta^{\nu\sigma} \eta^{\tau\rho} \eta^{\lambda\iota} \eta^{\kappa\mu})]. \quad (2.7)$$

3pt graviton Feynman vertex

4pt graviton Feynman vertex

4pt graviton amplitude

$$M(1^- \ 2^- \ 3^+ \ 4^+ \ ) \quad = \quad \frac{\langle 12 \rangle^4 [34]^4}{stu}$$

DeWitt (1967)

Note: simplification is a nice target for *machine learning*.

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Learning the Simplicity of Scattering Amplitudes

Clifford Cheung[a] Aurélien Dersy[b,c] Matthew D. Schwartz[b,c]

[a] *Walter Burke Institute for Theoretical Physics,*
California Institute of Technology, 91125 Pasadena, CA, USA
[b] *Department of Physics, Harvard University , 02138 Cambridge, MA, USA*
[c] *NSF Institute for Artificial Intelligence and Fundamental Interactions*
E-mail: clifford.cheung@caltech.edu, adersy@g.harvard.edu,
schwartz@g.harvard.edu

https://spinorhelicity.streamlit.app/

Cheung, Dersy, Schwartz (2408.04720)

Note: simplification is a nice target for *machine learning*.

input an
equation

Input Equation

(-(ab(1,2)**2*ab(1,5)**2*ab(2,3)*ab(3,4)

$\langle 12\rangle^2 \langle 15\rangle^2 \langle 34\rangle \, [13]^2 \, [15] \, [24] \, [25] - \langle 12\rangle^2 \langle 15\rangle^2 \langle 34\rangle \, [$

go get
a coffee

```
Starting the simplification of 53 terms !

We have 39 terms left in the numerator

We have 23 terms left in the numerator

We have 4 terms left in the numerator
```

replace all
humans

```
Went from 53 to 1 terms in 34 simplification steps and 113.6 seconds
```

Simplified form: $-\dfrac{\langle 12\rangle^3}{\langle 15\rangle \langle 23\rangle \langle 34\rangle \langle 45\rangle}$

Why the unnecessary complexity?

This complexity arises from gauge "symmetries" of the action which are in truth redundancies.

$$V \to V - \frac{\partial \theta}{\partial t}$$

$$\vec{A} \to \vec{A} + \vec{\nabla}\theta$$

High Voltage Differential!

But the physically observable fields are invariant.

$$\vec{E} \to \vec{E} \qquad \vec{B} \to \vec{B}$$

To manifest locality and Lorentz invariance we introduce a four-vector field.

$$A_\mu = (V, \vec{A})$$

four degrees
of freedom

two polarizations

We are *forced* to invent a gauge symmetry in order to remove the extra degrees of freedom.

A byproduct of gauge symmetry is complexity that worsens at higher orders in perturbation theory.

n-pt	4	5	6	7	8
Feynman diagrams	4	25	220	2485	34300
modern methods	1	1	3	6	20

A byproduct of gauge symmetry is complexity that worsens at higher orders in perturbation theory.

n-pt	4	5	6	7	8
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modern methods	1	1	3	6	20

On-shell amplitudes are field basis invariant.

$$Z[J] \sim \int [d\phi]\, e^{iS[\phi] + i\int J\phi}$$

So many Lagrangians yield the same observables!

$$\mathcal{L}_1 \qquad \mathcal{L}_2 \qquad \cdots \qquad \mathcal{L}_\infty$$

$$A$$
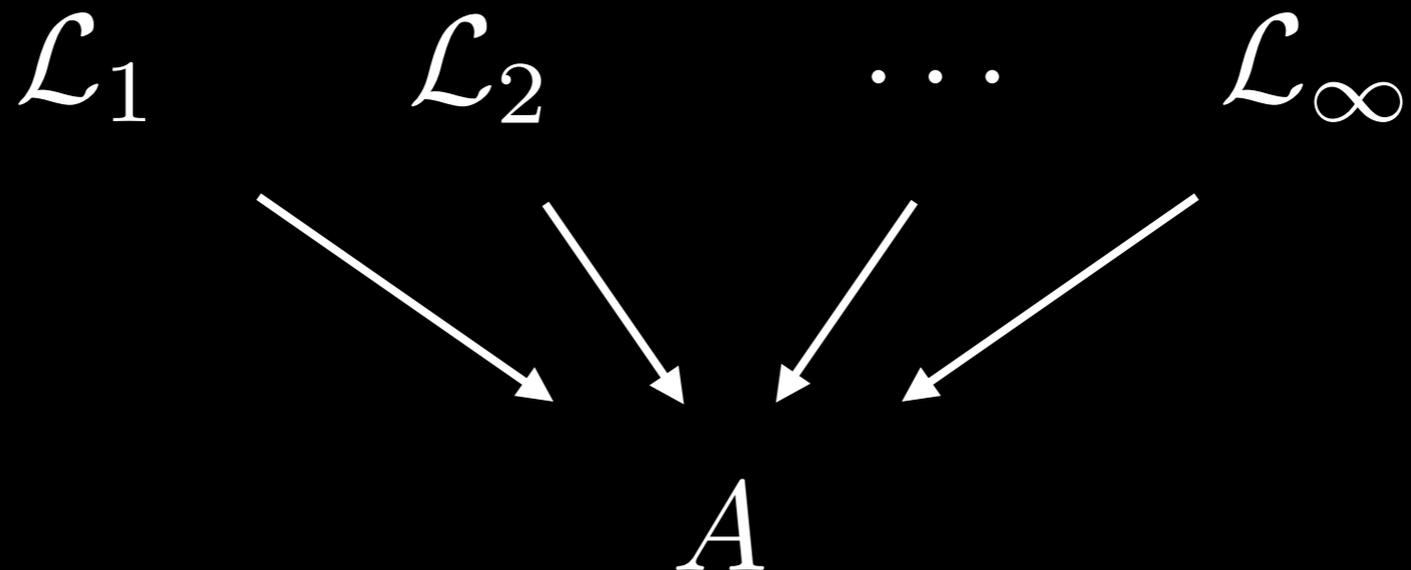
Bottom line: the action obscures important physics.

- rigidity in the laws of nature

  "only of all possible worlds…"

- hidden unity of gluons, pions, gravitons

  "gravity as the mother of all theories…"

- new perspectives on old questions

  "scattering from LHC to LIGO…"

rigidity in the
laws of nature

Nature conforms to at least two physical criteria:

- Special Relativity

- Locality

These principles *uniquely* define certain theories!

self-interacting massless spin two → gravity !

The magic ingredient in scattering theory is the most famous equation in physics.

$$E = mc^2 + \frac{\vec{p}^2}{2m} + \cdots$$

$$= \sqrt{\vec{p}^2 c^2 + m^2 c^4}$$
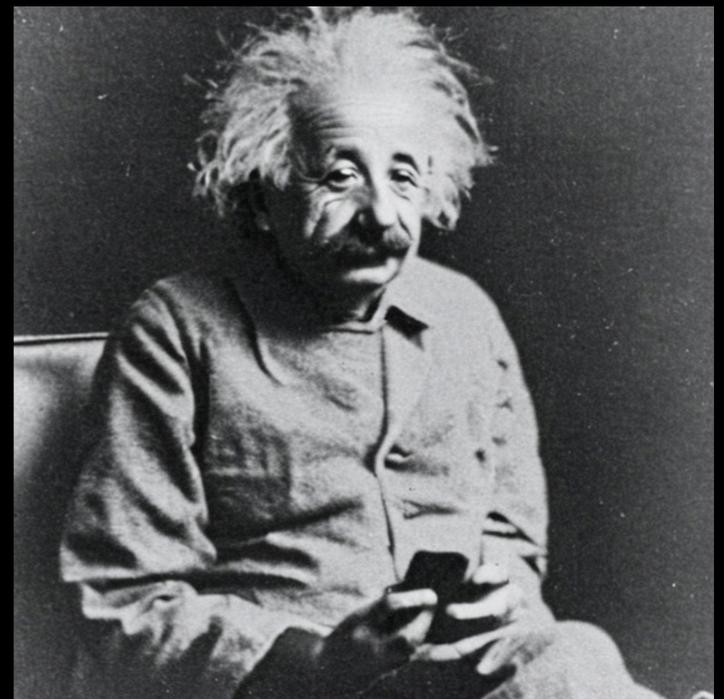
"on-shell condition"

Relativity dictates *every* massless 3pt amplitude!
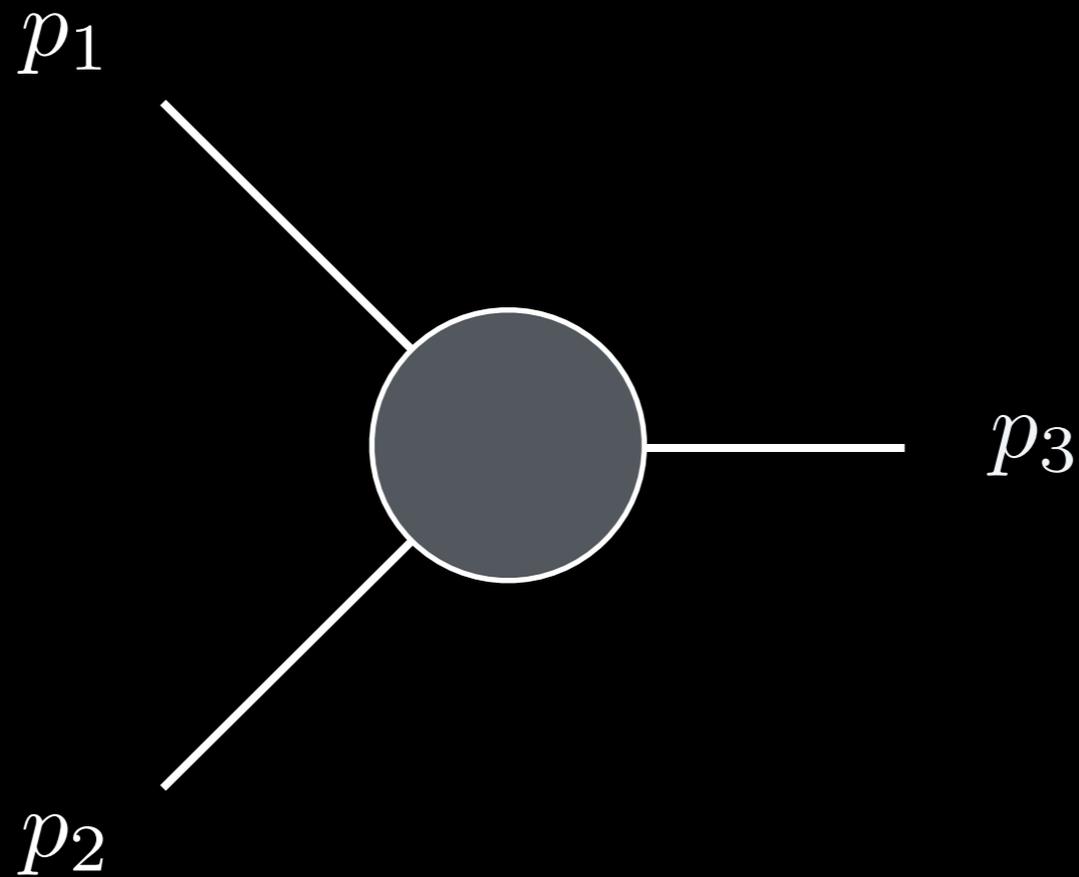
$$A(123) = \langle 12 \rangle^{\#} \langle 23 \rangle^{\#} \langle 31 \rangle^{\#} \quad \text{or} \quad [12]^{\#}[23]^{\#}[31]^{\#}$$

So the 3pt gluon and graviton scattering is fixed.

gluon

graviton

$$A(1^-2^-3^+) = \frac{\langle 12 \rangle^3}{\langle 13 \rangle \langle 32 \rangle}$$

$$M(1^-2^-3^+) = \frac{\langle 12 \rangle^6}{\langle 13 \rangle^2 \langle 32 \rangle^2}$$

$$A(1^+2^+3^-) = \frac{[12]^3}{[13][32]}$$

$$M(1^+2^+3^-) = \frac{[12]^6}{[13]^2[32]^2}$$

So the 3pt gluon and graviton scattering is fixed.

