Soft-theorem constraints on EFT's

Congkao Wen

Queen Mary University of London

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work with H. Luo; M. Bianchi, A. Guerrieri,
Y.-t. Huang, C.-J. Lee; C. Cheung,
K. Kampf, J. Novotny, C.-H. Shen, J. Trnka

Symmetries of S-matrix and Infrared Physics
Higgs centre, University of Edinburgh

Introduction

- Applications of scattering amplitude program (here focus on soft theorems) to effective field theories:
  - Soft theorems are efficient tools to implement the symmetry constraints on EFT's. Systematical tools are the soft recursion relations.
  - Maybe more interestingly, soft theorems as first principle input to discover (new) EFT's.

Introduction

- Amplitudes of Goldstone bosons or fermions satisfy soft theorems, reflecting the (spontaneously breaking) symmetries of theories.
  - Global internal symmetries: pions of chiral symmetry breaking, scalars in extended supergravity.
  - Spacetime symmetries: dilatons of conformal symmetry breaking, scalars of DBI for breaking of Poincare symmetry. Typically lead to soft theorems with higher orders.
- EFT's of Goldstone bosons or fermions are highly constrained. For many well-known theories, the tree-level S-matrices are on-shell constructible.

Outline

- Soft recursion relations.
- Applications to the EFT of $\mathcal{N} = 4$ SYM on the Coulomb branch.
- New (multi) soft theorems, and the uniqueness of Born-Infeld theory.
- Conclusion and outlook.

Recursion relations from soft theorems

- The usual BCFW recursion relations cannot apply to EFT's, because of the bad large-z behavior.
- Soft theorems provide additional information, and lead to new on-shell recursion relations.
- Recursion relations will not only provide efficient computational tools, but also systematical ways of constraining EFT's.

Recursion relations from soft theorems

- Soft BCFW shifts:

$$p_i \to (1 - a_i z) p_i\,, \quad \text{for} \;\; \text{all} \;\; i\,.$$

  - Preserves massless condition and momentum conservation, with constraints of $\sum_i a_i p_i = 0$.
  - The limit $z \to 1/a_i$ probes the soft limits.
- The residue theorems lead to on-shell recursion relations,

$$A_n(0) = \oint_{z=0} \frac{dz}{z} \frac{A(z)}{F_\sigma(z)} = \sum_i R_i$$

$F_\sigma(z) = \prod_i (1 - a_i z)^\sigma$.

Recursion relations from soft theorems

- If all the residues are determined in terms of lower-point amplitudes, we have a recursion relation.
- Introducing $F_\sigma(z)$ is to kill the pole at $z \sim \infty$ if $A_n(z) \sim z^m$, with $m < n\,\sigma$.
- $F_\sigma(z)$ introduces additional poles, whose residues are known if A_n satisfies soft theorems,

$$A_n(\tau p_n)\big|_{\tau\to 0} = \sum_{i=q}^{\sigma-1} \tau^i (\mathcal{S}_i A_{n-1})\,,$$

where some soft factors $\mathcal{S}_i$ may just be 0.

Recursion relations from soft theorems

- With this setup, the reside theorems tell us

$$A_n = \sum_L A_L \frac{1}{P_L^2} A_R + \sum_i R_i^s \,.$$

  Namely a higher-point amplitude is determined in terms of lower-point on-shell amplitudes from factorizations and soft limits.

- Many interesting EFT's are single-soft constructible: NLSM, special Galileon, DBI, conformal DBI, Volkov-Akulov theory, but not BI theory!

- Recursion relations are systematic ways of constraining EFT's: applications to $\mathcal{N}=4$ SYM on the Coulomb branch.

non-renormalization from SUSY

- $\mathcal{N}=4$ SYM on Coulomb branch: Consider the breaking $U(N+1)\to U(N)\times U(1)$, and focus on the low-energy EFT of $U(1)$ sector,

$$\mathcal{L}_{\mathcal{N}=4\,\mathrm{SYM}} = -\frac{1}{4}F^2 + f_4(g_{\mathrm{YM}},N)\frac{F_-^2F_+^2}{m_W^4} + f_6(g_{\mathrm{YM}},N)\frac{F_-^2F_+^4+F_-^4F_+^2}{m_W^8}+\dots$$

- What are the functions $f_4(g_{\mathrm{YM}},N), f_6(g_{\mathrm{YM}},N),\dots$
- SUSY constrains via amplitudes lead to non-renormalization: the "MHV" operators $F_-^2F_+^{2\ell}$ are ℓ-loop exact.
- $\ell=1$ was a statement in [Dine, Sieberg, 97'], $\ell=2,3$ were conjectured in [Buchbinder, Petrov, Tseytlin, 01'].

SUSY non-renormalization theorems

- For this particular MHV sector of the EFT,

$$\mathcal{L}_{\mathrm{MHV}} = \sum_{q=1}^{\infty} 4^{q-1} \left(-\frac{\lambda}{2(4\pi)^2}\right)^q \frac{F_-^2 F_+^{2q}}{m_W^{4q}}.$$

  It is an exact result, perturbatively and non-perturbatively.

