

# Short-Flow-Time Expansion of the Low Energy Effective Field Theory

Òscar Lara Crosas

Collaborators =

$\frac{\alpha_s}{4\pi}$  (Peter Stoffer) +  $(\frac{\alpha_s}{4\pi})^2$  (Nils Hermansson-Truedsson + Fabian Lange + Peter Stoffer)

University of Zürich and Paul Scherrer Institute

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- SFTEs relevant for the nEDM: generalities
- SFTE of the Low Energy Effective Field Theory
- SFTEs relevant for the nEDM: 2-loops

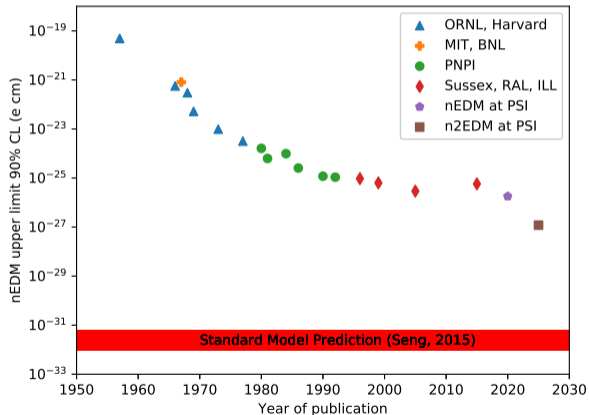
## Motivation: Neutron Electric Dipole Moment

- Baryon asymmetry of the universe  $\implies$  BSM CP violation.
- EDMs  $\simeq (\bar{\psi}\sigma_{\mu\nu}\gamma_5\psi) F^{\mu\nu}$  (e.g. nEDM) are interesting probes of BSM CP violation.
- If we allow for heavy BSM physics

$$d_N \sim \sum_i L_i \langle N | \mathcal{O}_i^{\text{MS}} | N \gamma \rangle$$

- These matrix elements are non-perturbative quantities  $\implies$  Lattice QCD.
- Lattice scheme has to be converted to MS!
- Ultimate goal: Translate the bounds on low-energy observables (nEDM) to constraints on heavy new physics.

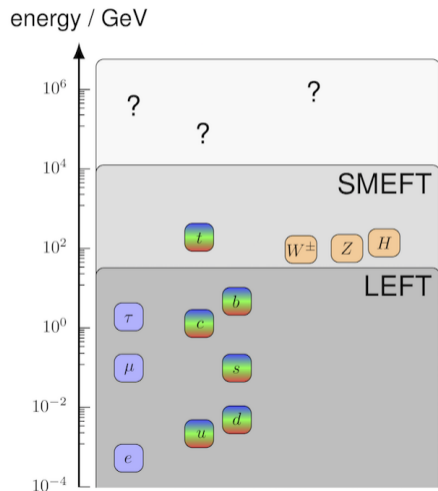
# Neutron Electric Dipole Moment



Two possibilities:

- Detecting a signal in the unexplored region  $\implies$  CP violating New Physics.
- Not detecting a signal, lowering the bound. Still very interesting! Constrains the shape of New Physics Models.

# Effective Field Theories



- Effective Field Theories contain only the relevant degrees of freedom at a certain energy scale.
- Model independent way to encode the effects of all heavy particles (both BSM and SM).

$$\mathcal{L}_{\text{LEFT}} = \mathcal{L}_{\text{QCD+QED}} + \sum_{d \geq 5} \sum_{i=1}^{n_d} L_i^{(d)} \mathcal{O}_i^{(d)}$$

$$d_N \sim \sum_i L_i \langle N | \mathcal{O}_i | N \gamma \rangle \quad (1)$$

$$\begin{aligned} d_N = & -(1.5 \pm 0.7) \cdot 10^{-3} \bar{\theta} e \text{ fm} - (0.20 \pm 0.01) d_u \\ & + (0.78 \pm 0.03) d_d + (0.0027 \pm 0.0016) d_s - (0.55 \pm 0.28) e \tilde{d}_u \\ & - (1.1 \pm 0.55) e \tilde{d}_d + (50 \pm 40) \text{ MeV} e \tilde{d}_G . \end{aligned} \quad (2)$$

