Some Remarks on

IR-physics

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Is gauge invariance fundamental?

Weinberg 1965

S- matrix theory
with $m=0$ $j=1$
$j=2$

(Lorentz invariance
and
analytic properties
of
Amplitudes)

$\Rightarrow$

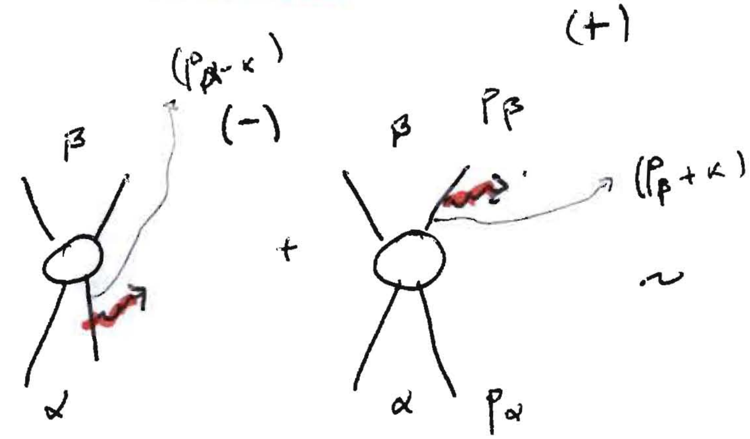
- Conservation of charges
- $m_g = m_i$
(Equivalence Principle)

$\boxed{\text{Soft Theorems}}$

Soft Photon theorem.

$m = 0 \qquad j = 1$

$$\lim_{K \to 0} S_{\alpha\beta K} =$$



$$\sim$$

$$\simeq S_{\alpha\beta}\left[\sum_\beta \frac{(\bar{\epsilon}(K)\cdot \bar{p}_\beta)\, q_\beta}{p_\beta \cdot K} - \sum_\alpha \frac{(\bar{\epsilon}(\kappa)\cdot p_\alpha)\, q_\alpha}{p_\alpha \cdot K}\right] \equiv$$

$$\equiv S_{\alpha\beta}\, \vec{\varepsilon}\cdot\left(f^\mu_\beta - f^\mu_\alpha\right)$$

Lorentz invariance:

$$\left(f^\mu_\beta - f^\mu_\alpha\right)\cdot K = 0 \Rightarrow$$

$$\left(\sum q_\alpha - \sum q_\beta\right) = 0 \qquad \boxed{\text{Charge Conservation.}}$$

In addition:

Lorentz invariance $\Longrightarrow$ "on shell gauge symmetry"

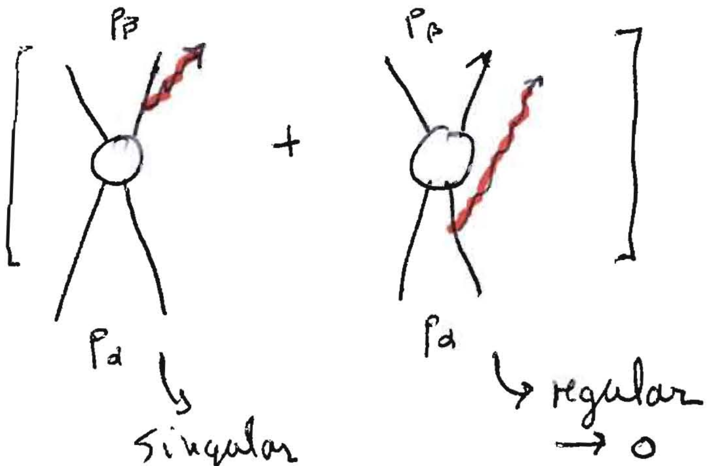
$$\vec{\varepsilon} \Rightarrow \vec{\varepsilon} + \lambda(\kappa)\vec{\kappa}$$

$$S_{\alpha\beta\kappa} \Longrightarrow S_{\alpha\beta\kappa} + \lambda(\kappa)\left[\sum q_\alpha - \sum q_\beta\right] = S_{\alpha\beta\kappa}$$

Comment: Massless charged electrons

collinear radiation is also singular.

$$\lim_{K\parallel p_\beta} S_{\alpha\beta K} = \lim_{K\parallel p_\beta}$$



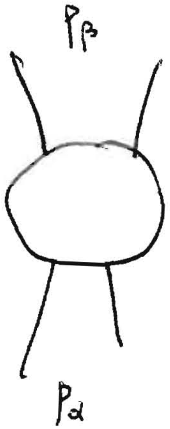
$$\sim S_{\alpha\beta}\,\frac{\varepsilon\cdot p_\beta}{p_\beta\cdot K}$$

No Lorentz invariant.