gluon

$$A(1^-2^-3^+) = \frac{\langle 12 \rangle^3}{\langle 13 \rangle \langle 32 \rangle}$$

$$A(1^+2^+3^-) = \frac{[12]^3}{[13][32]}$$

graviton

$$M(1^-2^-3^+) = \frac{\langle 12 \rangle^6}{\langle 13 \rangle^2 \langle 32 \rangle^2}$$

$$M(1^+2^+3^-) = \frac{[12]^6}{[13]^2[32]^2}$$

simplest example of "gluon^2 = graviton"
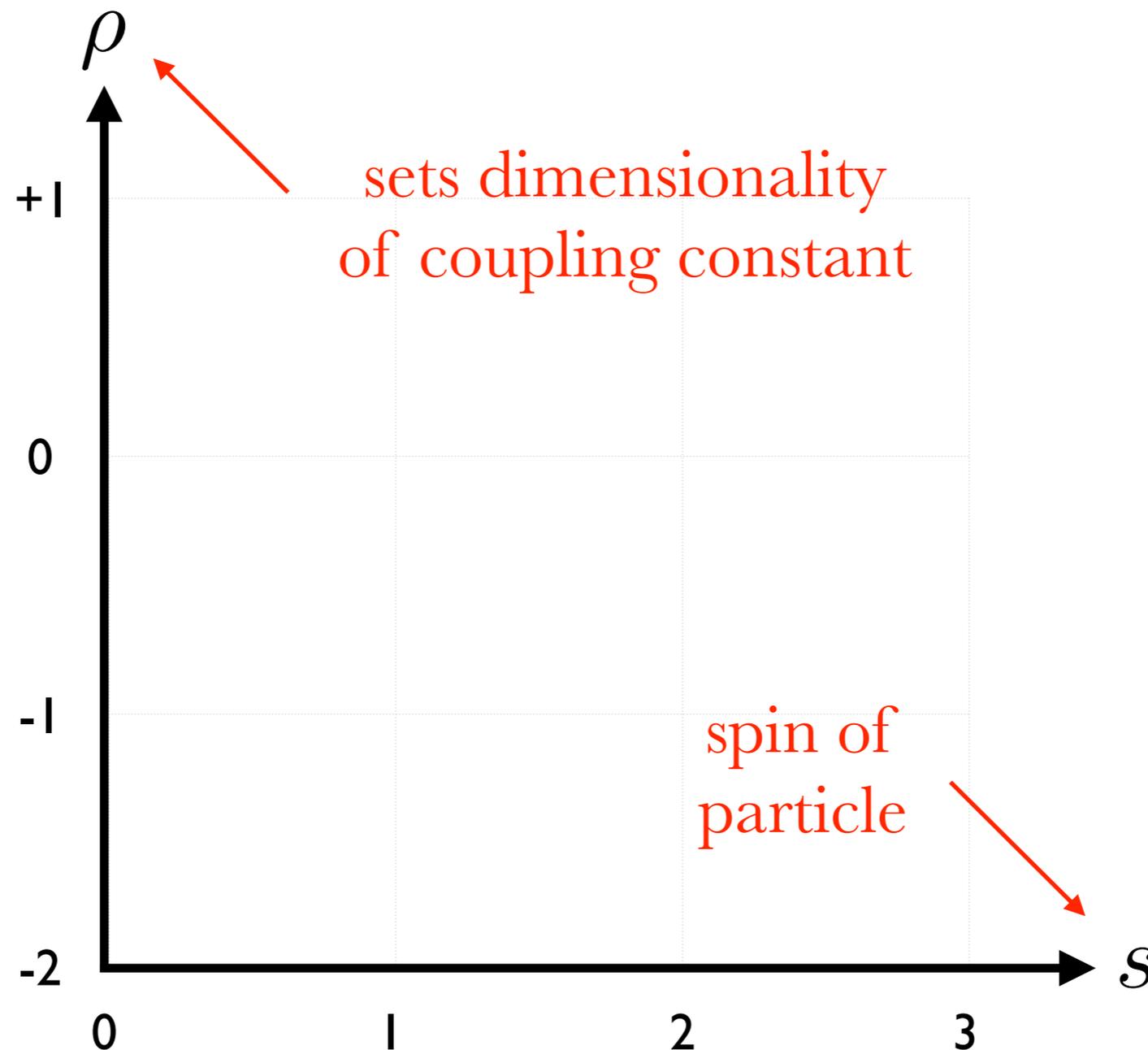
The 4pt amplitudes and up are fixed by locality.

$$(p_1 + p_2)^2 \to 0$$

$$A_4 \qquad A_3 \frac{1}{(p_1 + p_2)^2} A_3$$
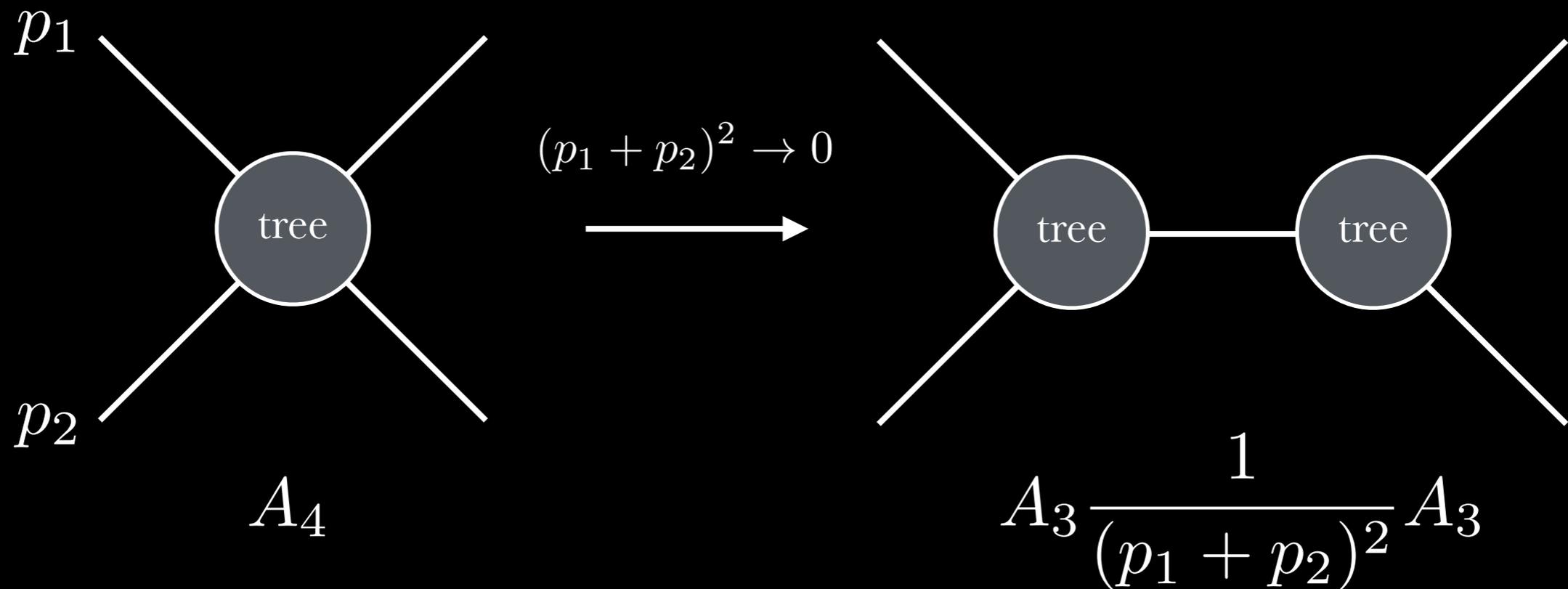
Recursion relations automate this construction.

Britto, Cachazo, Feng, Witten (hep-th/0412308, hep-th/0501052)
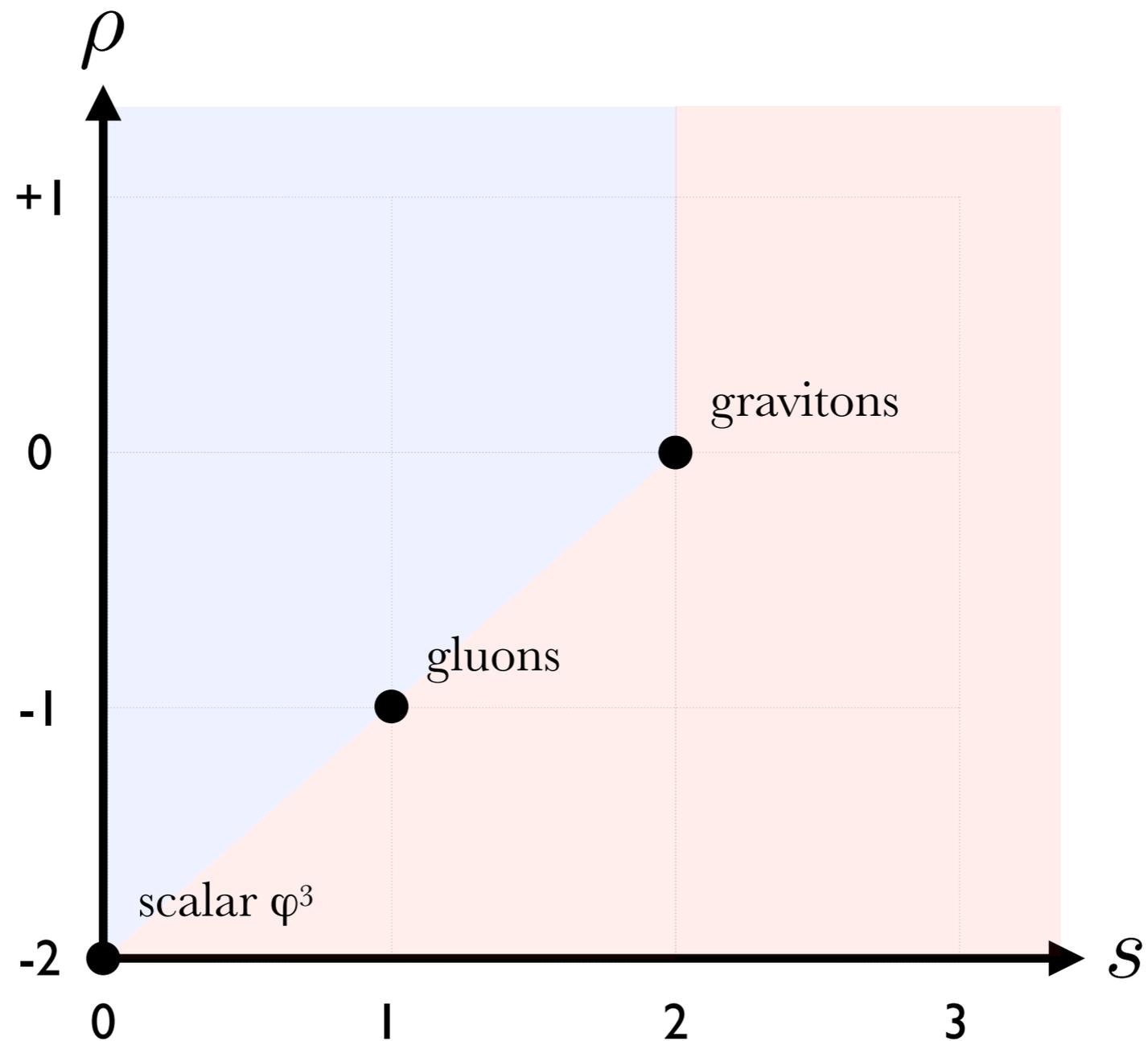
Let us draw a theory "map" for massless particles.
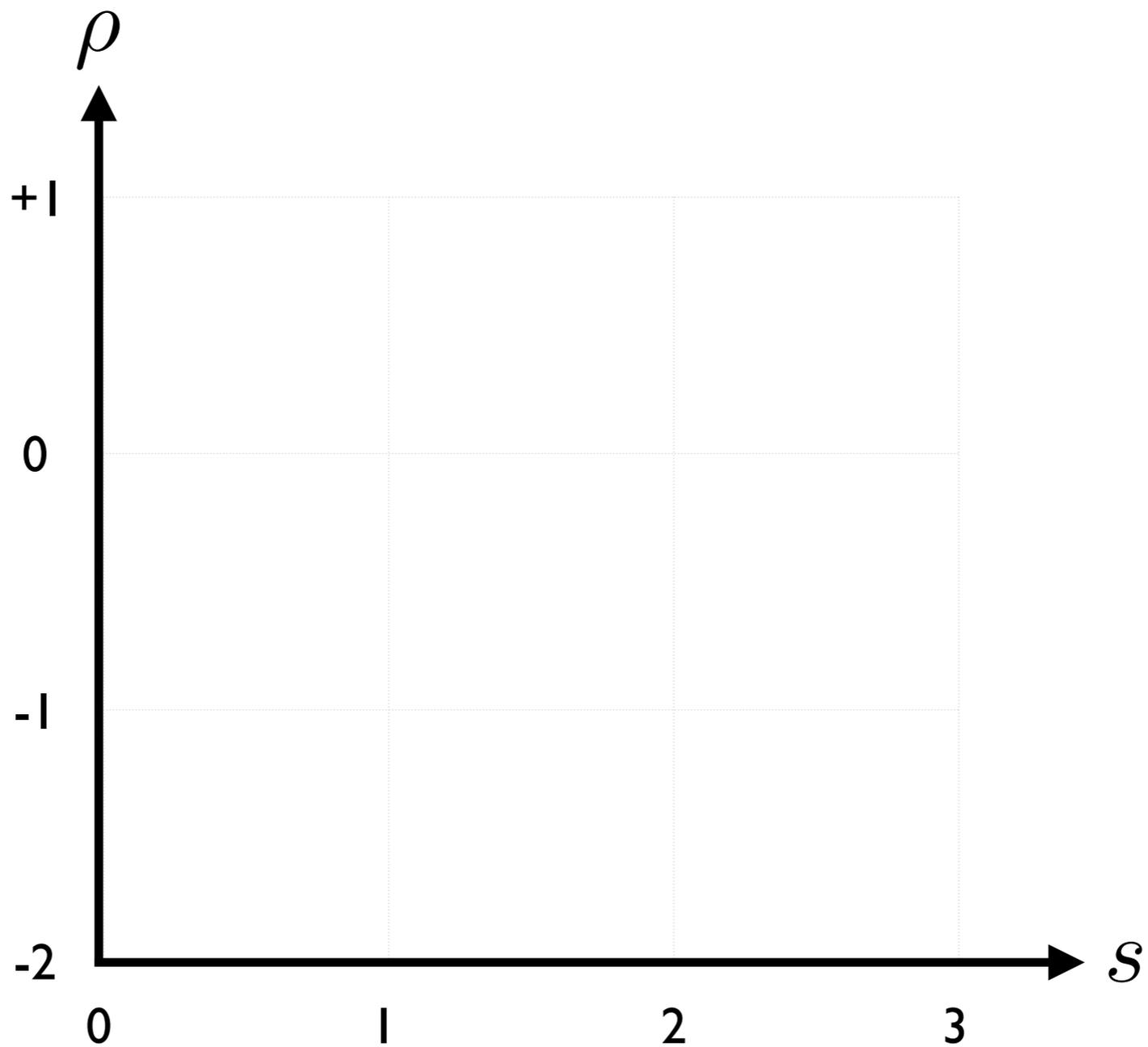
Let us draw a theory "map" for massless particles.

