Quark Masses from Gradient Flow using HISQ Ensembles

Akhil Chauhan

University of Illinois Urbana Champaign

Outline

- Background
- Gradient Flow
- Staggered Fermions
- Current Status

Gradient Flow Working Group

- Akhil Chauhan (University of Illinois Urbana-Champaign)
- Mingwei Dai (University of Illinois Urbana-Champaign)
- Anna Hasenfratz (University of Colorado Boulder)
- Nathan Mackey (University of Colorado Boulder)
- Curtis T. Peterson (Michigan State University)

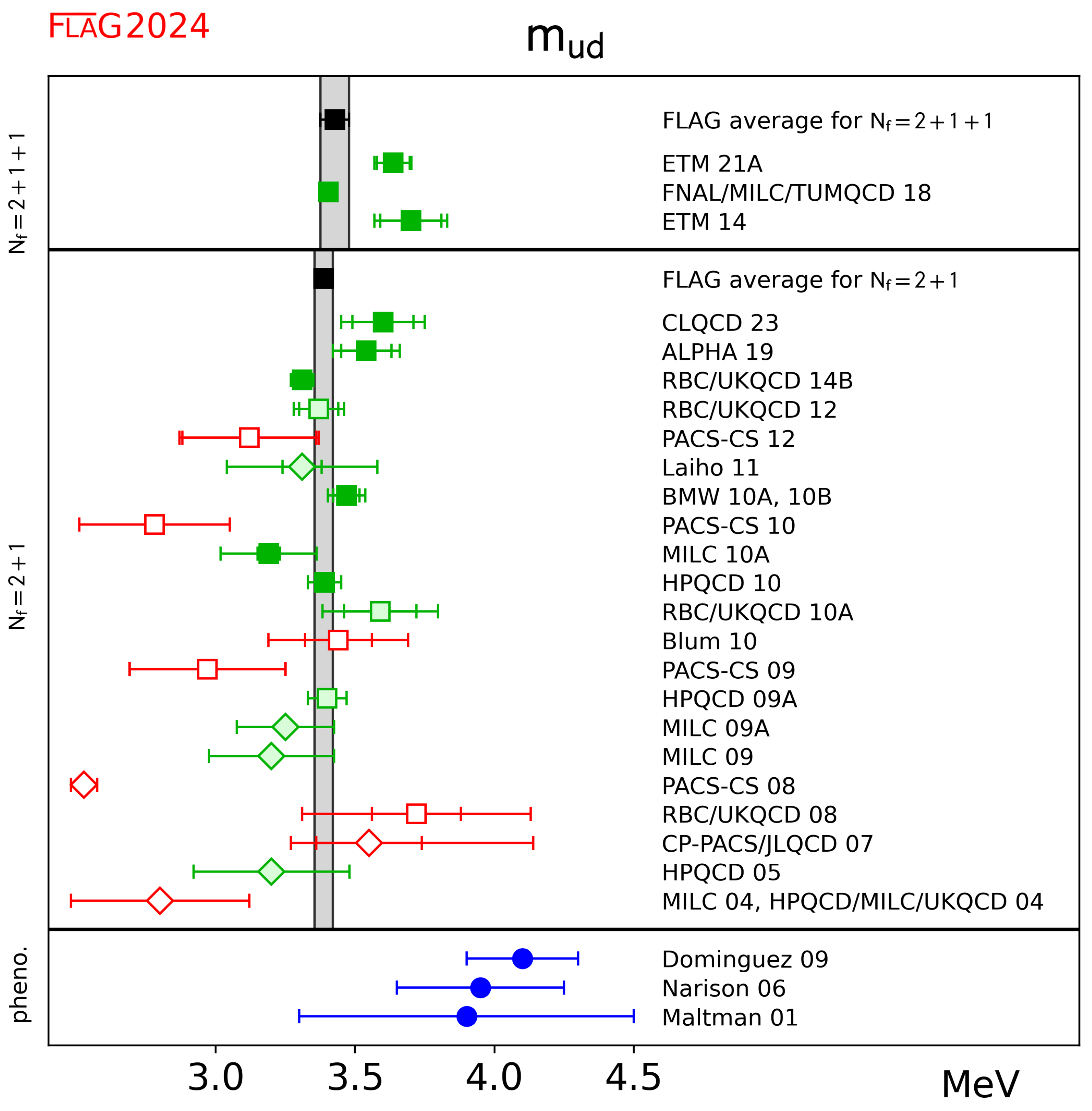
Quark Masses

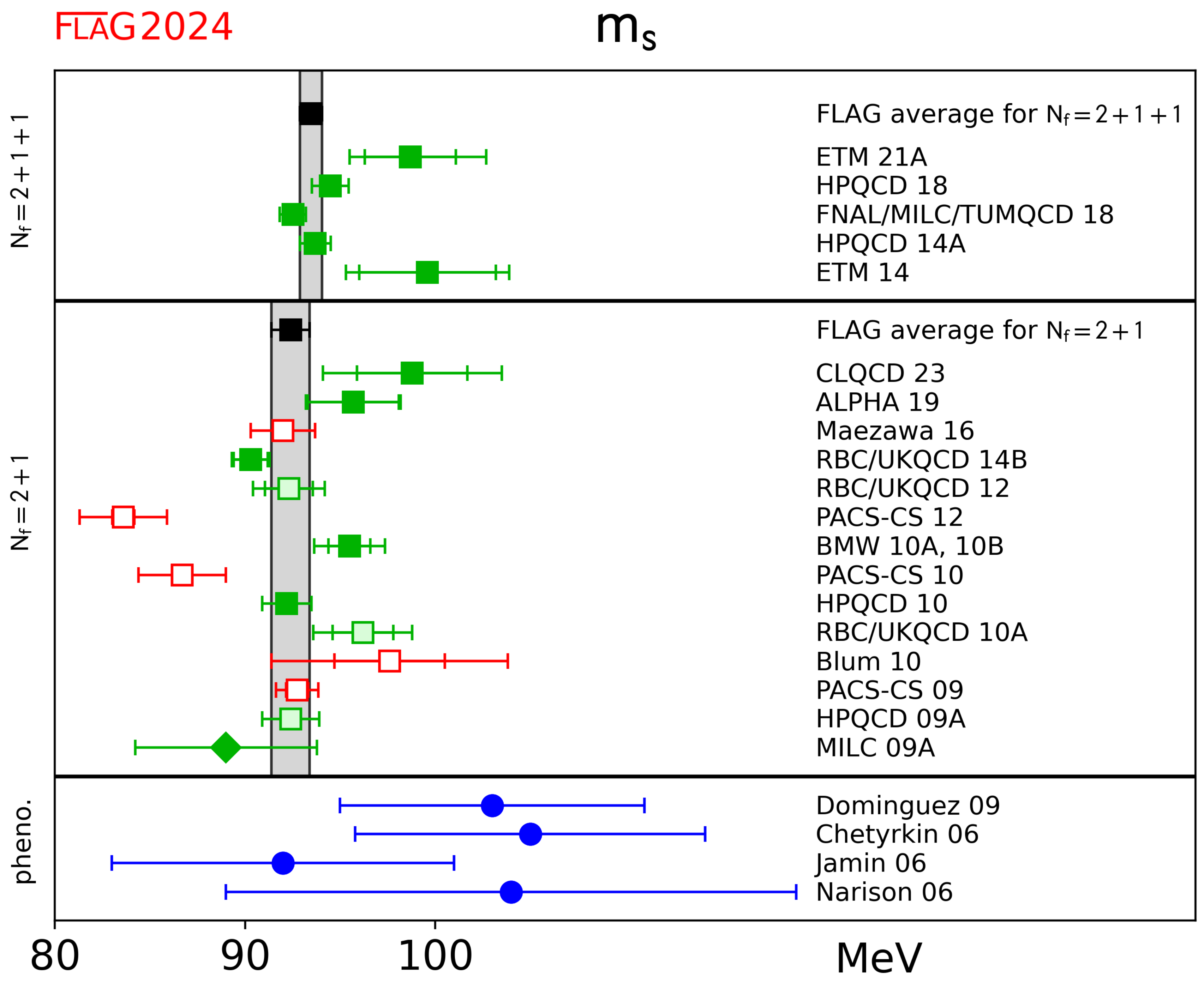
- Fundamental parameters in the Standard Model
- Depend on renormalization scheme and scale
- Many different determinations from the lattice
  - Current-current correlator moments
  - Fit spectrum to EFT
  - Gradient Flow

