Detecting a quantum spin liquid in the cuprate superconductors

New Directions in Theoretical Physics 5
Higgs Centre For Theoretical Physics
University of Edinburgh
January 9, 2026

Subir Sachdev

PHYSICS
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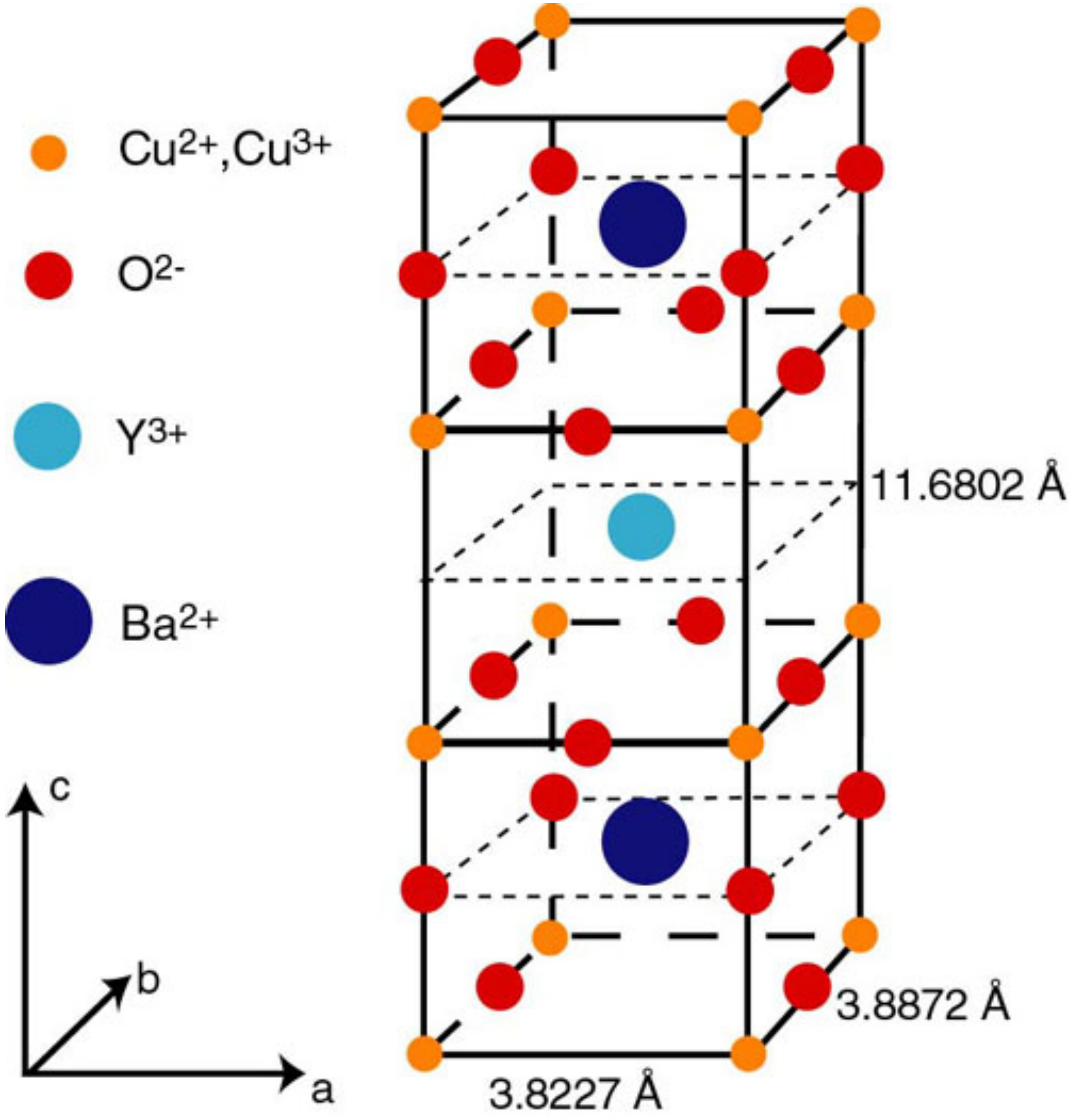
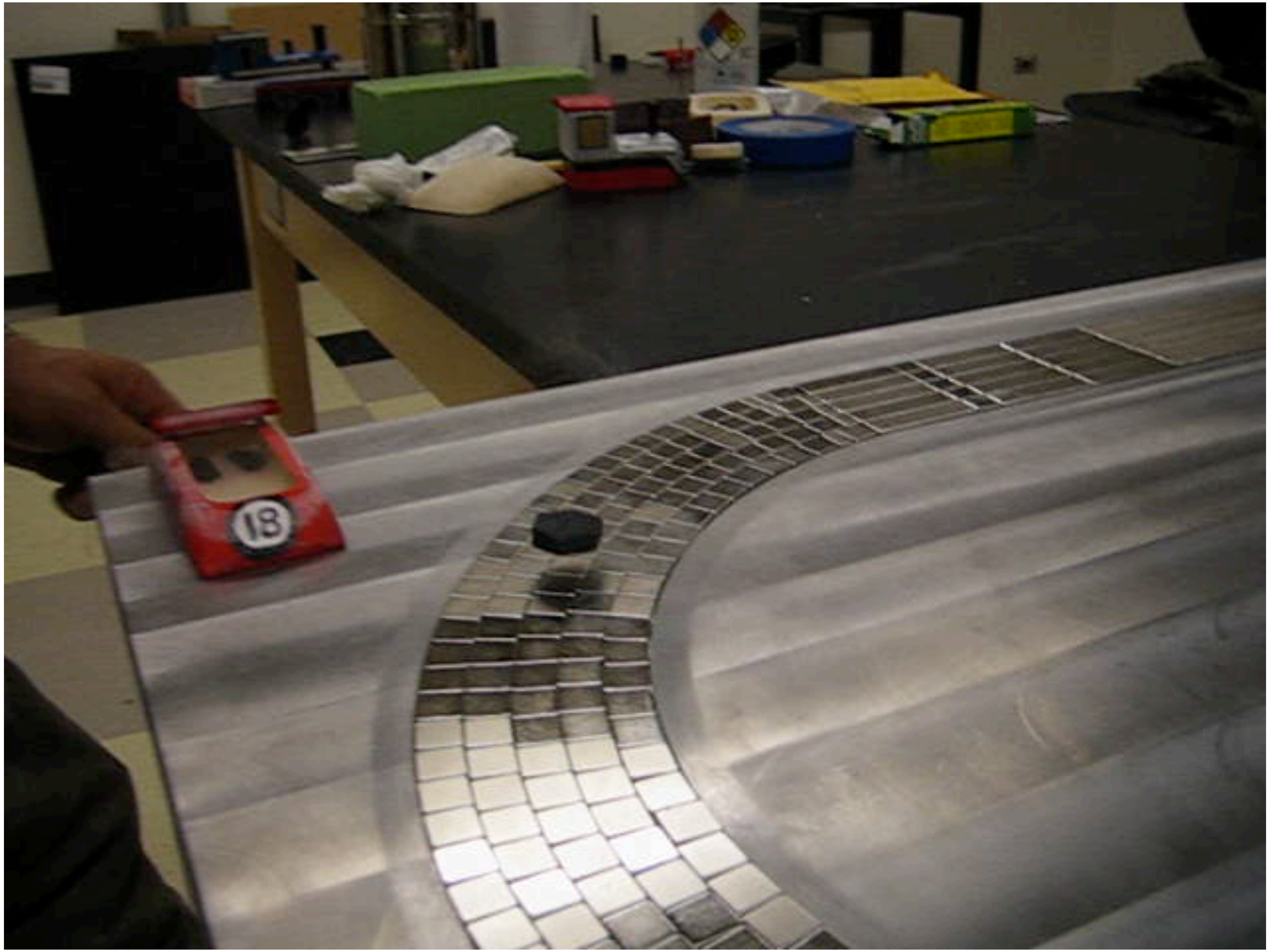
HARVARD

Cuprate high temperature superconductors



$$YBa_2Cu_3O_{6+x}$$



Nd-Fe-B magnets, YBaCuO superconductor

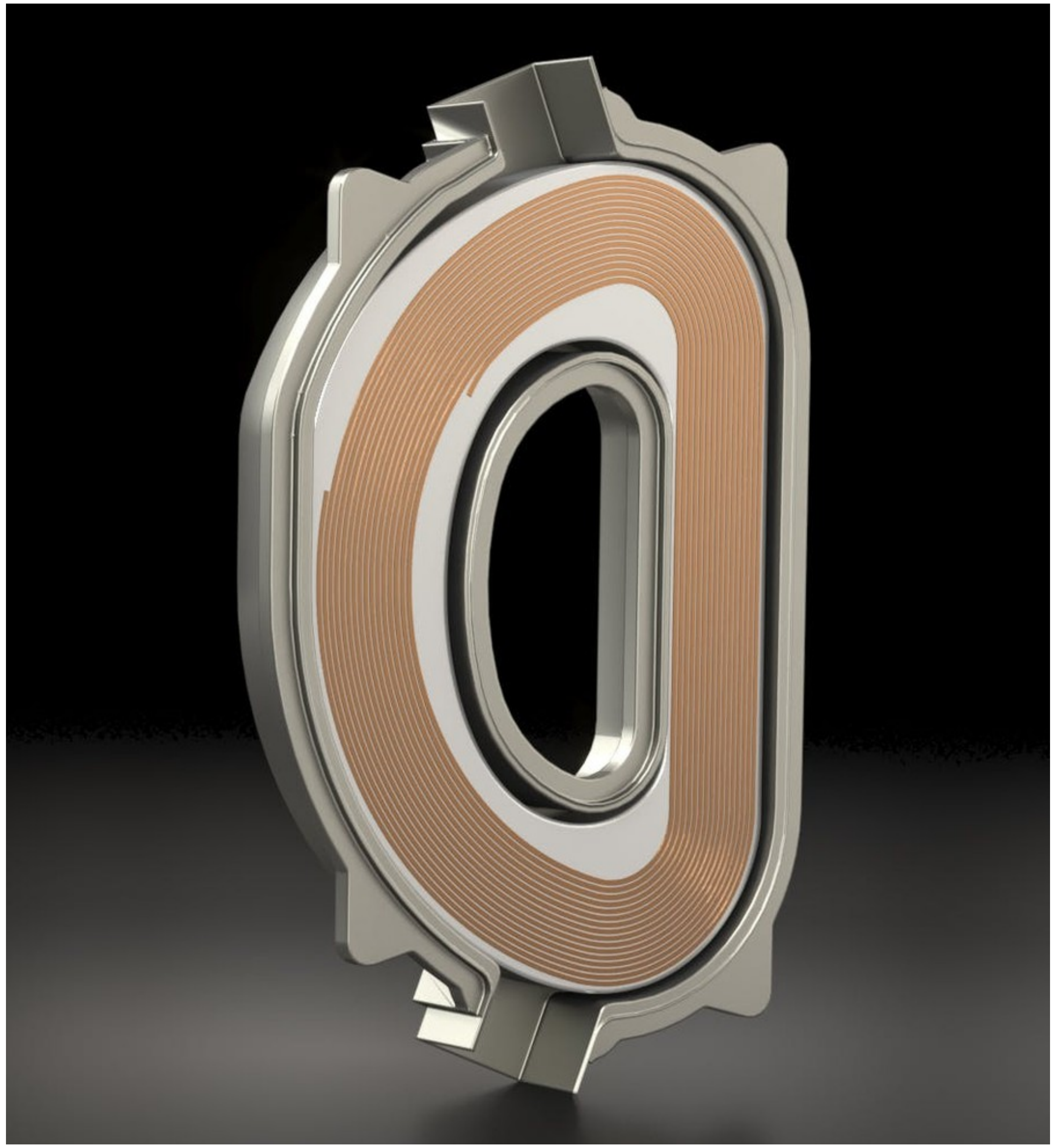
Julian Hetel and Nandini Trivedi, Ohio State University

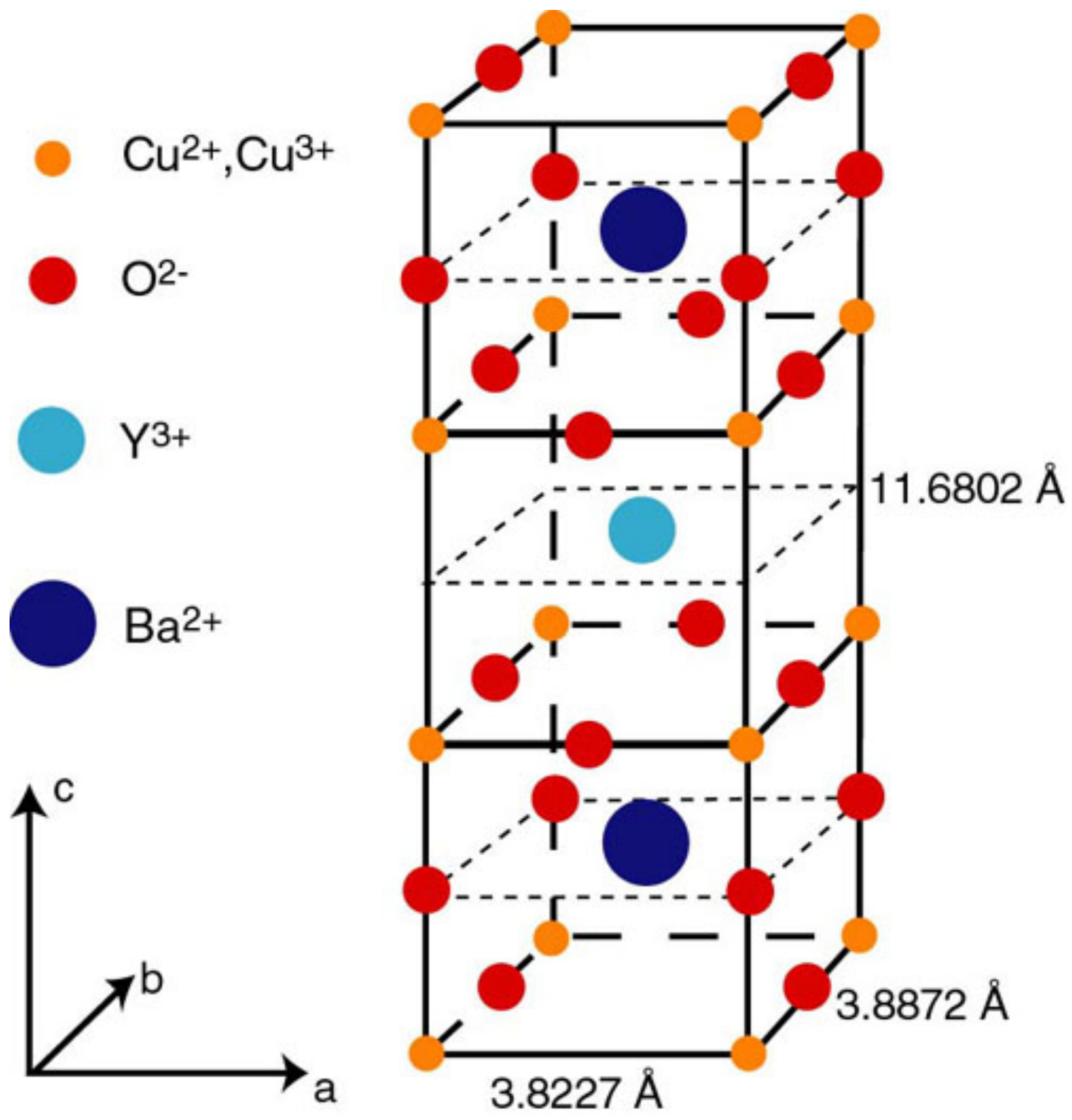
HTS Magnets: Enabling Technology

The surest path to limitless, clean, fusion energy

YBCO magnets allow for smaller, faster, and less expensive tokamaks for plasma fusion

Commonwealth Fusion Systems





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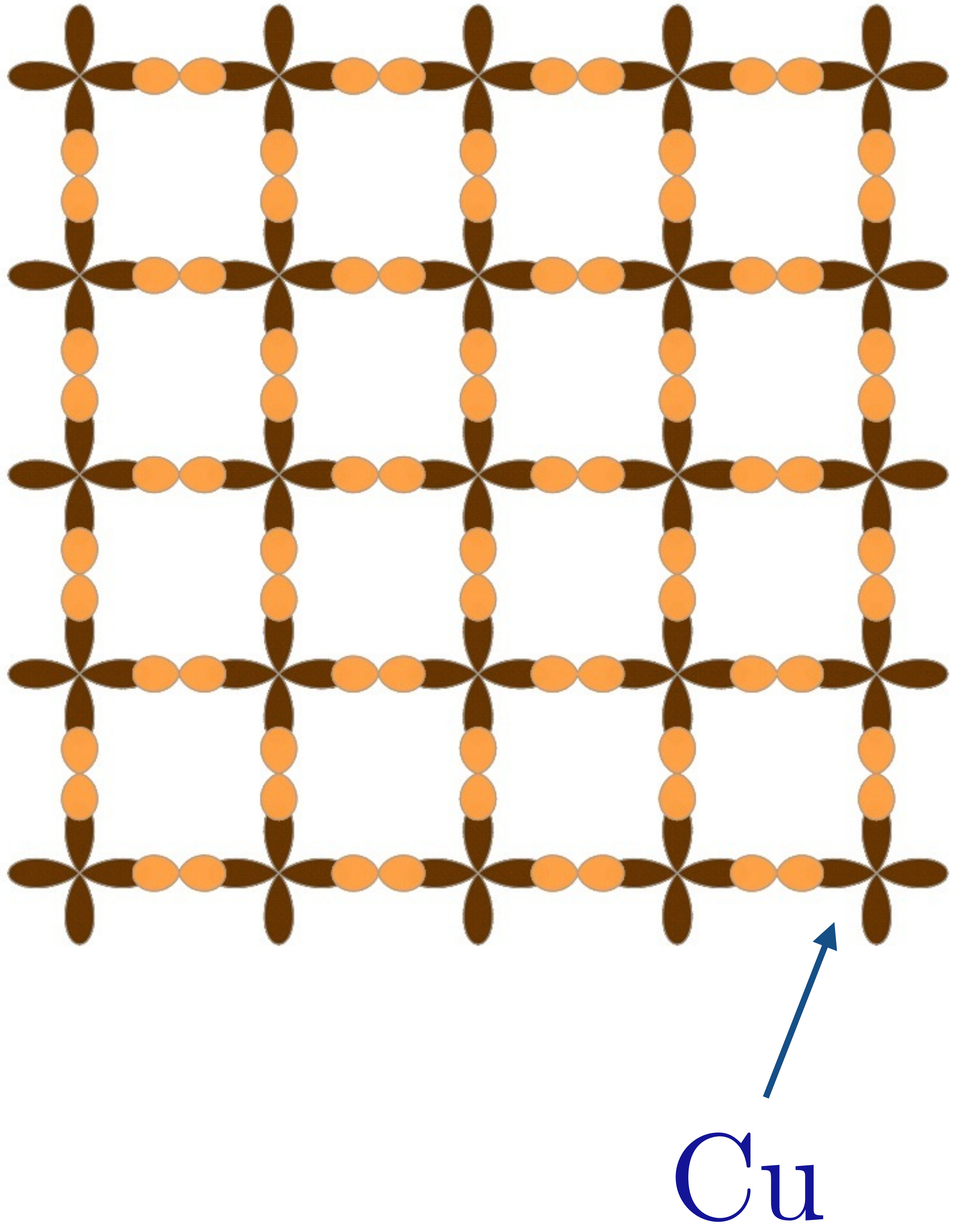

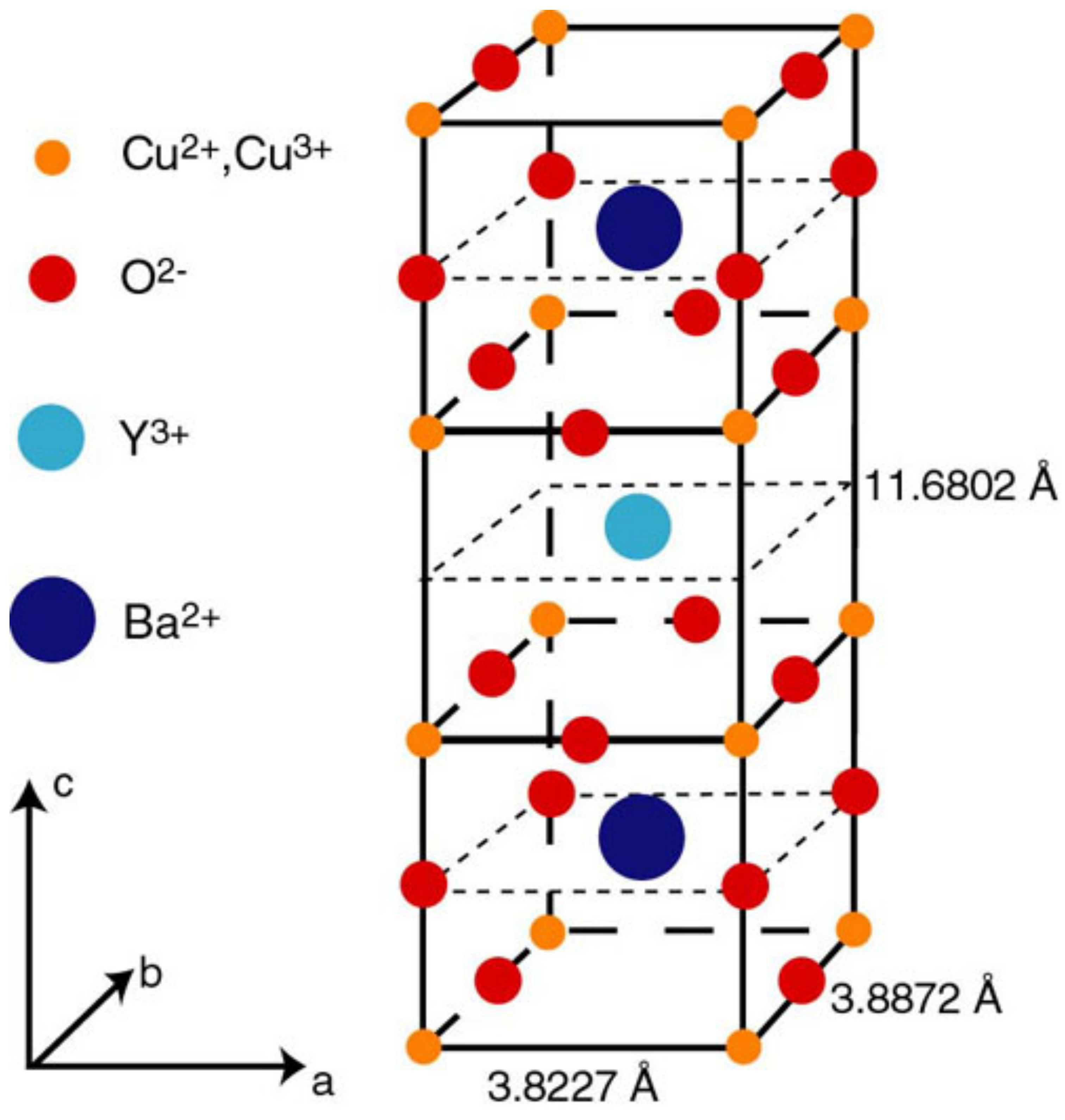

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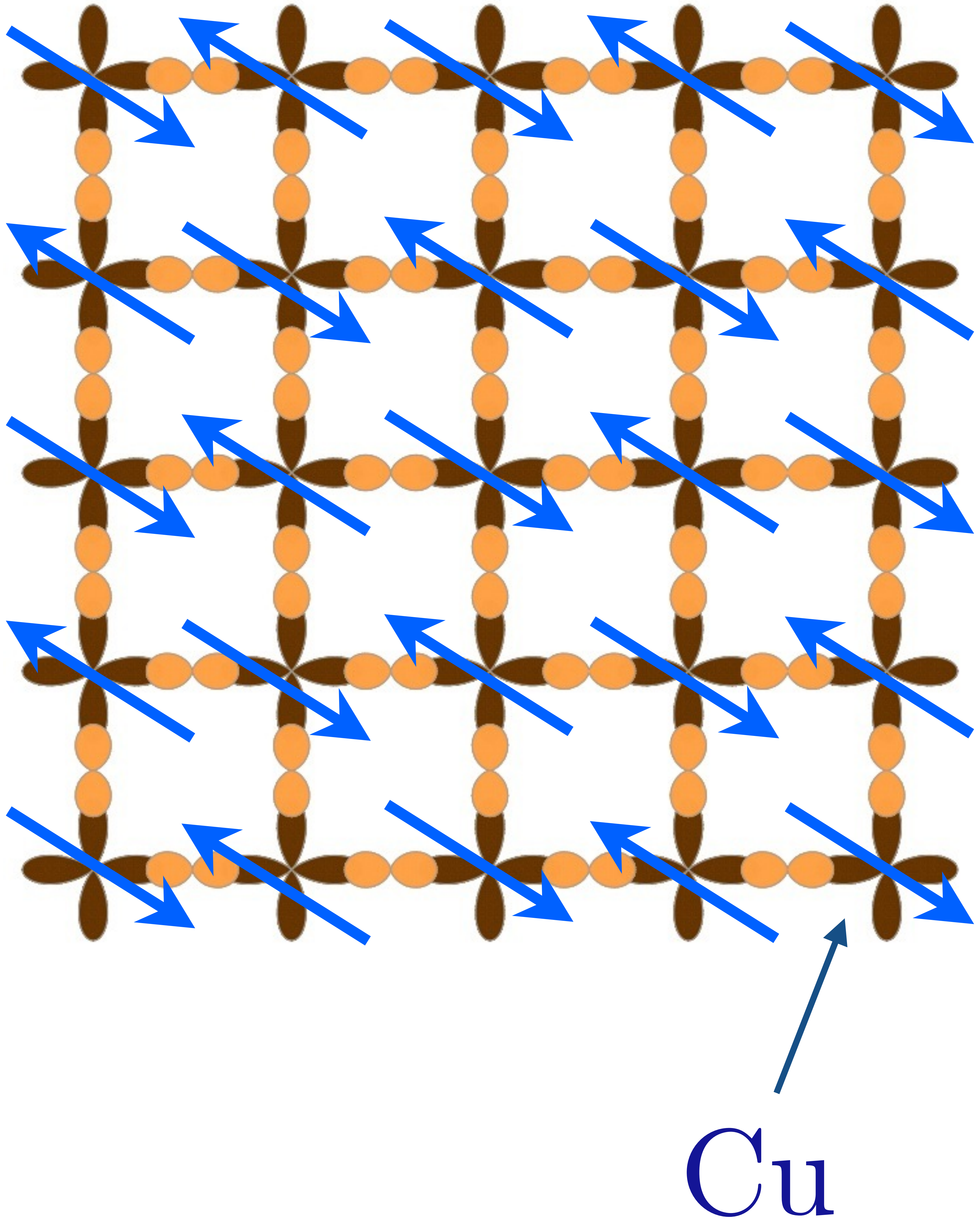


$$\mathcal{H} = J \sum_{\langle ij \rangle} \boldsymbol{S}_i \cdot \boldsymbol{S}_j$$

$S = 1/2$ on each site

$$|\uparrow\rangle, \, |\uparrow\rangle$$

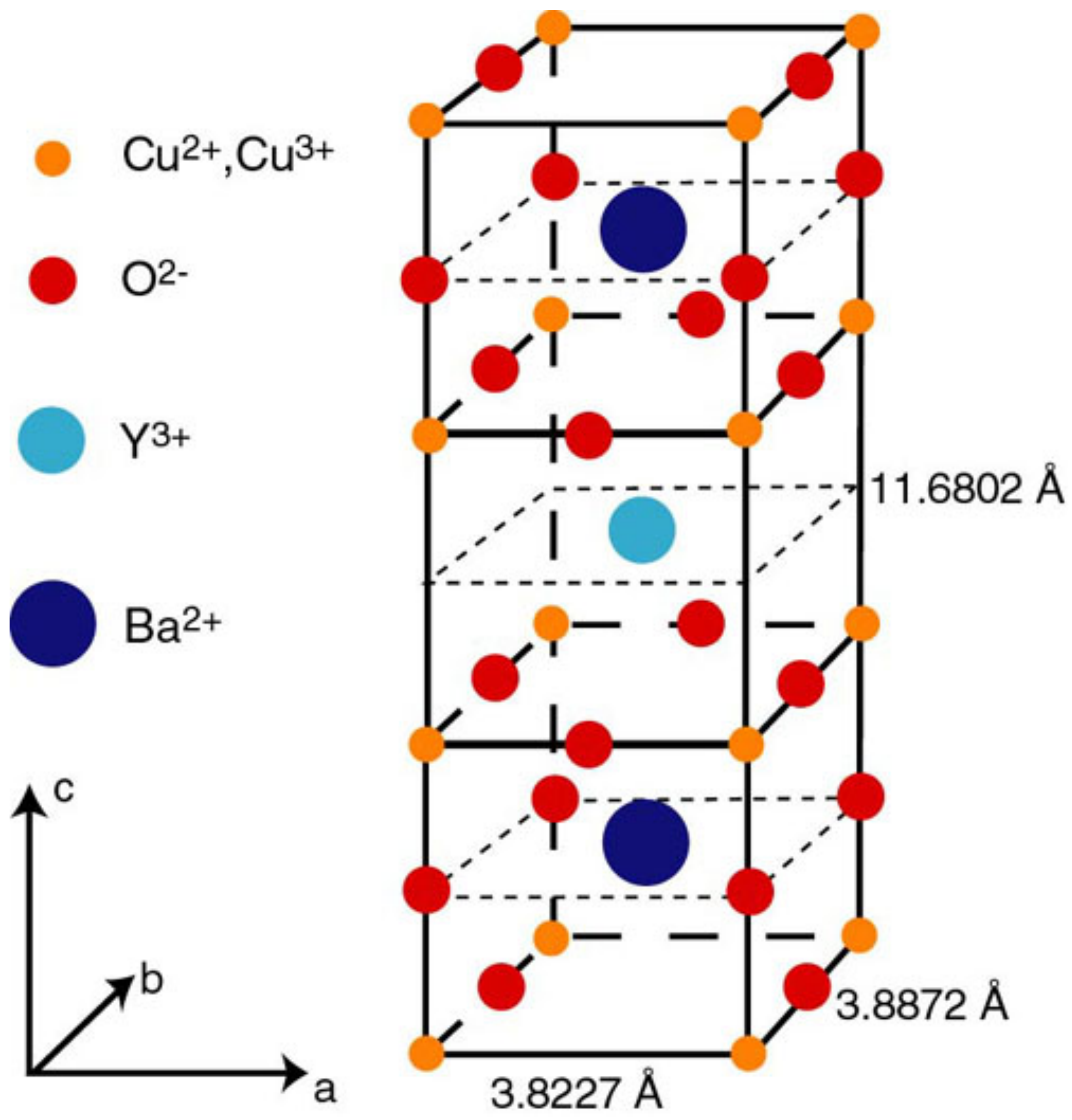
$$S_z \, |\uparrow\rangle = (1/2) \, |\uparrow\rangle$$

$$S_z \, |\downarrow\rangle = -(1/2) \, |\downarrow\rangle$$

$$(S_x + iS_y) \, |\downarrow\rangle = |\uparrow\rangle$$

$$(S_x - iS_y) \, |\uparrow\rangle = |\downarrow\rangle$$

Insulating antiferromagnet with one electron per site



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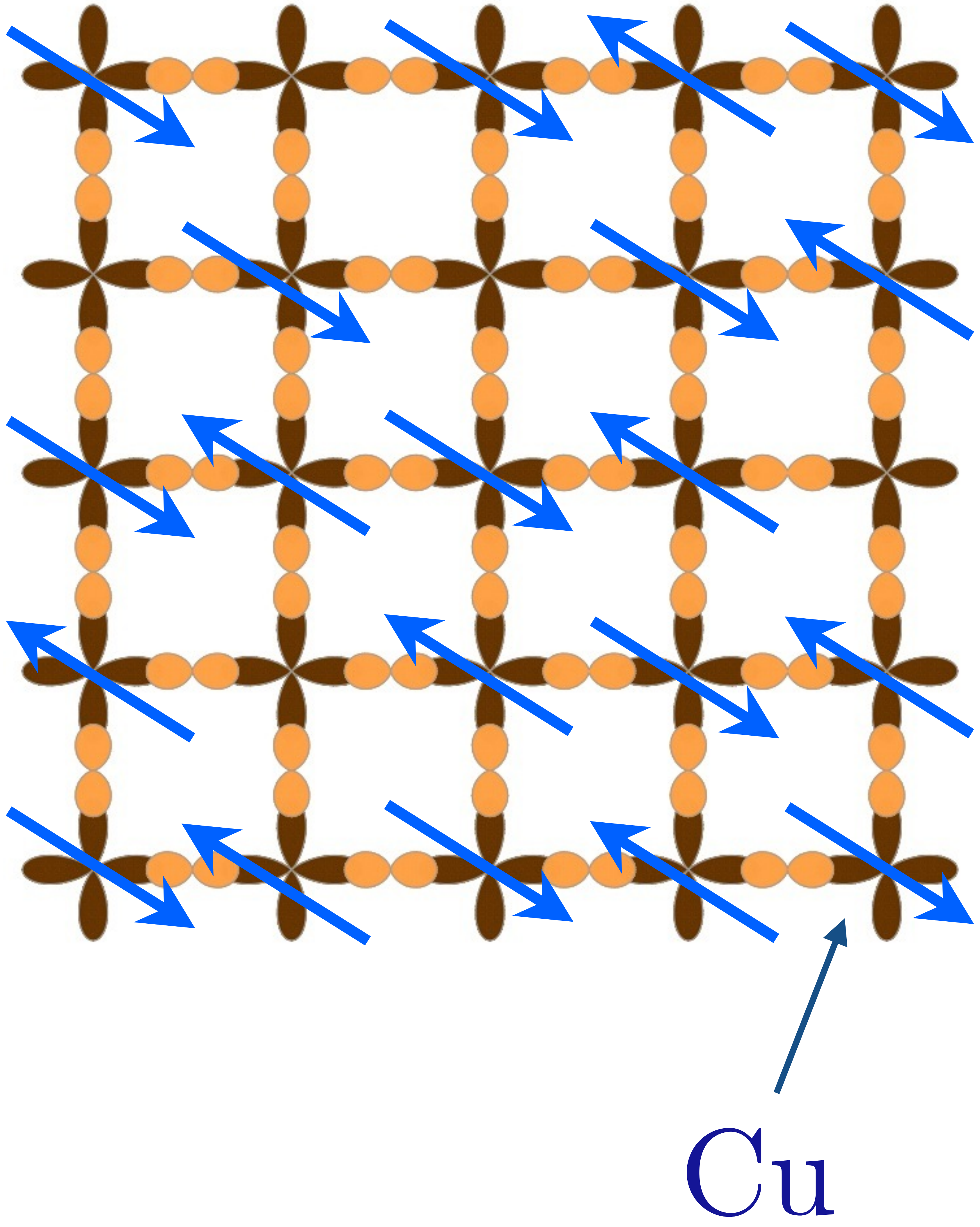
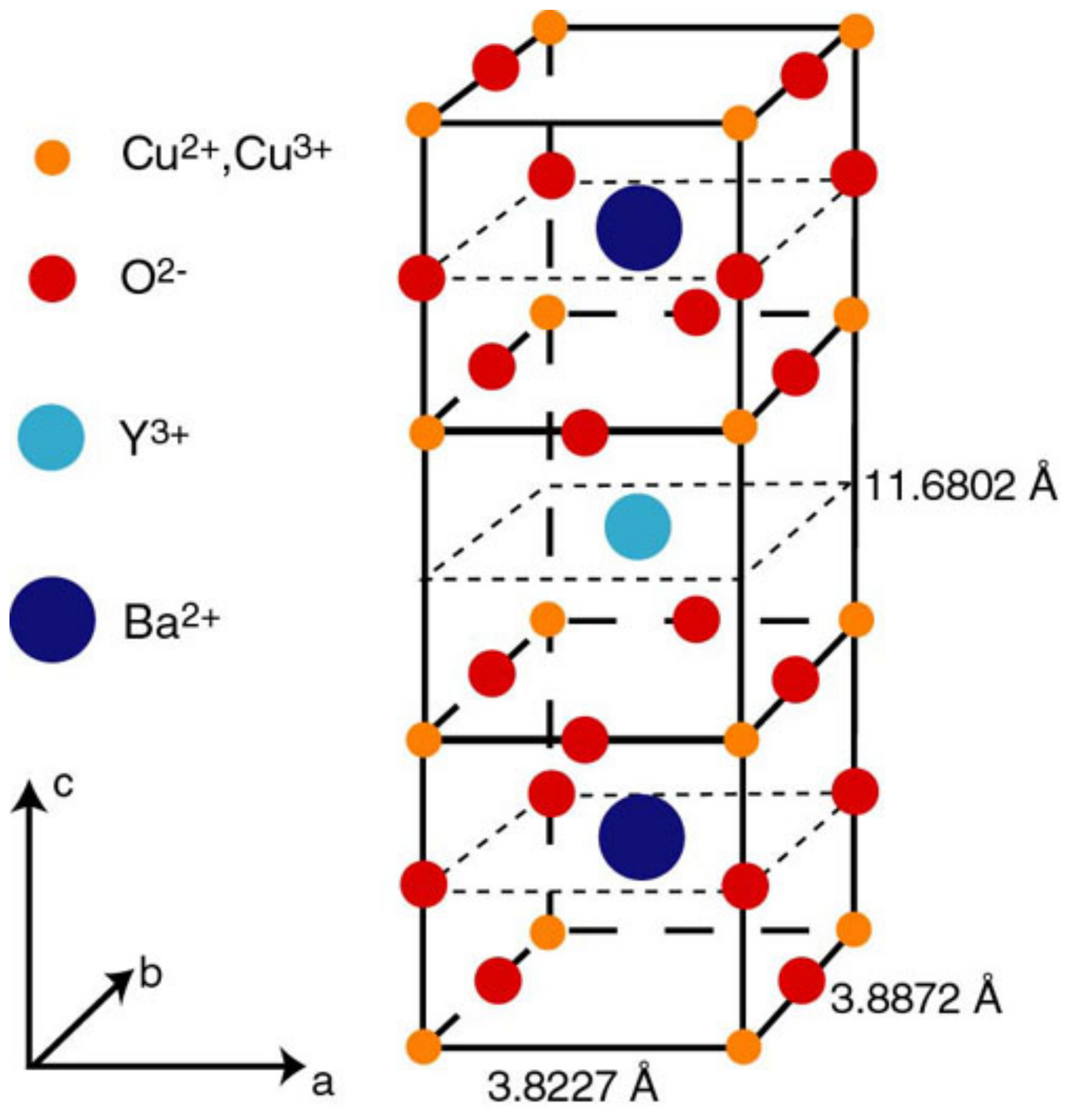
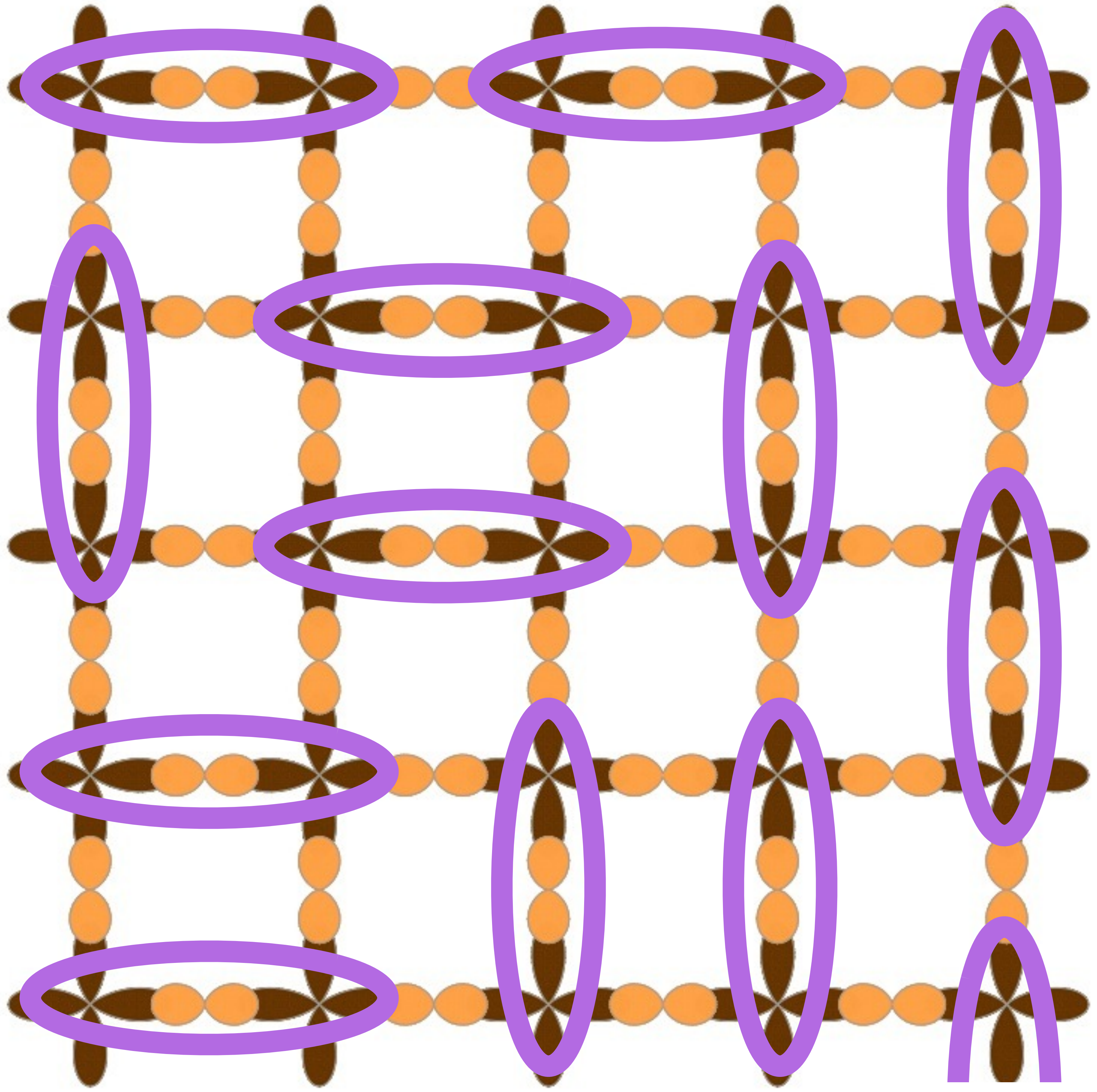


High temperature superconductor obtained upon doping the antiferromagnet with density p holes.





$$|G\rangle = \sum_{\mathcal{D}} c_{\mathcal{D}} \, |\mathcal{D}\rangle$$

$\mathcal{D} \rightarrow$ dimer covering of lattice

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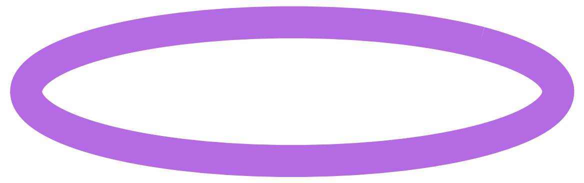
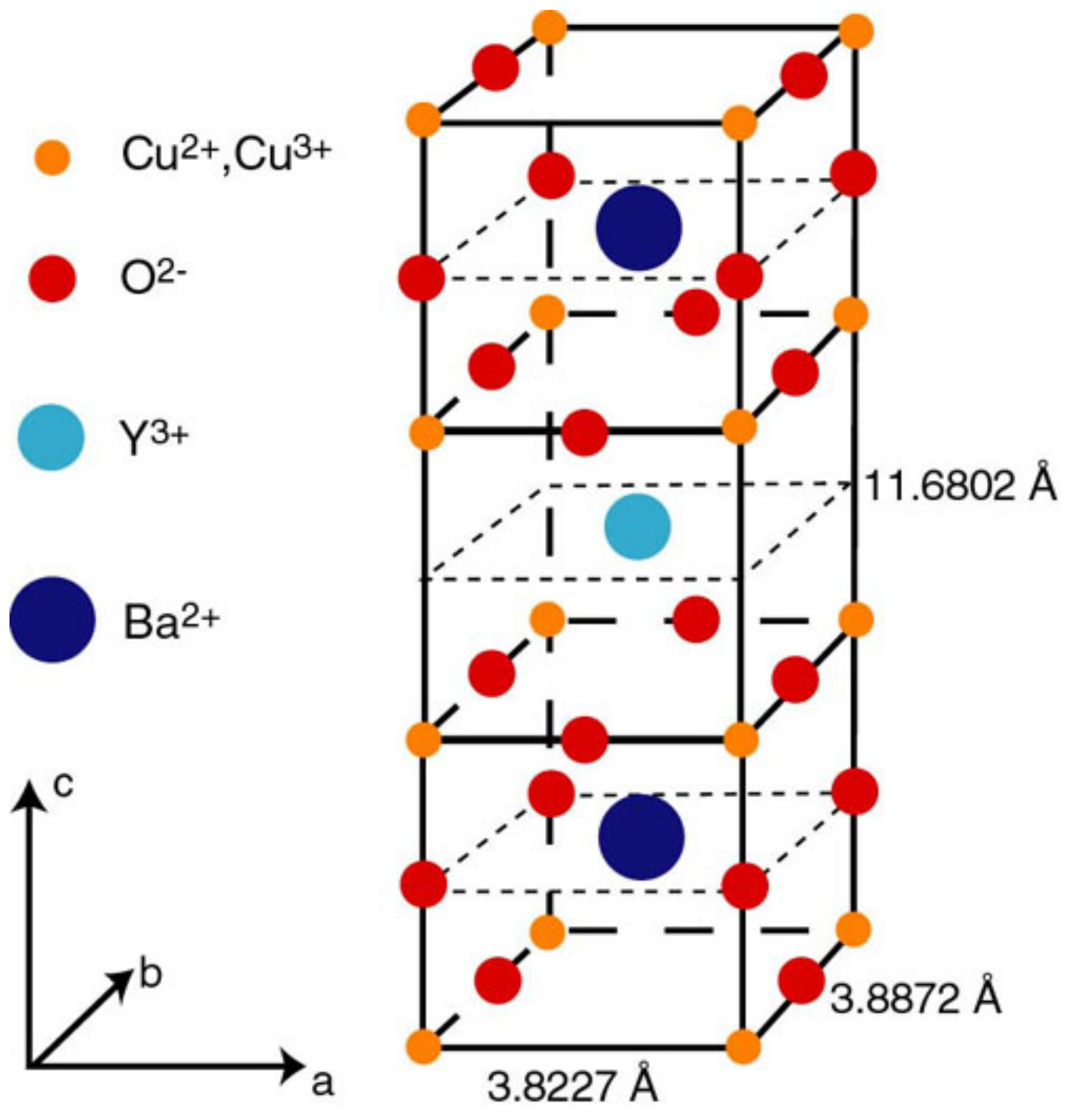
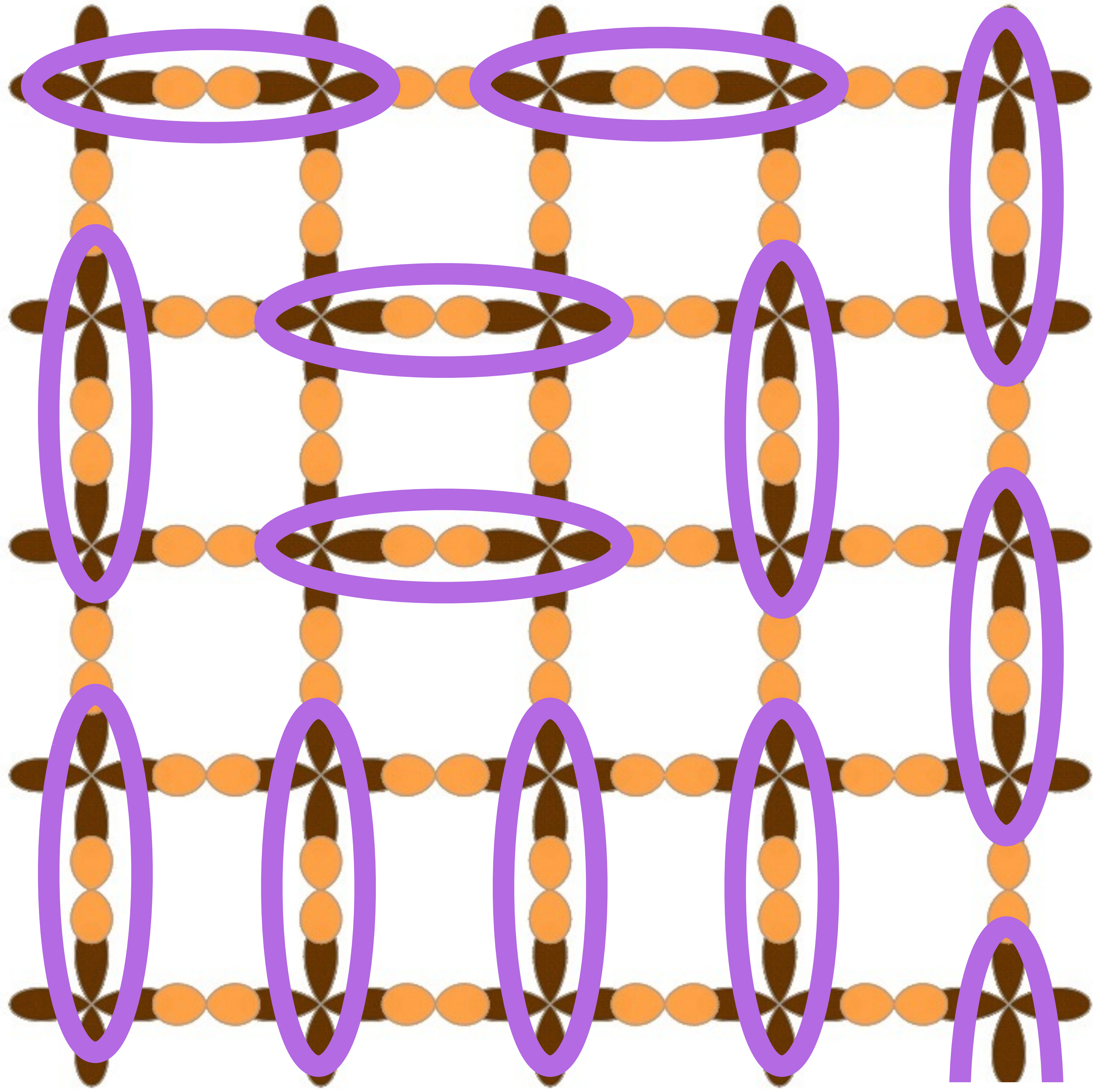
 $= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$

P.W. Anderson and G. Baskaran (1988): The key to high temperature superconductivity is the formation of a "resonating valence bond state" (a type of **quantum spin liquid**) which entangles the electrons on Cu





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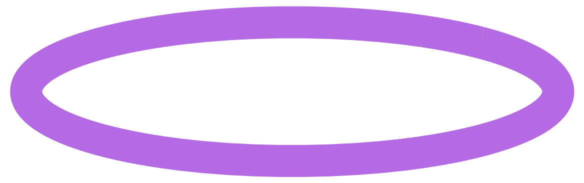 $= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$

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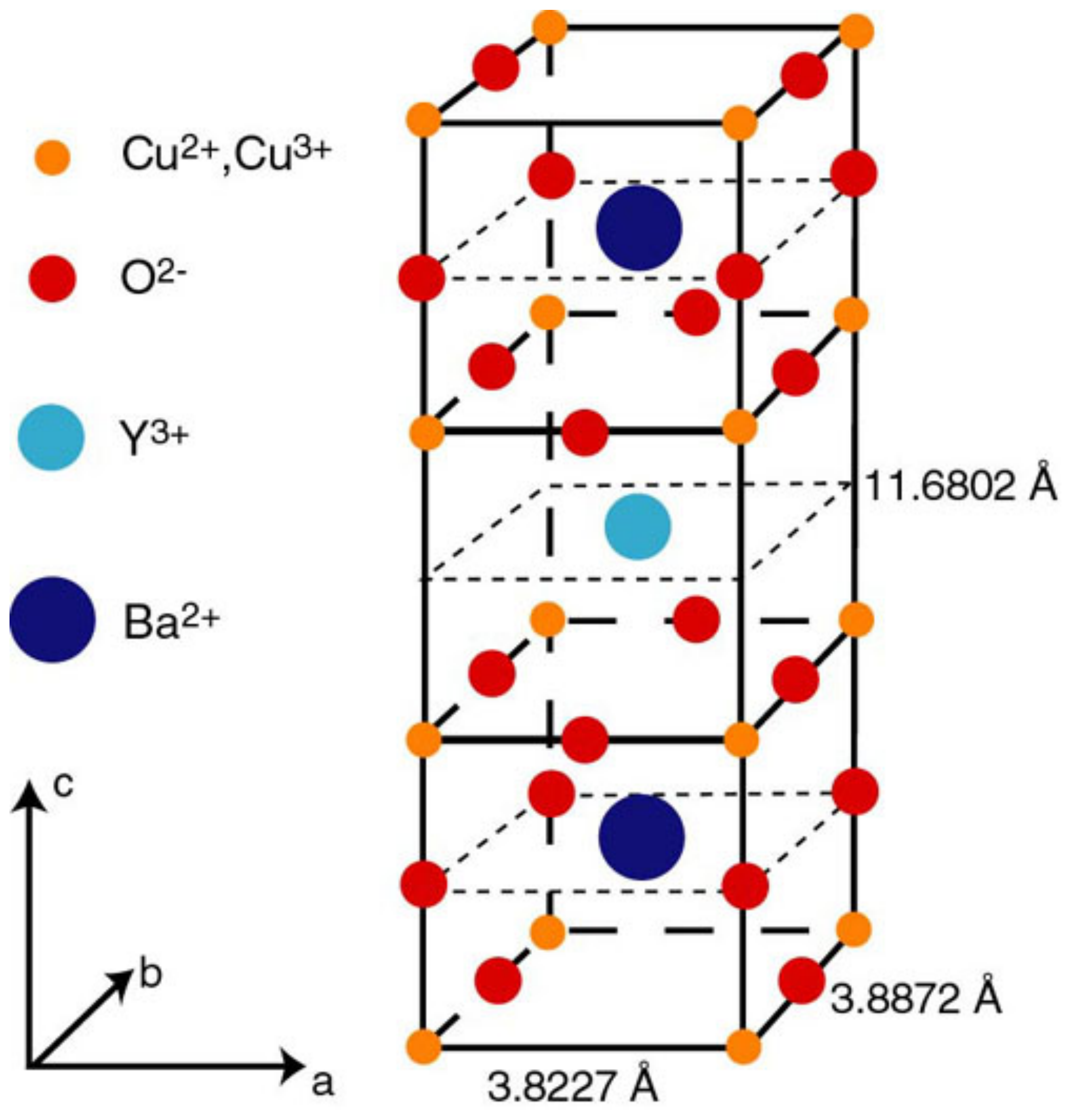

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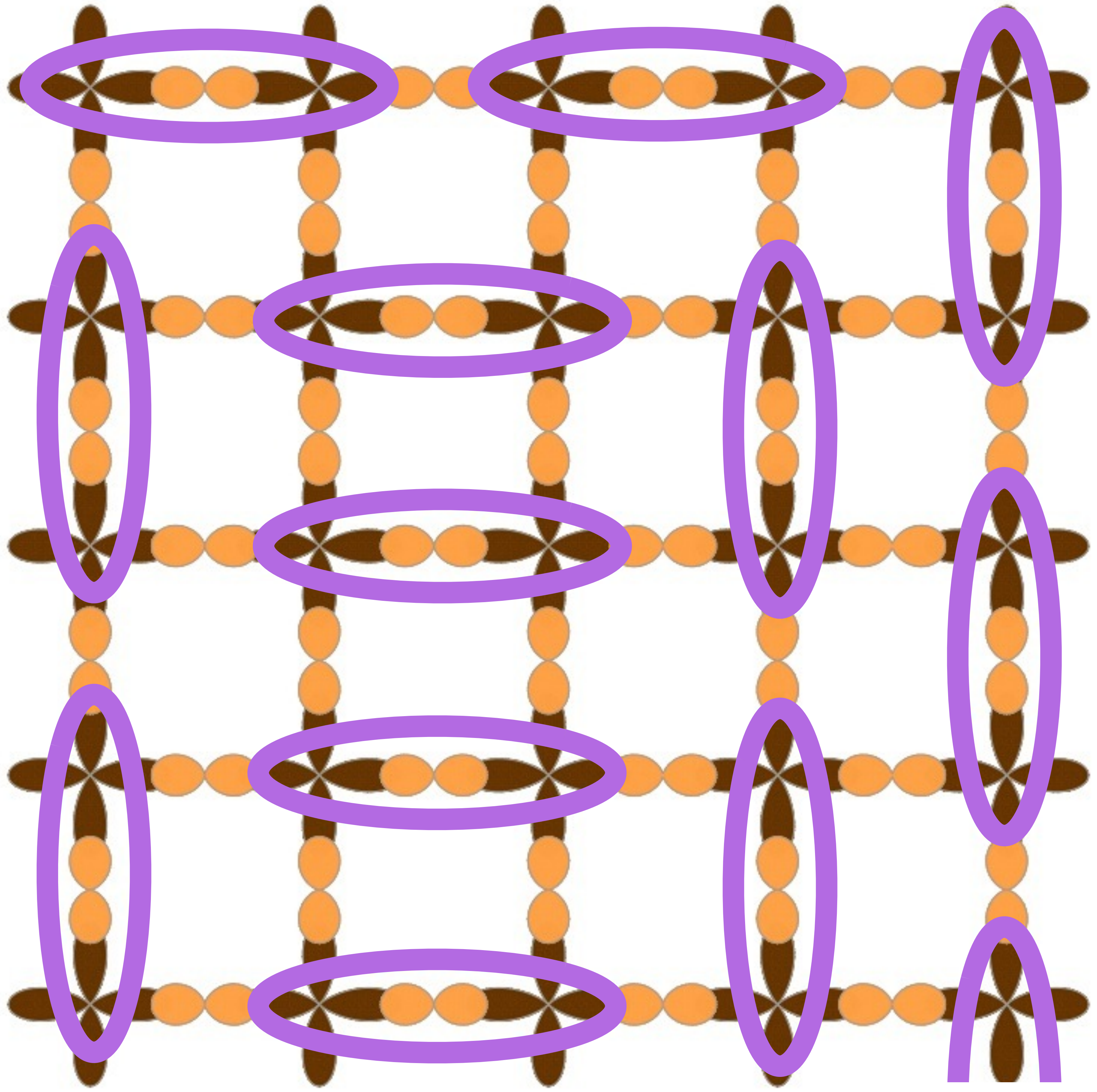

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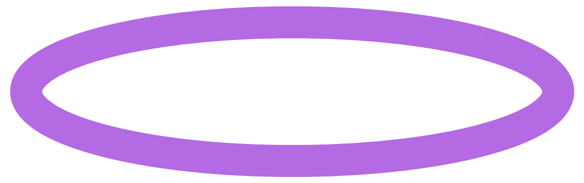 $= \frac{1}{\sqrt{2}} \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right)$

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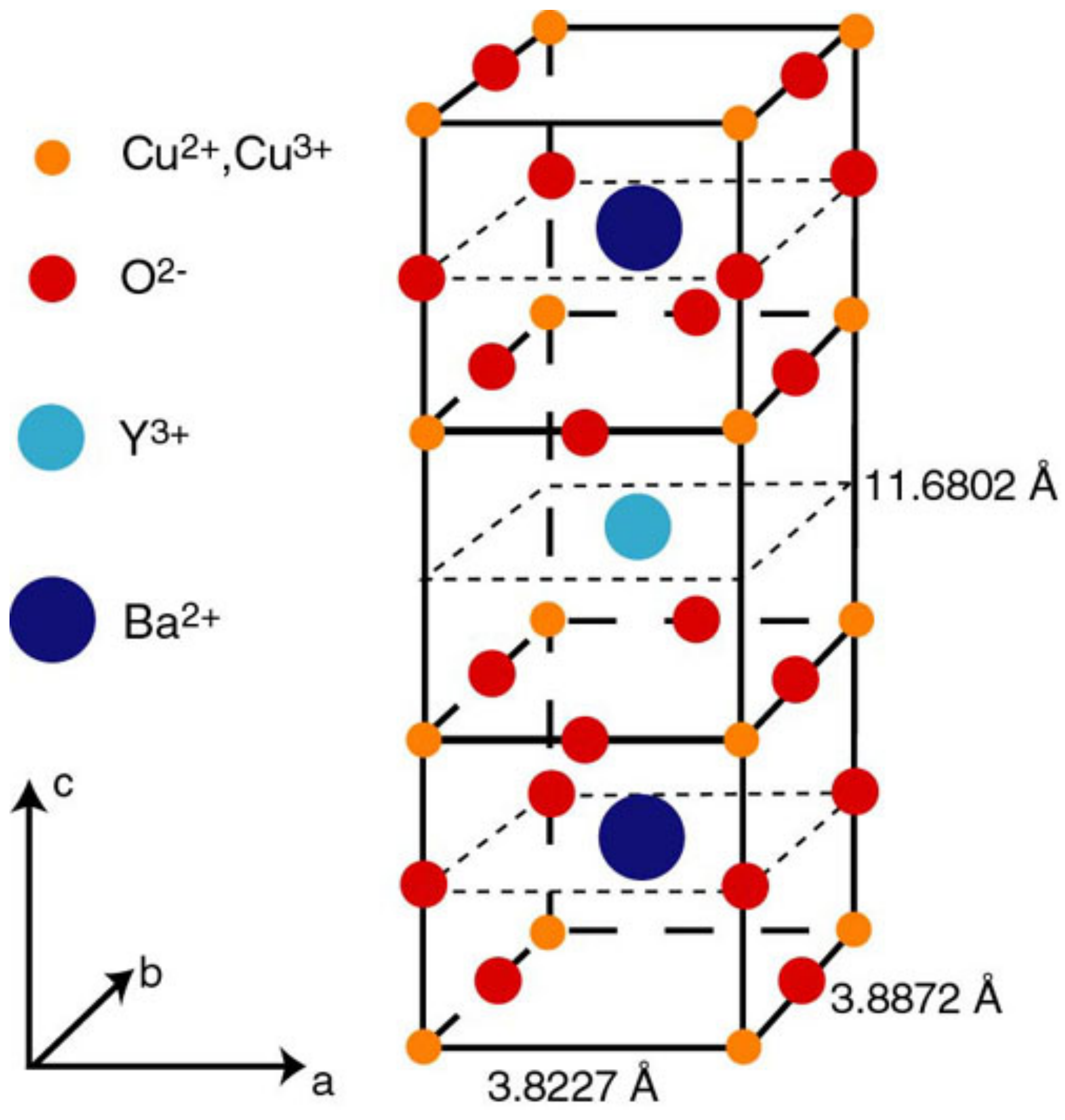

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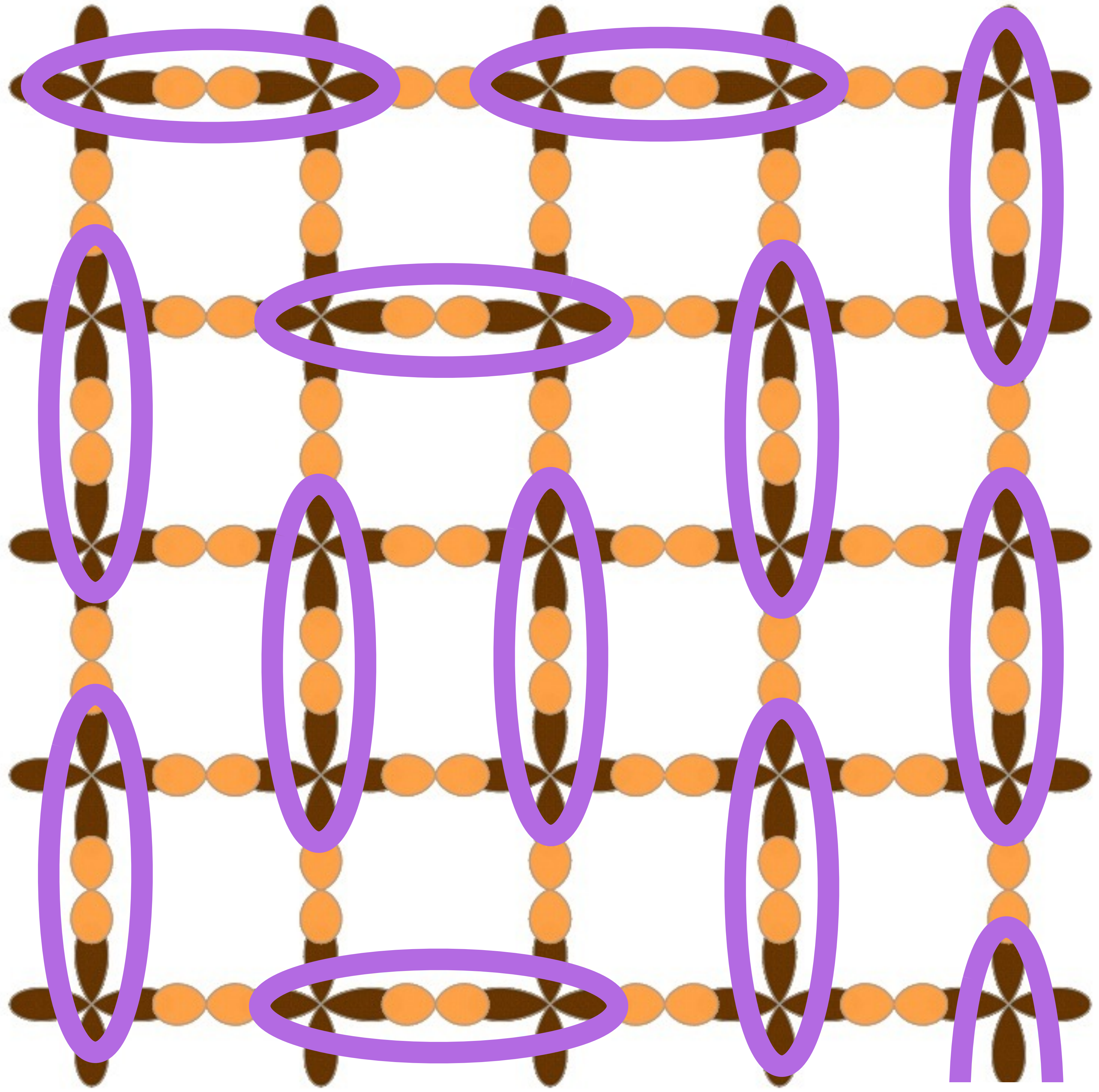

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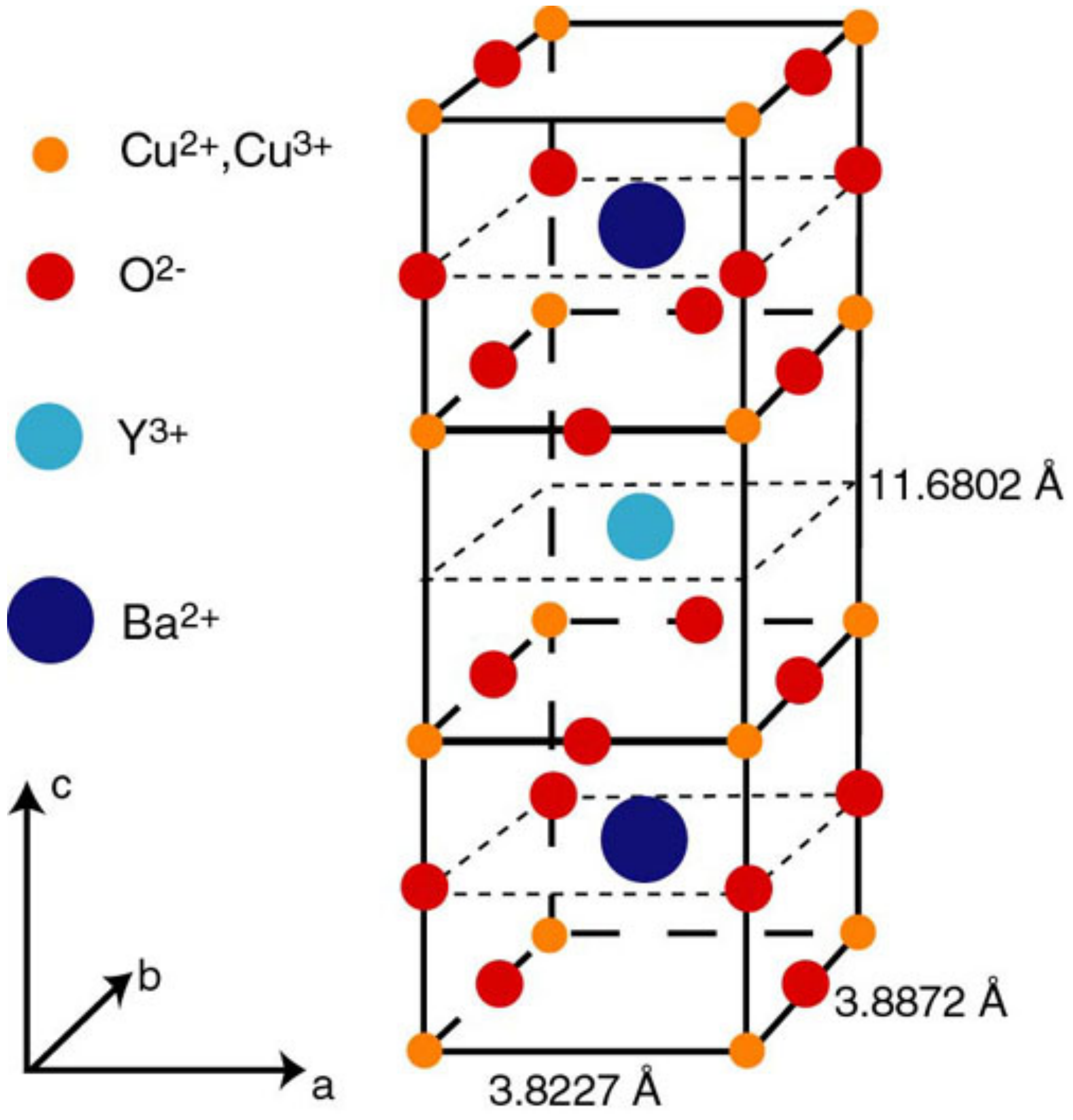

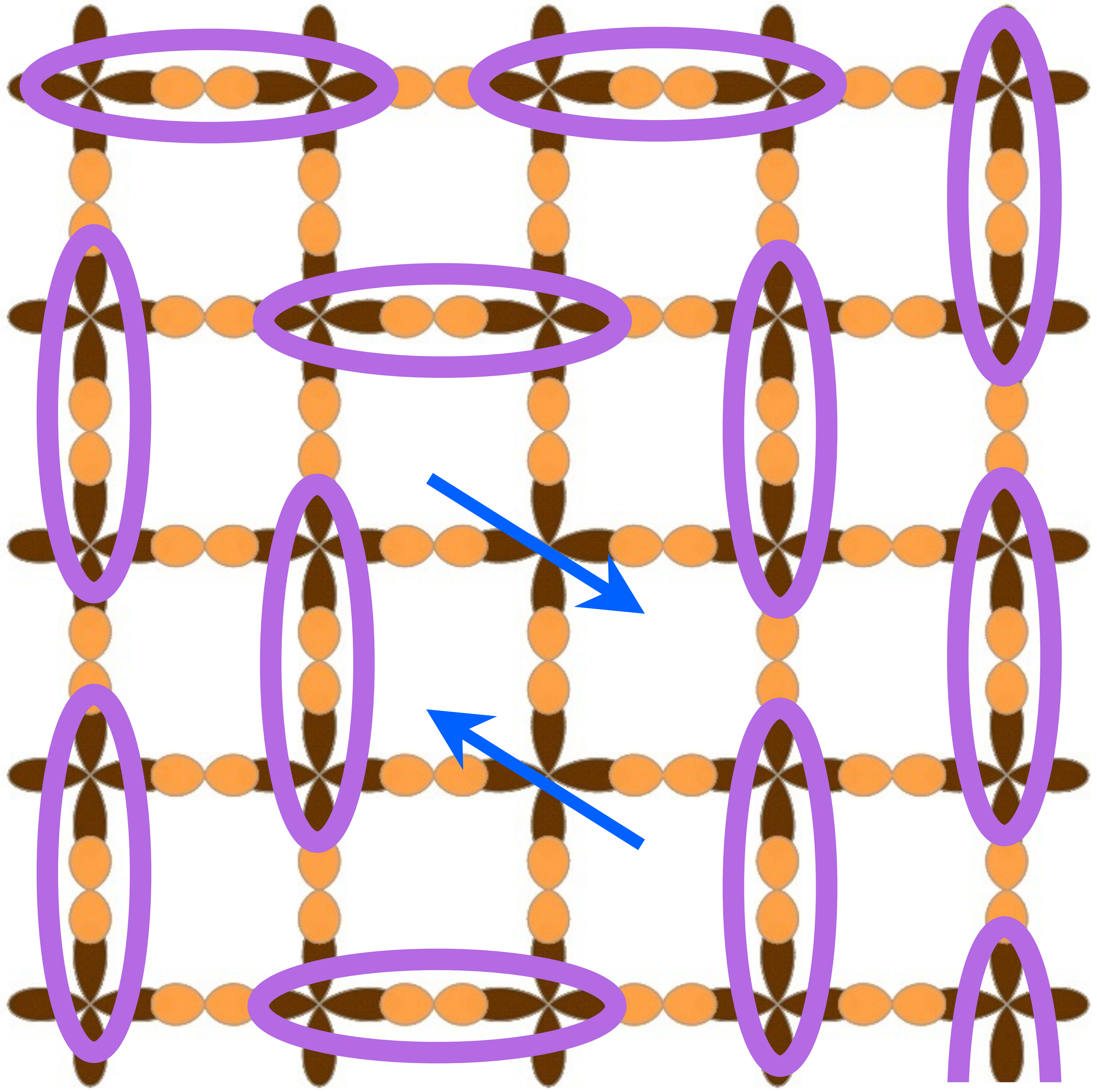

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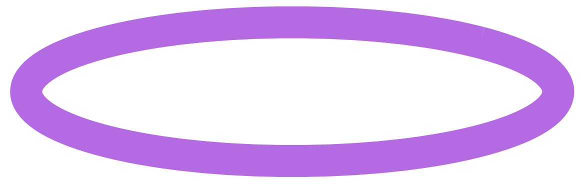
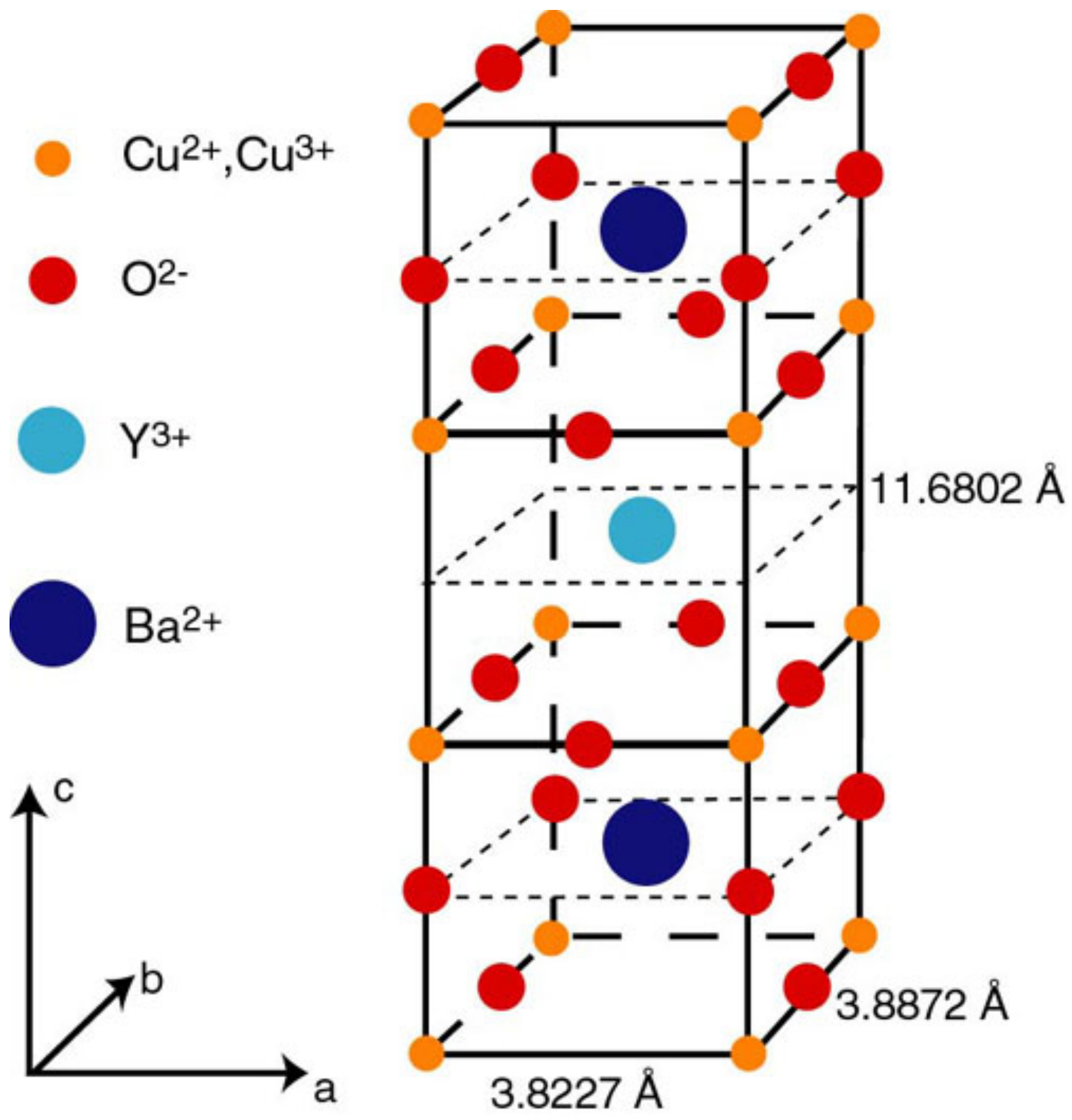
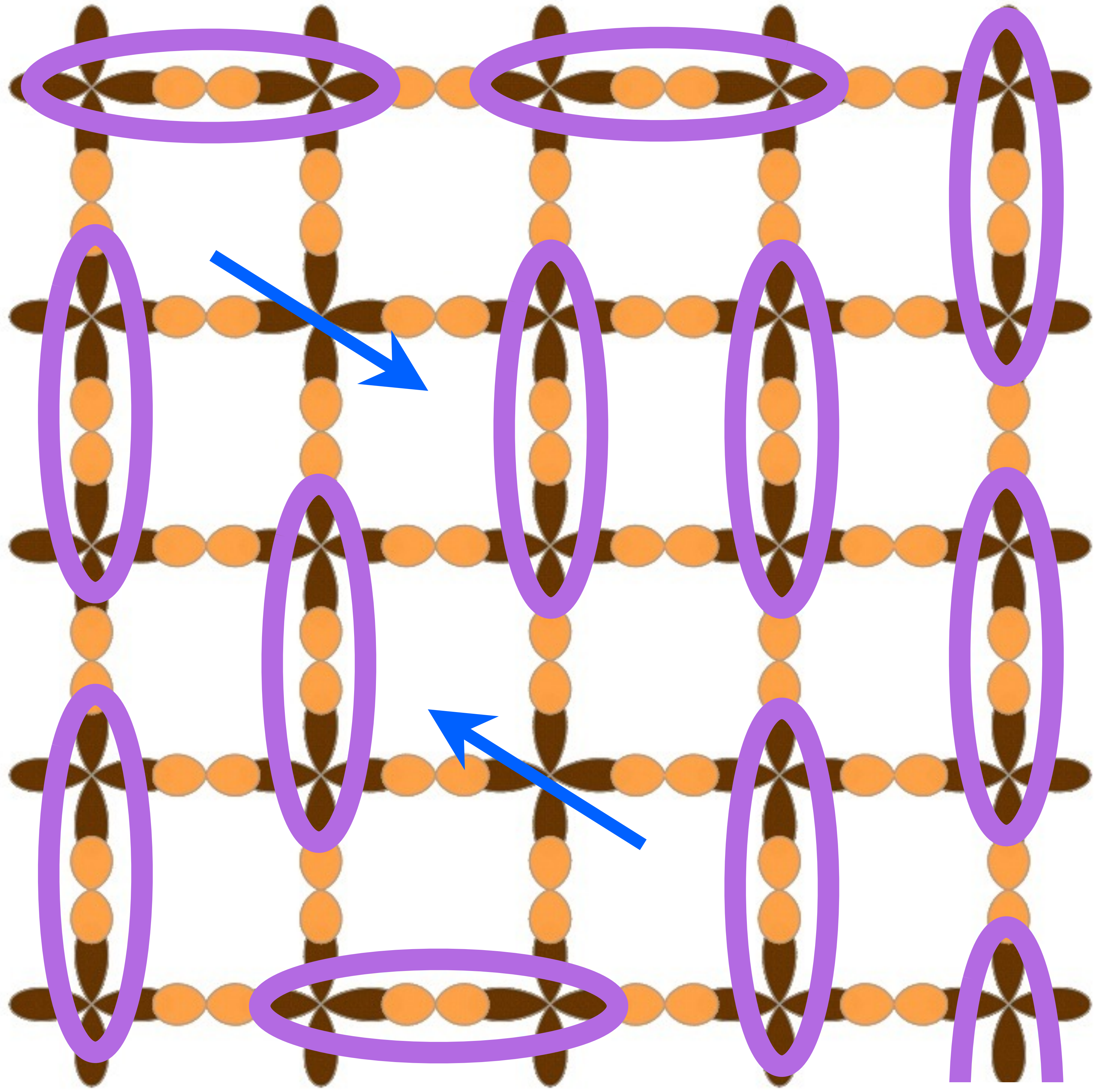
 $= \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

Key feature: fractionalization. Excitations are particle-like, but cannot be created by local operators: they are classified under distinct superselection/anyon sectors.





