S-Matrix Uniqueness from Soft Theorems

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Motivation

- Probe the power of soft theorems, taking some inspiration from three topics: holography, the S-matrix, black hole information paradox
- The S-matrix is naturally a holographic object
- "Holographic S-matrix" from asymptotic symmetries (= soft theorems)?
- Black hole information from soft hair? Superficial question: how much information can soft particles actually carry?
- Concrete question: how much of an amplitude can be fixed by soft theorems?

IR vs UV information

- Naive answer: IR and UV are disjoint; soft theorems go to finite order so can only constrain some IR part of an amplitude

$$\text{A=IR(soft theorem satisfying)+UV(soft theorem avoiding)}$$

- In fact soft theorems (together with locality) fully determine a wide range of amplitudes (YM, GR, NLSM, DBI, dilaton effective theories, extended theories like NLSM$\oplus\phi^3$, mixed theories like EYM; also including some low lying higher derivative corrections, like F^3 and F^4 for YM)
- Soft theorems can be relaxed

Three types of soft behavior

- Start with local ansatz B_n, take soft limit as $p_n^\mu = zp_n^\mu$, $z \to 0$
- Soft theorems
$$B_n \to (\frac{1}{z}S_0 + z^0 S_1 + \ldots)A_{n-1}$$
- Soft operators
$$B_n \to (\frac{1}{z}S_0 + z^0 S_1 + \ldots)B_{n-1}$$
- "Soft gauge invariance"
$$B_n \to \frac{1}{z}B_n^{-1} + z^0 B_n^0 + \ldots$$
with B_n^{-1} and B_n^0 gauge invariant only in particle n.

Soft theorem avoiding terms and enhanced soft limits

- Imposing soft theorems up to order $\mathcal{O}(z^k)$ on an ansatz:

$$B_n = \mathrm{IR}[A_n] + [\mathcal{O}(z^{k+1}) \text{ terms}]$$

- At low enough mass dimension, the soft theorem avoiding terms cannot exist, so the amplitude is completely fixed
- Proof not immediate from power counting because of momentum conservation
- Some special objects have enhanced soft limit behavior compared to what power counting suggests [Cheung 16', Arkani-Hamed 16']

$$\begin{aligned} \text{Non-linear sigma model} &= s_{13} \sim \mathcal{O}(z) \\ \text{Dirac-Born-Infeld} &= s_{12}^2 + s_{13}^2 + s_{14}^2 \sim \mathcal{O}(z^2) \\ \text{Special Galileon} &= s_{12}s_{13}s_{14} \sim \mathcal{O}(z^3) \end{aligned}$$

Soft theorem avoiding terms and enhanced soft limits

- Imposing soft theorems leads to one of three outcomes:

$$
\begin{aligned}
B_n =& A_n, \\
B_n =& \mathrm{IR}[A_n] + f(e, 1/K) A_{\text{enhanced}}, \\
& \text{or} \\
B_n =& \mathrm{IR}[A_n] + [\text{trivial objects}],
\end{aligned}
$$

- Can easily determine which answer you get from propagator structure, mass dimension, etc.

Yang-Mills from gauge invariance

- Consider a general (ordered) local function at four points, with mass dimension matching the expected amplitude:

$$B_4(p^2) = a_1 \frac{e_1.e_2\, e_3.p_1\, e_4.p_2}{p_1.p_2} + a_2 \frac{e_1.e_2\, e_3.e_4\, p_2.p_3}{p_1.p_2} + 60 \text{ terms}$$

- Impose gauge invariance in particle 1...4, solve linear system in the a_i's
- Unique solution which matches the amplitude! [Arkani-Hamed 16']
- Similar story for GR, and NLSM, DBI or special Galileon using the Adler zero
- Unitarity follows automatically
- Doesn't work for higher derivative corrections

Yang-Mills from soft theorems

- Consider the same ansatz, and impose soft theorems:

$$B_4(p^2) \to (S_0 + S_1)A_3$$

- Unique solution $B_4 = A_4$
- Easy exercise: any term you write at this mass dimension will have a $1/z$ or z^0 piece, so it is controlled by at least one soft theorem

$$\frac{e_1.e_2\; e_3.p_1\; e_4.p_2}{p_1.p_2}, \quad \frac{e_1.e_2\; e_3.e_4 p_1.p_2}{p_1.p_4}$$

