

EDINBURGH



Thank the organisers.

Collaborators - Sasha Haco, Stephen Hawking, Andy Strominger

Can soft hair account for black hole entropy?

↳ These are generalisations of the charges associated to asymptotic symmetries.

As asymptotic symmetries, these charge (partially) label the infinite number of degenerate vacua in gravity, electrodynamics and YM

Can they be generalised?

But also consider why black holes have entropy.

- No hair theorems. M, J, Q Black hole stationary spacetimes are unique and there are the only previously known charges. Ways of formulating given M, J, Q . Then 1st law $dM = \kappa dA + \mathcal{E} dJ + \mathcal{L} dQ$
2nd law. $dA > 0$

Let's start from covariant phase space (Reinhardt, Ashtekar, Witten + York, York, Zuckerman, Wald + Lee + Zoupas) [Lastly Gibbons + Hawking Euclidean]

Idea

Action \rightarrow Presymplectic Potential [1-form on phase space]



Presymplectic Form [2-form on phase space]



Symplectic Form ~~Charges~~ \longrightarrow Charges.

Illustrate for gravity

$$I = \frac{1}{16\pi} \int R + \dots$$

$$\delta I = \int_M (E/a)^ab \delta g_{ab} + \int_{\partial M} \theta^a \delta \Sigma_a$$

$\hookrightarrow g_{ab} \rightarrow g_{ab} + h_{ab}$

think $\Sigma p_i, \delta q_i$

$$\theta^a = \frac{1}{16\pi} (\nabla_b h^{ab} - \nabla^a h)$$

(Presymplectic potential.

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Presynthetic form

$$\omega^a(g, h, h') = \delta' \theta^a(g, h) - \delta \theta^a(g, h')$$

Synthetic form

$$\Omega(g, h, h') = \int_{\Sigma} \underbrace{\omega^a(g, h, h')}$$

$$\text{Think } \sum \delta p_i \delta q_i$$

Explicit formula is

$$\Omega =$$

but enlightening.

This Ω is going to reproduce the PB's, but it is different. Project out its kernel to the Dirac bracket. Quantization is accomplished of PB by $-i$ [Commutation] to. Suppose h' is pure gauge. Then

$$h'^{ab} = \nabla_a \tilde{h}^b + \nabla_b \tilde{h}^a \quad \text{and } h'^{ab} \text{ obeys linearised Einstein}$$

then ω^a is the divergence of a 2-form, and this defines a charge

$$\Delta Q_j(h) = \int_S F_{ab} dS^{ab}$$

where S is some ^{closed} surface in Σ

Explicitly

$$F_{ab} = \frac{1}{16\pi} \left[-\tilde{h}_a \nabla_b h + \tilde{h}_a \nabla_c h^c_b + \tilde{h}_c \nabla_b h^c_a - \frac{1}{2} h \nabla_a \tilde{h}_b + h a c \nabla^c \tilde{h}_b - (a \leftrightarrow b) \right]$$

ΔQ is the charge in the charge between the geometry g and $g+h$ conjugate to the vector field \tilde{h} .

Example: a) \tilde{h} is a Killing vector, then this is the Komar formula

b) \tilde{h} is a large gauge transformation, say a supertranslation, then Ω is the supertranslation charge.

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c) If $\int \partial g^a / \partial t$ somewhere in the interior then this defines quasi-local mass.

There can be a difficulty with this definition because the charge should depend on h , and not on the path from g to $g+h$.

One way to express this is

$$Q_S(h_1, g+h_2) = Q_S(h_2, g+h_1)$$

To make this true, one may have to add something to Q — but this reflects the ambiguity in deriving Q from I and fixes this ambiguity.

Diffeomorphism invariance

$$L_S, L_{S'} - L_S, L_{S'} = L_{[S, S']}$$

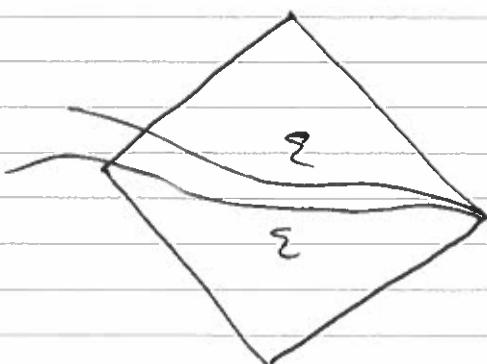
$$\Rightarrow Q_S, Q_{S'} - Q_{S_2} Q_{S_1} = Q_{[S_1, S_2]} + K_{S_1, S_2}$$

If $K \neq 0$, then diffeomorphism invariance is violated.
 K is some kind of surface charge.

K = Chern-Simons 2-cocycle for diffs.