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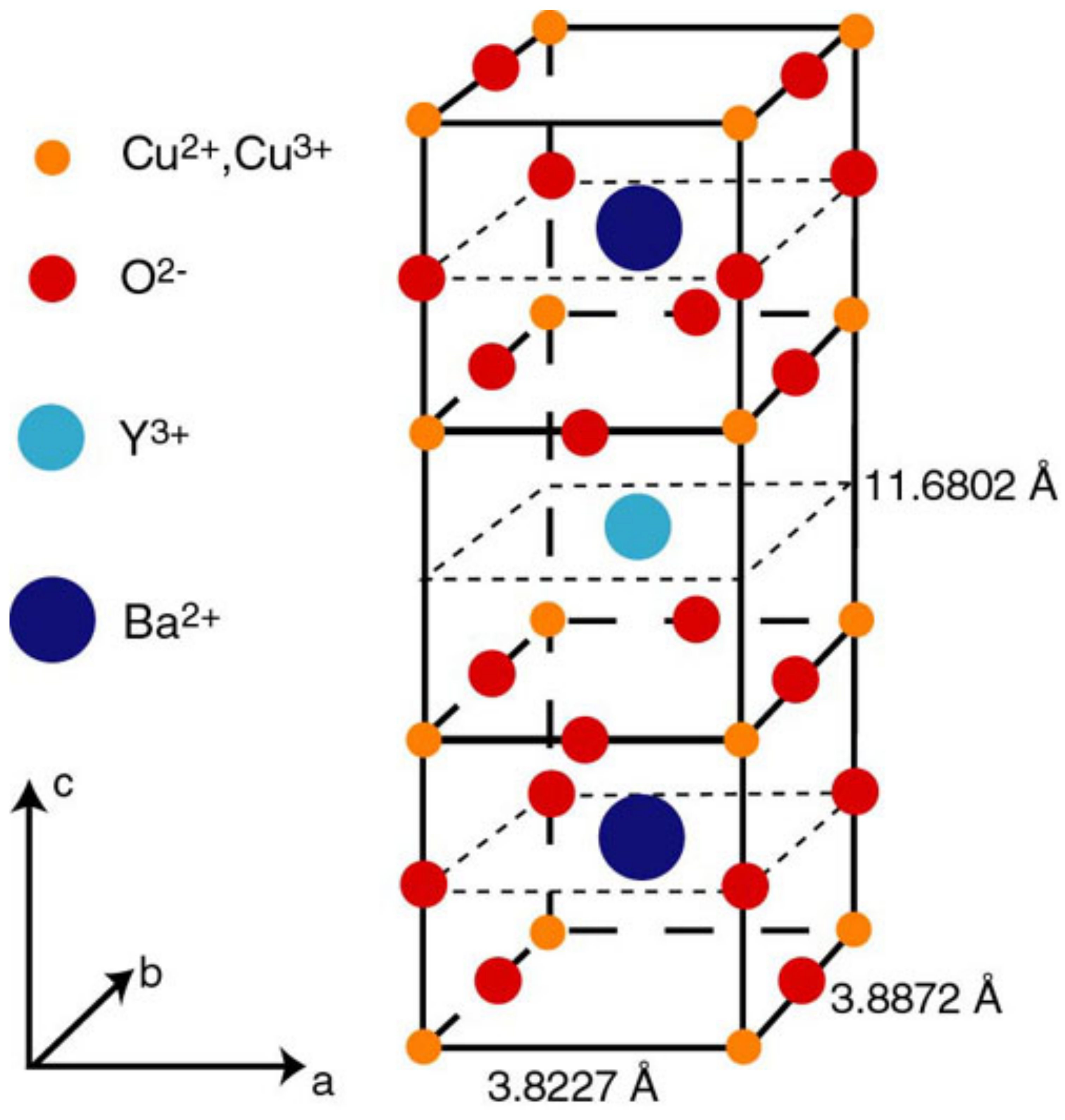

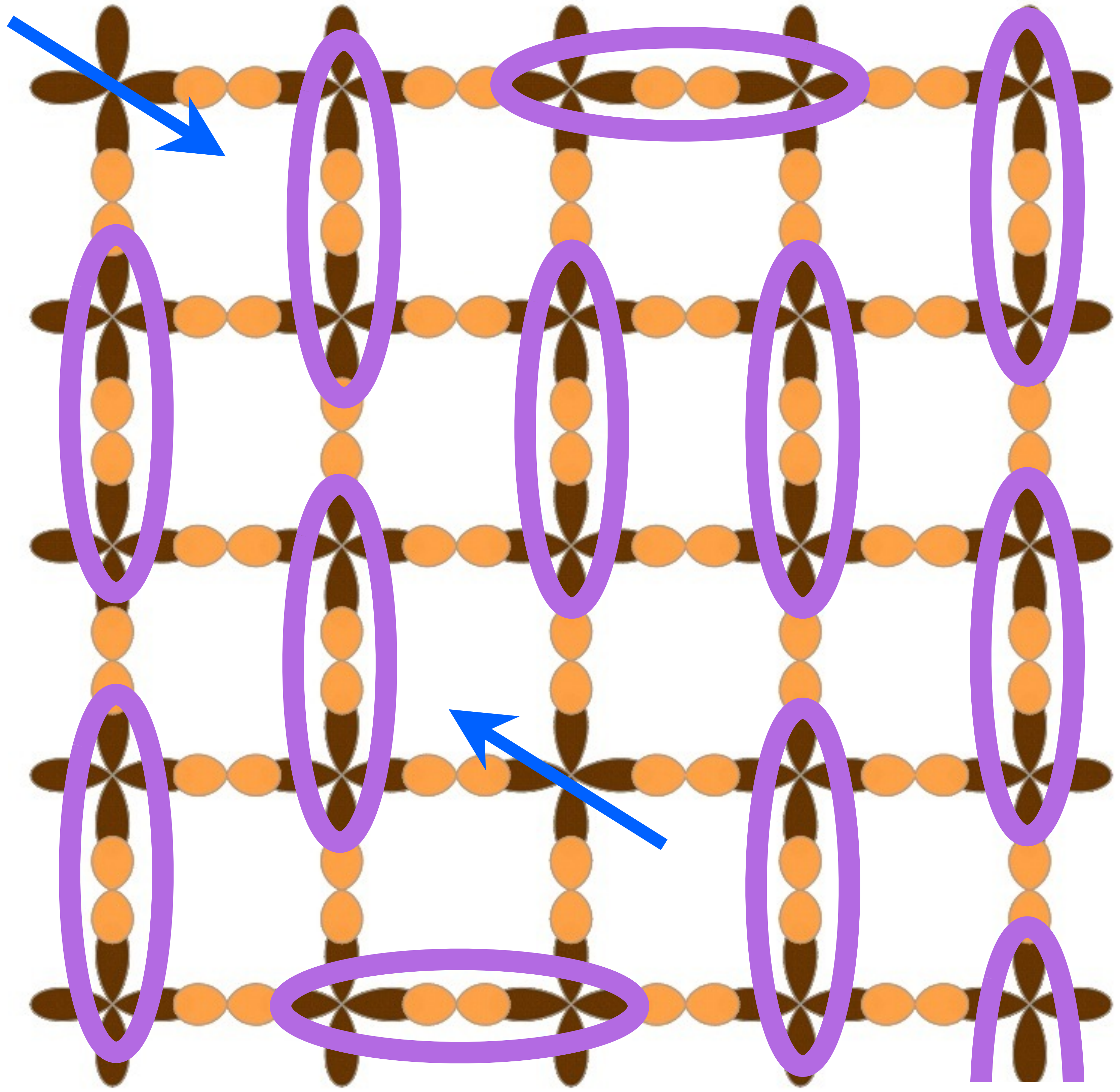

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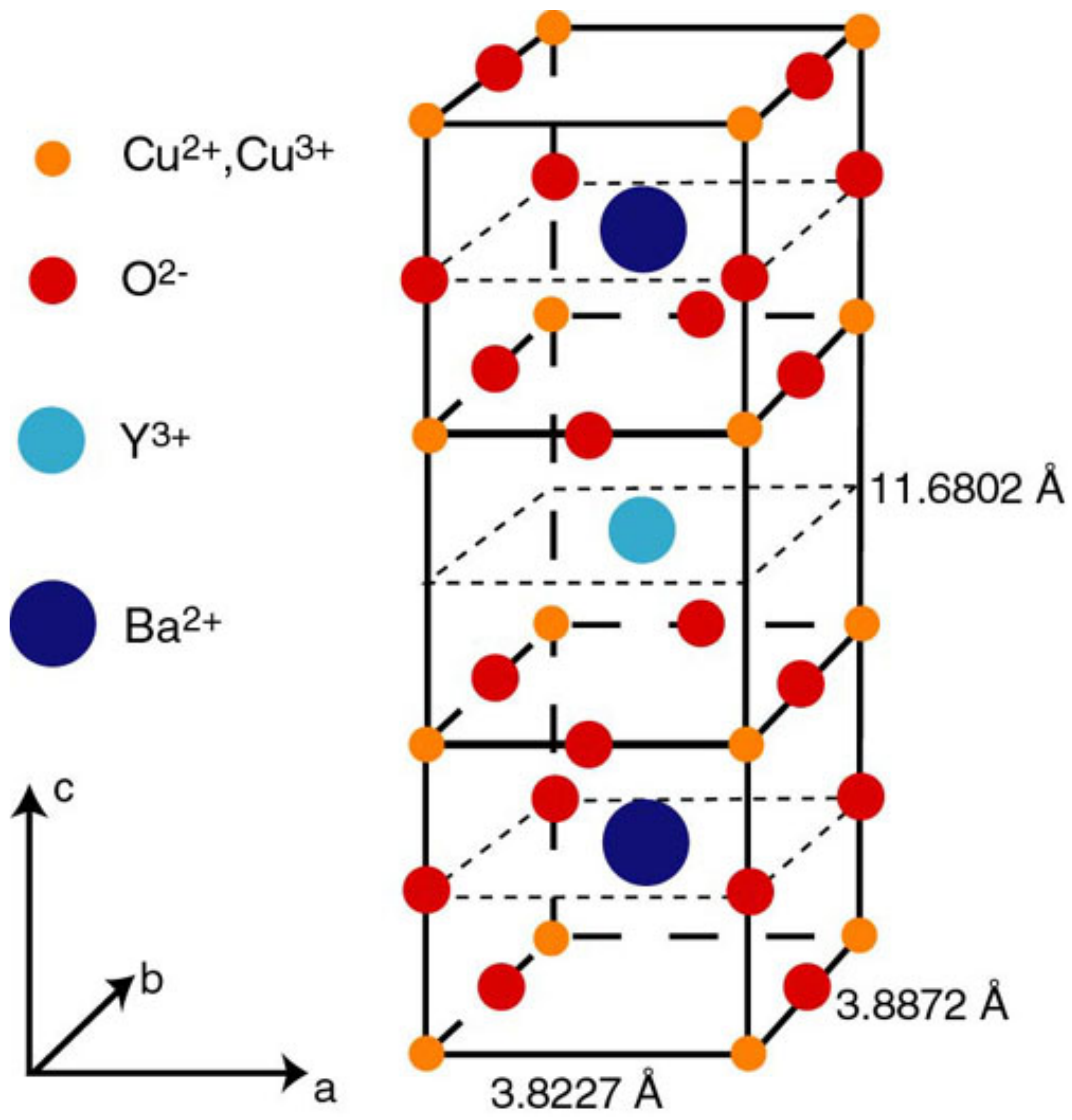

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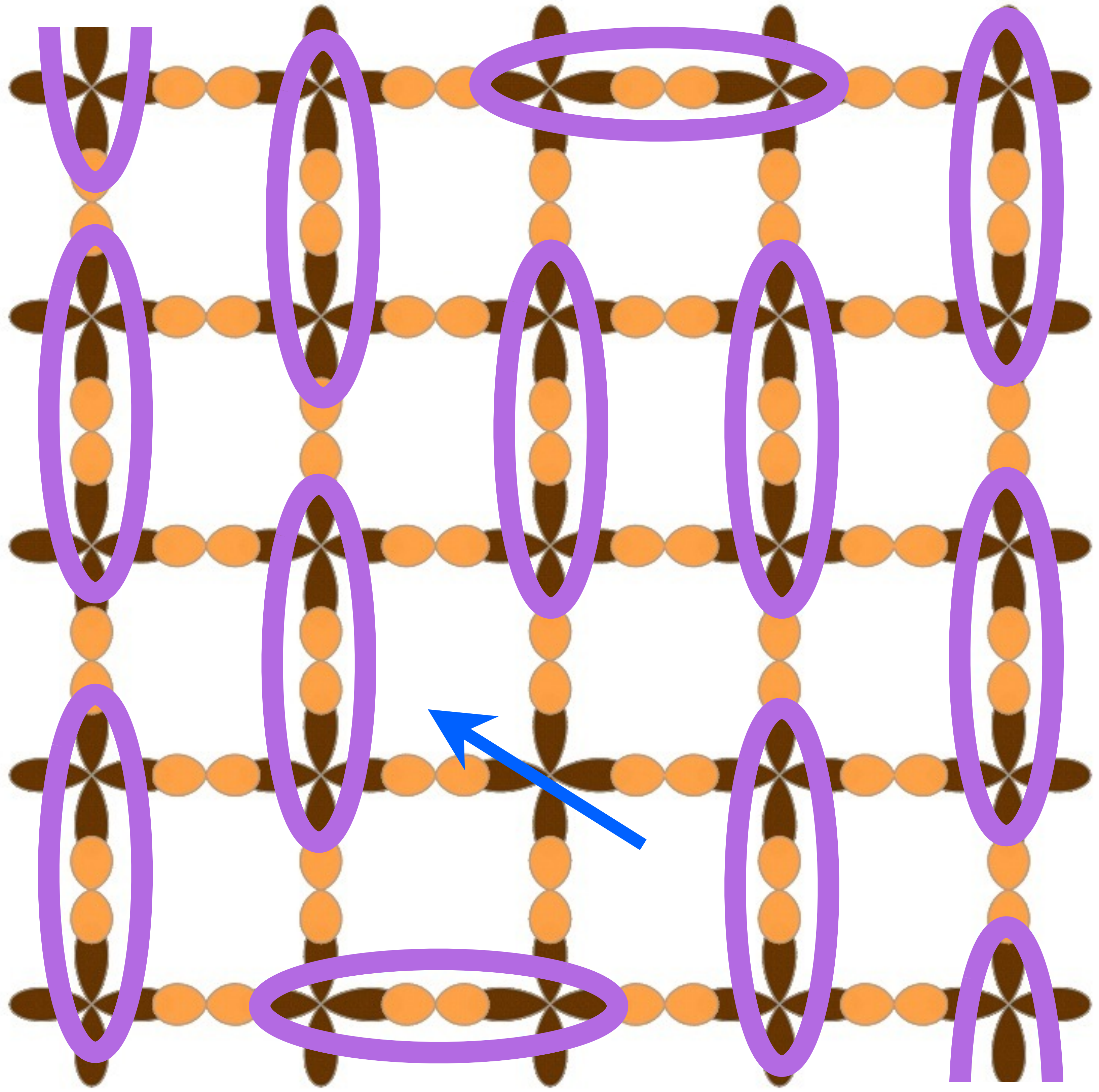

Spin S=1/2, charge neutral spinon

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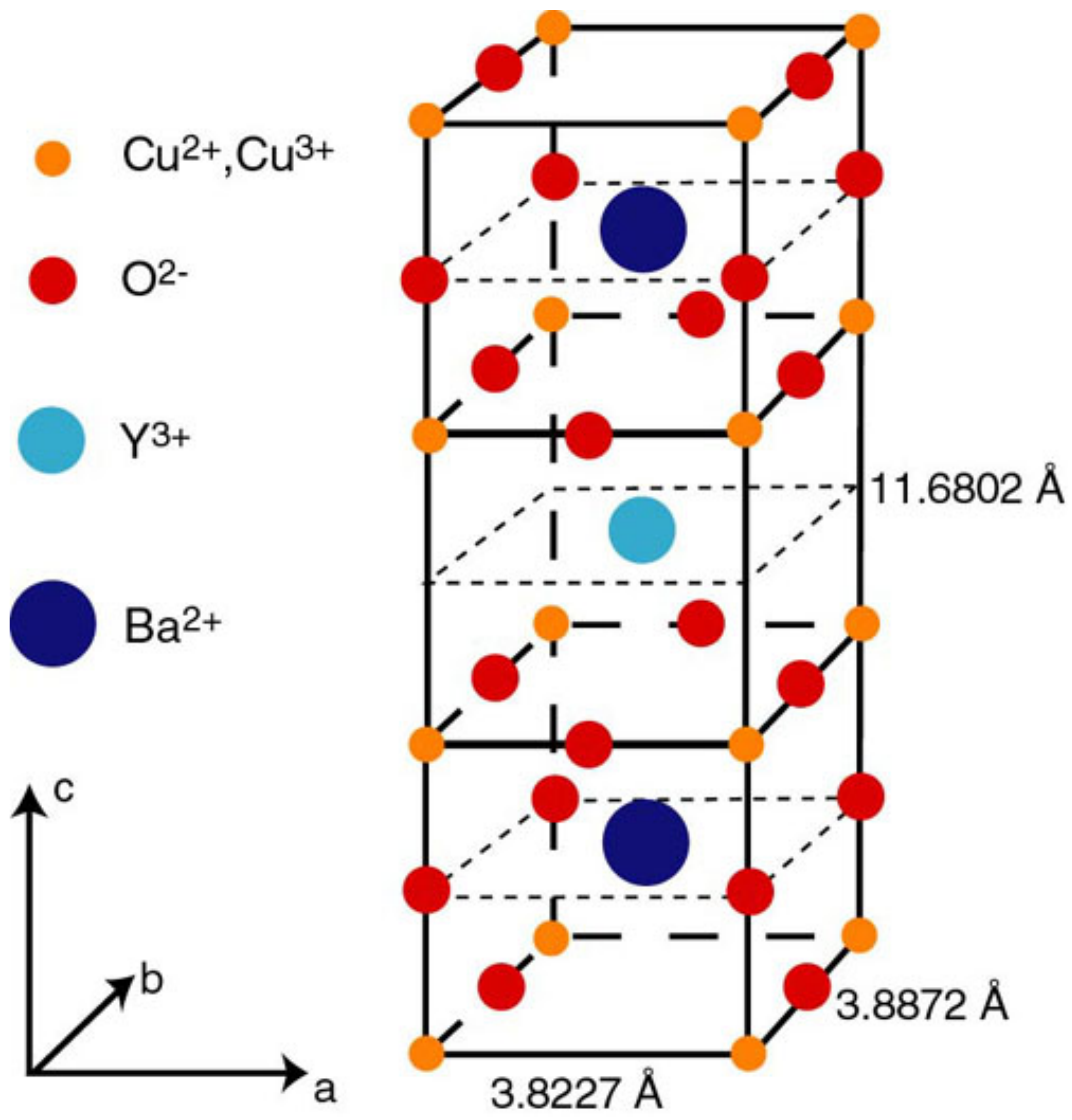

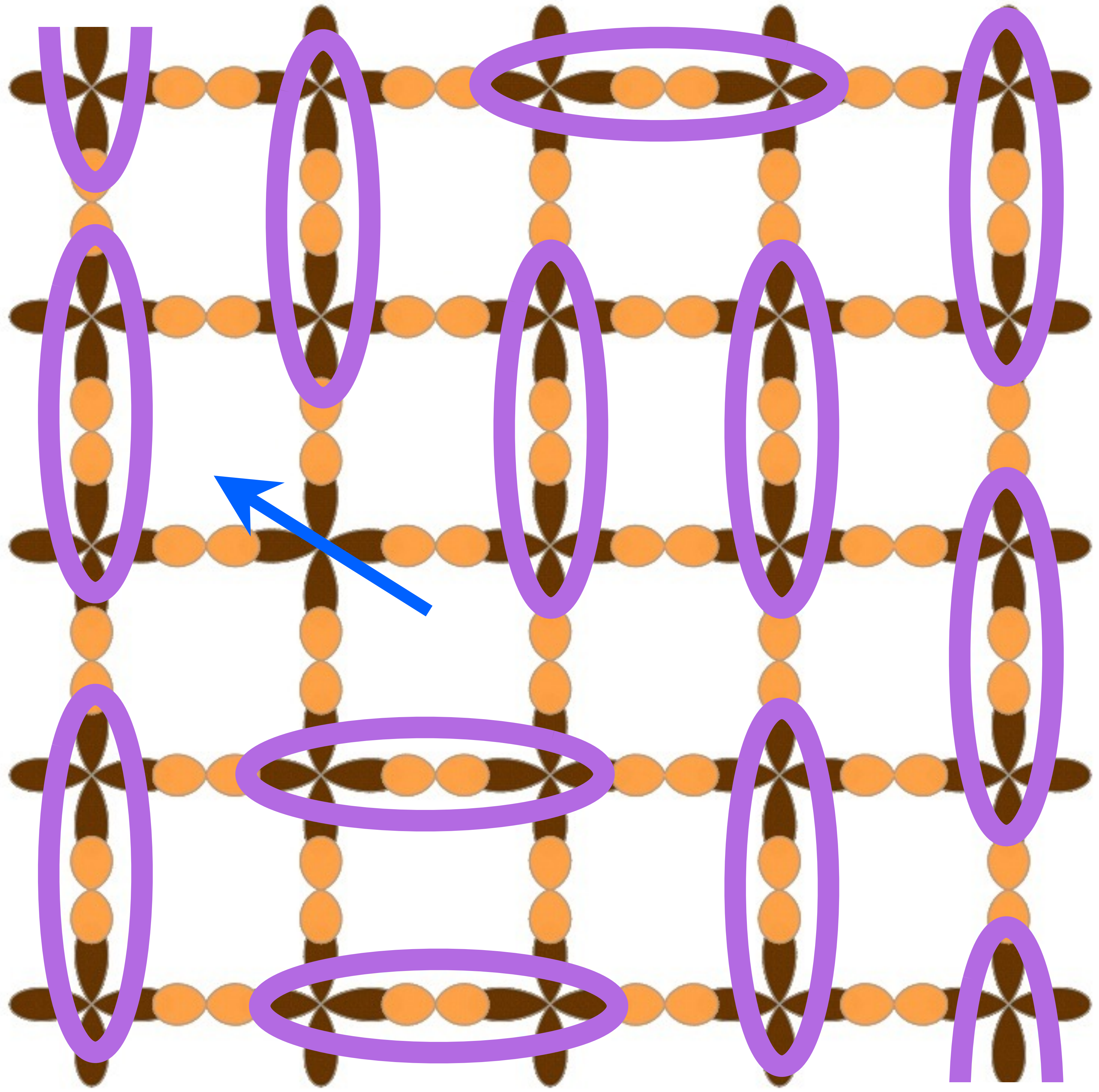

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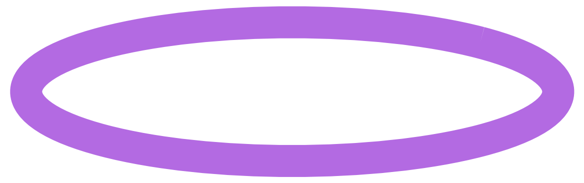 $= \frac{1}{\sqrt{2}}(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

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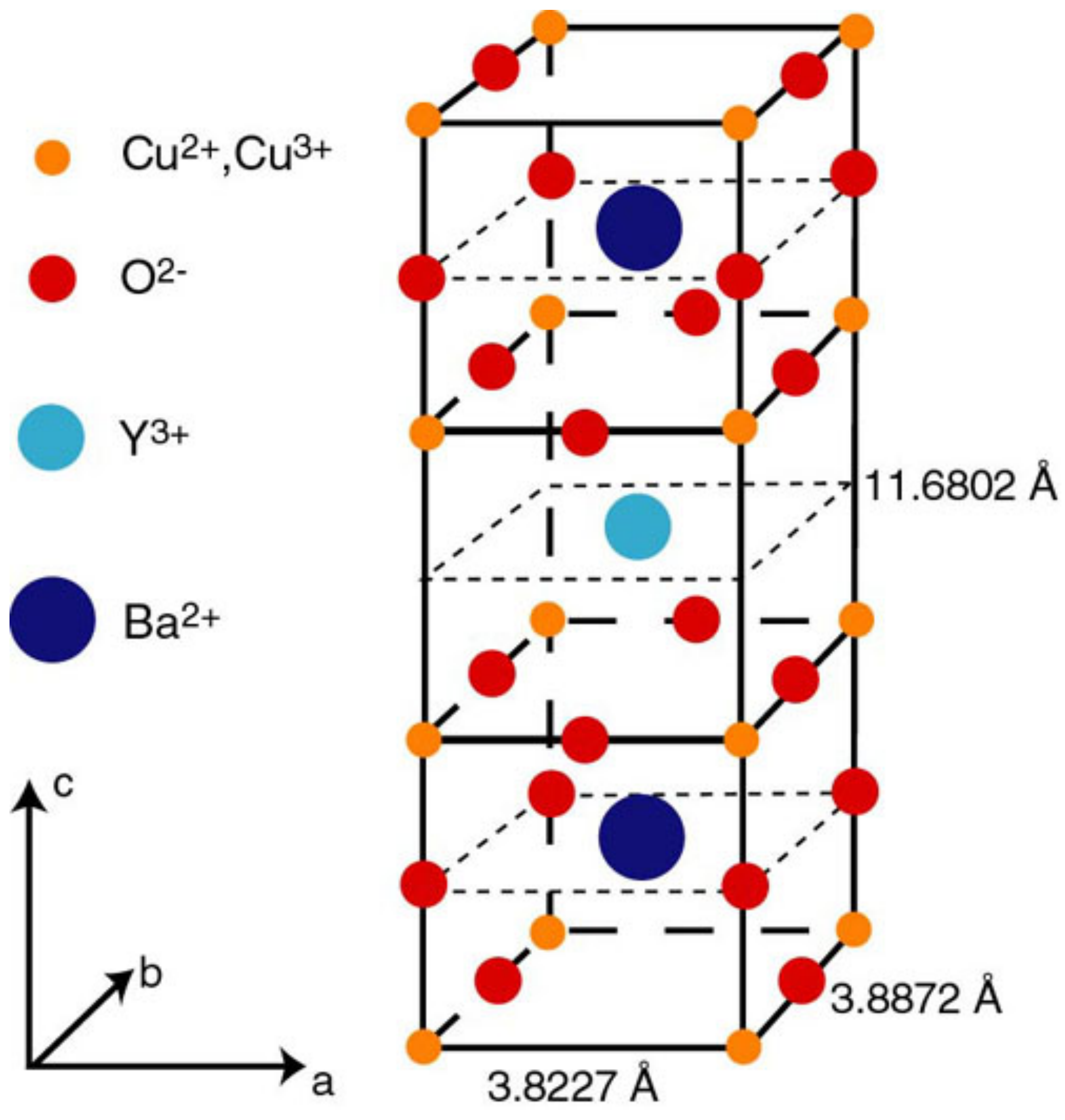

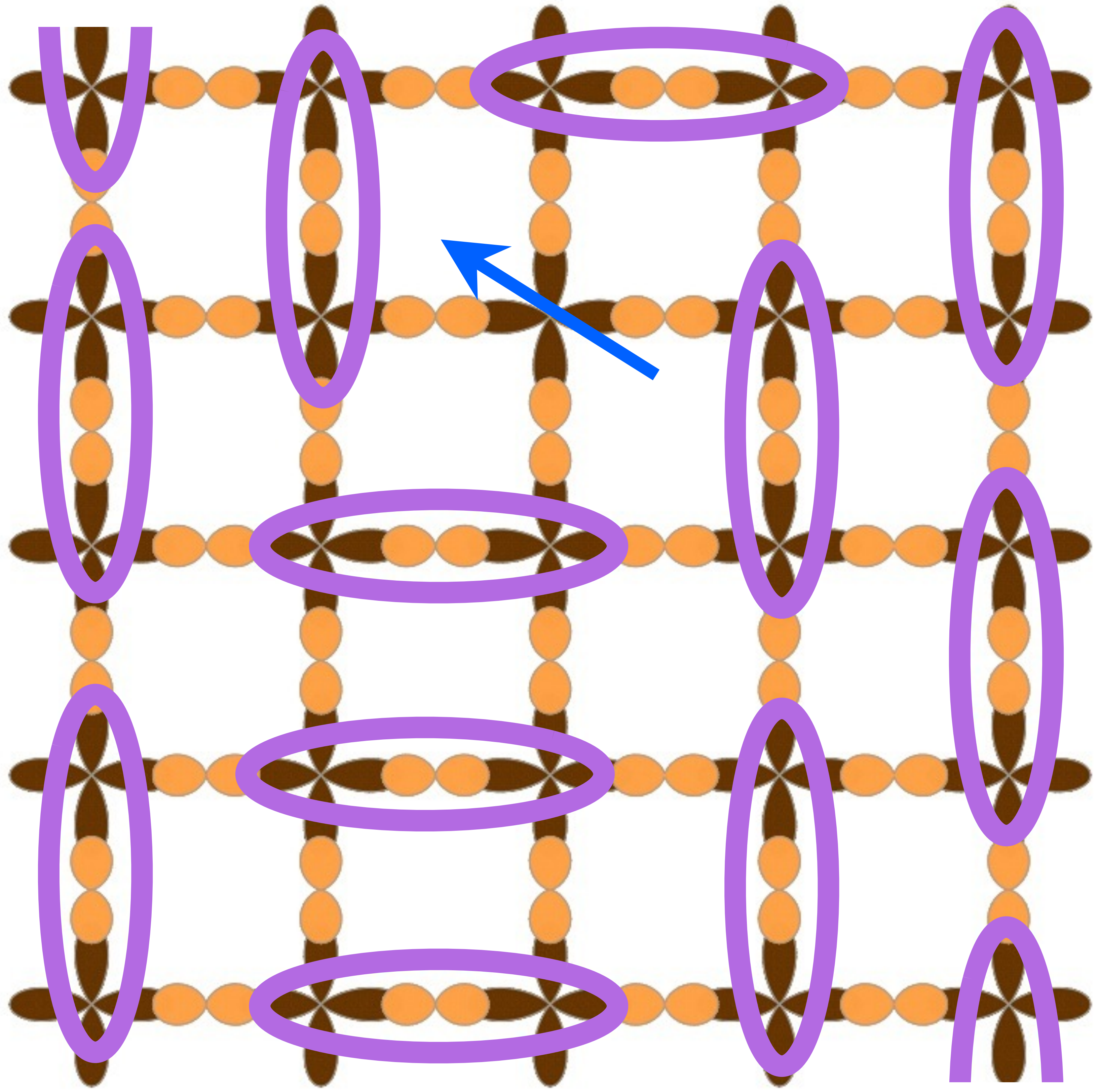

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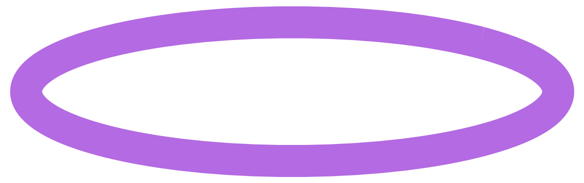 $= \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$

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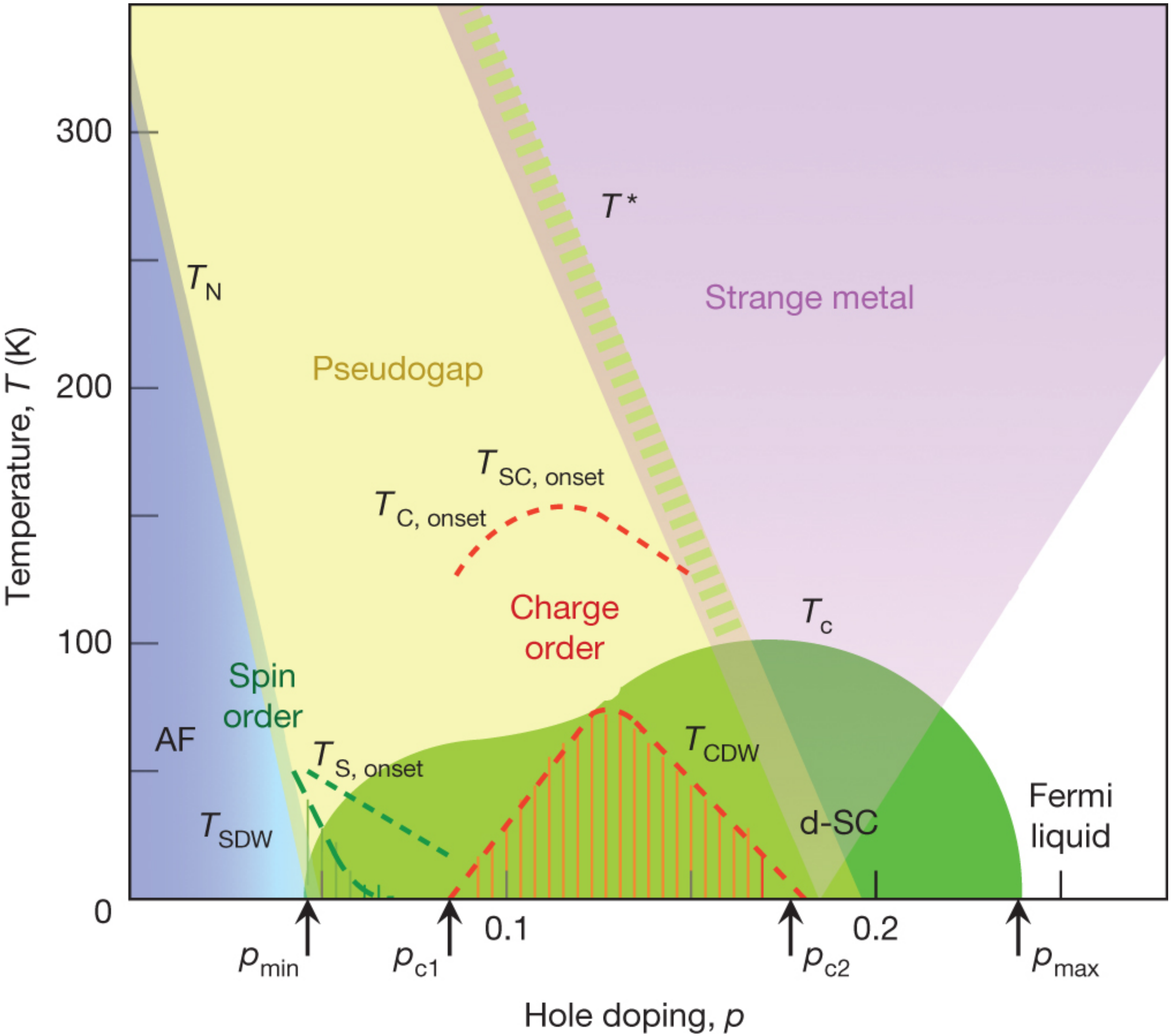
Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



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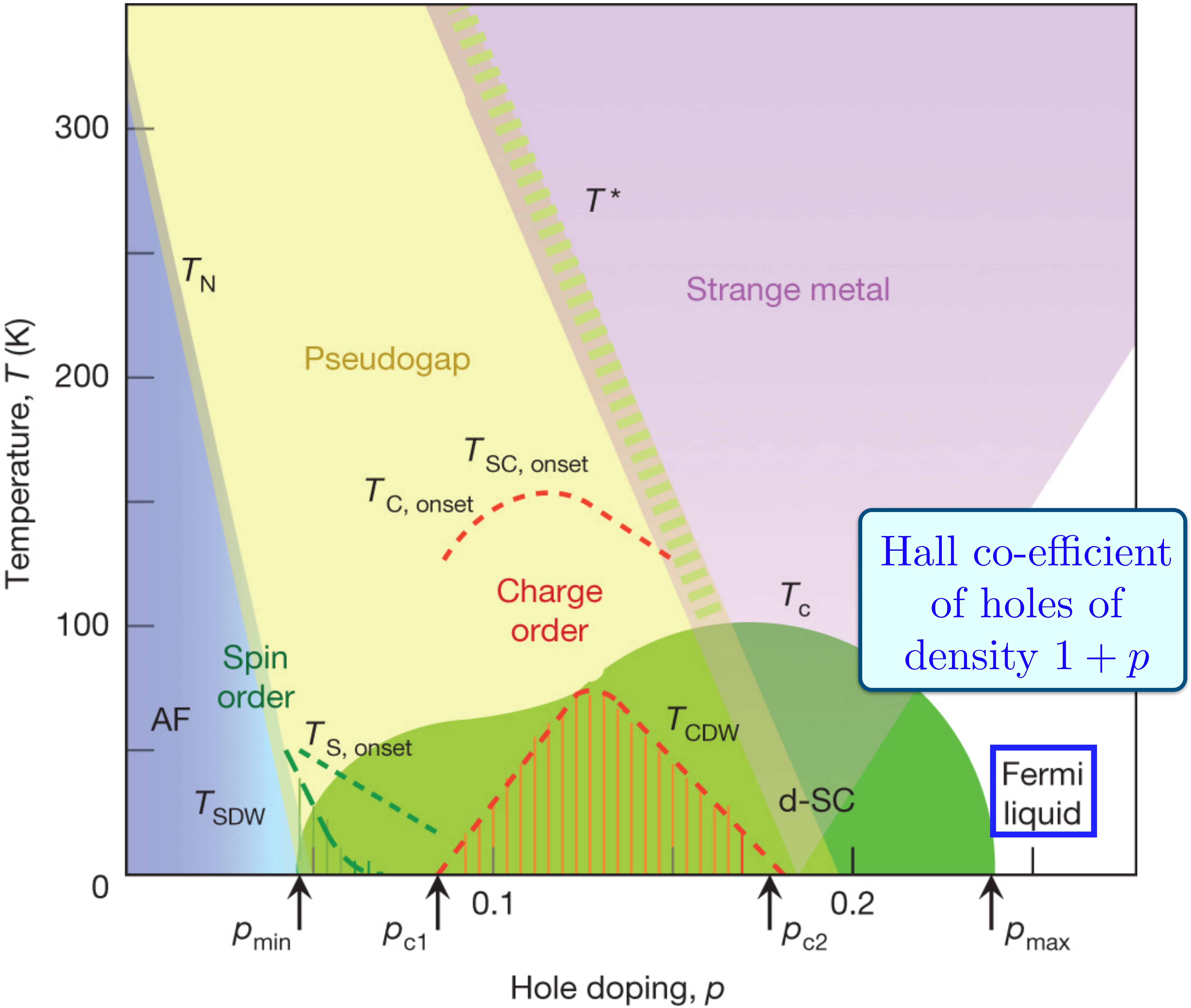

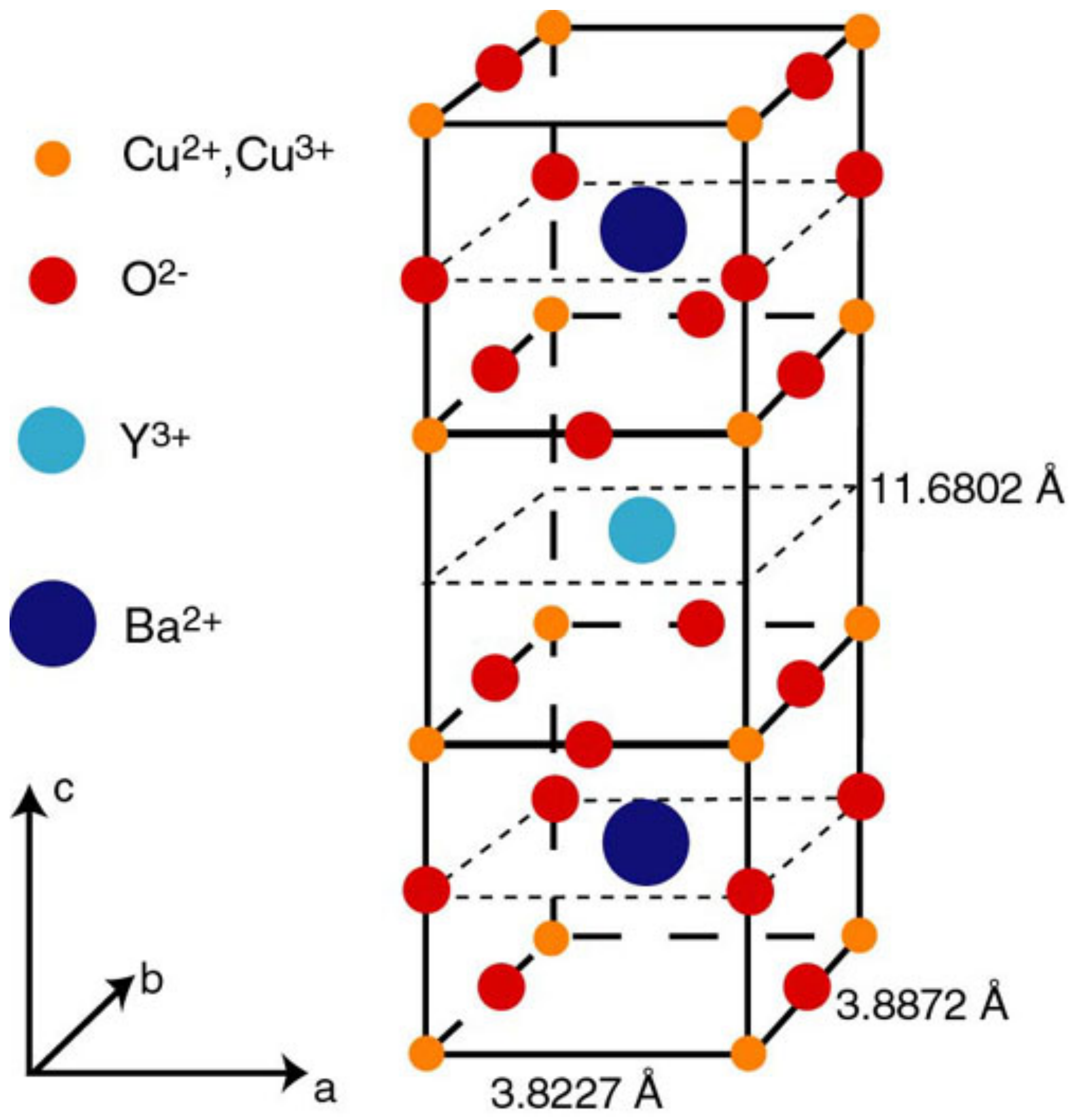

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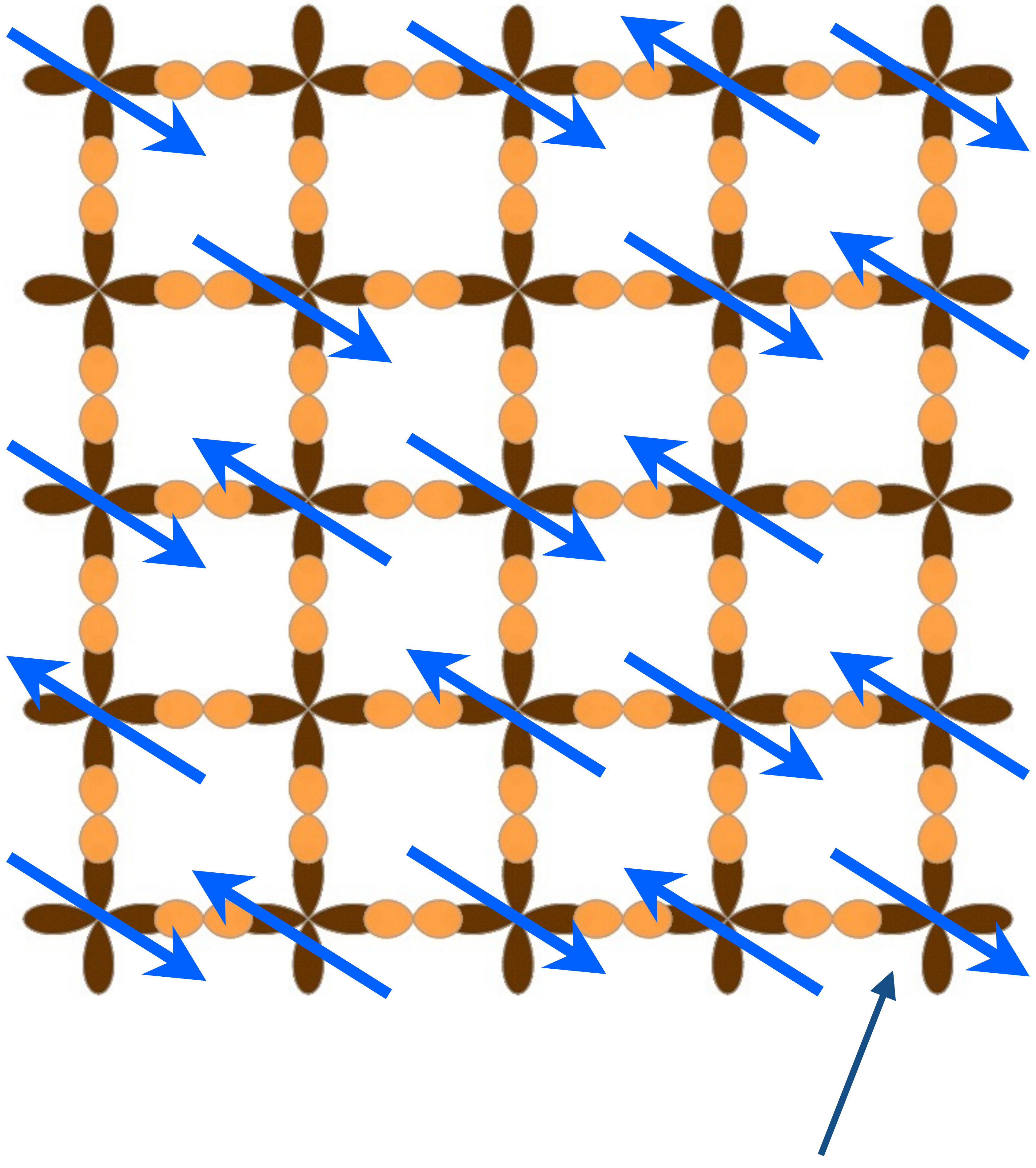
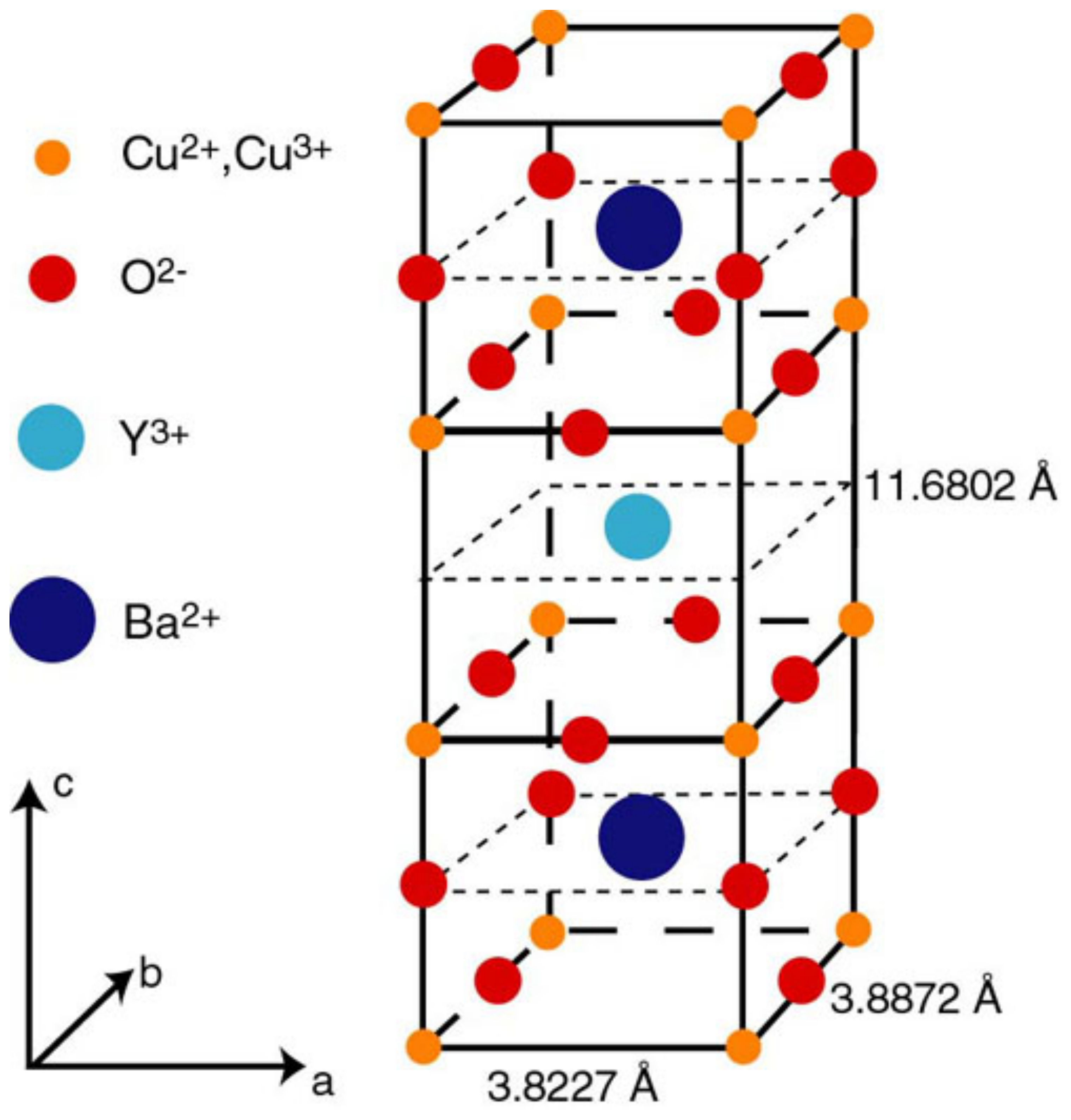


High temperature superconductor obtained upon doping the antiferromagnet with density p holes.



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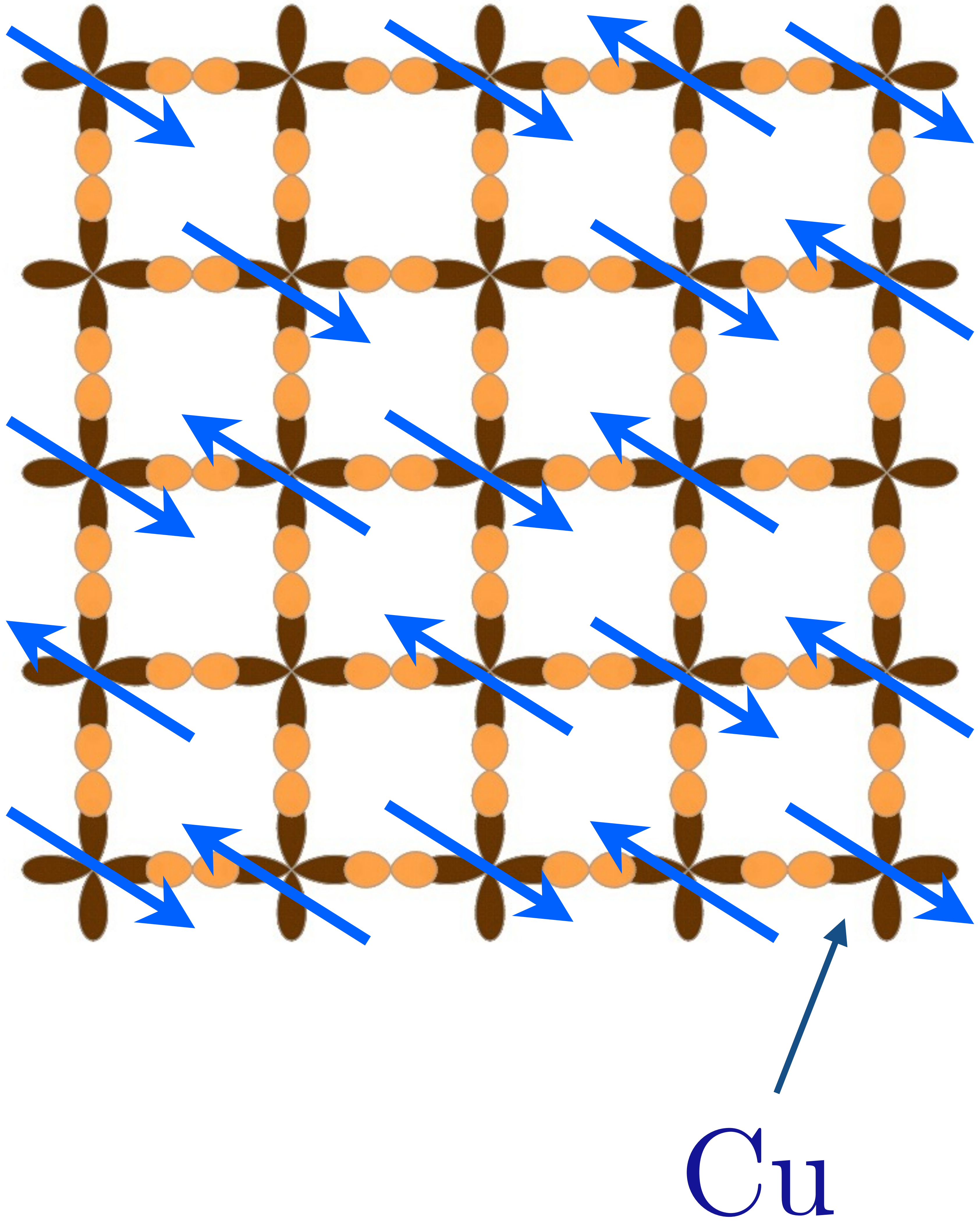

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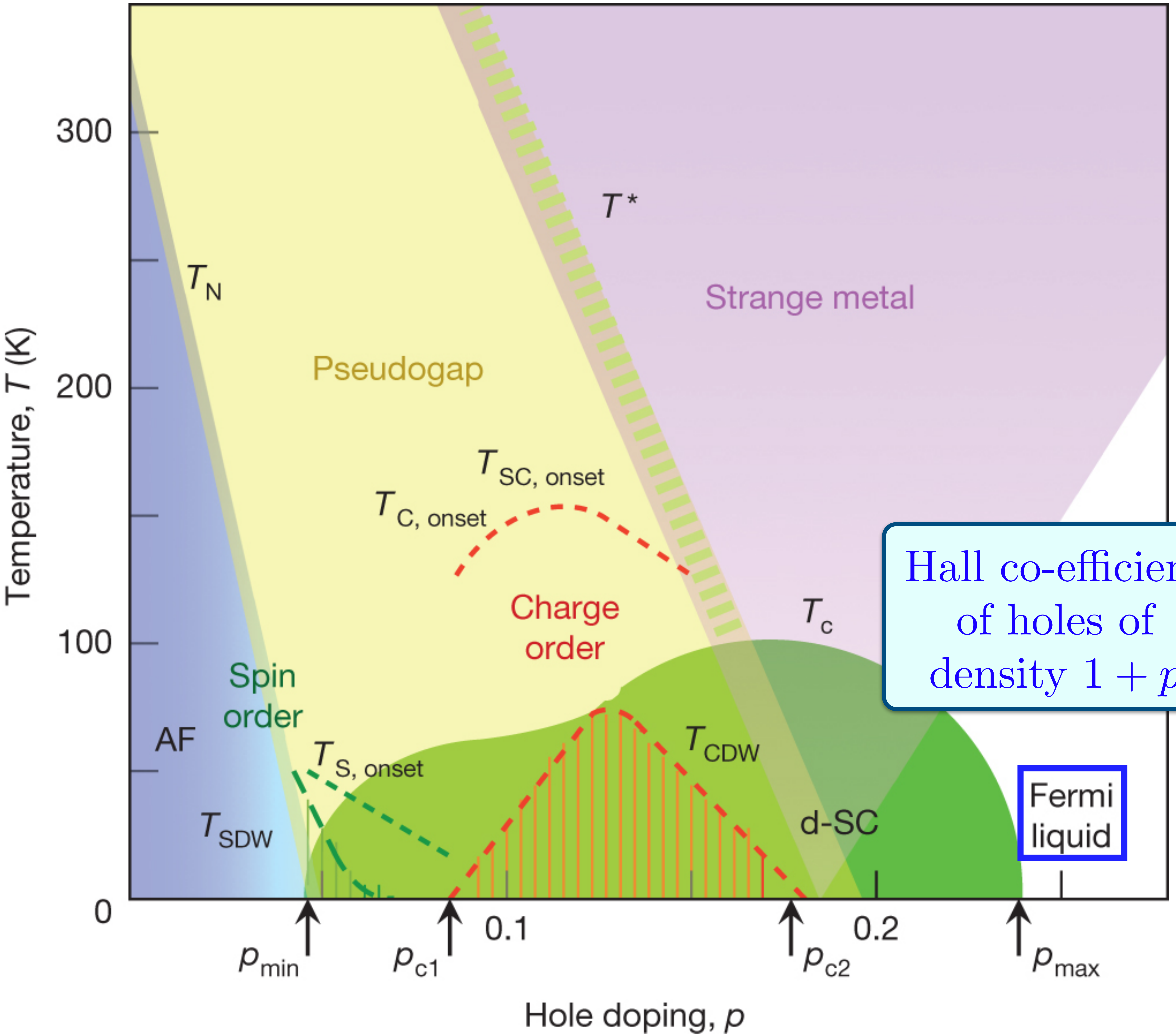
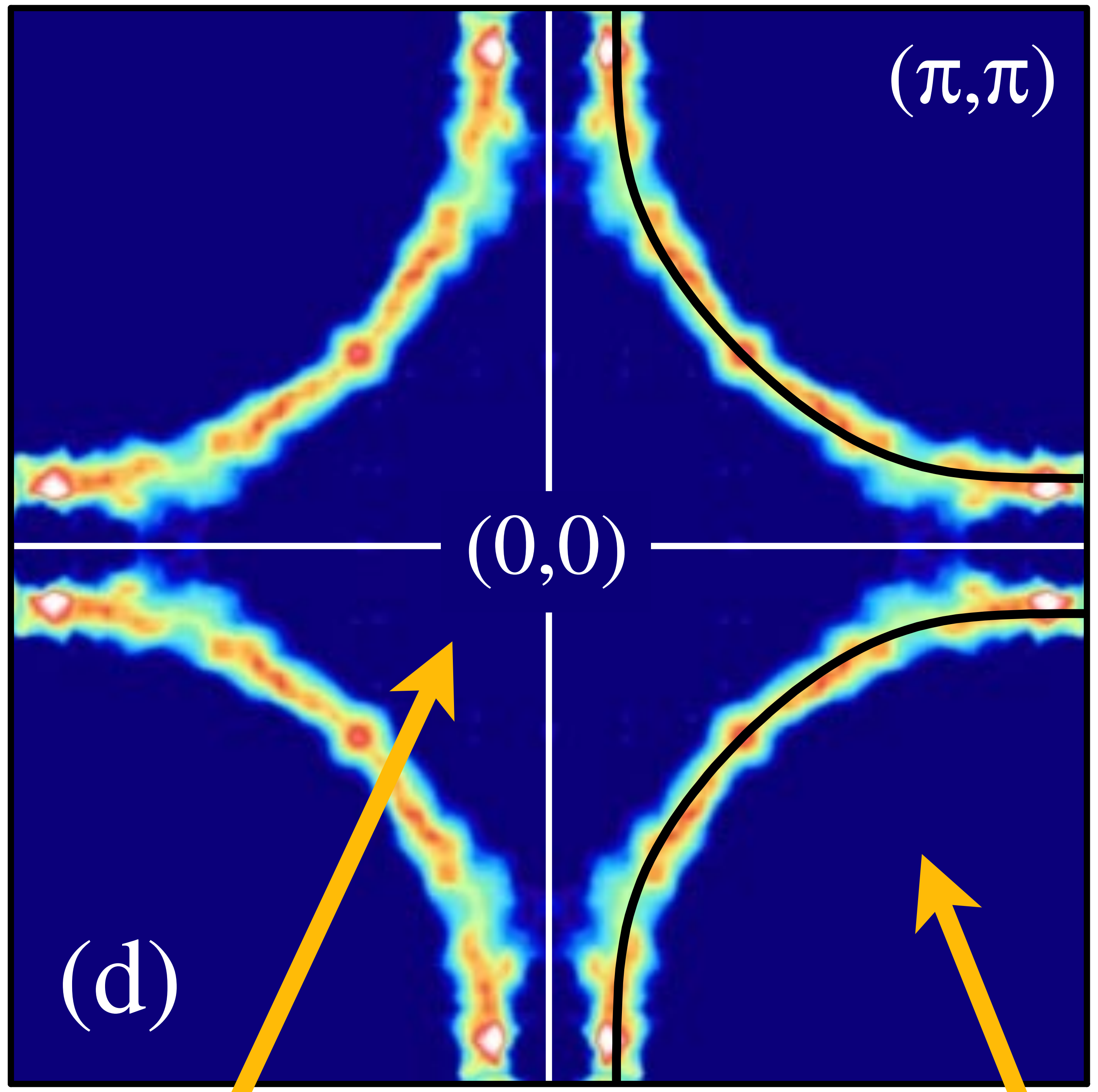
Hole density relative to the filled band $\rho = 1 + p$.

Electron density relative to the empty band $\rho_e = 1 - p$.

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



M. Platé *et al.*, PRL **95**, 077001 (2005)



Hall co-efficient of holes of density $1 + p$

1-p electrons

1+p holes

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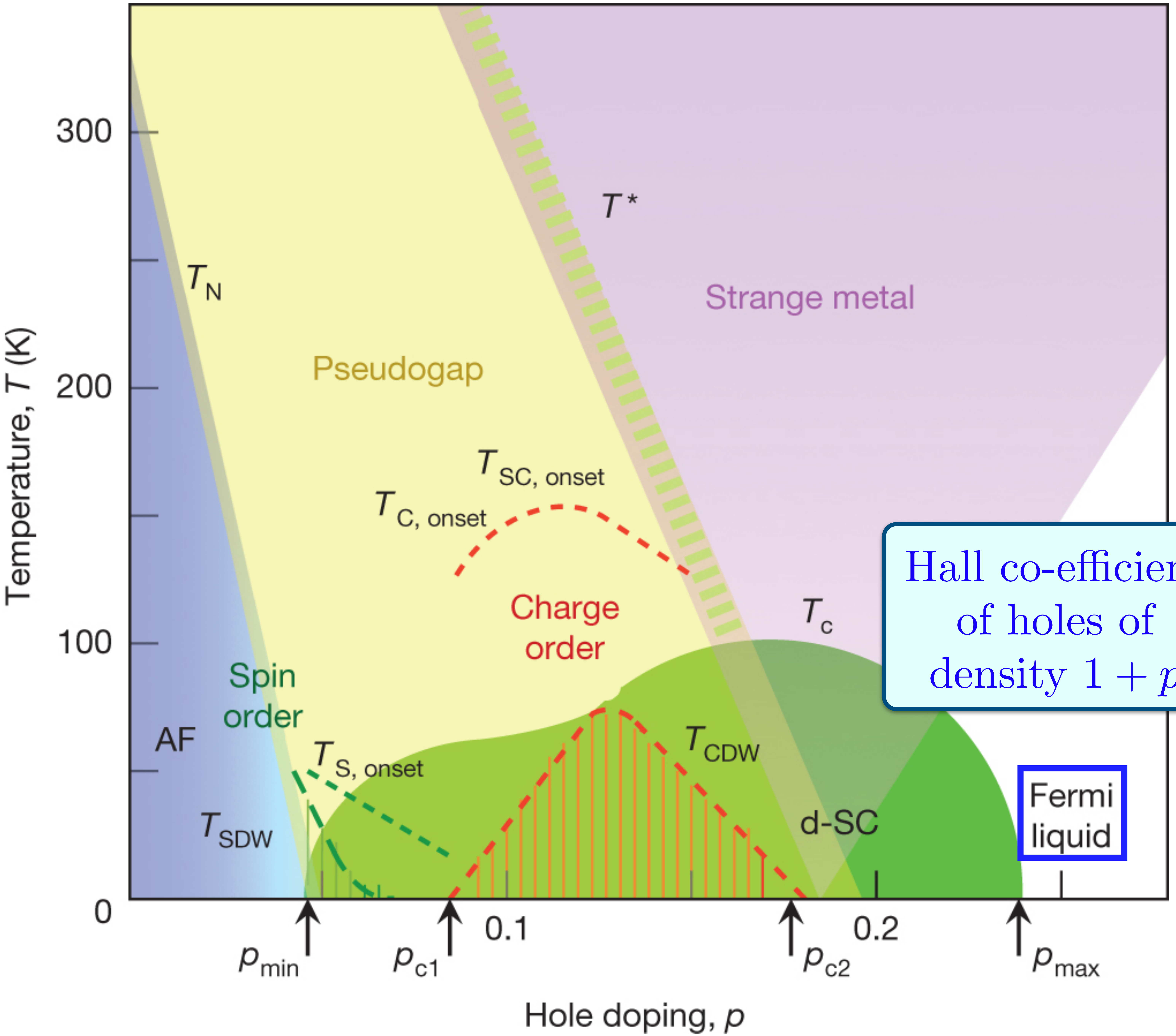

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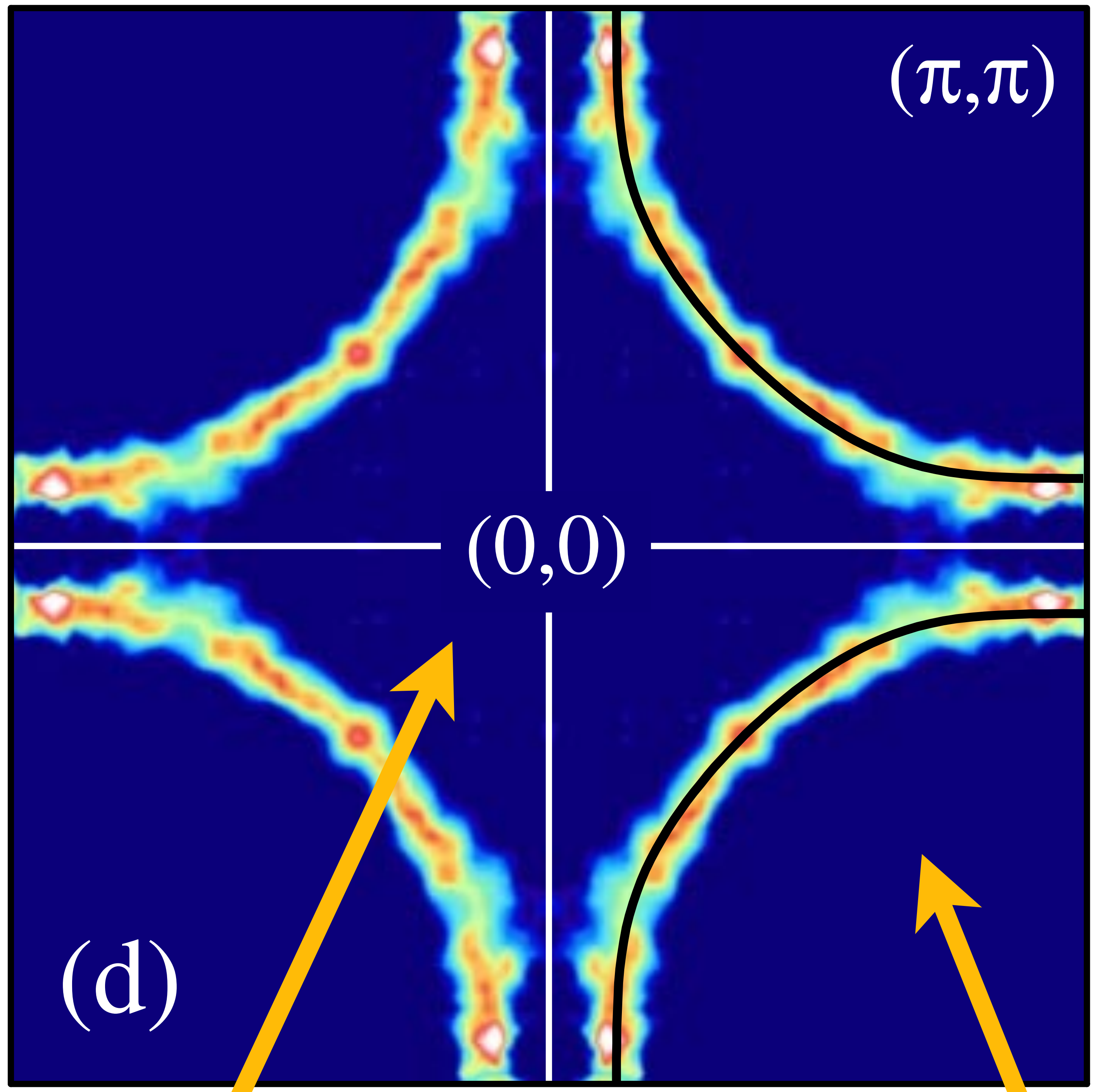

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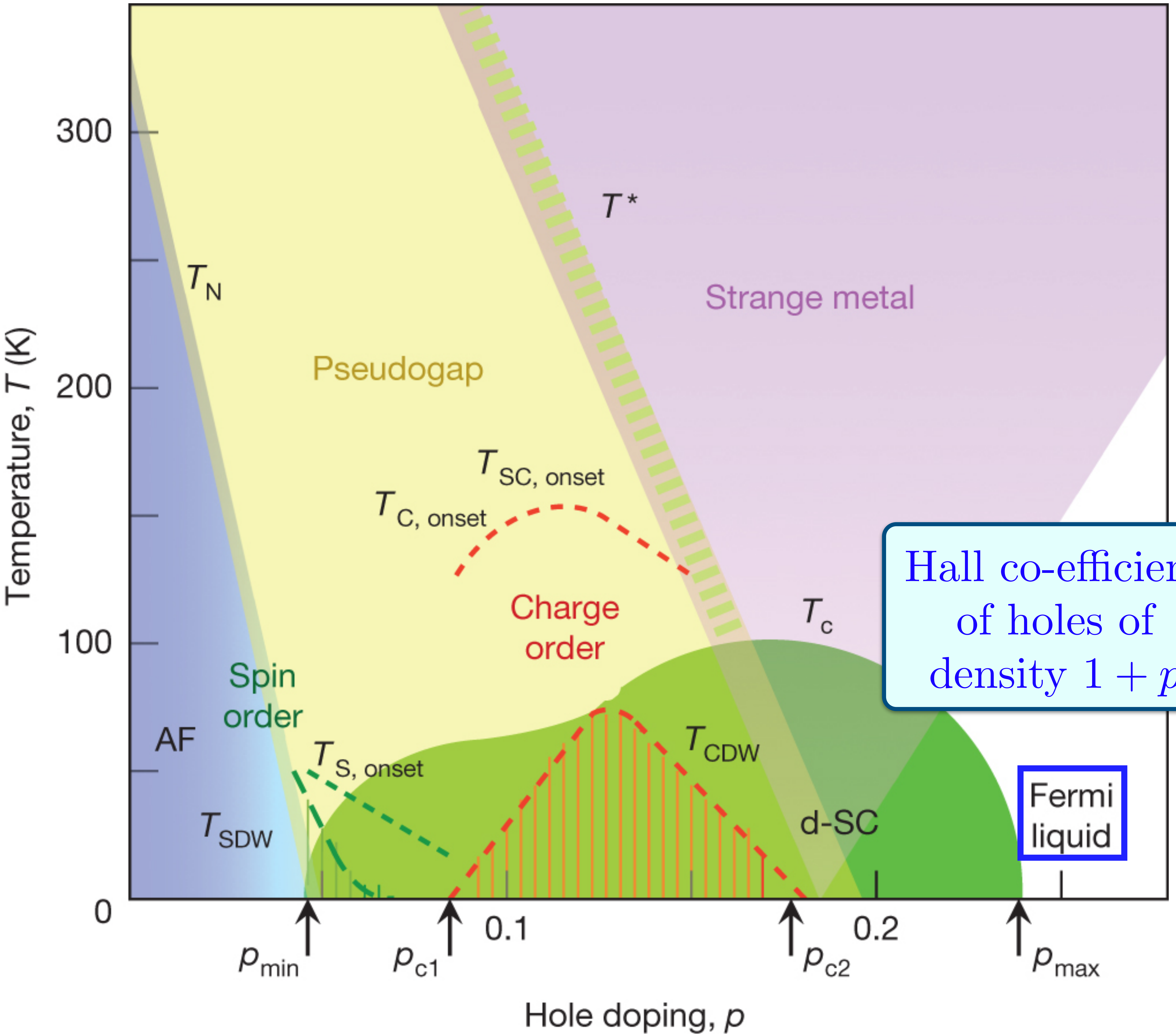

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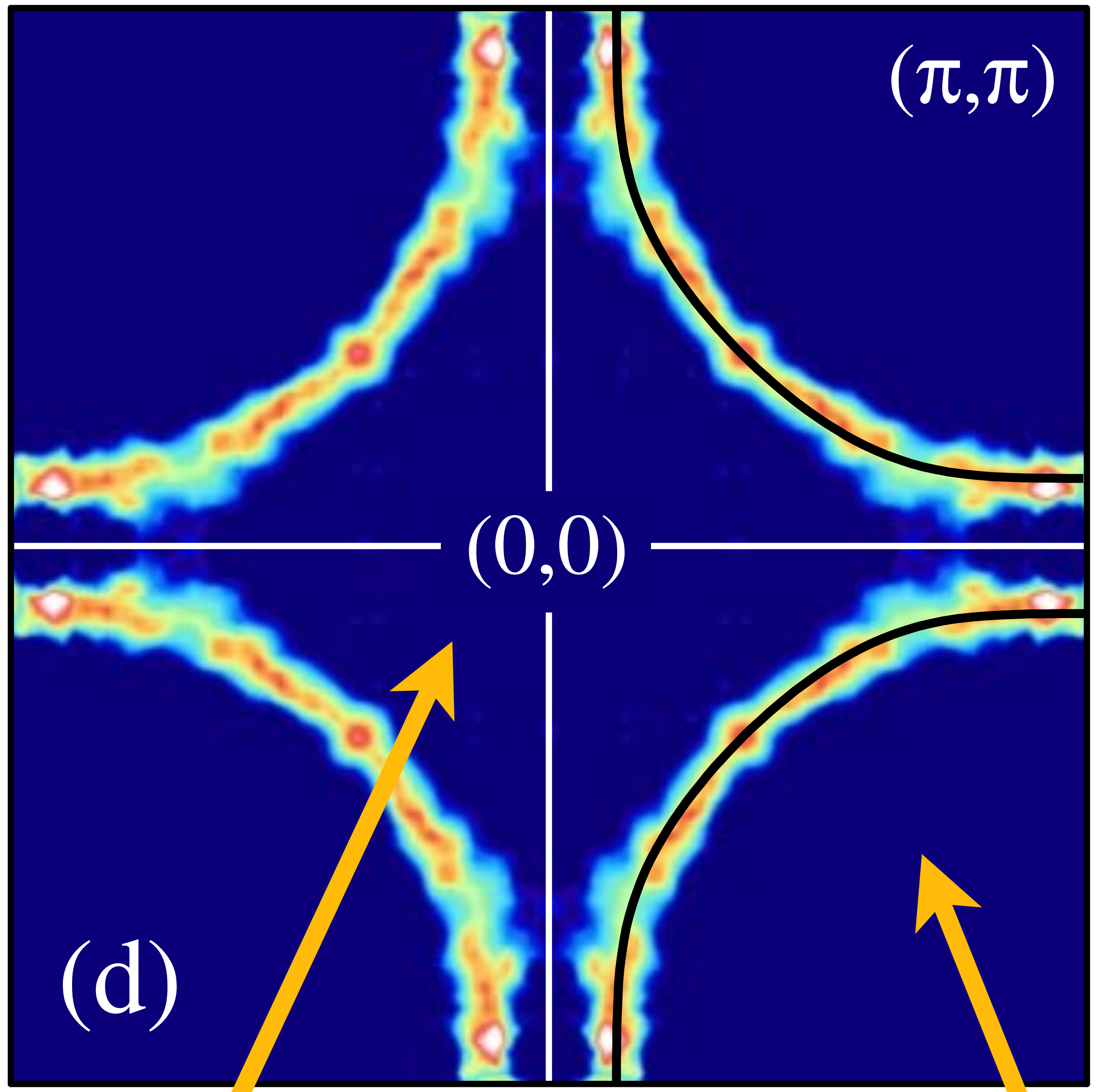

