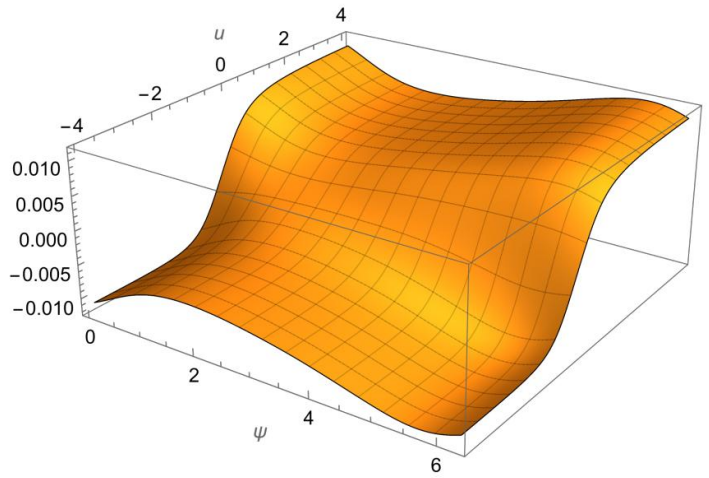
THE UNIVERSITY of EDINBURGH

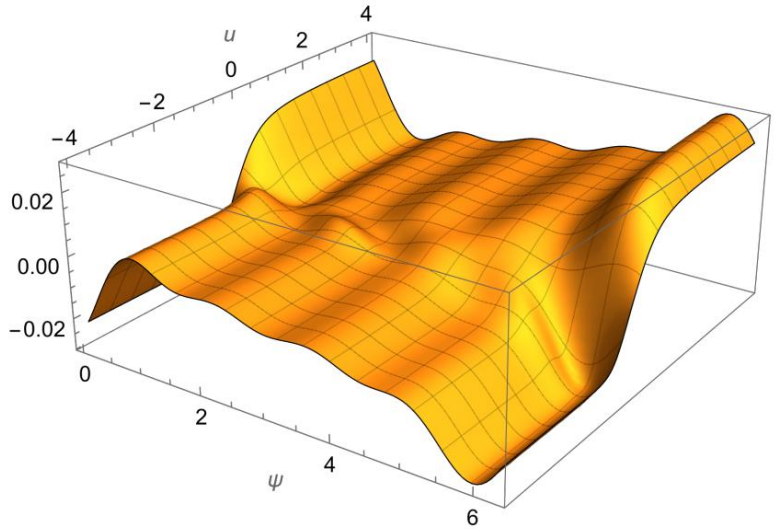
Black hole dynamics from scattering amplitudes