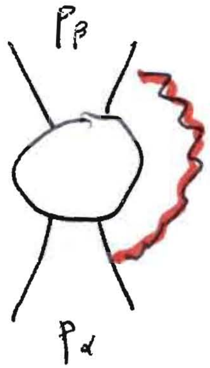
LN: add incoming "hard" photons.

Are massless charged particles Q.M consistent?

IR- Divergences.



$= \Gamma^0_{\alpha\beta}$



$$\sim \Gamma^0_{\alpha\beta}\left(\frac{\lambda}{\Lambda}\right)^{B_{\alpha\beta}/2} \longrightarrow 0$$

$\lim \lambda \to 0$

$B_{\alpha\beta}$ <u>Weinberg's IR-factor</u>

$$B_{\alpha\beta} = \sum \eta_n \eta_m e^2 \beta_{nm}^{-1} \ln\left(\frac{1+\beta_{nm}}{1-\beta_{nm}}\right) \qquad \beta_{nm} = \left[1 - \frac{m^4}{(p_n\cdot p_m)^2}\right]^{1/2}$$

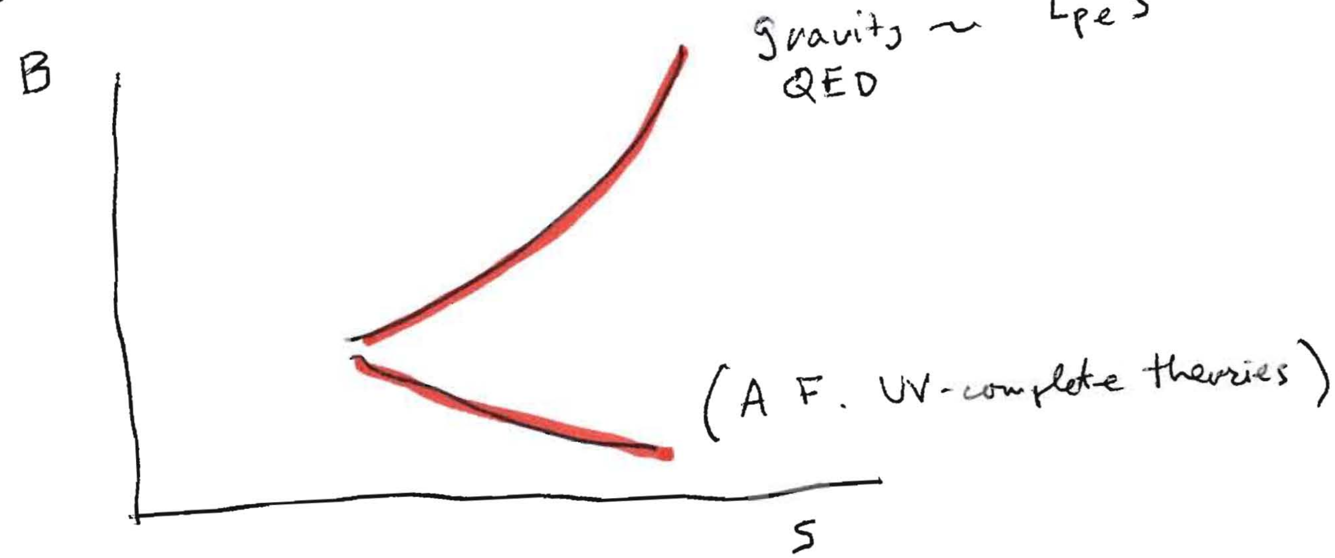
<u>gravity</u>:

$$\frac{G_N}{2\pi} \sum \eta_n \eta_m m_n m_m \frac{1+\beta_{nm}^2}{\beta_{nm}(1-\beta_{nm}^2)^{1/2}} \ln\left(\frac{1+\beta_{nm}}{1-\beta_{nm}}\right)$$