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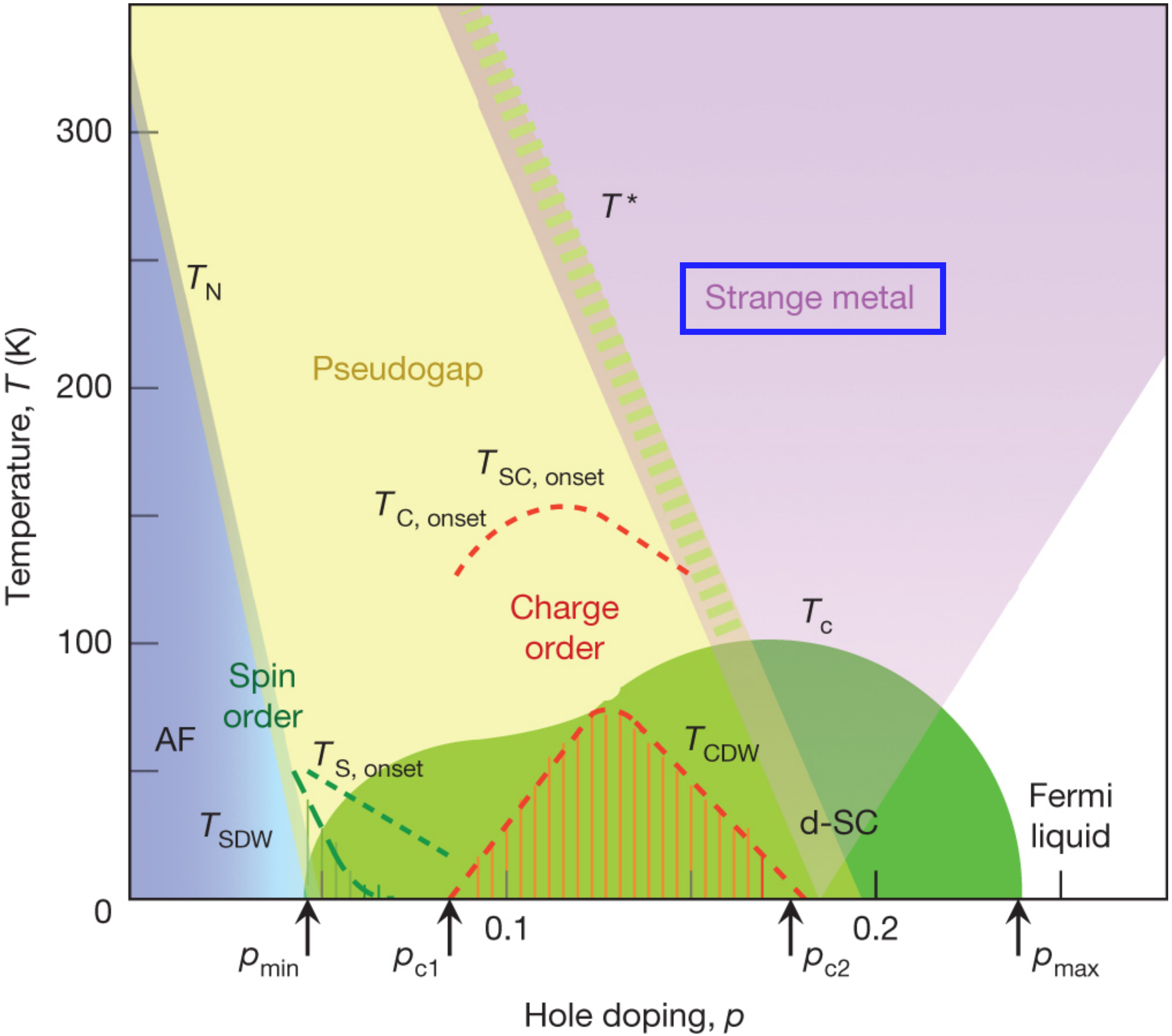


Non-Boltzmann

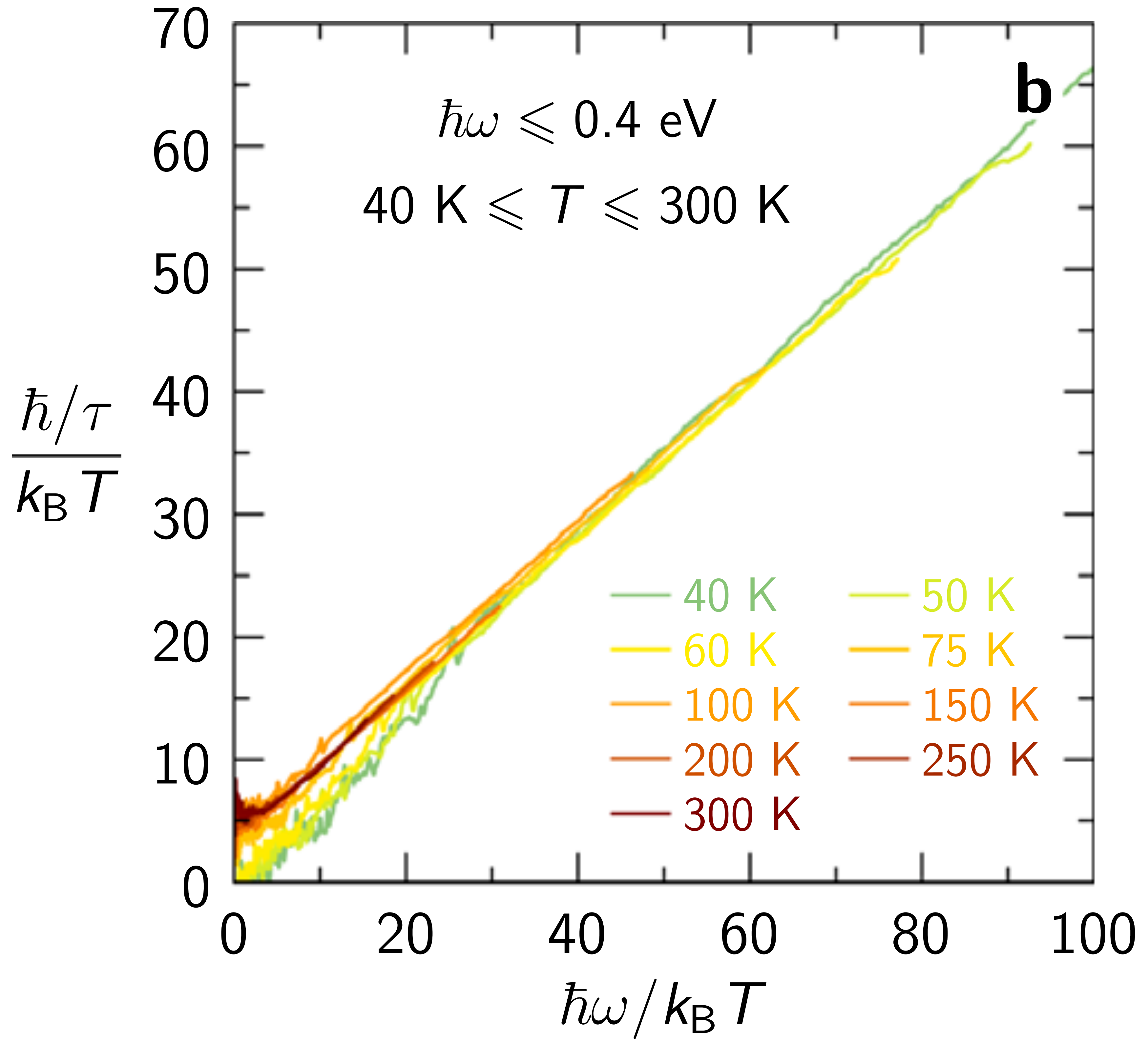
Planckian dynamics

of large Fermi surface

Electron scattering time τ

from optical conductivity

$$\tau(\omega) = \frac{\hbar}{k_B T} F\left(\frac{\hbar\omega}{k_B T}\right)$$



B. Michon, C. Berthod, C.W. Rischau, A. Ataei, L. Chen, S. Komiya, S. Ono, L. Taillefer, D. van der Marel, A. Georges, Nature Comm. **14**, 3033 (2023)

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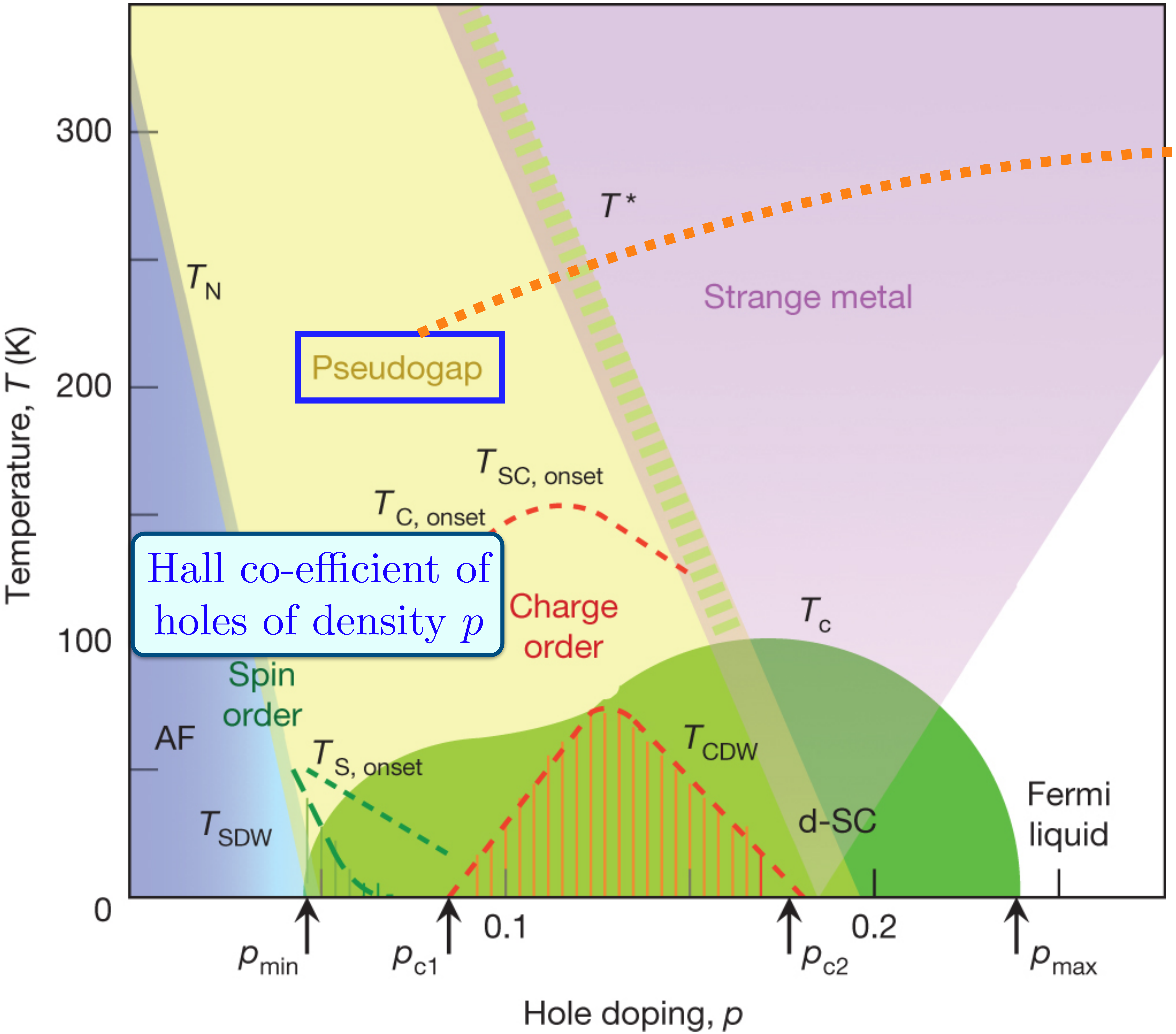

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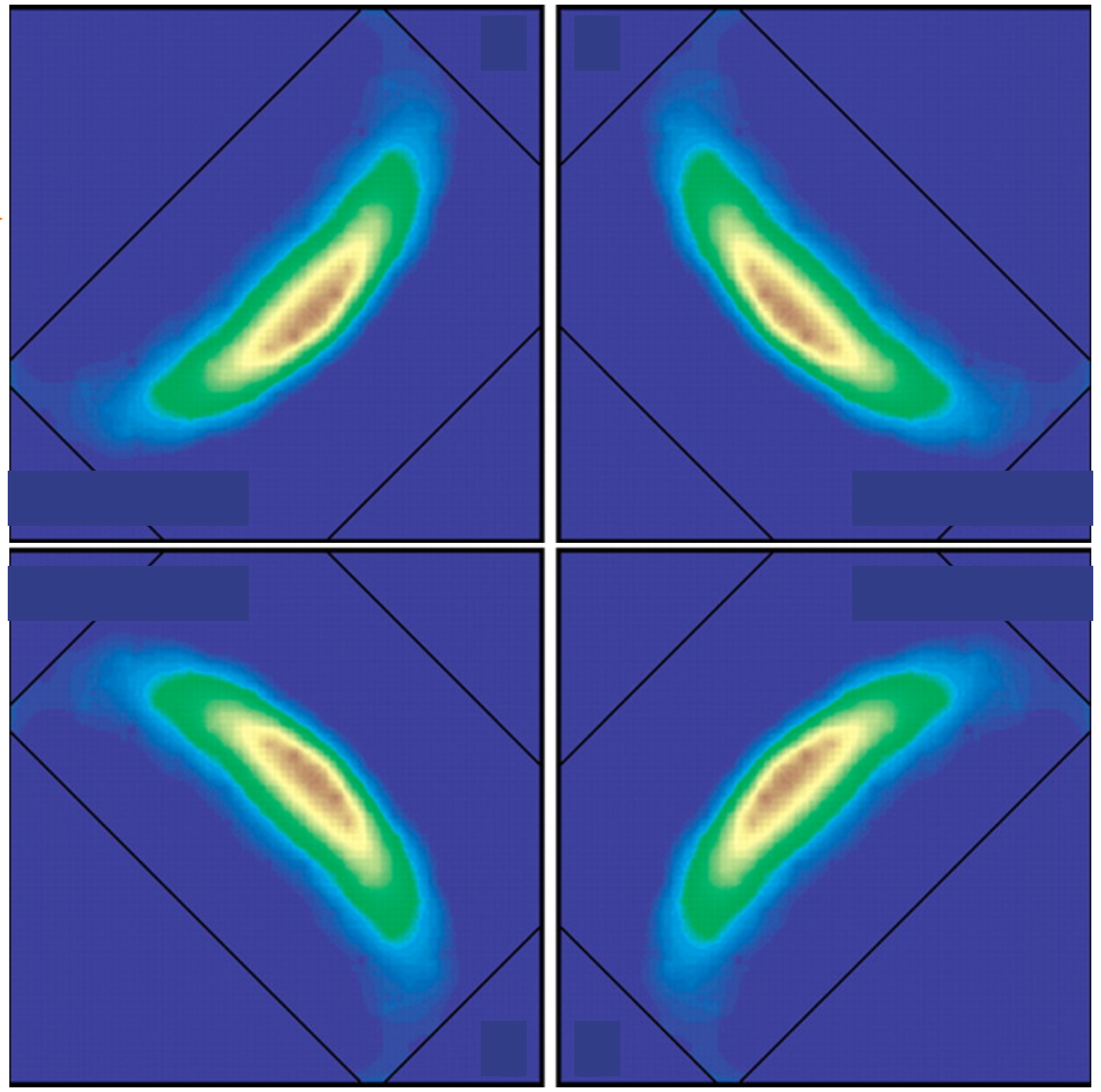

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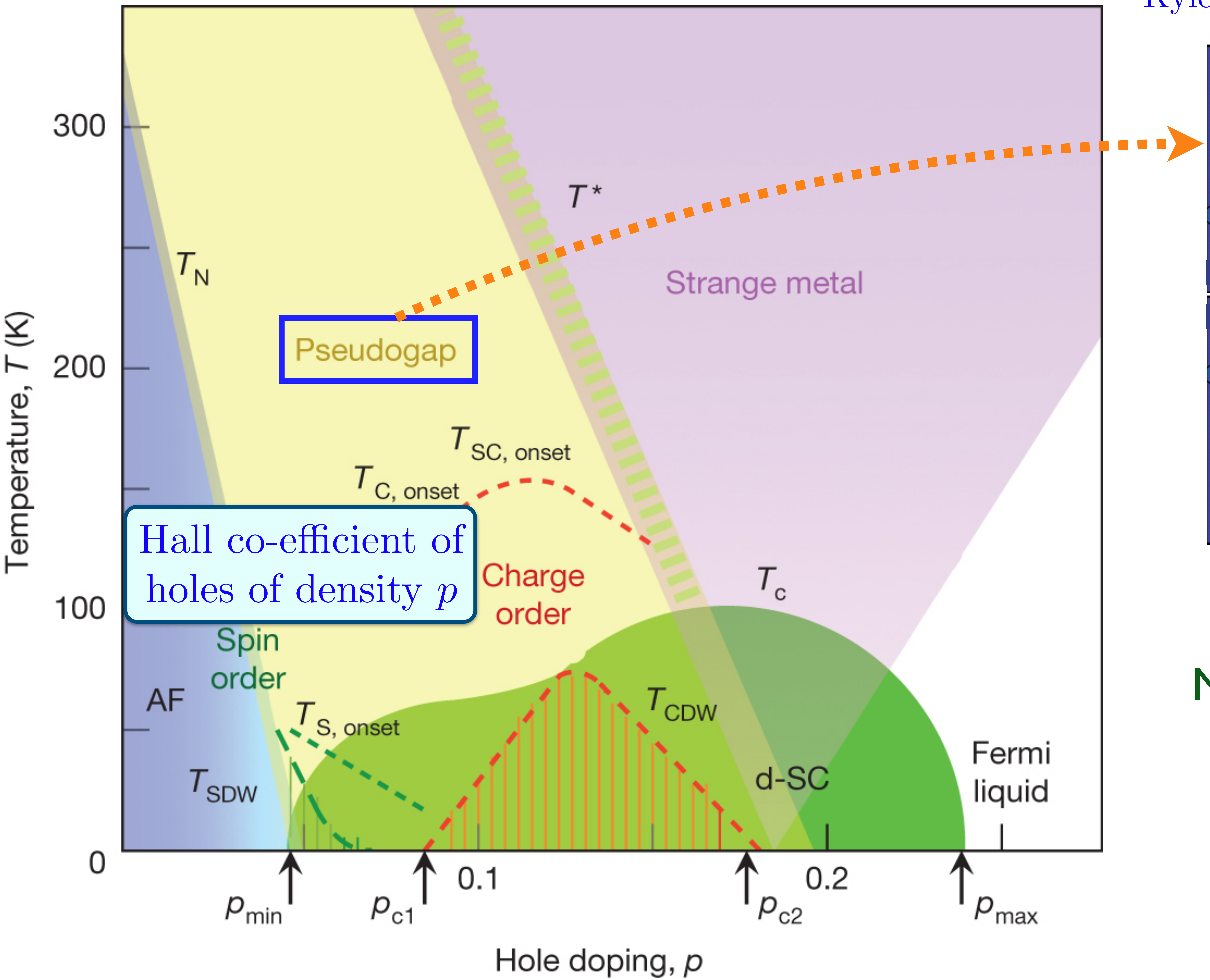
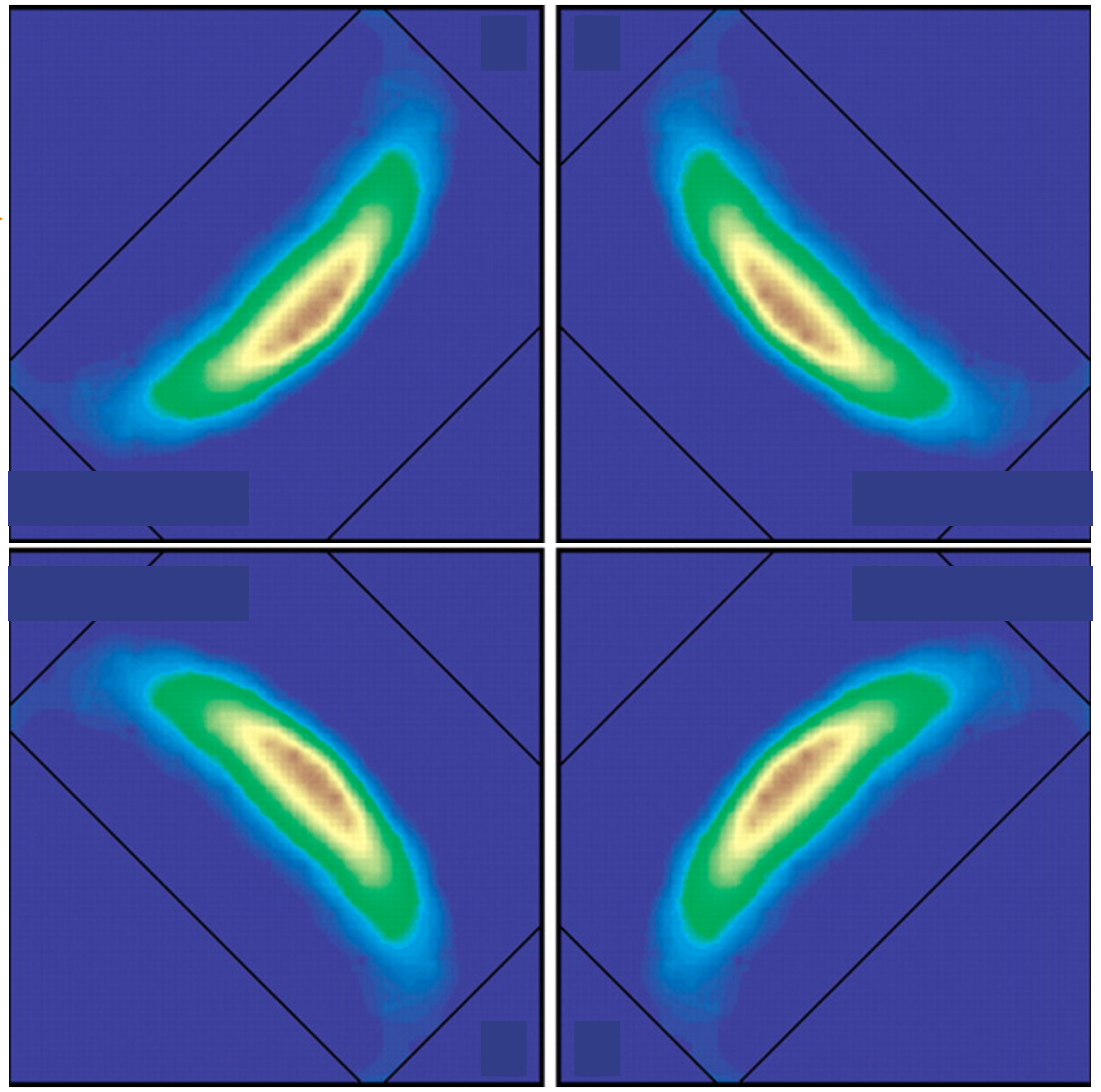
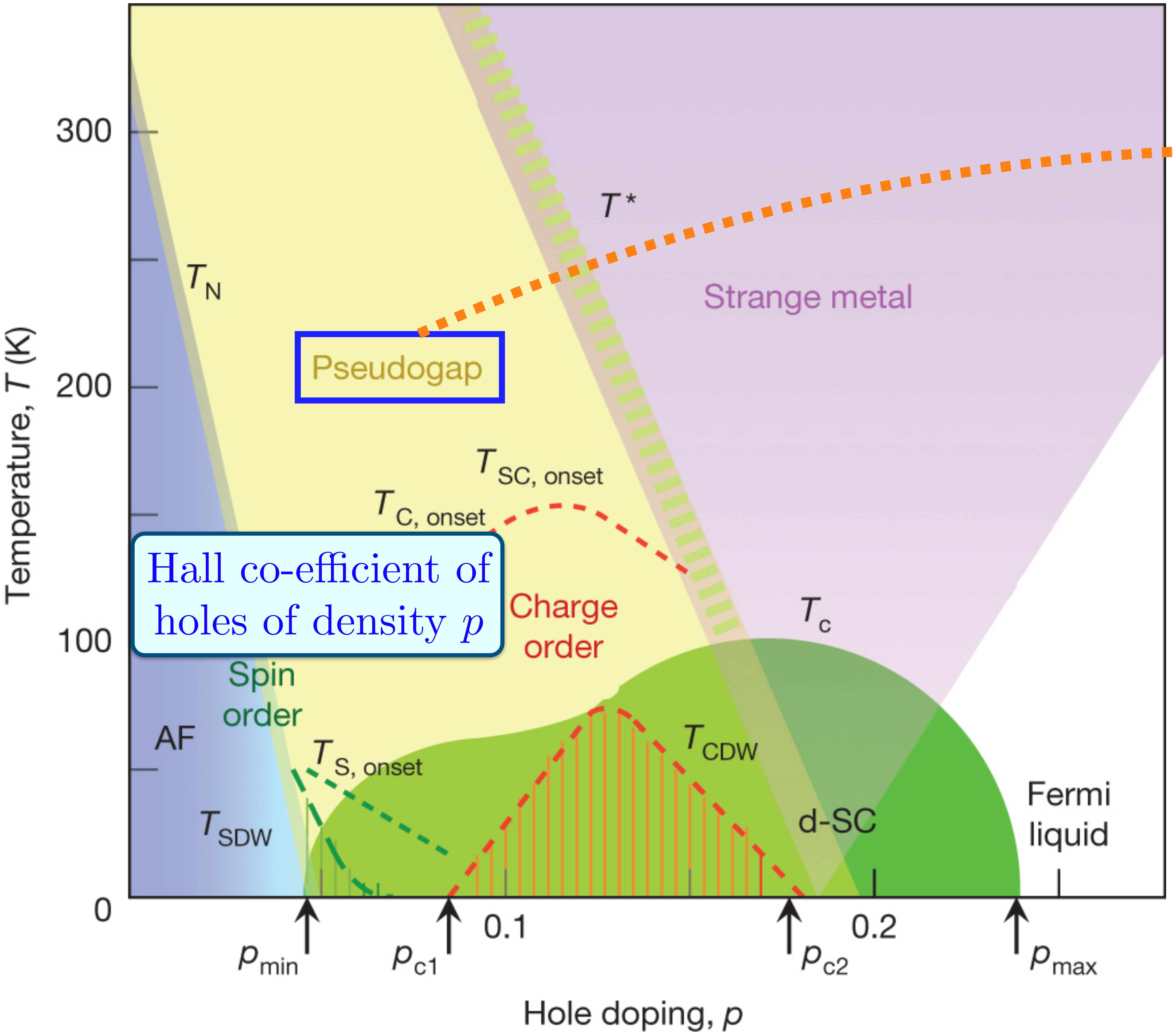




Many theories with fluctuating d-SC and charge orders

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)



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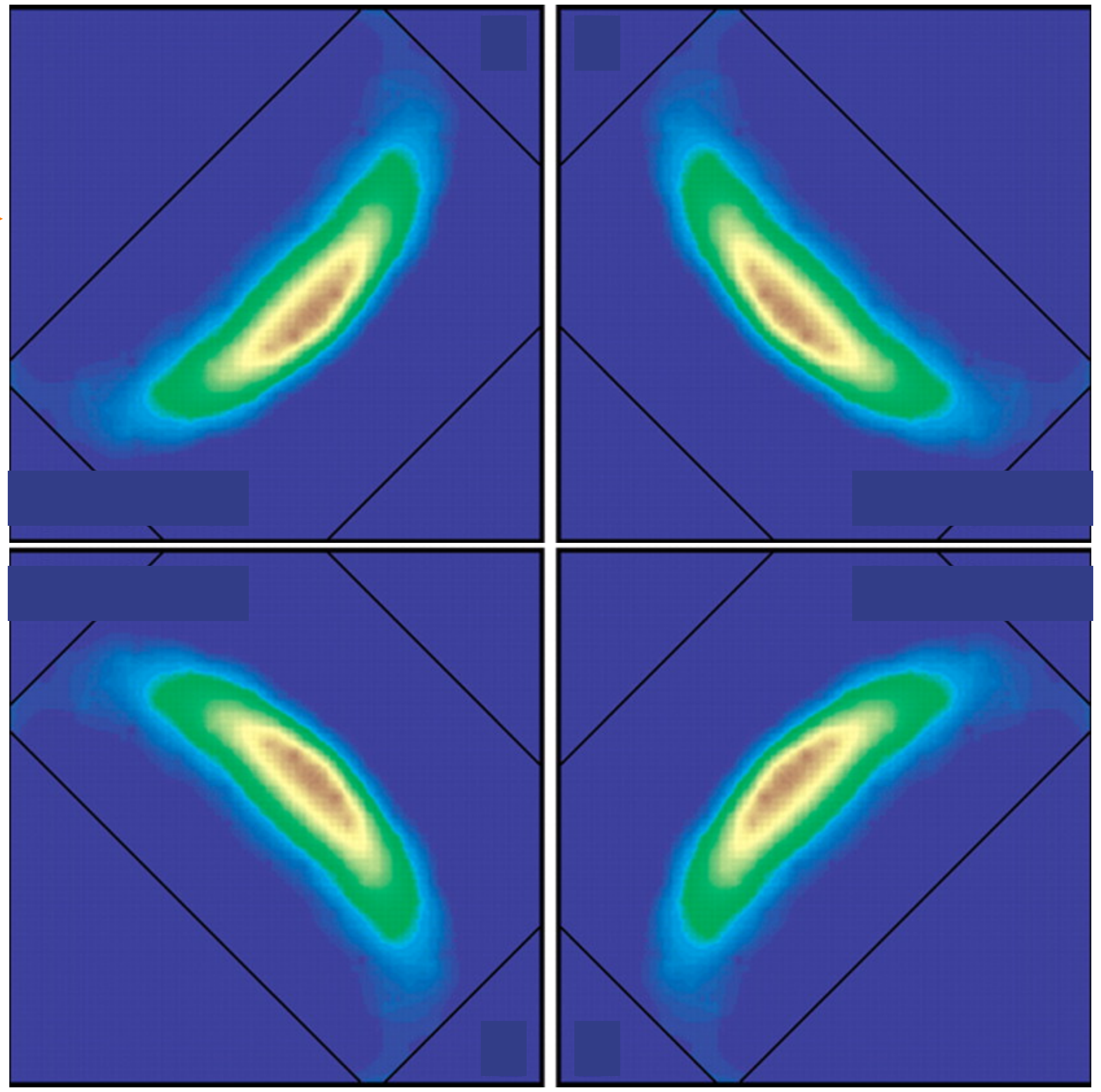
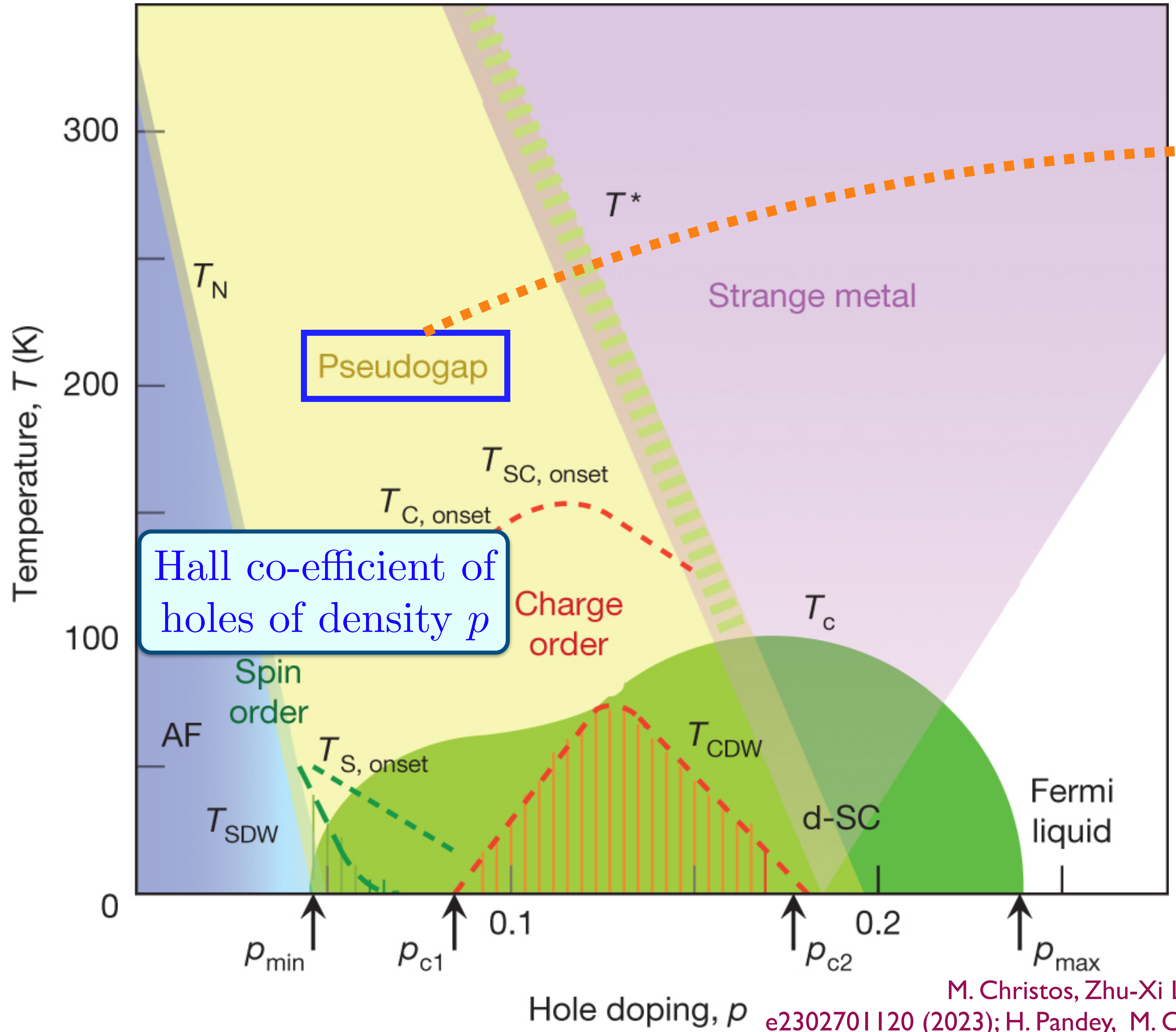


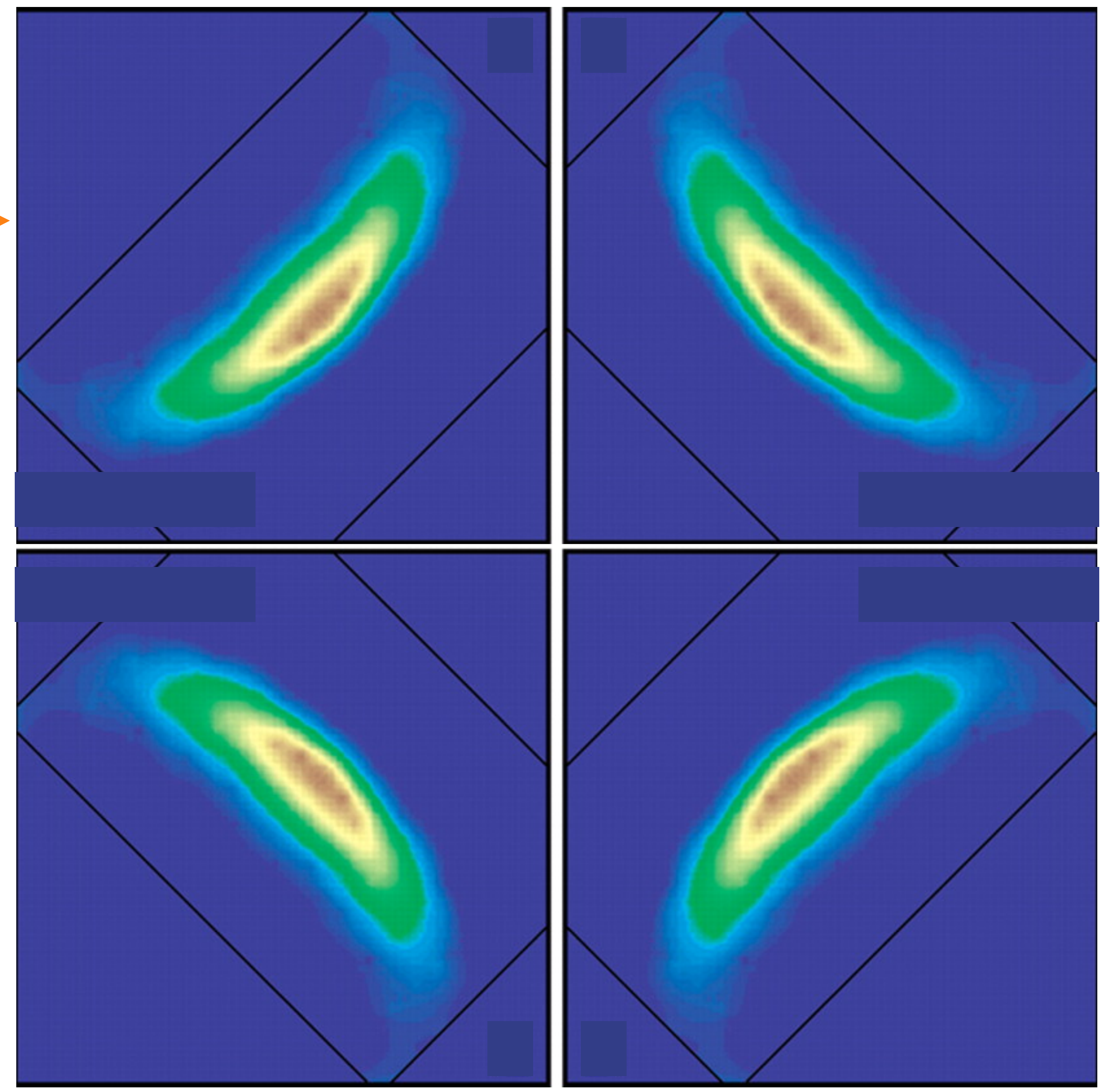
Will explain with a Fractionalized Fermi Liquid (FL*) which evades the Luttinger constraint, but satisfies the Oshikawa anomaly, by a *critical spin liquid*

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003)

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)

Kyle M. Shen *et al.*, Science **307**, 901 (2005)





Fluctuating orders appear as composites of a more fundamental fractionalized order parameter, B, which carries an emergent SU(2) gauge charge

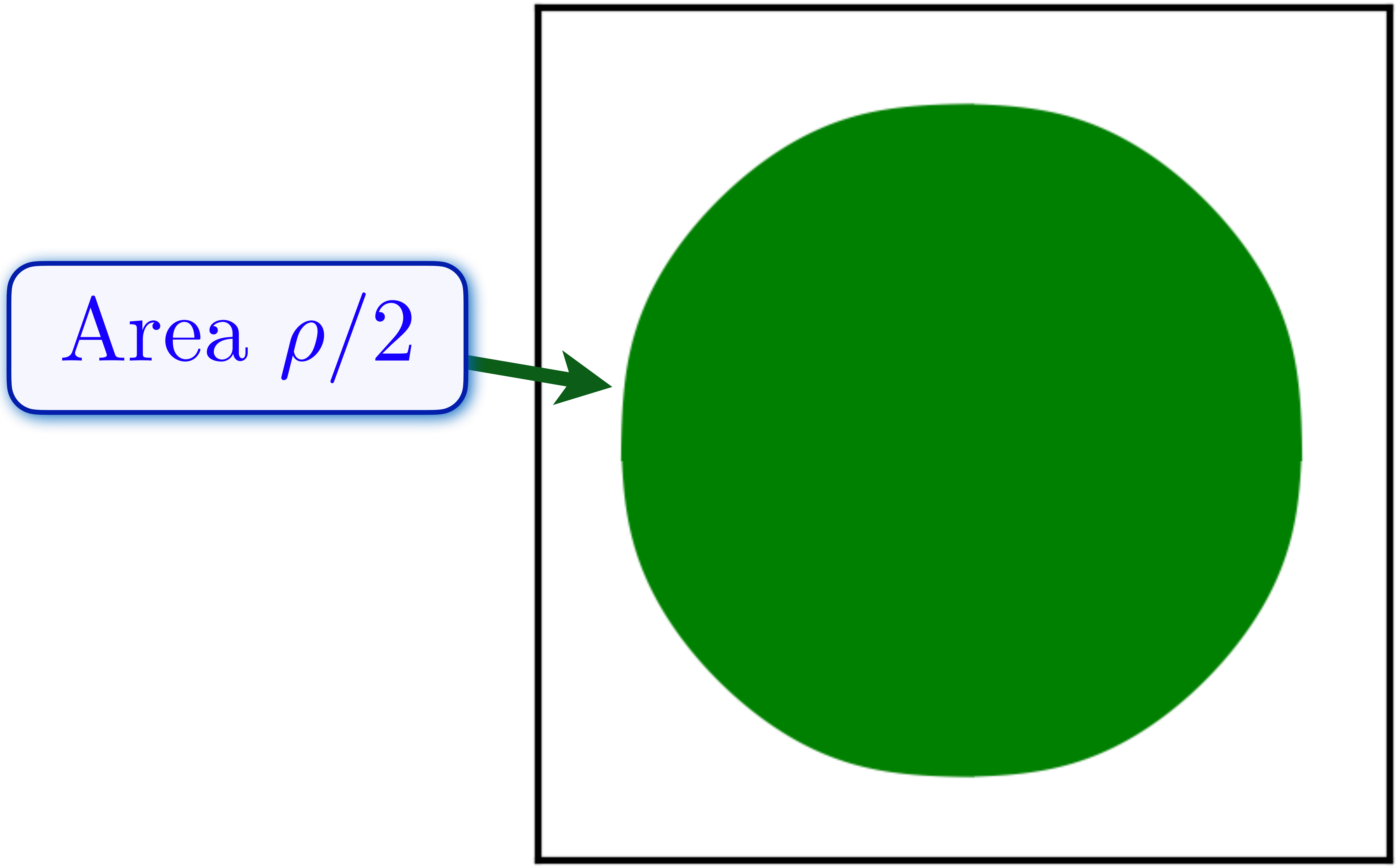
M. Christos, Zhu-Xi Luo, L. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120,** e2302701120 (2023); H. Pandey, M. Christos, P.M. Bonetti, R. Shanker, S. Sharma, S.S., arXiv:2507.05336

Fractionalized Fermi liquids (FL*)

Fermi liquid

Spin-1/2 holes of density
$\rho = 1 + p$

Positive Hall coefficient
of carrier density ρ



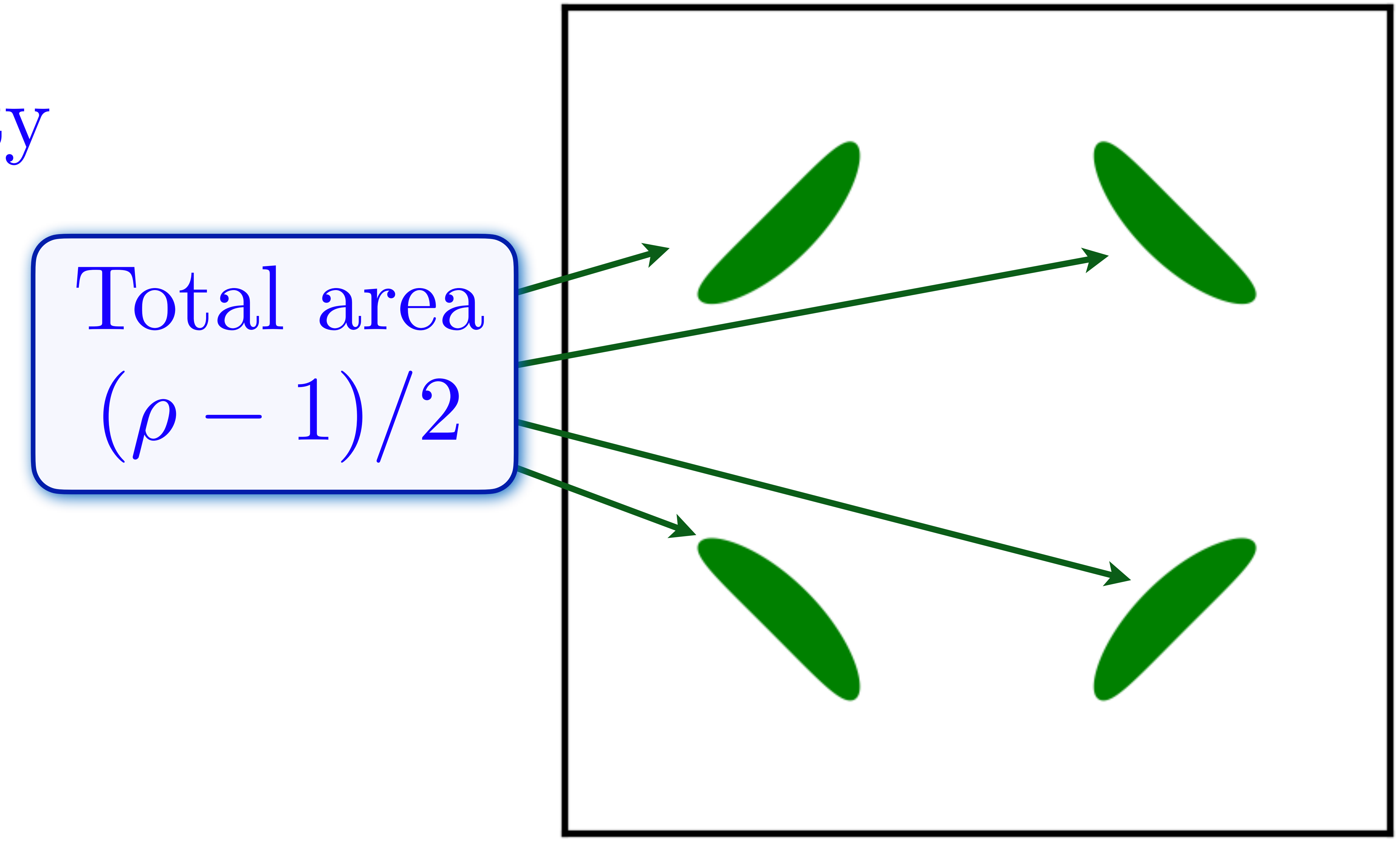
Luttinger, 1960: Area enclosed by the Fermi surface is the same as that for free fermions *with the same symmetry.*

Oshikawa, 2000: Area constrained by a ’t Hooft anomaly of global U(1) and translations

Fractionalized Fermi liquid (FL*)

Spin-1/2 holes of density
$\rho = 1 + p$

Positive Hall coefficient
of carrier density $\rho - 1$



Oshikawa anomaly is satisfied by the sum of
spin liquid (1) and
Fermi surface anomalies $(\rho - 1)$

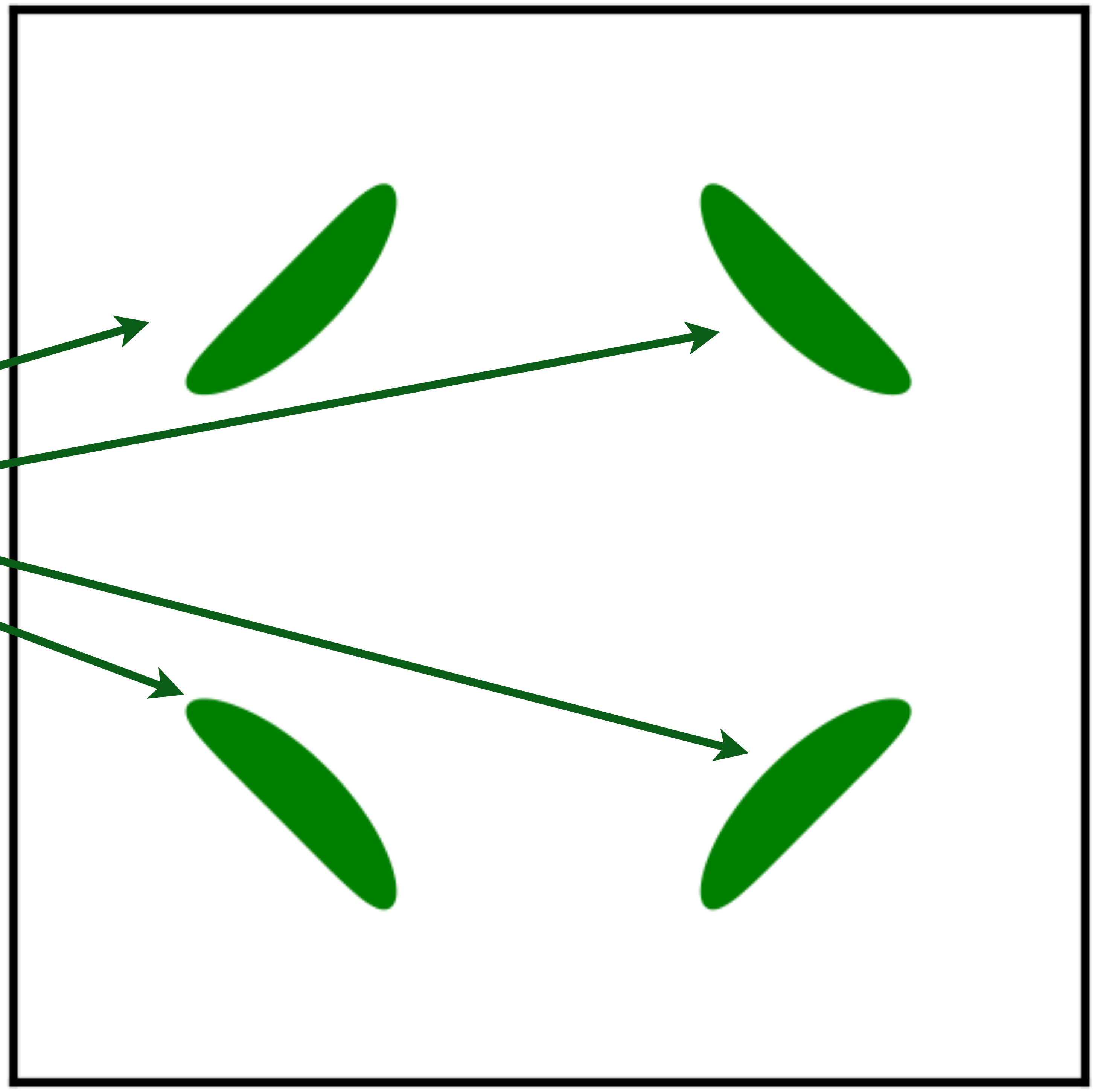
T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); T. Senthil, M. Vojta, and S. S., PRB **69**, 035111 (2004)

Fractionalized Fermi liquid (FL*)

Spin-1/2 holes of density
$$\rho = 1 + p$$

Positive Hall coefficient
of carrier density $\rho - 1$

Total area
$(\rho - 1)/2$



Area of
each
hole pocket
$= p/8$

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T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); T. Senthil, M. Vojta, and S. S., PRB **69**, 035111 (2004)

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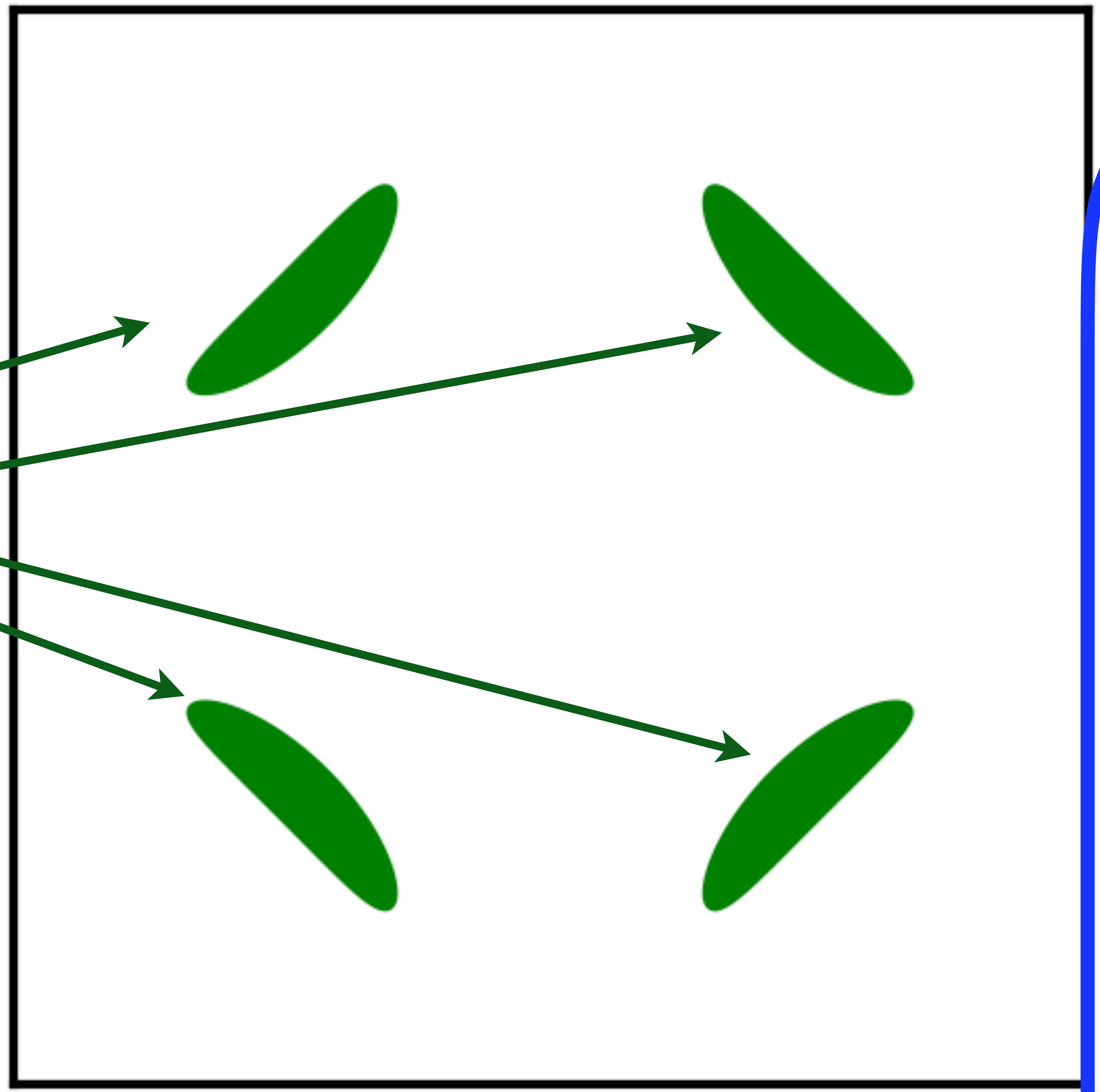


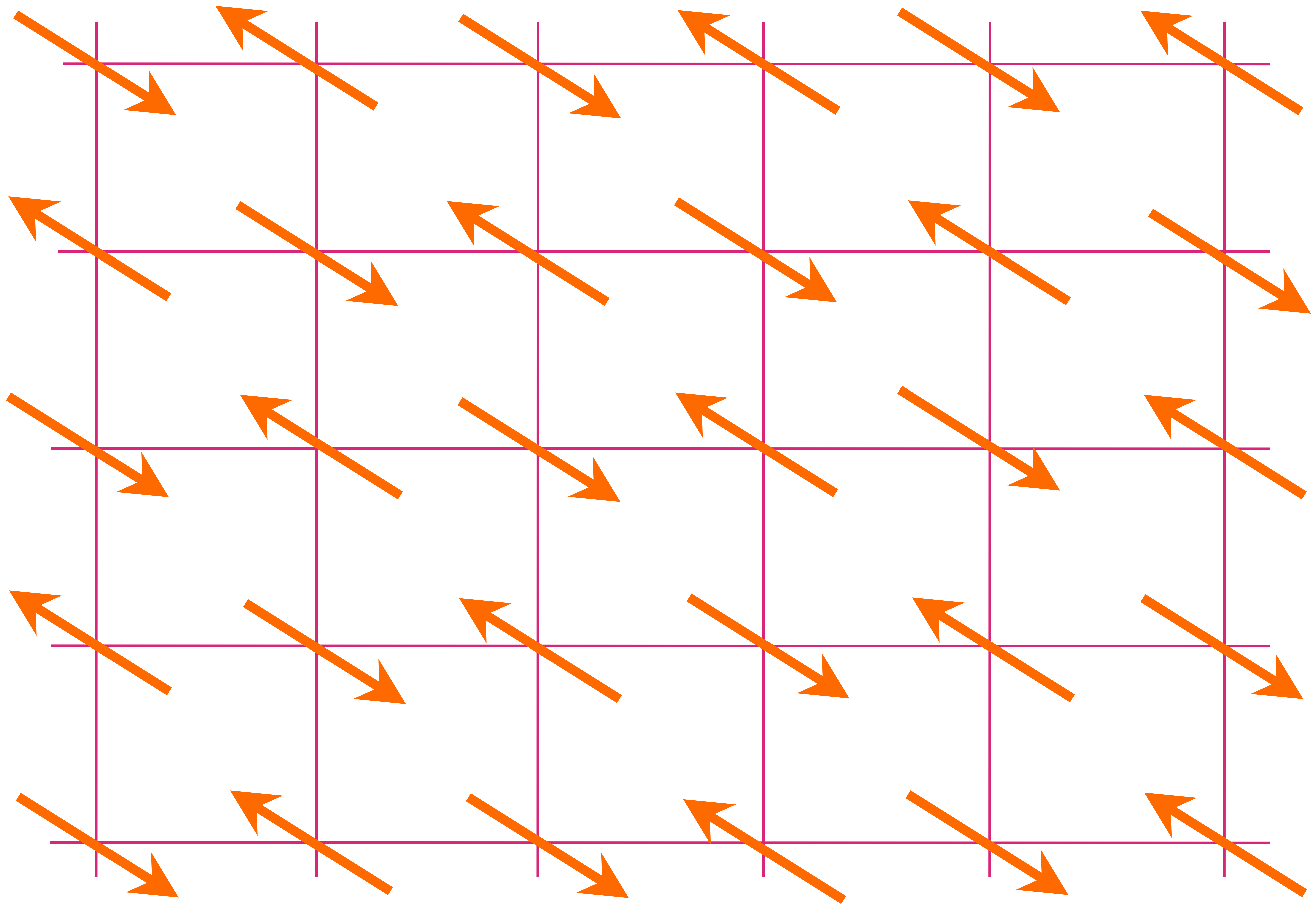
Measuring non-Luttinger Fermi surface area is direct evidence for spin liquid quantum entanglement.

Oshikawa anomaly is satisfied by the sum of spin liquid (1) and Fermi surface anomalies $(\rho - 1)$

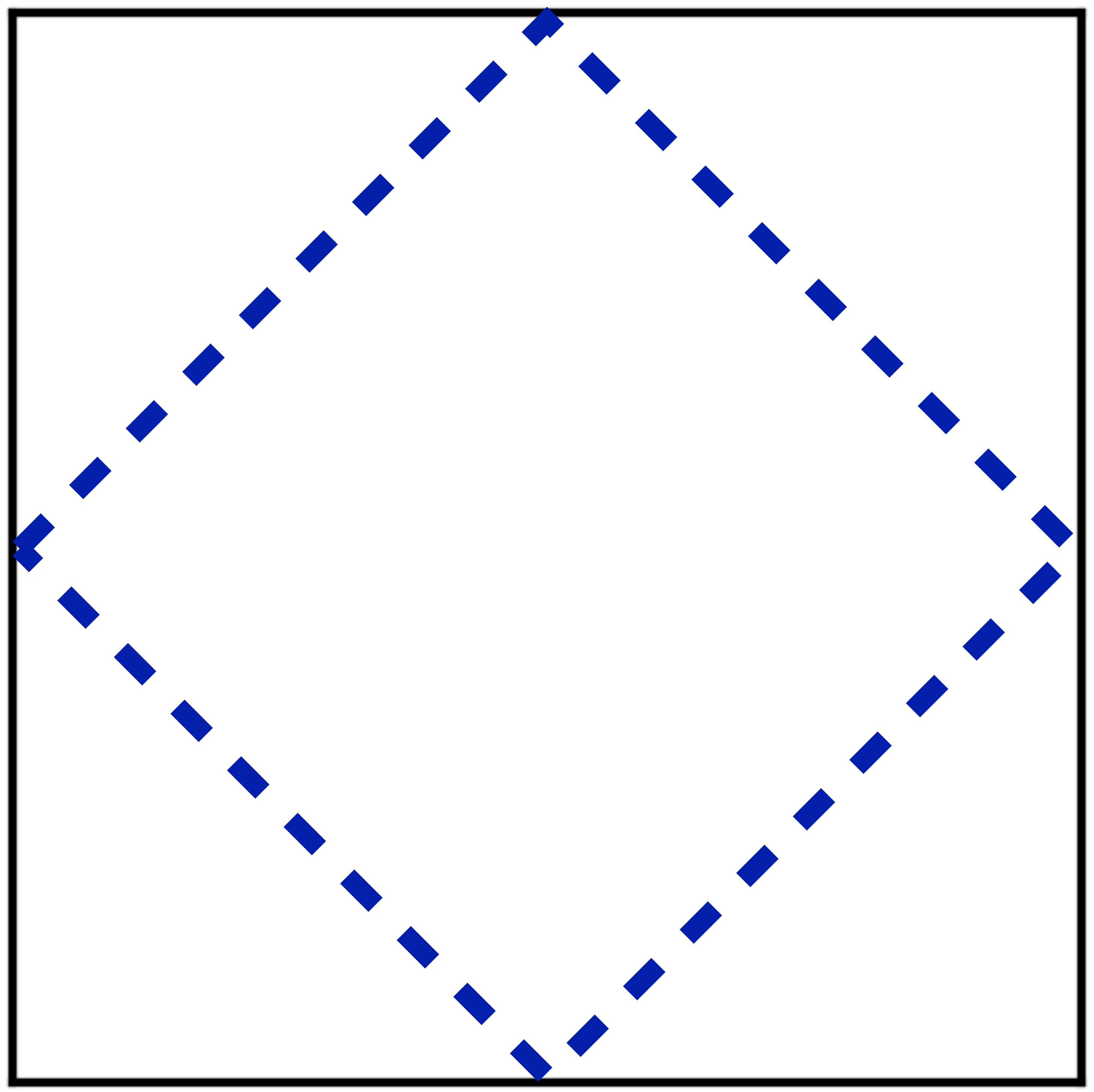
T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); T. Senthil, M. Vojta, and S. S., PRB **69**, 035111 (2004)

FL* in a single-band model on the square lattice

Insulating antiferromagnet



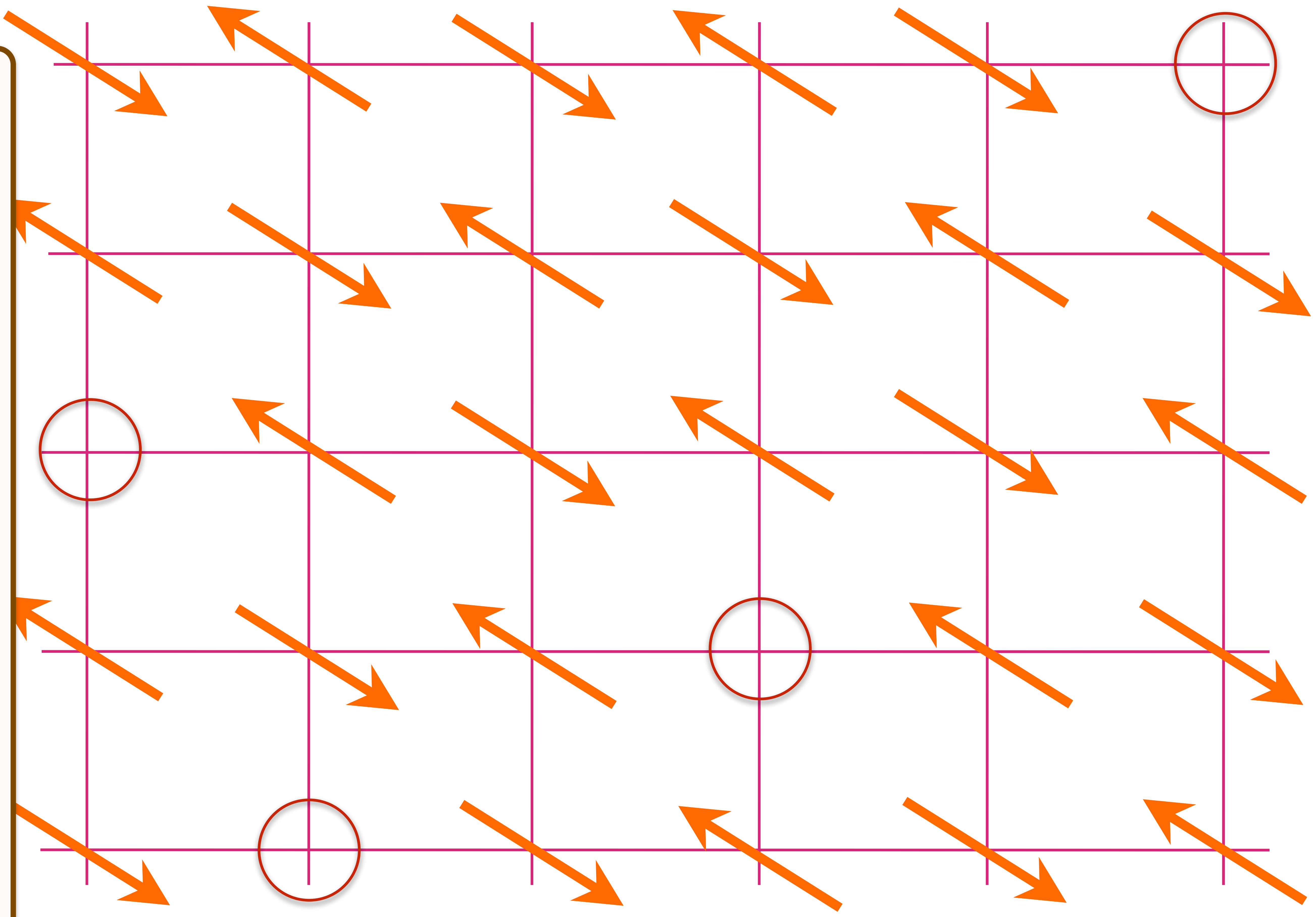
Reduced Brillouin Zone.
Broken symmetry



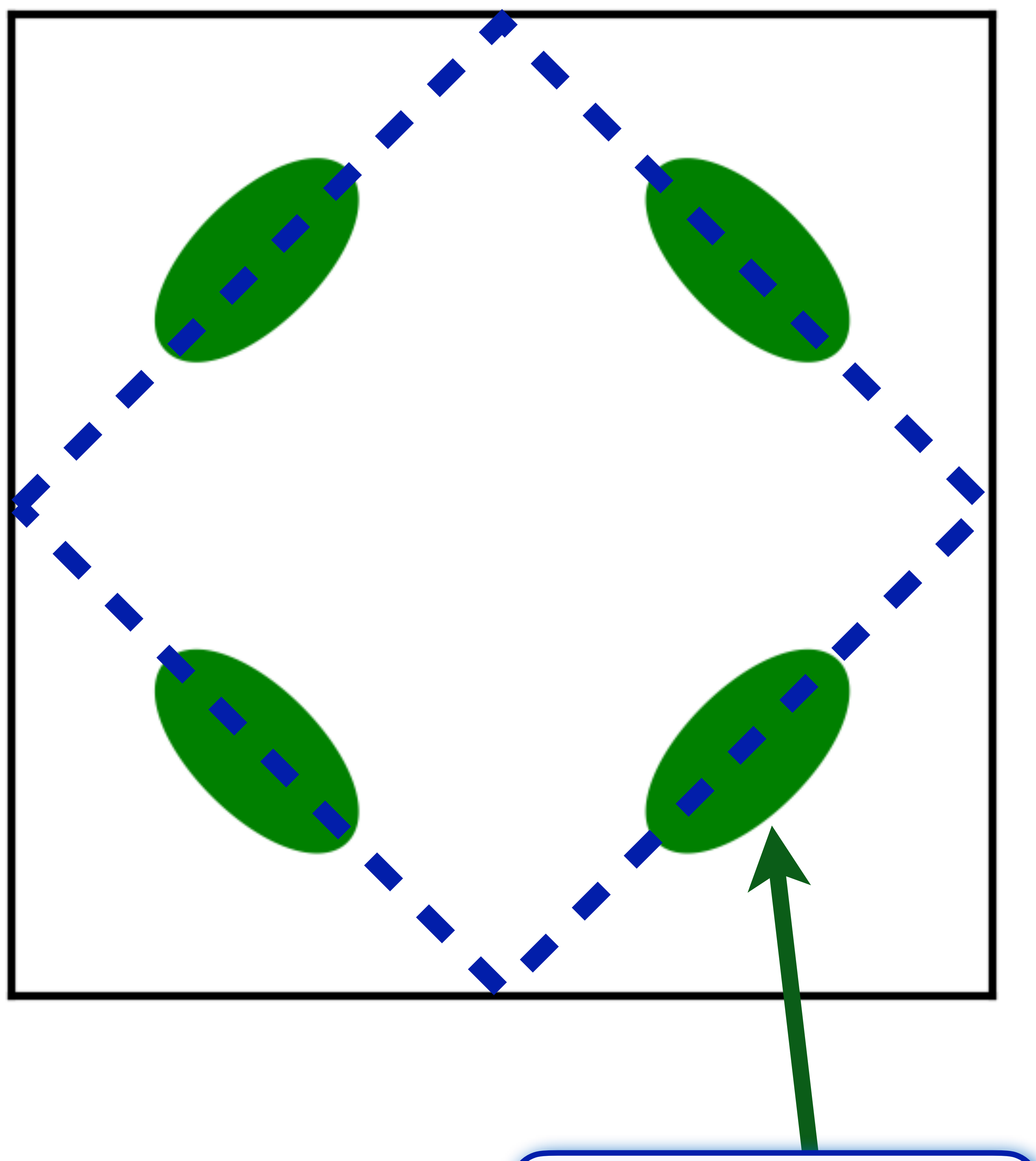
Doping an insulating antiferromagnet with holes of density p

AF metal

Fermi liquid with density p of spin 1/2, charge $+e$ holes. Coherent inter-layer transport requires inter-layer spin correlations.



Luttinger area. Broken symmetry



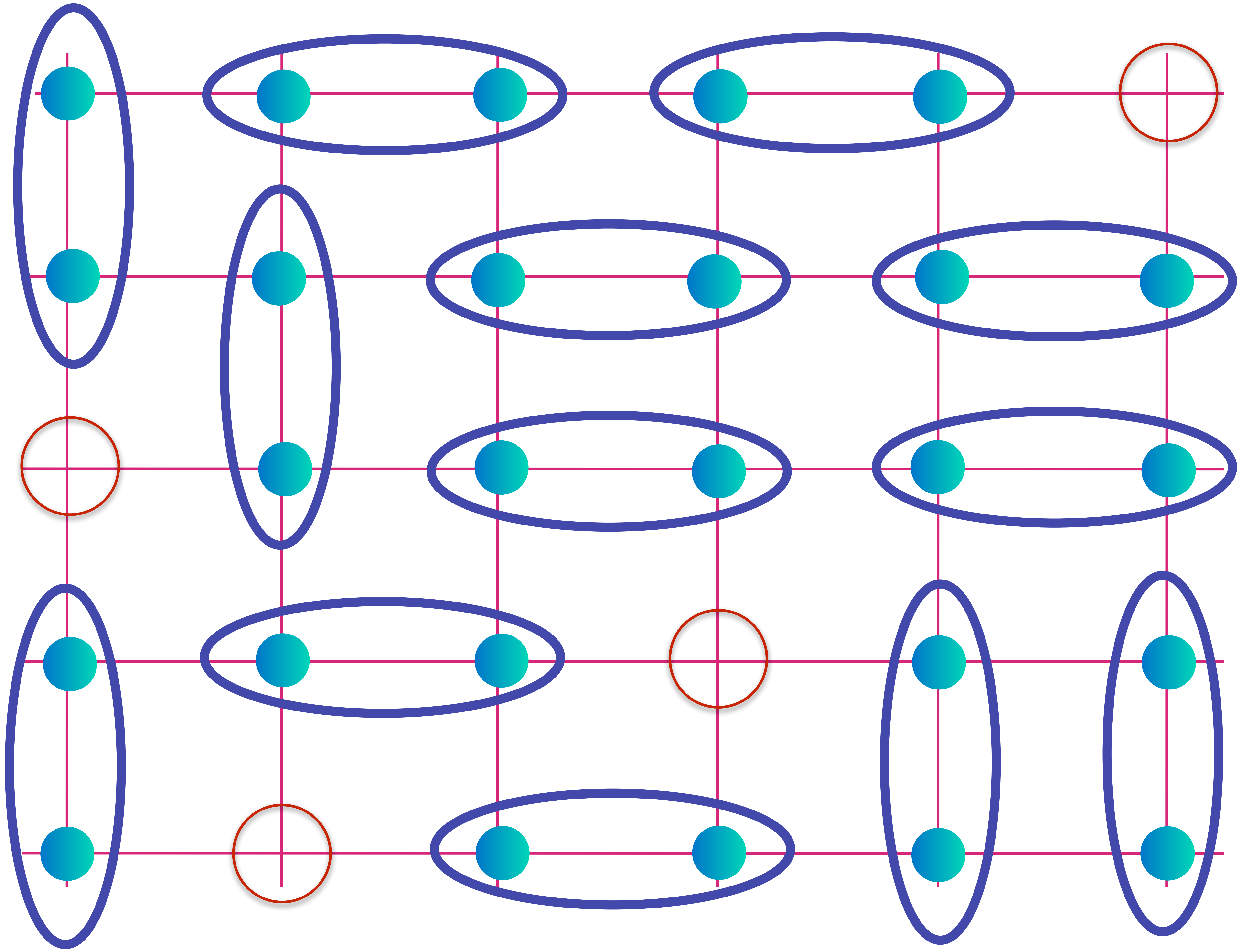
Area $p/4$

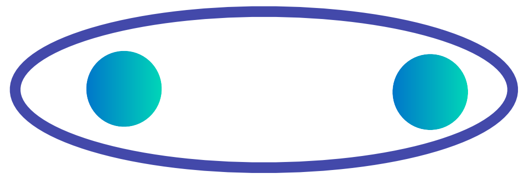
Doping an insulating antiferromagnet with holes of density p

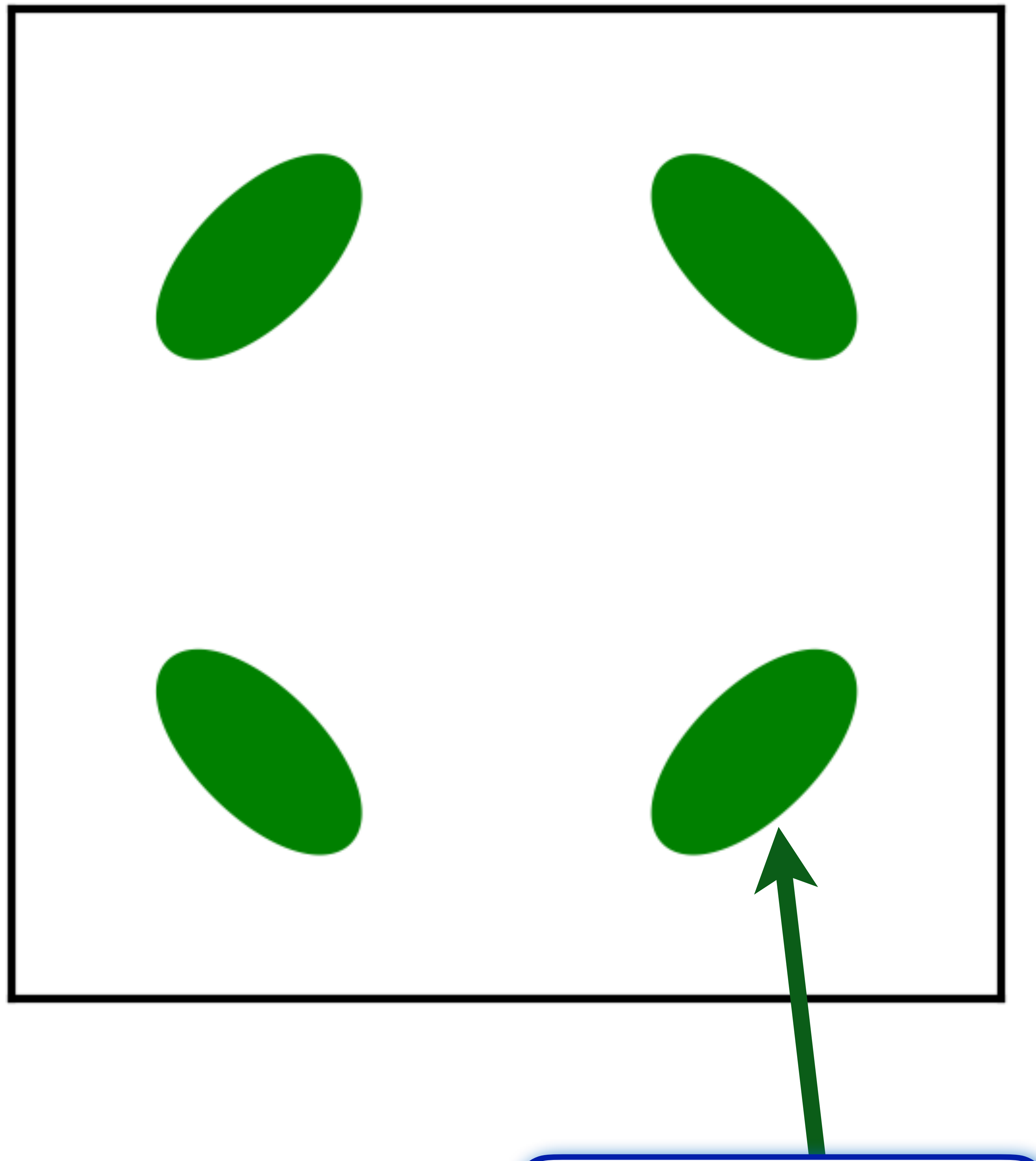
Holon metal

Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)

Spin liquid with density p of spinless, charge $+e$ holons.
No coherent inter-layer transport.