An explicit formula is

$$K = \frac{1}{16\pi} \int dS^{ab} \left[-4 \nabla_c \zeta_{1a} \nabla^c \zeta_{2b} + 2 \nabla_b \zeta_{1a} \nabla_c \zeta_{2}^c - 2 \nabla_c \zeta_1^c \nabla_b \zeta_{2a} - 2 R_{abcd} \zeta_1^c \zeta_2^d \right]$$



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lets look at Kerr.

Well-known in BL - coordinates. Lets define some new coordinates

$$w^+ = \sqrt{\frac{r-r_+}{r-r_-}} e^{2\pi T_R \phi}$$

$$w^- = \sqrt{\frac{r-r_+}{r-r_-}} e^{2\pi T_L \phi - t/2M}$$

$$y = \sqrt{\frac{r-r_-}{r-r_+}} e^{\pi(T_L + T_R)\phi - t/4M}$$

$$\begin{array}{ccc} \text{N.b.} & \text{N.b.} & w^+ \mapsto w^+ e^{4\pi T_R} \\ w^- \mapsto w^- e^{4\pi T_L} \\ y \mapsto y e^{2\pi(T_L + T_R)} \end{array}$$

$$\text{Then near } H \quad \frac{4\lambda^2}{y} dw^+ dw^- + \frac{16m^2 a^2 \sin^2 \theta}{y^2 \rho^2} dy^2 + \rho^2 d\theta^2$$

$$\rho^2 = r_+^2 + a^2 \cos^2 \theta \quad \text{which looks like} \\ \text{a piece of } AdS_3 \times S^1$$

$$\text{Consider } \xi_n^+ = e_n, \quad \xi_n^y = \frac{i}{2} y e_n' \quad \epsilon_n = (2\pi T_R)^{1+i\eta/2\pi T_R} (w^+)^n$$

$$\xi_n^- = \tilde{e}_n, \quad \xi_n^y = \frac{i}{2} y \tilde{e}_n' \quad \tilde{\epsilon}_n = (2\pi T_L)^{1+i\eta/2\pi T_L} (w^-)^n$$

$$\text{Obey } [\xi_n, \xi_m] = (n-m) \xi_{n+m}$$

$$[\tilde{\xi}_n, \tilde{\xi}_m] \rightarrow (n-m) \tilde{\xi}_{n+m}$$

$$[\xi_n, \tilde{\xi}_m] = 0$$

$$T_L = \frac{r_+ - r_-}{4\pi a}$$

$$m = \text{mass}$$

$$a = \text{rotation parameter}$$

$$T_R = \frac{r_+ - r_-}{4\pi a}$$

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What about \mathcal{Q} :

One needs to add a counterterm

$$\mathcal{Q}_{ct} = \frac{1}{16\pi} \int_S -2N_{cd} \nabla^d (\tilde{g}_{ab} h^{ab}) dS^{ab}$$

where N is the volume-form in the normal bundle of S .

Then, we find a central term

$$\frac{c}{12} m^3 S_{mn,0}$$

with for \tilde{g}

$$c = 12J$$

$$c_L^R = 12J$$

=

Now think of some prehistory of relativity.
Scattering from black holes

$$\text{Absorbtion prob. } \sim \left| \Gamma\left(1 + \frac{i\omega_L}{2\pi T_c}\right) \right|^2 \left| \Gamma\left(1 + \frac{i\omega_R}{2\pi T_R}\right) \right|^2$$

For quanta $\omega_L = 2m^3/J \delta E$

$$\omega_R = 2m^3/J \delta E - \delta J$$

carrying energy δE , angular momentum \parallel to that of the hole δJ ,

This formula also shows up in another area
physics

Nearly 2-d CFTs where it describes the absorption of particles at temperature T and energy ω

So, one can describe the behaviour of the hole as a $CFT_L \times CFT_R$

$$\downarrow \quad \quad \quad \text{Central charge } 1/L \quad - \quad \quad 1/L \\ \text{Top} \quad T_L \quad - \quad \quad T_R$$

A general formula for the entropy of such a CFT (due to Cardy) is

$$S = \frac{\pi^2}{3} (c_L T_L + c_R T_R)$$

* Not obviously valid,
but we think we can
fix this by identifying
the CFT

Stick in our answers and this agrees with Hawking.

We conclude that the states of this CFT are labelled by the soft (super-rotation) hair described by $\tilde{\gamma}, \tilde{\beta}$.

So this is an update to the no-hair theory (that assumed no gauge transforms with physical meaning)

Works for Kerr-Newman too,
" " de Sitter, but this is too much for now.

Said we have solved the information paradox.

No → Species problem.
Need to describe things in
+ Hawking radiation.
Collapse.
No decay. The singularity.

But it does look like a singularity.