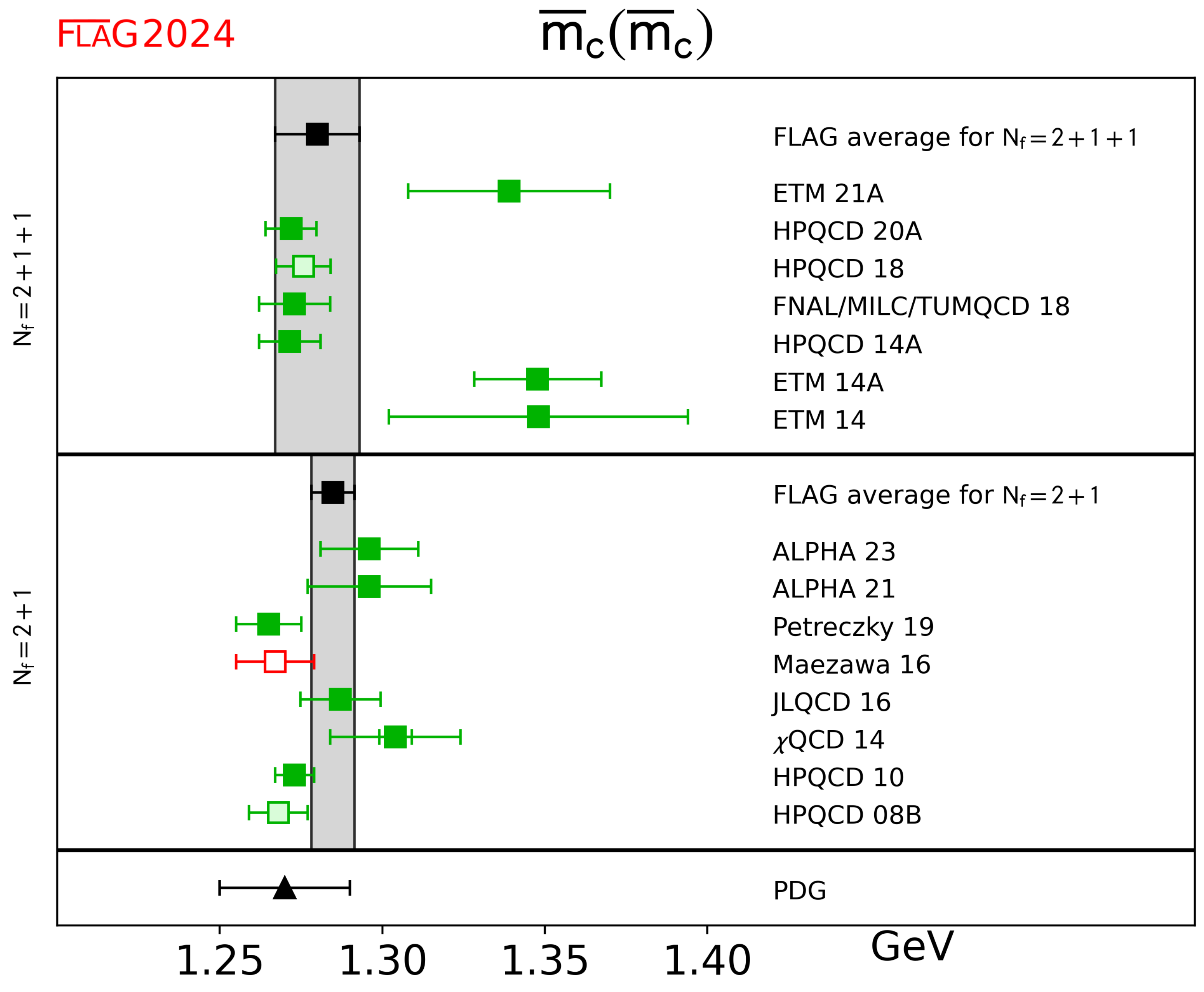
* B. Colquhoun, C. T. Davies, D. Hatton, G. P Lepage arXiv: 2508.02862
* A. Bazavov et al. PRD 98, 054517 arXiv:1802.04248