 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



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R. K. Kaul, Y. B. Kim, S. S., T. Senthil, Nature Physics **4**, 28 (2008); S.S. M.A. Metlitski, Y. Qi, and C. Xu, PRB **80**, 155129 (2009)

Doping an insulating antiferromagnet with holes of density p

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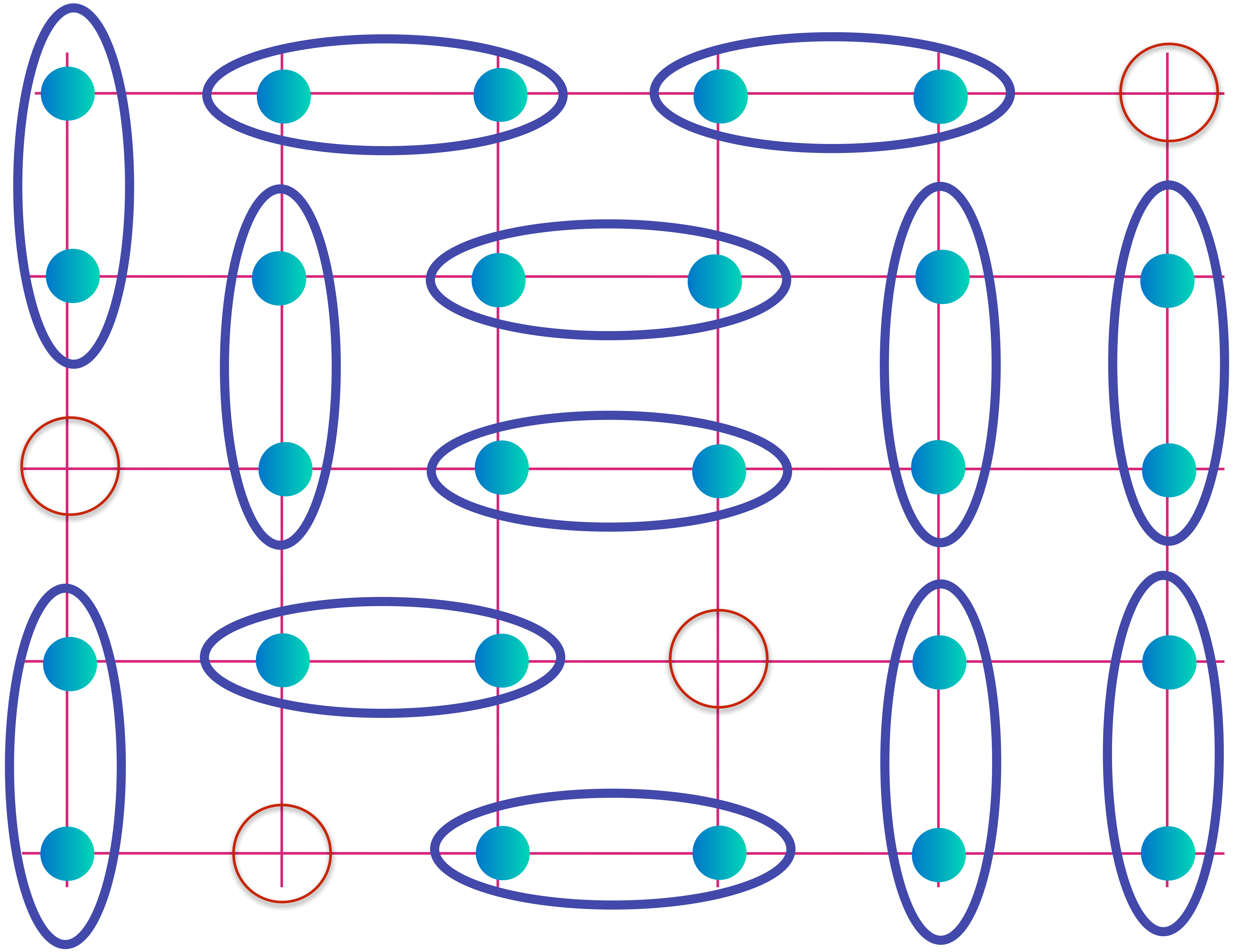
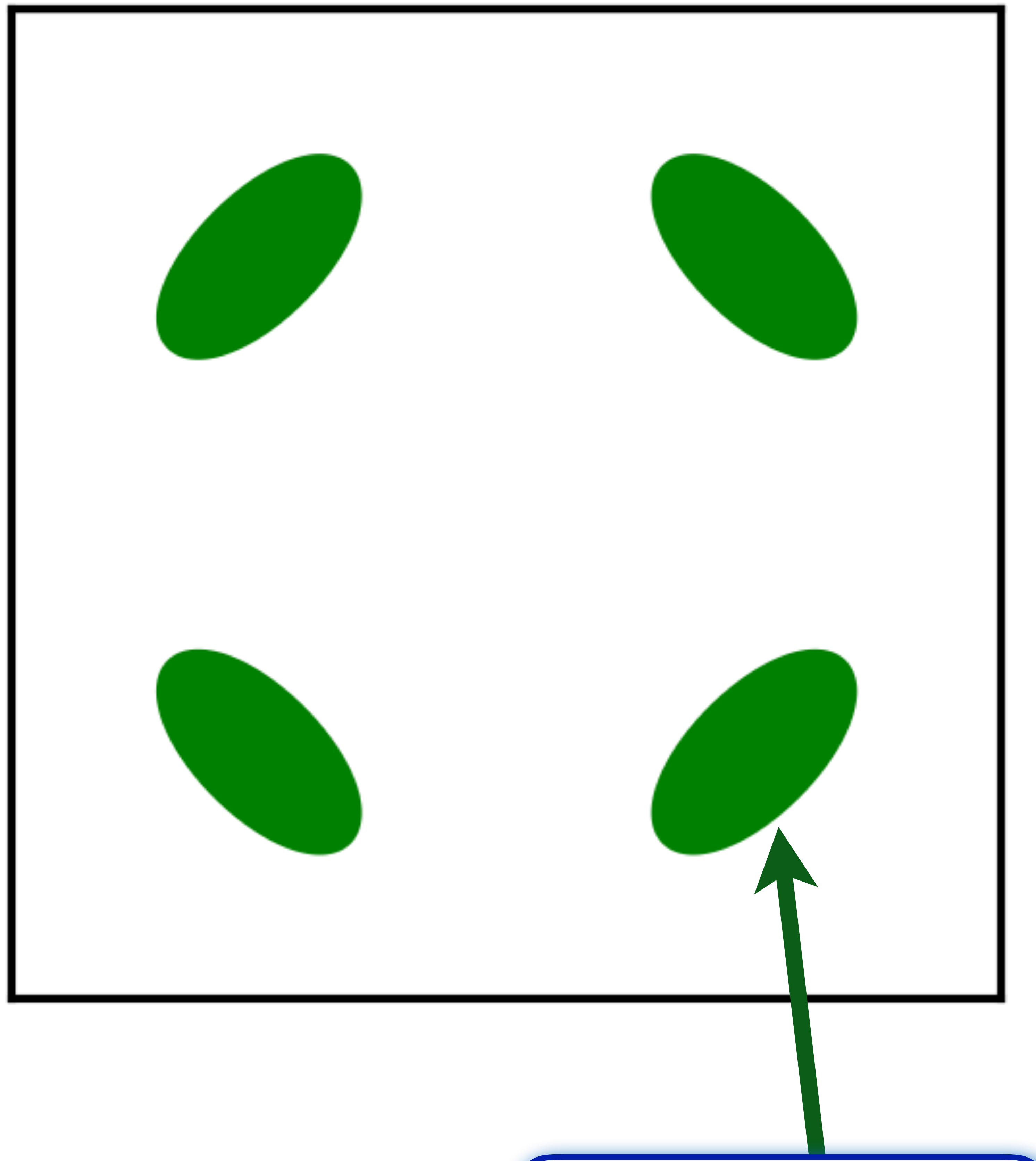


$$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$$



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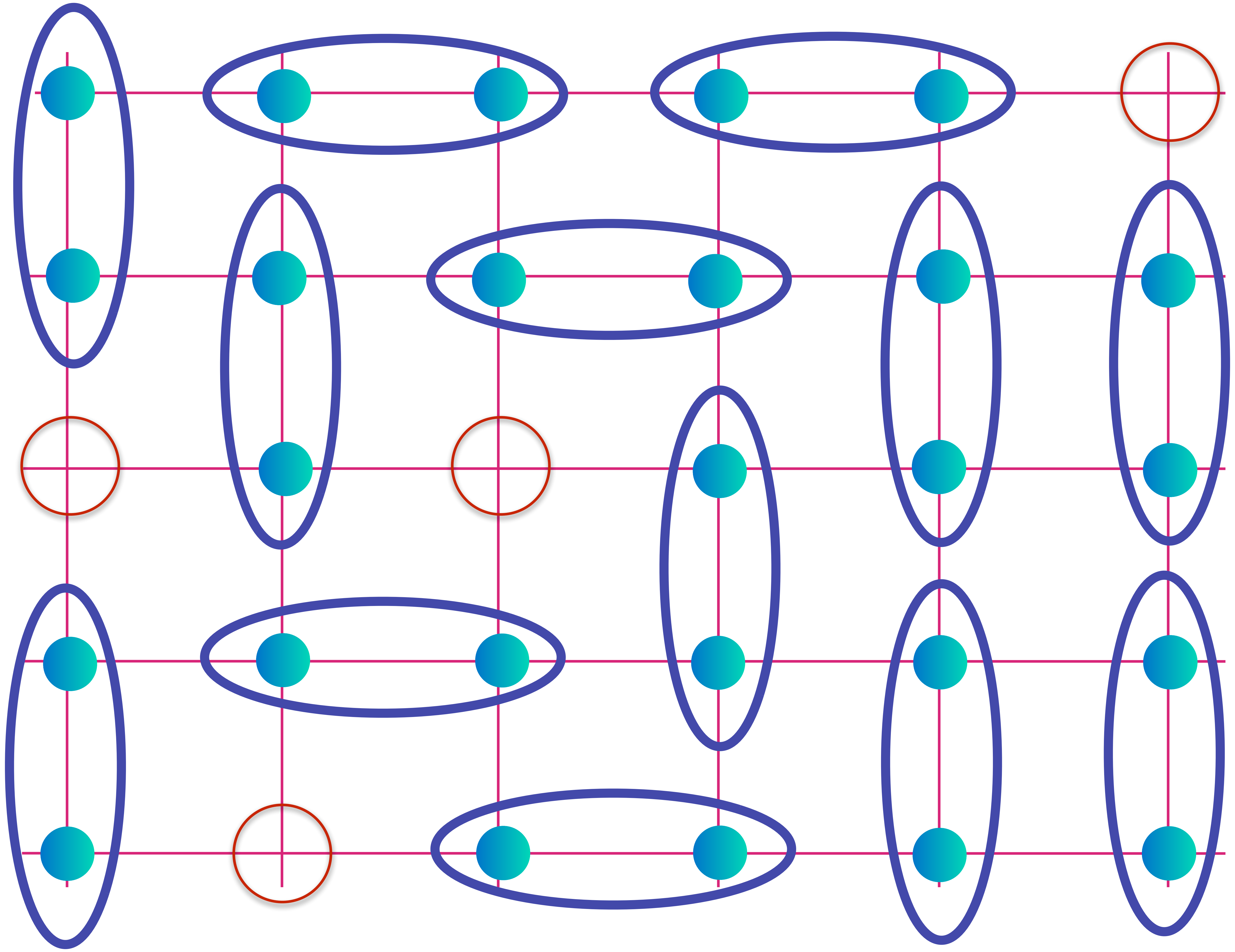
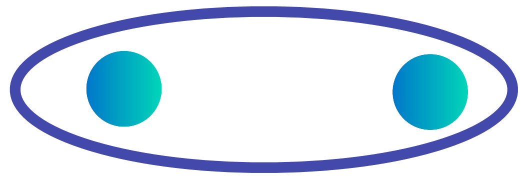
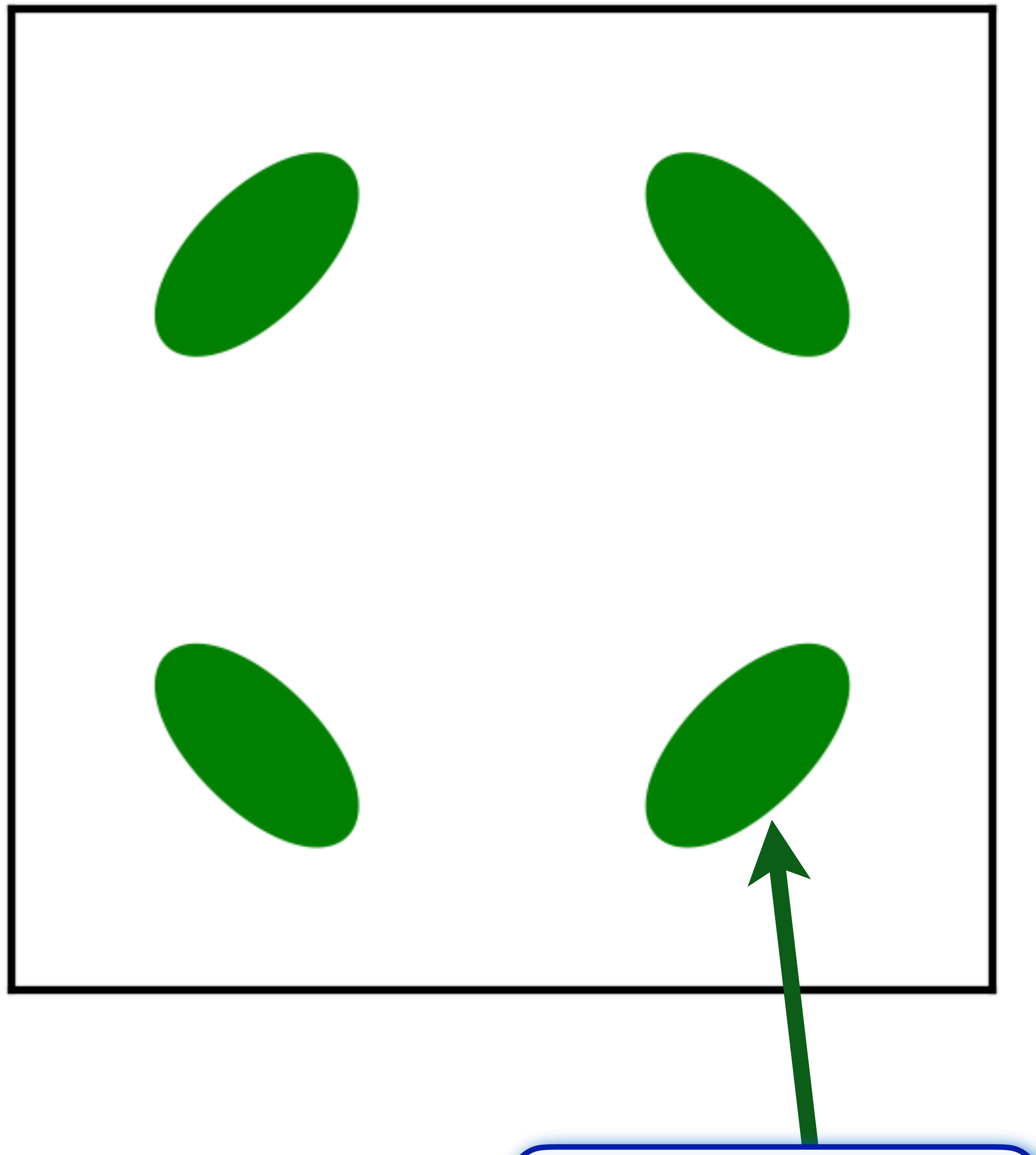


 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



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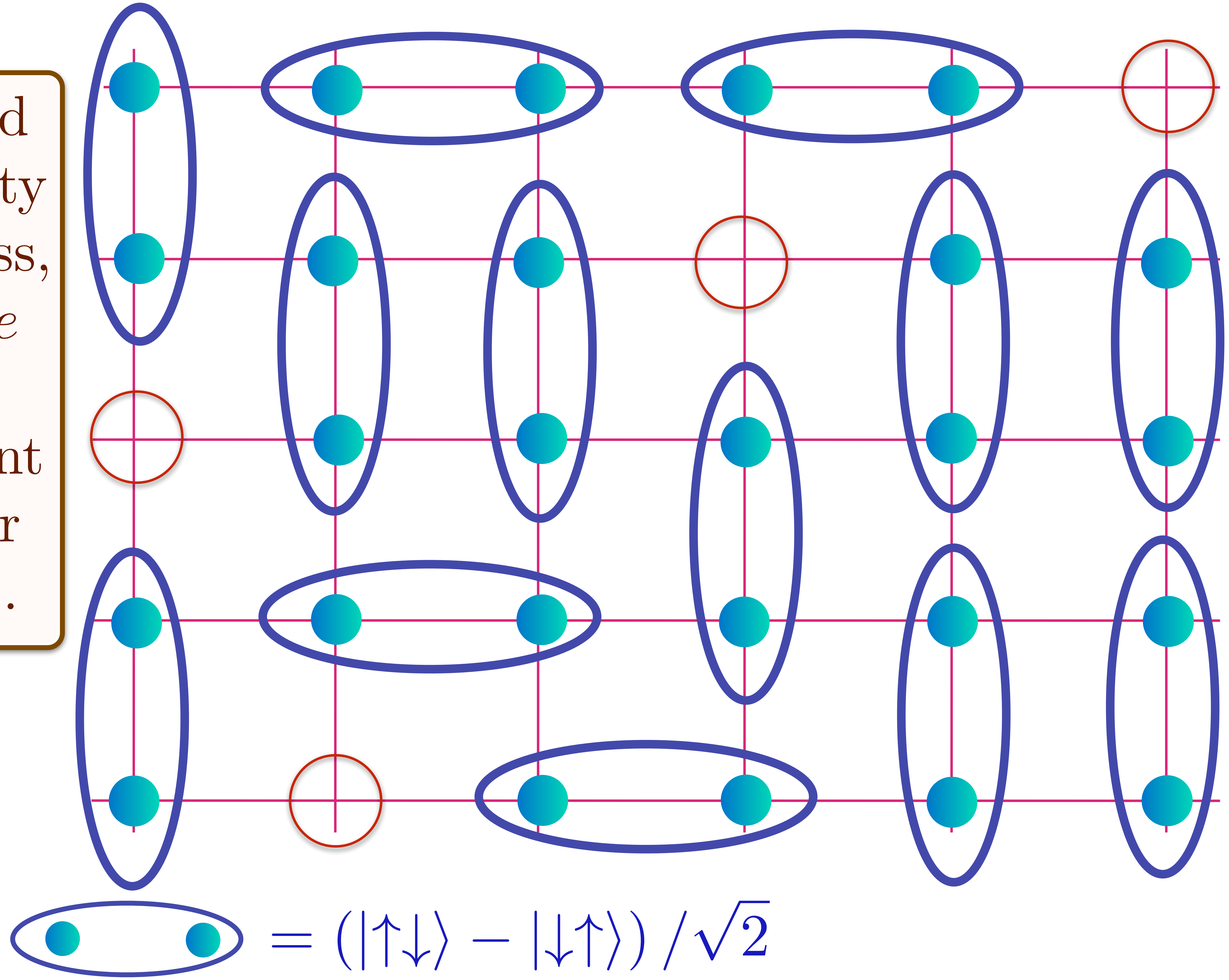

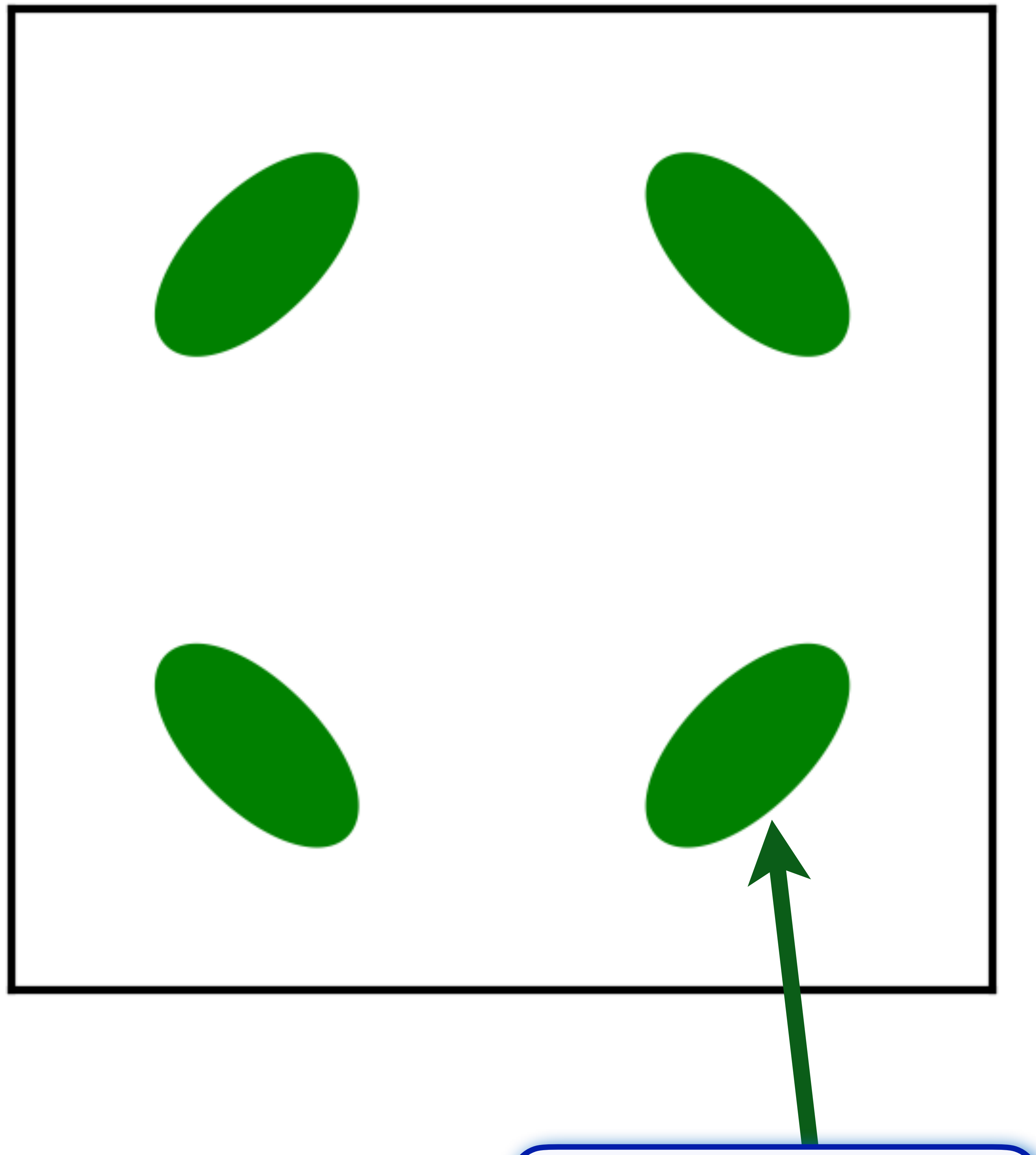

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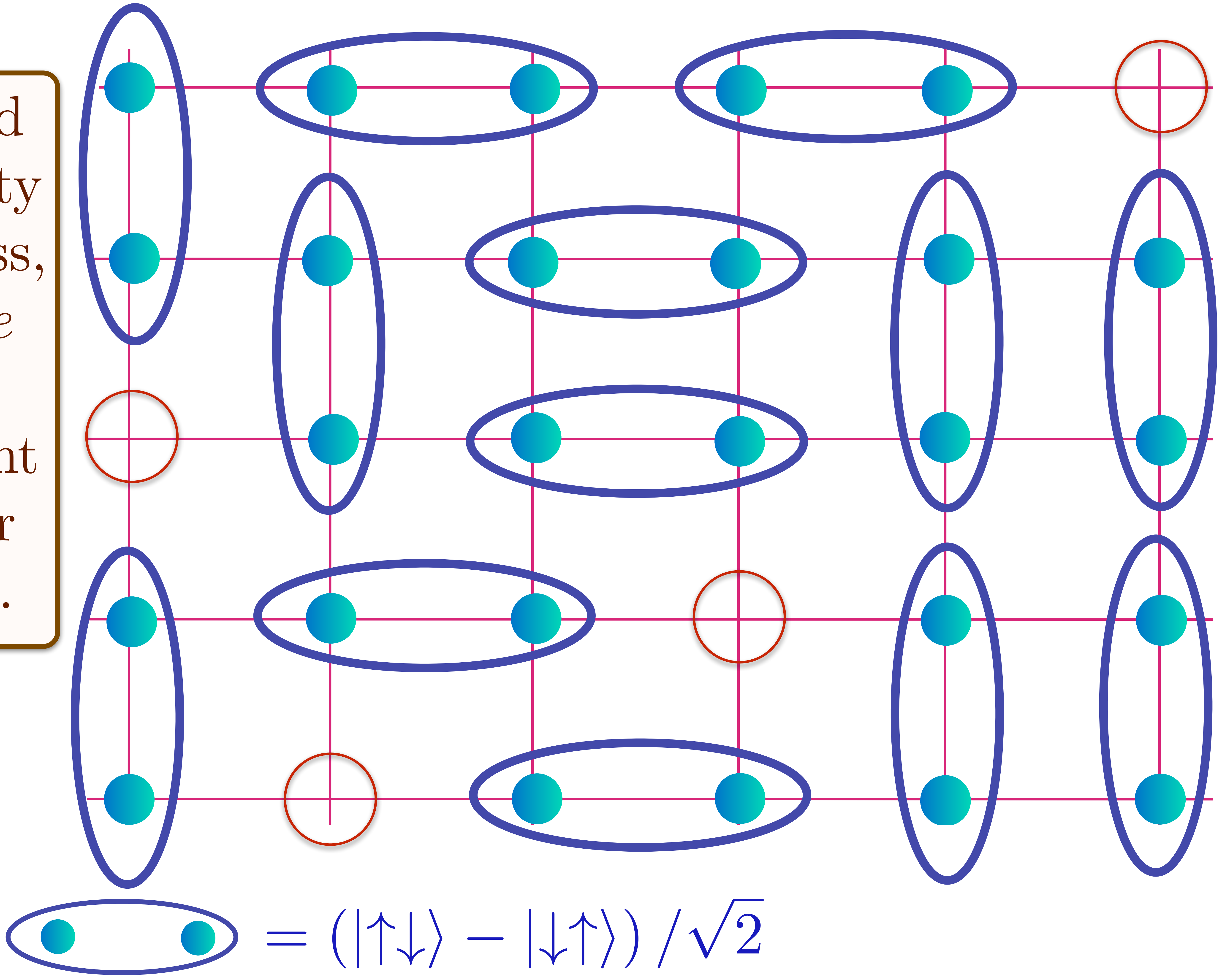

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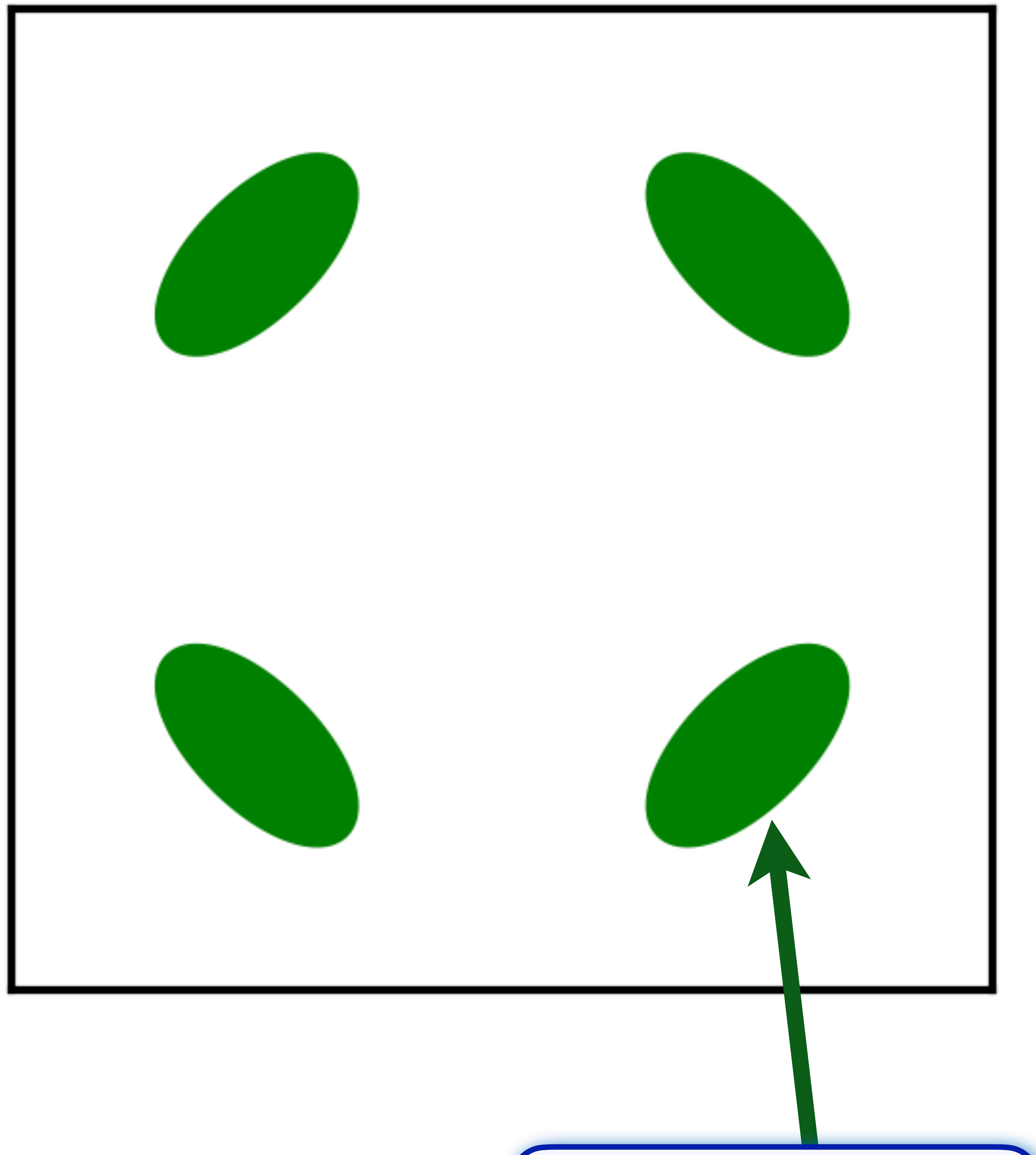

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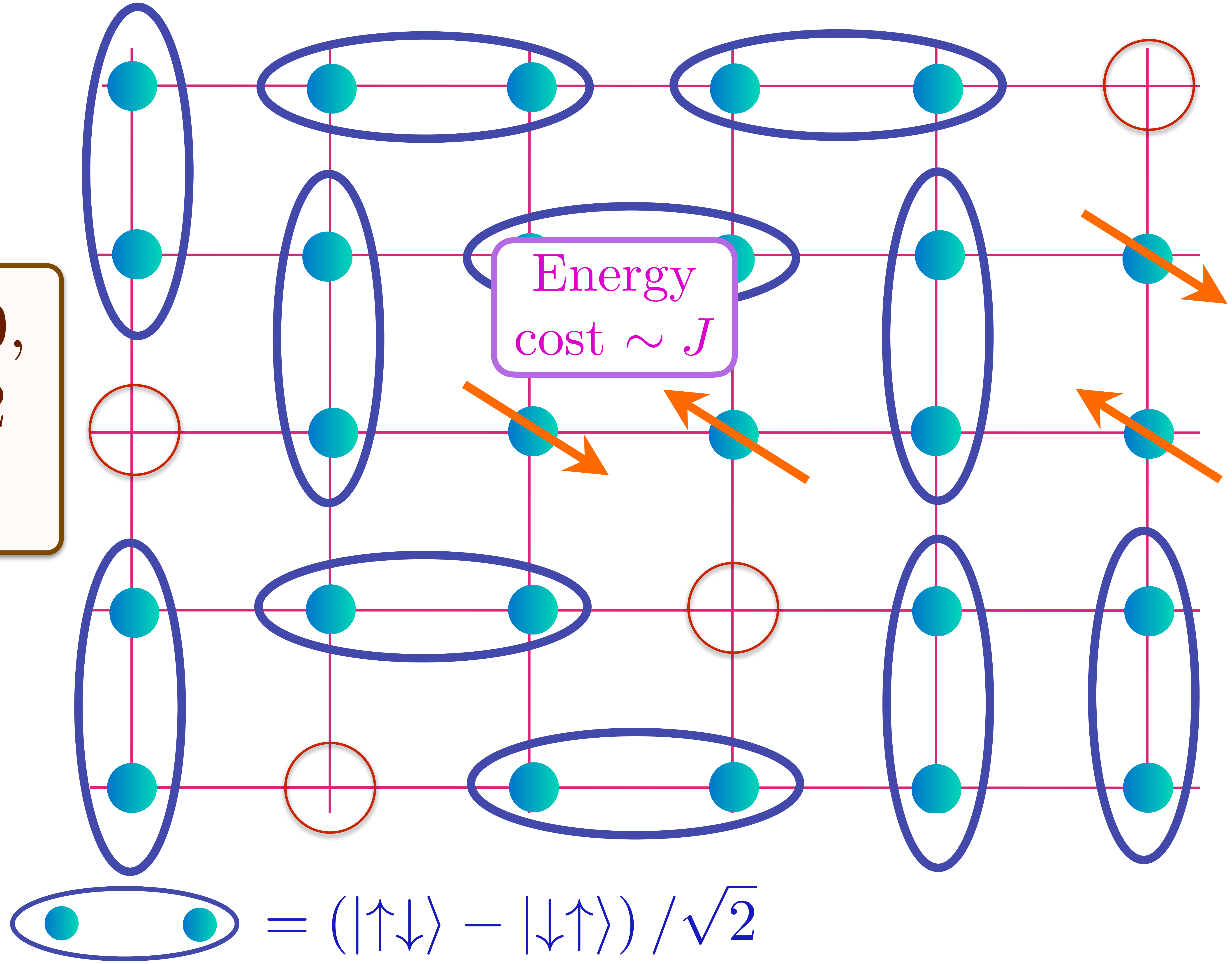
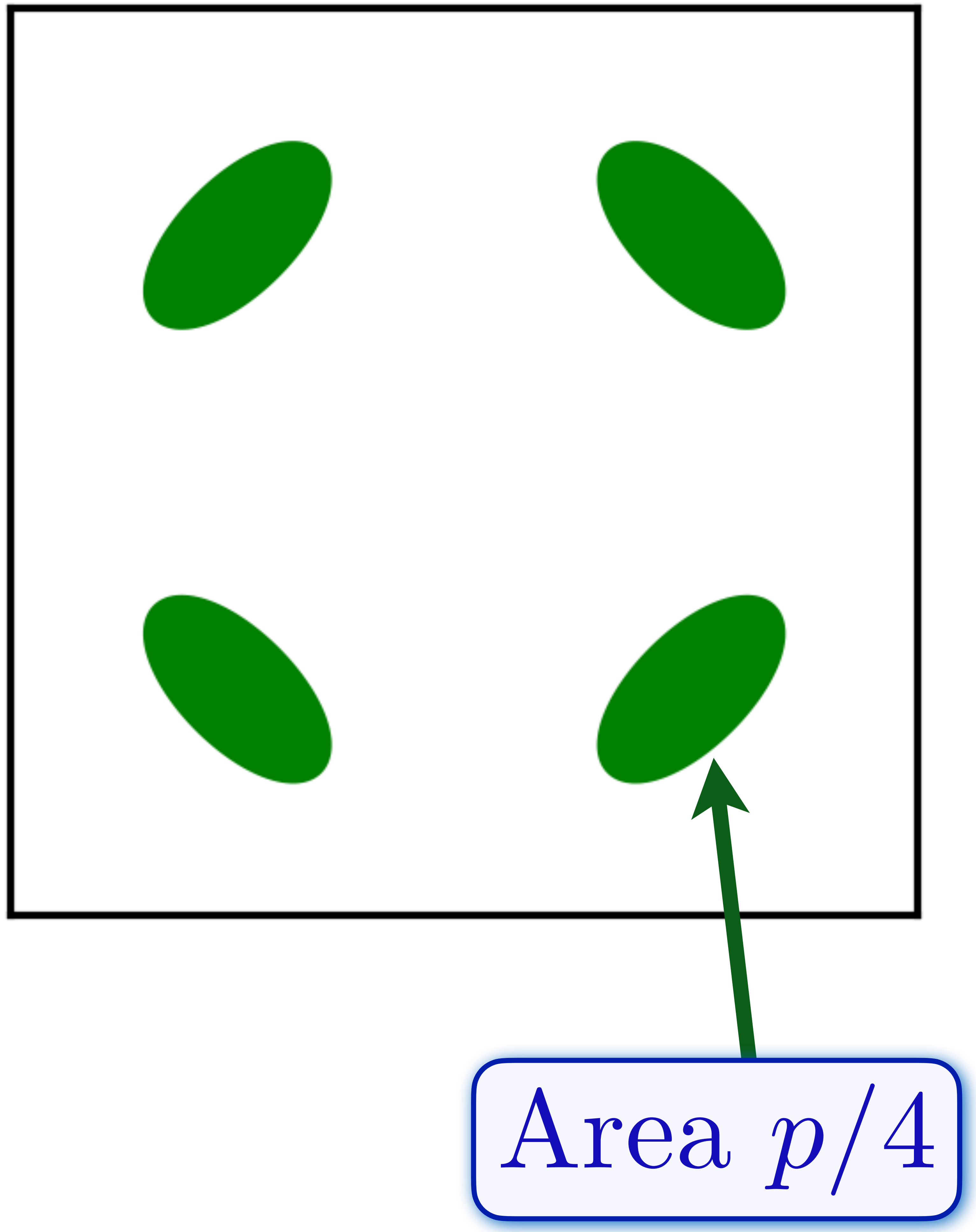
Doping an insulating antiferromagnet with holes of density p

Holon metal excited states

Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)

Charge 0, spin-1/2 spinons

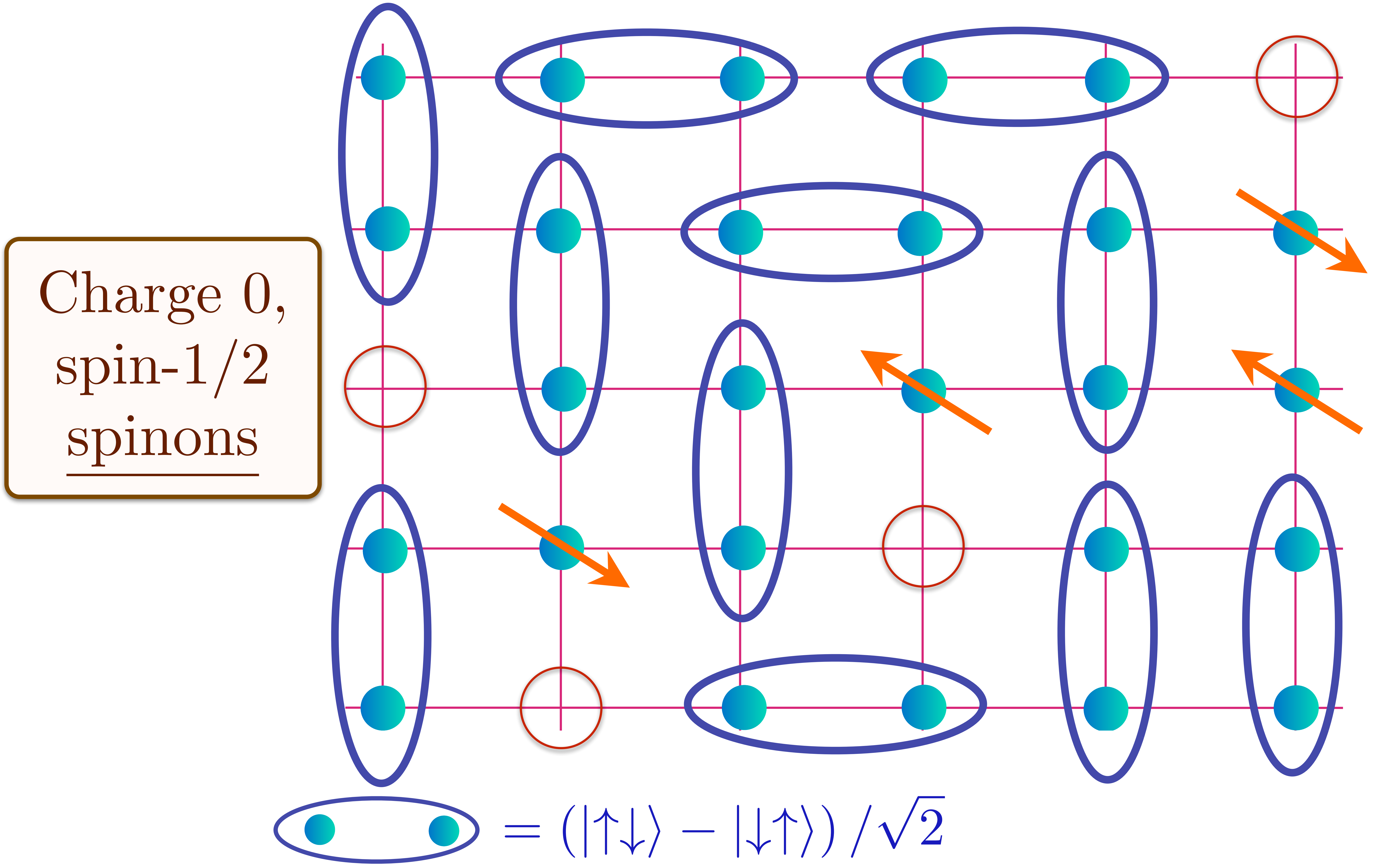


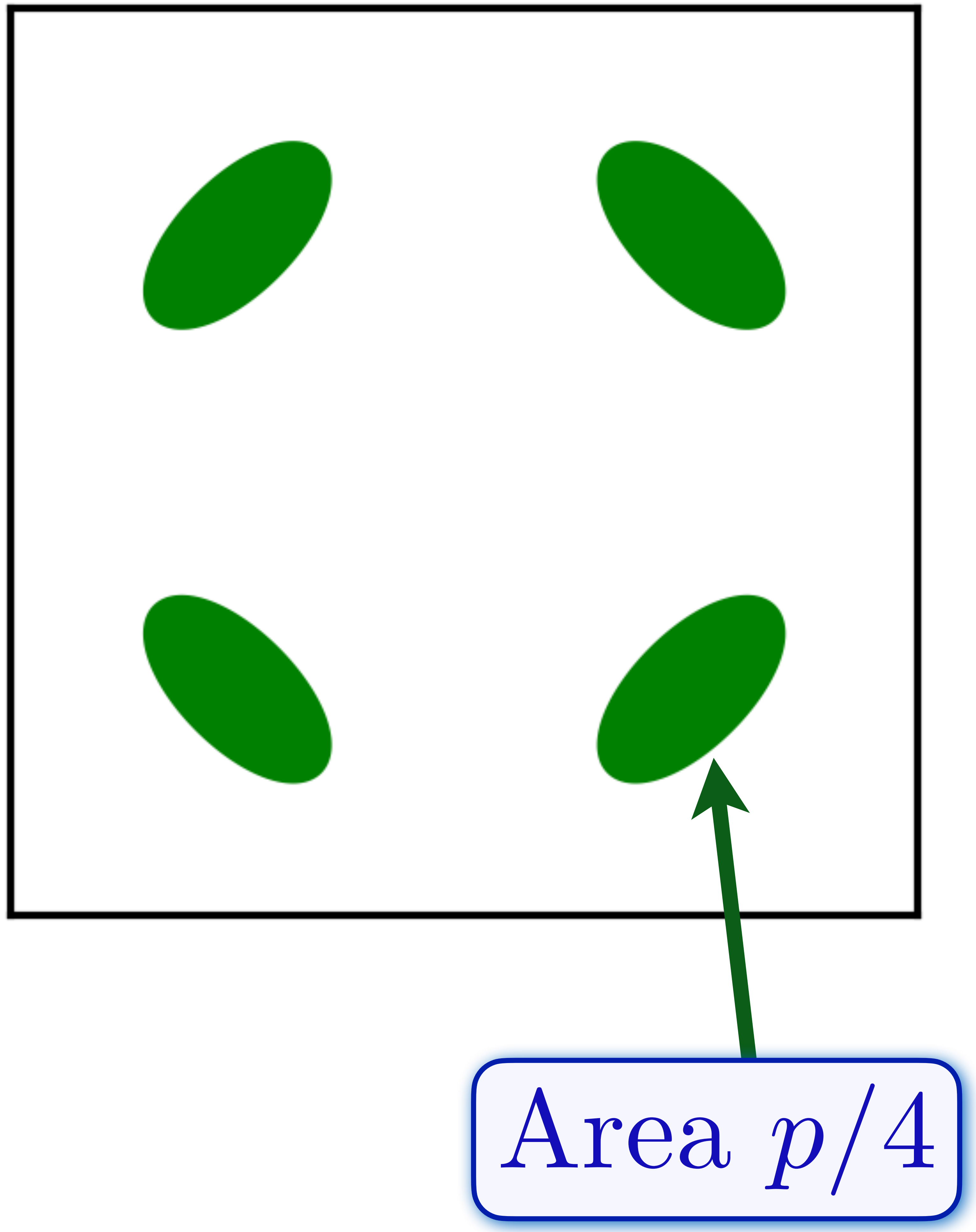


Doping an insulating antiferromagnet with holes of density p

Holon metal excited states

Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)





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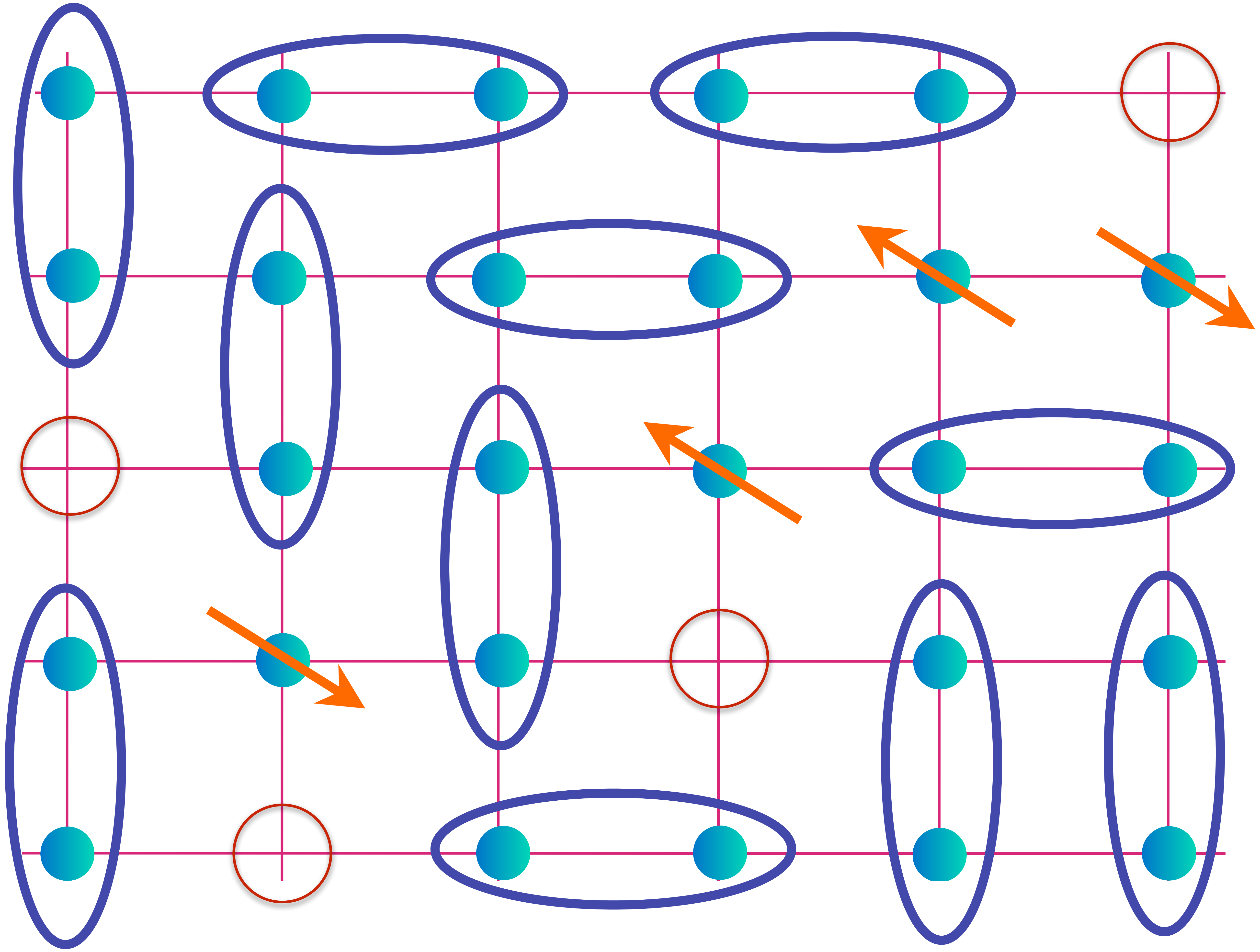
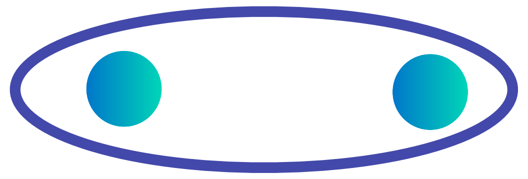
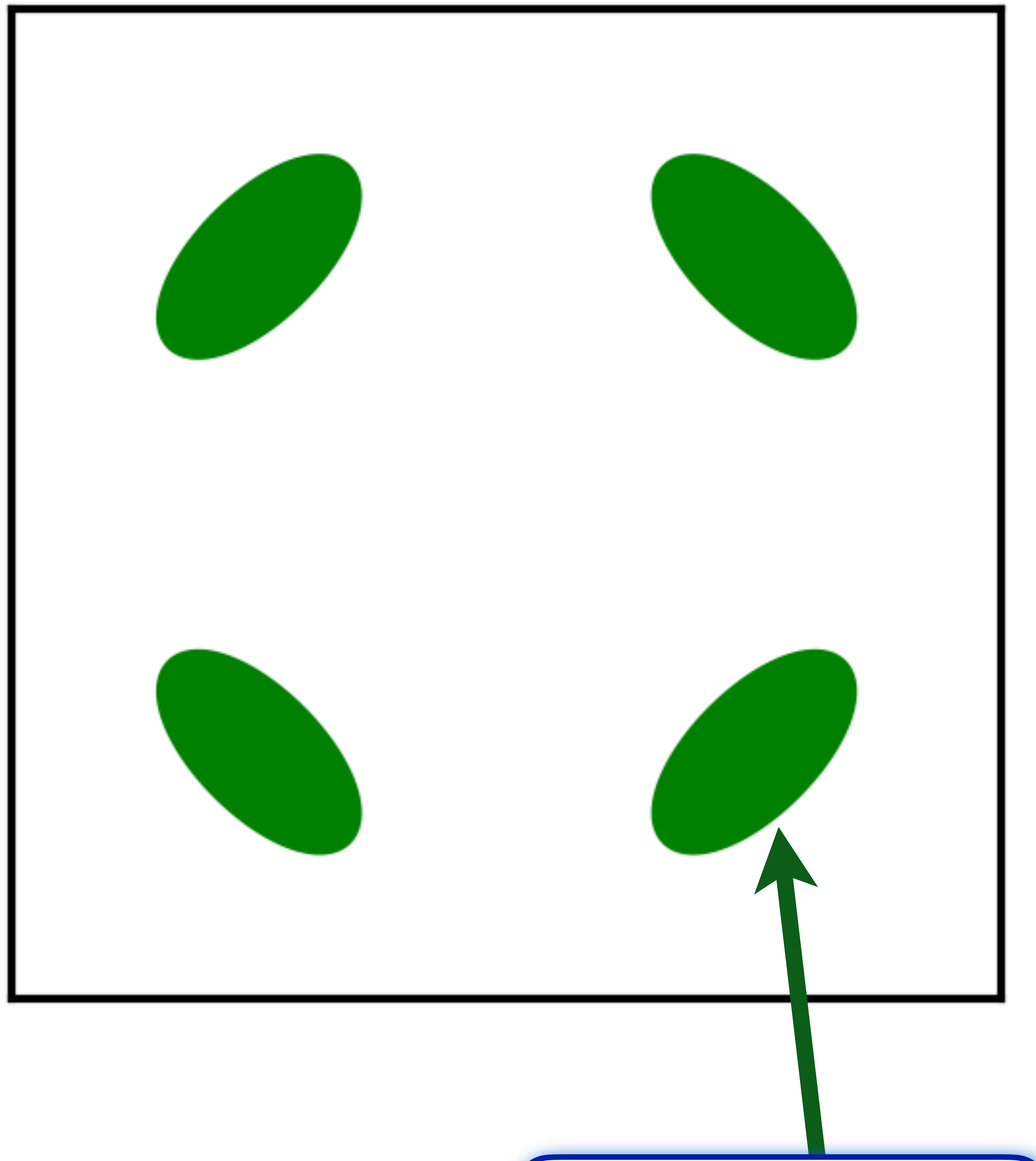


 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



Area $p/4$

Doping an insulating antiferromagnet with holes of density p

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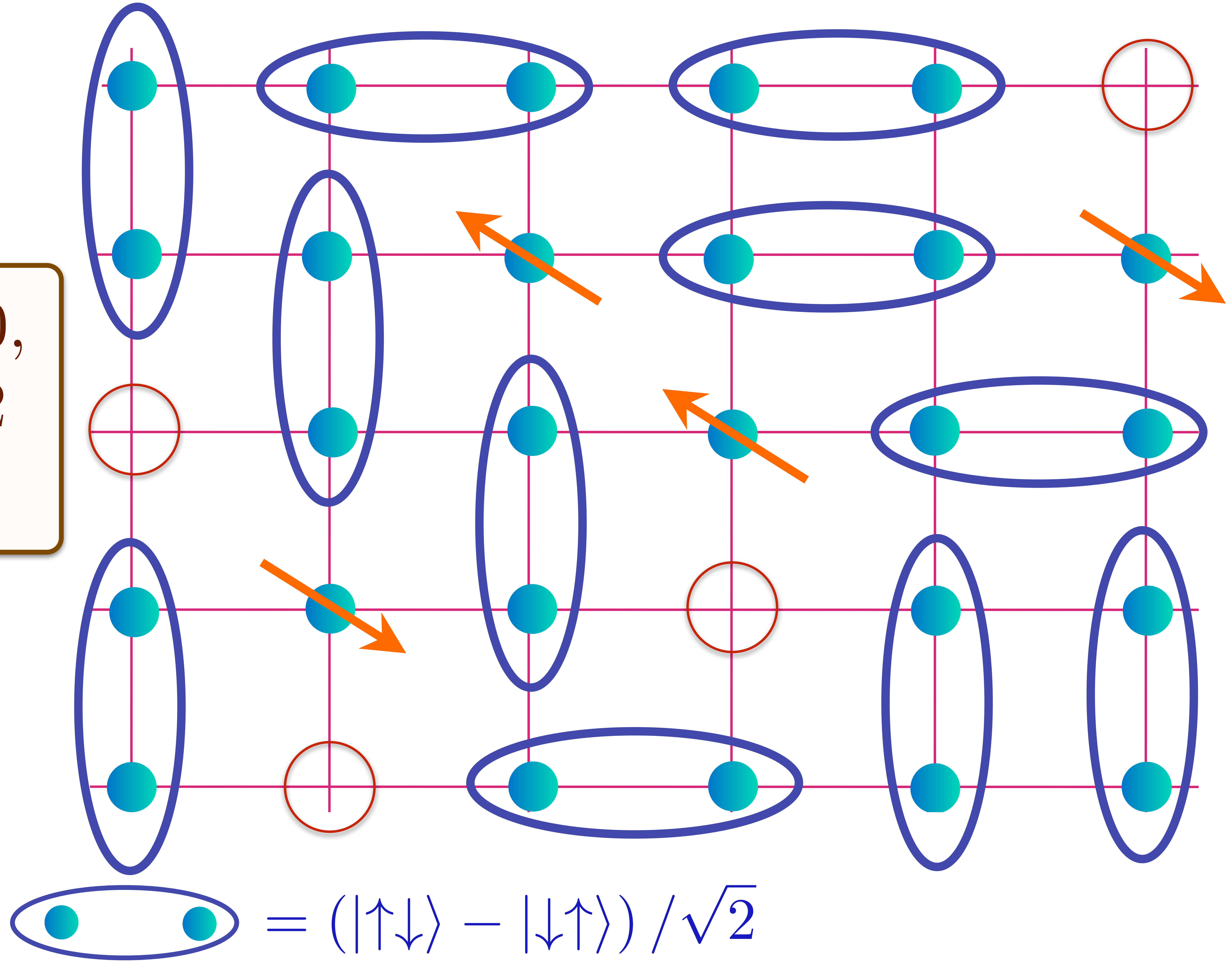
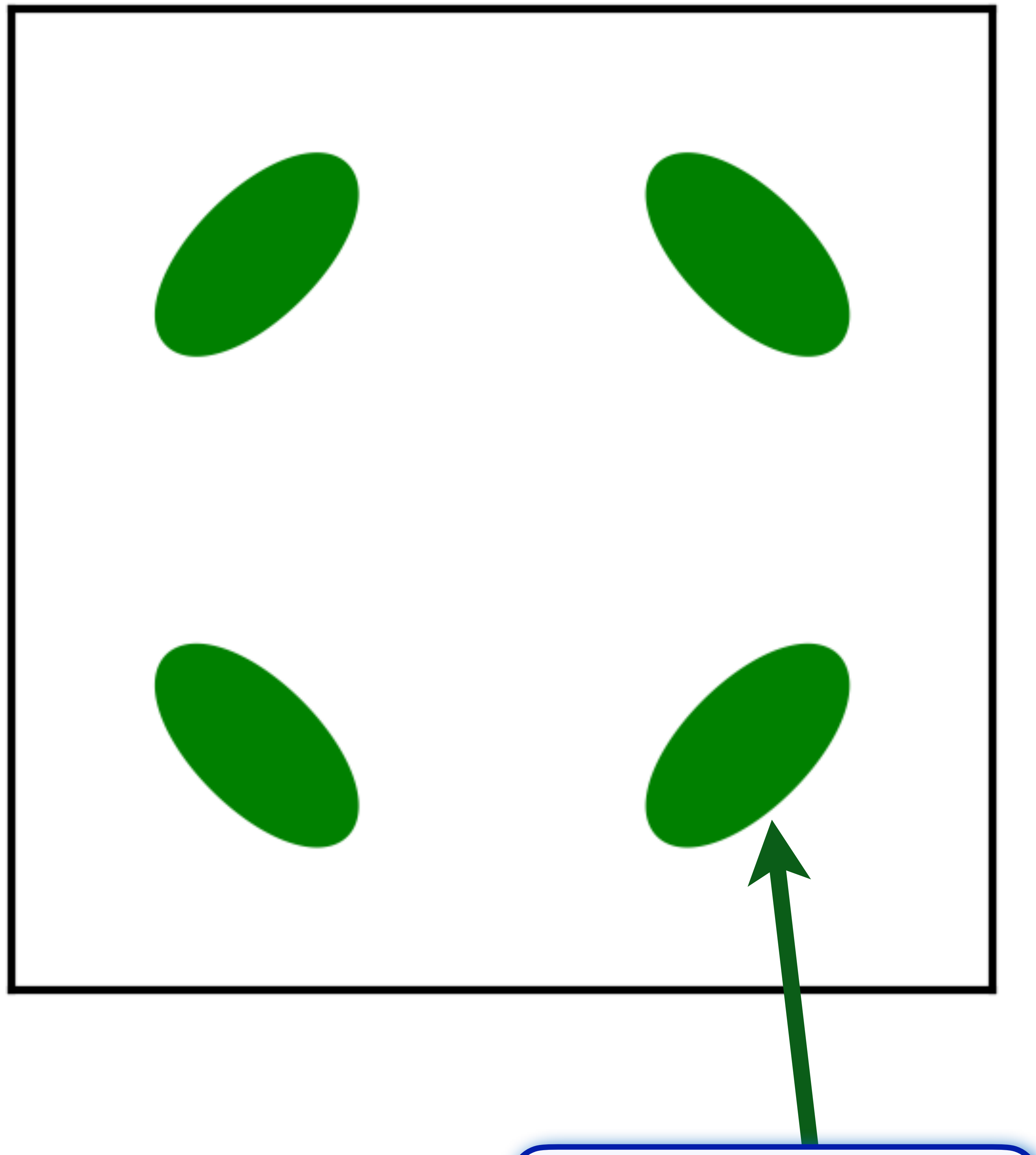


$= (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)/\sqrt{2}$



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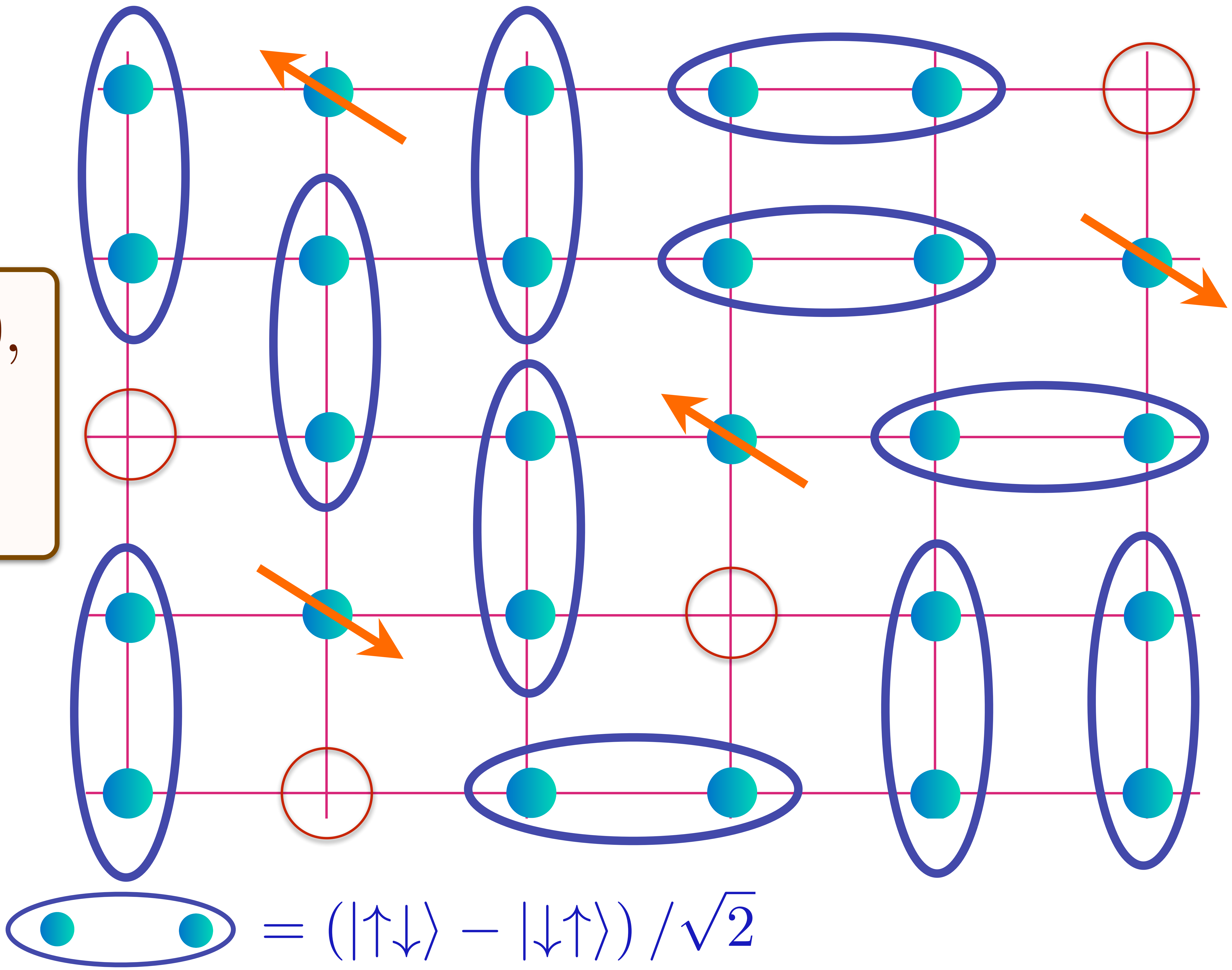

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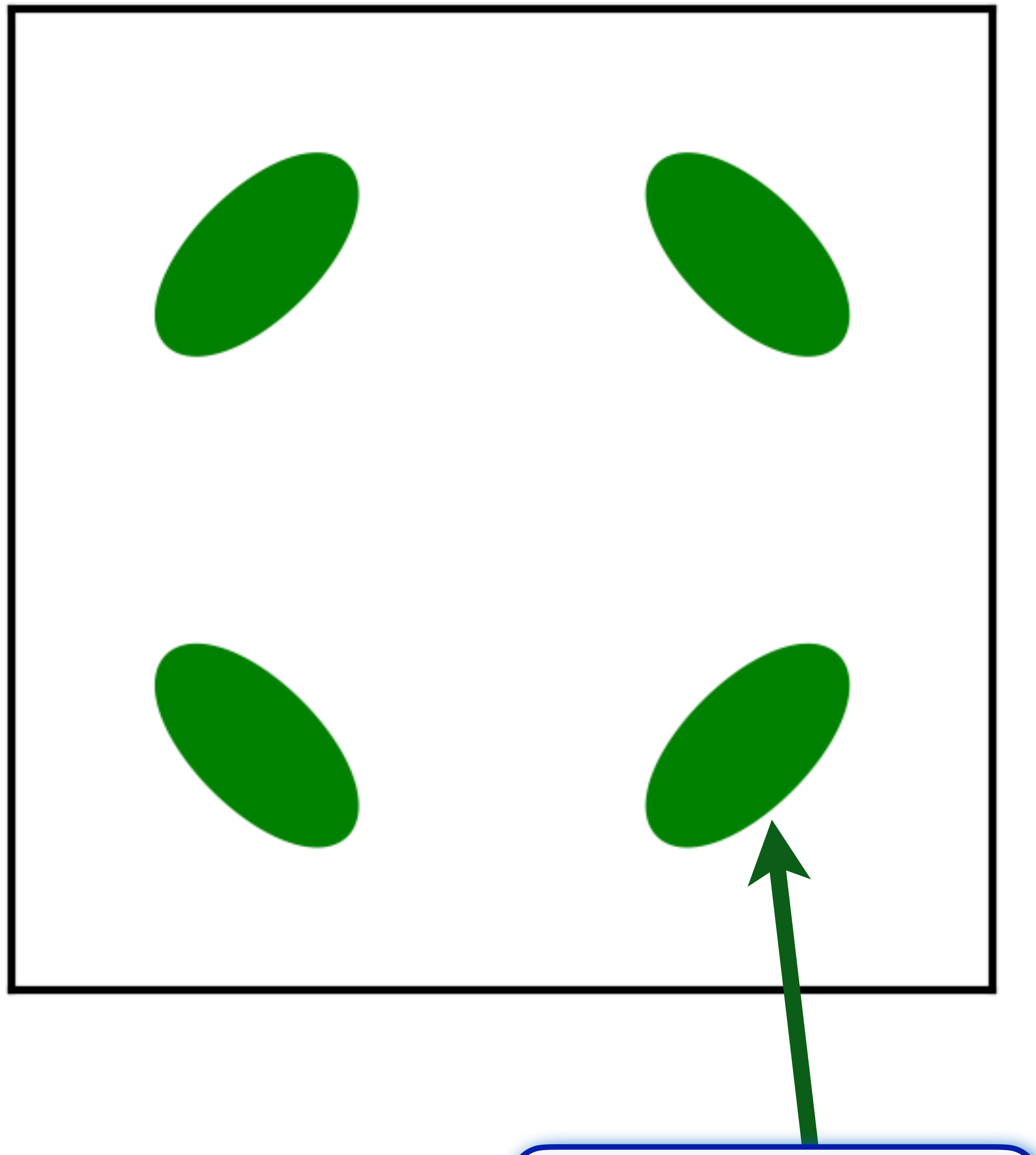

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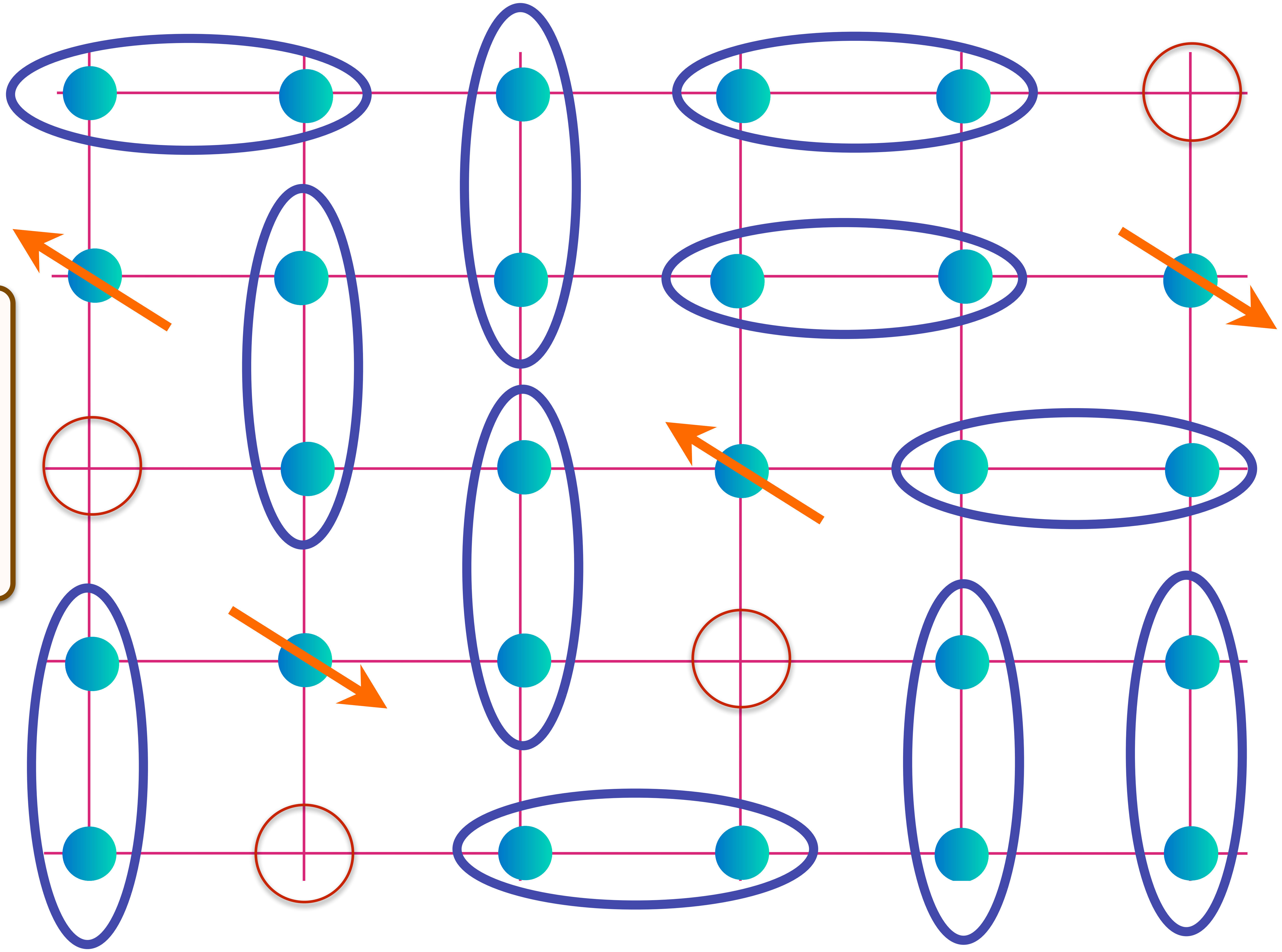
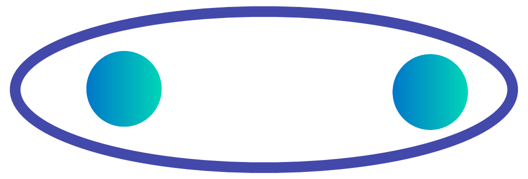
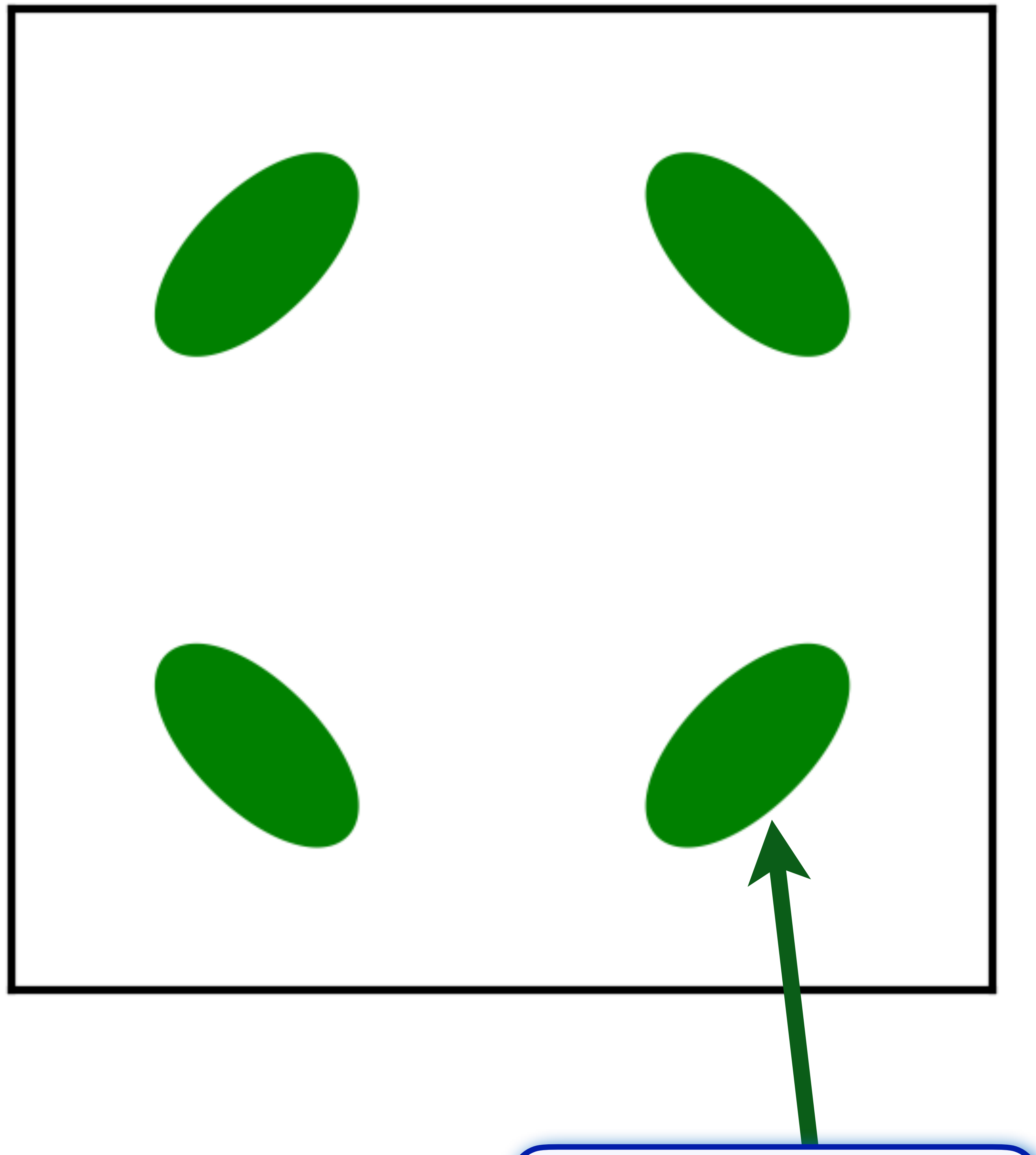


 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$



Area $p/4$

Doping an insulating antiferromagnet with holes of density p

Holon metal excited states

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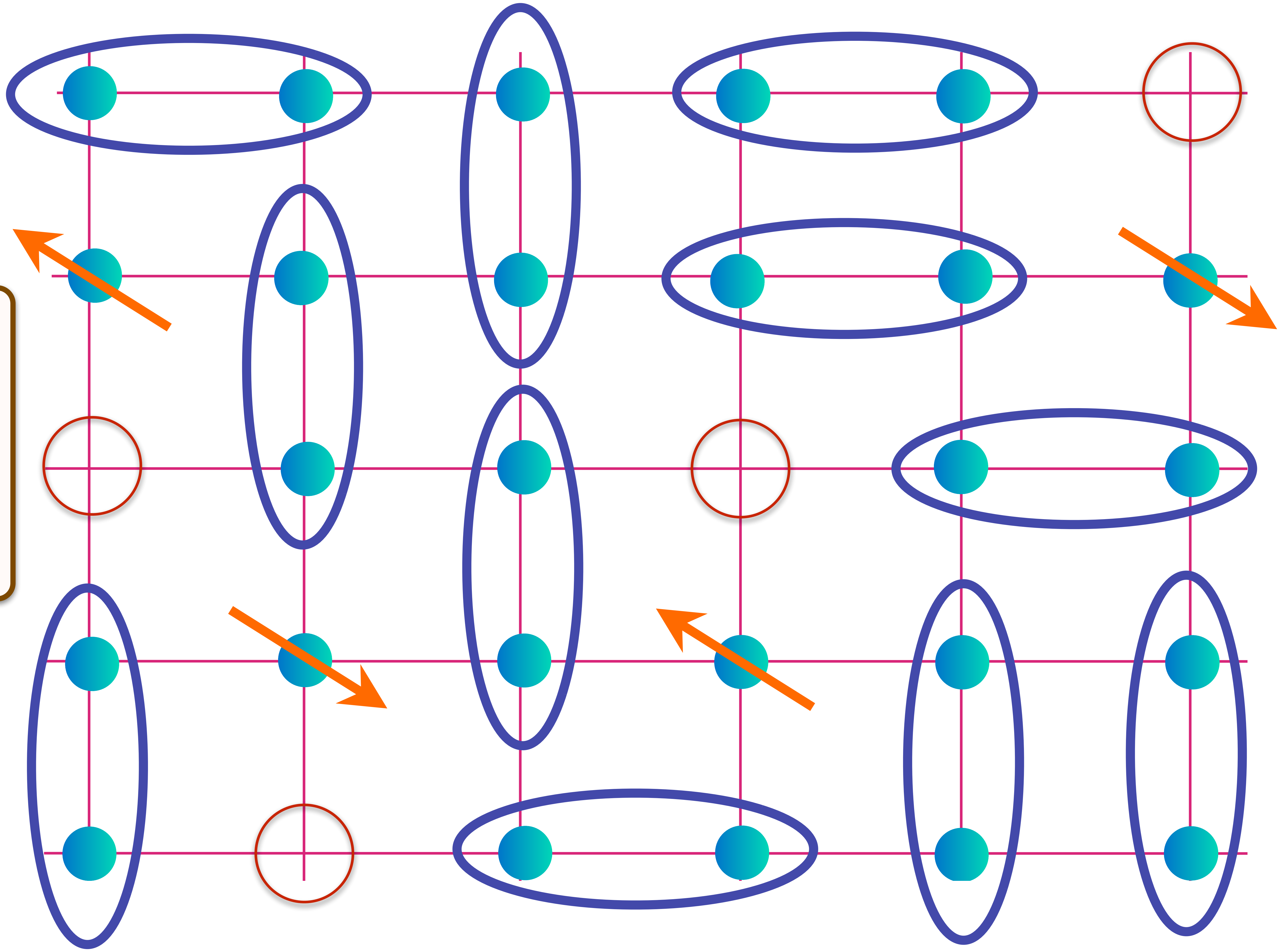