Yang-Mills higher derivative corrections

- Label the extra powers of momenta in the numerator by κ (so $\kappa = 0$ corresponds to the usual amplitude)
- Take a higher mass dimension ansatz, $\kappa = 2$, use F^3 as "seed":

$$B_n^{+2} \to (S_0 + S_1) A_{n-1}^{F^3}$$

- Still true for $\kappa = 4$, using the 5 possible amplitudes $(1 \times (F^3)^2 + 4 \times F^4)$:

$$B_n^{+4} \to (S_0 + S_1) A_{n-1}^{F^4}$$

- One low point exception: $\kappa = 4$ at $n = 6$, where you find

$$B_6^{+4} = \mathrm{IR}[A_6^{F^4}] + f(e) A_6^{\mathrm{NLSM}}$$

Uniqueness
Soft operators

- Soft theorems contain lots of info through the lower point amplitude, so maybe this is not so surprising. Can we get away with less?
- Instead of full soft theorem, only require:

$$B_n \to (S_0 + S_1) B_{n-1}$$

- Fixes both B_n and B_{n-1} (and still true for higher corrections*)
- Crucially this even fixes the low point amplitude, so all the information is contained in the soft operator

Uniqueness
"Soft" gauge invariance

- If we got this far, how about using even less info?
- Just impose gauge invariance up to sub-leading order in the soft particle.

$$B_n \to \frac{1}{z} B_n^{-1} + z^0 B_n^0 + \ldots \tag{1}$$

- Impose B_n^{-1}, B_n^0 gauge invariant in n; repeat for other particles
- Still unique solution
- Conclusion: soft particles carry enough information to fully constrain the amplitude

Gravity

- Same story, soft theorems fix amplitudes up to $\kappa = 4$
- Exception at $n = 6$ leads to DBI:

$$B_6^{+4} = \mathrm{IR}[A_6^{+4}] + f(e) A_6^{\mathrm{DBI}}$$

- Conjecture this is true for soft operators and soft gauge invariance

NLSM

- Impose double soft theorems (including non-adjacent one)
- Amplitudes up to $\kappa = 4$ are fixed
- Conjecture still true for soft operators
- Soft operator approach finds a new solution at $\kappa = 2$, missed by Z-theory [Carrasco 17']

$$B_6^{+2} \to (S_0 + S_1) B_4^{+2} \;\Rightarrow\; B_4 = s_{12} s_{14}$$

- Soft Adler zero also fixes amplitudes:

$$B_n \to \frac{1}{z} B_n^{-1} + z^0 B_n^0 + z B_n^1 + \dots$$

impose B_n^i scales as $\mathcal{O}(z)$ only in $p_n \to 0$ and $p_{n-1} \to 0$

Extended NLSM

- New mixed amplitude found via CHY [Cachazo 16']

$$A_n \to z \sum s A_{n-1}^{\mathrm{NLSM}\oplus\phi^3} + \mathcal{O}(z^2)$$

- Can fix $A^{\mathrm{NLSM}\oplus\phi^3}$ by imposing NLSM soft theorems

Scalar DBI

- Same story as before: up to $\kappa = 4$ amplitudes via soft theorems
- New feature: for $\kappa = 0$ amplitudes leading+subleading theorems are enough: subsubleading does not contain new info

Vector BI

- Strange fact: no subleading double soft theorem for BI? Leading not enough
- But can use the BI extended theory [Cachazo 16']

$$A_n \to \sum_i \sum_j s_{in} s_{jn} S_{0;j} A_{n-1}^{\mathrm{BI}\oplus\mathrm{YM}}$$

$$B_n = \mathrm{IR}[A_n^{\mathrm{BI}}] + f(e) A_n^{\mathrm{DBI}}$$

- Can fix by adding gauge invariance
- Not totally useless, $A_{n-1}^{\mathrm{BI}\oplus\mathrm{YM}}$ can be derived from soft behavior of gluons+BI photons.