where  $d_q$  denotes the EDM of a quark  $\sim (\bar{q} \sigma^{\mu\nu} \gamma_5 q) F_{\mu\nu}$ ,  $\tilde{d}_q$  denotes its chromo EDM  $\sim (\bar{q} \sigma^{\mu\nu} T^A \gamma_5 q) G_{\mu\nu}^A$ , and  $\tilde{d}_G$  denotes the gluon-chromo EDM  $\sim f^{ABC} \tilde{G}_{\mu\nu}^A G_{\nu\rho}^B G_{\rho\mu}^C$ . Lessons learned:

- We need to measure multiple EDMs.
- Uncertainties in matrix elements are too big, we should aim for at most 25% uncertainty.
- Lattice is key!

At one loop, the short-flow-time expansions for operators contributing to the nEDM are now known

- Quark dipoles  $\bar{q}qX$  (Mereghetti, Monahan, Rizik, Shindler, Stoffer, 2021)

- Weinberg operator  $GG\tilde{G}$  (ÒLC, Monahan, Rizik, Shindler, Stoffer, 2023)

- Four quark operators  $\bar{q}q\bar{q}q$  (Bühler, Stoffer, 2023)

$\mathcal{O}$  (10 – 25%) uncertainties are required for meaningful nEDM studies. (Alarcon et al., 2022)

- $\mathcal{O}$  (40%) uncertainties at one loop  $\implies$  we are not precise enough!

- Progress on the 2-loop dipole SFTE (Borgulat, Harlander, Rizik, Shindler, 2022)

# Nuisance operators for the LEFT SFTE

In a general off-shell matching

$$\mathcal{O}_i(t, x) = C_{ij} \underbrace{\mathcal{O}_j}_{\text{physical}} + C_{ik}^{\text{EOM}} \underbrace{\mathcal{O}_k^{\text{EOM}}}_{\text{EOM}} + C_{il}^{\mathcal{E}} \underbrace{\mathcal{E}_l}_{\text{evanescent}} + C_{is}^{\mathcal{N}} \underbrace{\mathcal{N}_s}_{\text{gauge variant}} .$$

The basis of nuisance operators  $\mathcal{N}$  relevant for the nEDM is known.

(Cirigliano, Mereghetti, Stoffer, 2020)

However:

- Basis of nuisance operators is large, computationally expensive to compute at higher orders!
- Basis of nuisance operators not known for the whole Low Energy Effective Field Theory (LEFT) basis.

# Background Field Method applied to the Gradient Flow

In a general off-shell matching, we mix with gauge-variant operators.

To avoid that, we apply the Background Field Method and split the conventional flow equation

$$\partial_t B_\mu(t, x) = D_\nu G_{\nu\mu}(t, x) + \alpha_0 D_\mu \partial_\nu B_\nu(t, x)$$

into two flow equations

(Suzuki, PTEP 2015 (2015) 10, 103B03)

$$\begin{aligned}\partial_t B_\mu(t, x) &= D_\nu G_{\nu\mu}(t, x) + \alpha_0 D_\mu \hat{D}_\nu B_\nu(t, x), \\ \partial_t \hat{B}_\mu(t, x) &= \hat{D}_\nu \hat{G}_{\nu\mu}(t, x)\end{aligned}$$

where both flow equations transform covariantly under background gauge transformations!

We made public the full set of Feynman rules for the GF Background Field Method ([ØLC, Stoffer, 2026](#))

# LEFT SFTE

- Goal: SFTE of all the operators in the LEFT containing quark and gluons  $\rightarrow$  connection to lattice and extraction of RGEs.
- Basis of physical, EOM and evanescent operators is known.

(Naterop, Stoffer, 2023)

- There are 3701 physical operators

Dipoles:  $(\bar{q}q)X$

Gluonic:  $X^3$

4-fermi:  $(\bar{\psi}_1\Gamma_1\psi_2)(\bar{\psi}_3\Gamma_2\psi_4)$  (3)

- The background field method lets us avoid nuisance operators.
- Flowed calculations at one-loop are "easy" and automatizable: master integrals + method of projectors.
- $\implies$  all ingredients ready for the LEFT SFTE at one-loop!