Constraints on scattering will exclude regions!
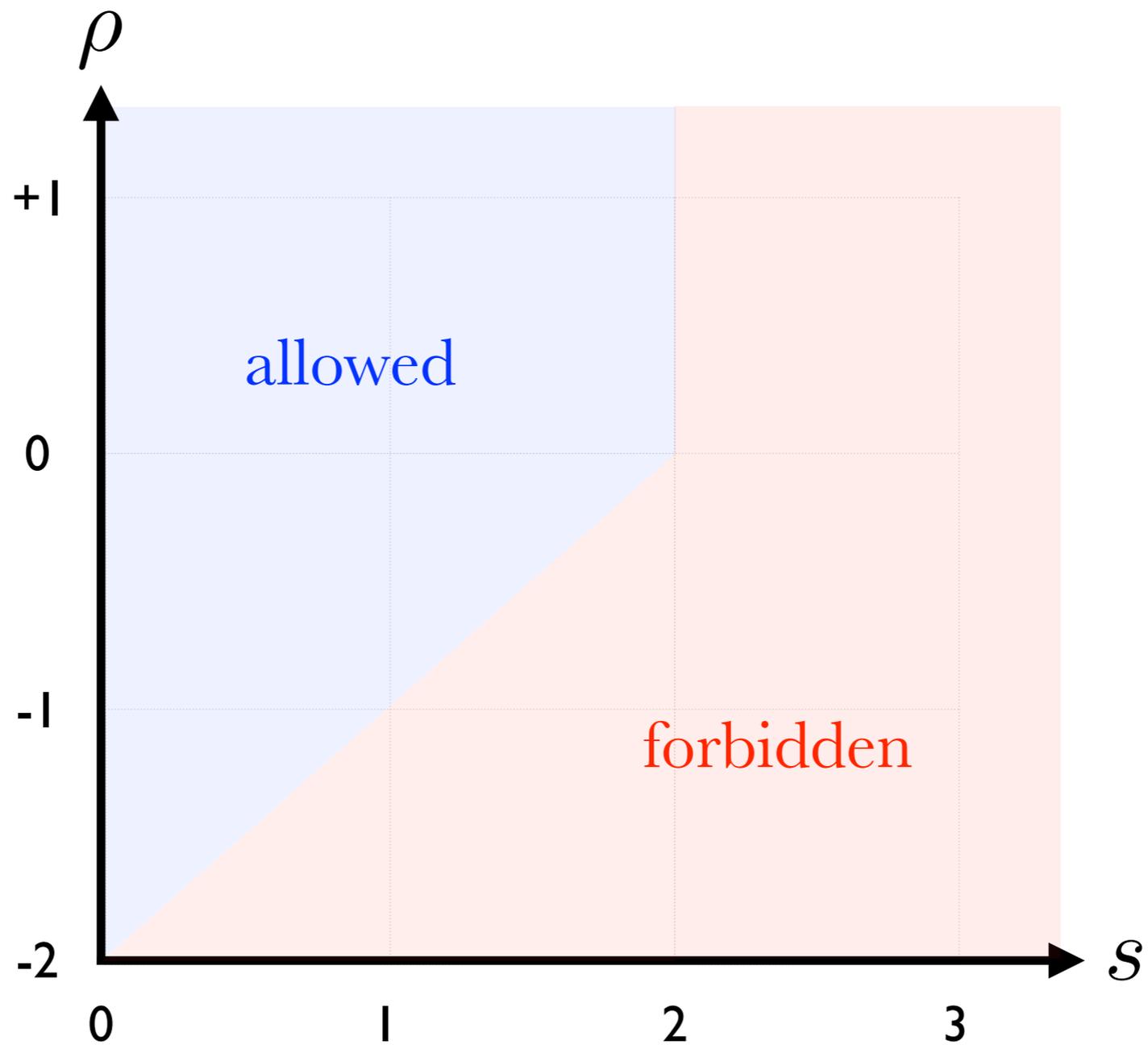
# Theories of nature live on the edge of consistency.

Amplitudes in a huge class of theories can now be bootstrapped to various degrees, in particular:

- massive particles
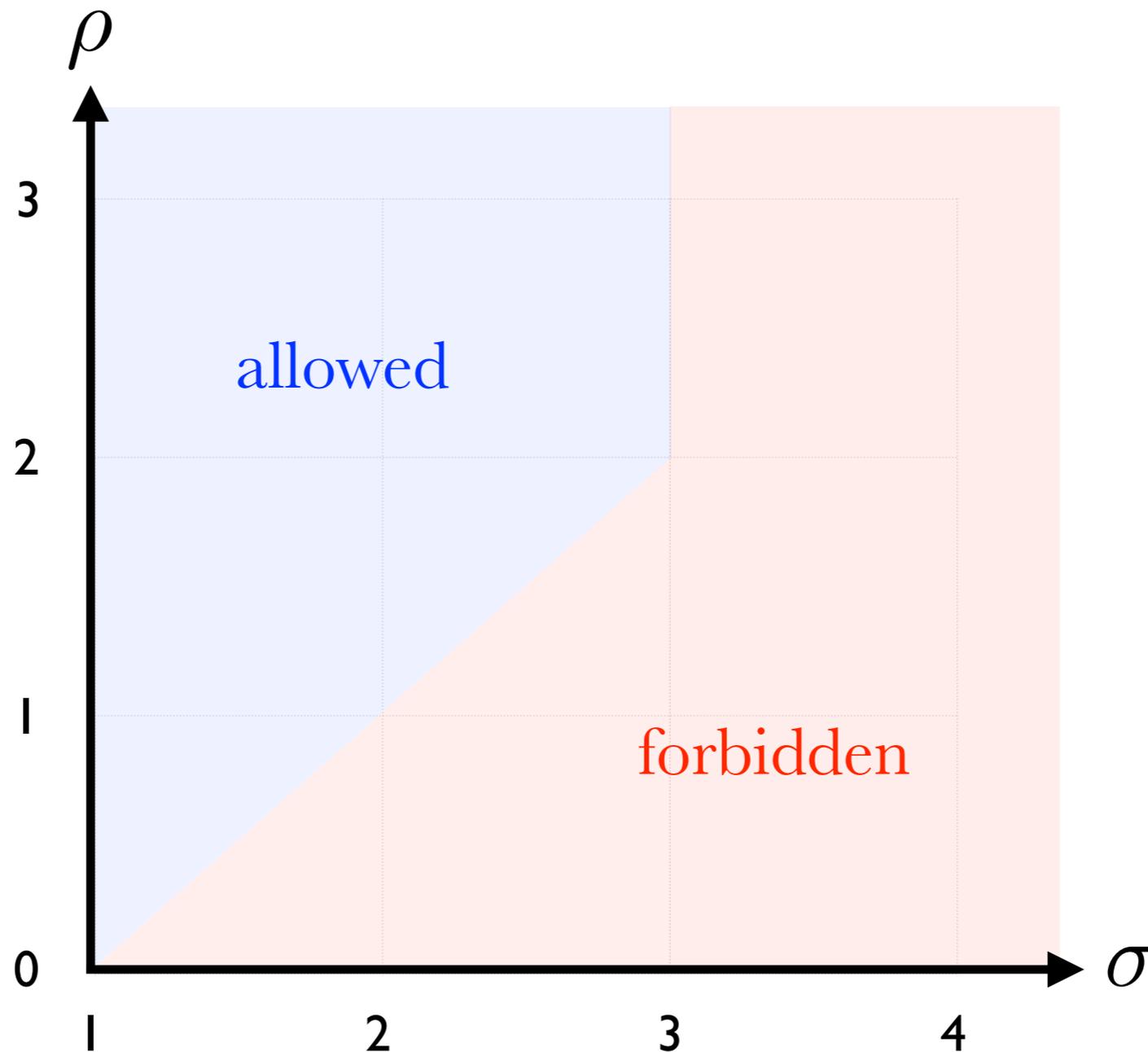
- higher loops

- diverse dimensions

But what about effective field theories (EFTs)?
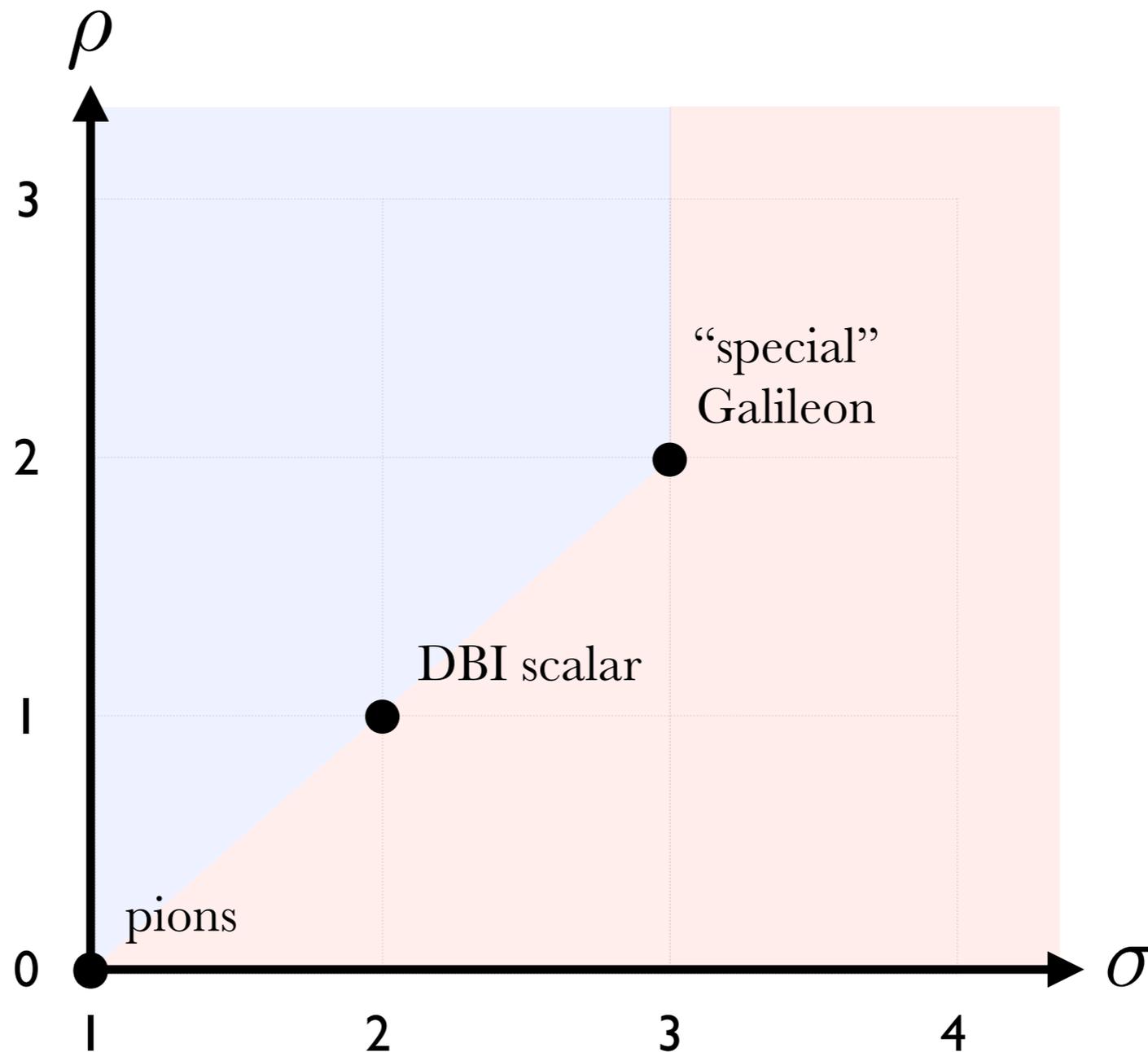
EFTs are practical, powerful, and ubiquitous!

- superconductivity

- strong interactions

- cosmic inflation

- large scale structure

- fluid dynamics
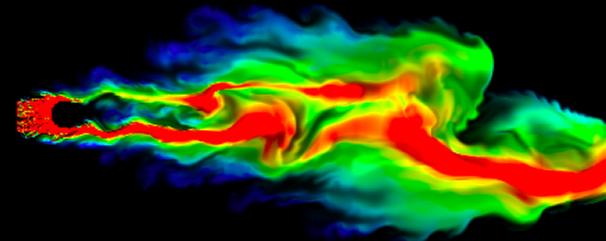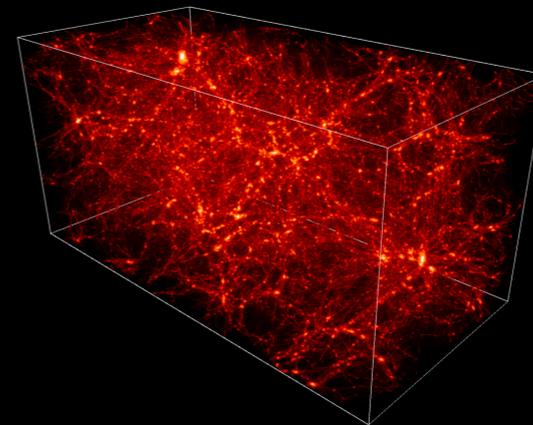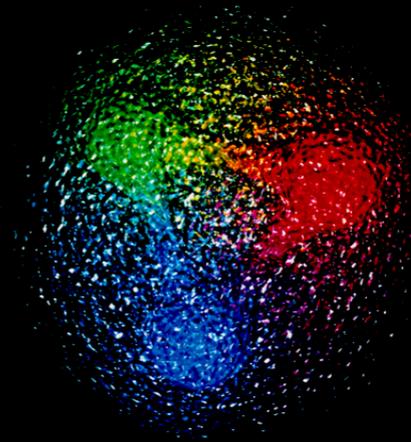
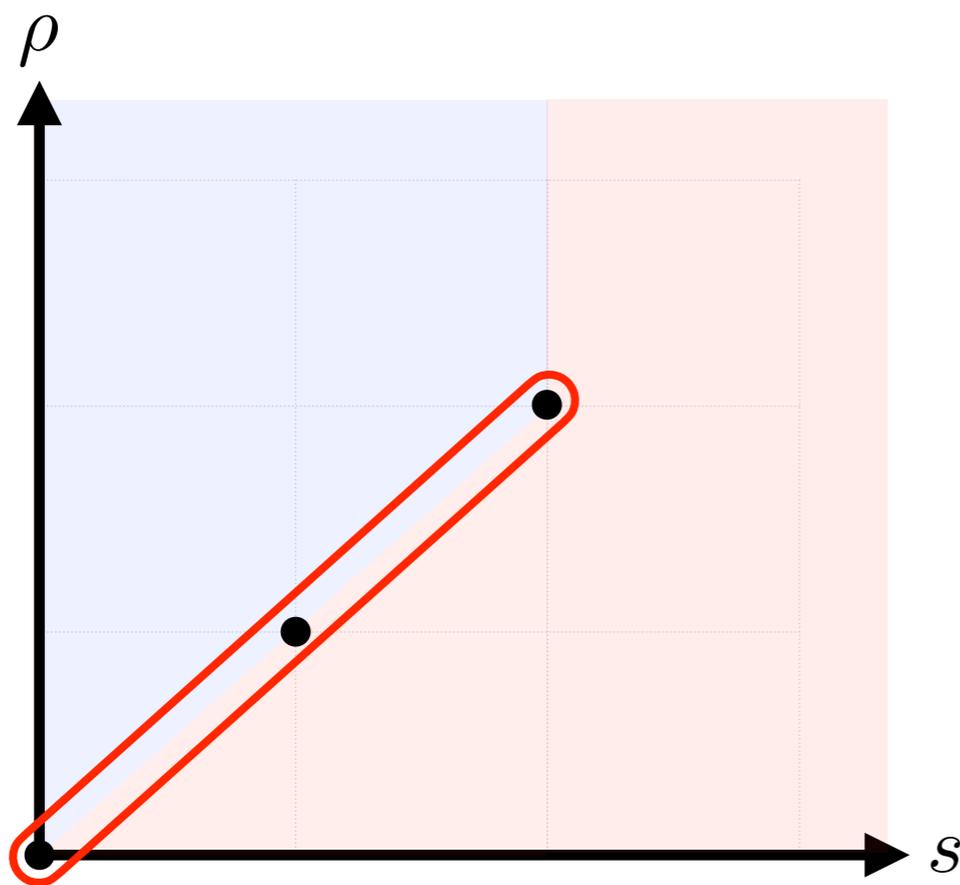# Repeating this procedure yields a "map" of EFTs!

Repeating this procedure yields a "map" of EFTs!

The "edge" theories are *maximally constrained* and moreover *secretly connected* across maps!

gauge/gravity

EFTs
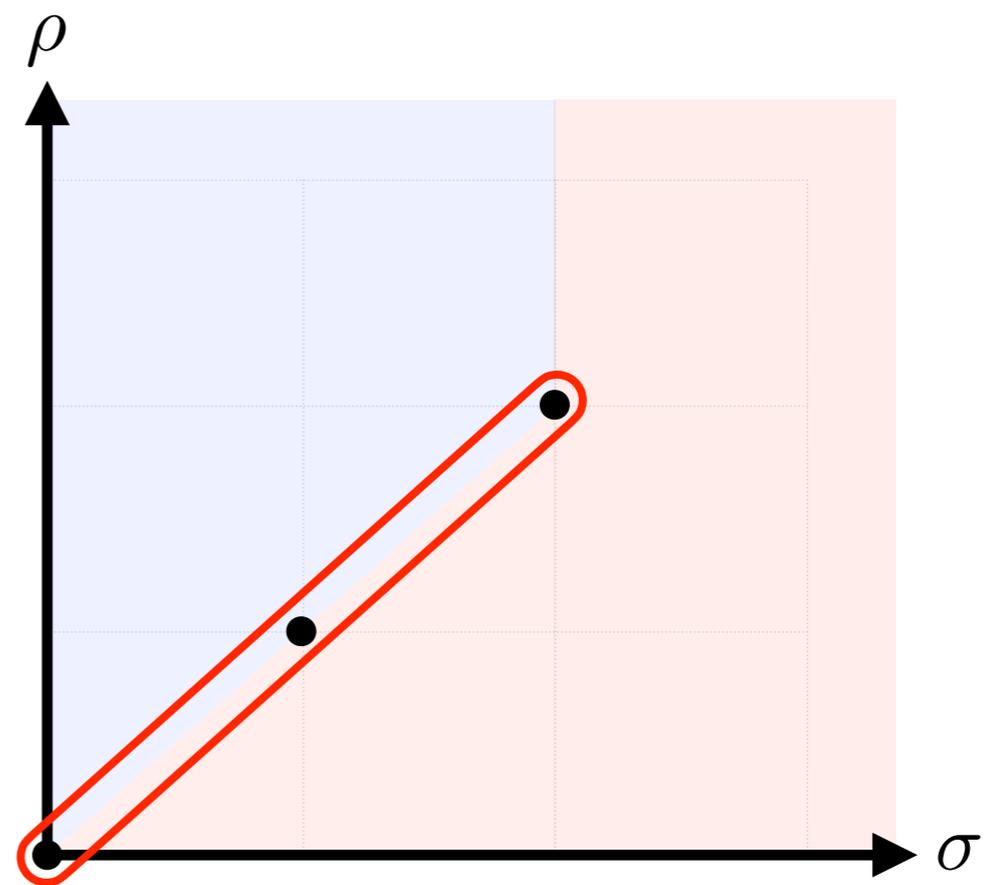
hidden unity of
gluons, pions, gravitons
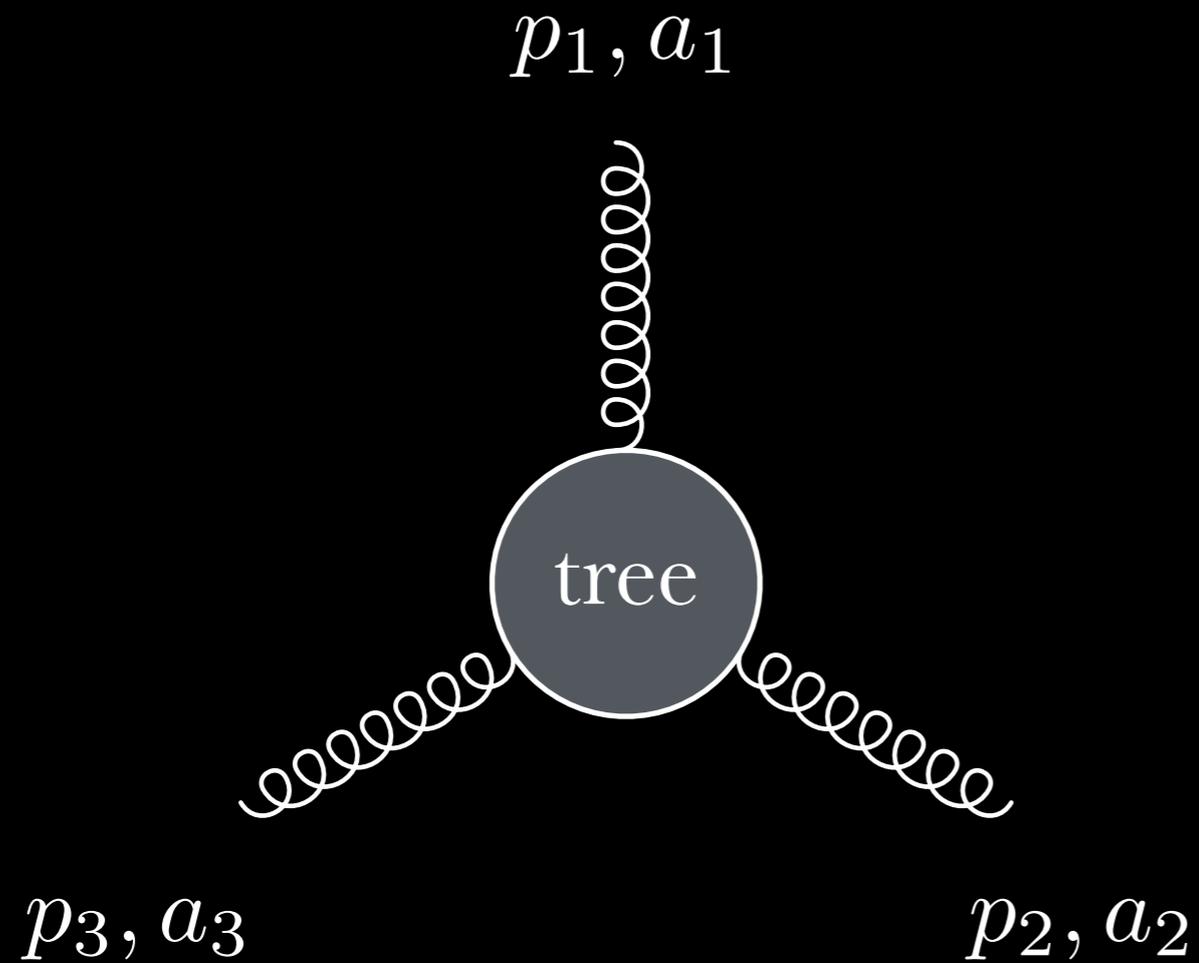
Gluons come in various types known as "colors".