Graham R. Brown

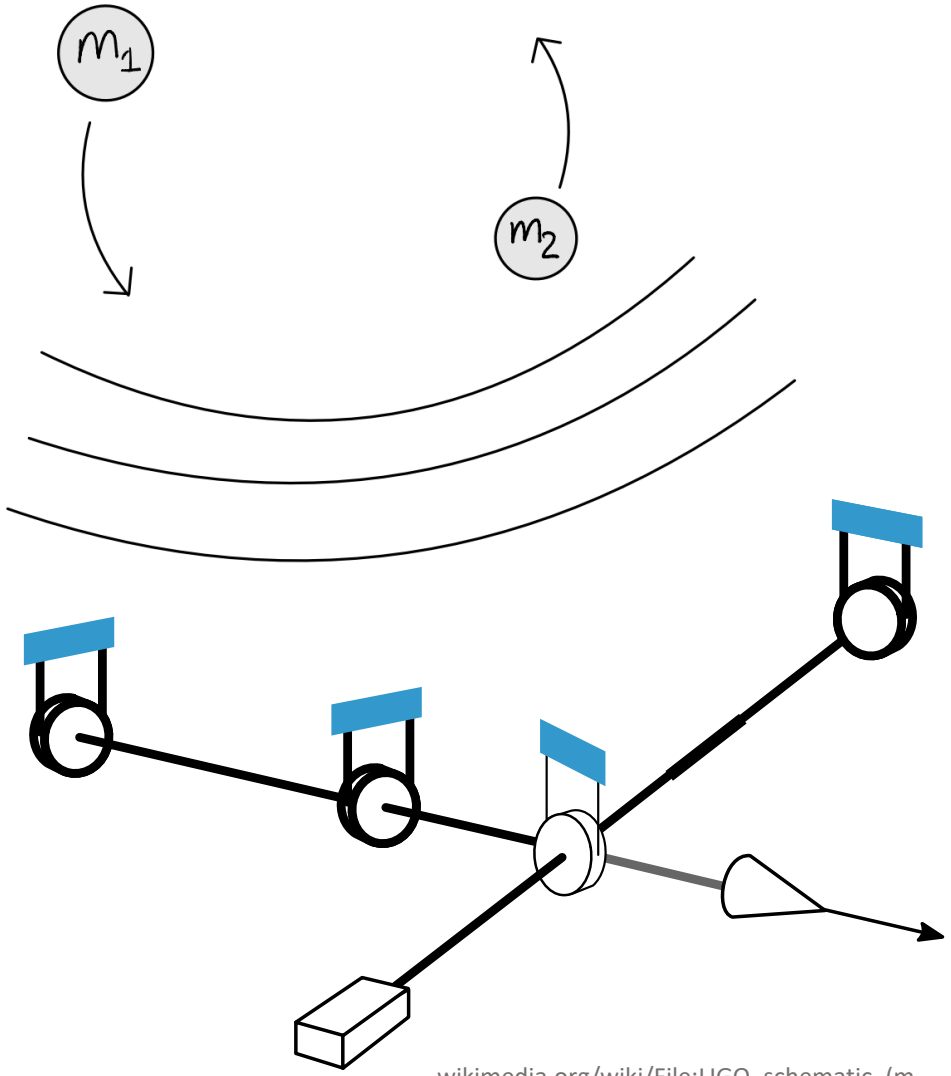
14.10.2025-Edinburgh

All-IPNP Meeting

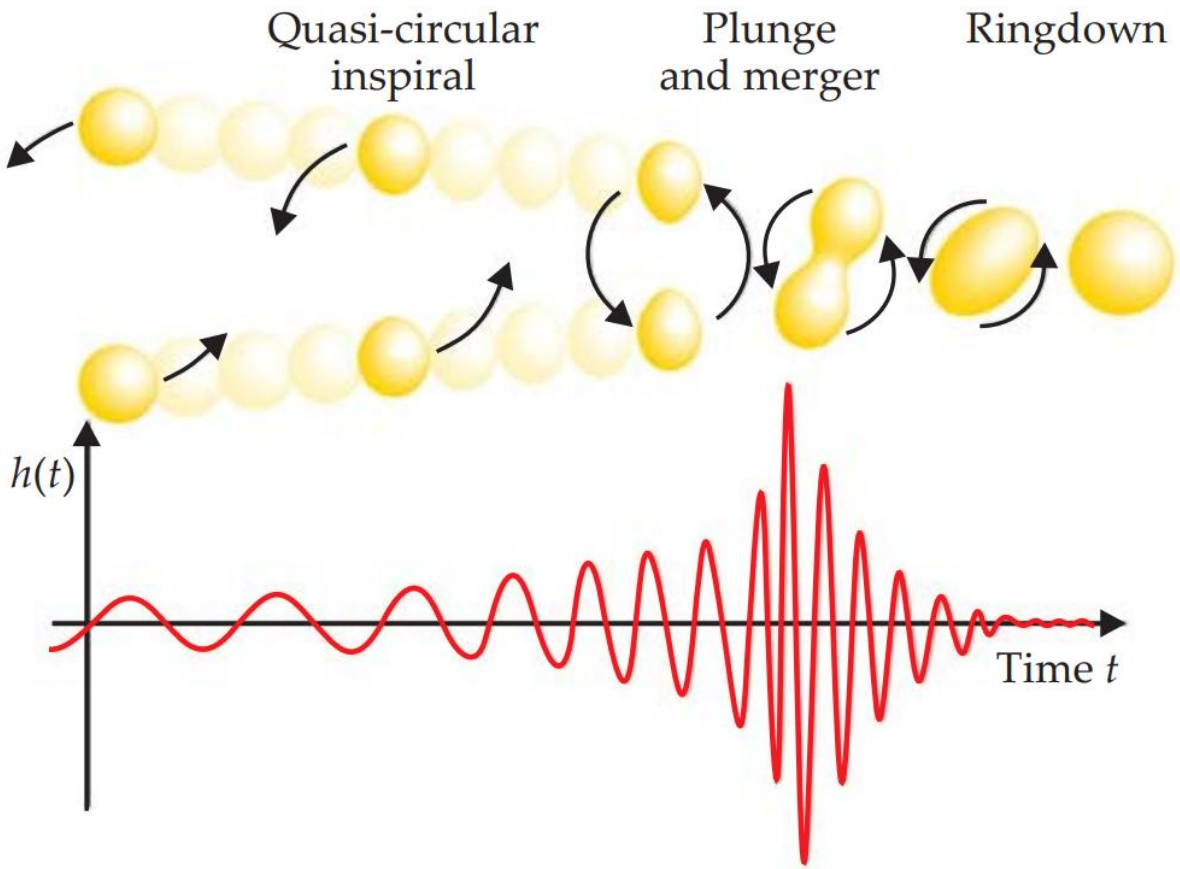




Motivation- Gravitational Waves