Quark Mass Overview

FLAG 2024 Review
arXiv: 2411.04268







Goal: Percent-level determination of quark masses

Gradient Flow

- Renormalization scheme that can be implemented on the lattice or pertubatively
- Gauge fields evolve as

*M. Lüscher arXiv: 1302.5246

$$\partial_\tau B_\mu = D_\nu G_{\nu\mu}.$$

- Fermion fields evolve as

$$\partial_\tau \chi = \Delta \chi, \qquad \partial_\tau \bar{\chi} = \bar{\chi}\,\overleftarrow{\Delta}, \qquad \Delta_{\mathrm{stag}} = -D^\dagger_{\mathrm{stag}} D_{\mathrm{stag}}$$

Quark Masses

- We can use gradient flow and the short flow time expansion for a precise determination of the quark masses

*M. Black, R. Harlander, A.H, A. Rago, O. Witzel arXiv: 2506.16327

- Compute anomalous dimensions nonperturbatively to run from lattice to UV scale

*A. Hasenfratz ,E. Neil, Y. Shamir, B. Swetitsky, O. Witzel, arXiv: 2306.07236

- We explore two approaches
  - PCAC: $2m_r A_0 = MP$
  - $m_R = Z_m m_{\text{bare}}$

*A. Carosso, A. Hasenfratz, E. Neil, PRL 121(2018)20

*A. Hasenfratz, C. Monahan, M. Rizik, A. Shindler, O. Witzel, 2201.09740

PCAC

- Use PCAC to obtain the renormalized quark masses directly

$$\partial_\mu A_\mu^{(R)}(x) = (m_1^{(R)} + m_2^{(R)}) P^{(R)}(x)$$
$$\partial_\mu A_\mu(\tau)(x) = \left(m_1^{(R)}(\tau) + m_2^{(R)}(\tau) \right) P(\tau)(x)$$

- On the lattice we compute two point correlation function and form ratios

$$\partial_t^- C_{A_0 P}(x_4, \tau) = 2 m_{\mathrm{GF}}(\tau) C_{PP}(x_4, \tau) \quad \rightarrow \quad m_{\mathrm{GF}}(\tau) = \frac{\partial_t^- C_{A_0 P}(x_4, \tau)}{2 C_{PP}(x_4, \tau)}$$

Mass Renormalization

- We can also compute the mass renormalization from the pseudoscalar and vector correlators

$$\widetilde{Z}_P(\tau) = \widetilde{Z}_\chi(\tau)\,\frac{\langle P(\tau)\,P(0)\rangle}{\langle P(0)\,P(0)\rangle} \qquad \widetilde{Z}_\chi(\tau) = Z_V\,\frac{\langle V_k(0)\,V_k(0)\rangle}{\langle V_k(\tau)\,V_k(0)\rangle}$$