(ÒLC, Stoffer, 2026)

# Chiral Symmetry Breaking in the 't Hooft Veltman scheme

- Chiral symmetry broken in the LEFT by mass terms and higher dimensional operators

$$\mathcal{L}_{\text{mass}} = -\bar{\psi}_R M \psi_L - \bar{\psi}_L M^\dagger \psi_R.$$

- We can preserve chiral symmetry by promoting masses and Wilson coeffs. to be spurions

$$M \rightarrow U_R M U_L^\dagger, \quad M^\dagger \rightarrow U_L M^\dagger U_R^\dagger.$$

- HV breaks chiral symmetry

$$\mathcal{L}_{\text{kin}} = \bar{\psi} i \not{D} \psi = \bar{\psi}_L i \overline{\not{D}} \psi_L + \bar{\psi}_R i \overline{\not{D}} \psi_R + \bar{\psi}_L i \hat{\not{D}} \psi_R + \bar{\psi}_R i \hat{\not{D}} \psi_L.$$

- Breaking of spurious chiral symmetry due to HV can be cured by finite counterterms.  
(Naterop, Stoffer, 2023)

$$d_N \sim \sum_i L_i \langle N | \mathcal{O}_i | N \gamma \rangle \quad (4)$$

$$\begin{aligned} d_N = & -(1.5 \pm 0.7) \cdot 10^{-3} \bar{\theta} e \text{ fm} - (0.20 \pm 0.01) d_u \\ & + (0.78 \pm 0.03) d_d + (0.0027 \pm 0.0016) d_s - (0.55 \pm 0.28) e \tilde{d}_u \\ & - (1.1 \pm 0.55) e \tilde{d}_d + (50 \pm 40) \text{ MeV} e \tilde{d}_G . \end{aligned} \quad (5)$$

- Uncertainties in matrix elements are too big, we should aim for at most 25% uncertainty.
- One-loop SFTEs typically have a 10 – 25% uncertainty, but this does not include any lattice uncertainty
- We started working on the 2-loop SFTE of the LEFT at  $D = 5$  ([ØLC](#), [Hermansson-Truedsson, Lange, Stoffer, 202X](#))

- One-loop SFTEs relevant for the neutron electric dipole moment are done.
- Background Field Method applied to the gradient flow lets us avoid mixing into gauge variant operators.
- One-loop SFTE of the QCD sector of the Low Energy Effective Field Theory is done.
- We need more precision  $\implies$  2-loop SFTEs!

Thank you! 😊

Back up slides 

# Nuisance operators for the LEFT SFTE

In a general off-shell matching

$$\mathcal{O}_i(t, x) = C_{ij} \underbrace{\mathcal{O}_j}_{\text{physical}} + C_{ik}^{\text{EOM}} \underbrace{\mathcal{O}_k^{\text{EOM}}}_{\text{EOM}} + C_{il}^{\mathcal{E}} \underbrace{\mathcal{E}_l}_{\text{evanescent}} + C_{is}^{\mathcal{N}} \underbrace{\mathcal{N}_s}_{\text{gauge variant}} .$$

The basis of nuisance operators  $\mathcal{N}$  relevant for the nEDM is known.

(Cirigliano, Mereghetti, Stoffer, 2020)

However:

- Basis of nuisance operators is large, computationally expensive to compute at higher orders!
- Basis of nuisance operators not known for the whole Low Energy Effective Field Theory (LEFT) basis.

## nEDM in terms of LEFT Wilson Coefficients

$$d_N \sim \sum_i L_i \langle N | \mathcal{O}_i | N \gamma \rangle$$

$$\begin{aligned} d_N = & -(1.5 \pm 0.7) \cdot 10^{-3} \bar{\theta} e \text{ fm} - (0.20 \pm 0.01) d_u \\ & + (0.78 \pm 0.03) d_d + (0.0027 \pm 0.0016) d_s - (0.55 \pm 0.28) e \tilde{d}_u \\ & - (1.1 \pm 0.55) e \tilde{d}_d + (50 \pm 40) \text{ MeV} e \tilde{d}_G . \end{aligned}$$

where  $d_q$  denotes the EDM of a quark  $q$ ,  $\tilde{d}_q$  denotes its chromo EDM, and  $\tilde{d}_G$  denotes the gluon-chromo EDM. (Alarcon et al., 2022)

Lessons learned:

- We need to measure multiple EDMs.
- Uncertainties in matrix elements are too big, we should aim for at most 25% uncertainty.
- Lattice is key!