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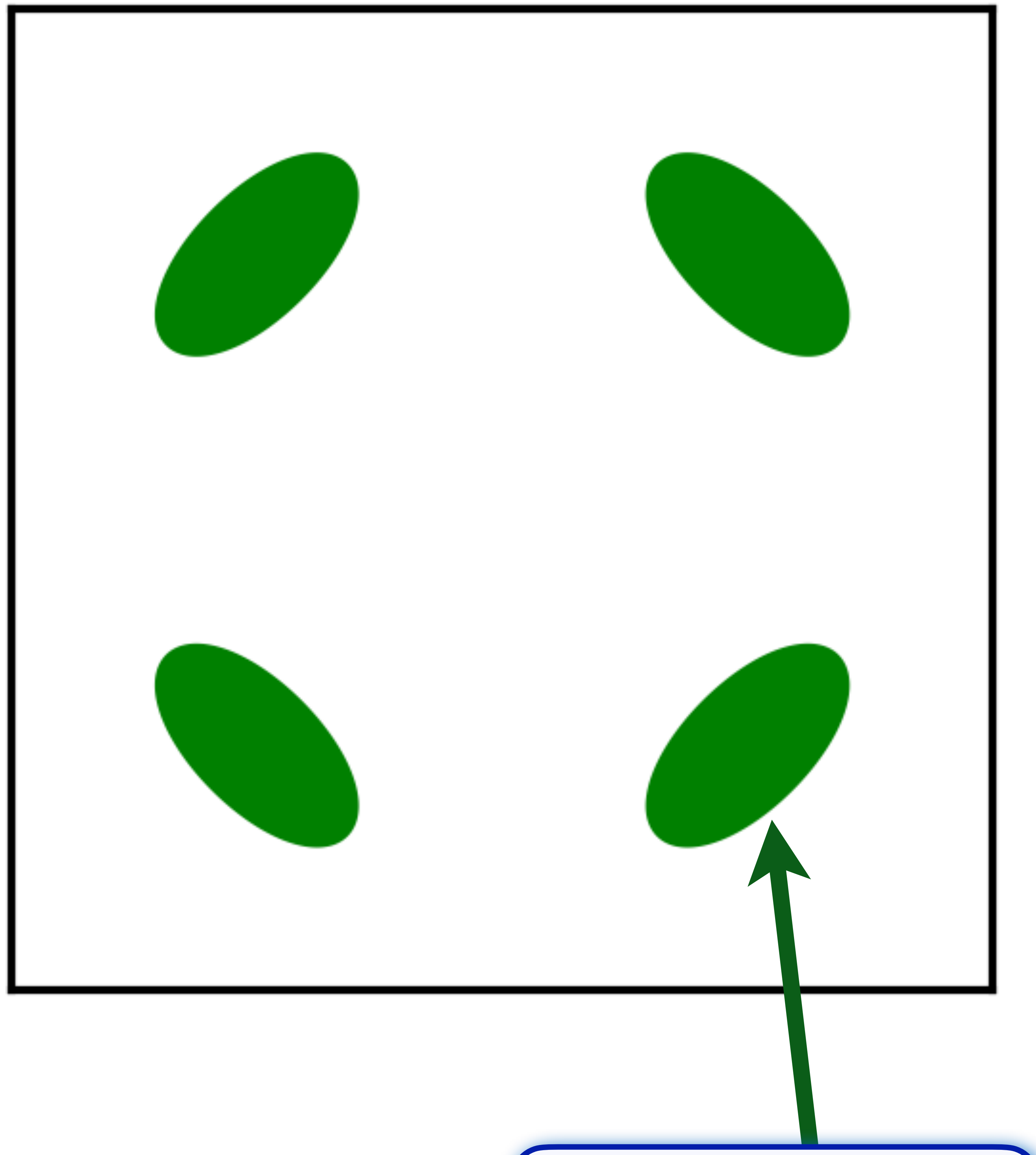

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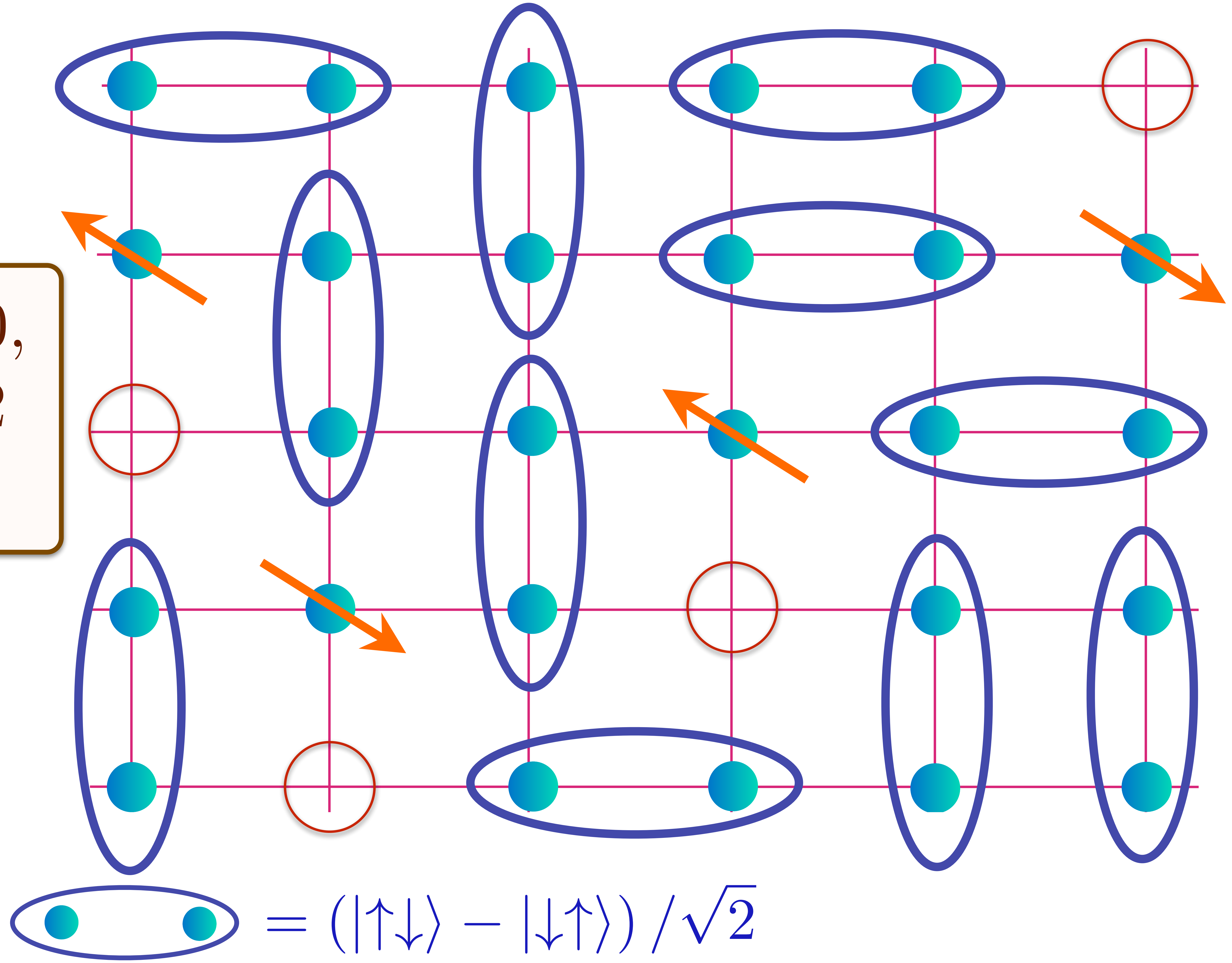

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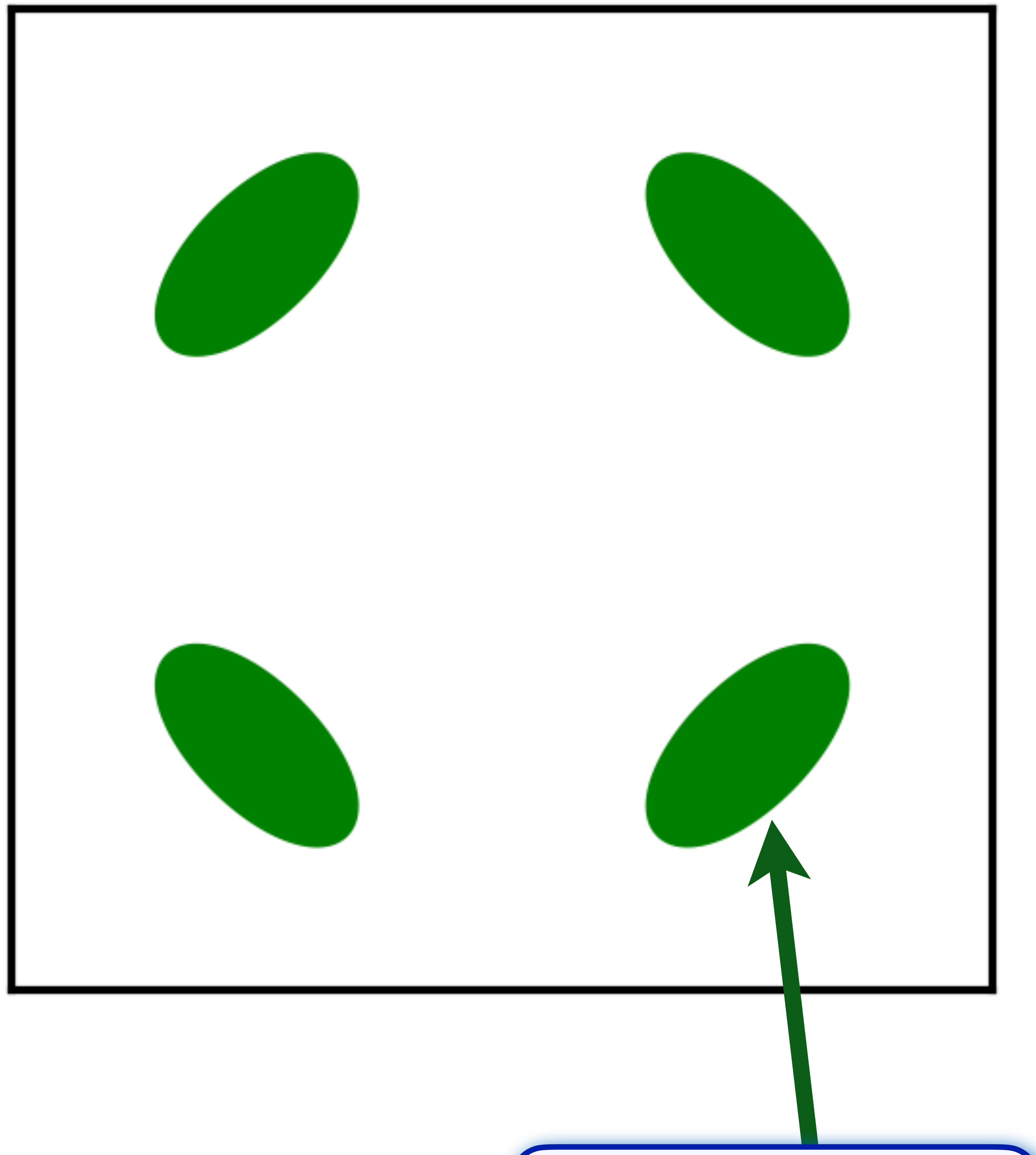

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FL*

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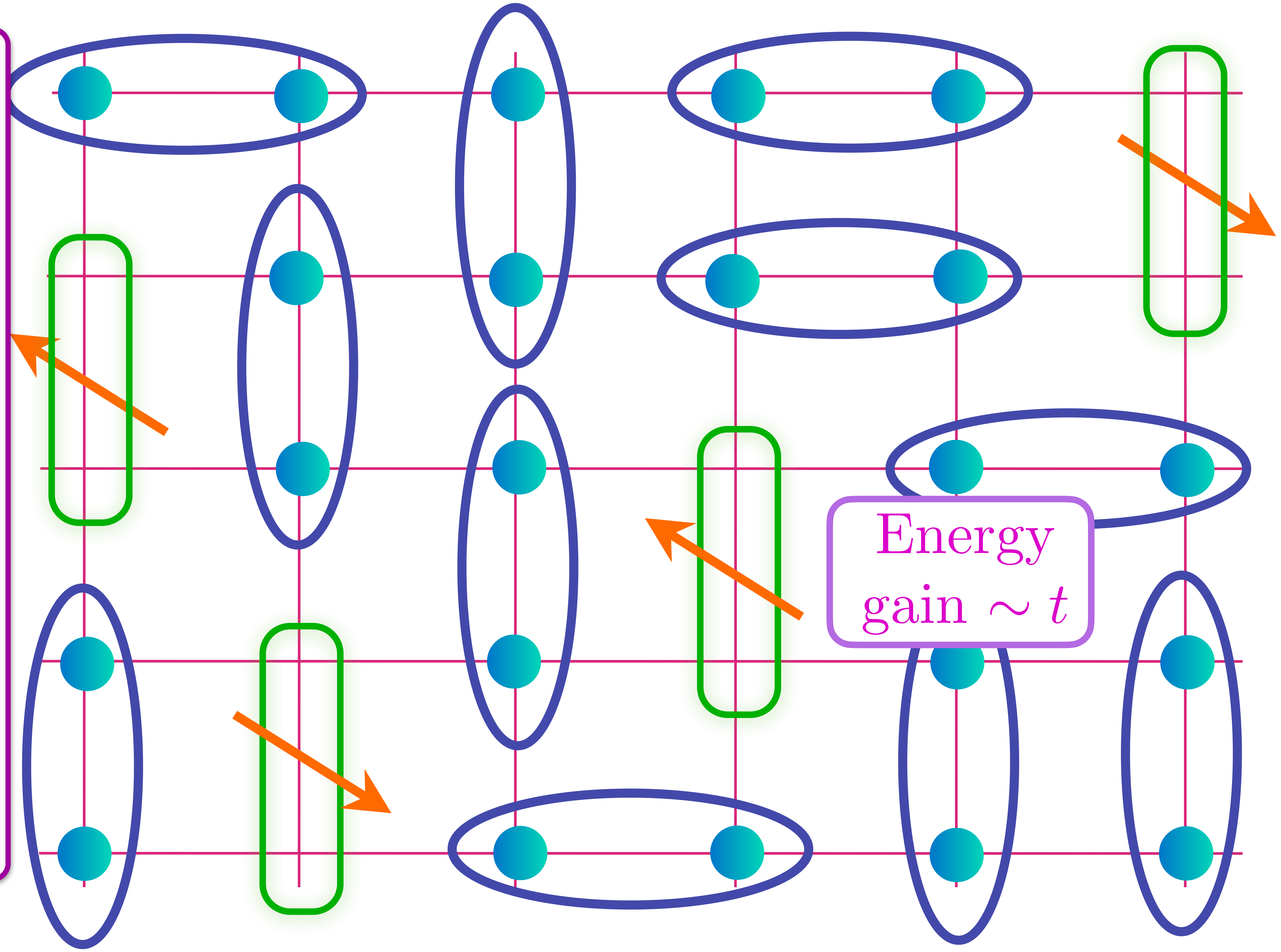
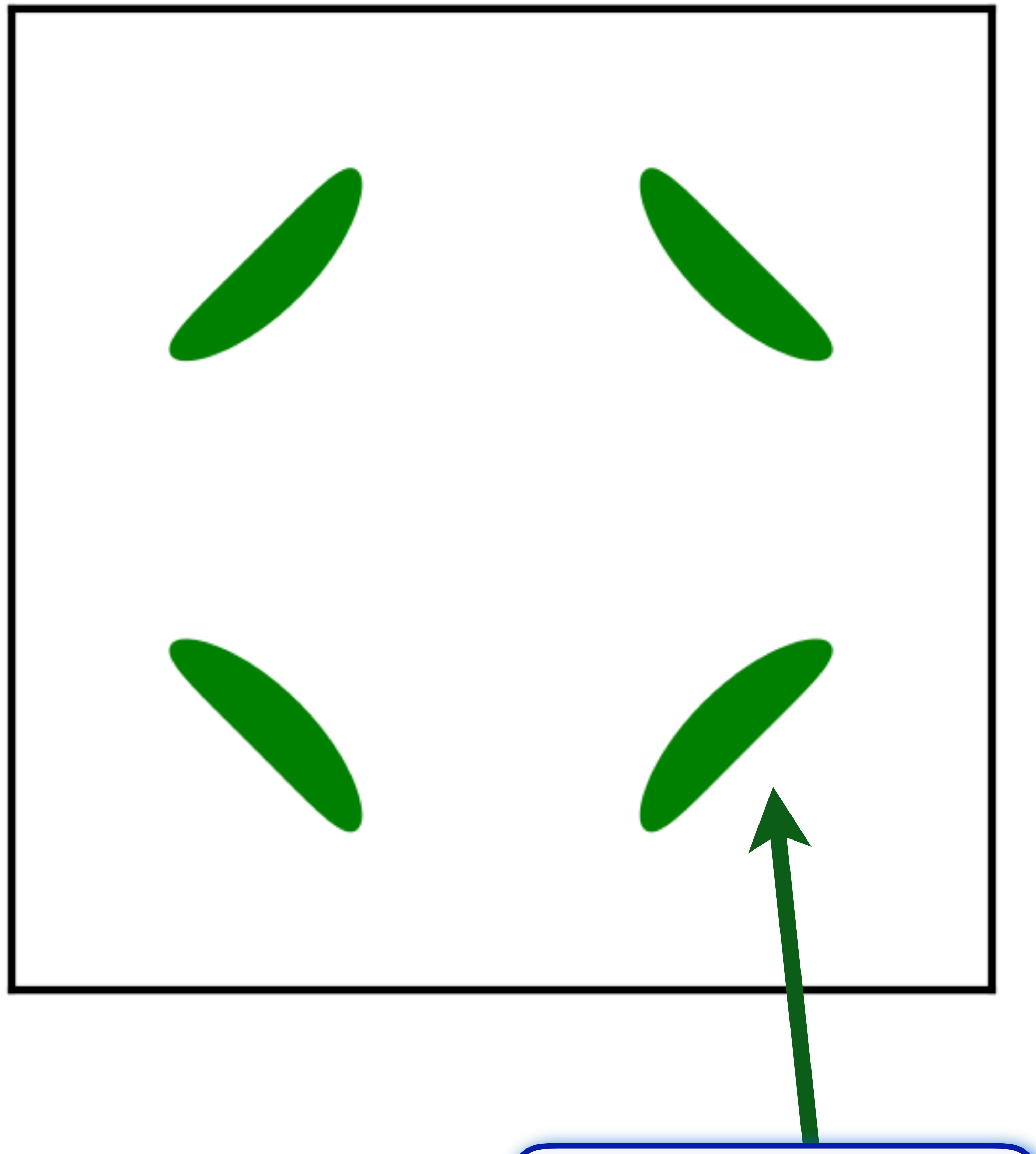
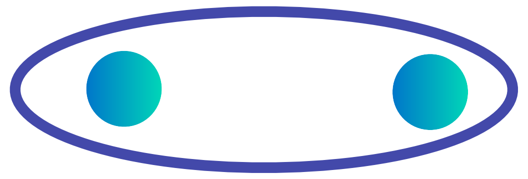
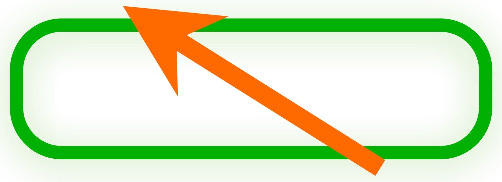




Area $p/8$

 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$  $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

Doping an insulating antiferromagnet with holes of density p

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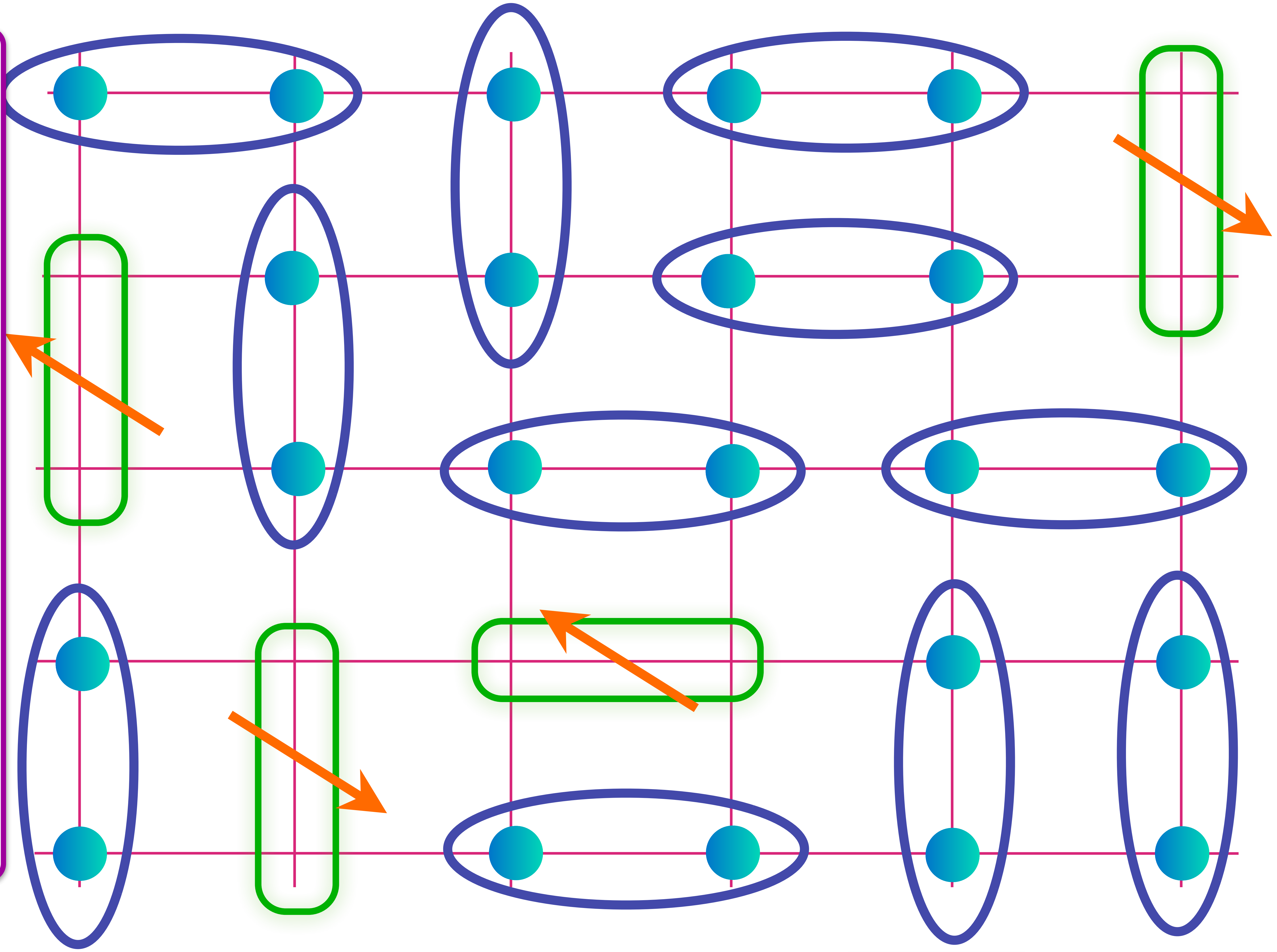

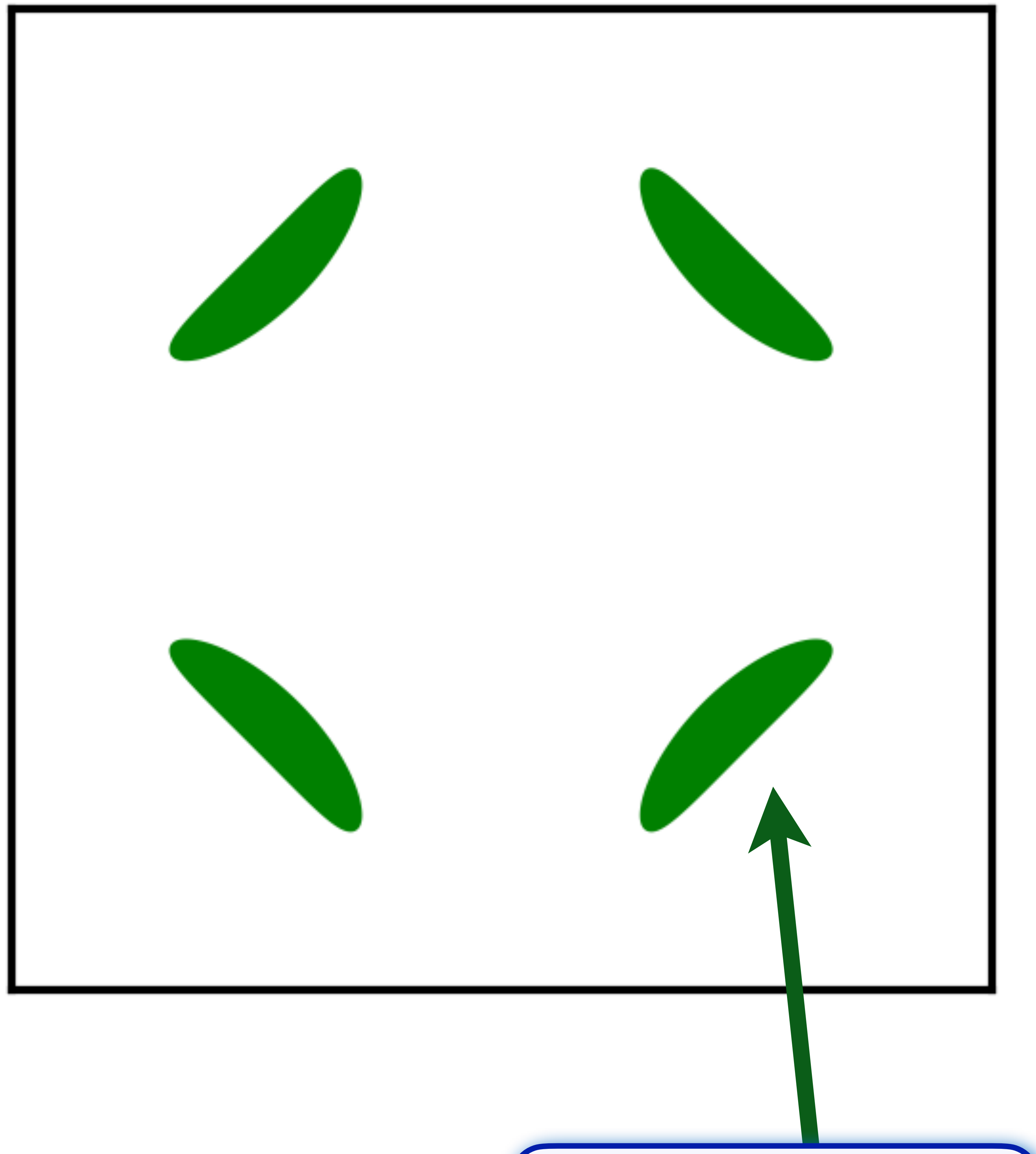

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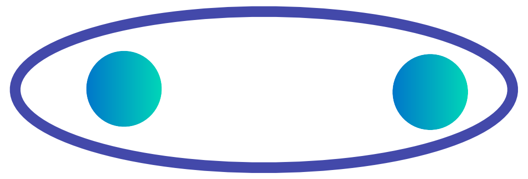
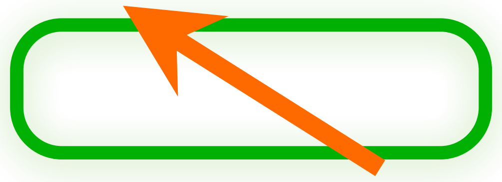
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

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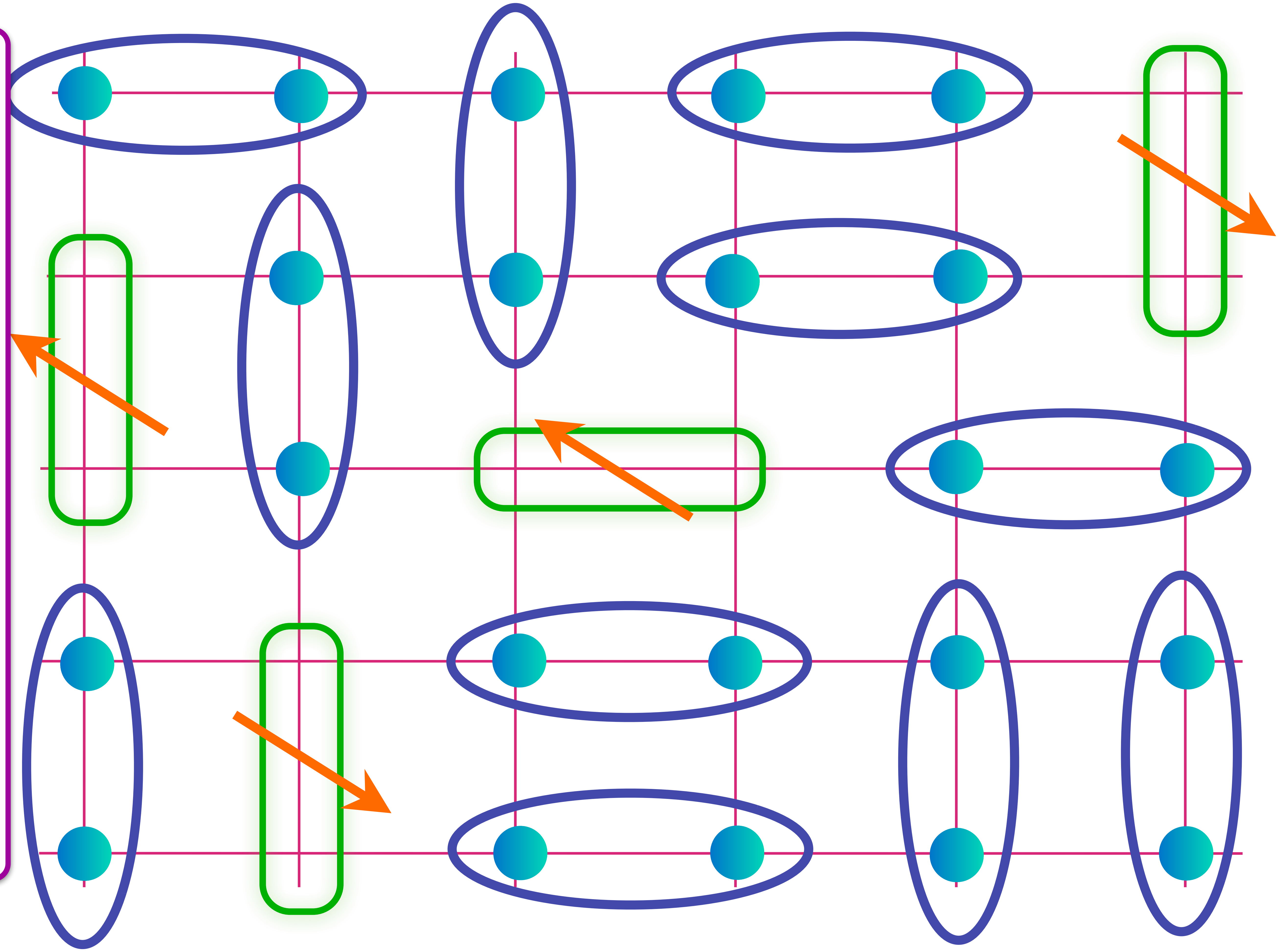

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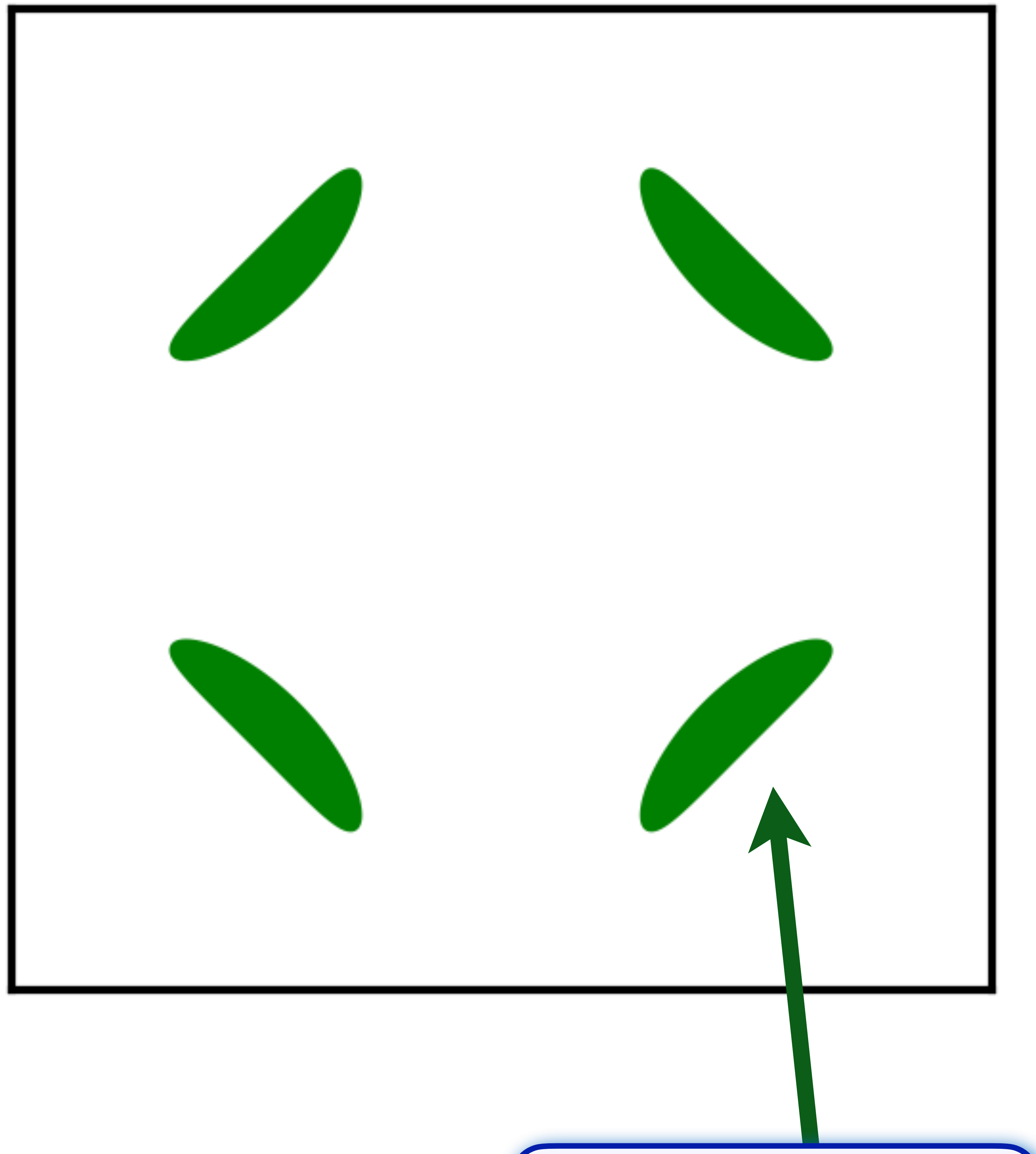

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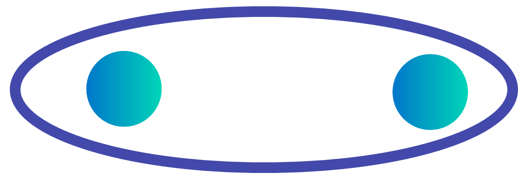
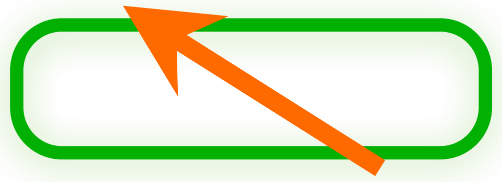
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$  $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

T. Senthil, S. S., M. Vojta, PRL **90**, 216403 (2003); R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
M. Punk, A. Allais, and S. Sachdev, PNAS **112**, 9552 (2015)

Doping an insulating antiferromagnet with holes of density p

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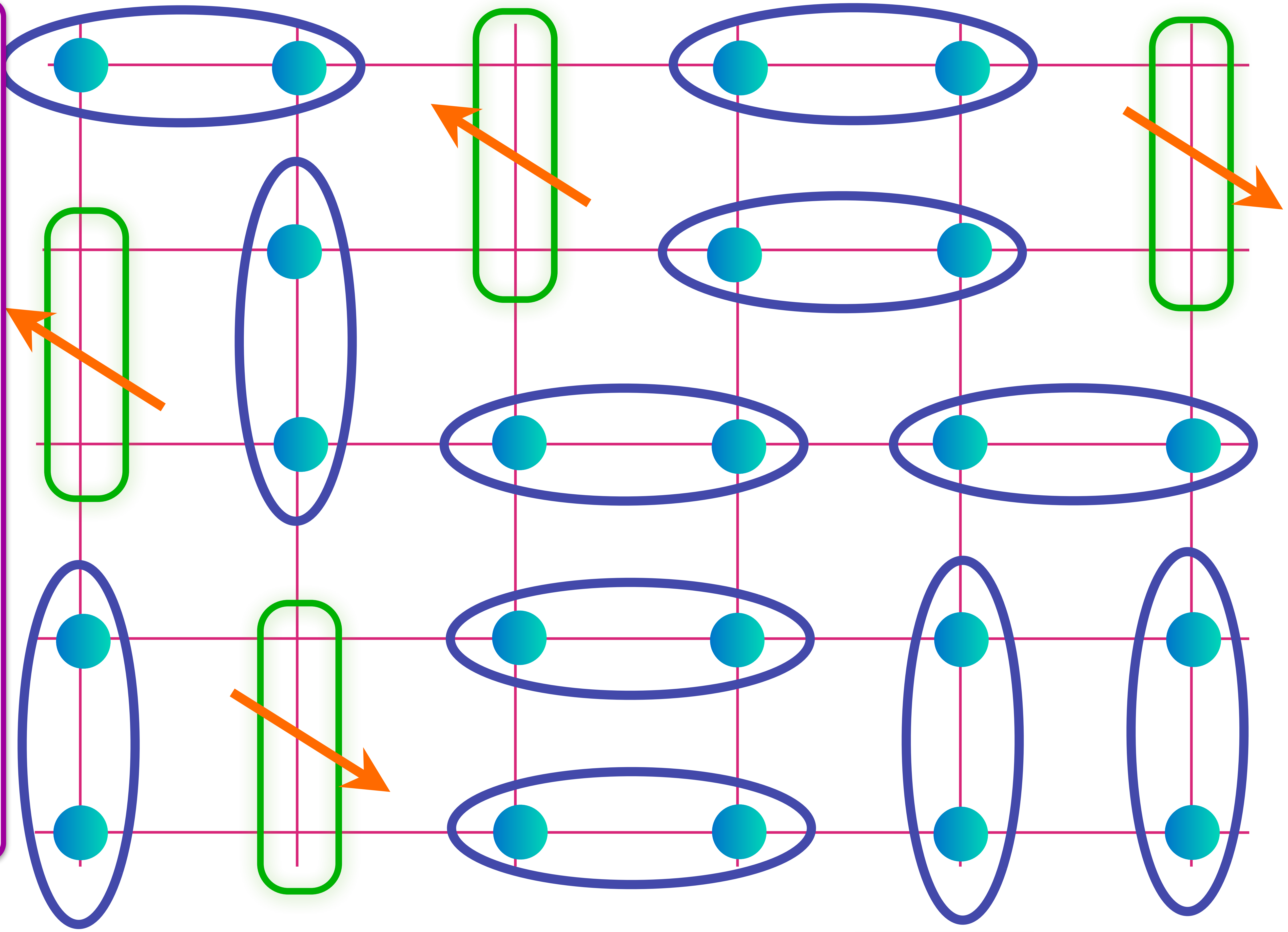
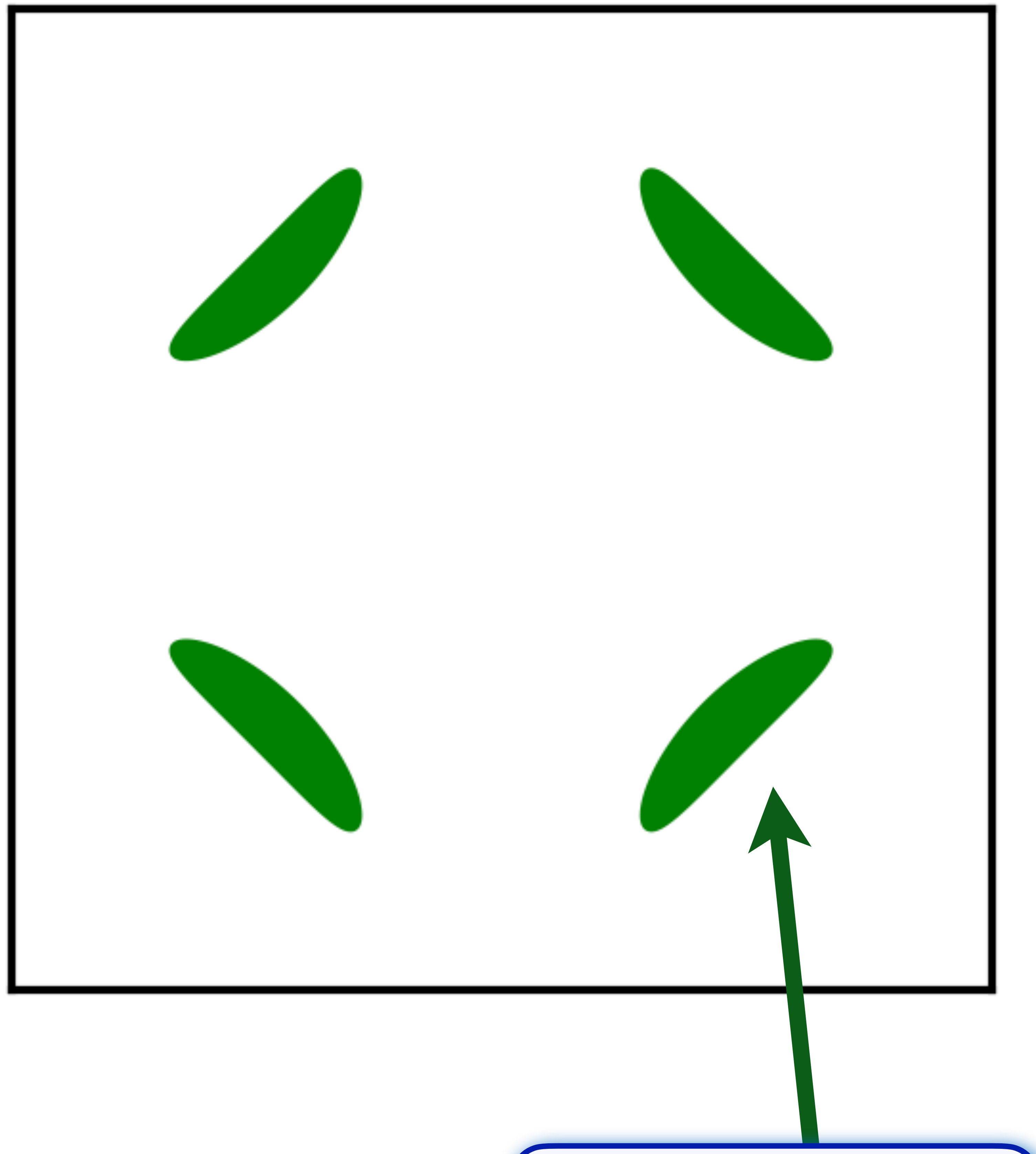


Oshikawa anomaly is satisfied by sum of spin liquid (1) and Fermi surface anomalies (p)



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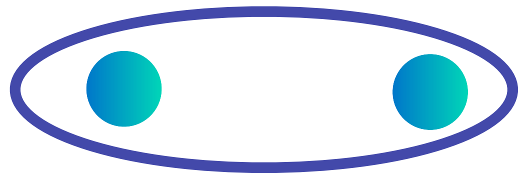
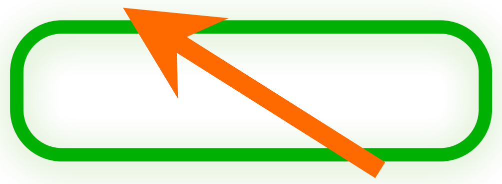
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$

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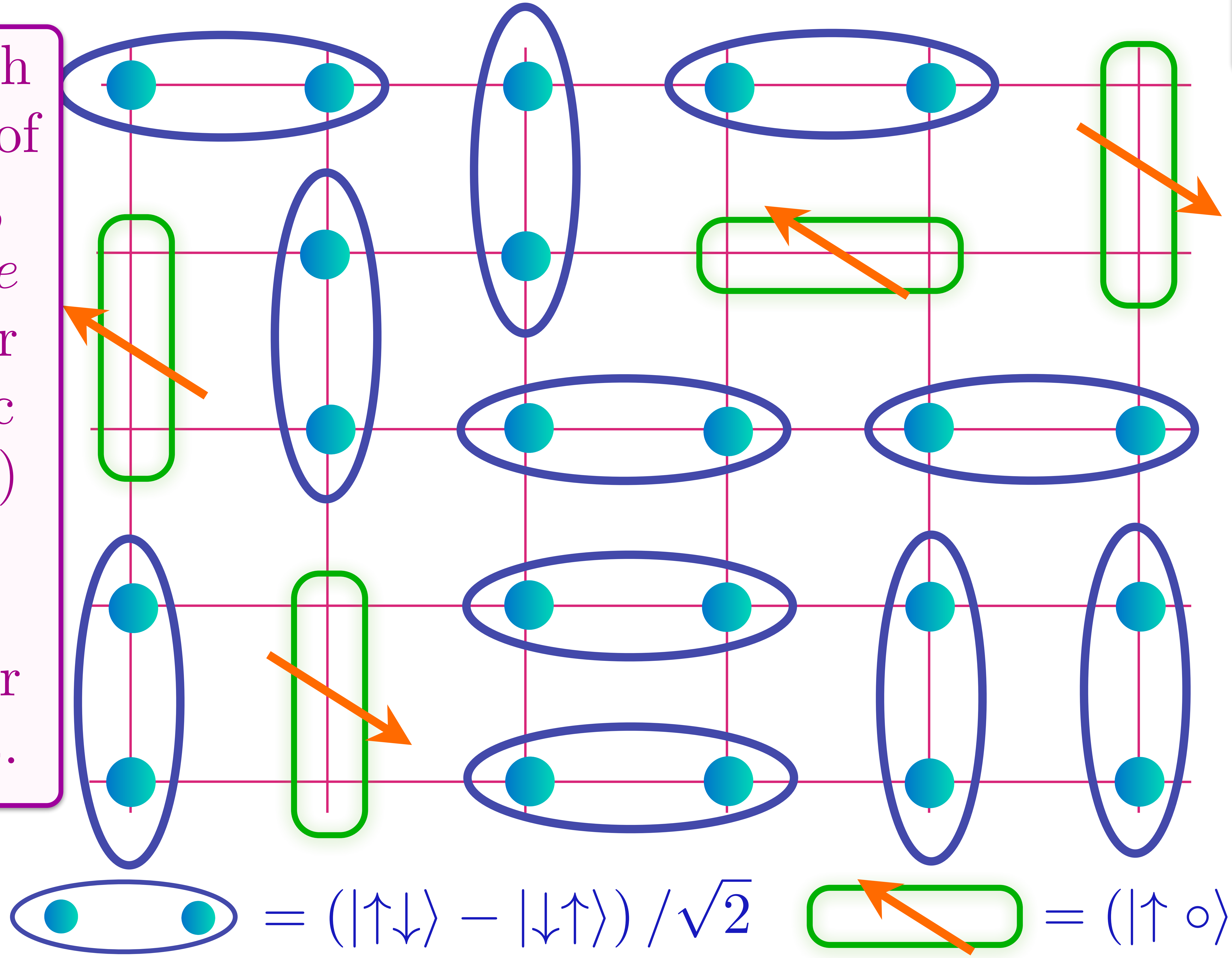

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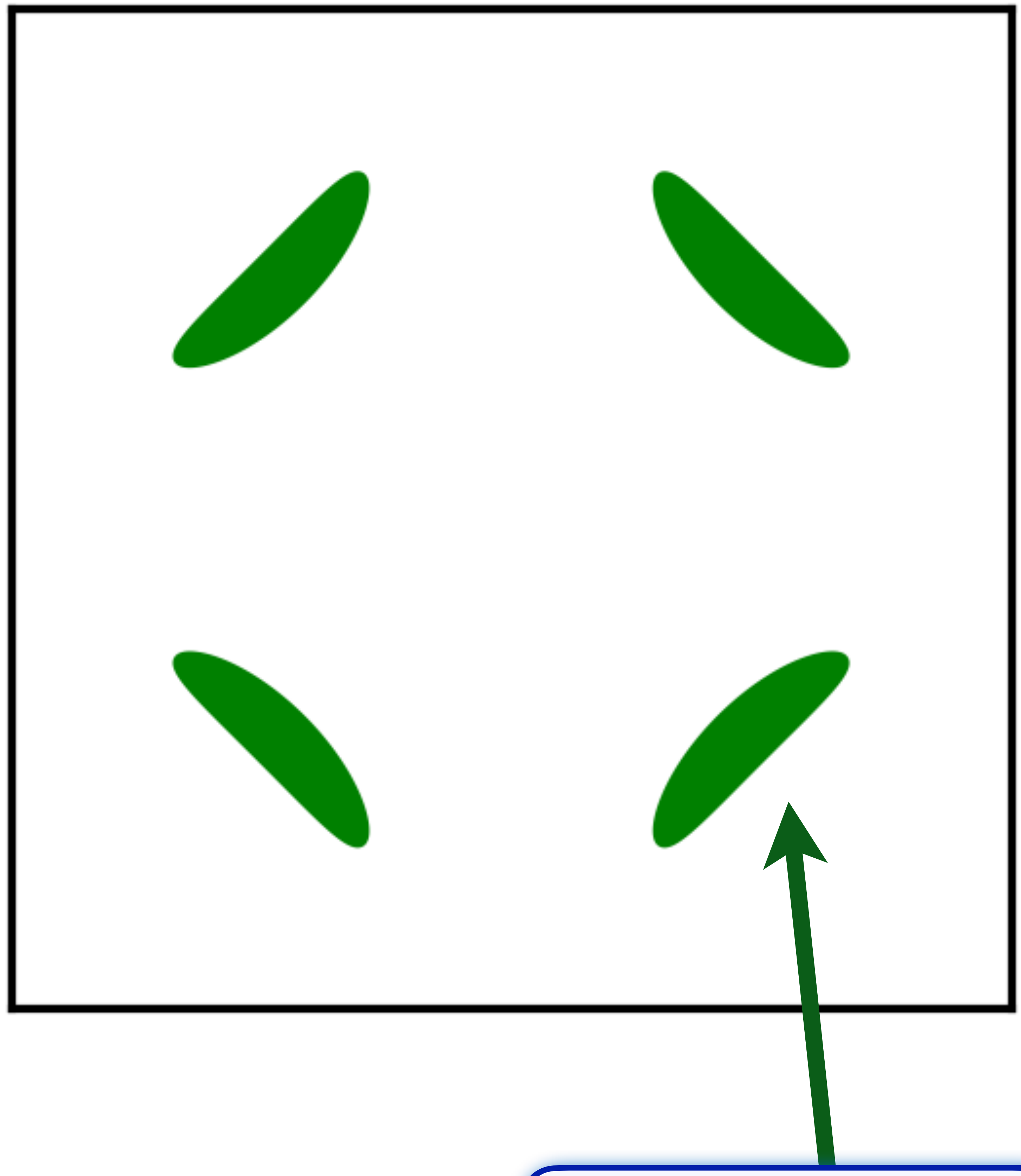

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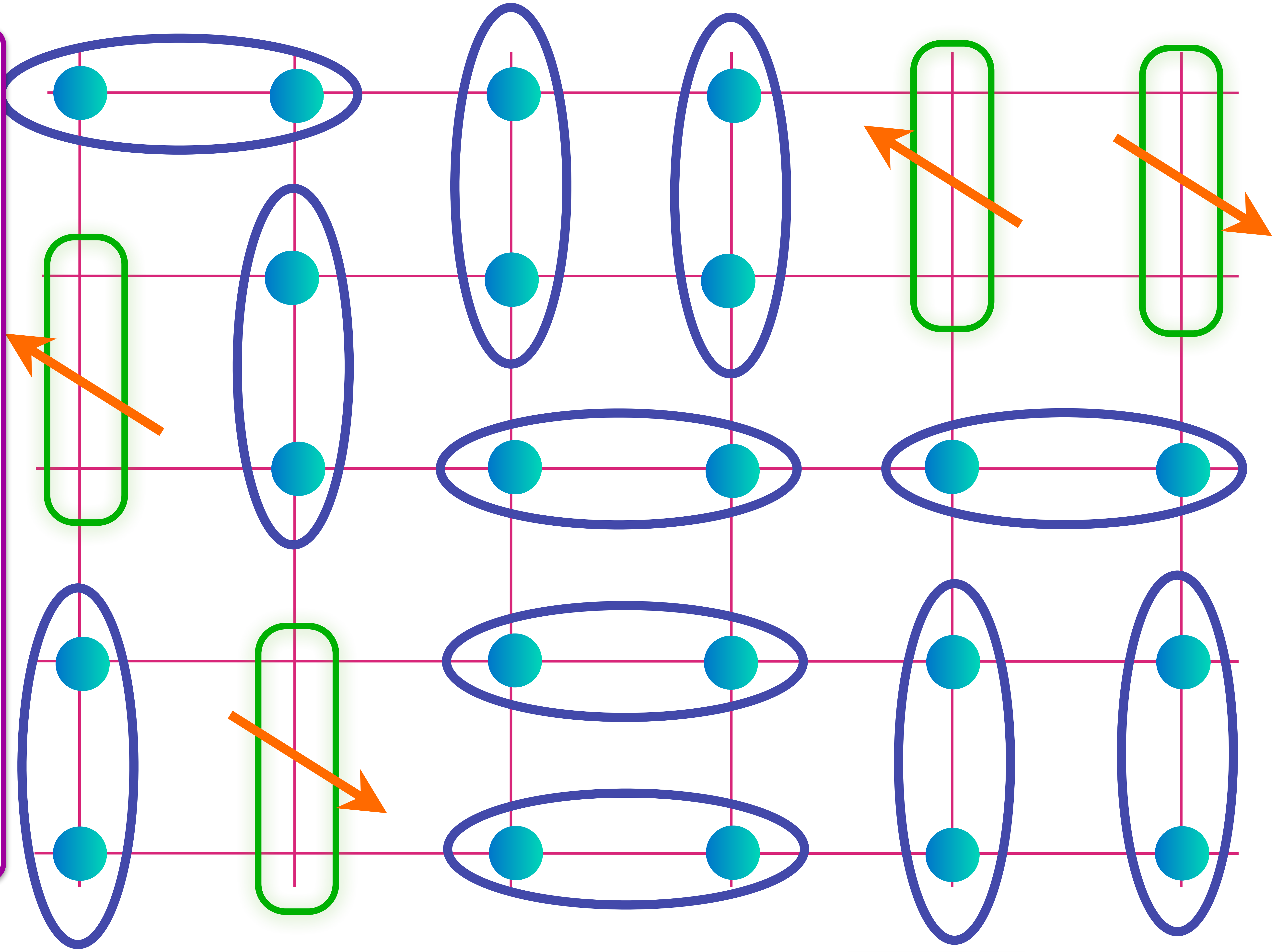

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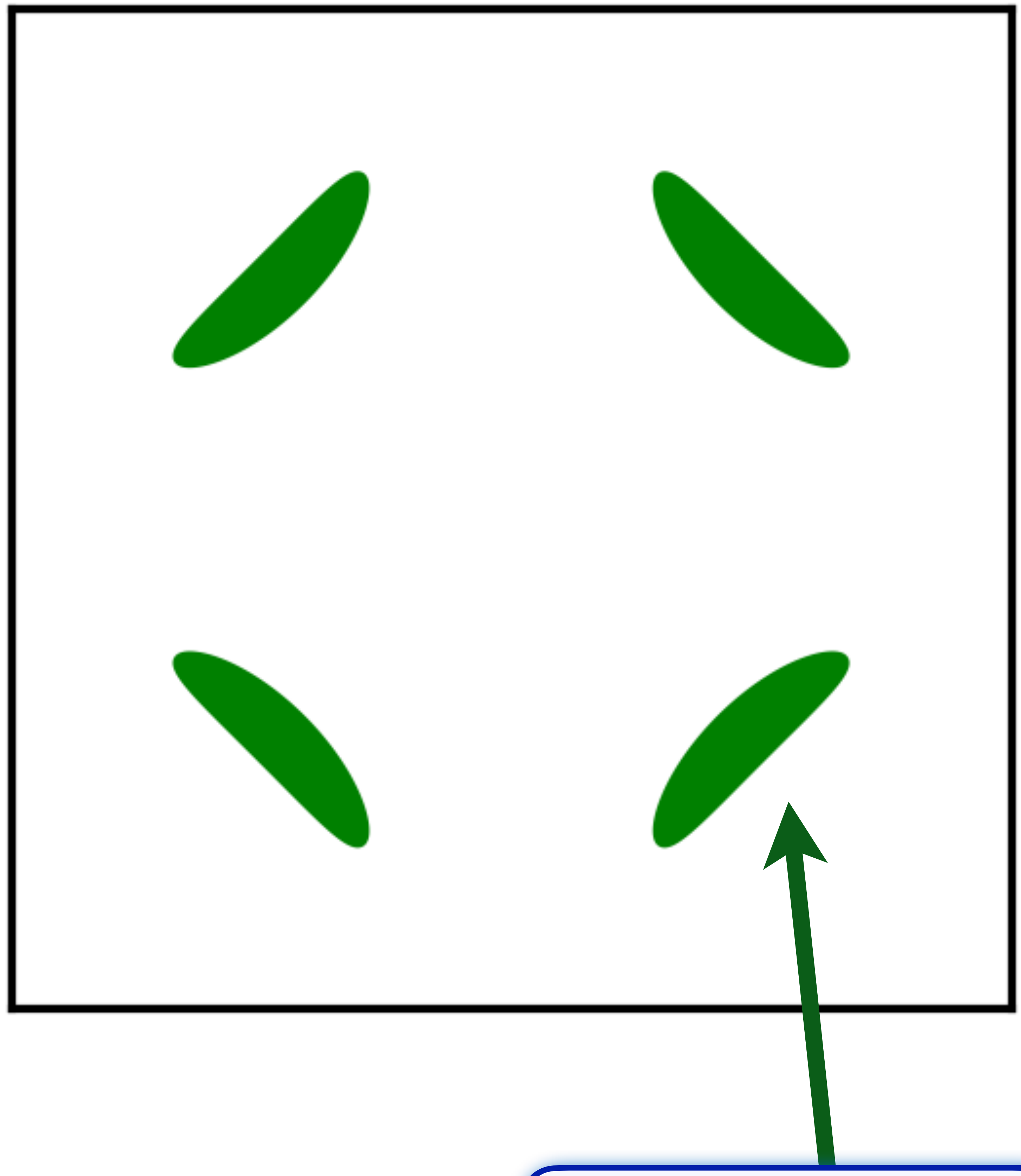

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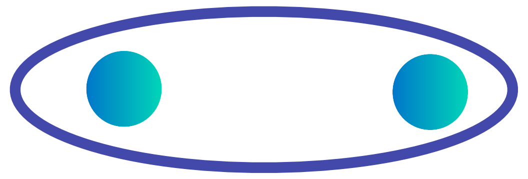
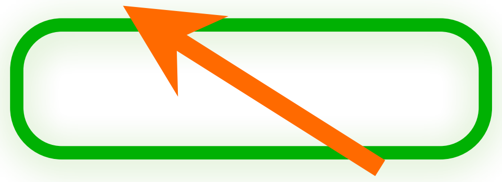
 $= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$

 $= \left(|\uparrow\circ\rangle + |\circ\uparrow\rangle\right)/\sqrt{2}$

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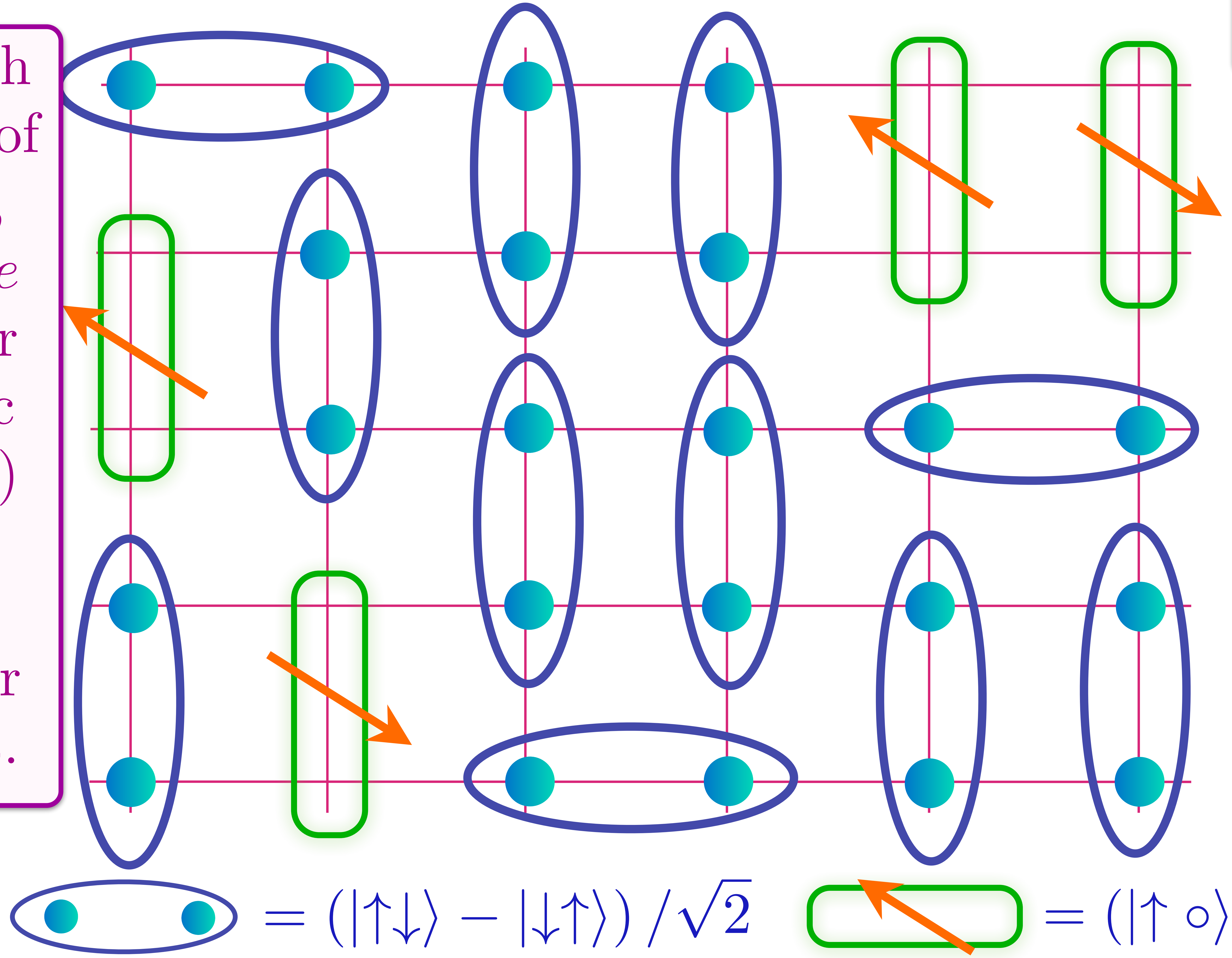

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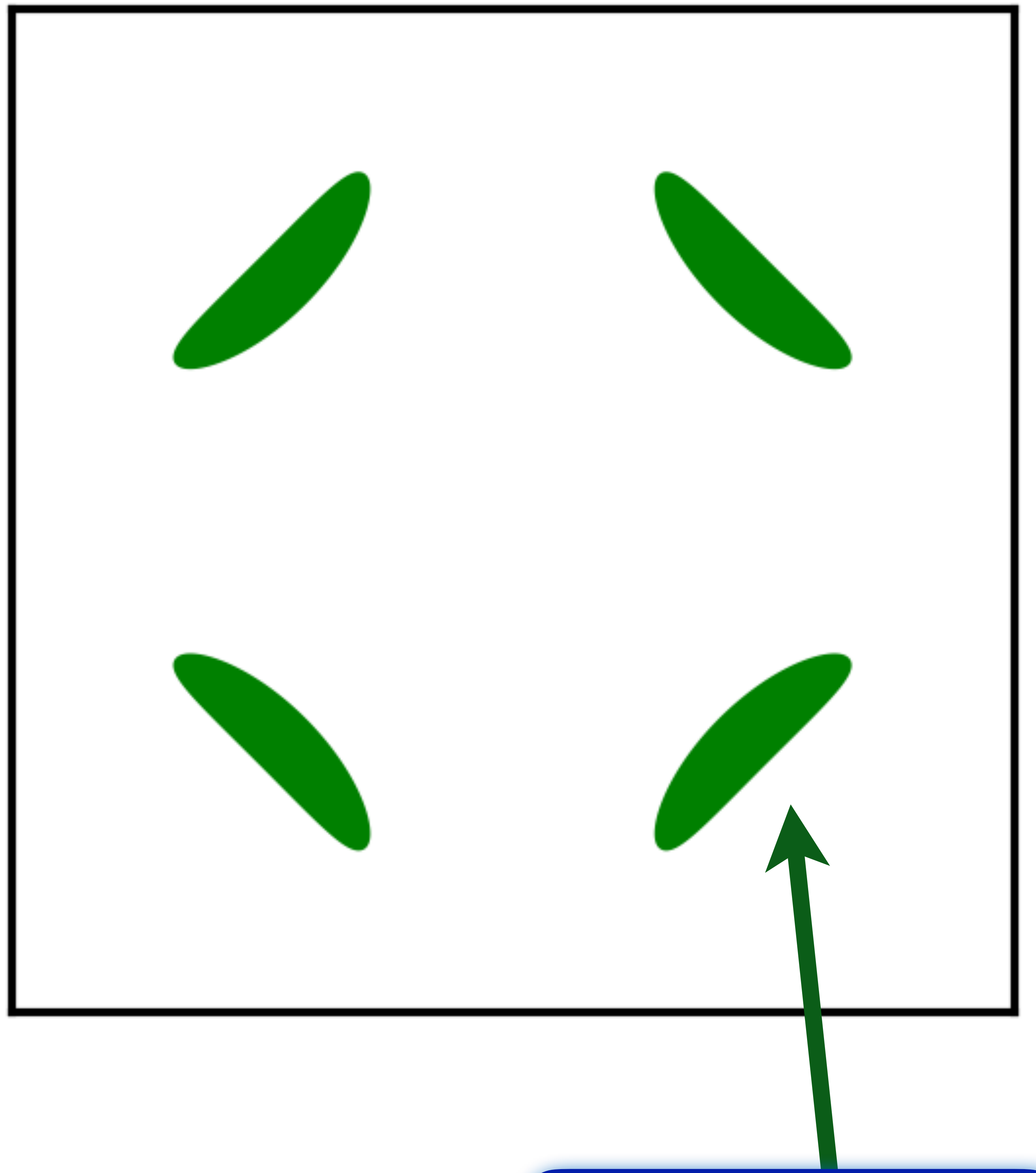

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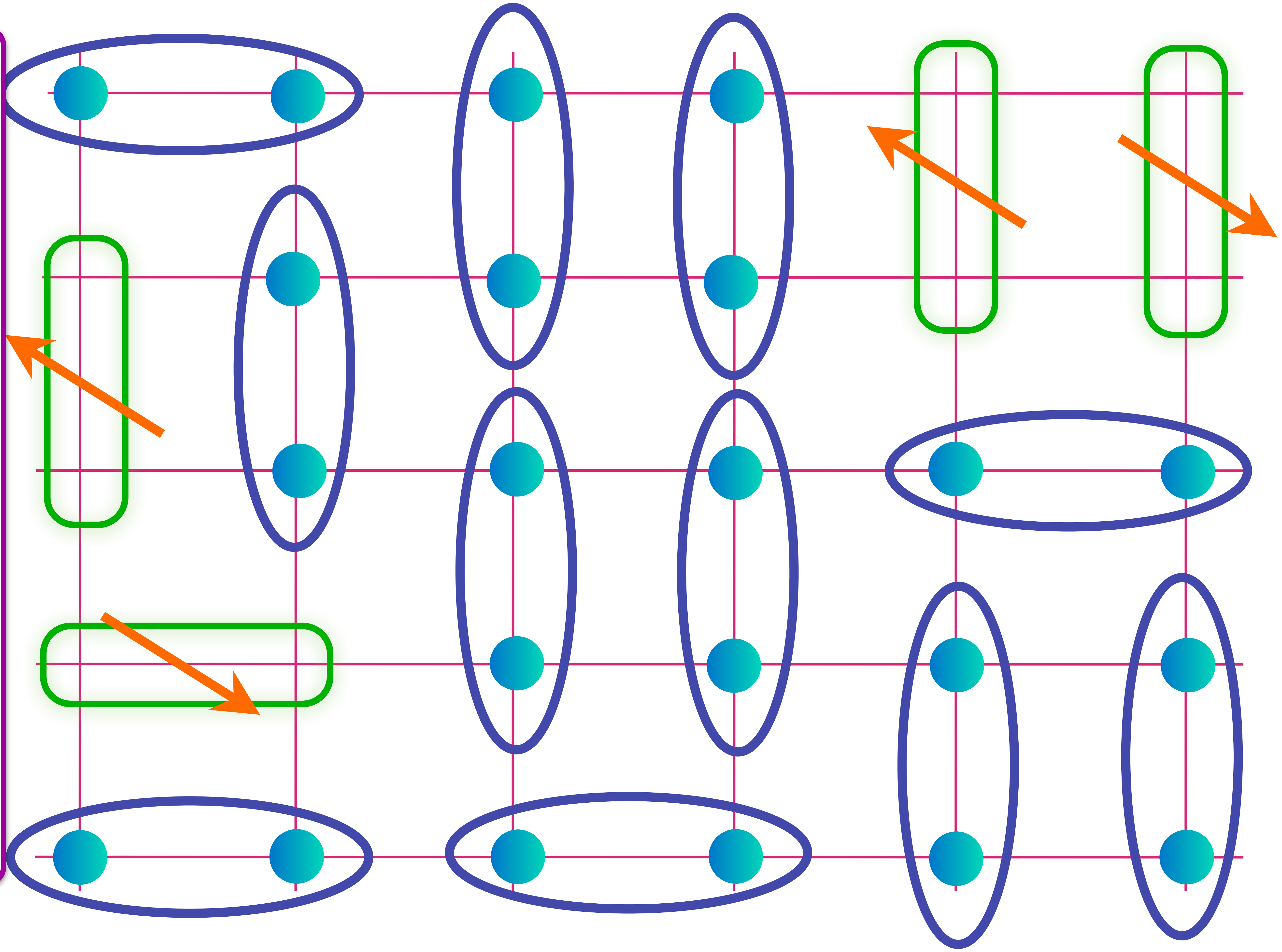

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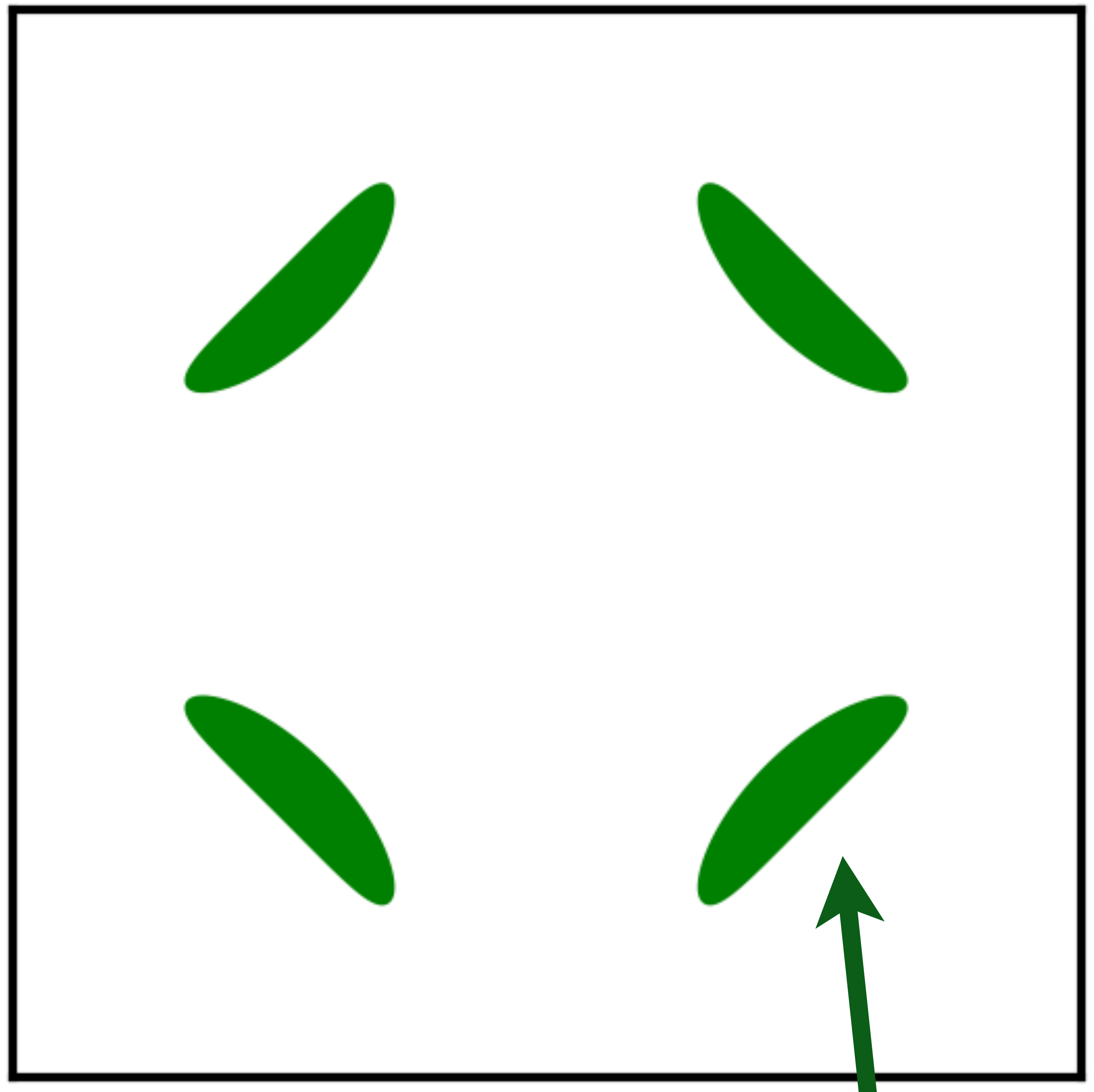