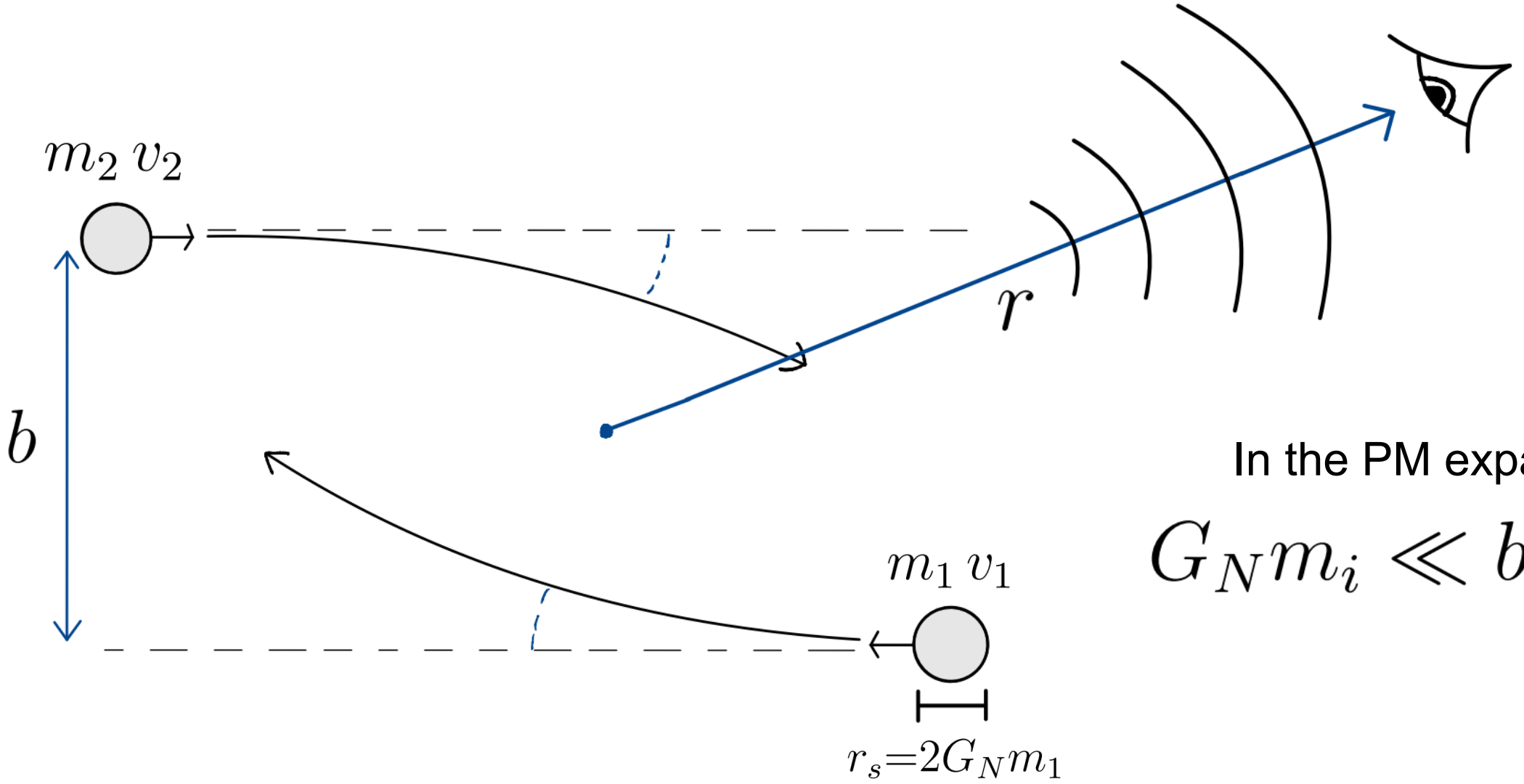


wikimedia.org/wiki/File:LIGO_schematic_(multilang).svg



Baumgarte, Shapiro,
DOI:10.1063/pt.3.1294

We will focus on the scattering case



In the PM expansion:

$$G_N m_i \ll b \ll r$$

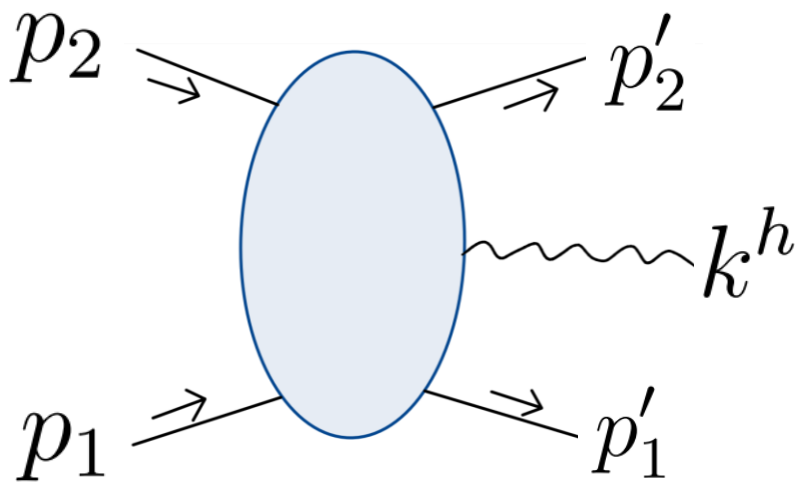
How to approach perturbation theory?

Solve perturbatively in G_N .

$$G_{\mu\nu} = 8\pi G_N T_{\mu\nu}$$

Vs.

