Solutions to problems from 30th of May

1 Problem 1 - new physics bound from the neutrino masses

1.1 Problem

Consider the Weinberg operator

$$\frac{1}{\Lambda}\left(\epsilon^{\alpha\beta}H^iH^jL^i_\alpha L^j_\beta\right) \tag{1}$$

where H is the Higgs field, L is the left-handed lepton field, i and j are $SU(2)$ indices and α and β are spinor indices. Derive the bound on the scale of new physics Λ to ensure that the neutrino mass is below 0.1 eV.

1.2 Solution

Below electroweak symmetry breaking scale we can replace $H \to v + h$, where $v \approx 246$ GeV is the vacuum expectation value of the Higgs and h is the Higgs boson field. The mass term will arise then both H are replaced by v. We can then read off the neutrino mass from the Lagrangian and we see that it will be proportional to

$$m_\mu \propto \frac{v^2}{\Lambda}. \tag{2}$$

Solving for Λ gives

$$\Lambda \gtrsim 10^{15}\text{GeV}. \tag{3}$$

2 Problem 2 - proton decay

2.1 Problem

Consider a baryon number violating operator

$$\frac{1}{\Lambda^2}\epsilon^{IJK}\epsilon^{ij}(Q^T_{Ii}C\sigma^\mu u_J)(L^T_j C\sigma_\mu d_K) \tag{4}$$

Using the constraint on the proton lifetime $\tau \gtrsim 10^{34}$ years, derive a bound on the new physics scale Λ.

2.2 Solution

We can see that the single insertion of the operator will induce proton decay by converting uud to $\bar{d}d$ and e^+. The amplitude is then proportional to $1/\Lambda^2$ and the decay width will be proportional to amplitude squared giving:

$$\Gamma(p \to e^+\pi^0) \propto \frac{m_p^5}{\Lambda^4} \tag{5}$$

where the proton mass, m_p, which is the characteristic energy scale of this process, was inserted on dimensional grounds (decay width has dimension 1). Proton lifetime is then the inverse of its decay width:

$$\tau_p = \frac{1}{\Gamma(p \to X)} = \frac{1}{\Gamma(p \to e^+\pi^0)} = \frac{\Lambda^4}{m_p^5} \tag{6}$$

The rest is a matter of substitution:

$$1 = \hbar = 6.582 \times 10^{-25}\mathrm{GeVs} \tag{7}$$
$$m_p \approx 1\mathrm{GeV} \tag{8}$$
$$1\mathrm{year} \approx \pi \times 10^7\mathrm{s} \tag{9}$$

This results in $\Lambda \gtrsim 10^{16}$ GeV.

3 Problem 3 - elecric dipole moment of the electron

3.1 Problem

Consider the operator in the Lagrangian:

$$\frac{1}{\Lambda^2}\bar{L}^i Y_e \sigma^{\mu\nu} e_R (\sigma^a_{ij}) H^j W^a_{\mu\nu} \tag{10}$$

where the indices i and j are SU(2) indices in the fundamental representation and a - in the adjoint representation and Y_e is the Yukawa coupling matrix of the electron.

Using the experimental result for the electron's electric dipole moment

$$d_e \lesssim 10^{-28} e\mathrm{cm} \tag{11}$$

obtain the constraint on the scale Λ.

3.2 Solution

The electric dipole moment is defined from the effective Lagrangian:

$$\mathcal{L}_{eff} = -\frac{i}{2} d_e \bar{\psi}\gamma^5 \sigma^{\mu\nu} F_{\mu\nu}\psi \tag{12}$$

and it can be read off from our effective operator after replacing the Higgs field with its vev. We can then see that

$$d_e \approx \frac{(Y_e)_{11}\, v}{\Lambda^2} \tag{13}$$

Recalling that $(Y_e)_{11} \sim m_e/v$ and using the values from the previous problem together with

$$1 = c \approx 3 \times 10^{10} \text{cms}^{-1} \tag{14}$$

$$m_e \approx 0.5\text{MeV} \tag{15}$$

we end up with

$$\Lambda \approx 10^5 \text{GeV} \tag{16}$$

4 Problem 4 - K-$\bar{K}$ mixing

4.1 Problem

Find the dependence on the Yukawa couplings of K^0-$\bar{K}^0$ mixing amplitude.

4.2 Solution

The leading order diagram has 4 Yukawa insertions as shown in figure. This is necessary because:

- In flavour basis, W interactions are diagonal in flavour basis
- W couples only to left-handed fields
- H couples to one left-handed and one right-handed field

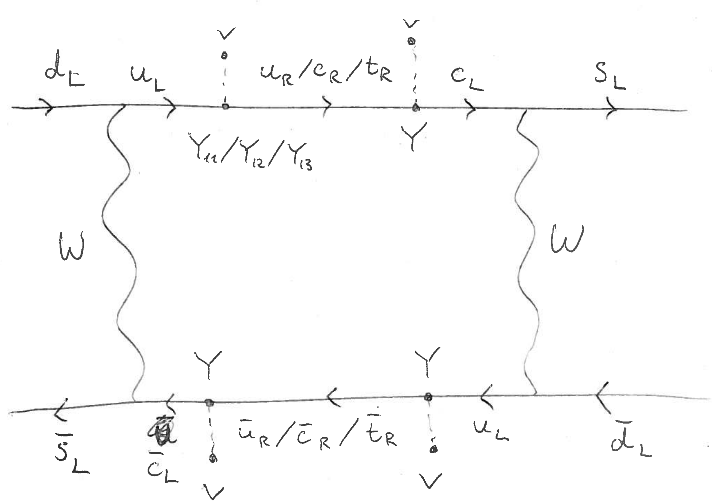

Figure 1: Leading Standard Model contribution to $K\bar{K}$ mixing

The remaining loop integral, taking large M_W limit so the W-propagators get contracted to a point, will have 6 fermion propagators and is IR divergent in the massless quark limit, behaving as:

$$\int \frac{d^4p}{p^6} \sim \frac{1}{m_c^2}. \tag{17}$$

Furthermore, we choose the basis of the Yukawa couplings to be related to the diagonal (mass) basis by

$$Y_d = Y_d^{\text{diag}} \tag{18}$$

$$Y_u = V_{CKM}^{\dagger} Y_u^{\text{diag}} \tag{19}$$

Adding the $1/(16\pi^2)$ loop factor, g^4 for W vertices, $1/m_W^4$, replacing $Y_u^{diag} = M/v$ and recalling that $(V_{CKM})_{11} \approx (V_{CKM})_{22} \approx 1$ and $(V_{CKM})_{12} \approx \lambda \approx 0.2$ we get the following order-of-magnitude estimate:

$$\sim \frac{g^4 \lambda^2 m_c^2}{16\pi^2 m_W^4} = \frac{G_F^2}{2\pi^2} m_c^2 \lambda^2 \tag{20}$$

Using kaon decay constant f_K to compensate for the dimensions we find

$$\frac{\Delta m_K}{m_K} \sim \frac{G_F^2}{2\pi^2} m_c^2 \lambda^2 f_K^2 \approx 10^{-15} \tag{21}$$