Dilaton

- Two types of dilatons - "gravity" dilaton, and a "conformal" dilaton [Di Vecchia][Wen]
- Conformal dilaton satisfies:

$$S_0 = D - \left(\sum_{i=1}^{n} k_i^\mu \frac{\partial}{\partial k_i^\mu} + \frac{D-2}{2} \right)$$

$$S_1 = -q^\lambda \sum_{i=1}^{n} \left(\frac{1}{2} \left(2k_i^\mu \frac{\partial^2}{\partial k_i^\mu \partial k_i^\lambda} - k_{i\lambda} \frac{\partial^2}{\partial k_{i\nu} \partial k_i^\nu} \right) + \frac{D-2}{2} \frac{\partial}{\partial k_i^\lambda} \right)$$

- Further two types of dilaton theories depending on how conformal invariance is broken
- We can get both by modifying our ansatz: w/o singularities for explicit breaking, w/ singularities for spontaneous breaking

- Again the same story works, for example at $\kappa = 2$ by imposing

$$B_7(p^6) \to (S_0 + S_1)B_6(p^6)$$

  we find

$$B_7(p^6) = f(D) \sum_{1\leq i<j\leq 7} s_{ij}^3 + g(D) \sum_{1\leq i<j<k\leq 7} s_{ijk}^3$$

- A new feature: more derivatives fixed as multiplicity increases
- Like for DBI, the subleading theorem follows from the leading one (without assuming unitarity) [Wen 16']

- Now allow poles
- $\kappa = 0$ is the same as before, but at $\kappa = 2$ the DBI amplitude can appear
- We obtain:

$$B_6(p^6) = A_6^{\mathrm{explicit}}(p^6) + A_6^{\mathrm{DBI}}(p^8)$$

- But now it's not a bug, it's a feature: DBI appears in the action of this theory

- Consider the following $[i,j\rangle$ D-dimensional BCFW shift:

$$e_i \to \hat{e}_i \qquad\qquad p_i \to p_i + z\hat{e}_i$$

$$e_j \to \hat{e}_j + zp_i \frac{\hat{e}_i . e_j}{p_i . p_j} \qquad p_j \to p_j - z\hat{e}_i$$

  where $\hat{e}_i = e_i - p_i \frac{e_i . p_j}{p_i . p_j}$.
- Claim 1: there are unique objects which have the usual BCFW scaling under this shift ($1/z$ for adjacent, $1/z^2$ for non-adjacent or permutation invariant functions - no bad shift)
- Claim 2: these objects are scattering amplitudes (for YM or GR)
- The first claim is provable via soft limits
- The second claim can only be proven now that we have uniqueness from soft theorems

Uniqueness from BCFW

- Unique object "C" order by order

$$C \to \frac{1}{z} C^{-1} + z^0 C^0 + z C^1 + \dots$$

- Can check that $(S_0 + S_1) A_n$ also has the usual scaling, so by uniqueness:

$$C \to \frac{1}{z} S_0 A_n + z^0 S_1 A_n + z C^1 + \dots$$

- By uniqueness from soft theorems the unique BCFW object must be the amplitude
- Can use the shift to build general dimension amplitudes

Conclusion

- Conceptually interesting that soft behavior is so constraining; gives a common purpose to all soft theorems
- Thoughts about soft theorem avoiding terms in YM, GR lead to NLSM, DBI
- Easy new method to compute amplitudes using maximal ignorance
- But we also find a few new things
- The $\kappa = 2$ derivative correction to the NLSM

$$A_4 = p_1.p_2\, p_1.p_4$$

- For DBI subsubleading theorem follows automatically from the others
- Can find arbitrarily high derivative corrections to dilaton
- Instance of scale invariance $\Rightarrow$ conformal invariance, even without assuming unitarity

Conclusion

Future questions

- Extra constraints from double, triple etc. soft limits?
- General inverse soft factor method?
- Does this work in 4D?
- Holographic S matrix directly from asymptotic symmetries?
- General D symmetry responsible for soft theorems?
- Soft particles contain all amplitude information: is this extra motivation for BH info in soft particles, or just similar sounding words?