Gluons come in various types known as "colors".
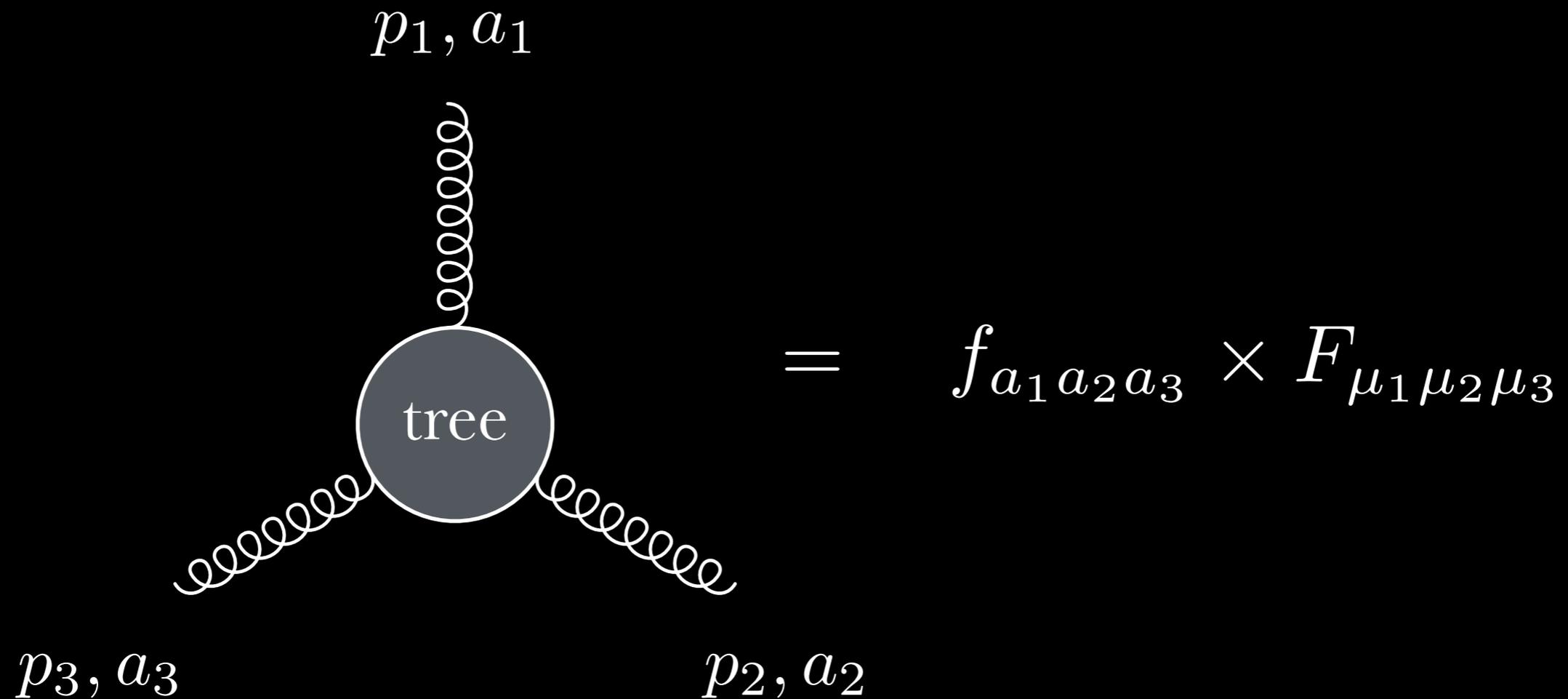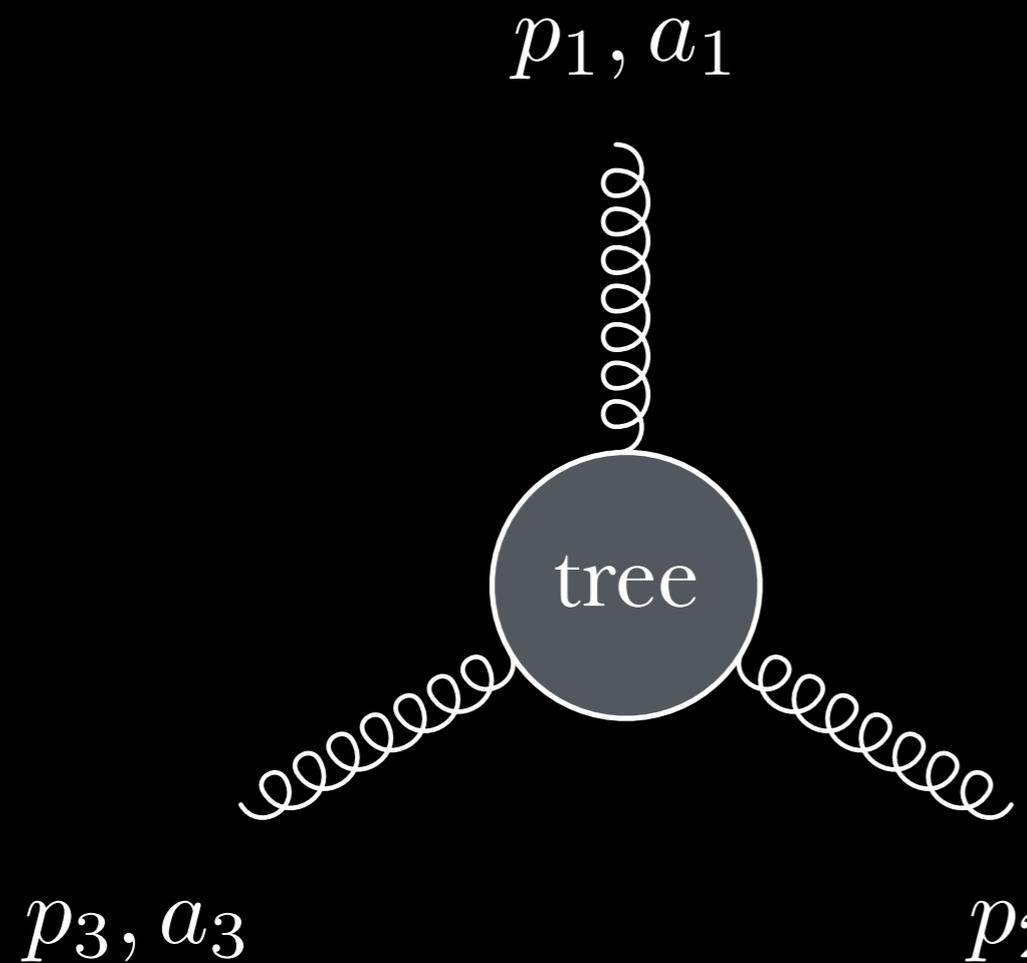
$$p_1, a_1$$

tree

$$p_3, a_3$$

$$p_2, a_2$$

momentum

Gluons come in various types known as "colors".

p_1, a_1

tree

p_3, a_3

p_2, a_2

"color" = 1, 2, 3, …

momentum

Gluons come in various types known as "colors".

$$= \quad f_{a_1 a_2 a_3} \times F_{\mu_1 \mu_2 \mu_3}$$

Gluons come in various types known as "colors".

$$= \quad f_{a_1 a_2 a_3} \times F_{\mu_1 \mu_2 \mu_3}$$

momentum dependence

Gluons come in various types known as "colors".

p_1, a_1

which colors couple
to which other colors

$$= \quad f_{a_1 a_2 a_3} \times F_{\mu_1 \mu_2 \mu_3}$$

tree

momentum
dependence

p_3, a_3

p_2, a_2

Gluons come in various types known as "colors".

p_1, a_1
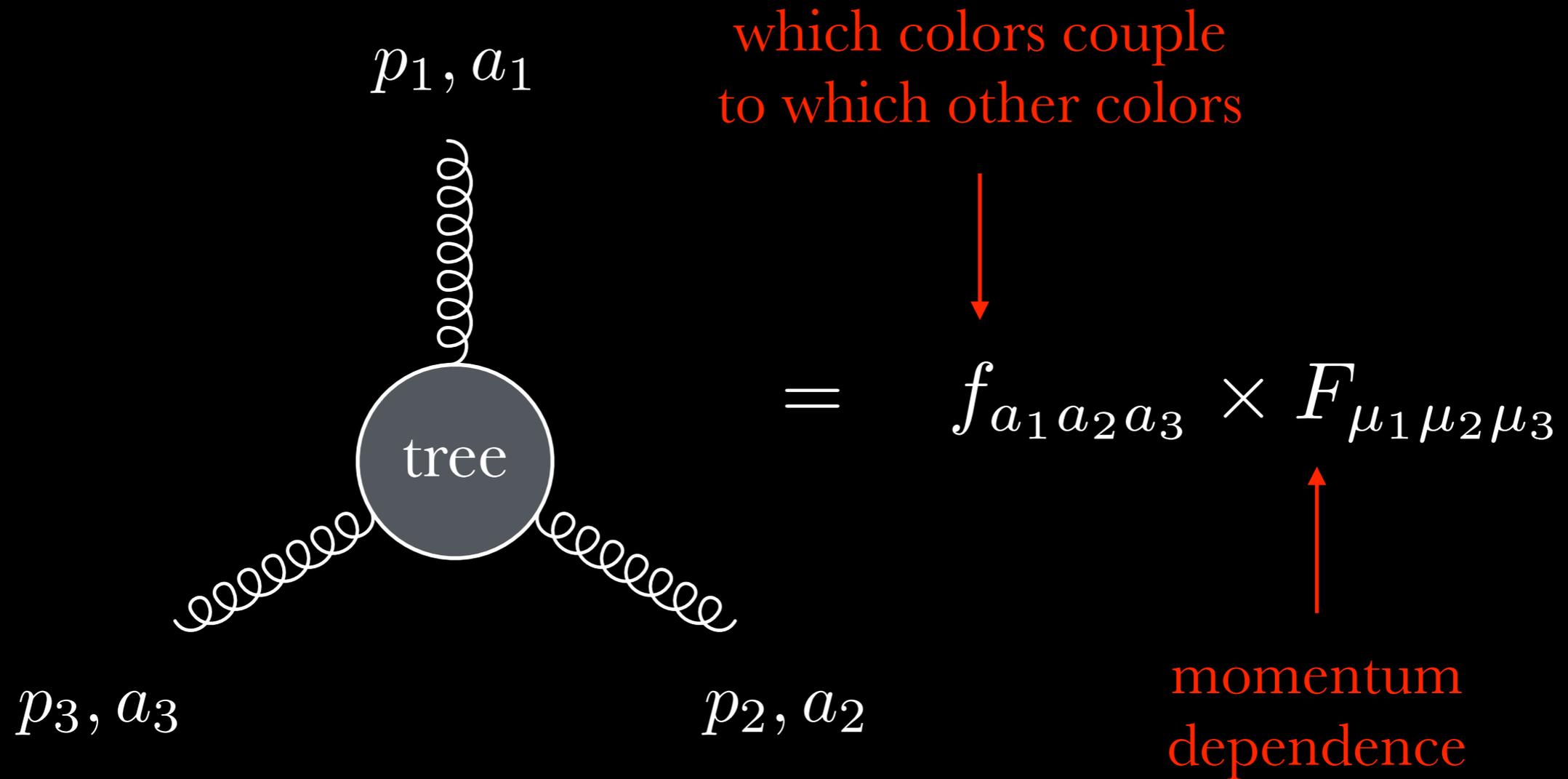
are color and kinematics interchangeable?