*Or any (partially) conserved current

$$Z_m(\tau) = 1/\widetilde{Z}_P(\tau) \quad \rightarrow \quad m_{\mathrm{GF}}(\tau) = Z_m(\tau)m_{\mathrm{bare}}$$

Match to $\overline{\mathrm{MS}}$

- Perform matching in the SFTX

$$m_{\overline{\mathrm{MS}}}(\mu_{\mathrm{UV}}) = \lim_{\tau\to 0} \zeta_{AP}^{-1}(\mu_{\mathrm{UV}}, \tau) m_{\mathrm{GF}}(\tau)$$

- Factors are computed perturbatively

$$\zeta_{AP}^{-1} = 1 + \alpha_s(\mu)\left(c_1 + c_2 \ln\left(\mu^2\tau\right)\right) + \alpha_s(\mu)^2(\dots) + \dots$$

  - α_s should be small
  - $ln(\mu^2\tau)$ should be small
  - $\sqrt{8\tau} \gg a$ to minimize cutoff effects

*There might not be a flow time that mutually satisfies

RG Running

- Run from large lattice τ to small PT τ
- γ_m^{GF} known perturbatively to NNNLO

$$m_{\overline{MS}}(\mu) = \zeta_{AP}^{-1}(\mu, \tau_\mu) \times \exp\left(-\int_{\tau_\mu}^{\tau} d\tau' \frac{\gamma_m^{GF}(\tau')}{\tau'} \right) m_R(\tau) \ , \ \tau_\mu = e^{\gamma_E}/(2\mu^2)$$

- Can compute anomalous dimensions nonperturbatively. Combine with nonperturbative β-function (See N. Mackey's & Y. Mandlecha talks)

$$m_{\overline{MS}}(\mu) = \zeta_{AP}^{-1}(\mu, \tau_\mu) \times \exp\left(-\int_{g^2(\mu)}^{g^2(\tau)} dg^2 \frac{\gamma_m^{GF}(g^2)}{\beta(g^2)} \right) m_R(\tau)$$

Fermilab-MILC Ensembles

- This analysis uses the Fermilab-MILC 2+1+1 physical point ensembles

a [fm]	L [fm]	$N_s^3 \times N_t$	am_l^{sea}	am_s^{sea}	am_c^{sea}	N_{cfg}
0.15	4.8	$32^3 \times 48$	0.002426	0.0673	0.8447	10026
0.12	5.8	$48^3 \times 64$	0.001907	0.05252	0.6382	10000
0.09	5.8	$64^3 \times 96$	0.001326	0.03636	0.4313	7208
0.08	5.8	$80^3 \times 144$	0.001135	0.03096	0.33656	2032
0.06	5.8	$96^3 \times 192$	0.0008	0.022	0.260	10065
0.04	5.8	$144^3 \times 288$	0.000569	0.01555	0.1827	2041

- Subsequent figures will show figures from 12 configurations from 0.09 physical ensemble for the η_s

Staggered Operators

- We construct taste singlets

Operator	**Spin $\otimes$ Taste**
P	$\gamma^5 \otimes \xi_5$
A^0 (one-link)	$\gamma^0\gamma^5 \otimes \xi_5$
A^0 (local)	$\gamma^0\gamma^5 \otimes \xi^0\xi^5$
V^i	$\gamma^i \otimes 1$