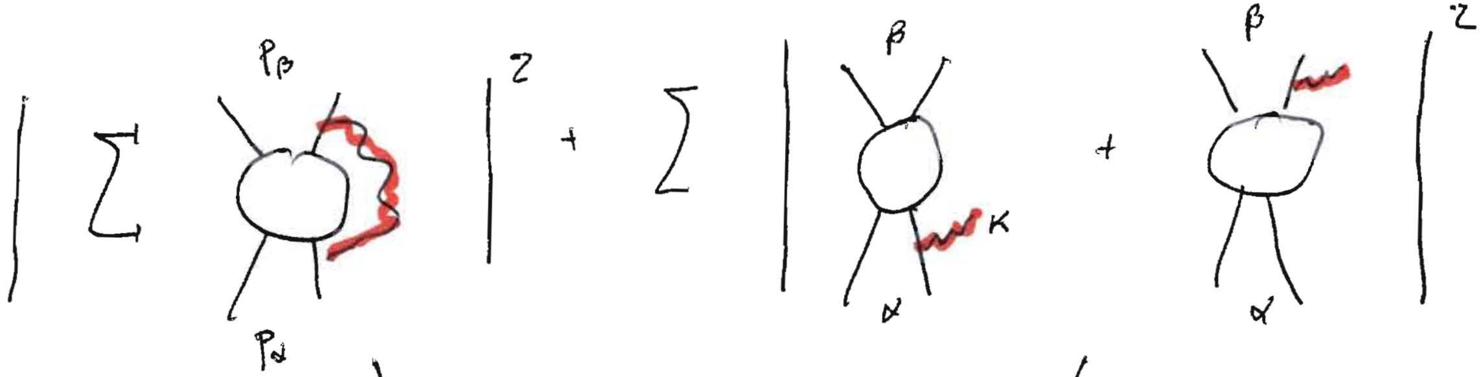
Note:

QED $\lim_{m \to 0} B_{\alpha\beta} \longrightarrow \ln(m_e)$ collinear problem.

Gravity $\lim_{m \to 0}$ good.



IR - Divergences and Soft theorem.



$$\left(\frac{\lambda}{\Lambda}\right)^{B_{\alpha\beta}} \qquad\qquad \left(\frac{\varepsilon}{\lambda}\right)^{\tilde{B}_{\alpha\beta}} \leftarrow \text{Determined by soft theorem.}$$

$$\sum_n \frac{1}{n!} \int^{\varepsilon} \frac{dk_1 \cdots dk_n}{k_1 \cdots k_n} \left(\tilde{B}_{\alpha\beta}\right)^n$$

inclusive.

Soft theorem $\Rightarrow$ $\boxed{B = \tilde{B}}$

$$\sigma \approx \sigma_0 \left(\frac{\varepsilon}{\Lambda}\right)^{B_{\alpha\beta}}$$

Note: i) In massless case

$$\underline{B_{\alpha\beta} \sim \ln\left(\frac{q^2}{m_e^2}\right)}$$ q - transfer momentum.

$$\sigma \sim \left(\frac{\varepsilon}{\Lambda}\right)^{\ln\left(\frac{q^2}{m_e^2}\right)} \to 0$$ no scattering!

ii) In cases where $B(s\to\infty) \to \infty$ gravity in certain kinematical region

IR - effects act as suppression factor $\left(\frac{\varepsilon}{\Lambda}\right)^{B}$ $\varepsilon \ll \Lambda$

Coherent States

$$S|\alpha\rangle = \sum_\beta S^0_{\alpha\beta}\, |e_{p_\beta}\rangle \otimes \sum_n \frac{1}{n!}\int_\lambda^\varepsilon dk_1 \dots dk_n\, S_{\alpha\beta k_i}\, a^\dagger(k_1)\dots a^\dagger(k_n)\, |0\rangle$$

$$|\alpha\rangle = |e_{p_\alpha}\rangle \otimes |0\rangle$$

$$= \sum_\beta S^0_{\alpha\beta}\, \underbrace{e^{\int_\lambda^\varepsilon S_{\alpha\beta}(k)\, a^\dagger(k)\, dk}\, |0\rangle}_{\text{coherent state}} \otimes |e_\beta\rangle$$