- $\mathcal{L}_{\mathrm{MHV}}$ is in fact identical to BI theory for this particular sector.

- Beyond SUSY constraints? (Broken) Conformal symmetry and R-symmetry.

non-renormalization from soft theorems

- Coulomb branch $\mathcal{N}=4$ SYM has two kinds of Godestones: conformal symmetry and R-symmetry. Two soft theorems.
  - Conformal symmetry [Di Vecchia, Marotta, Mojaza, Nohle]

    $$v\, A_n(\tau p_n) = \left(\mathcal{S}_n^{(0)} + \tau \mathcal{S}_n^{(1)}\right) A_{n-1} + \mathcal{O}(\tau^2),$$

    soft factors $\mathcal{S}_n^{(0)}$ from scaling and $\mathcal{S}_n^{(1)}$ from special conformal transformation.
  - R-symmetry

    $$v\, A_n(\ldots,\phi_n^I)\big|_{p_n\to 0} = A_{n-1}(\ldots,\delta_I\phi_i,\ldots) + \mathcal{O}(\tau)$$

    ϕ_i can be dilaton φ or R-symmetry Goldstone ϕ^J: $\delta^I\varphi = \phi^I$ and $\delta^I\phi^J = -\delta^{IJ}\phi^I$.

non-renormalization from soft theorems

- Explore the constraints systematically using soft recursion.
- The EFT of $\mathcal{N} = 4$ SYM on the Coulomb branch can be naturally separated:

$$\mathcal{L}_{\mathcal{N}=4\,\mathrm{SYM}} = \mathcal{L}_{\mathrm{CDBI}} + \mathcal{L}_{\mathrm{Quantum}}$$

$\mathcal{L}_{\mathrm{CDBI}}$ is the conformal DBI (D-brane in AdS background), which is uniquely fixed by the soft theorems of (breaking) conformal symmetry

$$\mathcal{L}_{\mathrm{CDBI}} = -\frac{1}{\phi^4}\left(\sqrt{1 + \partial\phi \cdot \partial\phi} - 1\right).$$

non-renormalization from soft theorems

- Combine with the SUSY non-renormalization theorems:
  - Less than 8-derivative terms (with any number of fields) in $\mathcal{L}_{\text{Quantum}} = 0$.
  - 8-derivative terms in $\mathcal{L}_{\text{Quantum}}$ are fixed up to one constant (which can be non-trivial function of coupling).
  - 10-derivative terms in $\mathcal{L}_{\text{Quantum}}$ are fixed up to two constants.

Born-Infeld theory

- Born-Infeld theory is the effective theory of a single D-brane in flat space

$$\mathcal{L}_{\mathrm{BI}} = 1 - \sqrt{\det(\eta_{\mu\nu} + F_{\mu\nu})}\,.$$

- Low-energy expansion of string amplitudes.
- It is closely related to DBI theory and Volkov-Akulov theory.
- CHY formulas, or twistor-string-like formulas.

Born-Infeld theory in 4D

- BI theory enjoys the U(1) duality symmetry. Satisfy Noether-Gaillard-Zumino (NGZ) identity,

$$\left((\partial_t L)^2-(\partial_z L)^2-1\right)z-\left(2(\partial_z L)(\partial_t L)\right)t=0$$

  $t=F^2/4$ and $z=F\tilde{F}/4$.
- In 4D, only helicity conserved amplitudes, $A(++\ldots+--\ldots-)$
- There are infinity many such kind duality-symmetric theories, such as Bossard-Nicolai model. All have only the helicity conserved amplitudes.
- What is special about BI theory? Soft theorems.

Born-Infeld theory in 4D

- In the single-soft limit, the amplitudes go as

$$A(p_1,\ldots,p_{n-1},\tau p_n)\sim\mathcal{O}(\tau)$$

  But $\mathcal{O}(\tau)$ behavior is trivial because one derivative per field.

- We find that in 4D, BI theory behaves non-trivially in the multi-chiral soft limits

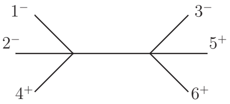
$$A^{\mathrm{BI}}(\tau\lambda_i,\tilde{\lambda}_J)\sim A^{\mathrm{BI}}(\lambda_i,\tau\tilde{\lambda}_J)\sim\mathcal{O}(\tau),$$

  for all $i\in P_+$ (positive photons) and $J\in P_-$ (negative photons).

- The multi-chiral soft theorems uniquely fix the vector theory (with this power counting) to be BI theory.

Born-Infeld theory in 4D

- 4pt amplitude: $\langle 12\rangle^2[34]^2$.
- 6pt helicity-non-conserved amplitudes vanish.
- 6pt helicity conserved amplitude from factorizations

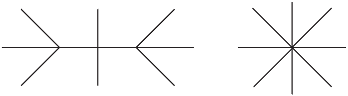


$$A_6 = \frac{\langle 12\rangle^2[56]^2\langle 3|1+2|4]^2}{s_{124}} + \dots$$

- A_6 already behaves as $\mathcal{O}(\tau)$ in the multi-chiral soft limit. It is consistent with the fact that no contact term at 6pts, $\langle 12\rangle\langle 23\rangle\langle 31\rangle + \mathrm{Perm.} = 0$.