Area $p/8$

$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle \right) / \sqrt{2}$

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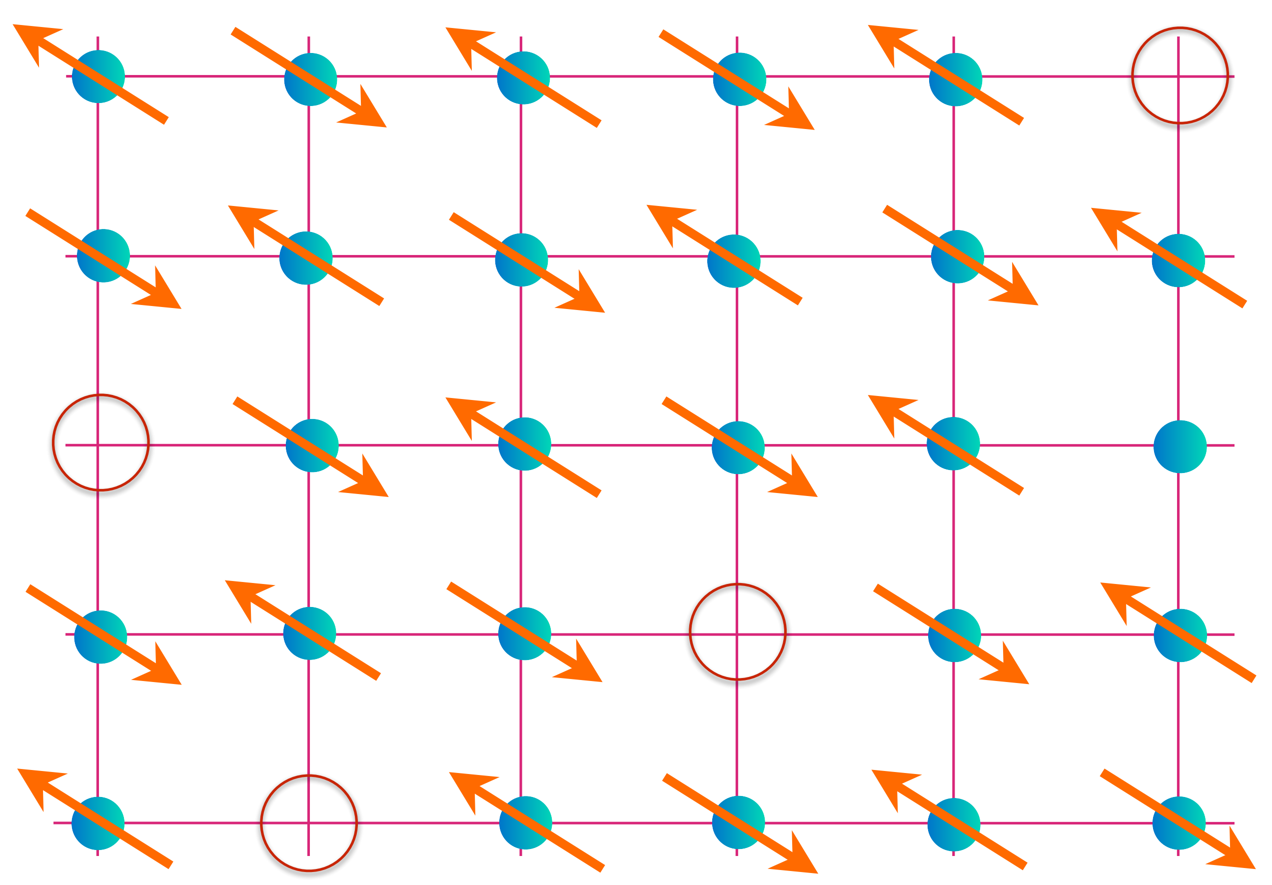
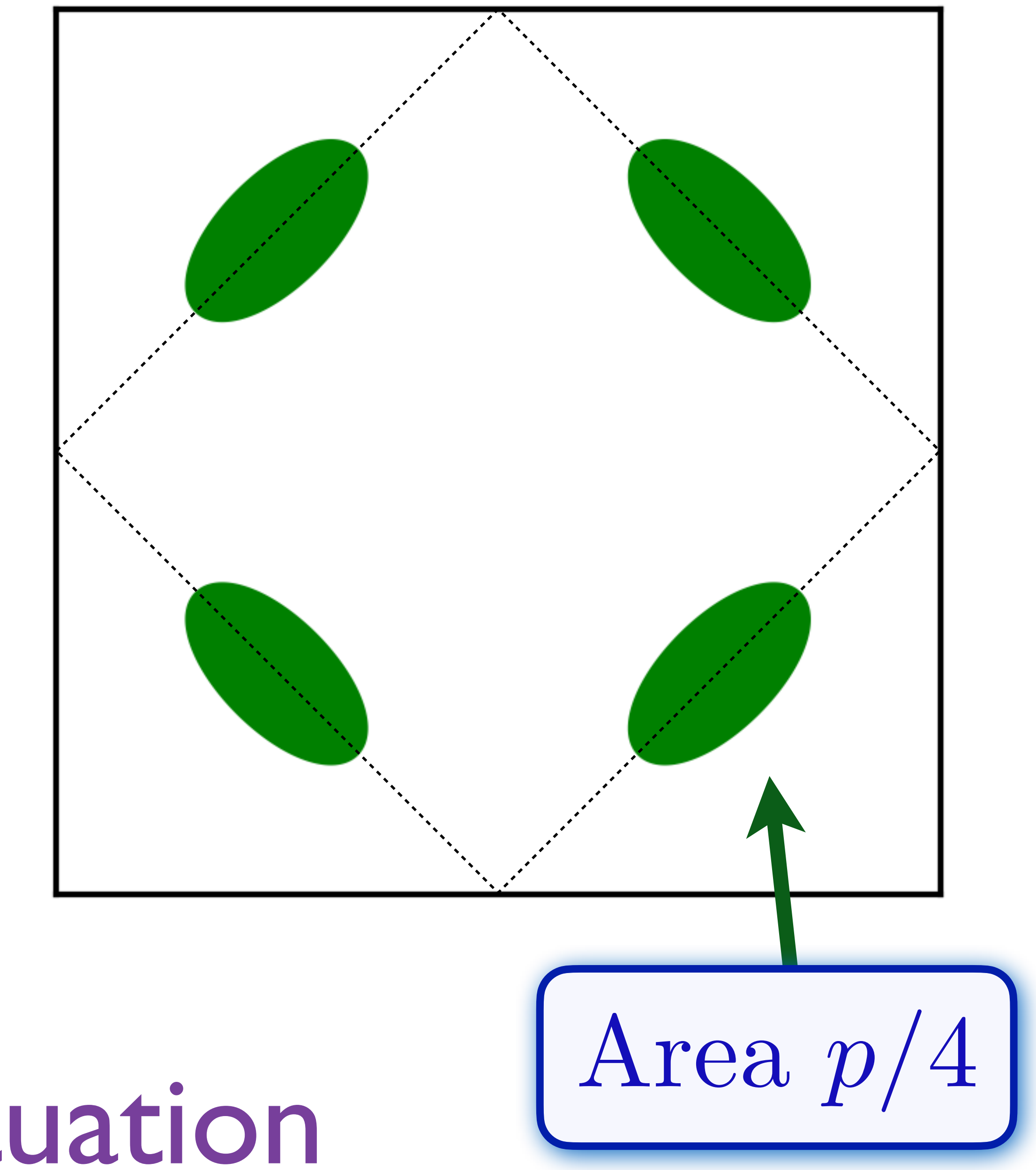
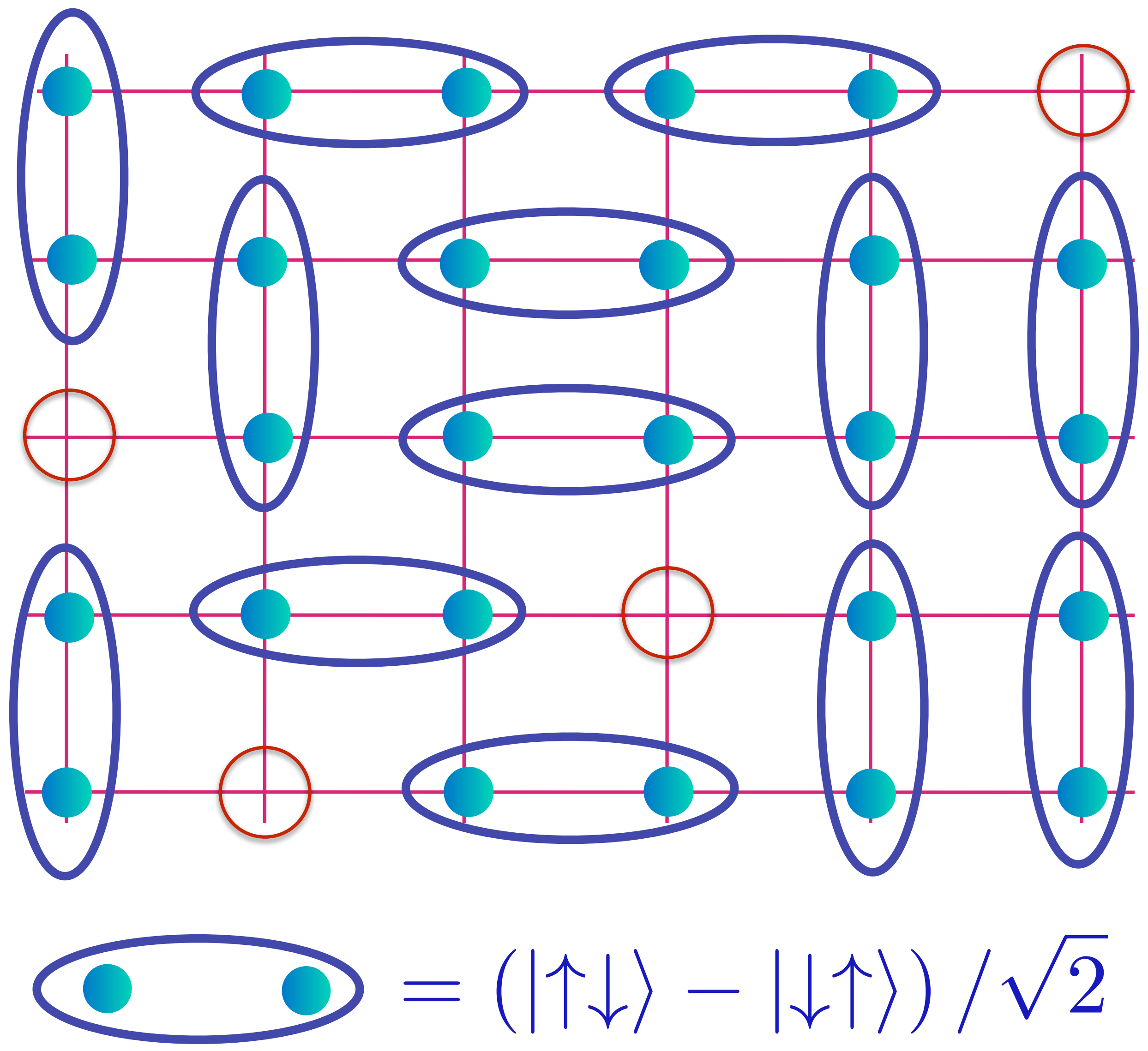
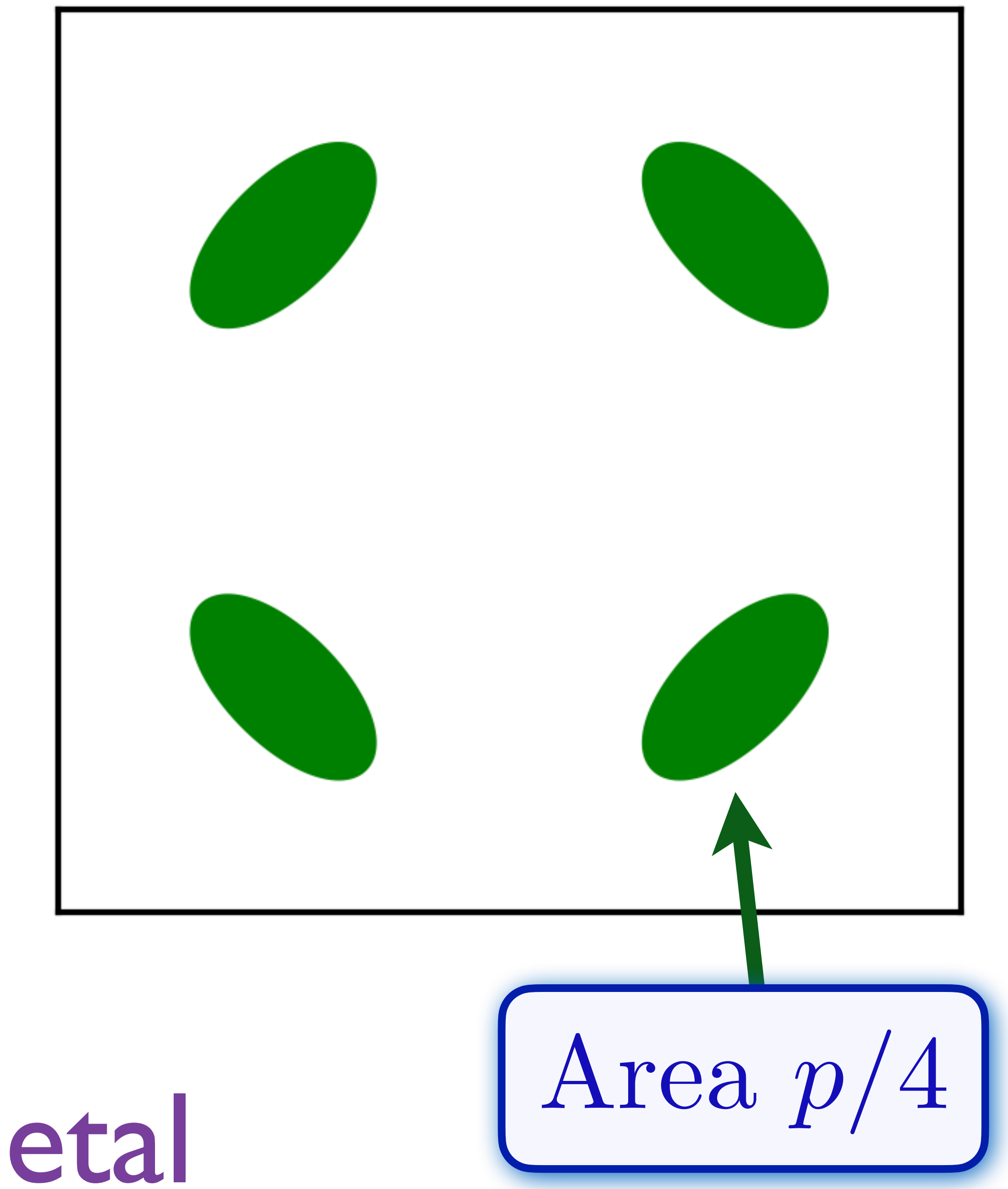




AF metal and SDW fluctuation





Holon metal

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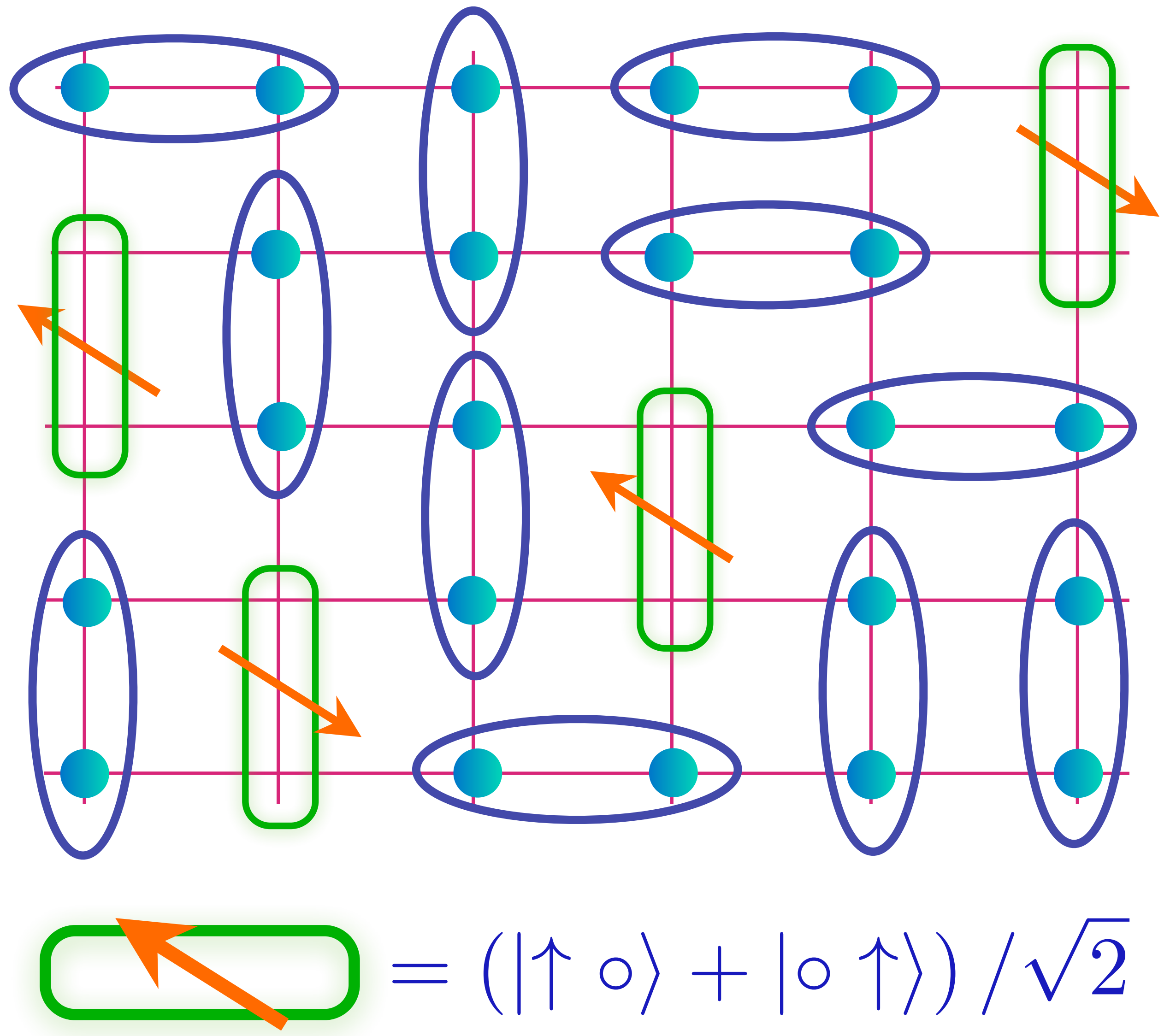
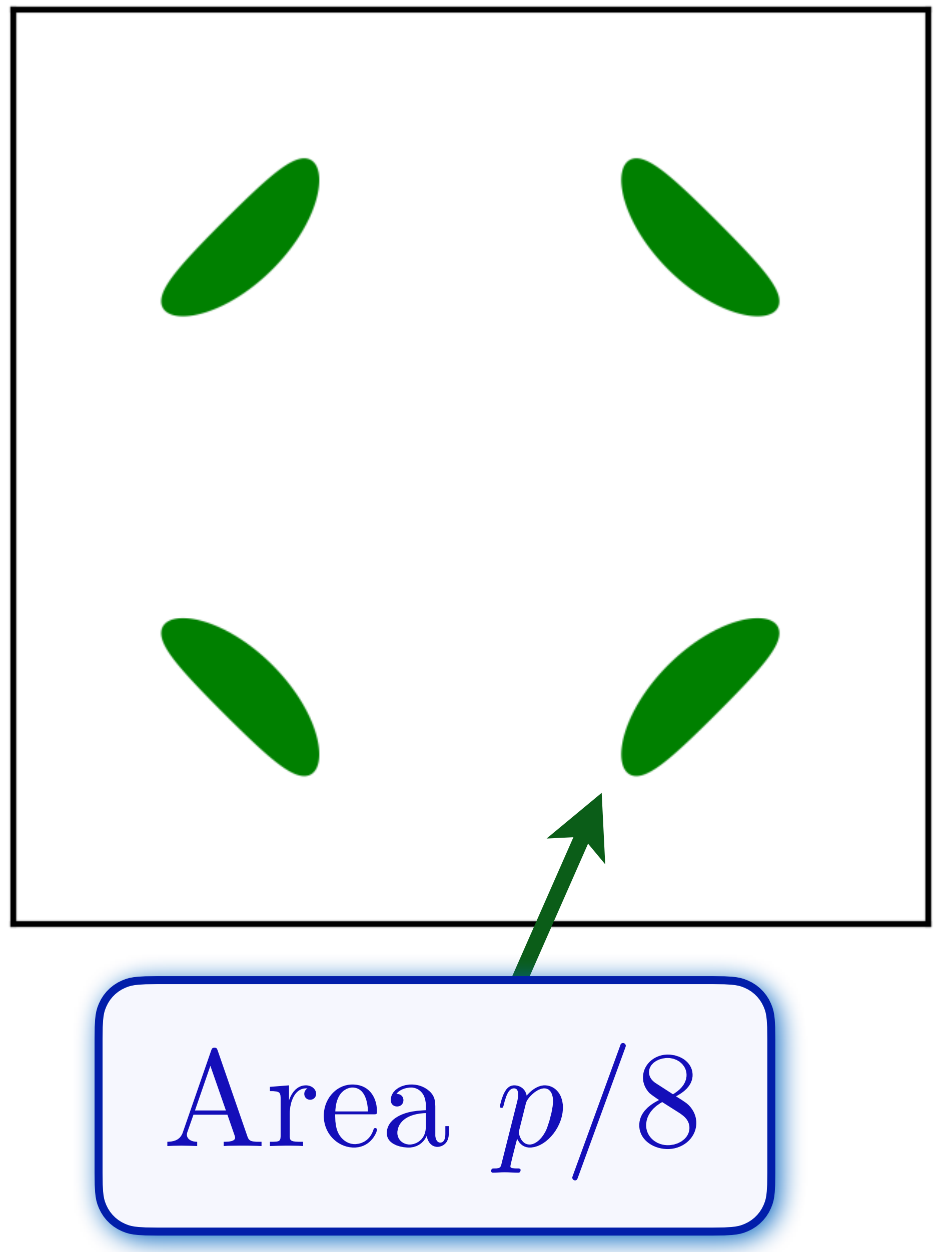
R. K. Kaul, A. Kolezhuk, M. Levin, S.S., T. Senthil, PRB **75**, 235122 (2007)
M. Punk, A. Allais, and S. S., PNAS **112**, 9552 (2015)
E. Mascot, A. Nikolaenko, M. Tikhanovskaya, Ya-Hui Zhang, D. K. Morr, S. S., PRB **105**, 075146 (2022)



FL*

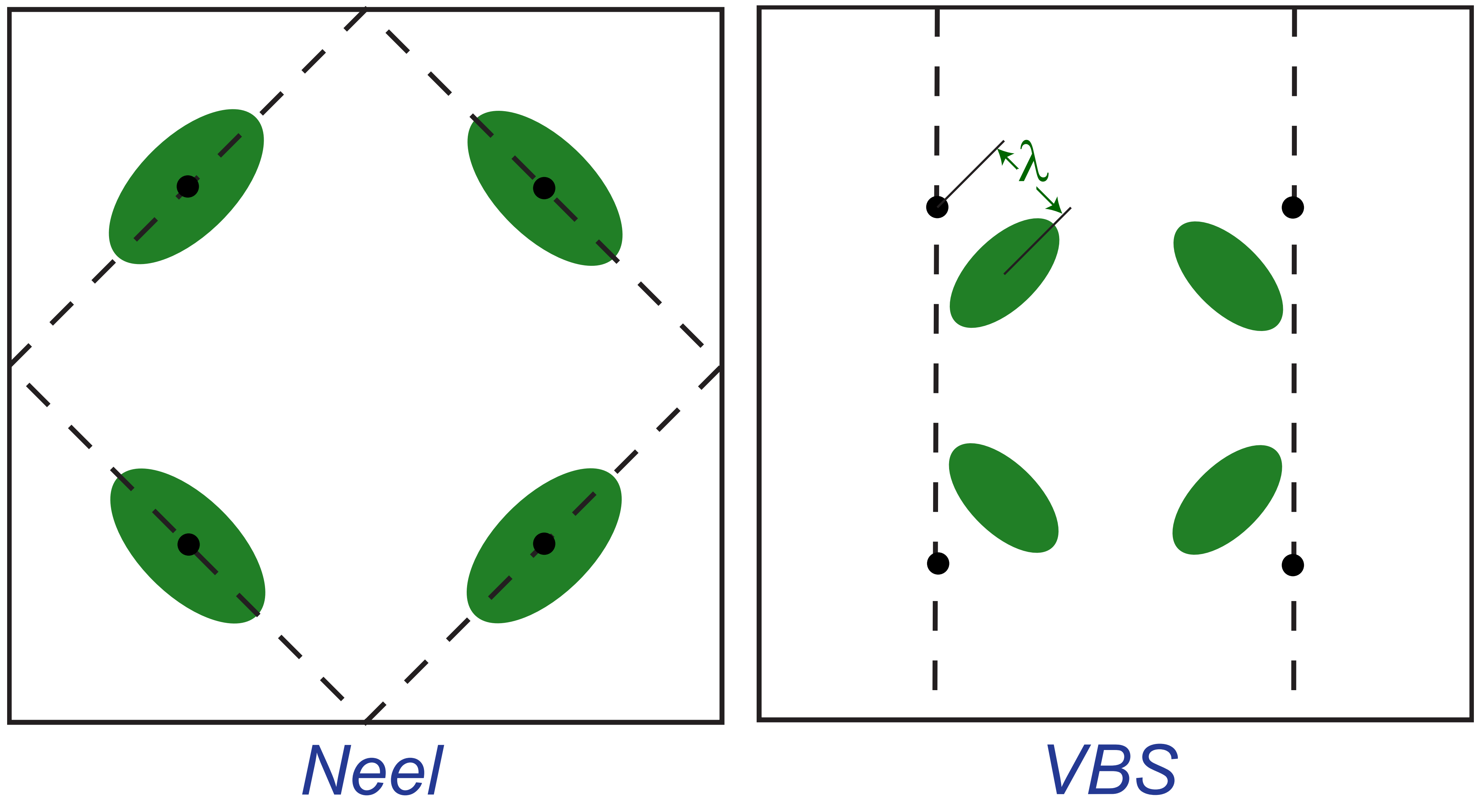


Quantization of spin liquid anomaly implies Fermi surface areas are also *quantized* and robust to all corrections.

PHYSICAL REVIEW B **75**, 235122 (2007)

Hole dynamics in an antiferromagnet across a deconfined quantum critical point

Ribhu K. Kaul,[1] Alexei Kolezhuk,[1,2] Michael Levin,[1] Subir Sachdev,[1] and T. Senthil[3,4]



The dashed line in the Néel phase indicates the boundary of the magnetic Brillouin zone. Only the Fermi surfaces within this zone contribute to the Luttinger counting, and so the area of each ellipse is $\mathcal{A}_F=(2\pi)^2\delta/4$. In the VBS phase, all four pockets are inequivalent, and so the area of each ellipse is $\mathcal{A}_F=(2\pi)^2\delta/8$.

Factor of 2 between SDW fluctuation and FL*

Observation of the Yamaji effect in the cuprate pseudogap

Keimer, Kivelson, Norman, Uchida, and Zaanen, *Nature* **518**, 179 (2015)

Observation of the Yamaji effect in a cuprate superconductor

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nature physics
21, 1753 (2025)
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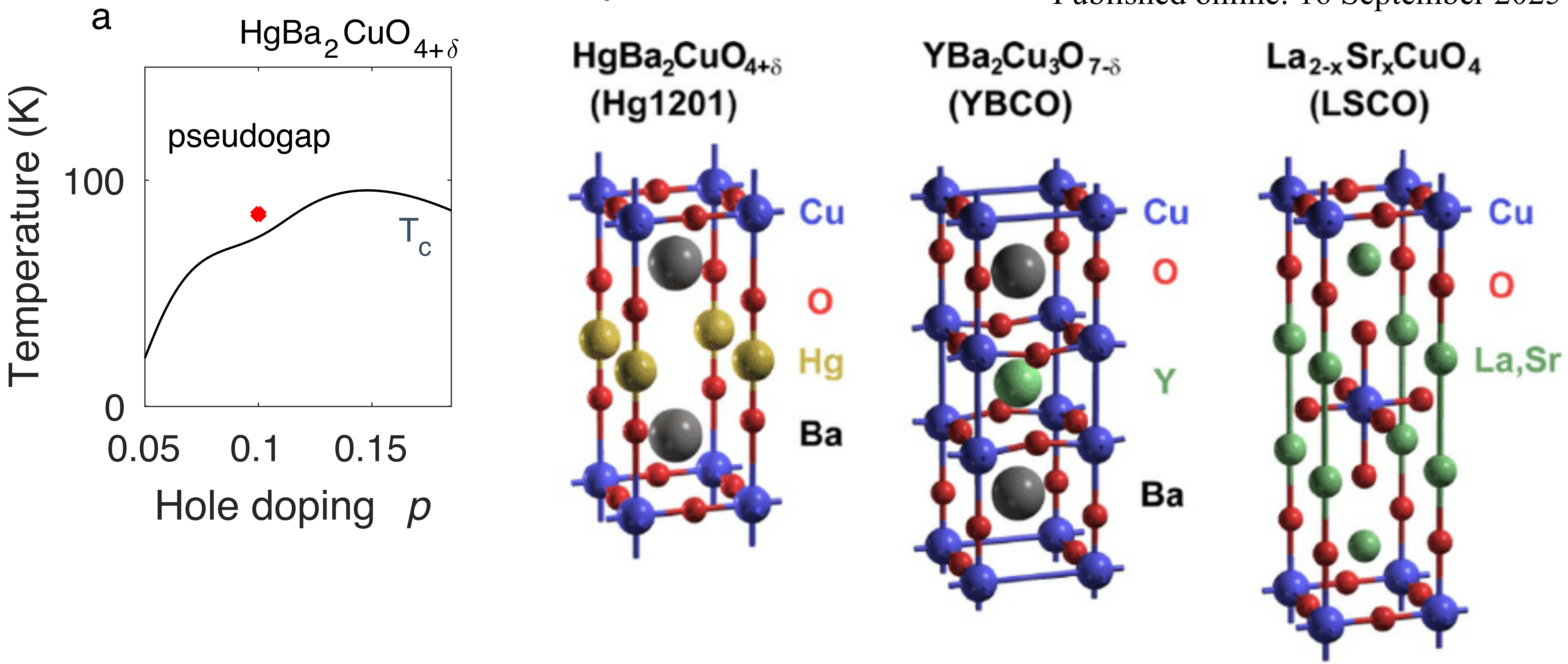

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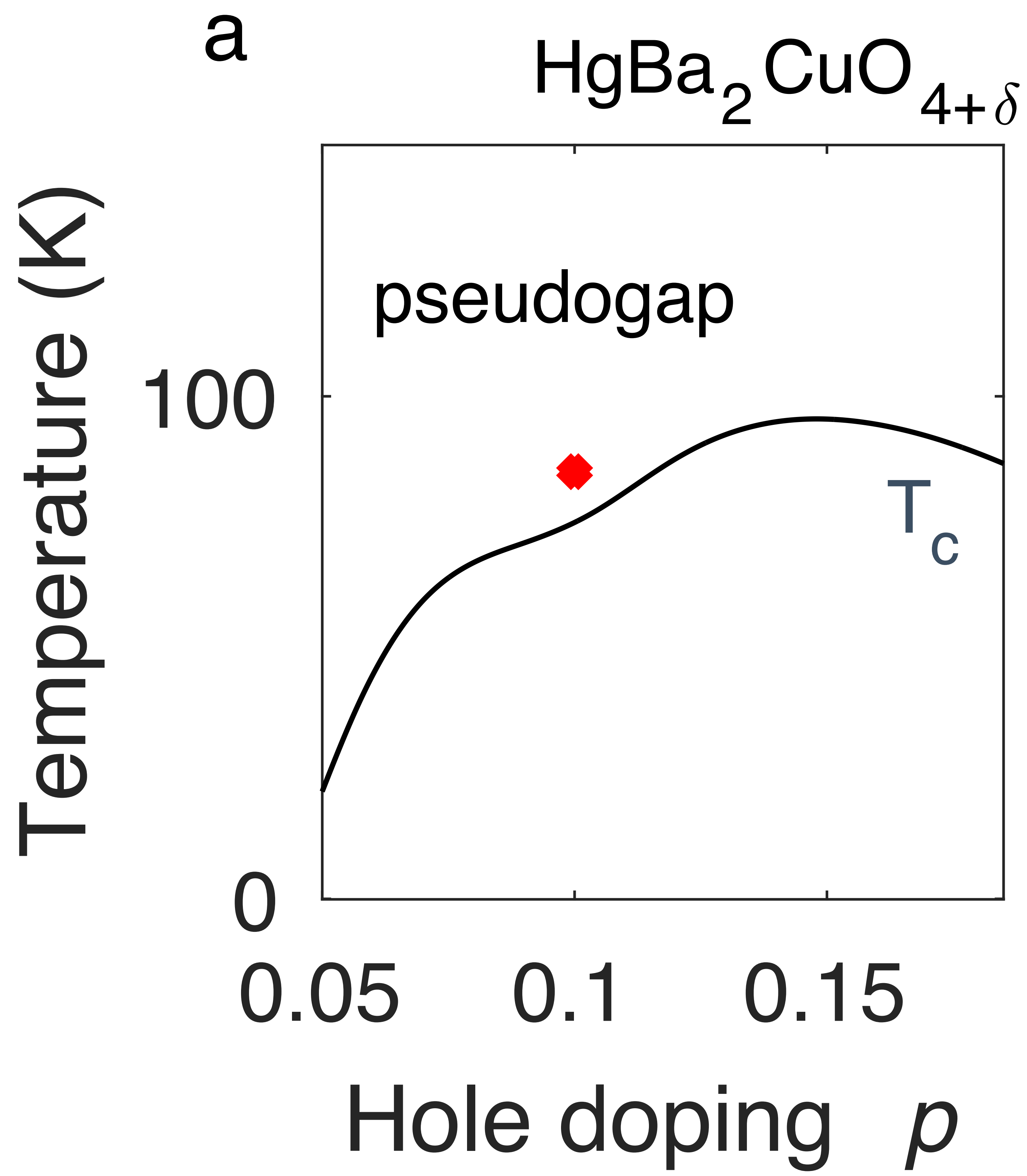

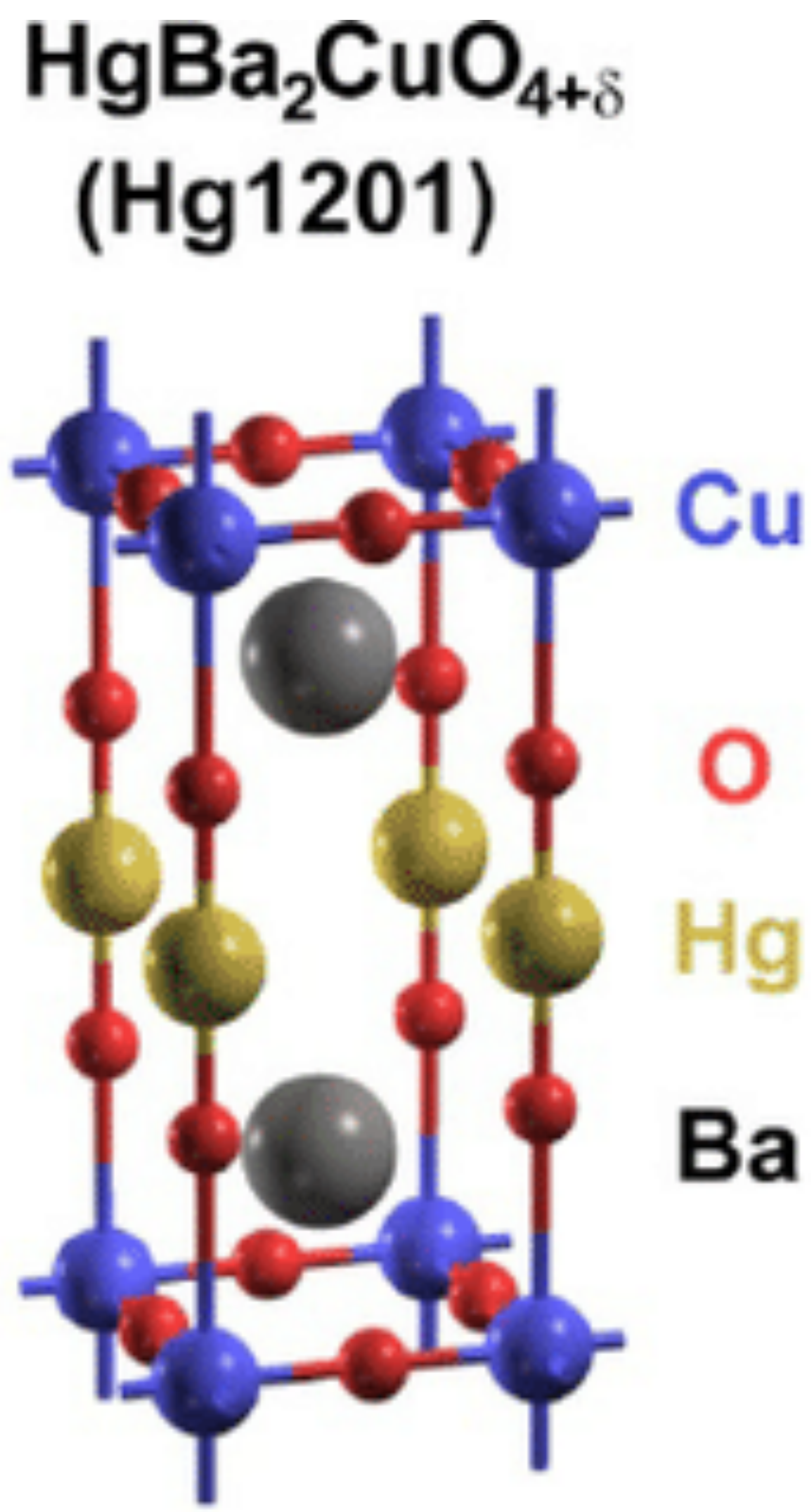

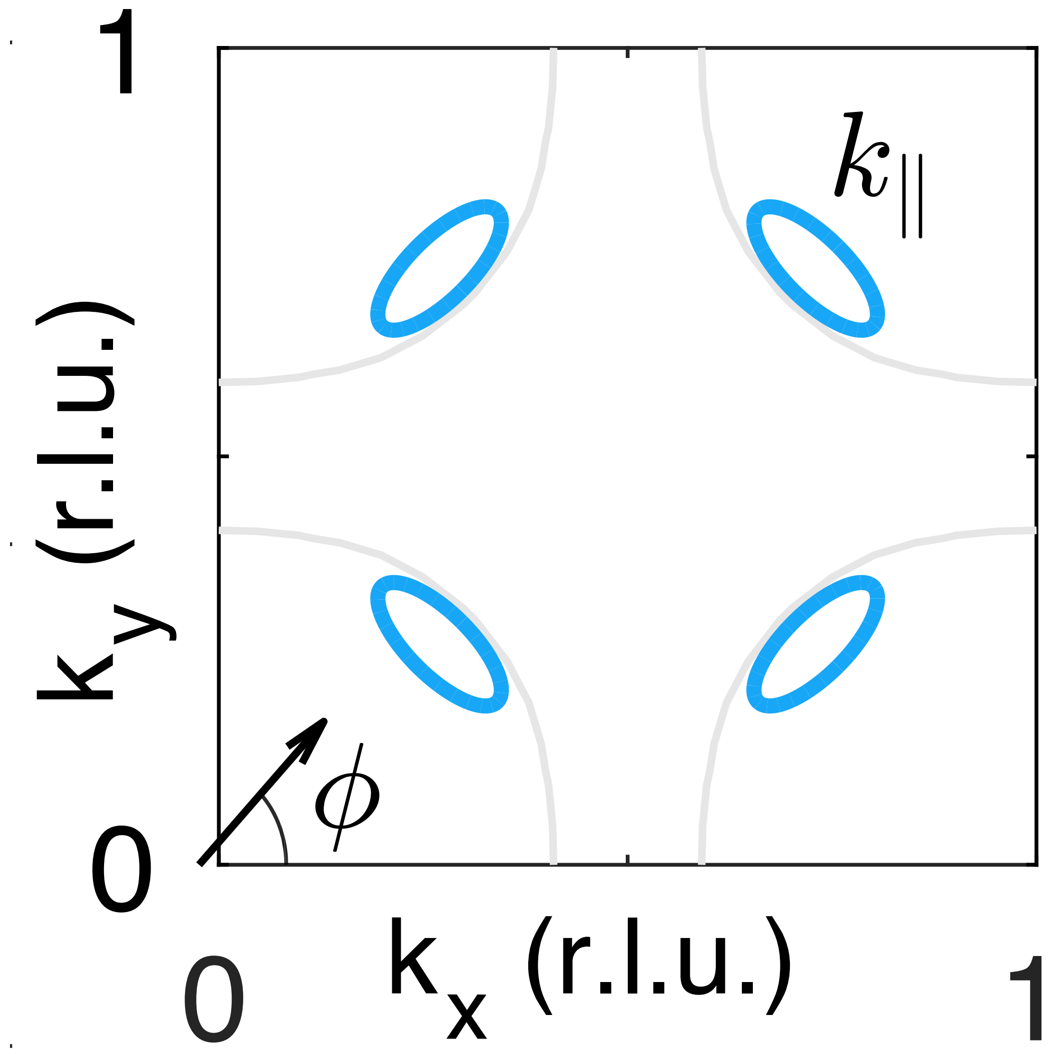

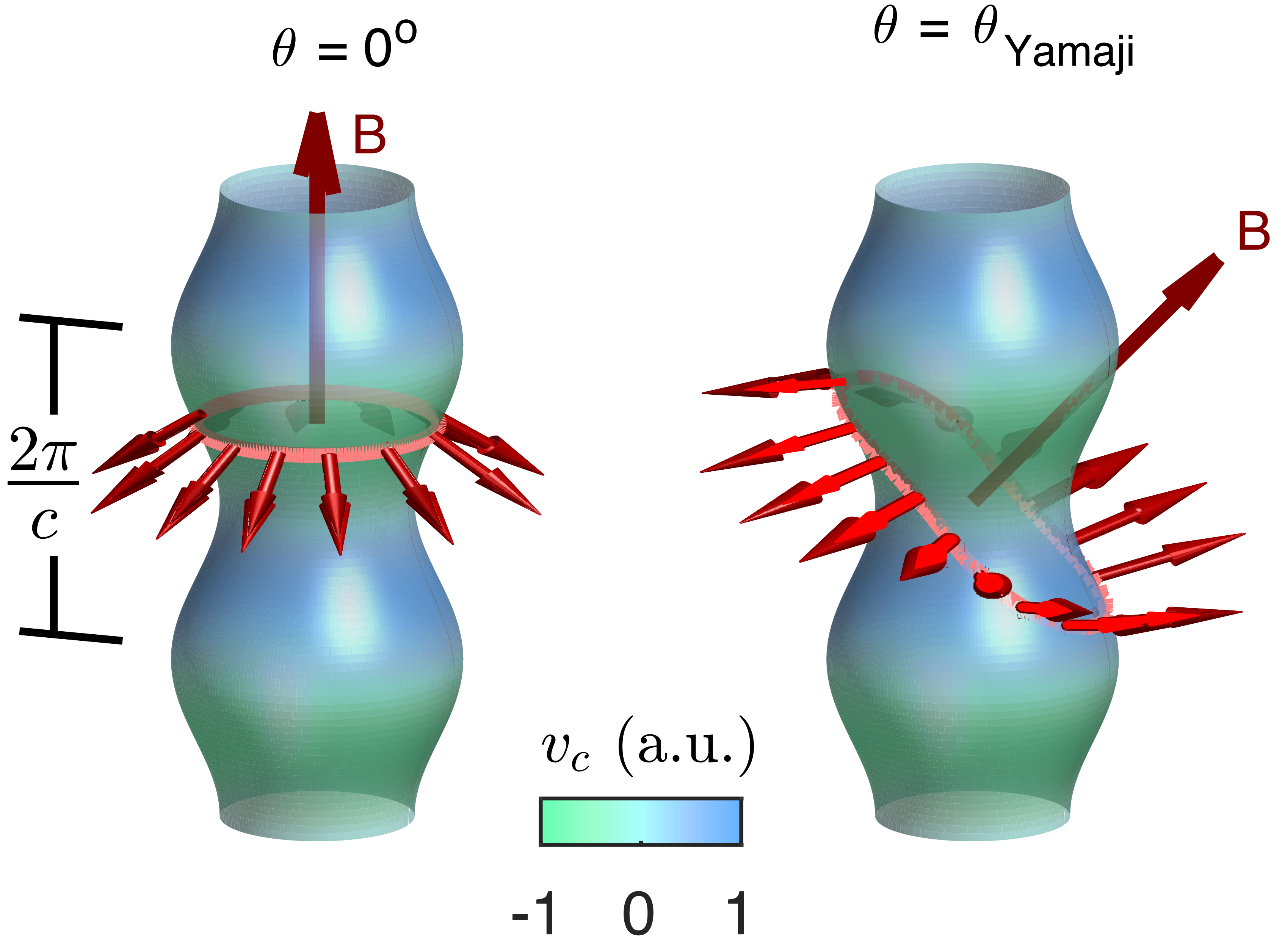

At the Yamaji angle, the orbits in the plane orthogonal to ***B*** have an area which is independent of momentum in the c direction, to first order in the hopping along the c direction.

K. Yamaji JPSJ **58**, 1520 (1989)

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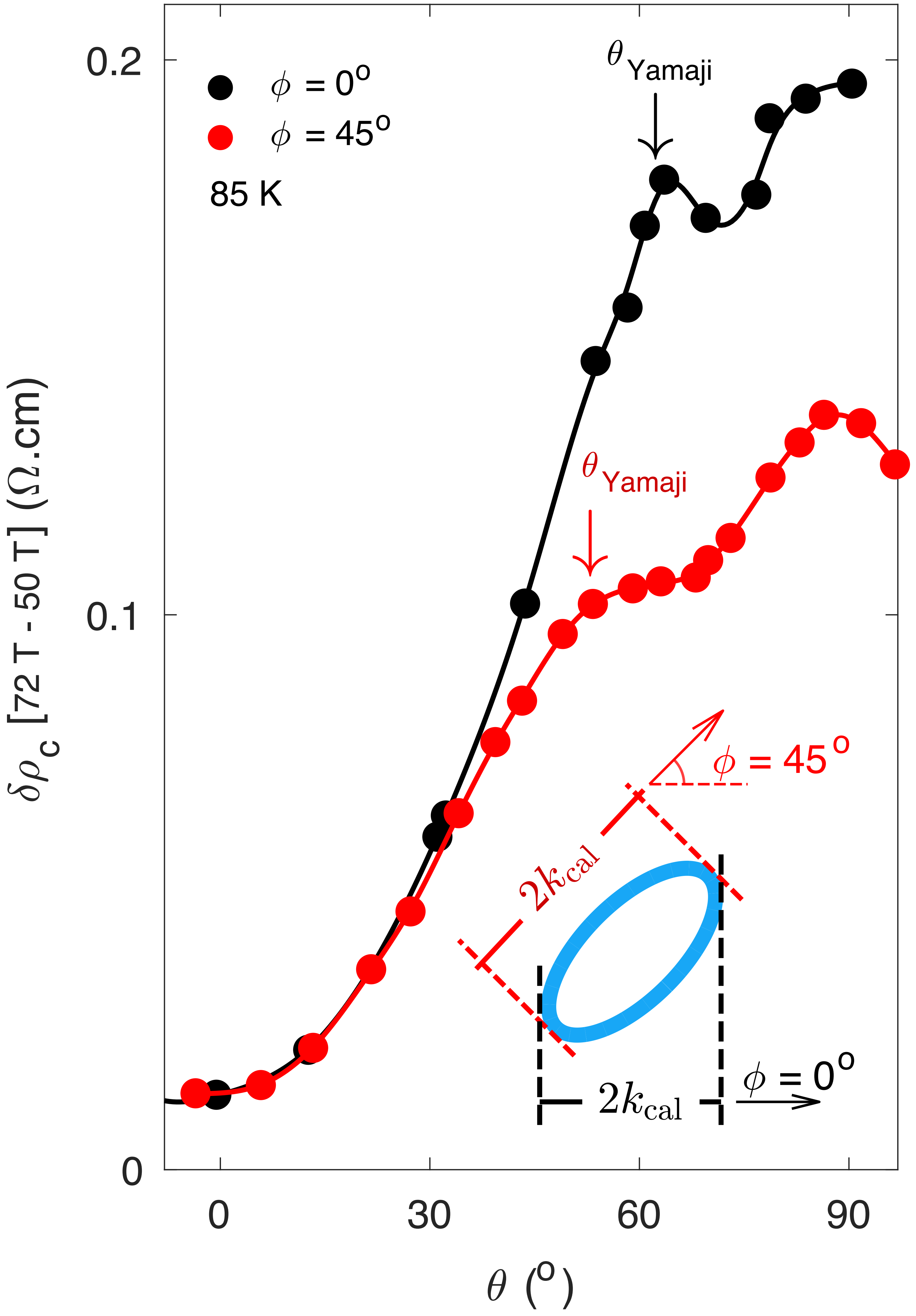

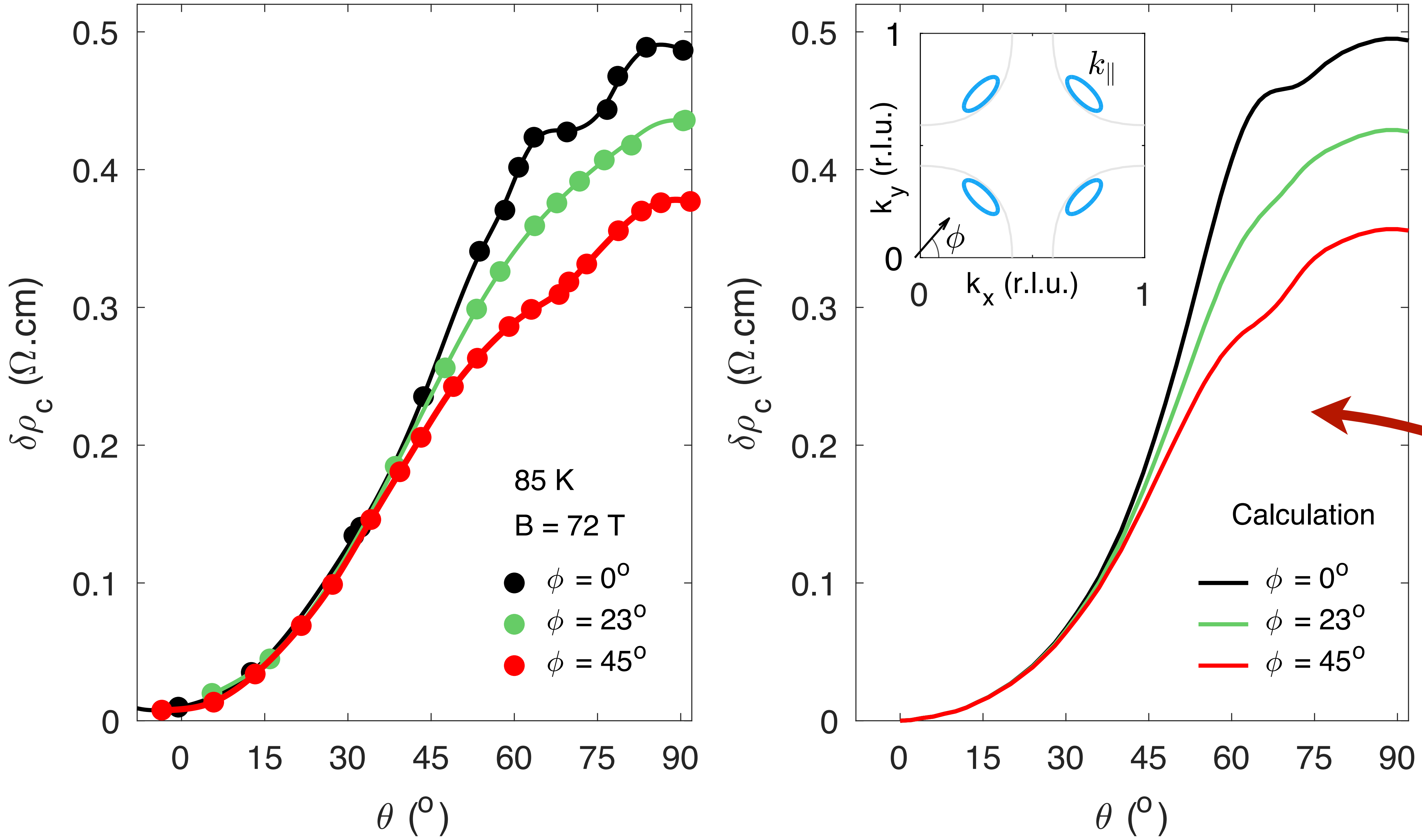

Doping
$p = 0.1$

The observation of the Yamaji peak is evidence for small Fermi-surface pockets in the normal state of the pseudogap phase.

$$\frac{\partial f}{\partial t} + e(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \cdot \mathbf{v} \left(-\frac{\partial f}{\partial \epsilon} \right) = -\frac{f - f_0}{\tau}$$

$$\mathbf{v} = \nabla_{\mathbf{k}} \epsilon(\mathbf{k}) \; ; \; f_0(\epsilon) = \frac{1}{e^{(\epsilon-\mu)/T} + 1} \; ; \; \epsilon(\mathbf{k}) = \frac{k_x^2}{2m_1} + \frac{k_y^2}{2m_2} - 2t_\perp \cos(k_z c)$$

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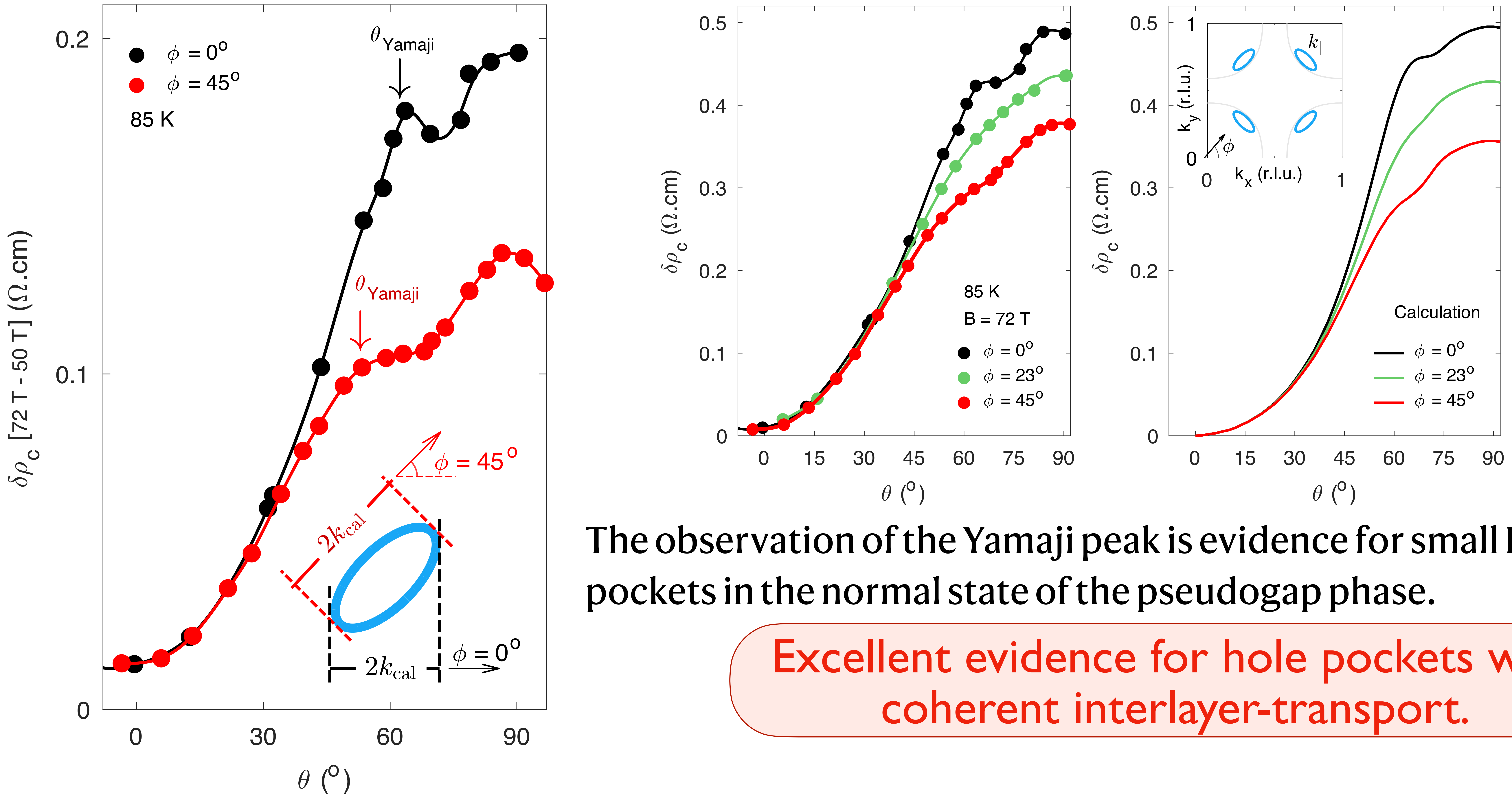

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Observation of the Yamaji effect in a cuprate superconductor

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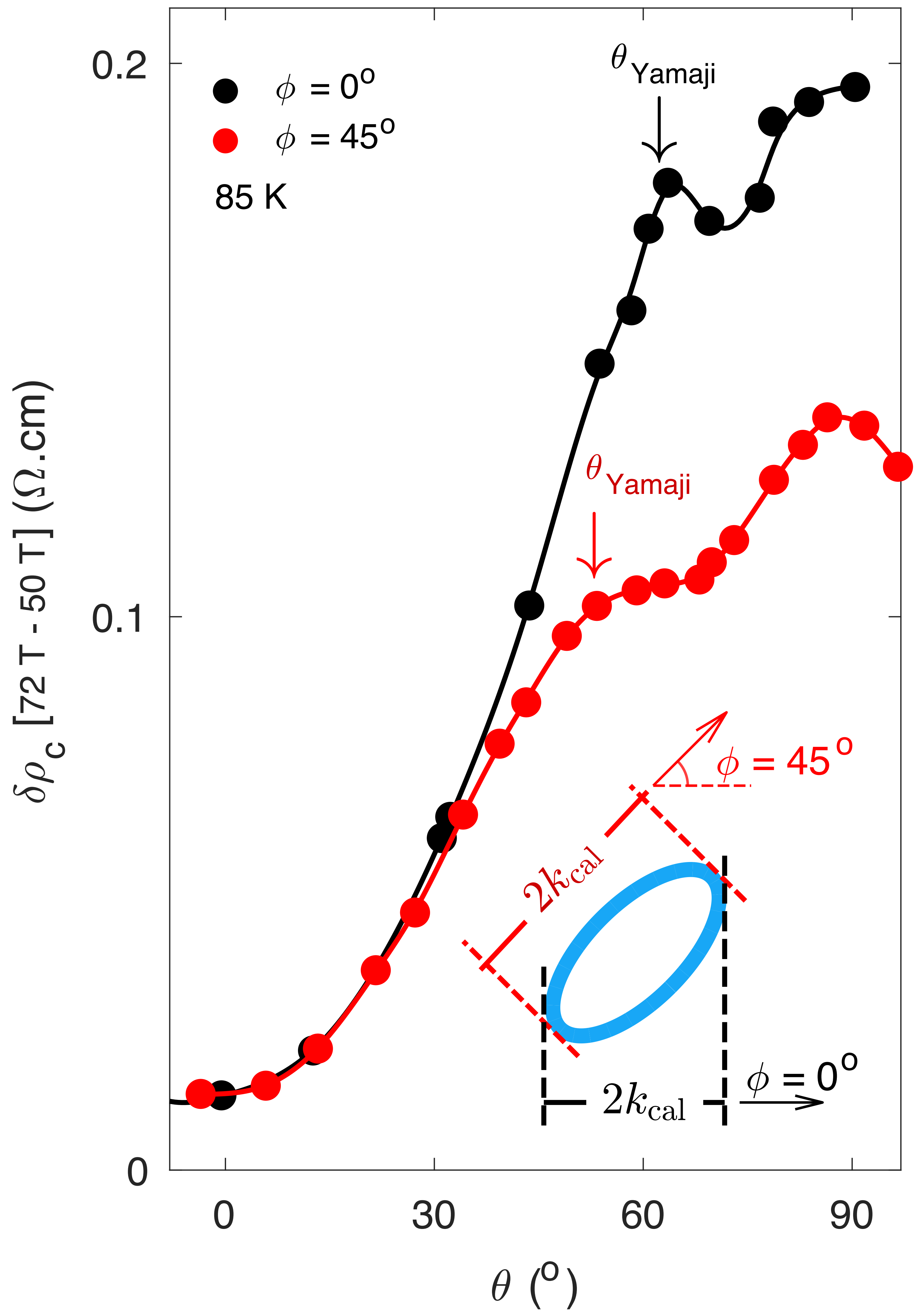

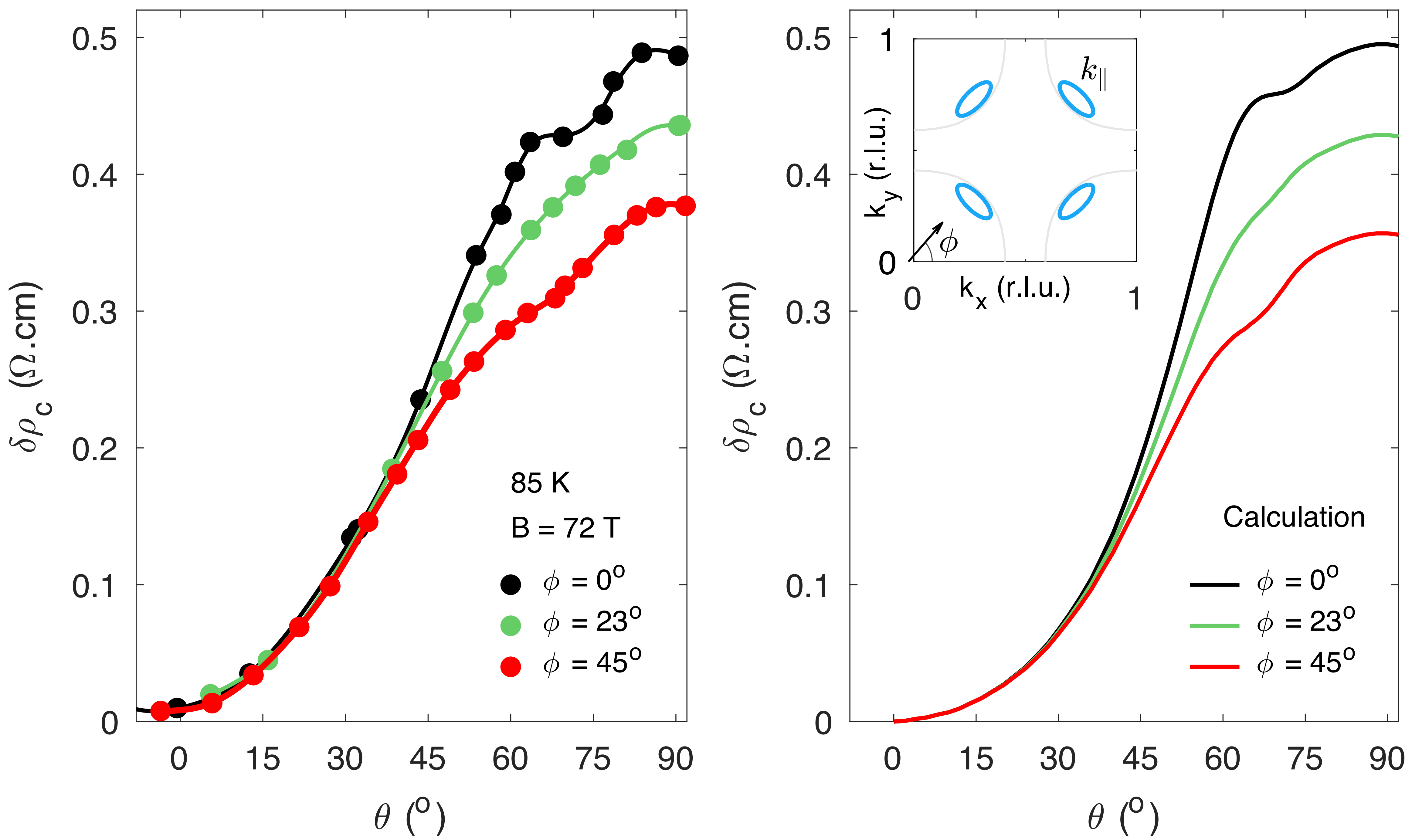

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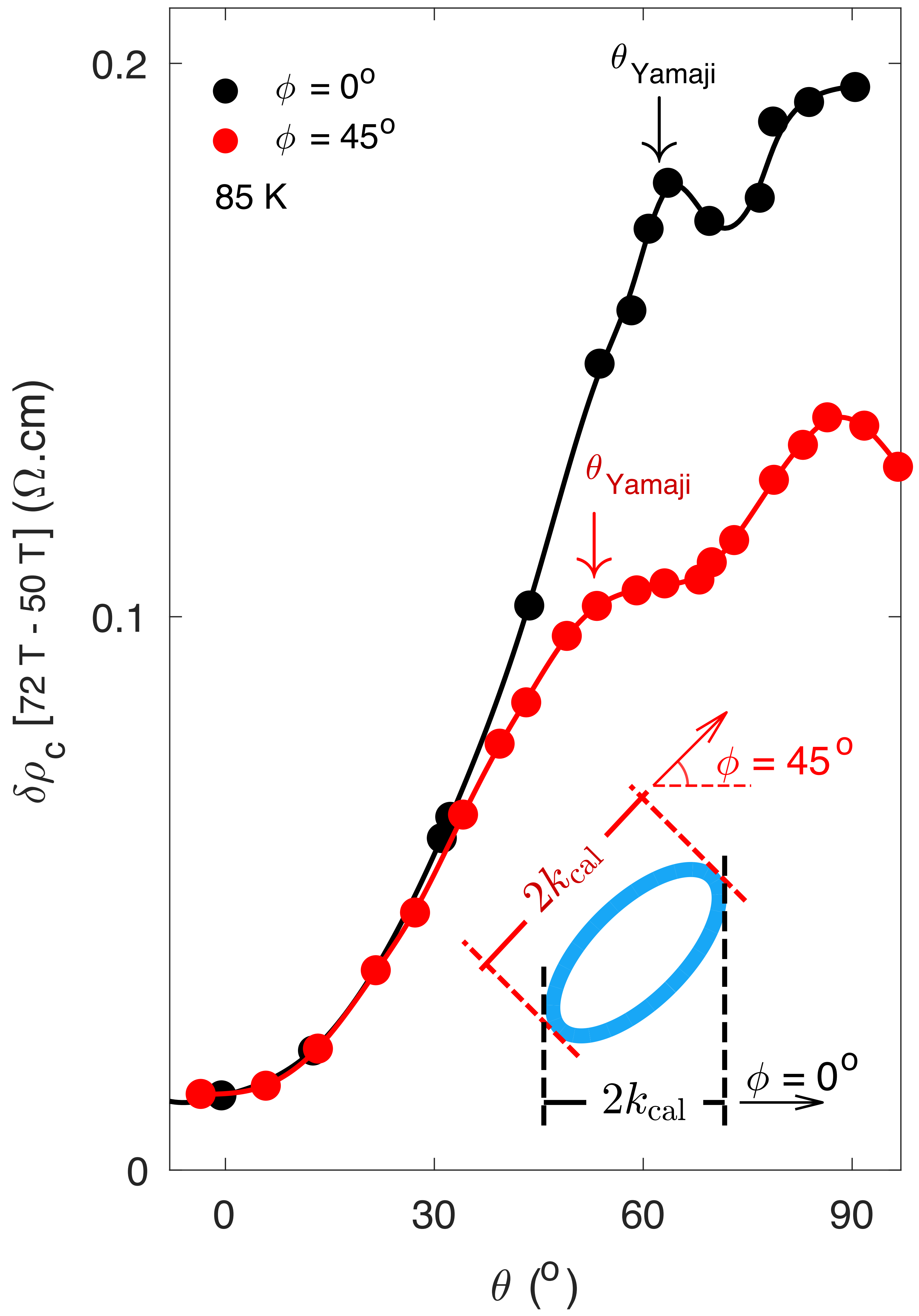

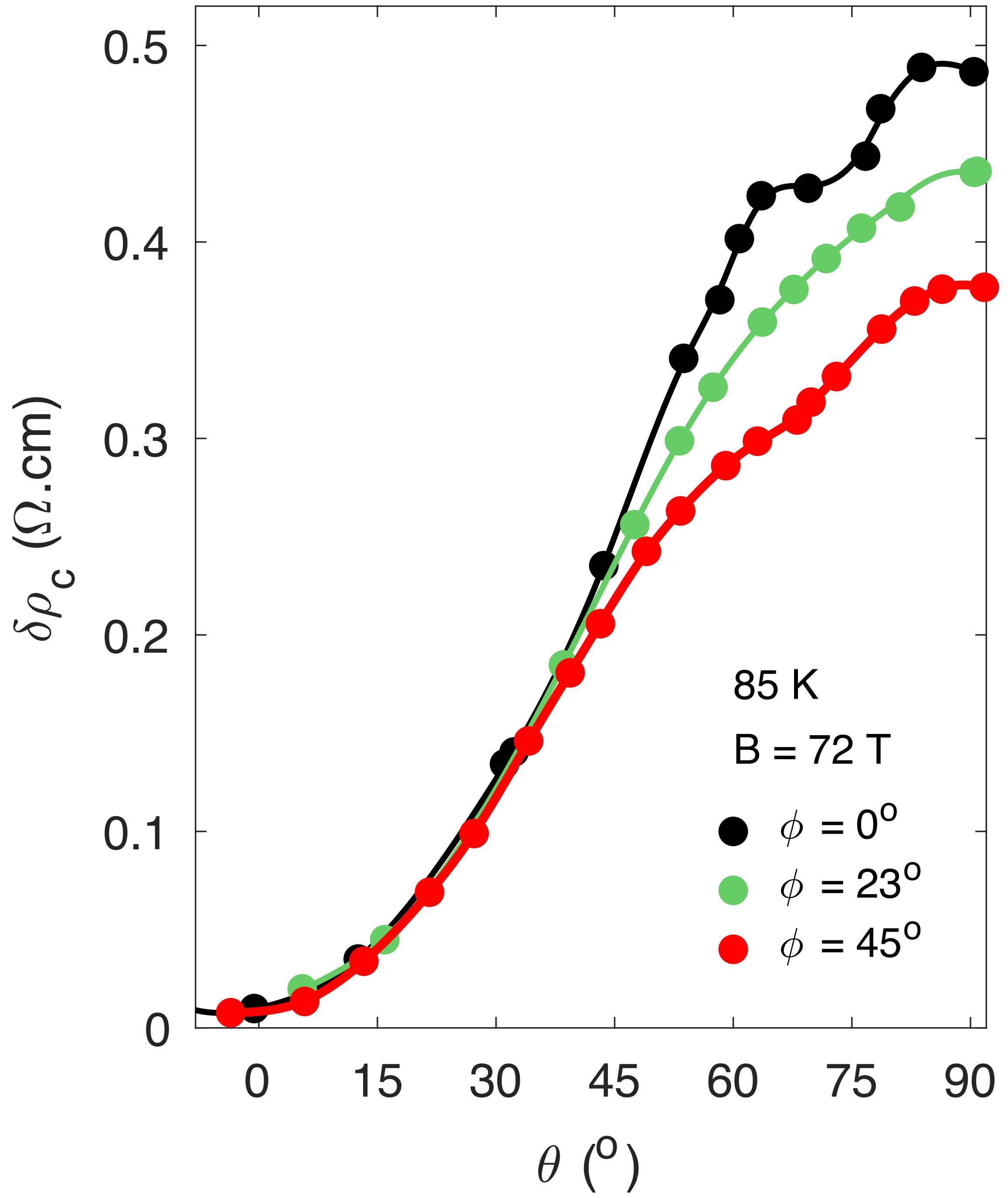

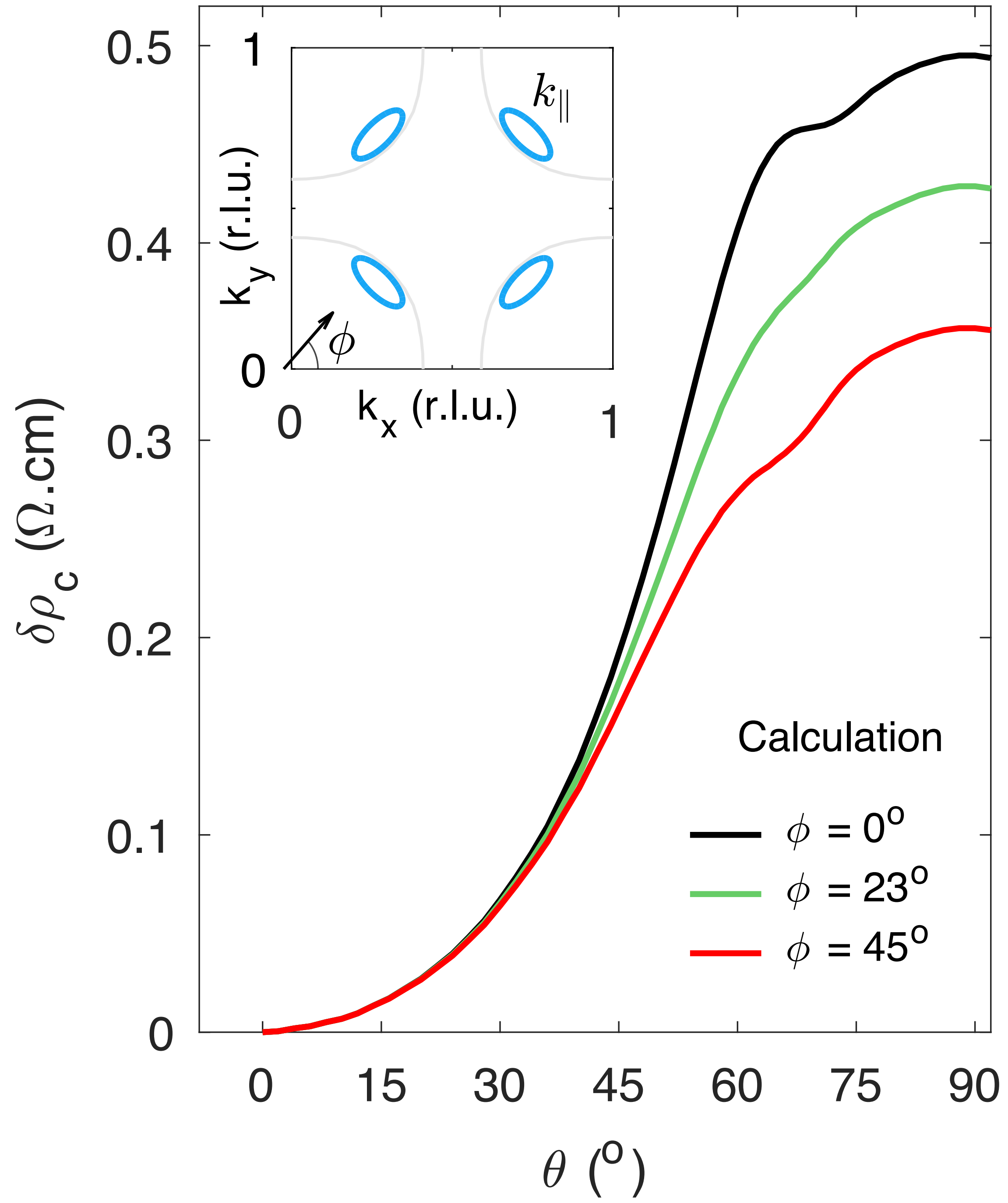

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Rules out holon metal
and possibly SDW metal

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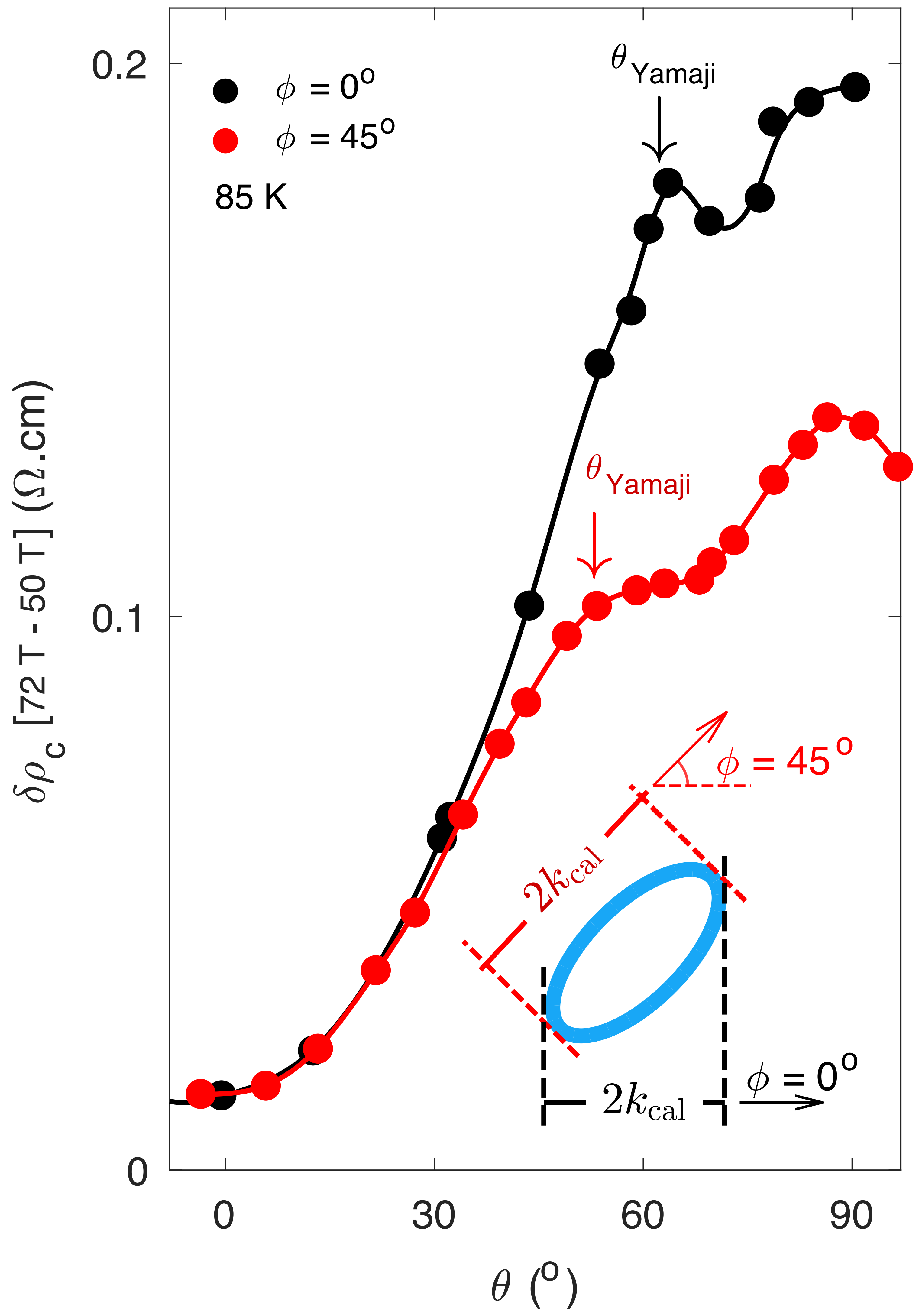

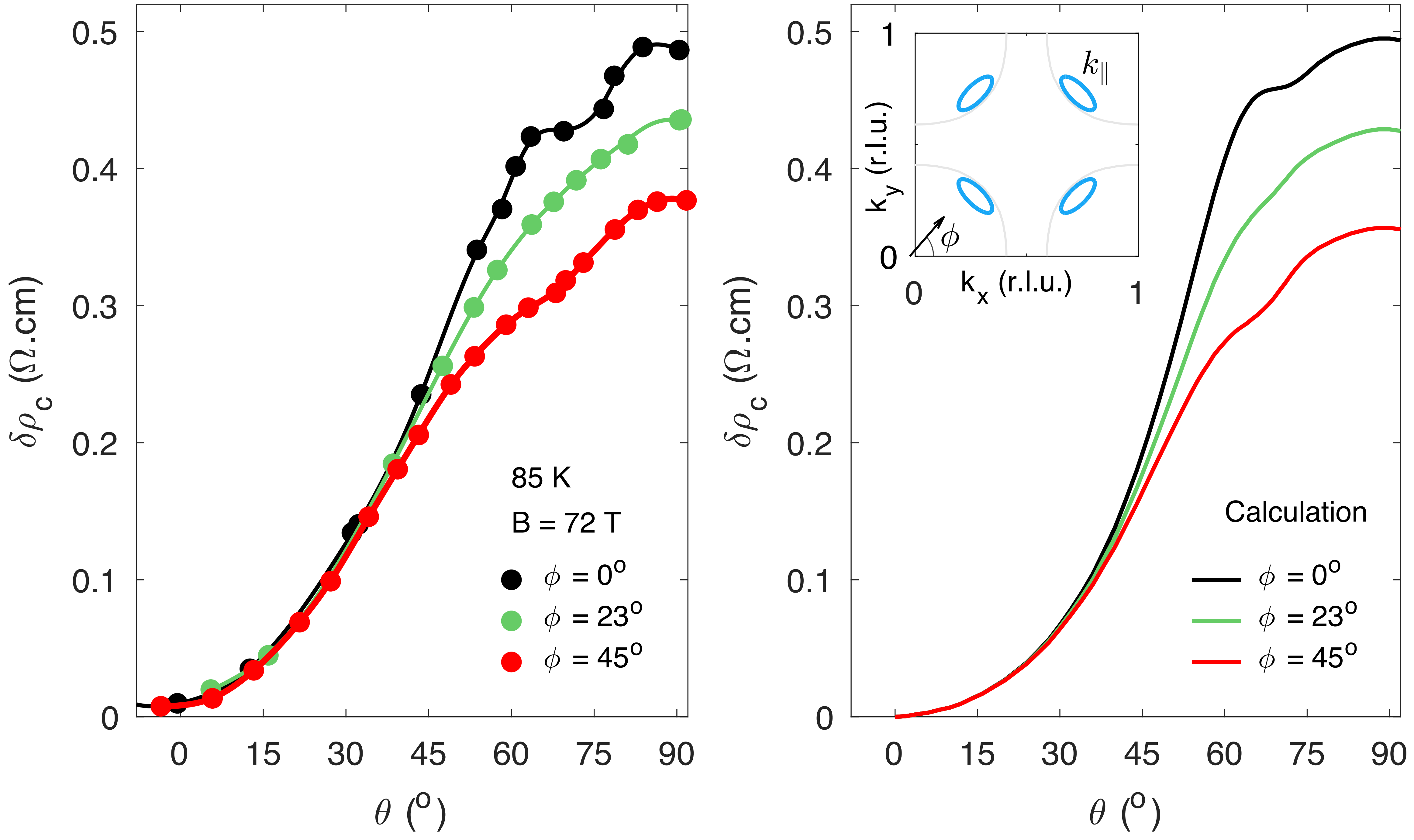

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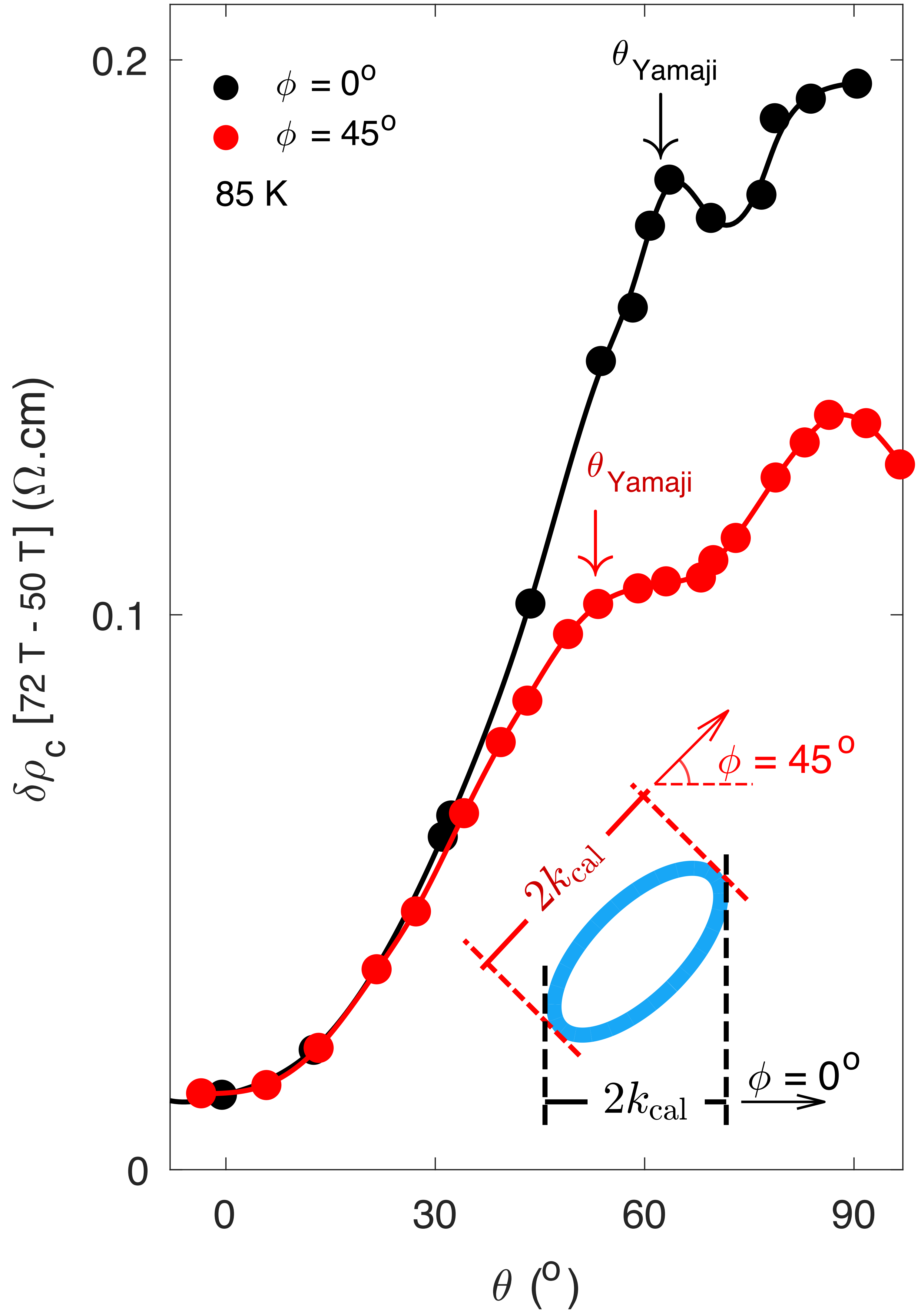

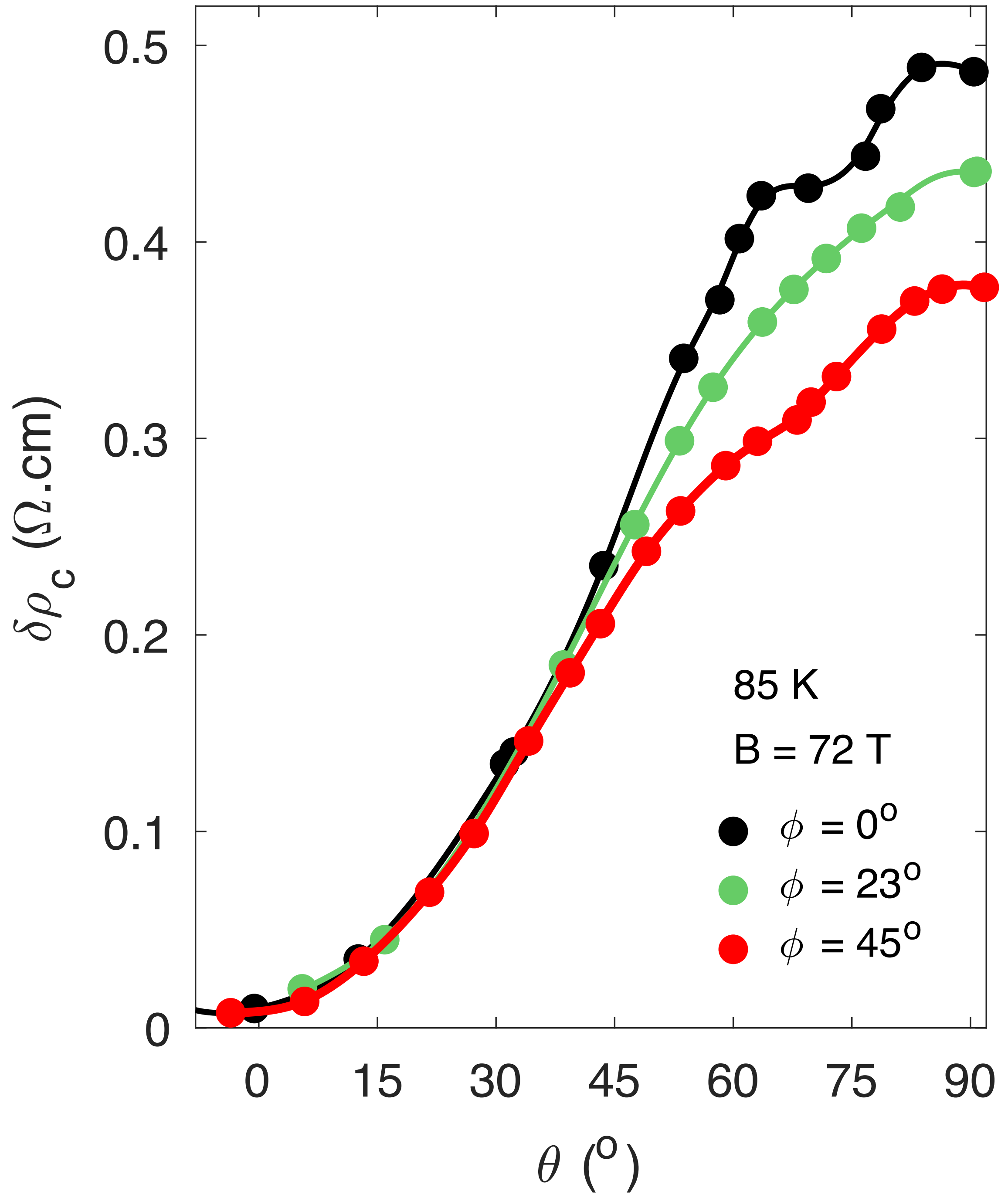

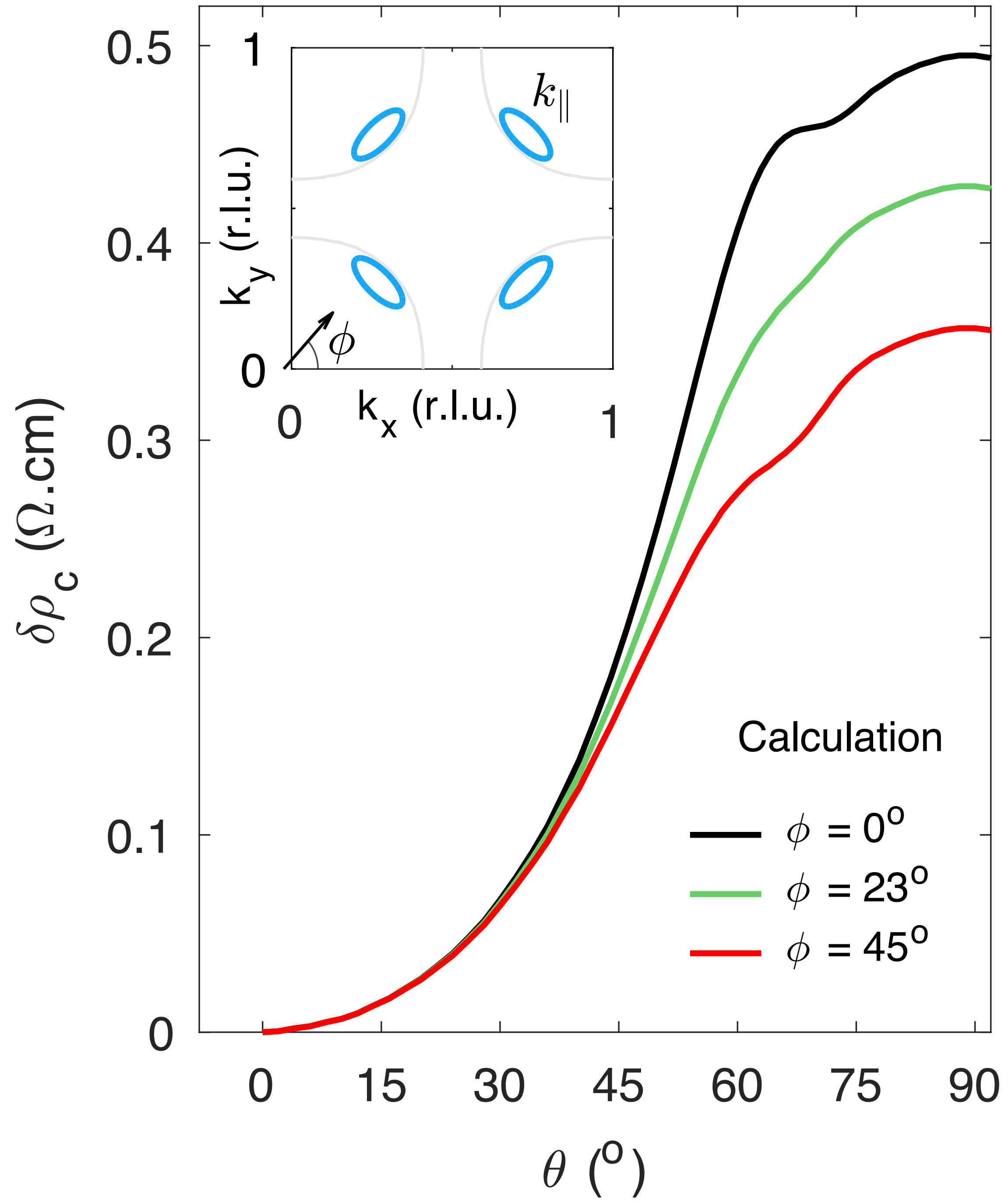

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(was expected by us!)