Build amplitudes perturbatively in G_N

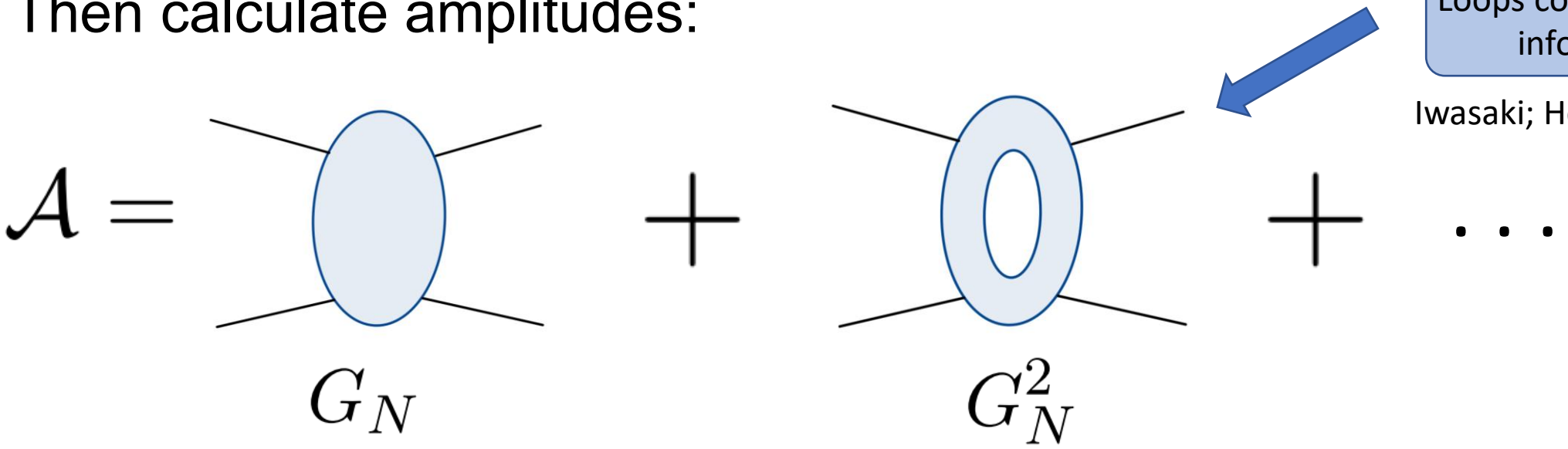


$$= \langle p_1' p_2' k^h | iT | p_1 p_2 \rangle$$

Basic Idea: Non-spinning black holes only have one parameter, their mass m. So describe them by a scalar field, treat GR as an EFT:

$$\mathcal{L} = \int d^D x \sqrt{-g} \left(\frac{R}{16\pi G_N} + \sum_{i=1,2} \left(\frac{1}{2} g^{\mu\nu} \partial_\mu \phi_i \partial_\nu \phi_i - m_i^2 \phi_i^2 \right) \right)$$

Then calculate amplitudes:

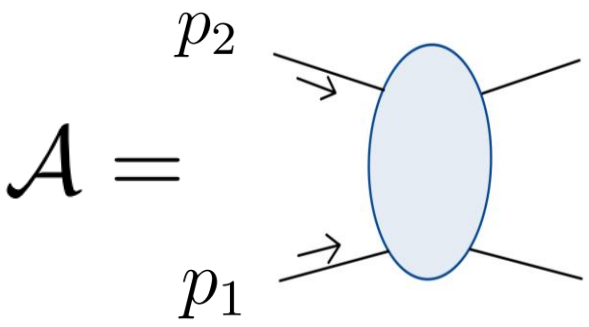


Loops contain classical information

Iwasaki; Holstein, Donoghue…

Amplitudes for Gravitational Dynamics

Full amplitude



Expand as $\hbar \to 0$

$\mathcal{A}_{Cl}$

Extract classical part

Match to a Hamiltonian:

$$H(p,r) = \sqrt{\underline{p}^2 + m_1^2} + \sqrt{\underline{p}^2 + m_2^2} - \frac{G_N m_1 m_2}{r} + \mathcal{O}(G_N^2)$$