tree

$= \quad f_{a_1 a_2 a_3} \times F_{\mu_1 \mu_2 \mu_3}$

p_3, a_3

p_2, a_2

Bern-Carrasco-Johansson (BCJ) showed gluon amplitudes can always be *put into* a special form.
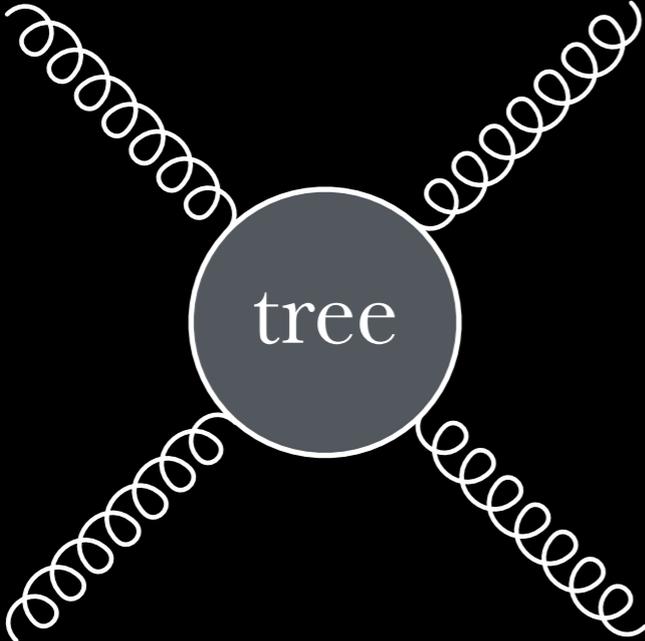
$$= \frac{c_s n_s}{s} + \frac{c_t n_t}{t} + \frac{c_u n_u}{u}$$

Bern-Carrasco-Johansson (BCJ) showed gluon amplitudes can always be *put into* a special form.

$$\text{tree} = \frac{c_s n_s}{s} + \frac{c_t n_t}{t} + \frac{c_u n_u}{u}$$

$f_{a_1 a_2 b} f_{a_3 a_4 b}$ $f_{a_2 a_3 b} f_{a_1 a_4 b}$ $f_{a_3 a_1 b} f_{a_2 a_4 b}$

$\sim (e_1 \cdot e_2)(e_3 \cdot e_4)(p_2 \cdot p_3) + \dots$

$$c_s + c_t + c_u = 0$$

(automatic)

$$n_s + n_t + n_u = 0$$

(not automatic)

Bern-Carrasco-Johansson (BCJ) showed gluon amplitudes can always be *put into* a special form.

$$= \quad \frac{c_s n_s}{s} + \frac{c_t n_t}{t} + \frac{c_u n_u}{u}$$

substitute color
for kinematics

$$\frac{n_s^2}{s} + \frac{n_t^2}{t} + \frac{n_u^2}{u}$$

Bern-Carrasco-Johansson (BCJ) showed gluon amplitudes can always be *put into* a special form.

$$= \quad \frac{c_s n_s}{s} + \frac{c_t n_t}{t} + \frac{c_u n_u}{u}$$

substitute color for kinematics

this is the 4pt graviton scattering amplitude!!! $\implies$ $\dfrac{n_s^2}{s} + \dfrac{n_t^2}{t} + \dfrac{n_u^2}{u}$

$$\left(\; \bullet \; \right) \otimes \left(\; \bullet \; \right) = $$

gluon
amplitude

gluon
amplitude

graviton
amplitude

The "double copy" is proven at tree-level, verified at loop-level, and applicable to numerous QFTs!

gluon $\otimes$ gluon $=$ graviton

pion $\otimes$ pion $=$ special Galileon

gluon $\otimes$ pion $=$ Born-Infeld photon

The graviton S-matrix encodes other S-matrices!
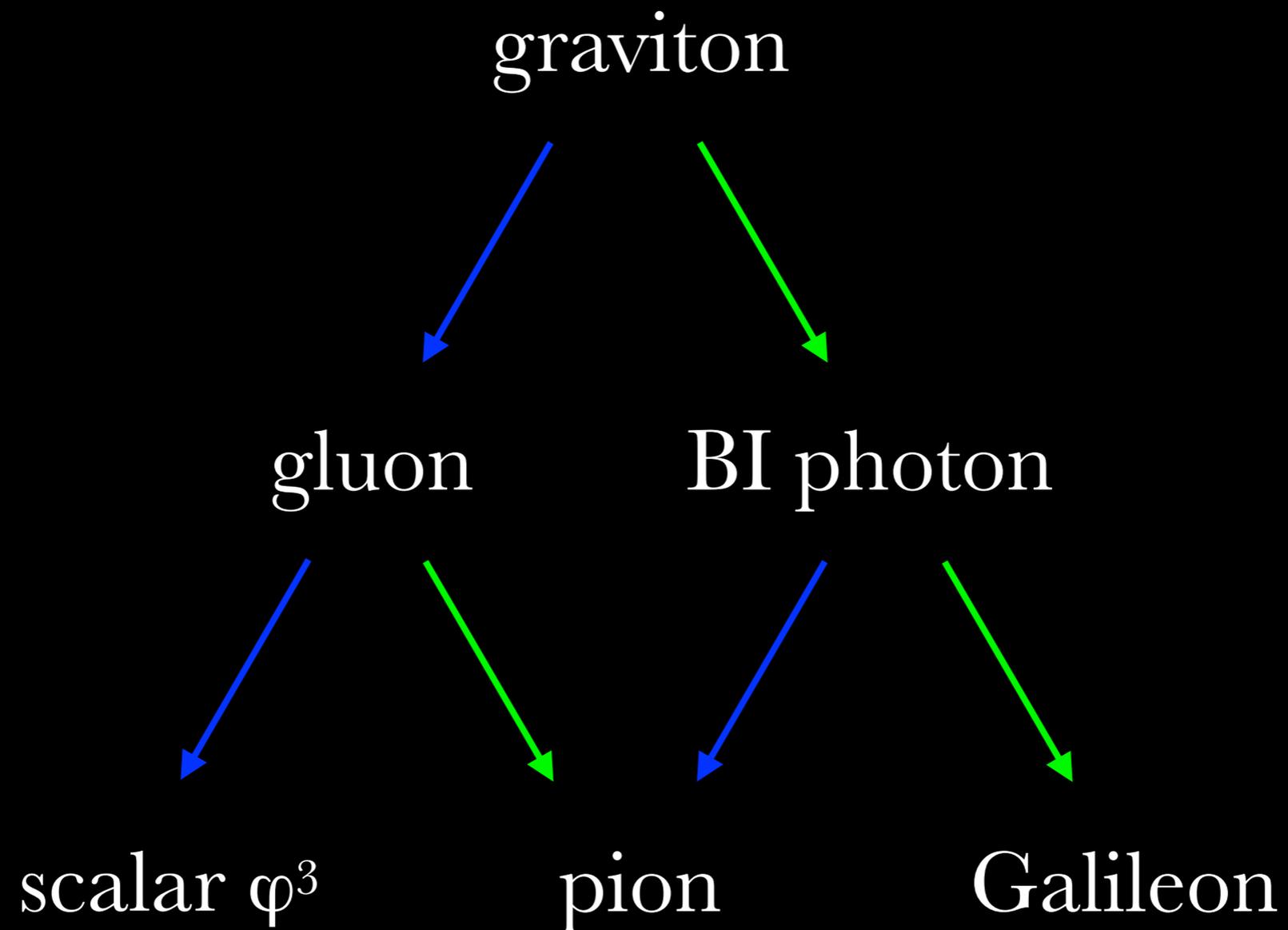
Can we learn anything new about *black holes*?

new perspectives on
black hole dynamics

Inspiral — Merger — Ringdown

Post − Newtonian Theory — Numerical Relativity — Perturbation Theory
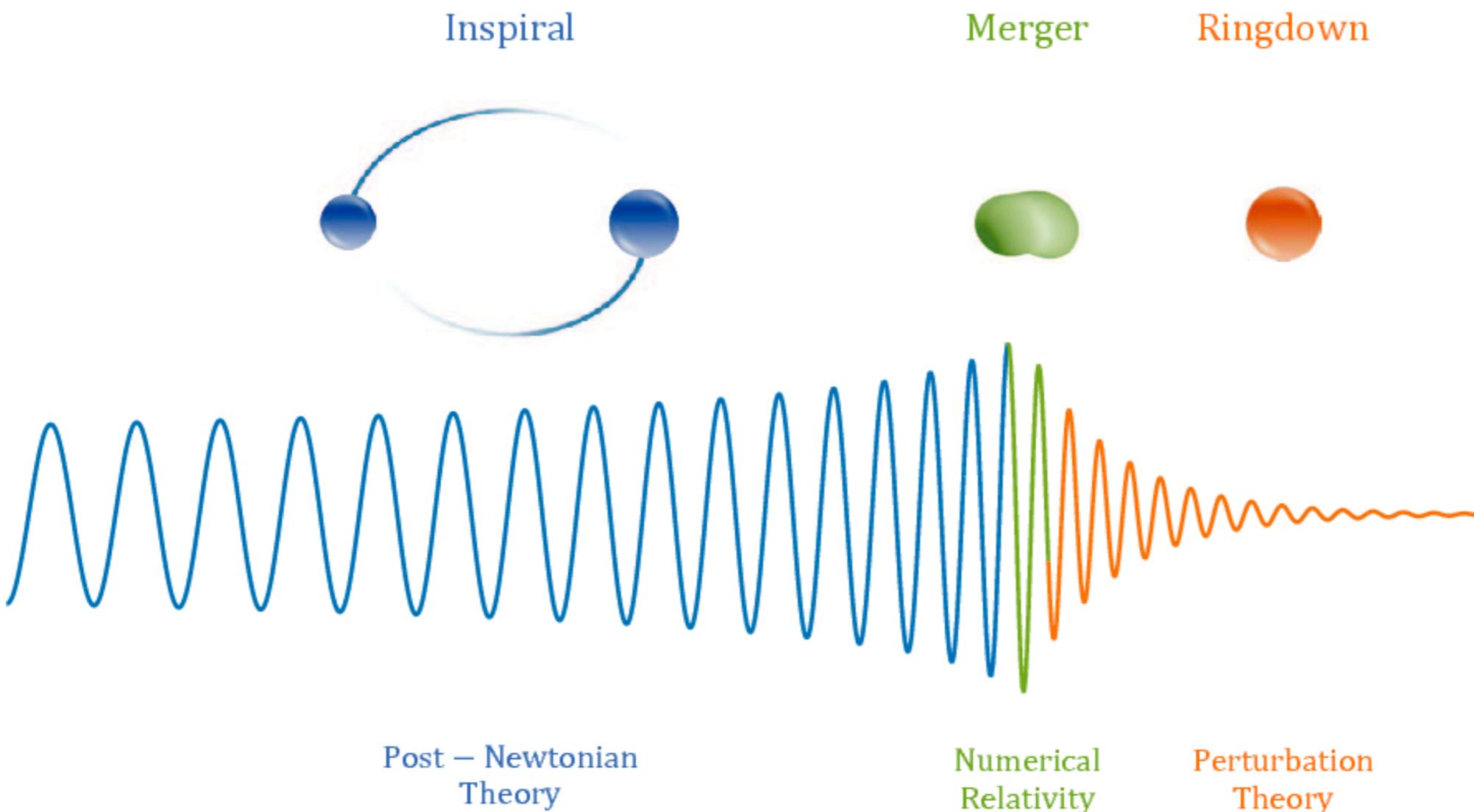
our focus is the conservative potential

(figure from 1610.03567)

The *post-Newtonian (PN)* approximation expands in powers of the virilized quantities,

$$v^2 \sim \frac{GM}{r} \ll 1$$

which are tiny and perturbatively calculable during the inspiral phase.

The *post-Minkowskian (PM)* approximation is an expansion in powers of G, aka perturbation theory!