Soft theorem

$S^0_{\alpha\beta} =$ 

$S_{\alpha\beta k} =$  $+$ 

$$= \underbrace{\sum S^0_{\alpha\beta}}_{\text{IR finite}}\, e^{1/2 \int_\lambda^\varepsilon |S_{\alpha\beta k}|^2}\, D(\alpha,\beta\rangle \otimes |e_\beta\rangle$$

normalization factor

normalized coh state

The soft photon function $S_{\alpha\beta\kappa} = \bar{\varepsilon}(\kappa)\cdot f^{\gamma}_{\alpha\beta}(\kappa) = \bar{\varepsilon}(\kappa)(f_\beta - f_\alpha)$

defines the coh states

$$f_\alpha \sim \frac{\bar{P}_\alpha}{P_\alpha \cdot K} \quad \text{singular at } K = 0$$

∞ # of zero energy $K=0$ photons.

To define these states requires to use

Von Neumann Hilbert space.

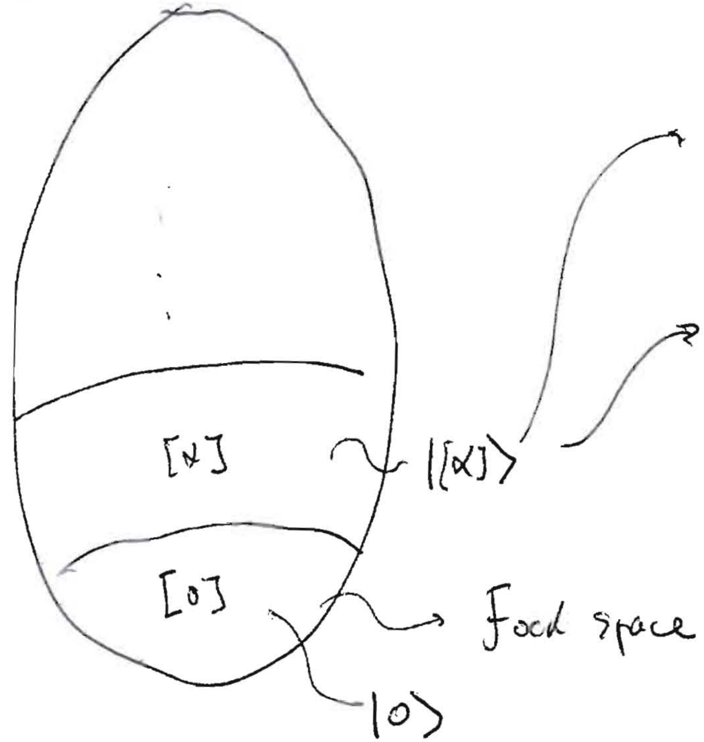
In a nutshell:

In QM ; $[p, q] = \hbar$ or $[a_k, a^+_k] = 1$

$\mathcal{H}$ all representations are unitarily equivalent.

VN - space:

$[a_k a^+_k] = 1 \rightsquigarrow$ different unitarily non equivalent representations in each equivalence class.