Eikonal phase/radial action:

$$\mathcal{A}_{Cl} = e^{i\delta_0 + i\delta_1 + \cdots} - 1$$

KMOC formalism

⋮

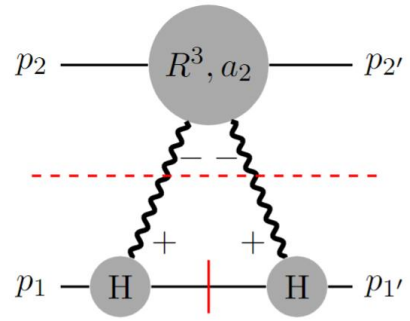
Why do this?:

- We can apply amplitudes techniques to the GR two body problem.
- Many correspondences between properties of amplitudes and classical observables: IR divergences, BMS symmetries, soft factors, memory…

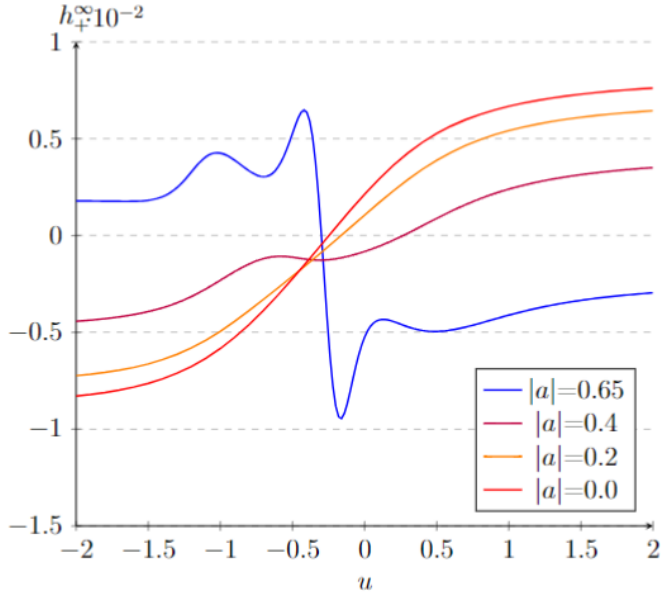
What to compute?

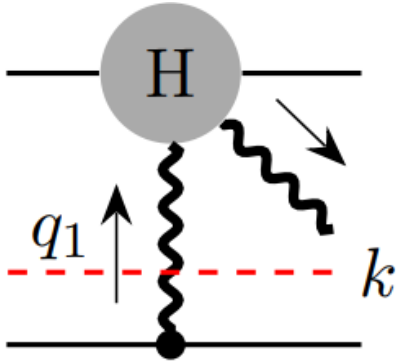
We are interested in some *classical* observables:

- Impulse: $\langle \Delta \mathbb{P}_1^\mu \rangle = p_1^\mu(t{=}\infty) - p_1^\mu(t{=}-\infty)$
- Spin-kick: $\langle \Delta \mathbb{S}_1^\mu \rangle = S_1^\mu(t{=}\infty) - S_1^\mu(t{=}-\infty)$



- Waveforms: $\langle h^\infty(x) \rangle =$





What are we up to now?

- Modified gravity: $S_{\rm grav} = -\frac{2}{\kappa^2}\int d^4x\sqrt{-g}\left[R + \beta_1\, I_1 + \beta_2\, G_3 + \tilde{\beta}_1\, \tilde{I}_1 + \tilde{\beta}_2\, \tilde{G}_3\right],$

- Integrability of black hole orbits from the amplitudes perspective.

$$\{\mathcal{Q}, I_r\}_{\rm DB} = 0 \to \langle \Delta \mathcal{Q} \rangle = 0$$

- Position operators in QFT, trajectories and supertranslations…

Collaborators and colleagues in Edinburgh:

- Mao Zeng, Dogan Akpinar, Rafael Aoude, Donal O'Connell, Anton Ilderton, Karthik Rajeev, Fabian Bautista, Tim Adamo, Raikhik Das…