Status of PN Theory (circa 2018)

	0PN	1PN	2PN	3PN	4PN	5PN	6PN	7PN

$$1\text{PM} \quad \left(1 \; + \; v^2 \; + \; v^4 \; + \; v^6 \; + \; v^8 \; + v^{10} + v^{12} + v^{14} + \cdots \right) G$$

$$2\text{PM} \quad \left(1 \; + \; v^2 \; + \; v^4 \; + \; v^6 \; + \; v^8 \; + v^{10} + v^{12} + \cdots \right) G^2$$

$$3\text{PM} \quad \left(1 \; + \; v^2 \; + \; v^4 \; + \; v^6 \; + \; v^8 \; + v^{10} + \cdots \right) G^3$$

$$4\text{PM} \quad \left(1 \; + \; v^2 \; + \; v^4 \; + \; v^6 \; + \; v^8 \; + \cdots \right) G^4$$

$$5\text{PM} \quad \left(1 \; + \; v^2 \; + \; v^4 \; + \; v^6 \; + \cdots \right) G^5$$

$$\vdots$$

Status of PN Theory (circa 2018)

(1686) (1938) (1974) (2000) (2014)

0PN 1PN 2PN 3PN 4PN 5PN 6PN 7PN

$$1\text{PM} \quad \left(1 \;+\; v^2 \;+\; v^4 \;+\; v^6 \;+\; v^8 \;+\; v^{10} + v^{12} + v^{14} + \cdots \right) G$$

$$2\text{PM} \quad \left(1 \;+\; v^2 \;+\; v^4 \;+\; v^6 \;+\; v^8 \;+\; v^{10} + v^{12} + \cdots \right) G^2$$

$$3\text{PM} \quad \left(1 \;+\; v^2 \;+\; v^4 \;+\; v^6 \;+\; v^8 \;+\; v^{10} + \cdots \right) G^3$$

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$$5\text{PM} \quad \left(1 \;+\; v^2 \;+\; v^4 \;+\; v^6 \;+\; \cdots \right) G^5$$

$$\vdots$$

Existing tools suffer from redundancy of QFT.
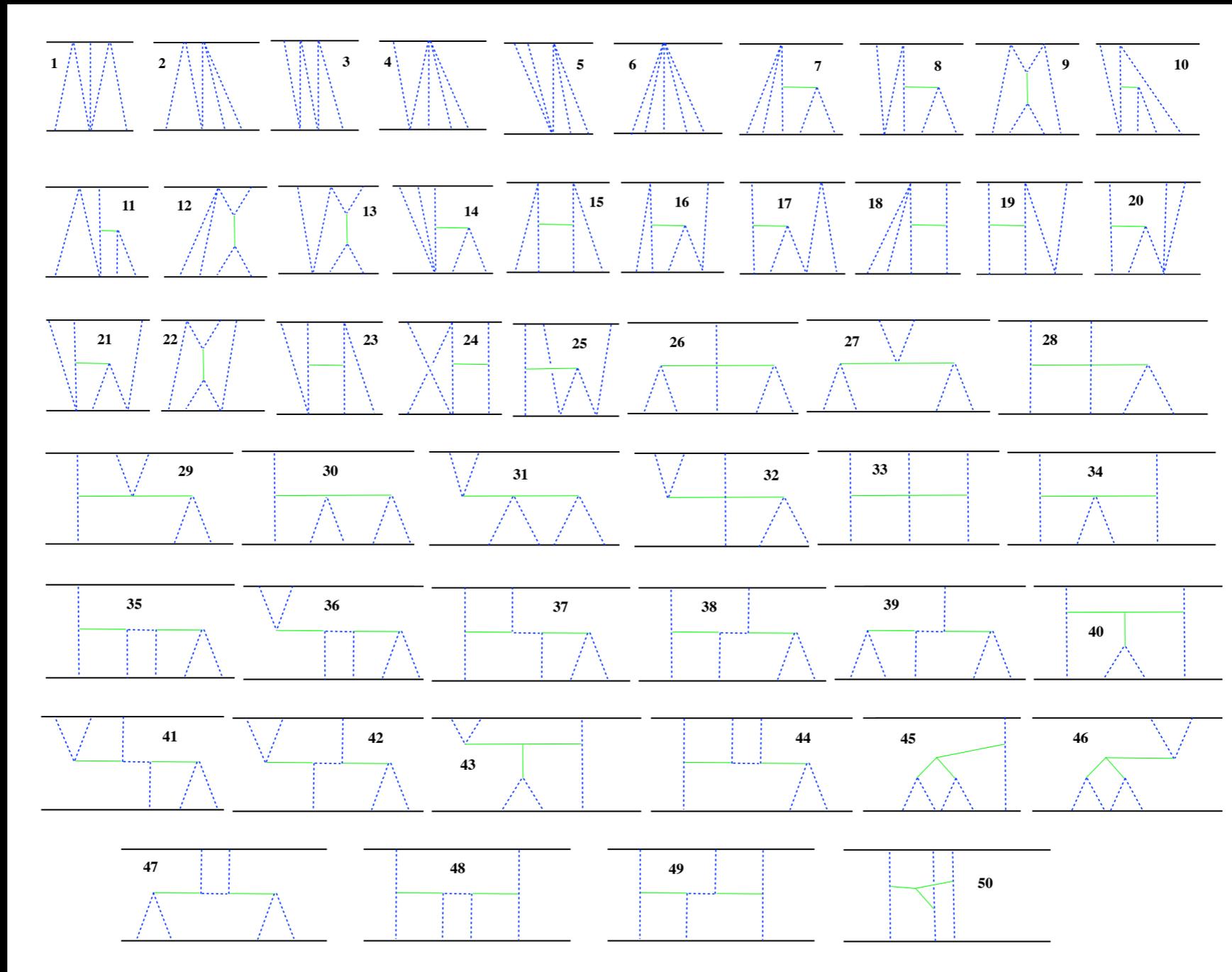
New PM results derived from double copy + EFT!

Scattering Amplitudes and the Conservative Hamiltonian for Binary Systems at Third Post-Minkowskian Order

Zvi Bern,[1] Clifford Cheung,[2] Radu Roiban,[3] Chia-Hsien Shen,[1] Mikhail P. Solon,[2] and Mao Zeng[4]

$$H(\boldsymbol{p}, \boldsymbol{r}) = \sqrt{\boldsymbol{p}^2 + m_1^2} + \sqrt{\boldsymbol{p}^2 + m_2^2} + V(\boldsymbol{p}, \boldsymbol{r}),$$

$$V(\boldsymbol{p}, \boldsymbol{r}) = \sum_{i=1}^{\infty} c_i(\boldsymbol{p}^2) \left(\frac{G}{|\boldsymbol{r}|} \right)^i, \tag{9}$$

$$c_1 = \frac{\nu^2 m^2}{\gamma^2 \xi} \left(1 - 2\sigma^2 \right), \qquad c_2 = \frac{\nu^2 m^3}{\gamma^2 \xi} \left[\frac{3}{4} \left(1 - 5\sigma^2 \right) - \frac{4\nu\sigma \left(1 - 2\sigma^2 \right)}{\gamma\xi} - \frac{\nu^2 (1-\xi) \left(1 - 2\sigma^2 \right)^2}{2\gamma^3 \xi^2} \right],$$

$$c_3 = \frac{\nu^2 m^4}{\gamma^2 \xi} \left[\frac{1}{12} \left(3 - 6\nu + 206\nu\sigma - 54\sigma^2 + 108\nu\sigma^2 + 4\nu\sigma^3 \right) - \frac{4\nu \left(3 + 12\sigma^2 - 4\sigma^4 \right) \operatorname{arcsinh}\sqrt{\frac{\sigma-1}{2}}}{\sqrt{\sigma^2 - 1}} \right.$$

$$- \frac{3\nu\gamma \left(1 - 2\sigma^2 \right) \left(1 - 5\sigma^2 \right)}{2(1+\gamma)(1+\sigma)} - \frac{3\nu\sigma \left(7 - 20\sigma^2 \right)}{2\gamma\xi} - \frac{\nu^2 \left(3 + 8\gamma - 3\xi - 15\sigma^2 - 80\gamma\sigma^2 + 15\xi\sigma^2 \right) \left(1 - 2\sigma^2 \right)}{4\gamma^3 \xi^2}$$

$$\left. + \frac{2\nu^3 (3 - 4\xi)\sigma \left(1 - 2\sigma^2 \right)^2}{\gamma^4 \xi^3} + \frac{\nu^4 (1 - 2\xi) \left(1 - 2\sigma^2 \right)^3}{2\gamma^6 \xi^4} \right], \tag{10}$$

Our calculations are now state of the art in PM.

	0PN	1PN	2PN	3PN	4PN	5PN	6PN	7PN

$$1\text{PM} \qquad \left(\, 1 \;+\; v^2 \;+\; v^4 \;+\; v^6 \;+\; v^8 \;+ v^{10} + v^{12} + v^{14} + \cdots \right) G$$

$$2\text{PM} \qquad \left(\, 1 \;+\; v^2 \;+\; v^4 \;+\; v^6 \;+\; v^8 + v^{10} + v^{12} + \cdots \right) G^2$$

$$3\text{PM} \qquad \left(\, 1 \;+\; v^2 \;+\; v^4 \;+\; v^6 \;+\; v^8 + v^{10} + \cdots \right) G^3$$

$$4\text{PM} \qquad \left(\, 1 \;+\; v^2 \;+\; v^4 \;+\; v^6 \;+\; v^8 + \cdots \right) G^4$$

$$5\text{PM} \qquad \left(\, 1 \;+\; v^2 \;+\; v^4 \;+\; v^6 \;+ \cdots \right) G^5$$

$$\vdots$$

Our calculations are now state of the art in PM.

	0PN	1PN	2PN	3PN	4PN	5PN	6PN	7PN

$$\left(\ 1\ +\ v^2\ +\ v^4\ +\ v^6\ +\ v^8\ +\ v^{10}+v^{12}+v^{14}+\cdots\right)G \qquad \text{1PM}$$

$$\left(\ 1\ +\ v^2\ +\ v^4\ +\ v^6\ +\ v^8\ +\ v^{10}+v^{12}+\cdots\right)G^2 \qquad \text{2PM}$$

$$\left(\ 1\ +\ v^2\ +\ v^4\ +\ v^6\ +\ v^8\ +\ v^{10}+\cdots\right)G^3 \qquad \text{3PM}$$

$$\left(\ 1\ +\ v^2\ +\ v^4\ +\ v^6\ +\ v^8\ +\cdots\right)G^4 \qquad \text{4PM}$$

$$\left(\ 1\ +\ v^2\ +\ v^4\ +\ v^6\ +\cdots\right)G^5 \qquad \text{5PM}$$

1808.02489

$$\vdots$$

Our calculations are now state of the art in PM.