- Different combinations affect analysis

Staggered Correlators

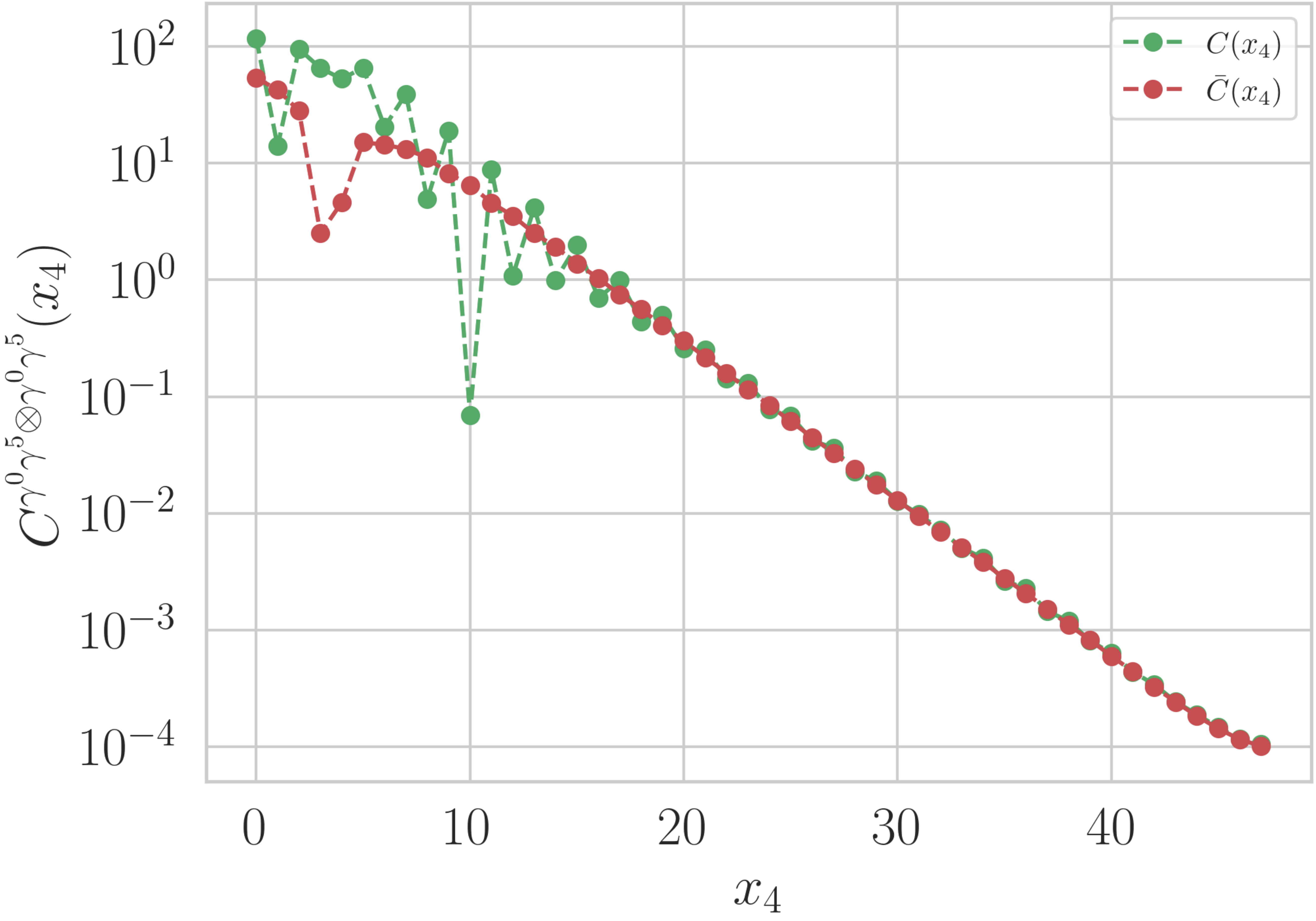
- Staggered fermions introduce oscillating states with opposite parity