Born-Infeld theory in 4D

- Diagrams and amplitude of 8pts



Now there is an 8pt contact term with an unfixed coefficient,

$$k\,\langle 12\rangle^2\langle 34\rangle^2[56]^2[78]^2+\dots\,.$$

- We find $A_8 \sim \mathcal{O}(\tau)$ in multi-chiral soft limits, iff $k=-1$.
- One can proceed similarly for higher points or systematically using recursion relations.

Born-Infeld theory in 4D

- On-shell recursion relations from multi-chiral soft theorems:

$$\widetilde{\lambda}_i \to \widetilde{\lambda}_i(1-z) \qquad \text{and} \qquad \lambda_k \to \lambda_k + z\eta_k,$$

for $i = 1, \ldots, \frac{n}{2}$ and $k = n-1, n$. To ensure momentum:

$$\eta_{n-1} = -\frac{1}{[n-1\, n]}\sum_{i=1}^{n/2}[i\, n]\lambda_i, \;\; \eta_n = \frac{1}{[n-1\, n]}\sum_{i=1}^{n/2}[i\, n-1]\lambda_i.$$

- Then Cauchy residue theorems

$$\int \frac{dz\, A(z)}{z(1-z)} = 0,$$

leads to on-shell recursion relations.

Born-Infeld theory in 4D

- There is no direct symmetry understanding of the multi-chiral soft theorem yet.
- Bonus relations: in fact $A_n \sim \mathcal{O}(\tau^2)$ under multi-chiral soft limits!
- The multi-chiral soft theorem can be understood by the fact that BI is supersymmetrically related to Volkov-Akulov theory or DBI theory.

Born-Infeld theory in 4D

- BI theory is the vector sector of breaking $\mathcal{N}=2$ to $\mathcal{N}=1$.
  SUSY Ward Identity:

$$\tilde{\lambda}_1^{\dot{\alpha}} A(1^-2^- \dots n/2^-(n/2+1)^+ \dots n^+)$$
$$= -\sum_{i=n/2+1}^{n} \tilde{\lambda}_i^{\dot{\alpha}} A(\psi_1^- 2^- \dots n/2^-(n/2{+}1)^+ \dots \psi_i^+ \dots n^+).$$

The amplitudes on the RHS go as $\mathcal{O}(\tau)$ term by term, as $\lambda_i \to \tau\lambda_i$ for $i = n/2+1, \dots, n$.

Born-Infeld theory in 4D

- BI theory actually has bigger symmetry, as $\mathcal{N}=4 \to \mathcal{N}=2$. SUSY Ward Identity:

$$
\begin{aligned}
&\tilde{\lambda}_n^{\dot\alpha}\tilde{\lambda}_n^{\dot\beta}\mathcal{A}_n(1^+\dots(\frac{n}{2})^+(\frac{n}{2}+1)^-\dots n^-) \\
&=\sum_{i=1}^{n/2}\tilde{\lambda}_i^{\dot\alpha}\tilde{\lambda}_i^{\dot\beta}\mathcal{A}_n(1^+\dots\bar{\phi}_i\dots(\frac{n}{2})^+(\frac{n}{2}+1)^-\dots\phi_n) \\
&+\sum_{i\neq j}^{n/2}\tilde{\lambda}_i^{\dot\alpha}\tilde{\lambda}_j^{\dot\beta}\mathcal{A}_n(1^+\dots\psi_i^1\dots\psi_j^2\dots(\frac{n}{2})^+(\frac{n}{2}+1)^-\dots\phi_n)
\end{aligned}
$$

It leads to $A_n \sim \mathcal{O}(\tau^2)$ behavior, for $\lambda_i \to \tau\lambda_i$ with $i=1,\dots,n/2$.

Born-Infeld theory in general D

Dimension reduction uniquely fixes the theory.

- At 4 points
$$\mathcal{L}_4 = c_1\langle FFFF\rangle + c_2\langle FF\rangle^2,$$
Requiring dimension reduced scalars behaves as $\mathcal{O}(\tau^2)$, relates $c_2 = c_1/4$.
- At 6 points
$$\mathcal{L}_6 = d_1\langle FFFFFF\rangle + d_2\langle FFFF\rangle\langle FF\rangle + d_3\langle FF\rangle^3.$$
All the coefficients are uniquely fixed by the requirement of $\mathcal{O}(\tau^2)$ behavior.

Conclusion and outlook

- On-shell recursion relations from soft theorems: soft bootstrap. [Elvang, Hadjiantonis, Jones, Paranjape, 18']
- Its application to EFT of $\mathcal{N} = 4$ SYM on the Coulomb branch.
- New soft theorems for BI theory, can be understood from SUSY breaking.
- Uniqueness of BI theory from dimension reduction.

Conclusion and outlook

- It would be interesting to understand the symmetry of multi-chiral soft theorem, without referring to SUSY.
- More new soft theorems, and search for and rule out new EFT's. Apply the ideas to fermionic theories, vector theories with higher derivatives, etc.
- Combining soft theorems with other constraints, such as SUSY.

Thank you!