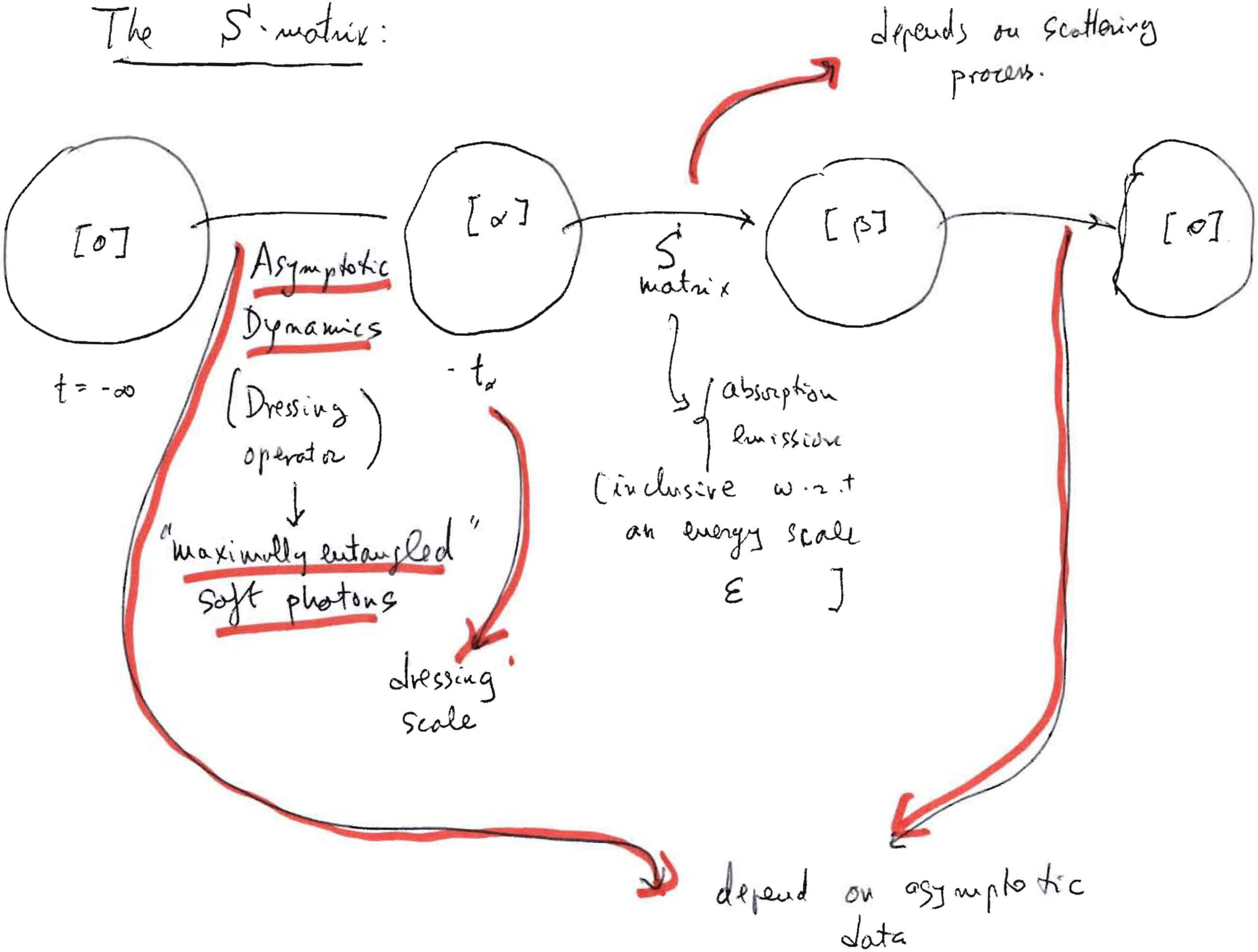
different "radiative vacua"

coh states

$$e^{\int dk\, \alpha(k)\, a^+_k + h.c}$$

$\int \alpha(k) = \infty$; $\alpha \sim \beta$ if $\int \alpha - \beta < \infty$

The S-matrix:


depends on scattering process.
[0]
[α]
[β]
[0]
Asymptotic Dynamics
(Dressing operator)
"maximally entangled" soft photons
S matrix
absorption
emission
(inclusive w.r.t an energy scale ε)
t = -∞
- t_α
dressing scale
depend on asymptotic data

$$S\,|\alpha\rangle = \sum_{[\beta]} S^{i}_{\alpha\beta}\,|[\beta], i\rangle$$

$\alpha \in [\alpha]$

i: basis of equivalence class $[\beta]$

Decoherence: Tracing over basis in $\mathcal{H}_{[\beta]}$.

$S \sim \ln\left(\frac{\varepsilon}{r}\right)$ → dressing scale.