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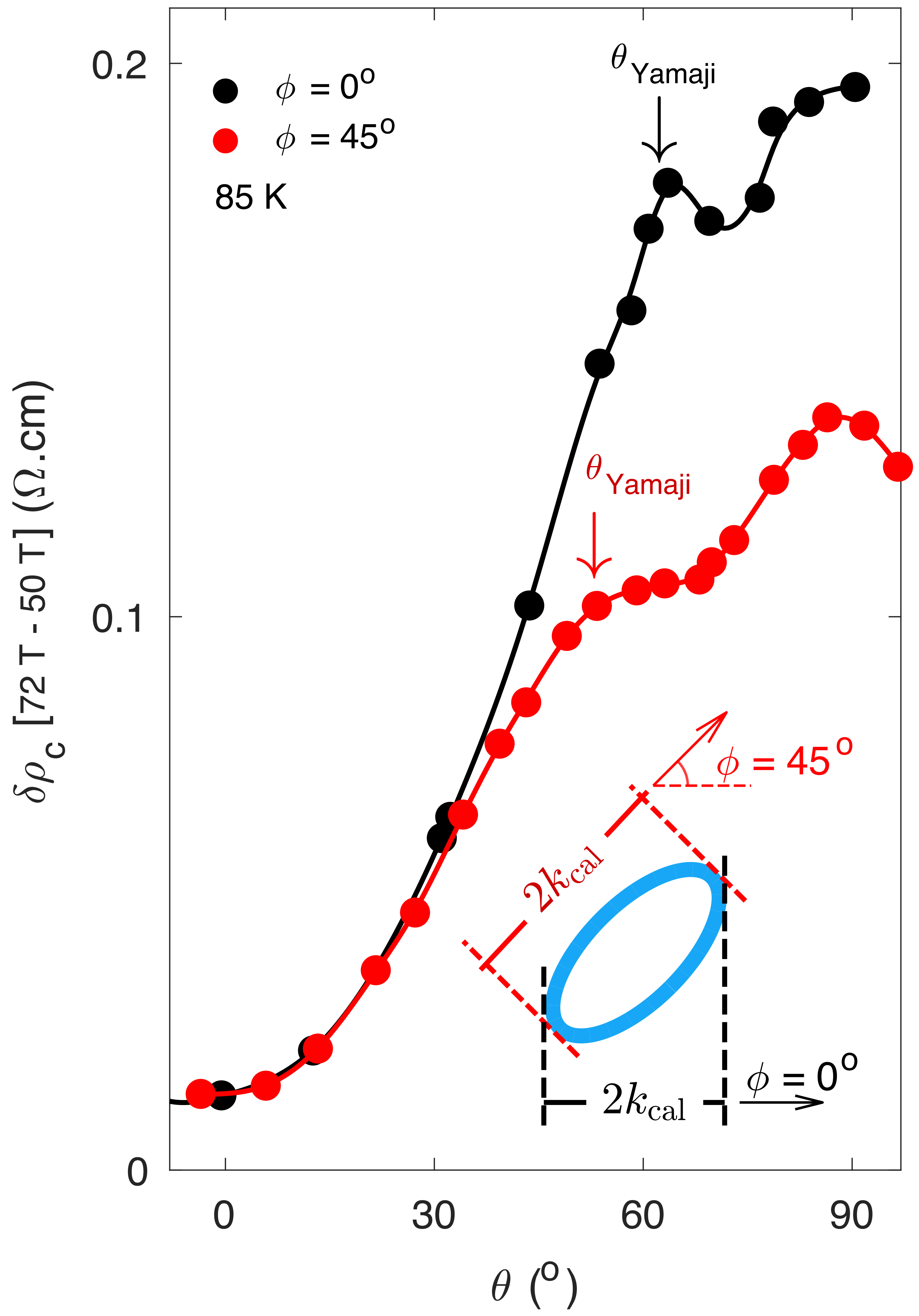

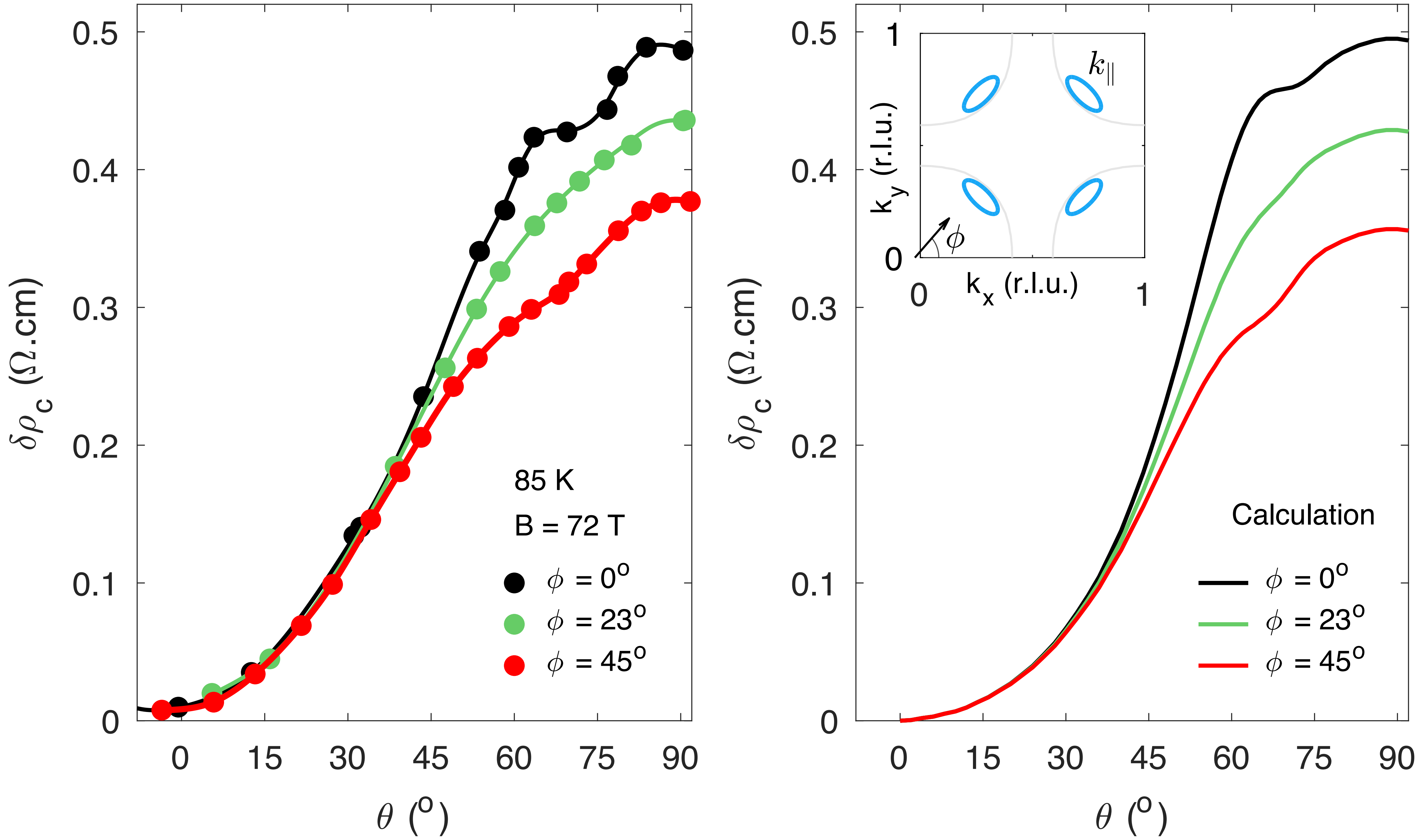

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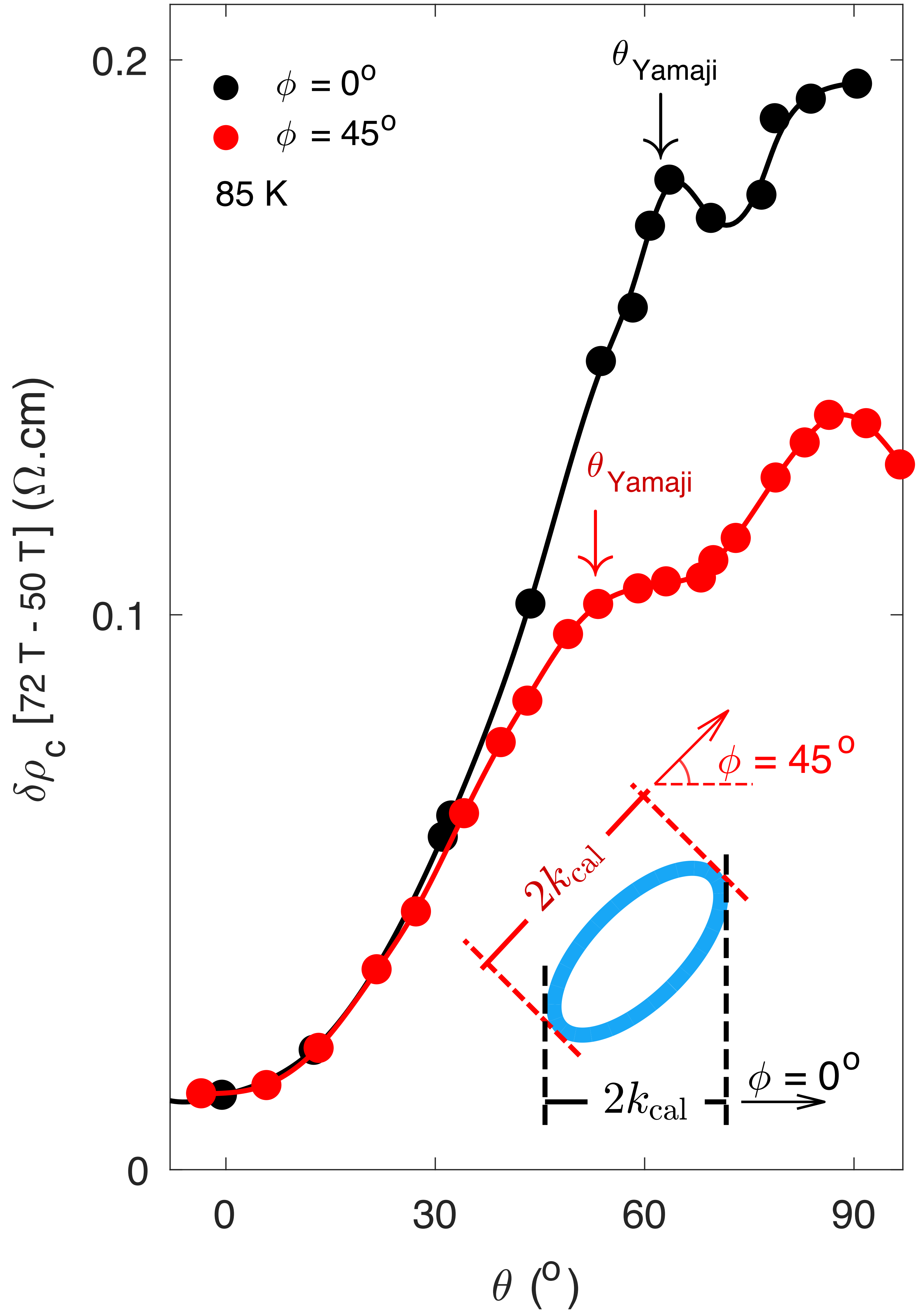
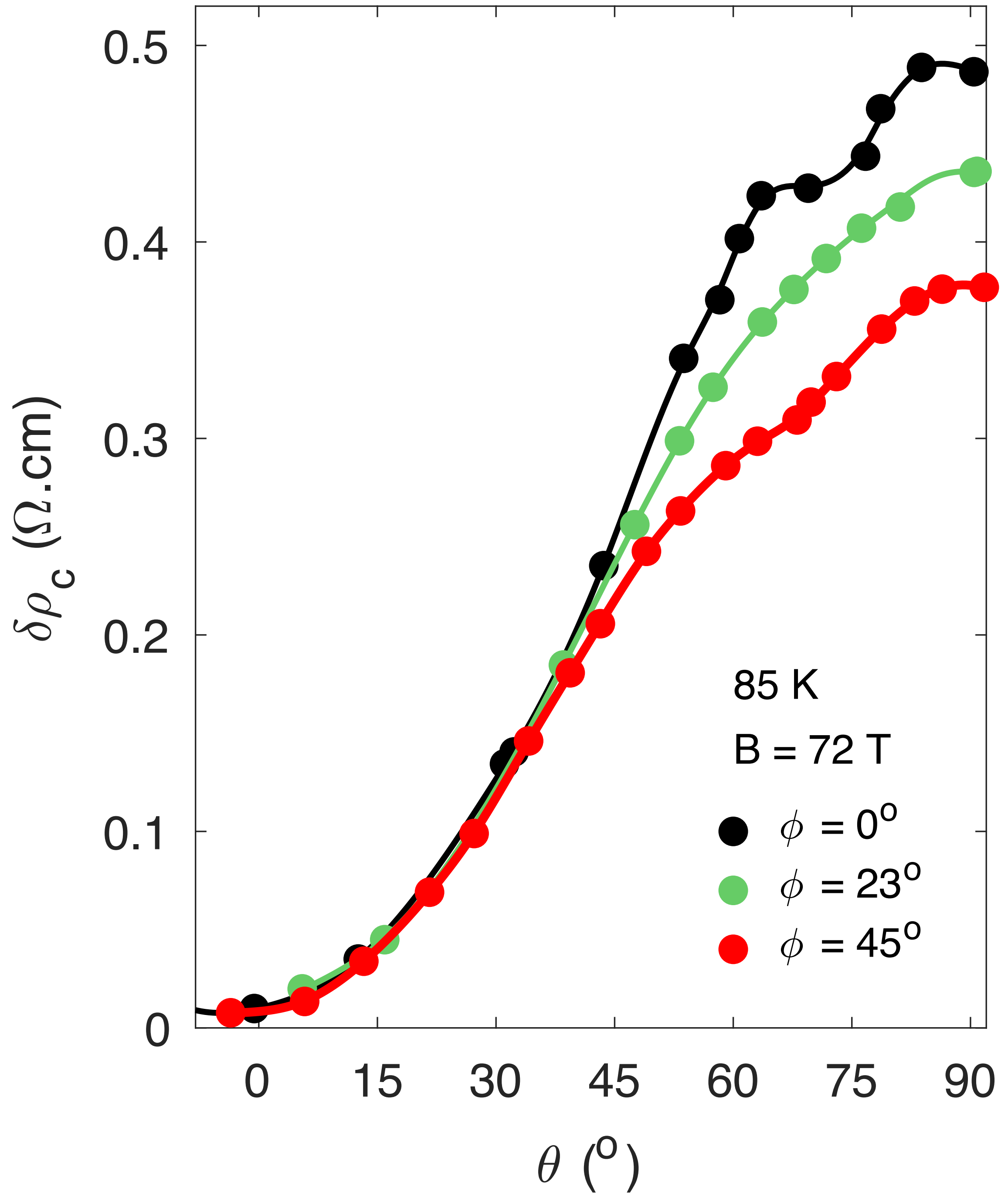
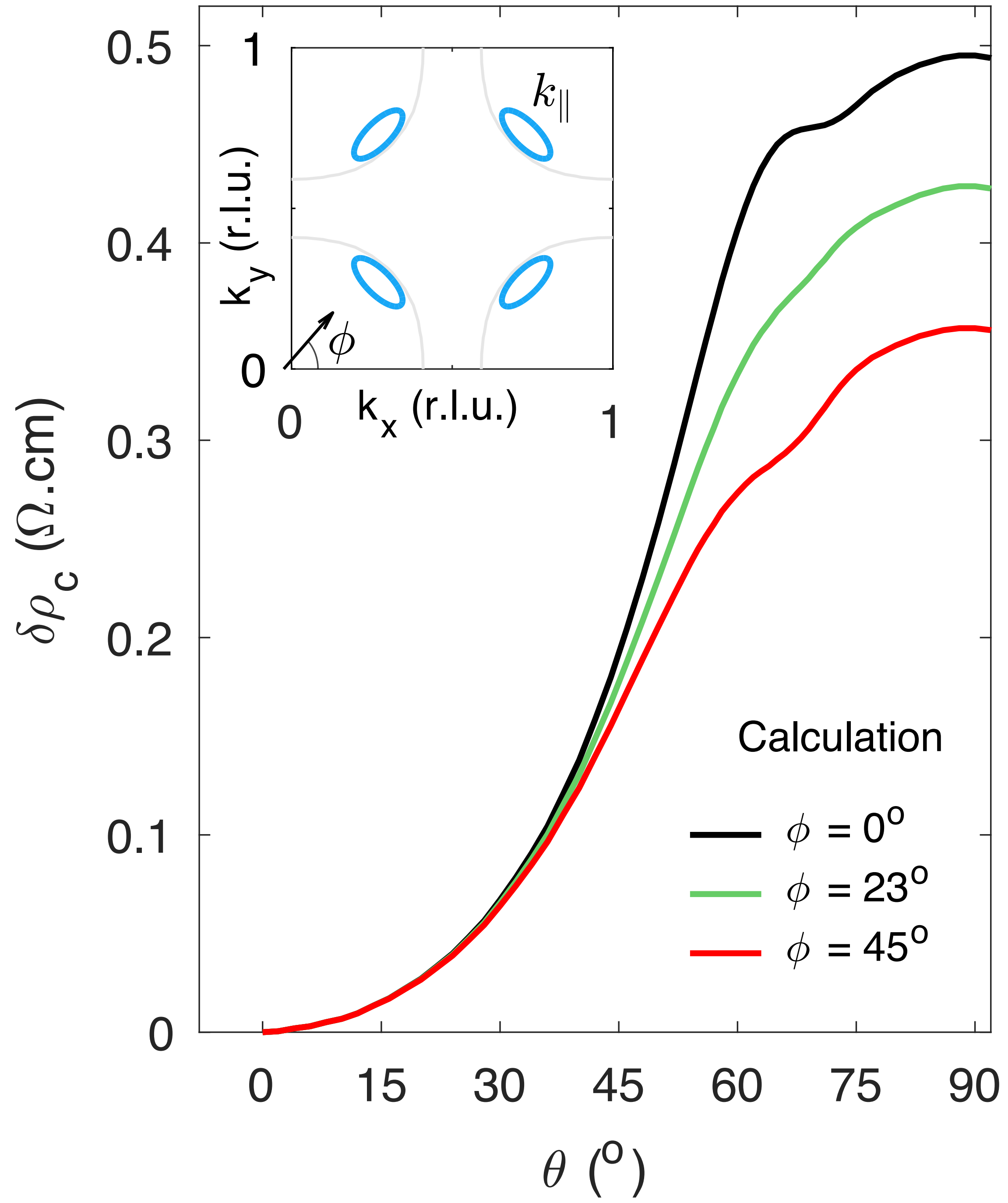






Doping
$p = 0.1$

The observation of the Yamaji peak is evidence for small Fermi-surface pockets in the normal state of the pseudogap phase. The small size of the pockets, each estimated to occupy only 1.3% of the Brillouin zone area, is not expected given the absence of long-range broken translational symmetry.

Predicted FL* pocket fraction $= p/8 = 1.25\%$!

Fluctuating AF metal fraction $= p/4 = 2.5\%$.

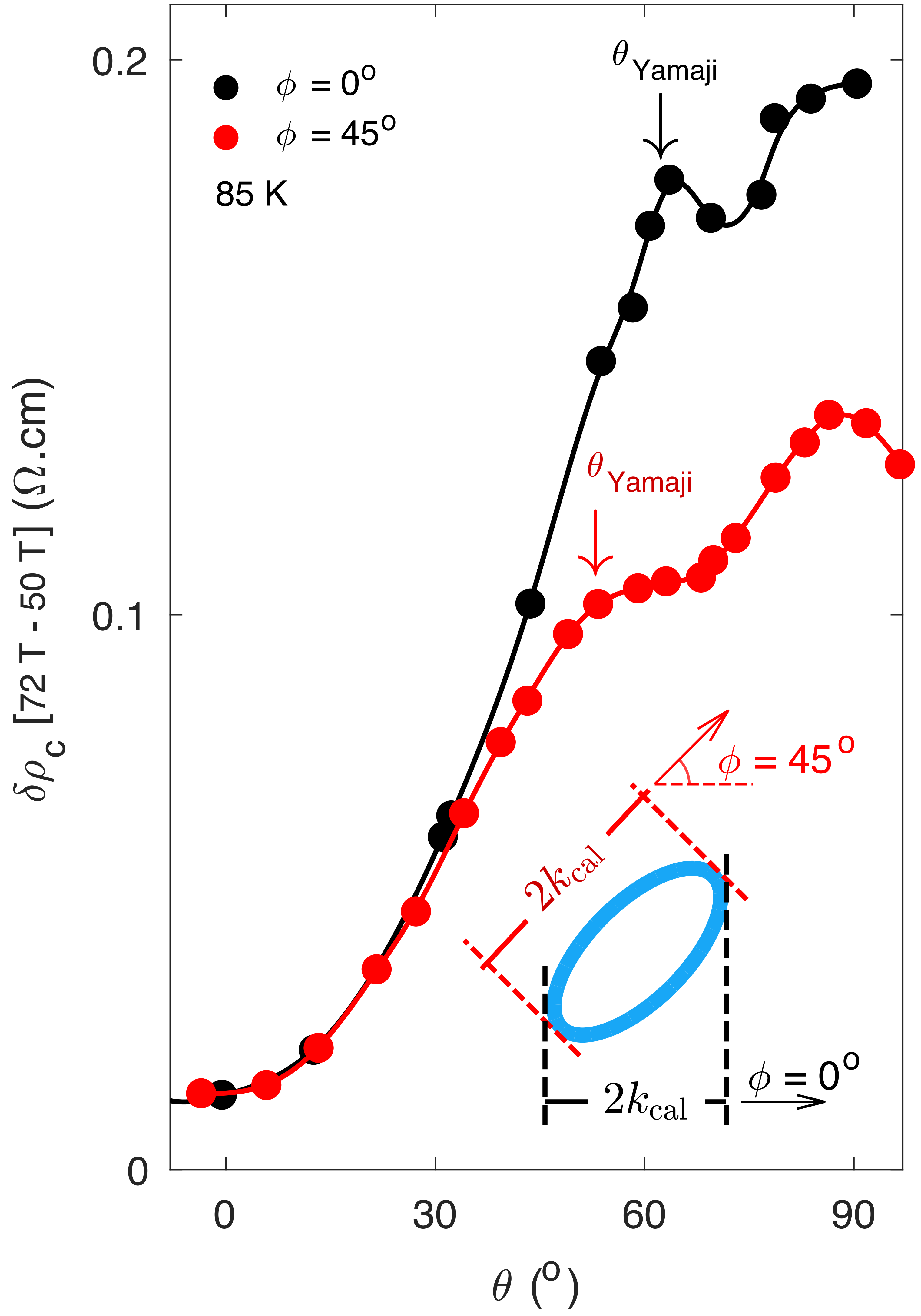
(p/8 also in YRZ ansatz, Peter Johnson photoemission, and Jenny Hoffman and Seamus Davis STMs; Stanescu-Kotliar)

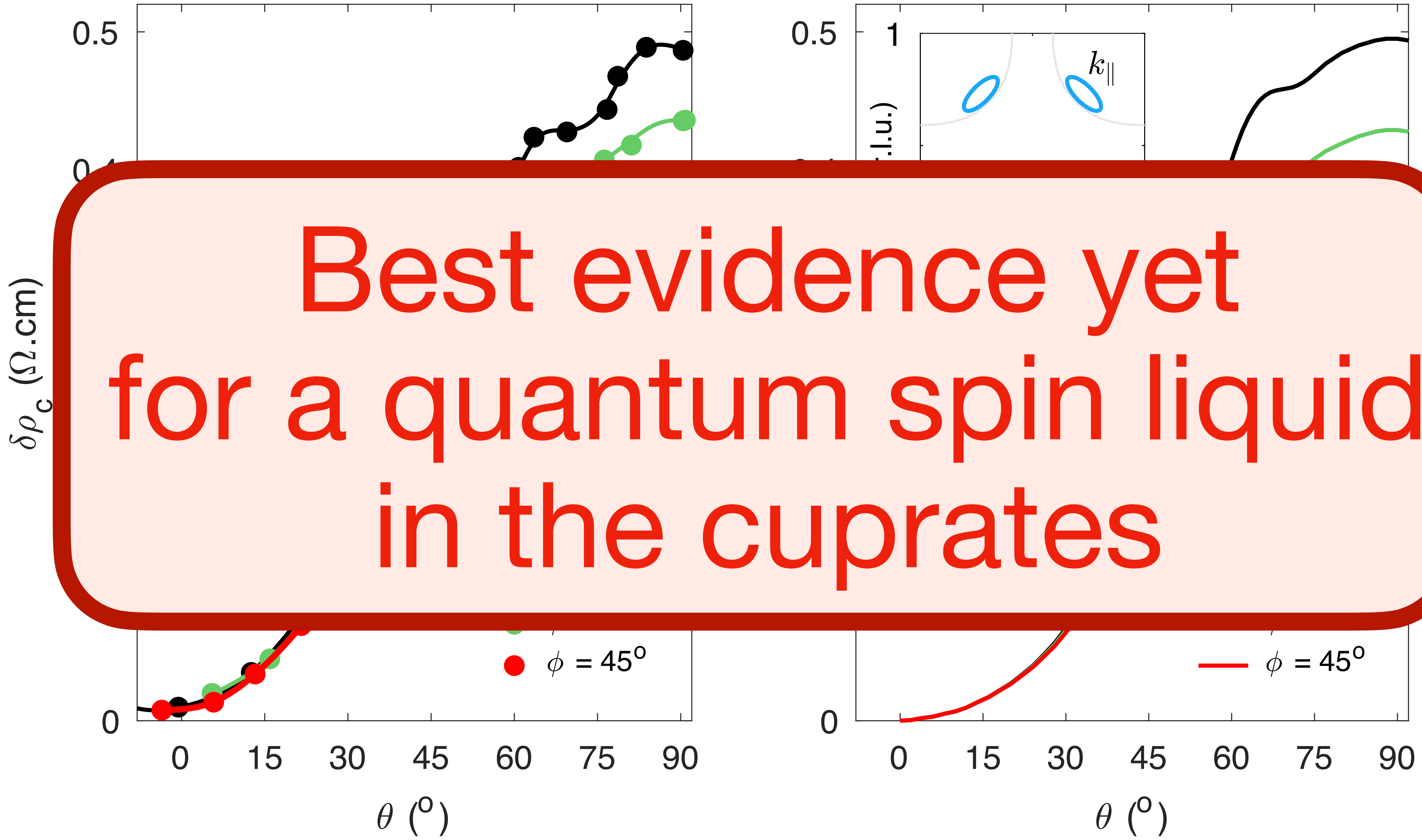
Jing-Yu Zhao, S. Chatterjee, S. S., Ya-Hui Zhang, arXiv:2510.13943

Observation of the Yamaji effect in a cuprate superconductor

Mun K. Chan[1], Katherine A. Schreiber[1], Oscar E. Ayala-Valenzuela[1], Eric D. Bauer[2], Arkady Shekhter[1] & Neil Harrison[1]

nature physics
21, 1753 (2025)
Published online: 16 September 2025





Best evidence yet for a quantum spin liquid in the cuprates

Doping $p = 0.1$

The observation of the Yamaji peak is evidence for small Fermi-surface pockets in the normal state of the pseudogap phase. The small size of the pockets, each estimated to occupy only 1.3% of the Brillouin zone area, is not expected given the absence of long-range broken translational symmetry.

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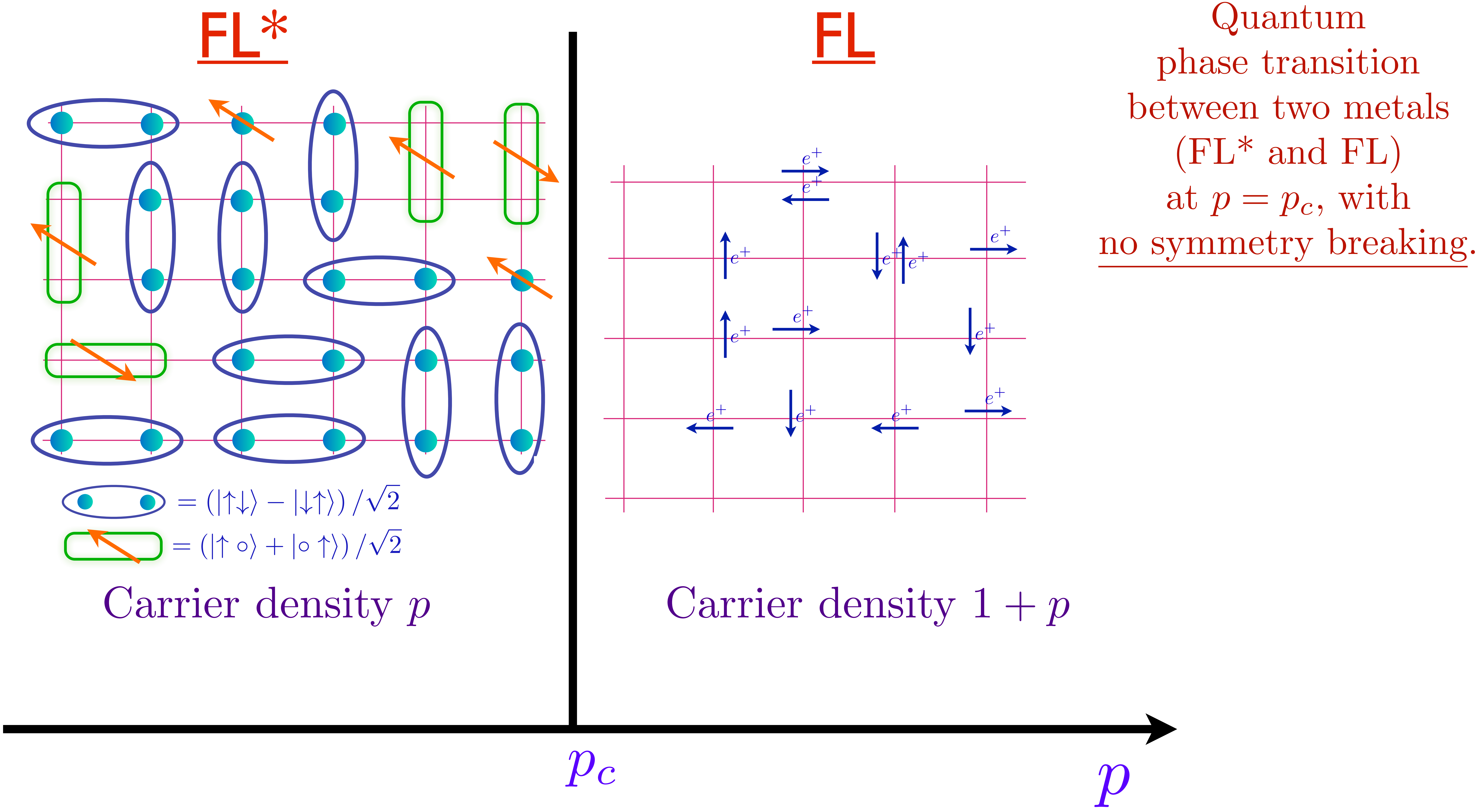
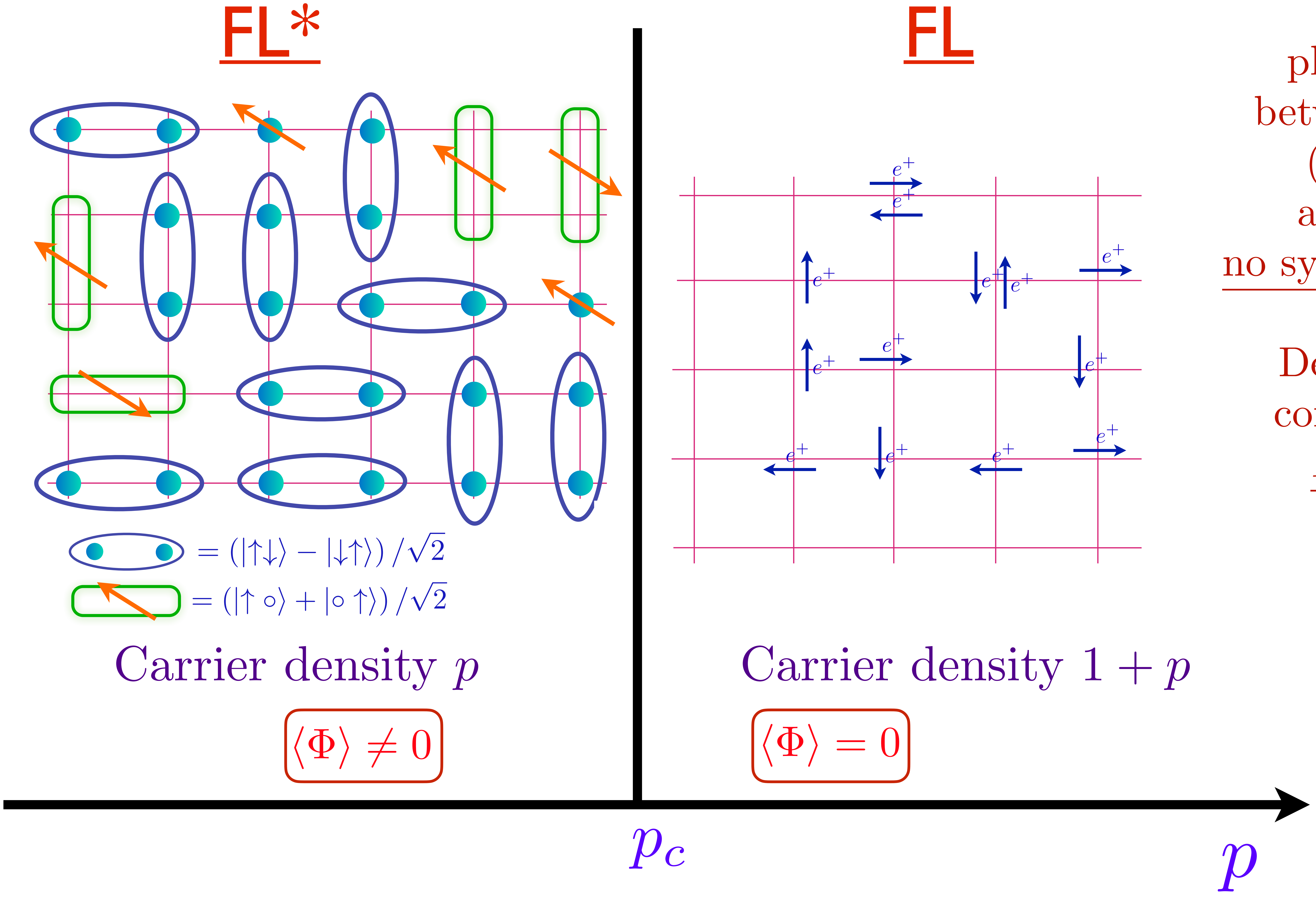
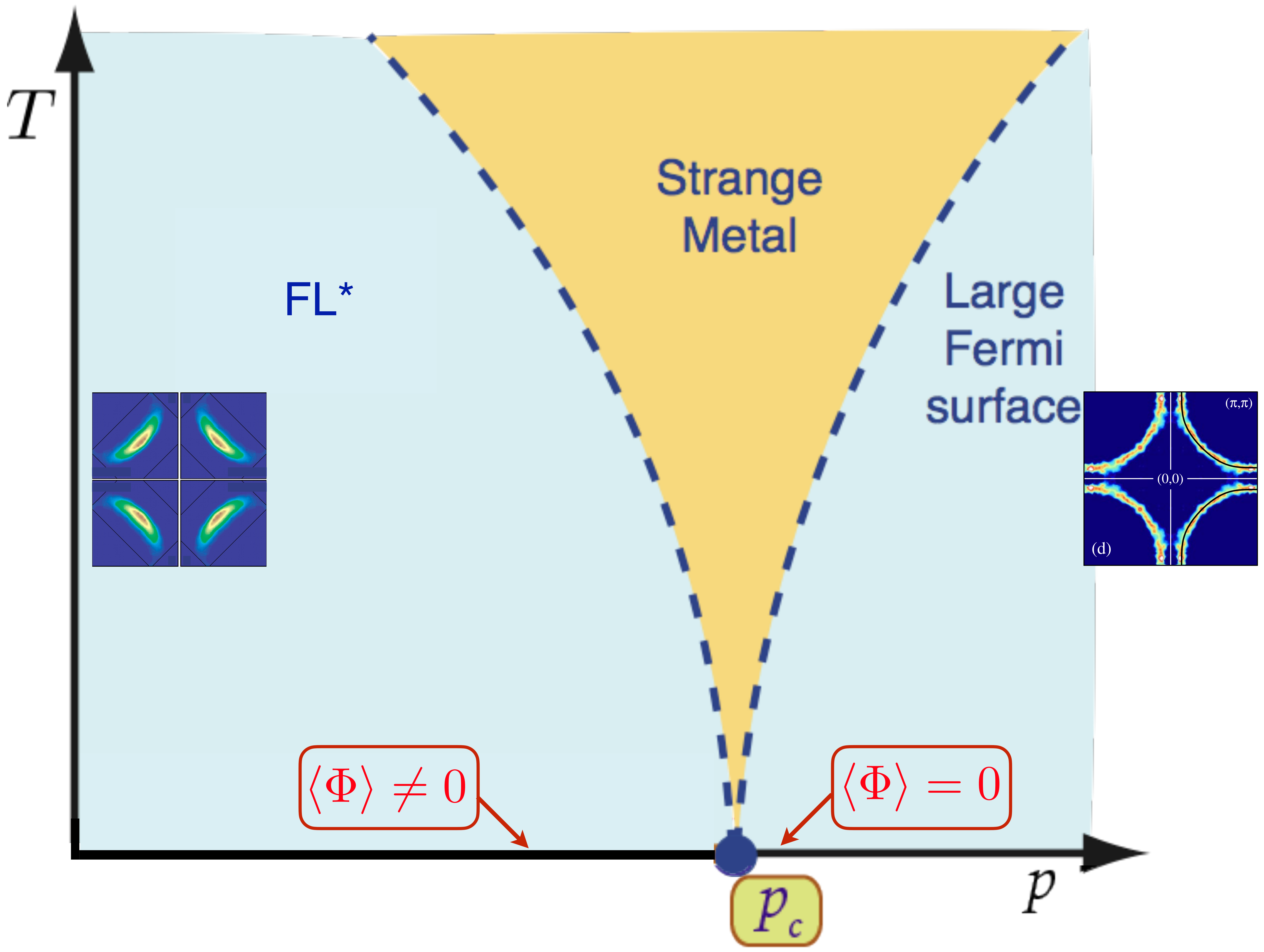
The cuprate phase diagram


FL*
FL
Quantum phase transition between two metals (FL* and FL) at $p = p_c$, with no symmetry breaking.
$= \left(|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle\right)/\sqrt{2}$
$= \left(|\uparrow \circ\rangle + |\circ \uparrow\rangle\right)/\sqrt{2}$
Carrier density p
Carrier density $1 + p$
e^+
p_c
p



Quantum phase transition between two metals (FL* and FL) at $p = p_c$, with no symmetry breaking.

Described by the condensation of a *Higgs* field Φ.

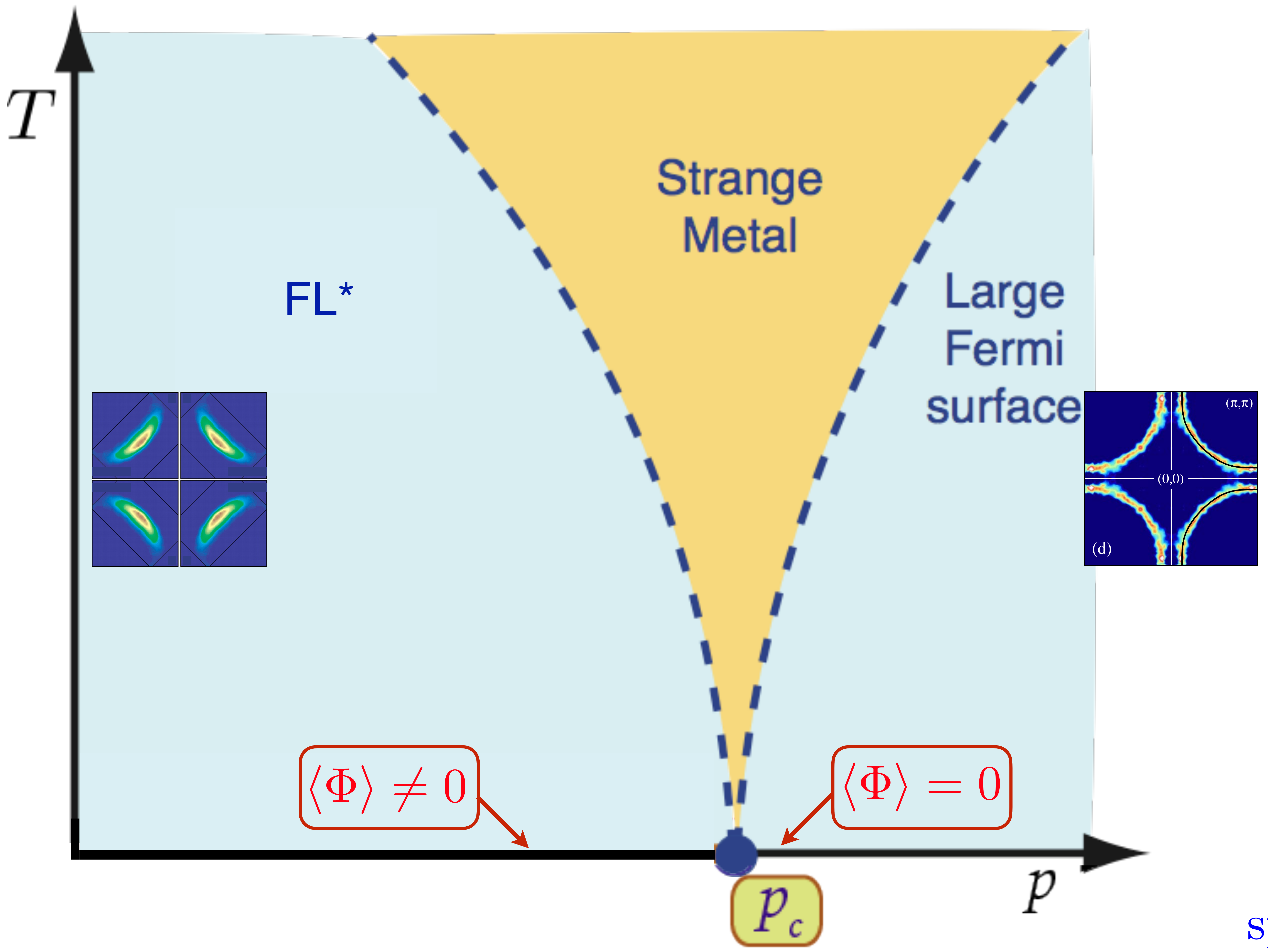


Quantum phase transition between two metals (FL* and FL) at $p = p_c$, with no symmetry breaking.

Described by the condensation of a *Higgs* field Φ.

Strange metal is obtained from the $T > 0$ quantum criticality of the FL-FL* transition, *provided* there is momentum relaxation.

Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)



Quantum phase transition between two metals (FL* and FL) at $p = p_c$, with no symmetry breaking.

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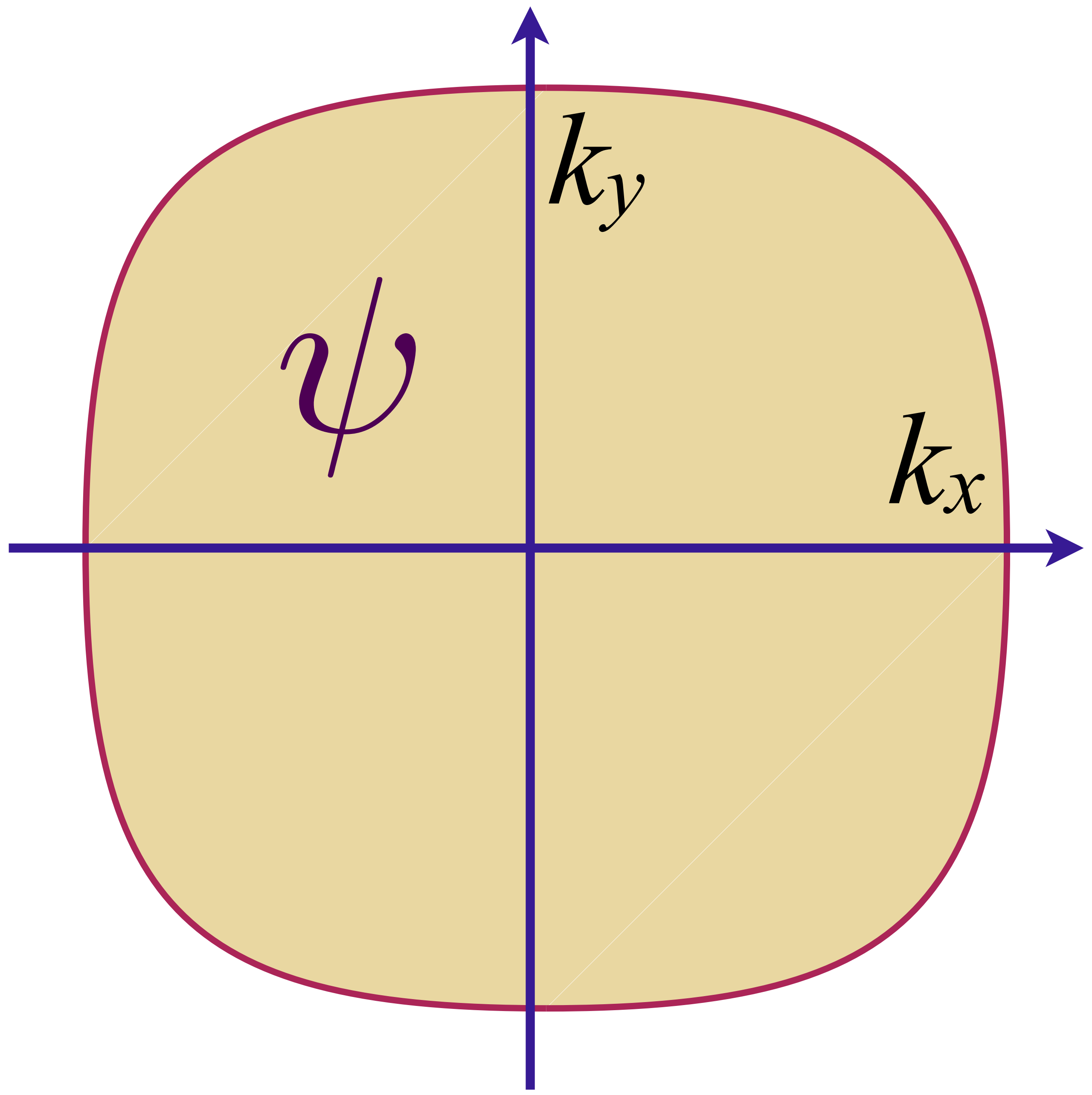
Strange metal is obtained from the $T > 0$ quantum criticality of the FL-FL* transition, *provided* there is momentum relaxation. At low T this requires spatial disorder: spatial variation in the value of p_c.

Ya-Hui Zhang and S. S., PRR **2**, 023172 (2020)

2d-YSYK model: Fermi surface + Higgs boson with interaction disorder

$$\mathcal{L}_\psi = \psi_{\boldsymbol{k}}^\dagger \left(\frac{\partial}{\partial \tau} + \varepsilon(\boldsymbol{k}) \right) \psi_{\boldsymbol{k}}$$



$$+ v(\boldsymbol{r}) \psi^\dagger(\boldsymbol{r}) \psi(\boldsymbol{r})$$

Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})} = 0$, $\overline{v(\boldsymbol{r}) v(\boldsymbol{r}')} = v^2 \delta(\boldsymbol{r} - \boldsymbol{r}')$

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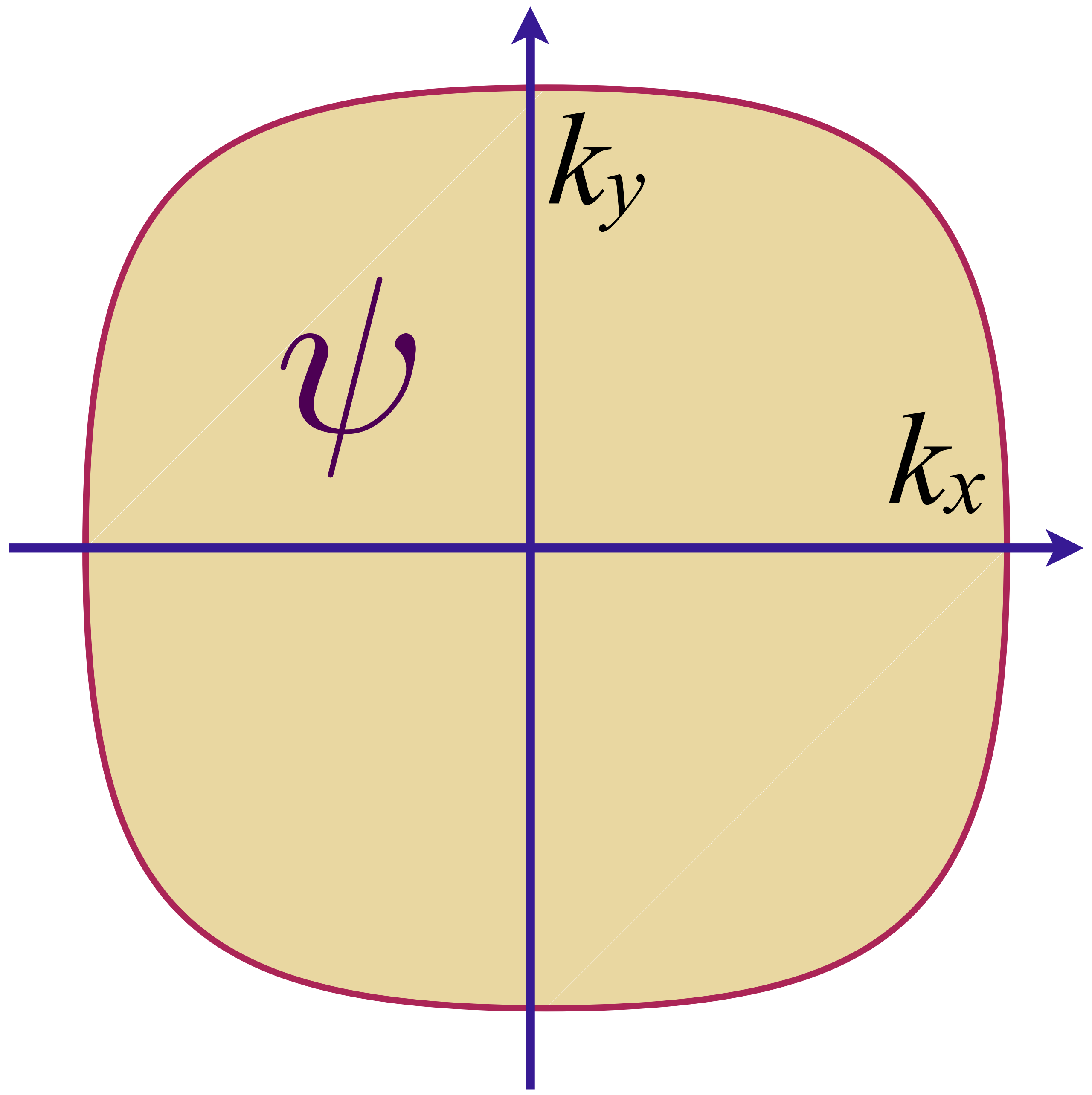

$$+ s\,[\Phi(\boldsymbol{r})]^2 + [g + g'(\boldsymbol{r})]\,\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})\,\Phi(\boldsymbol{r})$$

$$+ v(\boldsymbol{r})\psi^\dagger(\boldsymbol{r})\psi(\boldsymbol{r})$$

Φ^2 "mass" disorder $s \to s + \delta s(\boldsymbol{r})$ is strongly relevant; rescale Φ to move disorder to the Yukawa coupling.

Spatially random Yukawa coupling $g'(\boldsymbol{r})$ with $\overline{g'(\boldsymbol{r})} = 0$, $\overline{g'(\boldsymbol{r})g'(\boldsymbol{r}')} = g'^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

Spatially random potential $v(\boldsymbol{r})$ with $\overline{v(\boldsymbol{r})} = 0$, $\overline{v(\boldsymbol{r})v(\boldsymbol{r}')} = v^2\delta(\boldsymbol{r} - \boldsymbol{r}')$

Yukawa-Sachdev-Ye-Kitaev model

$$\mathcal{H} = -\mu \sum_i \psi_i^\dagger \psi_i + \sum_\ell \frac{1}{2}\left(\pi_\ell^2 + \omega_0^2 \phi_\ell^2\right) + \frac{1}{N} \sum_{ij\ell} g_{ij\ell} \psi_i^\dagger \psi_j \phi_\ell$$

with $g_{ij\ell}$ independent random numbers with zero mean.

W. Fu, D. Gaiotto, J. Maldacena, and S. Sachdev, PRD **95**, 026009 (2017)
J. Murugan, D. Stanford, and E. Witten, JHEP 08, 146 (2017)
A. A. Patel and S. Sachdev, PRB **98**, 125134 (2018)
E. Marcus and S. Vandoren, JHEP 01, 166 (2018)
Yuxuan Wang, PRL **124**, 017002 (2020)
I. Esterlis and J. Schmalian, PRB **100**, 115132 (2019)
Yuxuan Wang and A. V. Chubukov, PRR **2**, 033084 (2020)
E. E. Aldape, T. Cookmeyer, A. A. Patel, and E. Altman, PRB **105**, 235111 (2022)
Jaewon Kim, E. Altman, and Xiangyu Cao, PRB **103**, 081113 (2021)
W. Wang, A. Davis, G. Pan, Yuxuan Wang, and Zi Yang Meng, PRB **103**, 195108 (2021)
I. Esterlis, H. Guo, A. A. Patel, and S. Sachdev, PRB **103**, 235129 (2021).

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with $g_{ij\ell}$ independent random numbers with zero mean. The large N equations for the Green's functions and self energies of the fermions (G, Σ) and bosons (D, Π) are

$$G(i\omega_n) = \frac{1}{i\omega_n + \mu - \Sigma(i\omega_n)} \quad , \quad D(i\omega_n) = \frac{1}{\omega_n^2 + \omega_0^2 - \Pi(i\omega_n)}$$

$$\Sigma(\tau) = g^2 G(\tau) D(\tau) \quad , \quad \Pi(\tau) = -g^2 G(\tau) G(-\tau)$$

Make the low frequency ansatz

$$G(i\omega) \sim -i\,\mathrm{sgn}(\omega)|\omega|^{-(1-2\Delta)} \quad , \quad D(i\omega) \sim |\omega|^{1-4\Delta} \quad , \quad \frac{1}{4} < \Delta < \frac{1}{2}$$

A consistent solution exists for

$$\frac{4\Delta - 1}{2(2\Delta - 1)[\sec(2\pi\Delta) - 1]} = 1 \quad , \quad \Delta = 0.42037\ldots$$

I. Esterlis and J. Schmalian,
PRB **100**, 115132 (2019)
See also Yuxuan Wang,
PRL **124**, 017002 (2020)

Yukawa-Sachdev-Ye-Kitaev model

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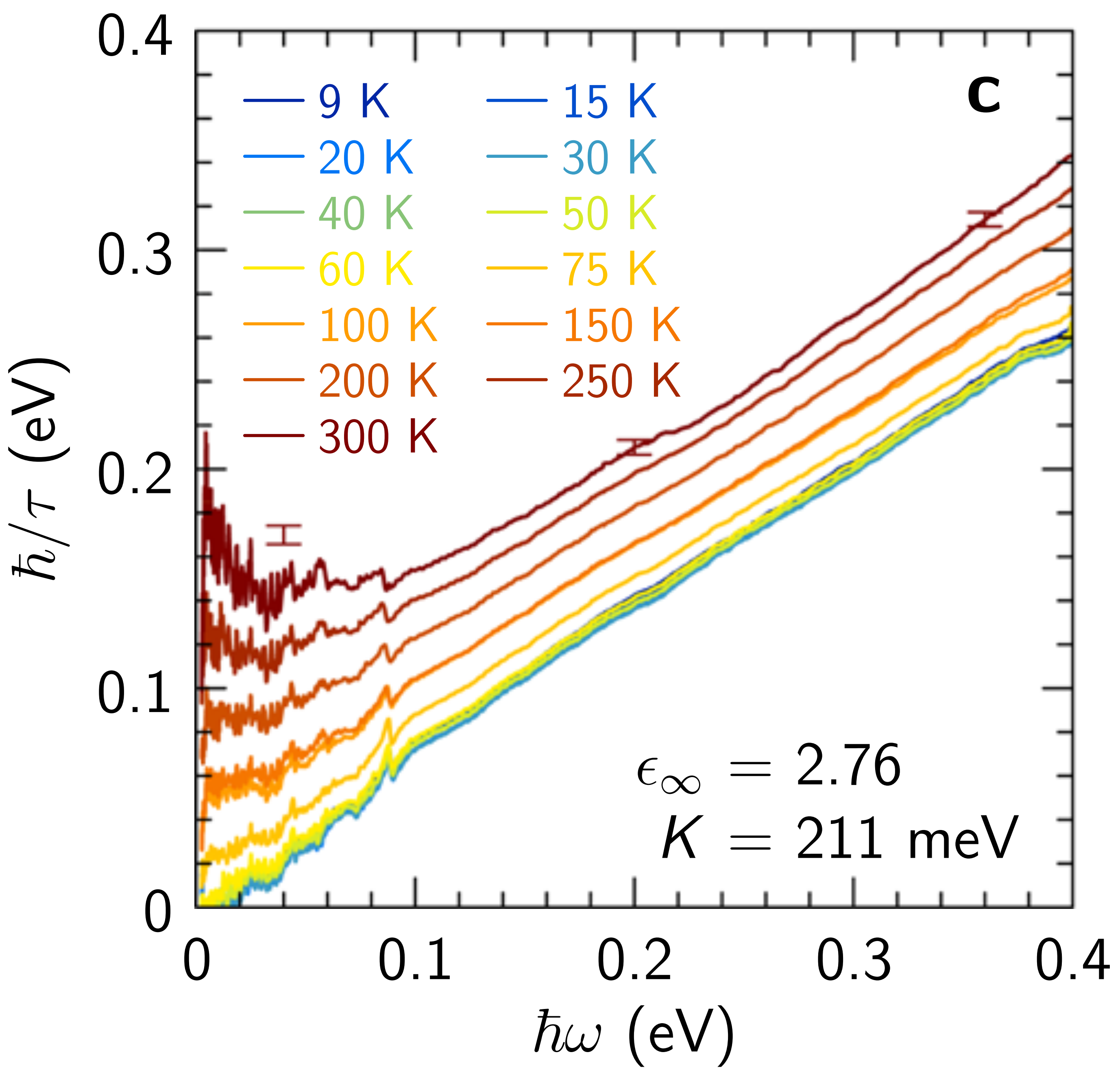
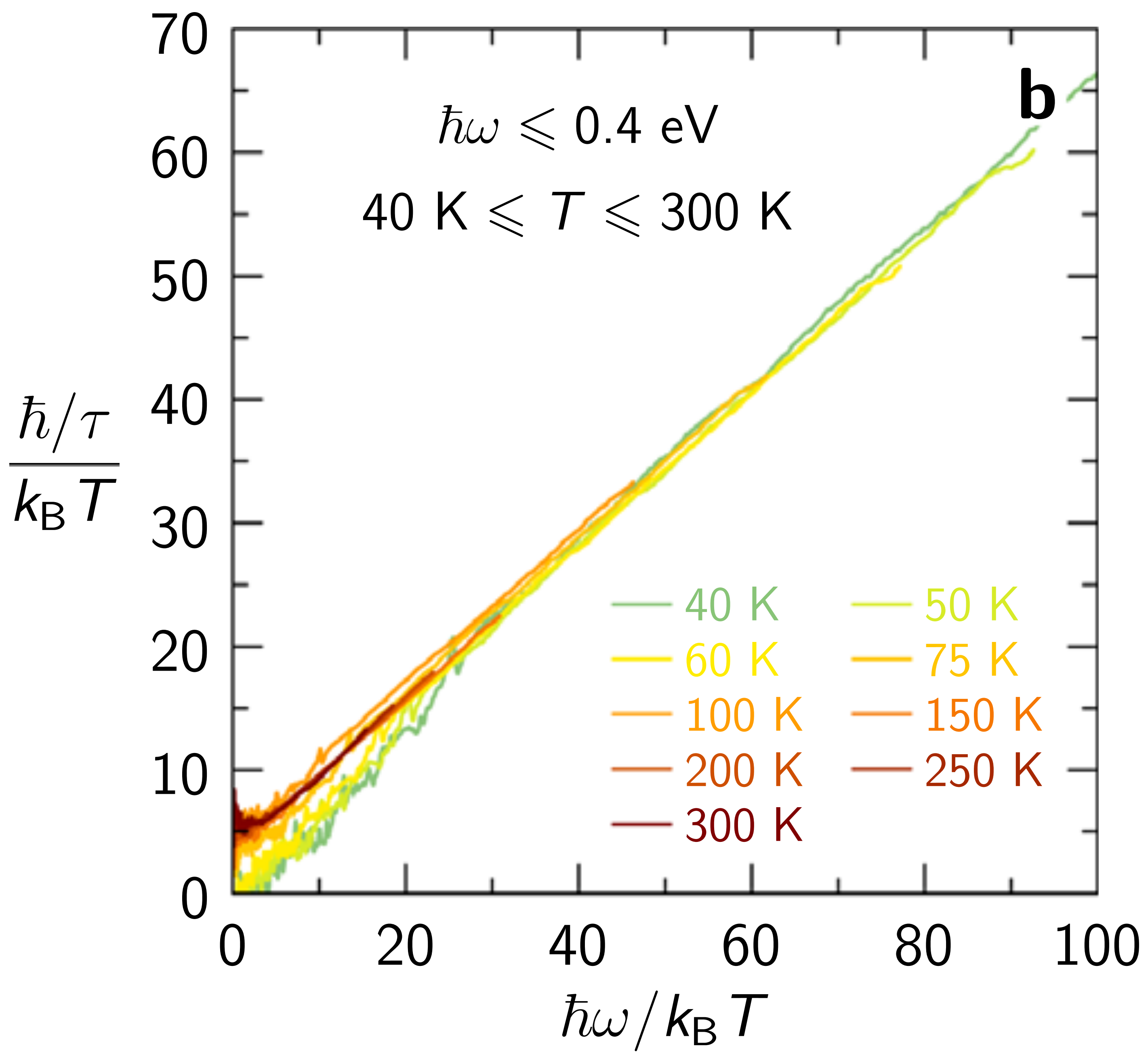
$$\Sigma(\tau) = g^2 G(\tau) D(\tau) \quad , \quad \Pi(\tau) = -g^2 G(\tau) G(-\tau)$$

At $T > 0$, solutions are fully characterized by a universal frequency-dependent relaxation time,

$$\frac{\hbar}{\tau(\omega)} = k_B T \, \Phi_\tau \left(\frac{\hbar\omega}{k_B T} \right)$$

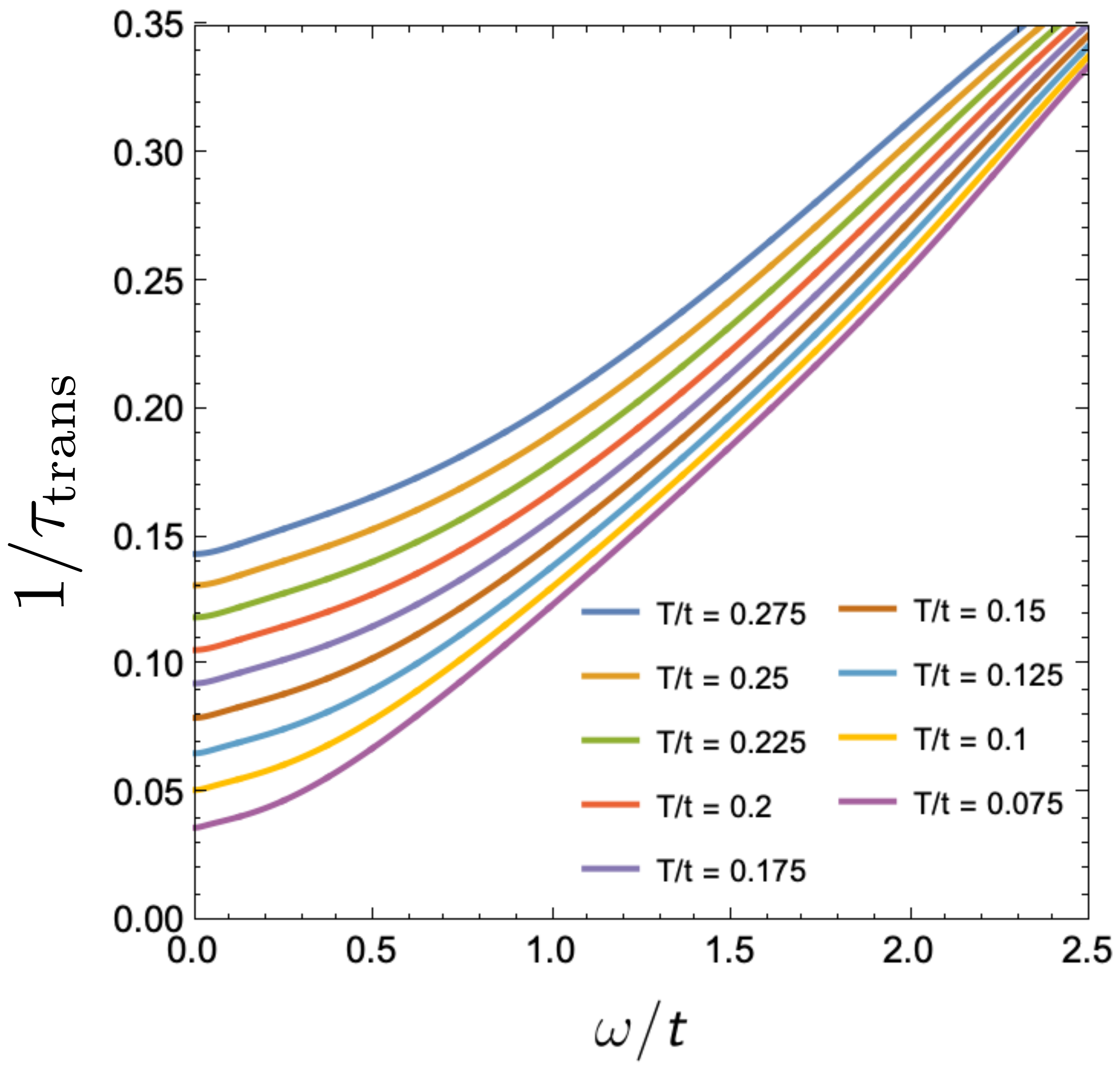
where Φ_τ is a known universal function.