	0PN	1PN	2PN	3PN	4PN	5PN	6PN	7PN

$$\text{1PM} \quad \left(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + v^{14} + \cdots \right) G$$

$$\text{2PM} \quad \left(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + v^{12} + \cdots \right) G^2$$

$$\text{3PM} \quad \left(1 + v^2 + v^4 + v^6 + v^8 + v^{10} + \cdots \right) G^3$$

$$\text{4PM} \quad \left(1 + v^2 + v^4 + v^6 + v^8 + \cdots \right) G^4$$

$$\text{5PM} \quad \left(1 + v^2 + v^4 + v^6 + \cdots \right) G^5$$

$$\vdots$$

1808.02489

1901.04424
1908.01493
2003.08351
2006.06665

Our calculations are now state of the art in PM.

| | 0PN | 1PN | 2PN | 3PN | 4PN | 5PN | 6PN | 7PN |

$$1\text{PM} \qquad \left(\ 1\ +\ v^2\ +\ v^4\ +\ v^6\ +\ v^8\ +\ v^{10} + v^{12} + v^{14} + \cdots\right) G$$

$$2\text{PM} \qquad \left(\ 1\ +\ v^2\ +\ v^4\ +\ v^6\ +\ v^8\ +\ v^{10} + v^{12} + \cdots\right) G^2$$

$$3\text{PM} \qquad \left(\ 1\ +\ v^2\ +\ v^4\ +\ v^6\ +\ v^8\ +\ v^{10} + \cdots\right) G^3$$

$$4\text{PM} \qquad \left(\ 1\ +\ v^2\ +\ v^4\ +\ v^6\ +\ v^8\ +\ \cdots\right) G^4$$

$$5\text{PM} \qquad \left(\ 1\ +\ v^2\ +\ v^4\ +\ v^6\ +\ \cdots\right) G^5$$

1808.02489

1901.04424
1908.01493
2003.08351
2006.06665

2101.07254

$\vdots$

Energetics of two-body Hamiltonians in post-Minkowskian gravity

Andrea Antonelli,[1] Alessandra Buonanno,[1,2] Jan Steinhoff,[1] Maarten van de Meent,[1] and Justin Vines[1]
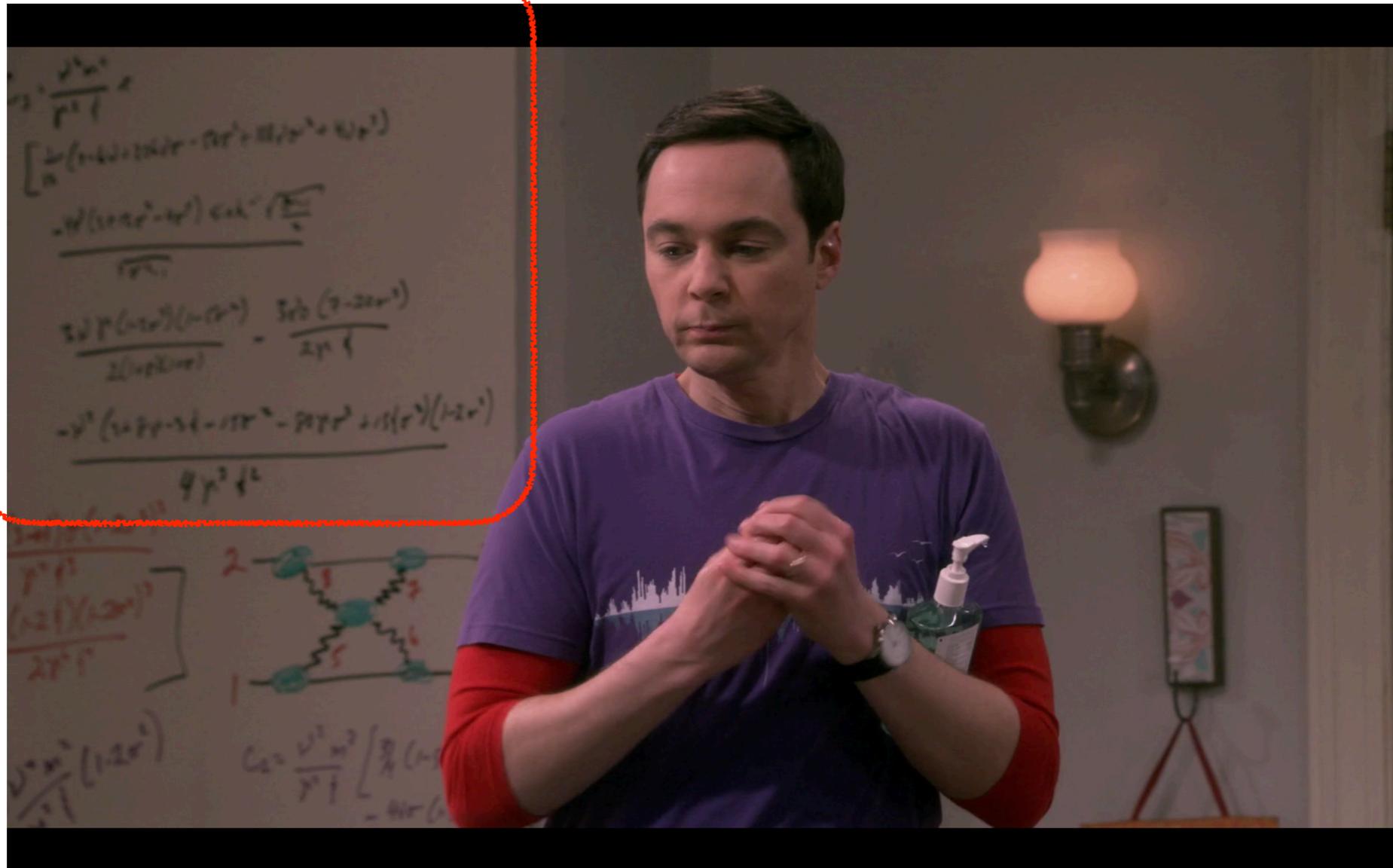
[1] Max Planck Institute for Gravitational Physics (Albert Einstein Institute), Am Mühlenberg 1, Potsdam 14476, Germany

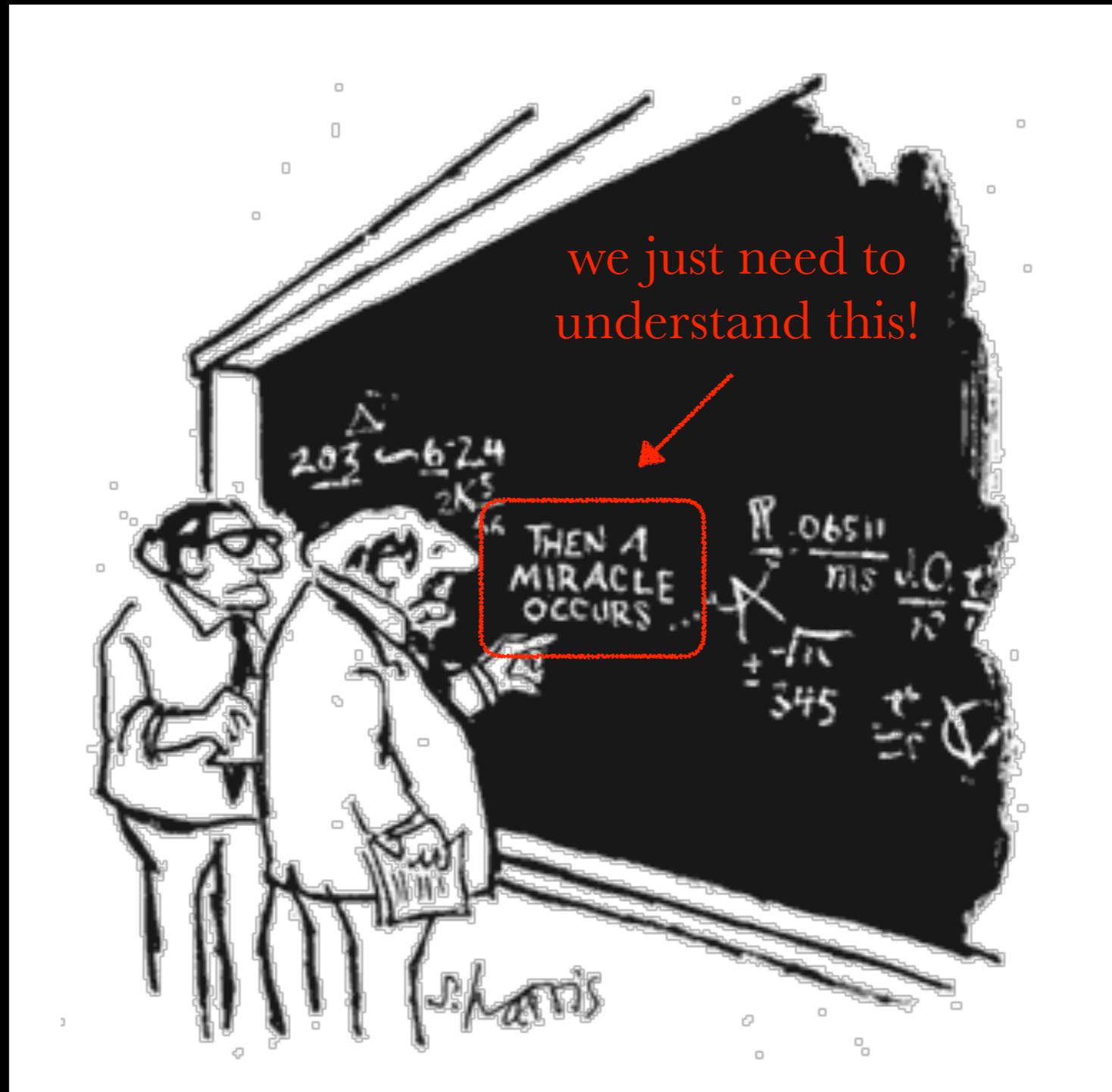[2] Department of Physics, University of Maryland, College Park, MD 20742, USA

(Dated: January 23, 2019)
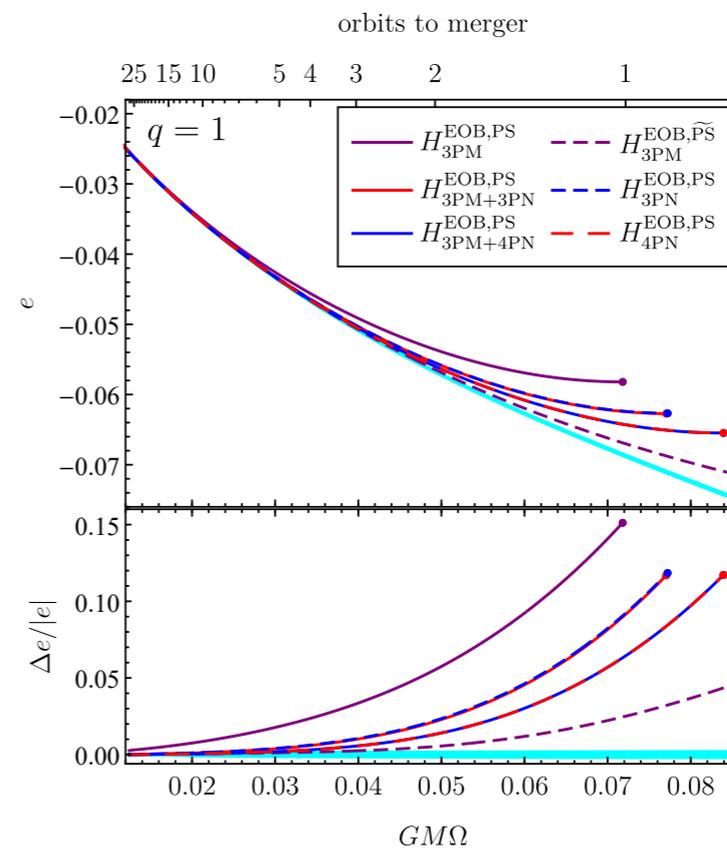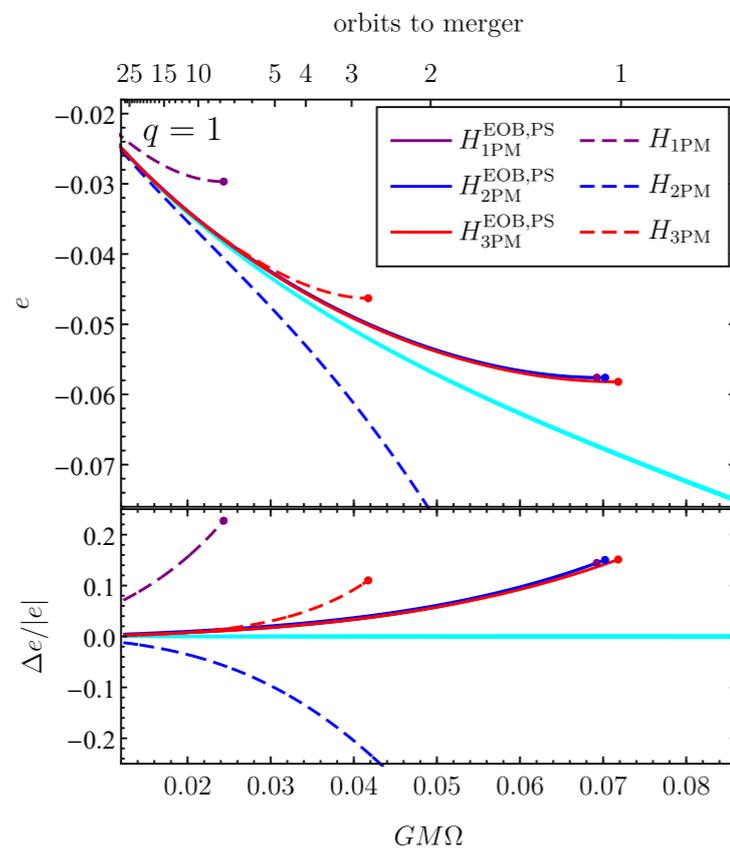
Citation #2: Sheldon Cooper, et al.

Despite immense progress, many question remain!

stay tuned!