Gauge Invariance and IR Dressing.

S-matrix approach:

"on shell" gauge transformations

$$\vec{\varepsilon}(k) \to \vec{\varepsilon}(k) + \lambda(k)\,\vec{k}$$

"Large gauge transformations"

$$\lambda(k=0) \neq 0.$$

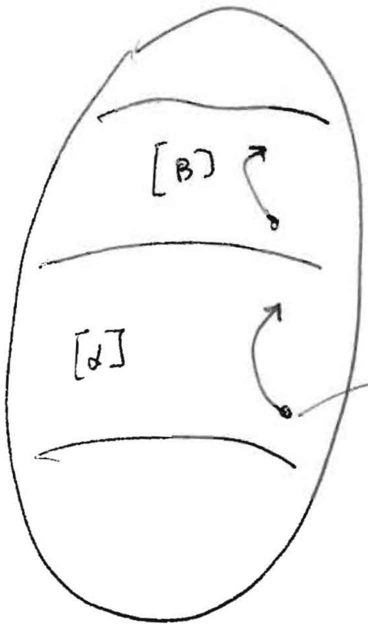
Why are important?:

After IR dressing:
$$|\alpha\rangle = \exp \int \hat{d}k\, f_\alpha(k)\, a^\dagger(k) + hc$$

$$f_\alpha(k) \sim \frac{\varepsilon \cdot P_\alpha}{P_\alpha \cdot K} \longrightarrow f_\alpha(k) + \lambda(k)$$

$\lambda(k\sim 0) \neq 0 \implies$ non trivial action on $|\alpha\rangle$.

VN-space



$$\mathcal{U}^{\beta}(\lambda) \sim \underline{Q^{\beta}}$$

$$\mathcal{U}^{\alpha}(\lambda) \sim \underline{Q_{\alpha}}$$

"generators"

$$\langle \beta | Q^{+}_{\beta} S - S Q_{\alpha} | \alpha \rangle = 0.$$

"weak" gauge invariance.

$$\bar{\varepsilon} \to \bar{\varepsilon} + \lambda(k)\bar{k} \qquad (1)$$

$$\lambda(k=0) \neq 0$$

$$\begin{tikzcd}
|\alpha\rangle \in [\alpha] \arrow[r, "S"] \arrow[d, "Q_\alpha"'] & |\beta\rangle \arrow[d, "Q_\beta"] \\
|\alpha'\rangle \arrow[r, "S"'] & |\beta'\rangle
\end{tikzcd}$$

commutative if the repr of (1) on different equiv classes are unit. equivalent!

Recall $\bar{\mathcal{E}} \to \bar{\mathcal{E}} + \bar{\kappa} \cdot \lambda(\kappa) \iff$ (S-matrix) Lorentz invariance.

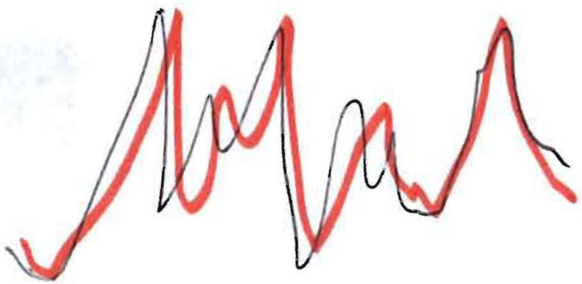
Different representations on different equivalence classes

$\overset{?}{\Rightarrow}$ Different asymp. Lorentz ? (BMS).

These questions strongly depend on dressing scale.

Marginal Comment:

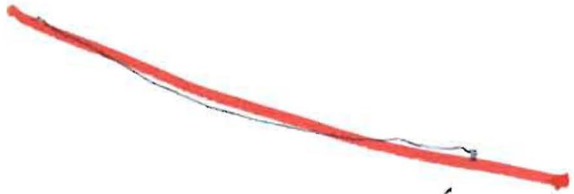
The IR - physics of Q - Gravity is what sets the vacuum structure.



after

$\longrightarrow$

Soft Q. Gravity physics



just "one point".

multiverse $\longrightarrow$ Swampland

(with G. Dvali).

Thank you