$$C(t) = A(e^{-Mt} + e^{-M(T-t)}) + (-1)^t B(e^{-M_o t} + e^{-M_o(T-t)})$$

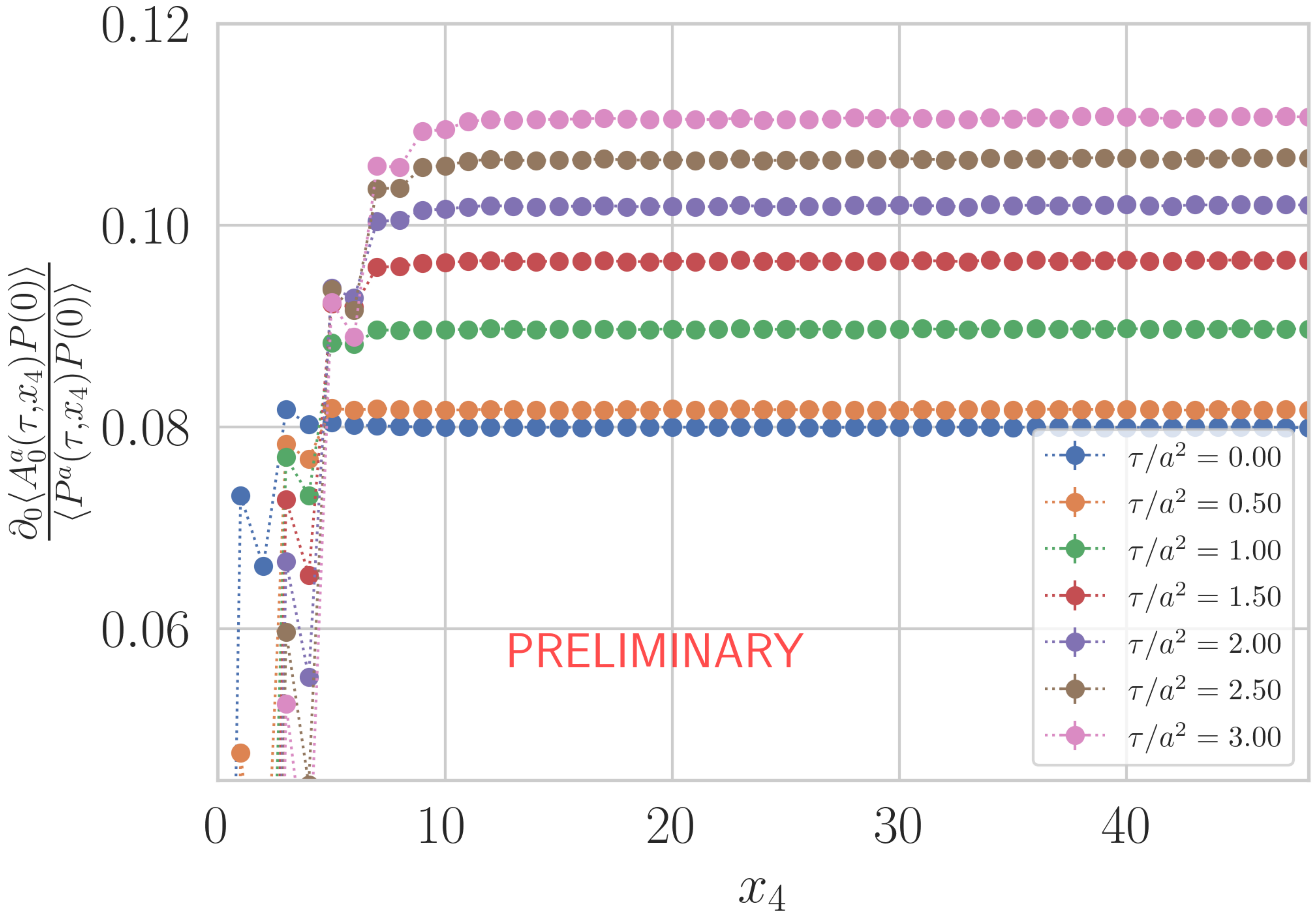
- We can project out oscillating state via

$$\bar{C}(t) = \frac{C(t+1) + C(t-1) + 2\cosh(M_o)C(t)}{2(\cosh M + \cosh M_o)}.$$