# Perturbative Short Flow-Time Expansion

Overview from the Zürich Gradient Flow Workshop 2025

$$\mathcal{O}_i(t) = C_{i,j}(t) \mathcal{O}_j^{MS}$$

$$C_{i,j} \sim \underbrace{\log}_{\text{RGEs}} + \underbrace{\text{finite}}_{\text{connection to lattice}}$$

RGEs

- LEFT SFTEs (ÖLC, Stoffer, 2026)
- Scalar QCD at 2-loops (talk by Robert H.)
- SMEFT SFTEs (talk by Janosch)

Connection to lattice

- Neutron electric dipole moment SFTEs (talk by Andrea)
- Neutral meson mixing and meson lifetimes (talks by Matthew and Jonas)
- Quark mass determination (talks by Hiromasa and Robert M.)

# Operator basis relevant for the nEDM: Physical (I)

Basis from (Cirigliano, Mereghetti, Stoffer, 2020):

$$\mathcal{O}_1^{(4)} = \frac{1}{g^2} \text{Tr}[G_{\mu\nu} \hat{G}_{\mu\nu}],$$

$$\mathcal{O}_2^{(4)} = \partial_\mu (\bar{q} \gamma_\mu \gamma_5 q),$$

$$\mathcal{N}_1^{(4)} = (\bar{q}_E \gamma_5 q + \bar{q} \gamma_5 q_E),$$

$$\mathcal{O}_1^{(5)} = \epsilon_{ijk} \epsilon_{lmn} \mathcal{M}_{mj} \mathcal{M}_{nk} \bar{q}^i i \gamma_5 q^l$$

$$\mathcal{O}_1^{(6)} = \frac{1}{g^2} \text{Tr}[G_{\mu\nu} G_{\mu\lambda} \hat{G}_{\nu\lambda}] = \mathcal{O}_{GG\hat{G}},$$

$$\mathcal{O}_2^{(6)} = (\bar{q} \tilde{\sigma}_{\mu\nu} \mathcal{M} t^a q) G_{\mu\nu}^a,$$

$$\mathcal{O}_3^{(6)} = (\bar{q} \tilde{\sigma}_{\mu\nu} \mathcal{M} Q q) F_{\mu\nu}.$$

## Operator basis relevant for the nEDM: Physical (II)

$$\mathcal{O}_4^{(6)} = \frac{1}{g^2} \text{Tr}[\mathcal{M}^2] \text{Tr}[G_{\mu\nu} \hat{G}_{\mu\nu}],$$

$$\mathcal{O}_5^{(6)} = \frac{1}{g^2} \partial_\nu \text{Tr}[(D_\mu G_{\mu\lambda}) \hat{G}_{\nu\lambda}]$$

$$\mathcal{O}_6^{(6)} = \partial_\mu (\bar{q} \gamma_\mu \gamma_5 \mathcal{M}^2 q) - \frac{1}{N_f} \mathcal{O}_7^{(6)},$$

$$\mathcal{O}_7^{(6)} = \text{Tr}[\mathcal{M}^2] \partial_\mu (\bar{q} \gamma_\mu \gamma_5 q),$$

$$\mathcal{O}_8^{(6)} = \partial_\mu (\bar{q} \gamma_\nu Q q \hat{F}_{\mu\nu}),$$

$$\mathcal{O}_9^{(6)} = \frac{1}{g^2} \square \text{Tr}[G_{\mu\nu} \hat{G}_{\mu\nu}],$$

$$\mathcal{O}_{10}^{(6)} = \square \partial_\mu (\bar{q} \gamma_\mu \gamma_5 q).$$