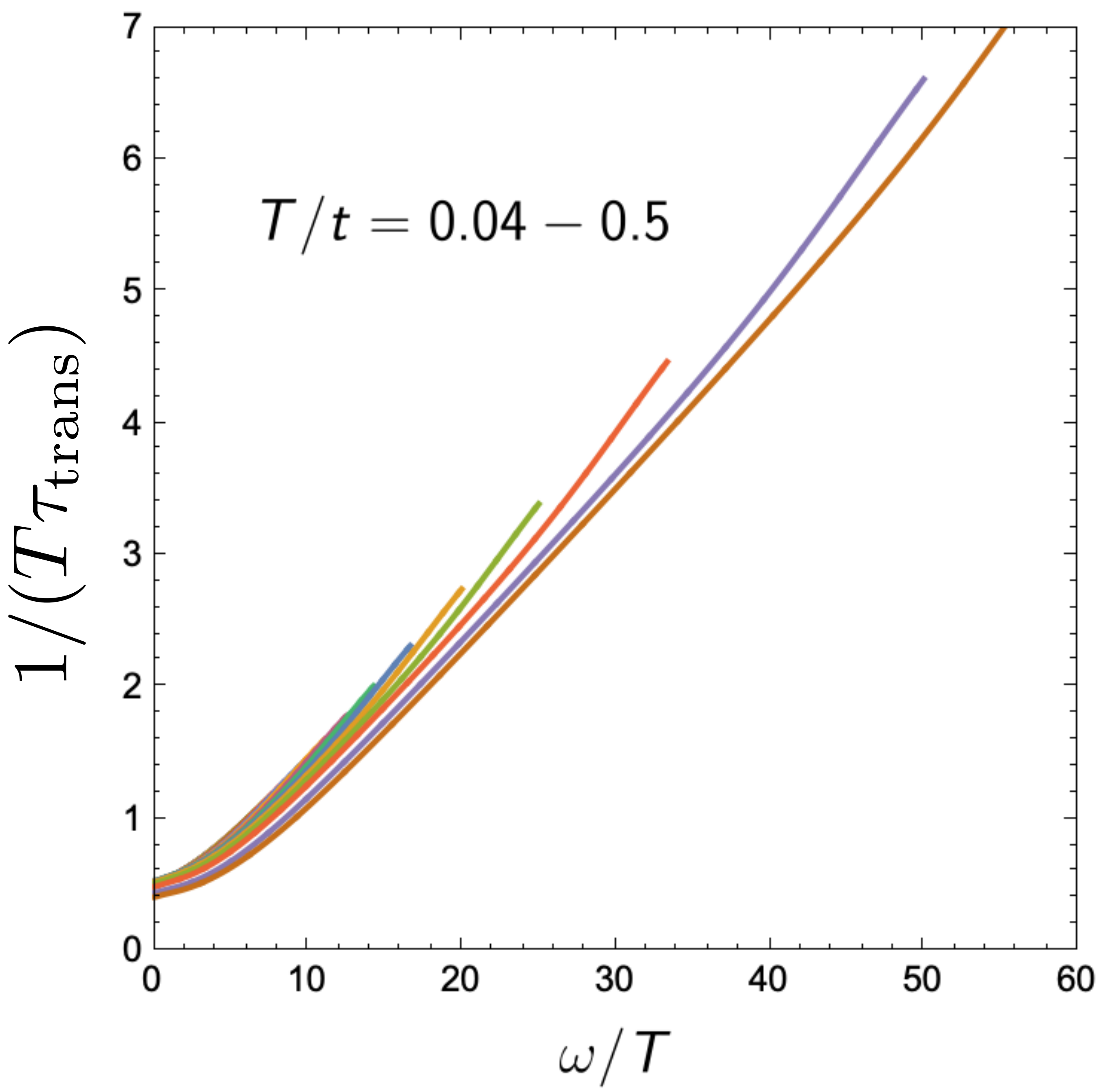




$$\sigma(\omega) = i\frac{e^2 K/(\hbar d_c)}{\hbar\omega\dfrac{m^*(\omega)}{m} + i\dfrac{\hbar}{\tau(\omega)}}$$

From optical conductivity data of Michon et al. (2023)



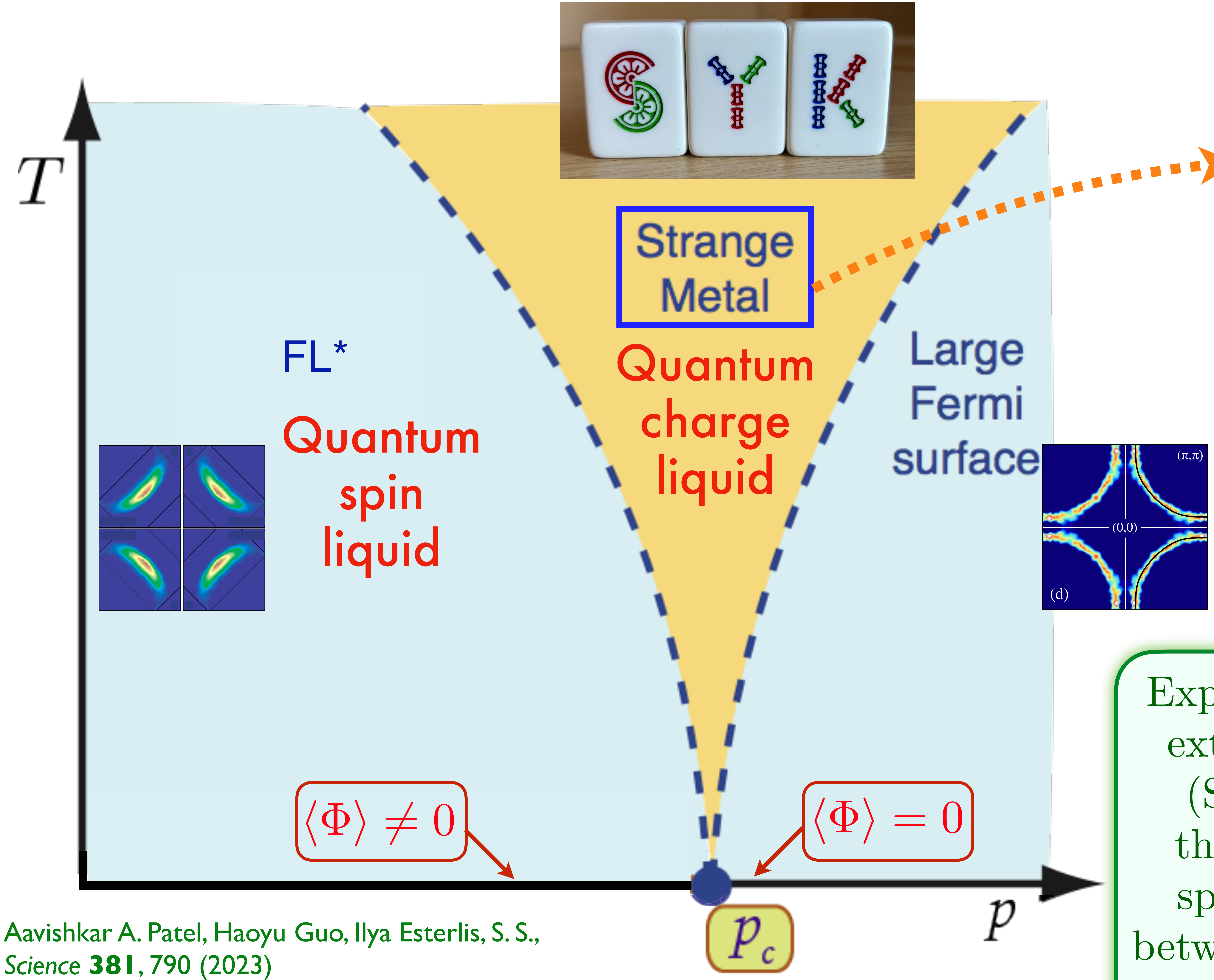


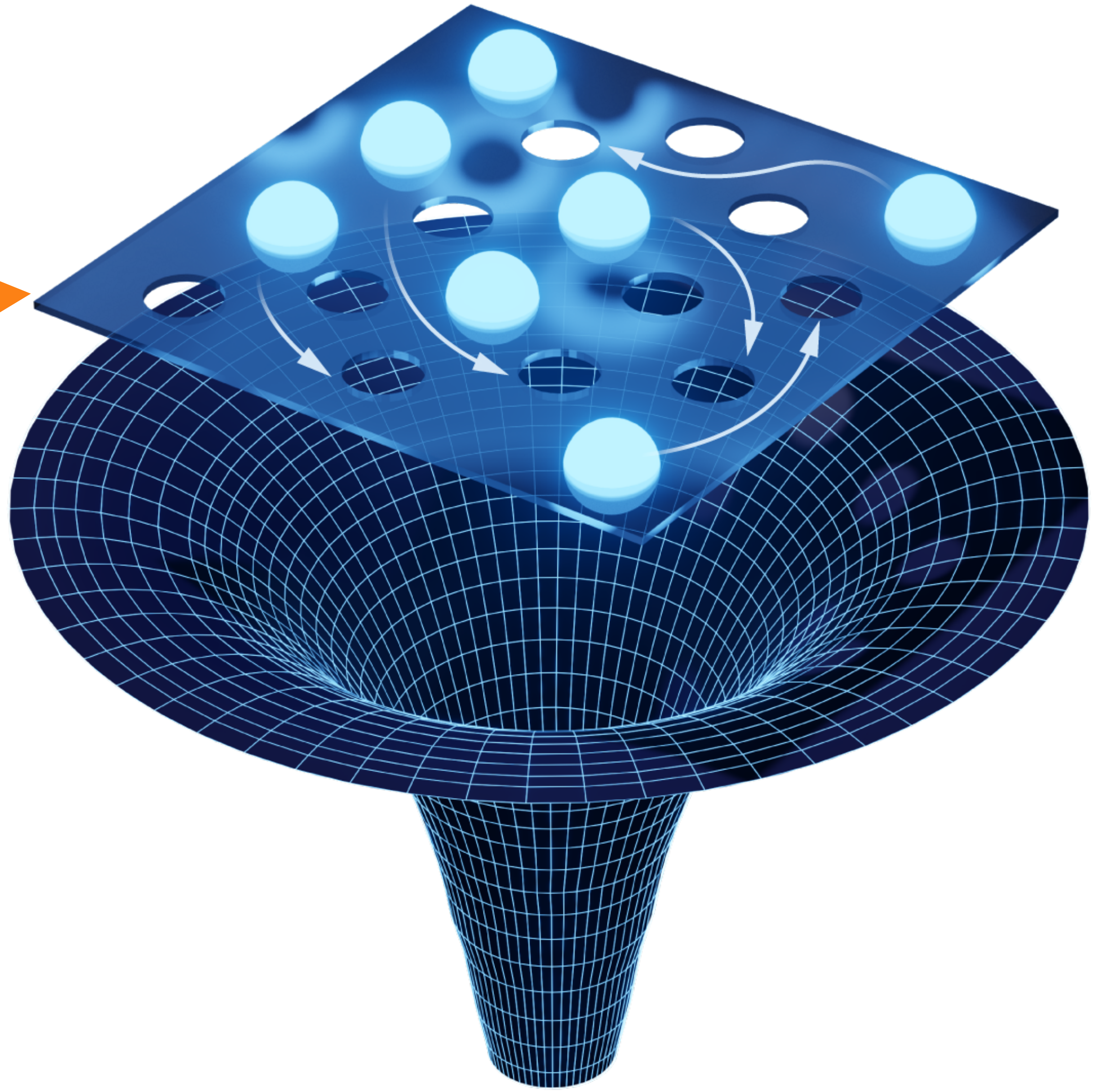
$$\frac{\hbar}{\tau(\omega)} = k_B T\,\Phi_\tau\left(\frac{\hbar\omega}{k_B T}\right)$$

2d-YSYK theory

Aavishkar A. Patel, Haoyu Guo, Ilya Esterlis, S. S., *Science* **381**, 790 (2023)

Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esterlis, PRL **133**, 186502 (2024)





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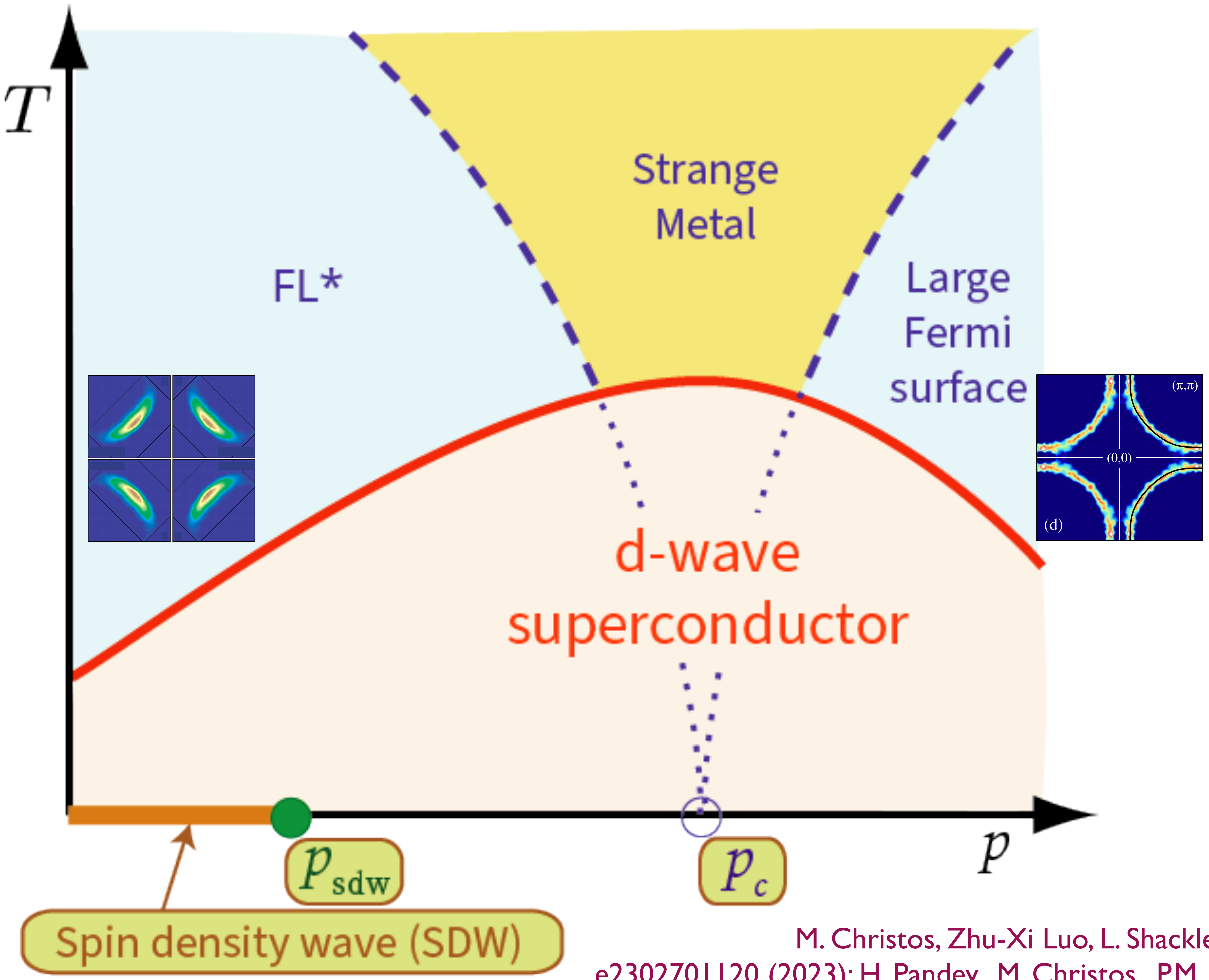
Chenyuan Li, Aavishkar A. Patel, Haoyu Guo, Davide Valentinis, Jorg Schmalian, S.S., Ilya Esterlis, PRL **133**, 186502 (2024)

Explain with a local, two-dimensional extension of the Sachdev-Ye-Kitaev (SYK) model of mobile electrons, the 2D-Yukawa-SYK model with a spatially random Yukawa coupling between the large Fermi surface and Φ: a critical charge liquid

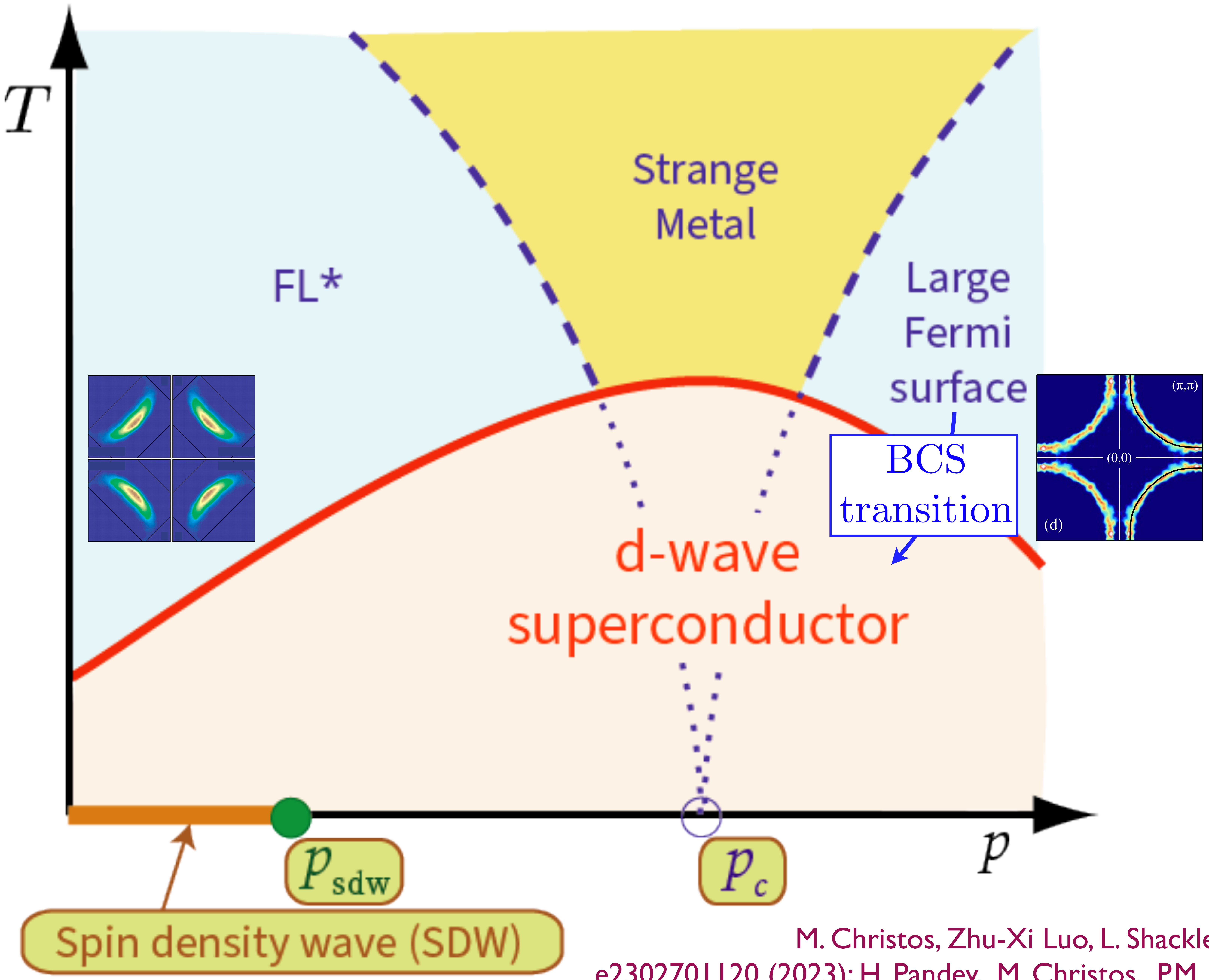
S. S., Physica Status Solidi B Basic Research **247**, 537 (2010)



Both metals lead to the same d-wave superconductor at lower temperatures, and so there is no transition at $p = p_c$ within the superconducting state.

M. Christos, Zhu-Xi Luo, L. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120,** e2302701120 (2023); H. Pandey, M. Christos, P.M. Bonetti, R. Shanker, S. Sharma, S.S., arXiv:2507.05336

S. S., Physica Status Solidi B Basic Research **247**, 537 (2010)



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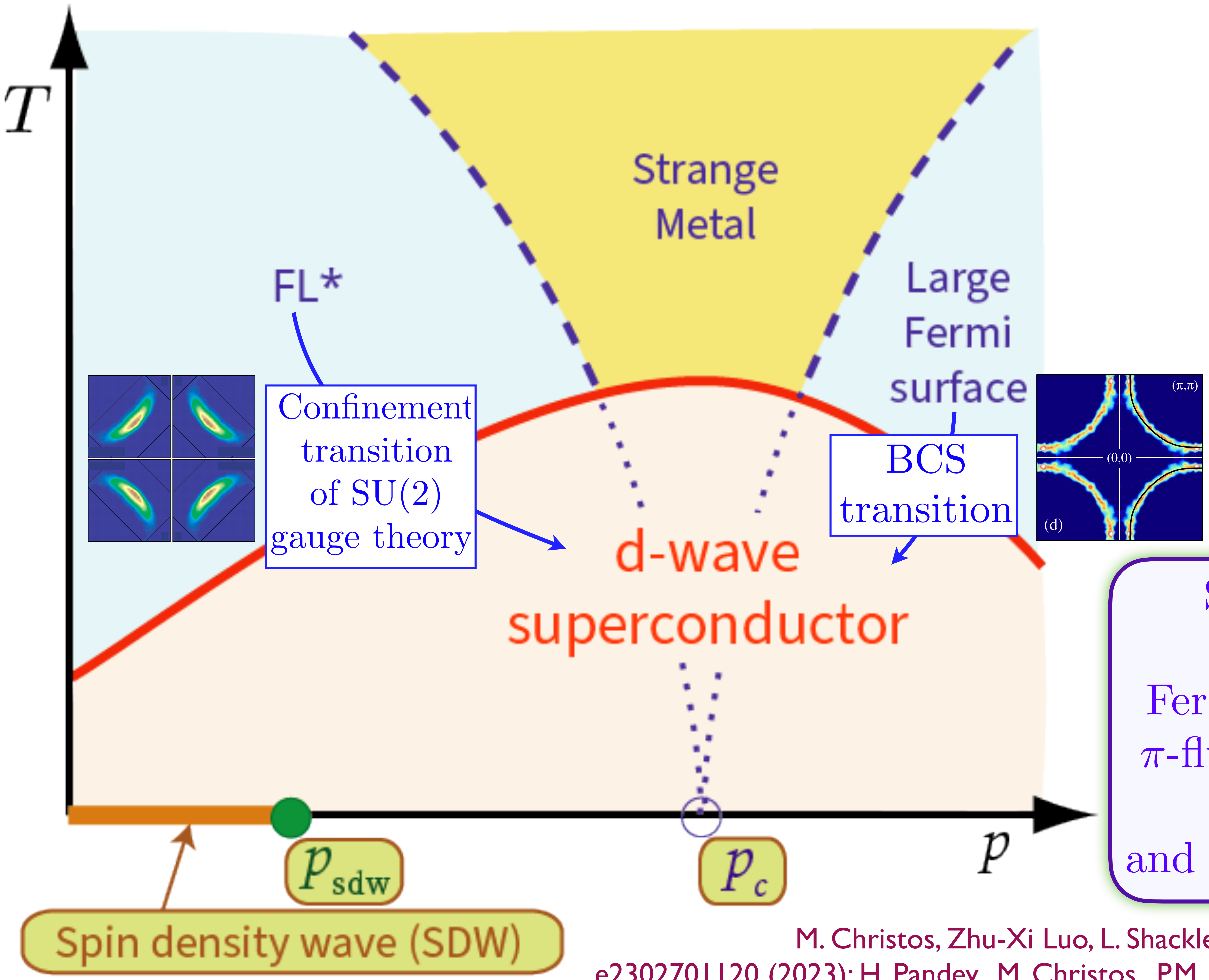

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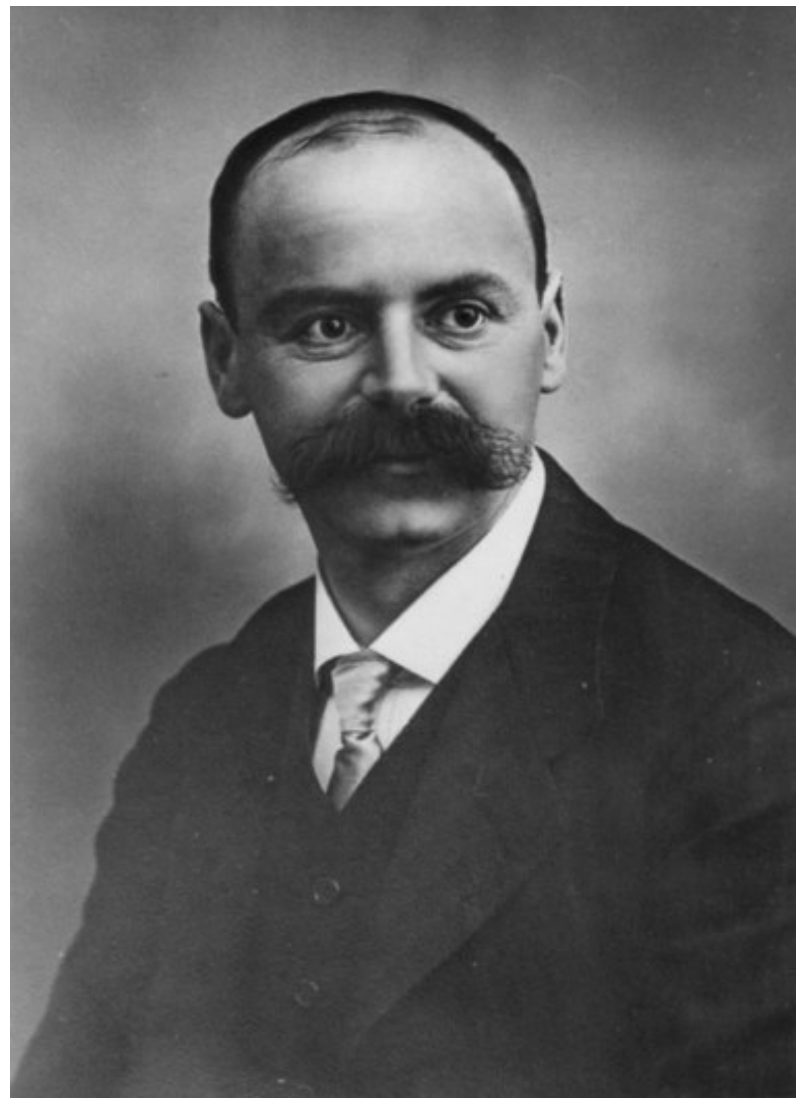
SU(2) gauge theory similar to Weinberg-Salam theory:
Fermionic spinons (*cf.* neutrinos) of π-flux state (Affleck-Marston, 1988),
electrons,
and SU(2) fundamental Higgs field B.

M. Christos, Zhu-Xi Luo, L. Shackleton, Ya-Hui Zhang, M. S. Scheurer, and S. S., PNAS **120**, e2302701120 (2023); H. Pandey, M. Christos, P.M. Bonetti, R. Shanker, S. Sharma, S.S., arXiv:2507.05336

The Sachdev-Ye-Kitaev model and black holes

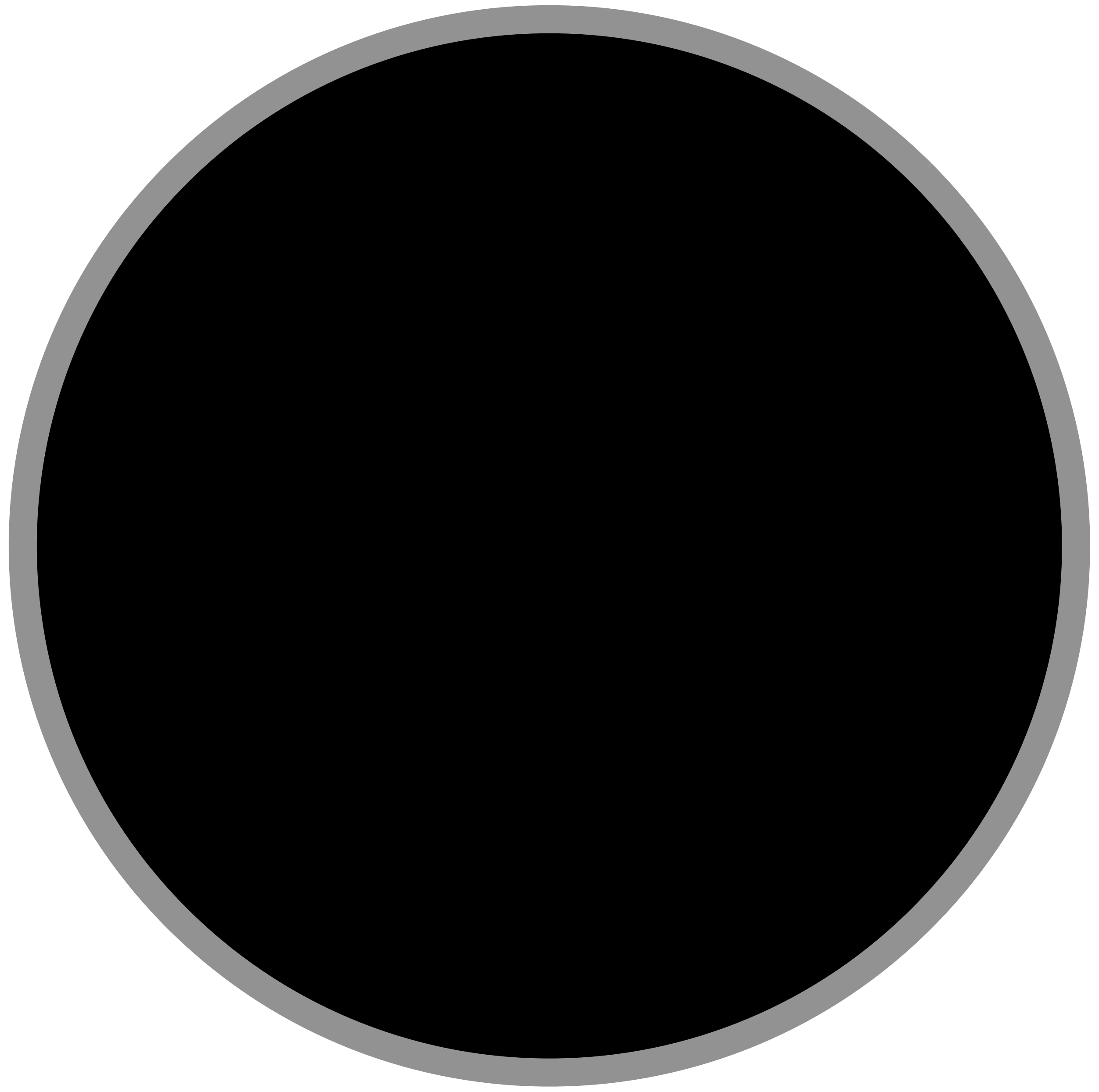
Black Holes

Objects so dense that light is gravitationally bound to them.



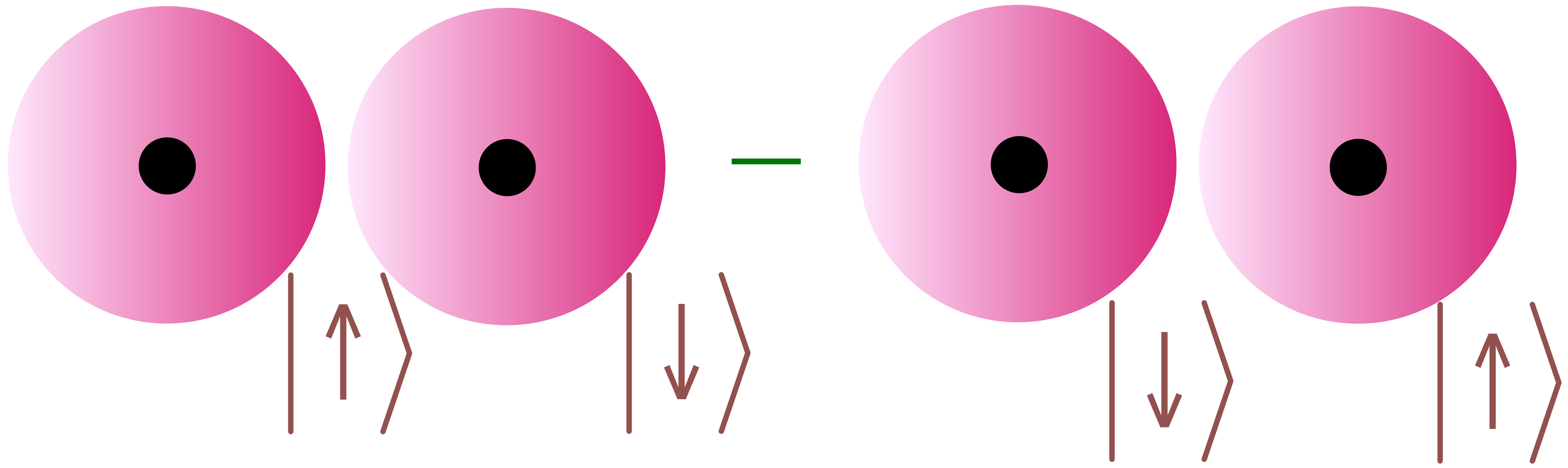
Karl Schwarzschild (1916)

Horizon radius $R = \frac{2GM}{c^2}$



G Newton's constant, c velocity of light, M mass of black hole
For $M =$ earth's mass, $R \approx 9\,mm$!

Quantum Entanglement across a black hole horizon



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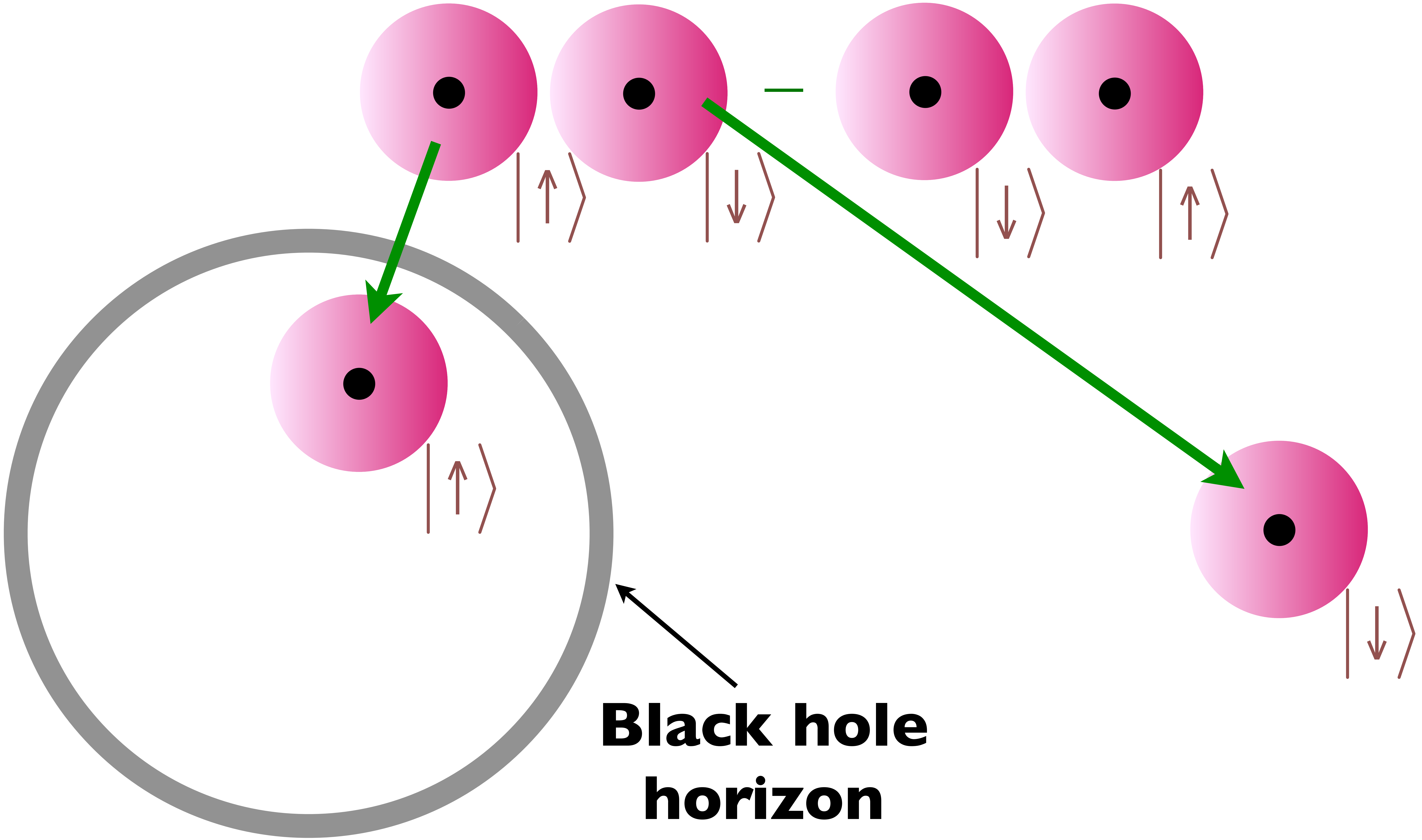

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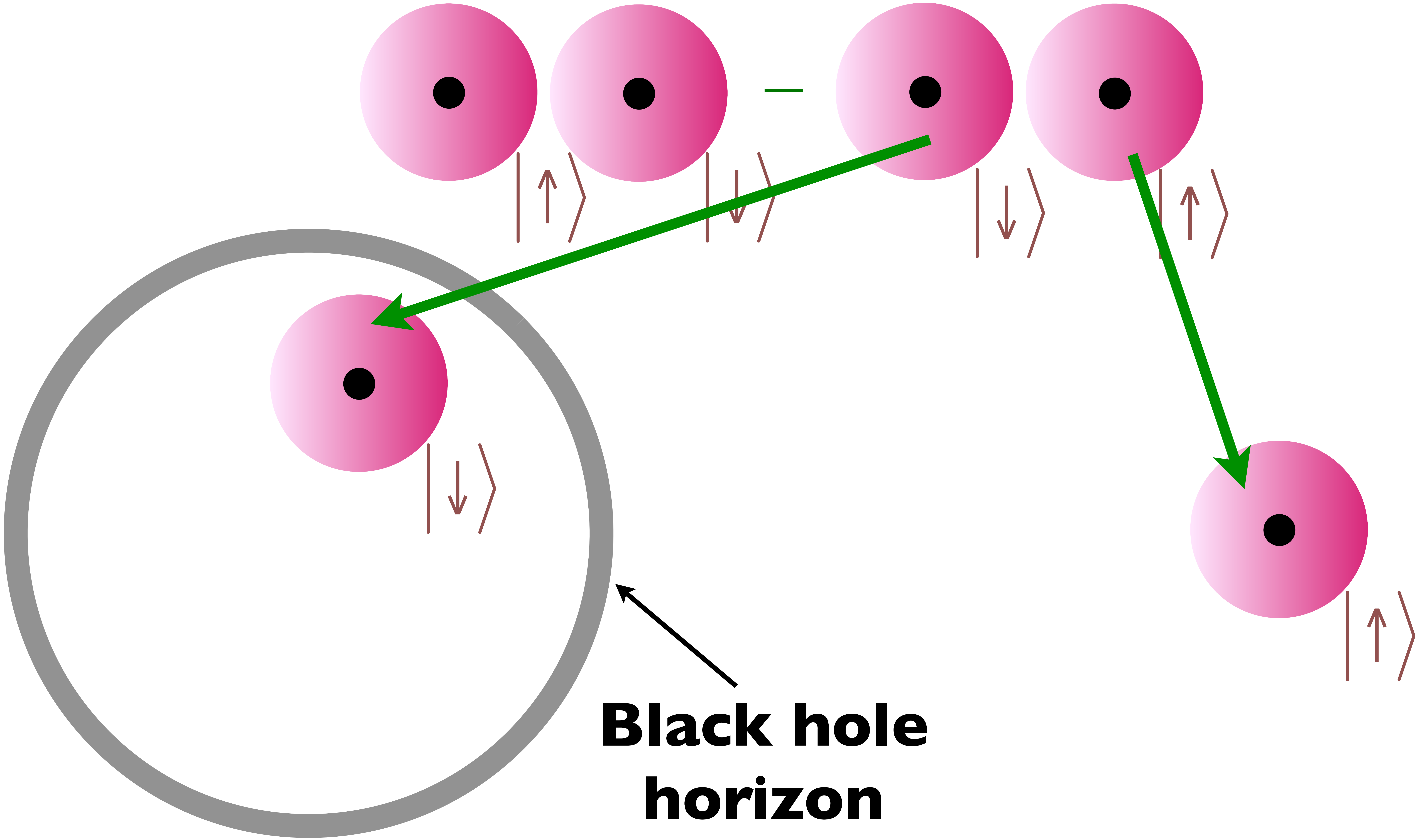
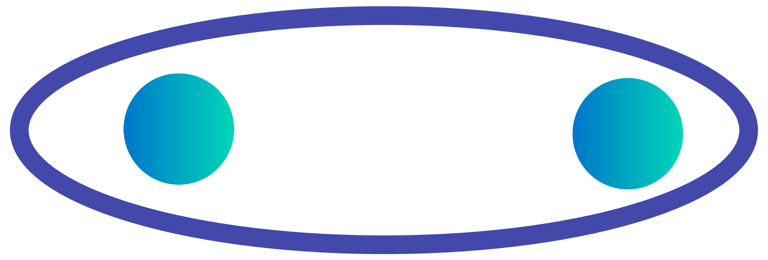
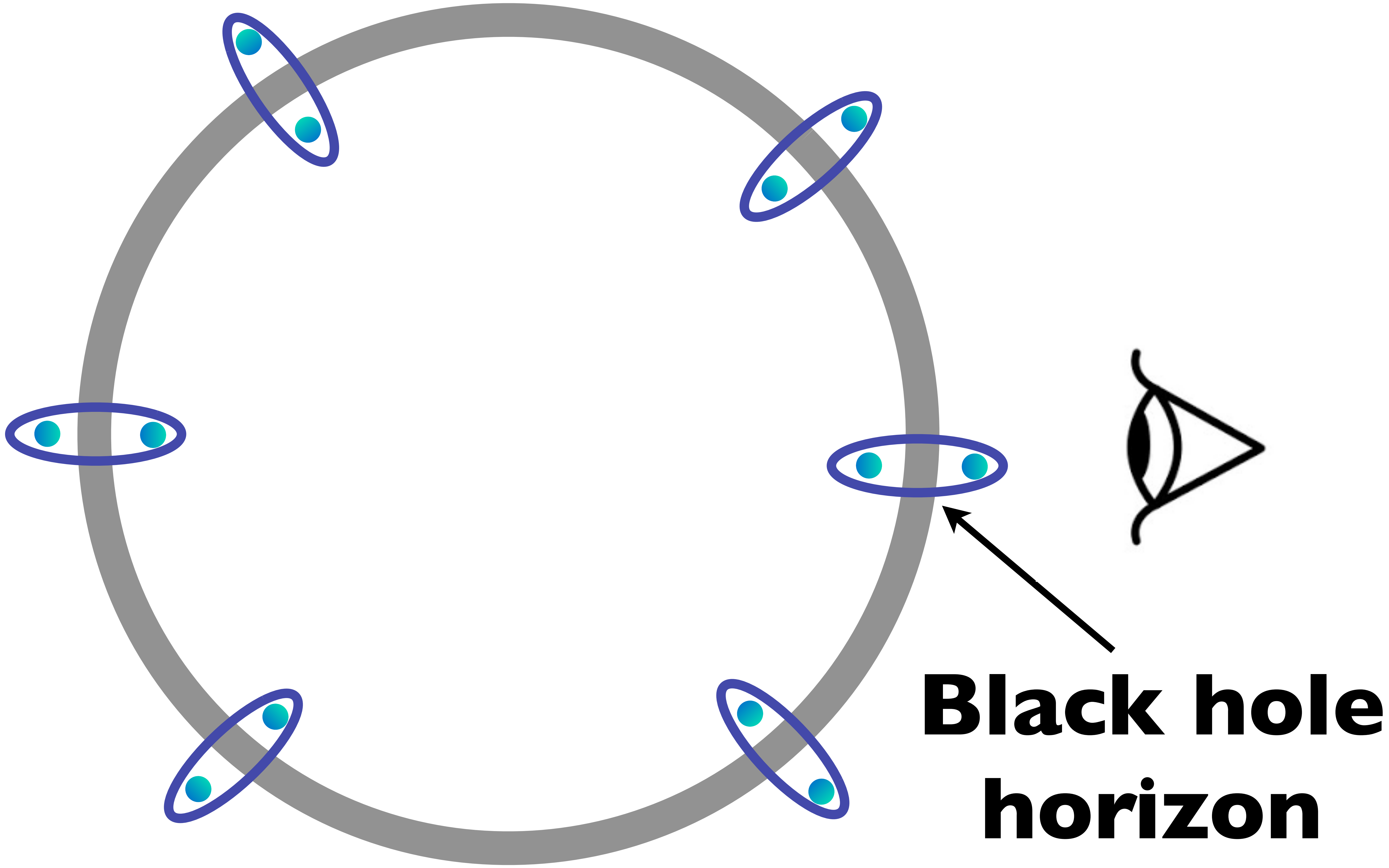


Quantum Entanglement across a black hole horizon

Quantum entanglement on the surface



$$= |\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle$$



By computations *outside* the black hole, Hawking obtained the black hole entropy

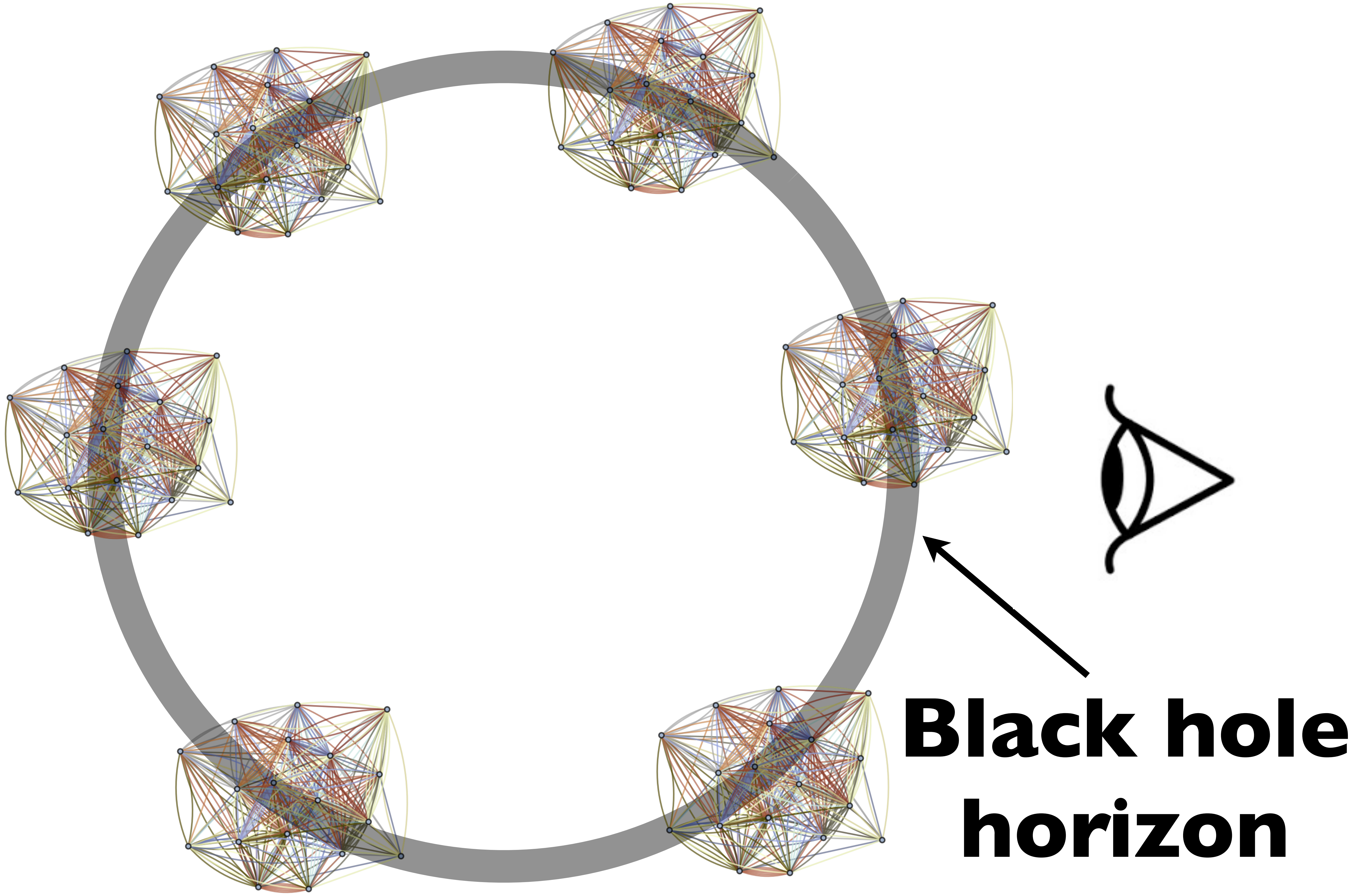
$$S = \frac{Ac^3}{4G\hbar}$$

where A is area of the black hole horizon.

All other systems have entropy proportional to their volume.

Quantum Entanglement across a black hole horizon

Quantum entanglement on the surface



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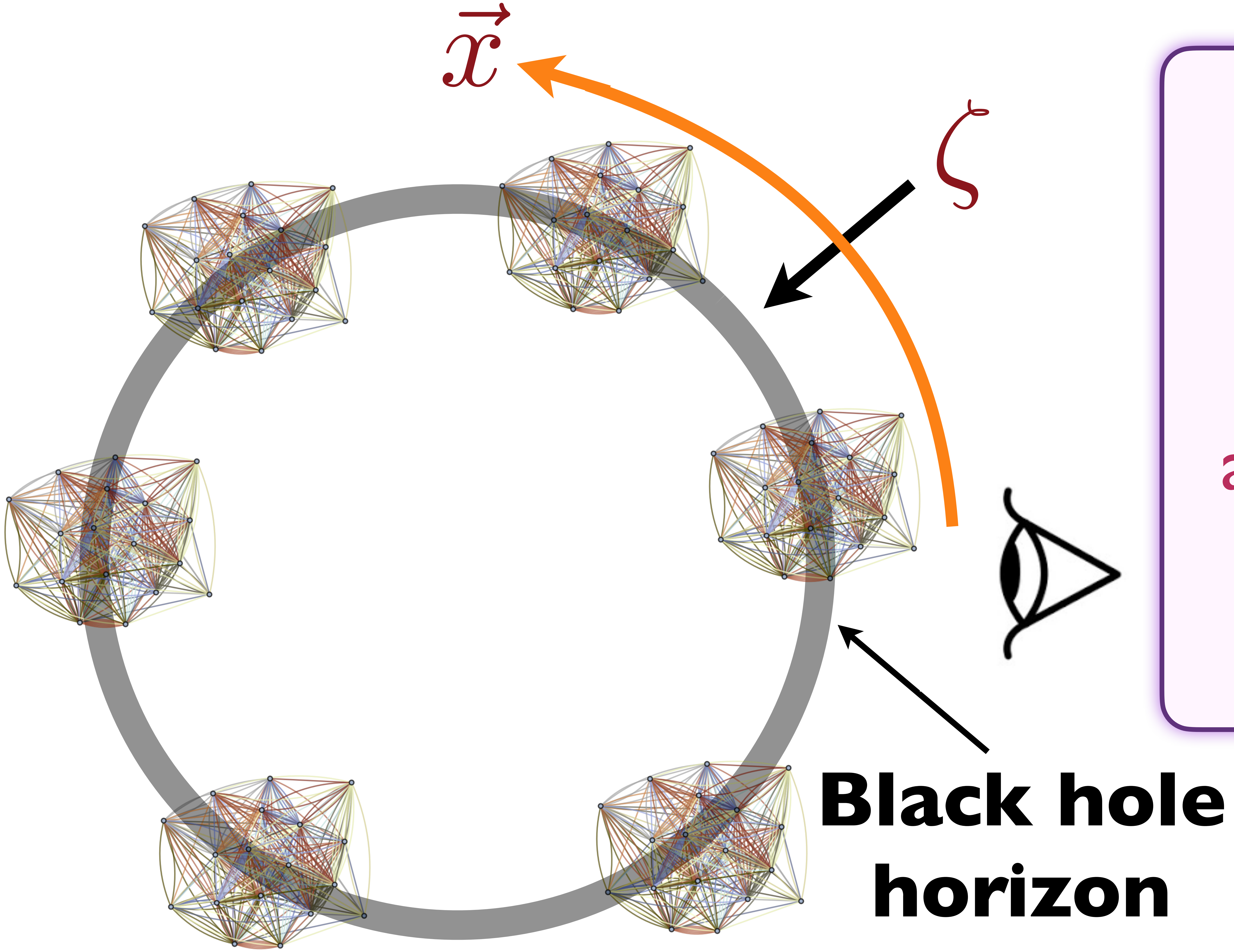
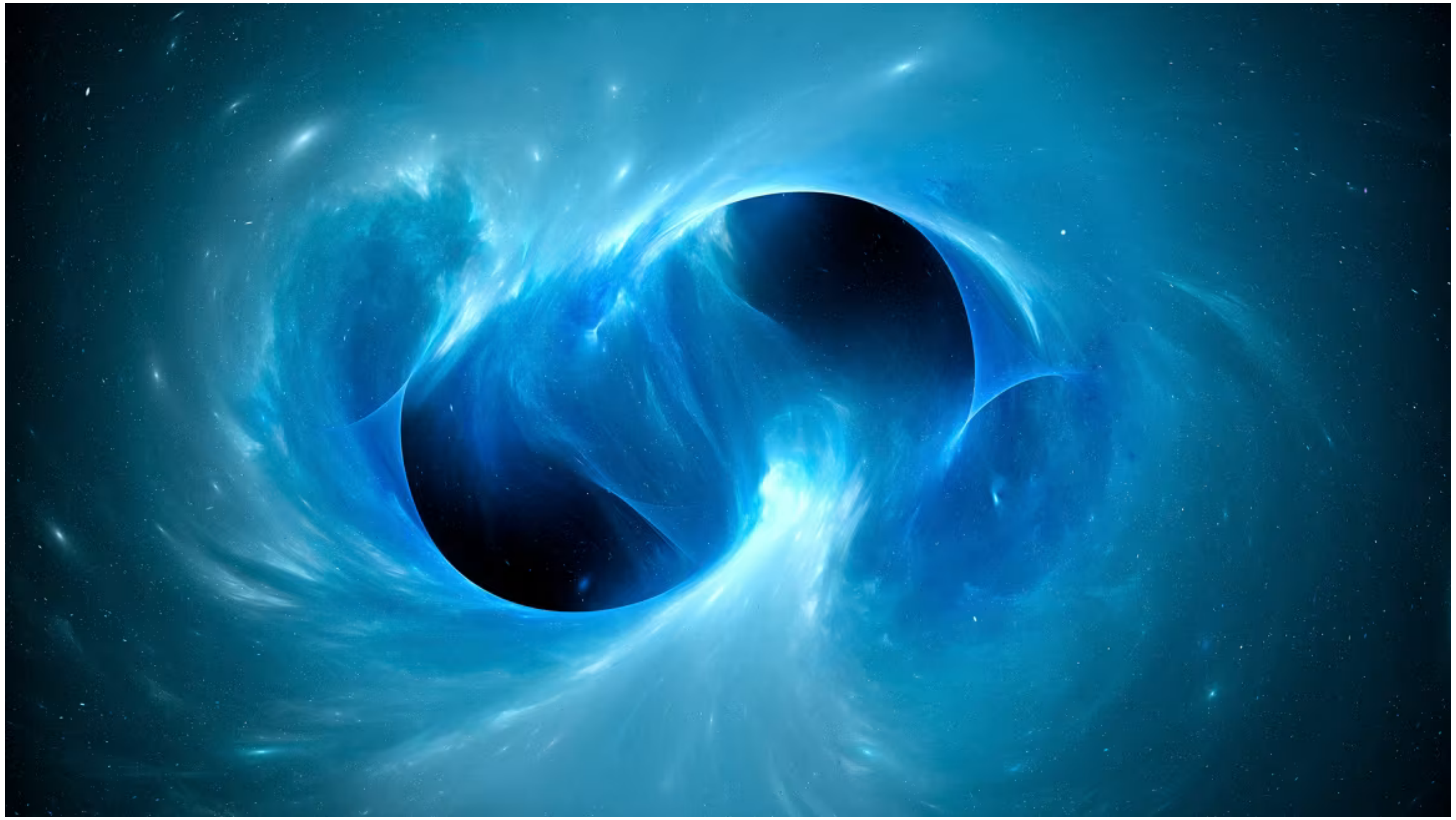
All other systems have entropy proportional to their volume.

Maxwell's electromagnetism
and Einstein's general relativity
allow black hole solutions with a net charge



The quantum versions of Maxwell's and Einstein's equations in ζ space and time are also the equations describing electron entanglement in the SYK model!

Quantum Entanglement across a black hole horizon



Sakkmesterke/Science Photo Library RF/Getty Images

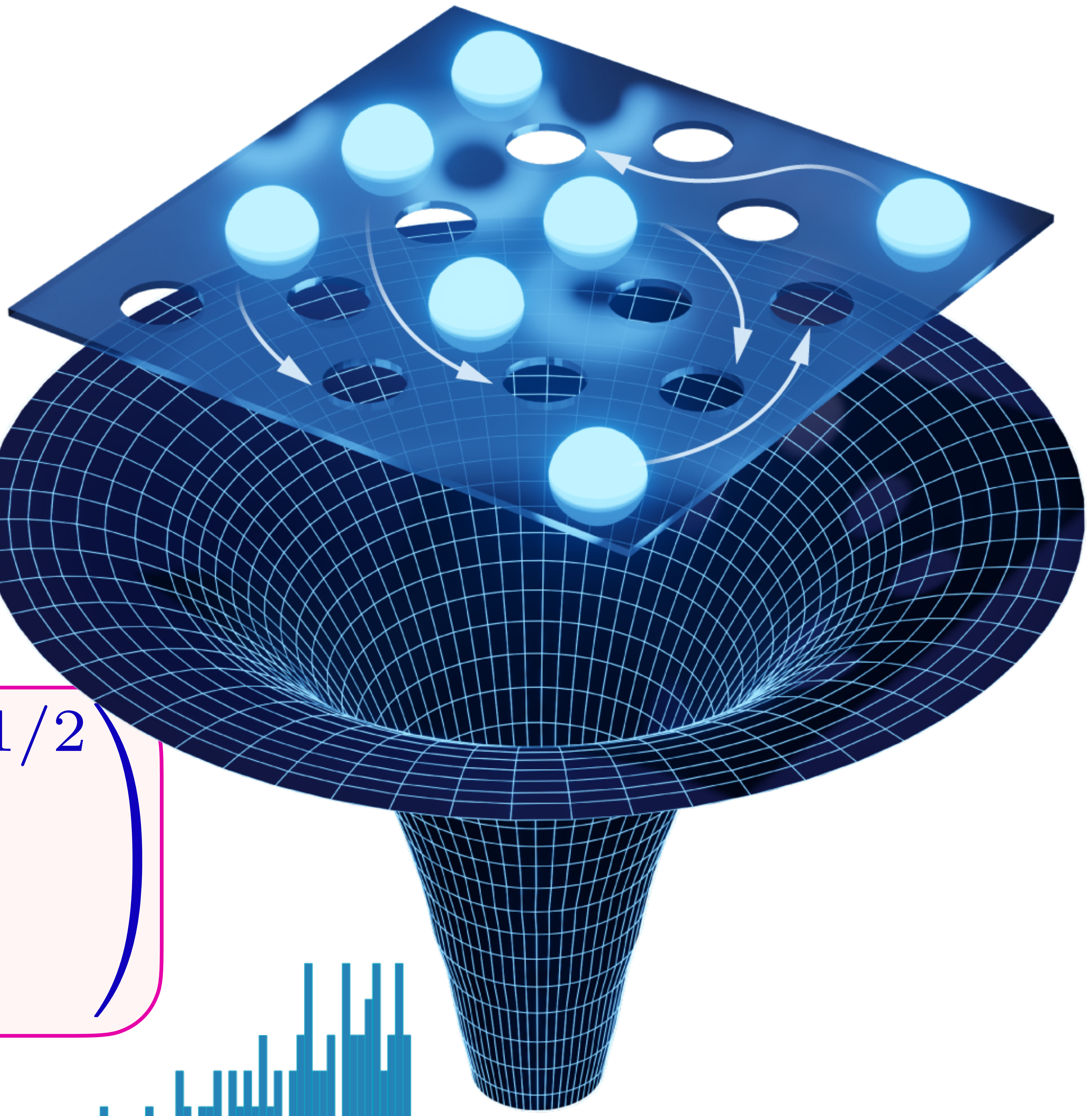
$$\tau_{\text{ring-down}} \sim \frac{\hbar}{k_B T}$$

Planckian dynamics!

T is the Hawking temperature of the black hole

D(E) of charged black holes from the SYK model

- For generic charged black holes in 3+1 dimensions with horizon area A_0 at $T = 0$ and fixed charge $\mathcal{Q}$ ($A_0 = 2G\mathcal{Q}^2/c^4$), the density of quantum states at small energy E is

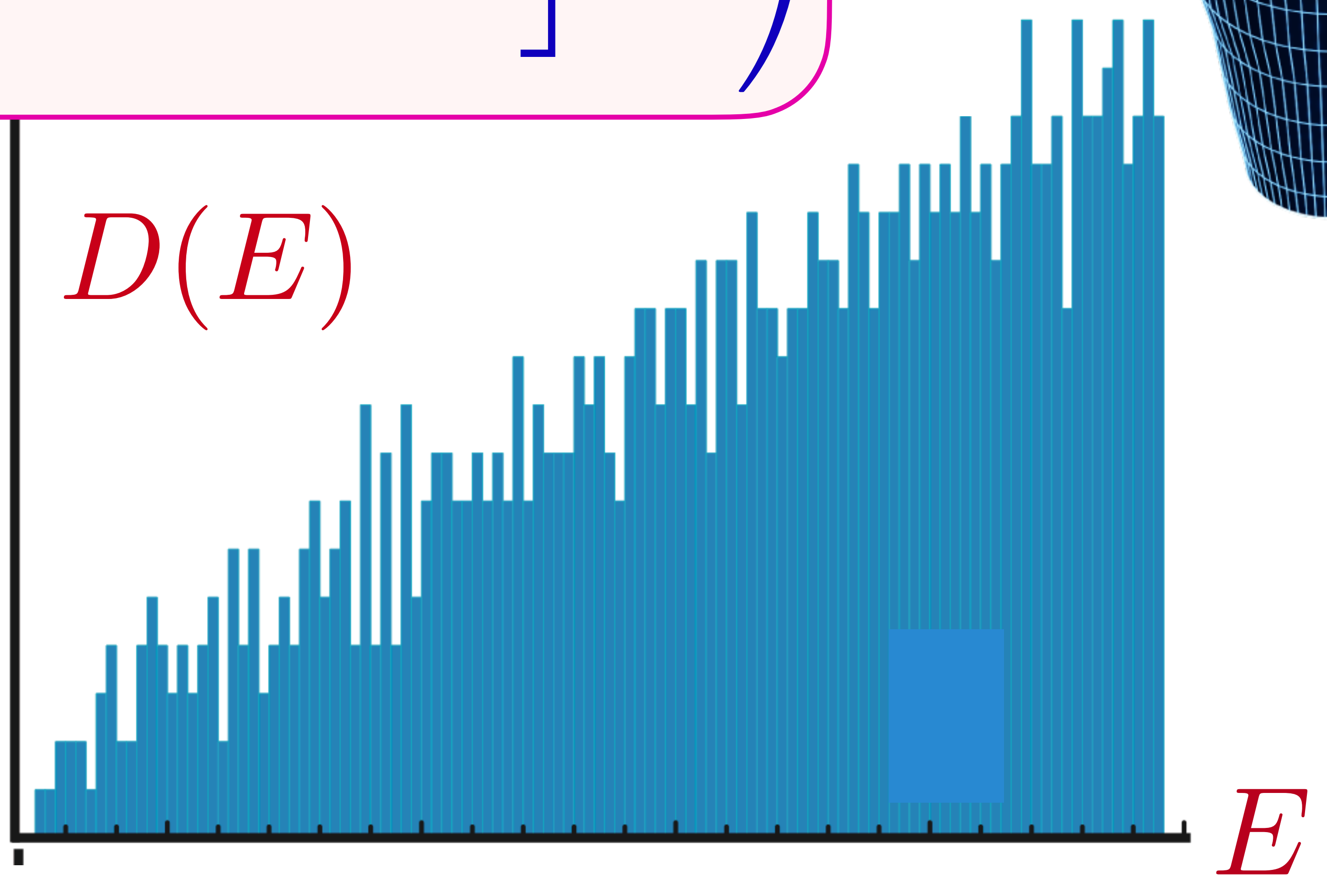


Bekenstein-Hawking

$$D(E) \sim \left(\frac{A_0 c^3}{\hbar G}\right)^{-347/90} \exp\left(\frac{A_0 c^3}{4\hbar G}\right) \sinh\left(\left[\frac{\sqrt{\pi} {A_0}^{3/2} c^2}{\hbar^2 G} E\right]^{1/2}\right)$$

Iliesiu, Murthy, Turiaci (2022)

$f_{\mathrm{smooth}}(E)$: developments from the SYK model

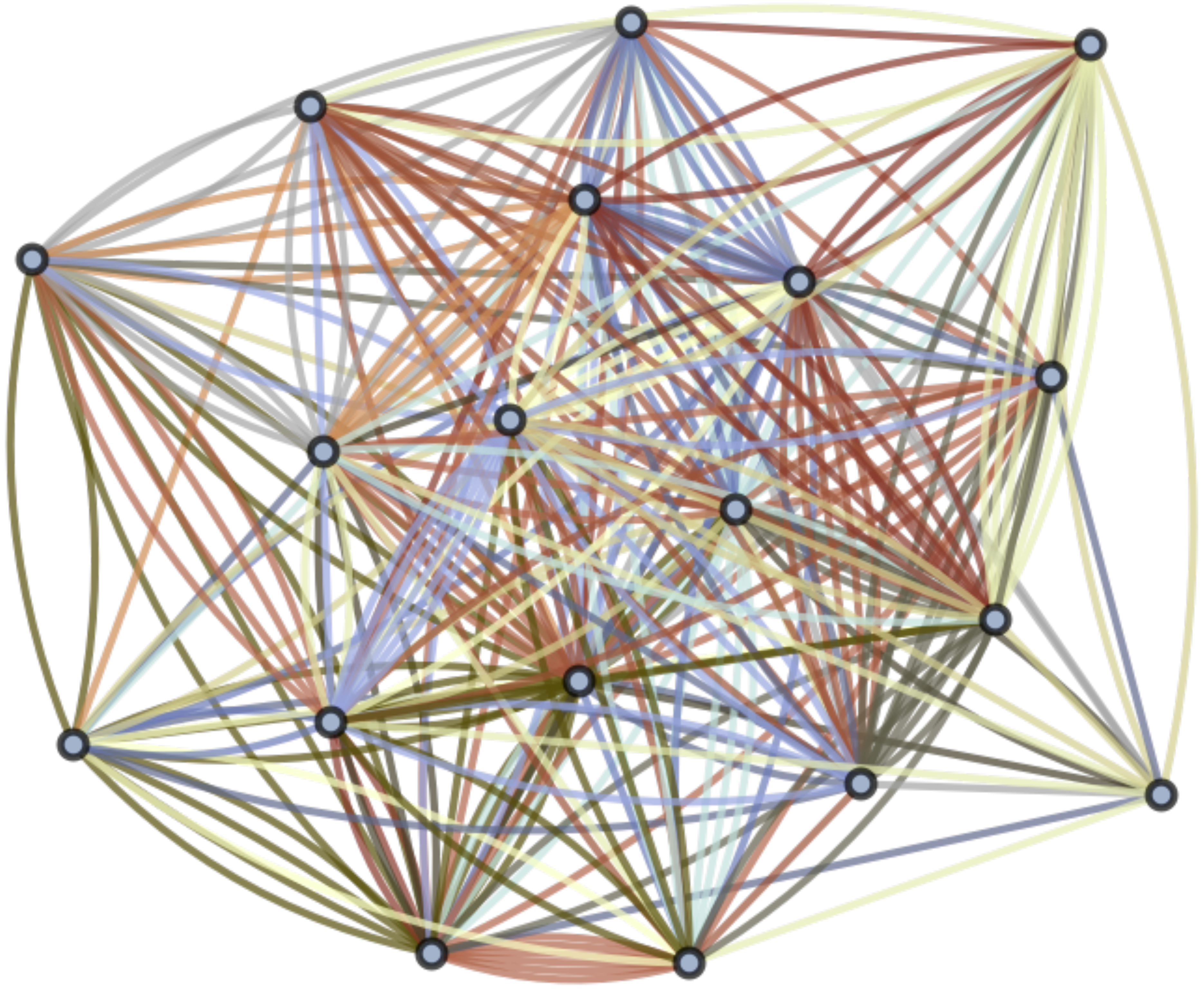


Similar remarks apply to rotating neutral black holes.

D. Chowdhury, A. Georges, O. Parcollet, and S. S., Rev. Mod. Phys. **94**, 035004 (2022)

The Sachdev-Ye-Kitaev (SYK) model

The SYK model describes multi-particle quantum entanglement resulting in the loss of identity of the particles



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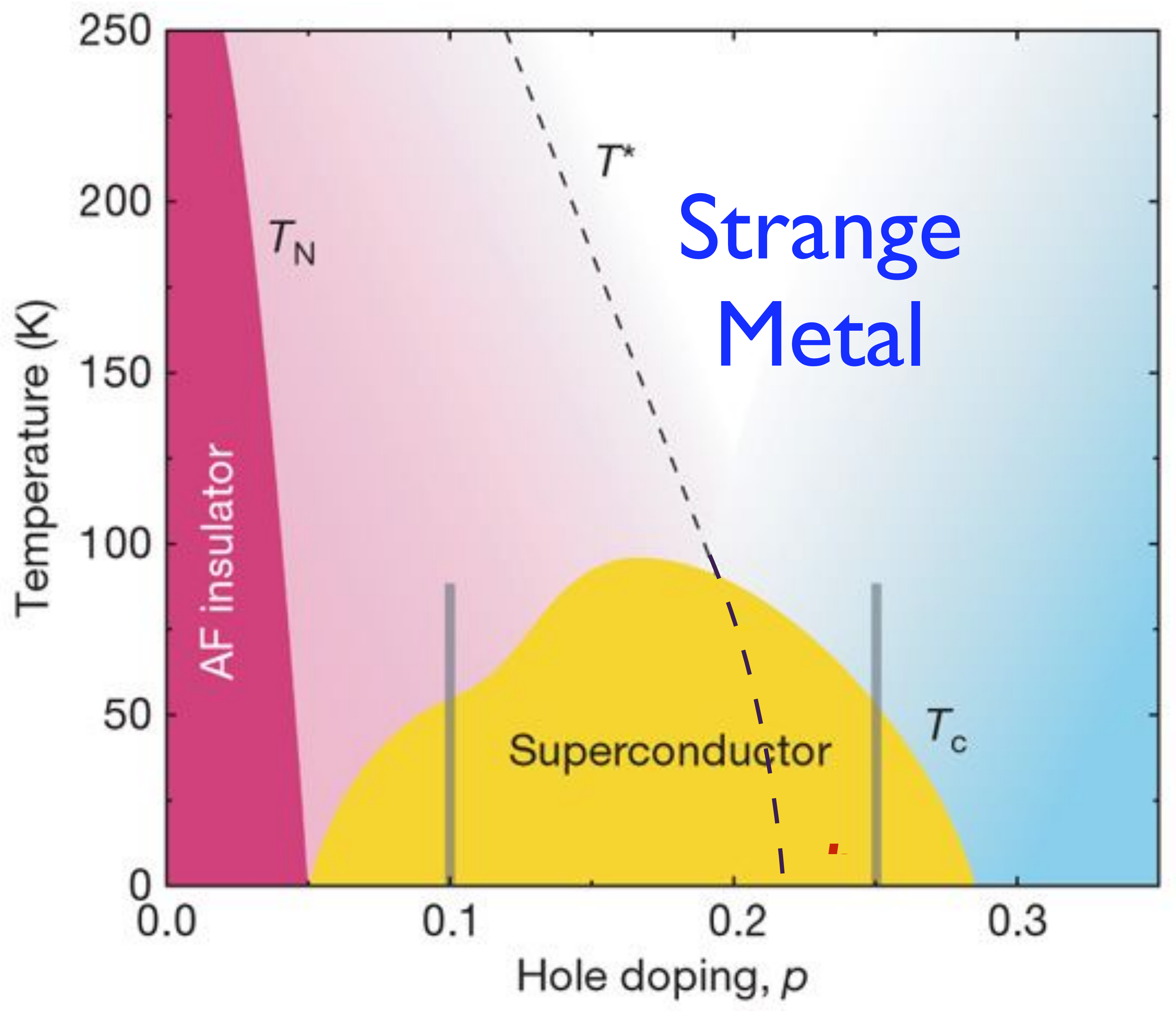
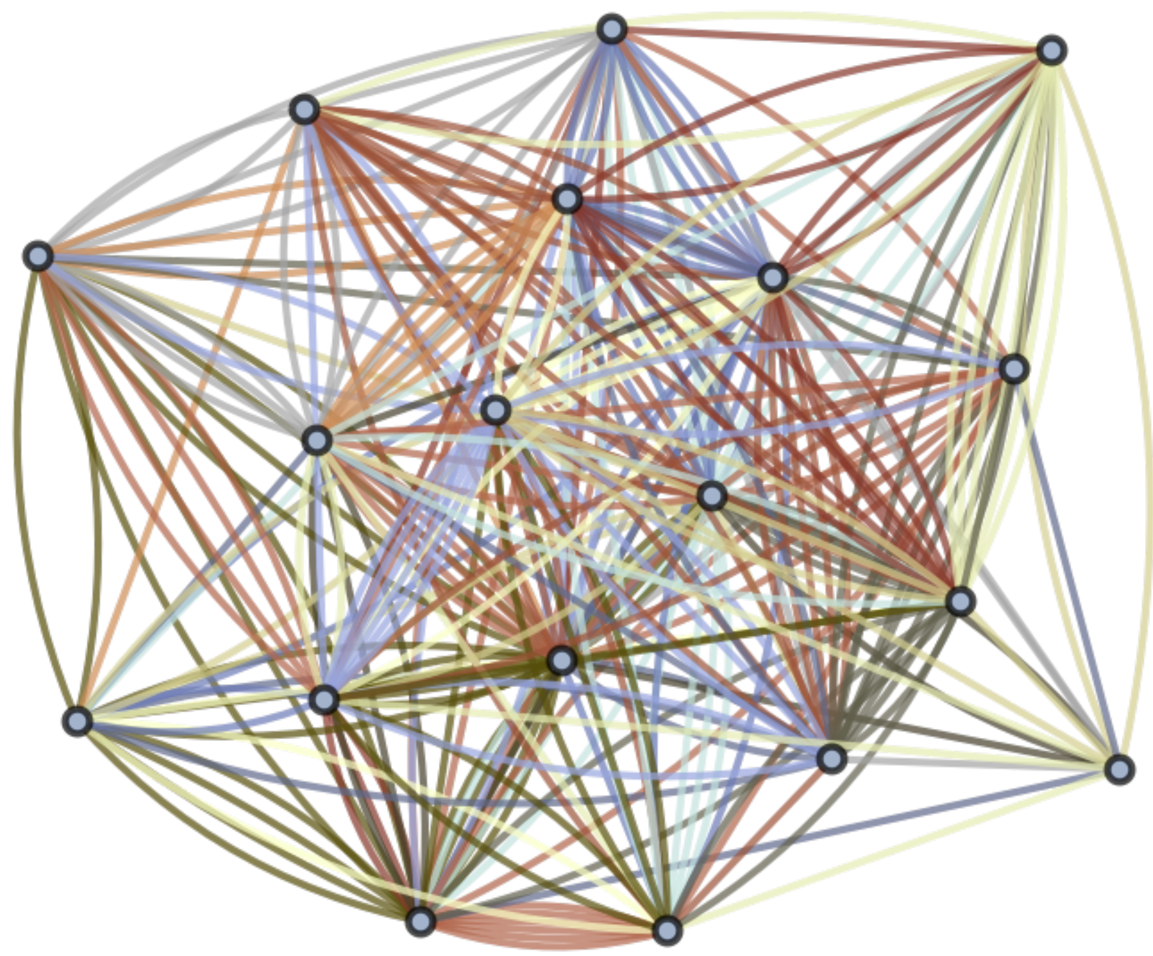


In one set of variables, it helps describe the ***strange*** electrical properties of YBCO

Sachdev, Ye (1993)



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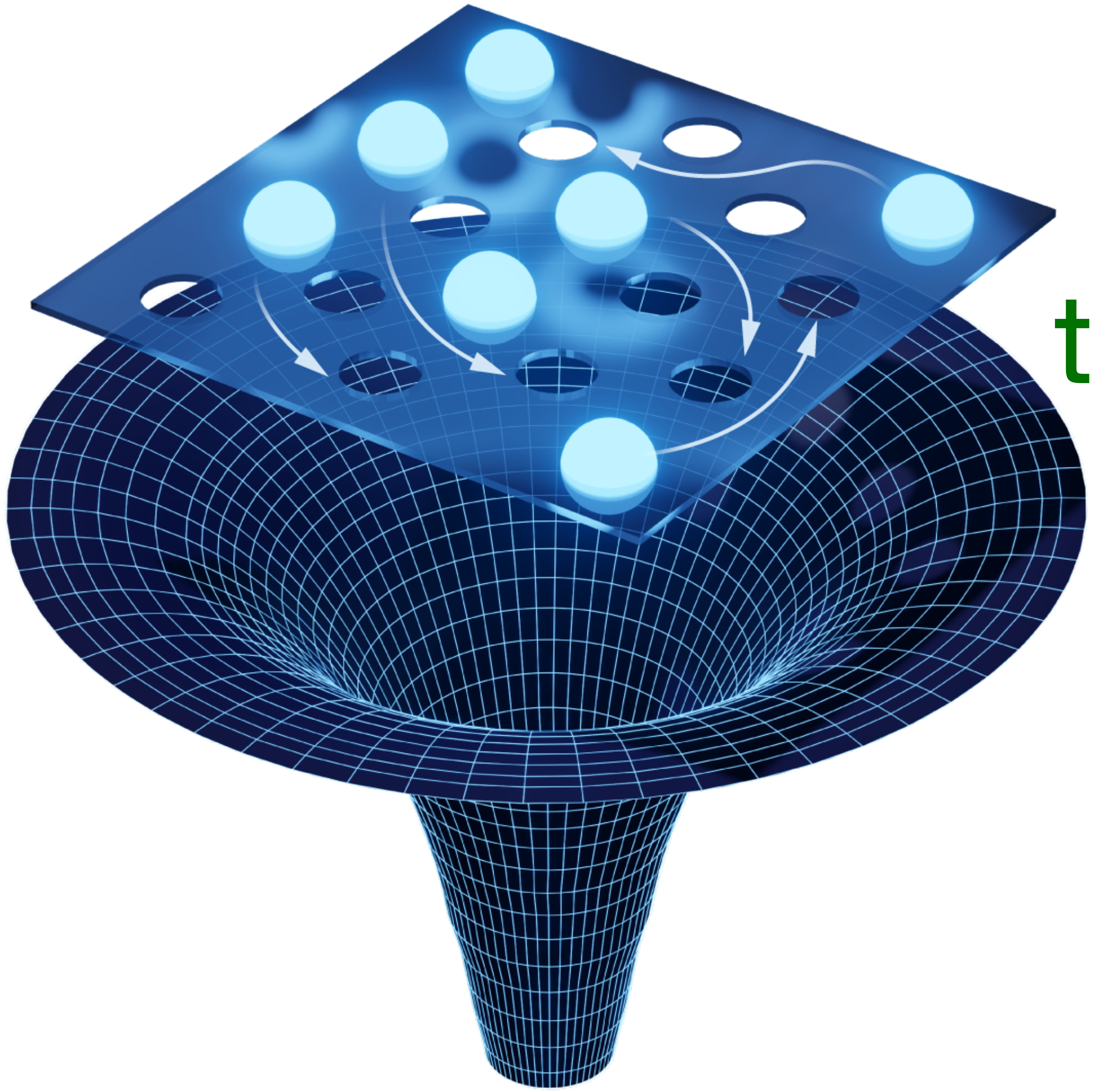

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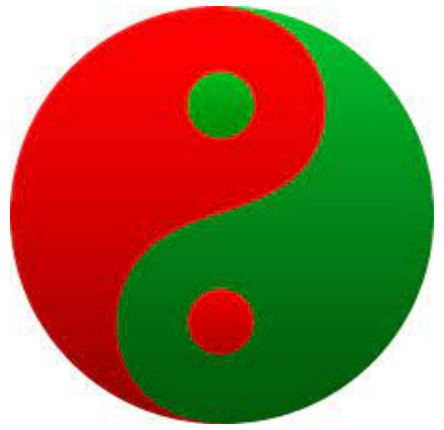
Sachdev, Ye (1993)



In a *dual* set of variables it describes the interior of ***charged black holes***

Sachdev (2010), Kitaev (2015), Maldacena Stanford (2015)