## Operator basis relevant for the nEDM: EOM

$$\mathcal{N}_1^{(6)} = (\bar{q}_E \tilde{\sigma}_{\mu\nu} t^a q + \bar{q} \tilde{\sigma}_{\mu\nu} t^a q_E) G_{\mu\nu}^a,$$

$$\mathcal{N}_2^{(6)} = (\bar{q}_E \tilde{\sigma}_{\mu\nu} Q q + \bar{q} \tilde{\sigma}_{\mu\nu} Q q_E) F_{\mu\nu},$$

$$\mathcal{N}_3^{(6)} = (\bar{q}_E \mathcal{M} \gamma_\mu \gamma_5 D_\mu q + \bar{q} \overleftarrow{D}_\mu \gamma_\mu \gamma_5 \mathcal{M} q_E),$$

$$\mathcal{N}_4^{(6)} = (\bar{q}_E \mathcal{M}^2 \gamma_5 q + \bar{q} \mathcal{M}^2 \gamma_5 q_E) - \frac{1}{N_f} \mathcal{N}_5^{(6)},$$

$$\mathcal{N}_5^{(6)} = \text{Tr}[\mathcal{M}^2] (\bar{q}_E \gamma_5 q + \bar{q} \gamma_5 q_E),$$

$$\mathcal{N}_6^{(6)} = \partial_\mu (\bar{q}_E \gamma_5 D_\mu q - \bar{q} \overleftarrow{D}_\mu \gamma_5 q_E),$$

$$\mathcal{N}_7^{(6)} = \partial_\mu (\bar{q}_E \tilde{\sigma}_{\mu\nu} D_\nu q - \bar{q} \overleftarrow{D}_\nu \tilde{\sigma}_{\mu\nu} q_E),$$

$$\mathcal{N}_8^{(6)} = \frac{1}{g^2} \partial_\lambda \left( G_{\mu\nu}^a \left( D_\rho G_{\rho\sigma}^a - g^2 \bar{q} t^a \gamma_\sigma q \right) \right) \epsilon_{\mu\nu\lambda\sigma},$$

$$\mathcal{N}_9^{(6)} = \partial_\lambda \left( \bar{q}_E \mathcal{M} \gamma_\lambda \gamma_5 q + \bar{q} \mathcal{M} \gamma_\lambda \gamma_5 q_E \right)$$

## Operator basis relevant for the nEDM: Gauge variant (I)

$$\mathcal{N}_{11}^{(6)} = \frac{1}{g^2} G_{\mu\nu}^a \left( \partial_\lambda \left( D_\rho G_{\rho\sigma}^a - g^2 \bar{q} t^a \gamma_\sigma q - g^2 f^{abc} (\partial_\sigma \bar{c}^b) c^c \right) \right) \epsilon_{\mu\nu\lambda\sigma},$$

$$\mathcal{N}_{12}^{(6)} = (\bar{q}_E \gamma_5 q + \bar{q} \gamma_5 q_E) G_\mu^a G_\mu^a,$$

$$\mathcal{N}_{13}^{(6)} = (\bar{q}_E \gamma_5 t^a q + \bar{q} \gamma_5 t^a q_E) G_\mu^b G_\mu^c d^{abc},$$

$$\mathcal{N}_{14}^{(6)} = (\bar{q}_E \gamma_5 t^a q - \bar{q} \gamma_5 t^a q_E) \partial_\mu G_\mu^a,$$

$$\mathcal{N}_{15}^{(6)} = (\bar{q}_E \gamma_5 t^a D_\mu q - \bar{q} \overleftarrow{D}_\mu \gamma_5 t^a q_E) G_\mu^a.$$

## Operator basis relevant for the nEDM: Gauge variant (II)

$$\mathcal{N}_{16}^{(6)} = (\bar{q}_E \tilde{\sigma}_{\mu\nu} t^a q + \bar{q} \tilde{\sigma}_{\mu\nu} t^a q_E) \partial_\mu G_\nu^a,$$

$$\mathcal{N}_{17}^{(6)} = (\bar{q}_E \mathcal{M} \gamma_\mu \gamma_5 t^a q - \bar{q} \mathcal{M} \gamma_\mu \gamma_5 t^a q_E) G_\mu^a,$$

$$\mathcal{N}_{18}^{(6)} = \frac{1}{g^2} \partial_\lambda \left( (\partial_\mu G_\nu^a) \left( D_\rho G_{\rho\sigma}^a - g^2 \bar{q} t^a \gamma_\sigma q - g^2 f^{abc} (\partial_\sigma \bar{c}^b) c^c \right) \right) \epsilon_{\mu\nu\lambda\sigma},$$

$$\mathcal{N}_{19}^{(6)} = \partial_\mu \left( (\bar{q}_E \gamma_5 t^a q - \bar{q} \gamma_5 t^a q_E) G_\mu^a \right),$$

$$\mathcal{N}_{20}^{(6)} = \partial_\mu \left( (\bar{q}_E \tilde{\sigma}_{\mu\nu} t^a q + \bar{q} \tilde{\sigma}_{\mu\nu} t^a q_E) G_\nu^a \right).$$

# Background field method

Background Field Method in QCD (QFT and the SM, Schwartz, 2014): Split gauge field in background and quantum parts

$$A_\mu^a \rightarrow \underbrace{\hat{A}_\mu^a}_{\text{bckg.}} + \underbrace{A_\mu^a}_{\text{quantum}} .$$

QCD Lagrangian is invariant under background gauge transformations:

$$\hat{A}_\mu^a \rightarrow \hat{A}_\mu^a + \frac{1}{g} \partial_\mu \alpha^a - f^{abc} \alpha^b \hat{A}_\mu^c, \quad A_\mu^a \rightarrow A_\mu^a - f^{abc} \alpha^b A_\mu^c .$$

We can now gauge fix  $A_\mu$  through Faddeev-Popov and choose the gauge condition  $\hat{D}_\mu A_\mu = 0$  to obtain

$$\mathcal{L}_{\text{QCD,BFM}} = \mathcal{L}_{\text{QCD}, A_\mu^a \rightarrow \hat{A}_\mu^a + A_\mu^a} - \frac{1}{2\tilde{\xi}} \left( \hat{D}_\mu A_\mu^a \right)^2 + \text{ghost terms} .$$

# Background field method applied to the GF: Fermions

We split the fermion fields in a quantum and background part

$$\chi(t, x) = \hat{\chi}(t, x) + k(t, x)$$

and change the flow equation from

$$\partial_t \chi = D_\mu D_\mu \chi - \alpha_0 (\partial_\mu B_\mu) \chi$$

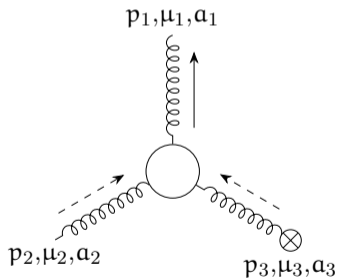
to

(Suzuki, 2015)

$$\begin{aligned}\partial_t \chi &= D_\mu D_\mu \chi - \alpha_0 (\hat{D}_\mu B_\mu) \chi, \\ \partial_t \hat{\chi} &= \hat{D}_\mu \hat{D}_\mu \chi.\end{aligned}$$

# Background field method applied to the GF: Feynman Rules

- Flow vertices involving only quantum gauge fields are equal to the ones of  $R_{\xi}$  gauge.
- Flow vertices involving only background gauge fields are equal to the ones of  $R_{\xi}$  gauge setting  $\alpha_0 = 0$ .
- Flow vertices involving both quantum and background fields involve new formulae:



$$\begin{aligned} &= -if^{a_1, a_2, a_3} \int_0^\infty dt \left( -g^{\mu_1 \mu_3} \left( 2p_3^{\mu_2} + (1 - \alpha_0) p_2^{\mu_2} \right) \right. \\ &\quad \left. - g^{\mu_1 \mu_2} \left( 2p_2^{\mu_3} + p_3^{\mu_3} \right) \right. \\ &\quad \left. - g^{\mu_2 \mu_3} \left( 2(\alpha_0 - 1) p_2^{\mu_1} + (\alpha_0 + 1) p_3^{\mu_1} \right) \right) \end{aligned}$$