Projected Correlators

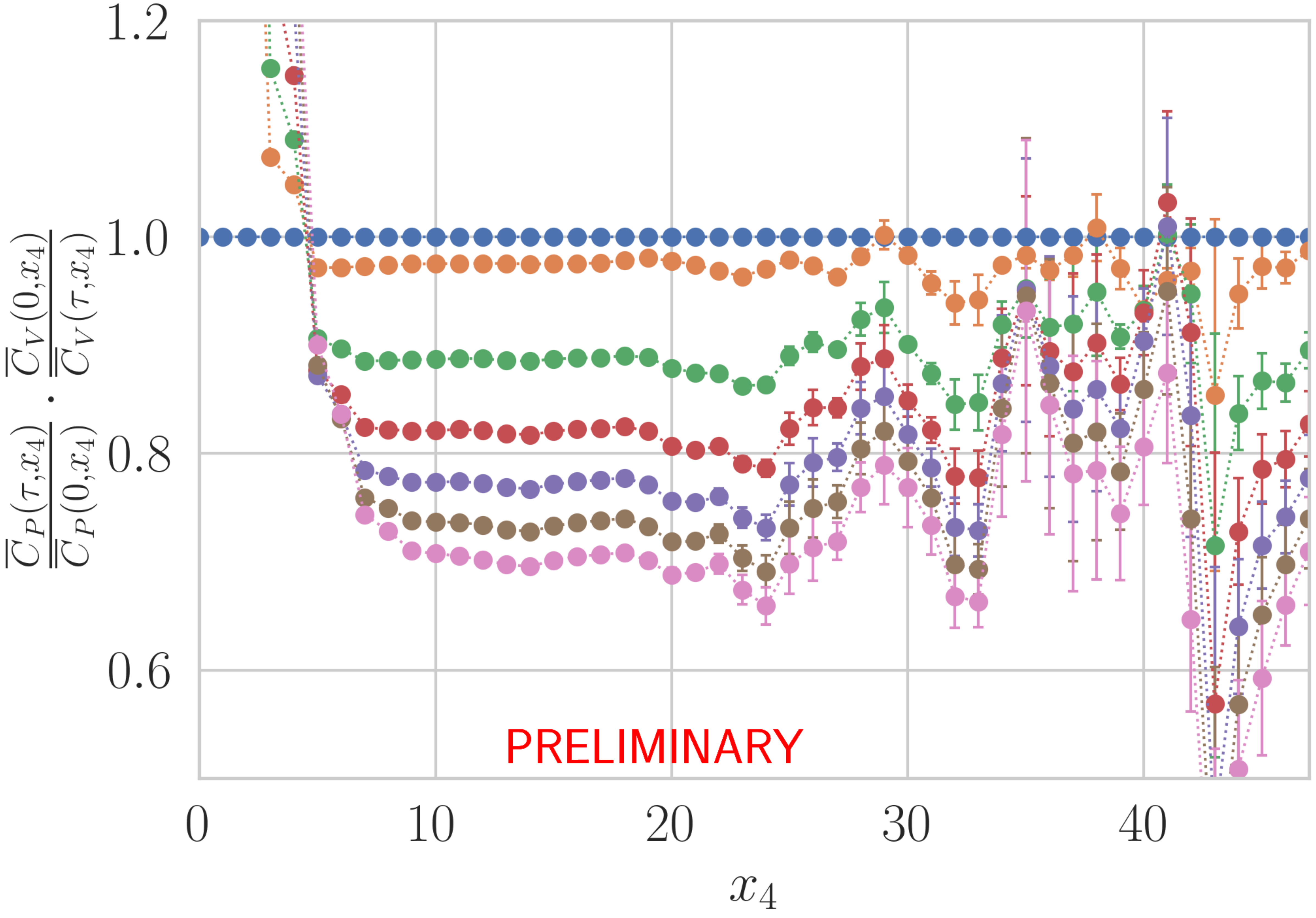


Flowed PCAC @ 0.09 fm

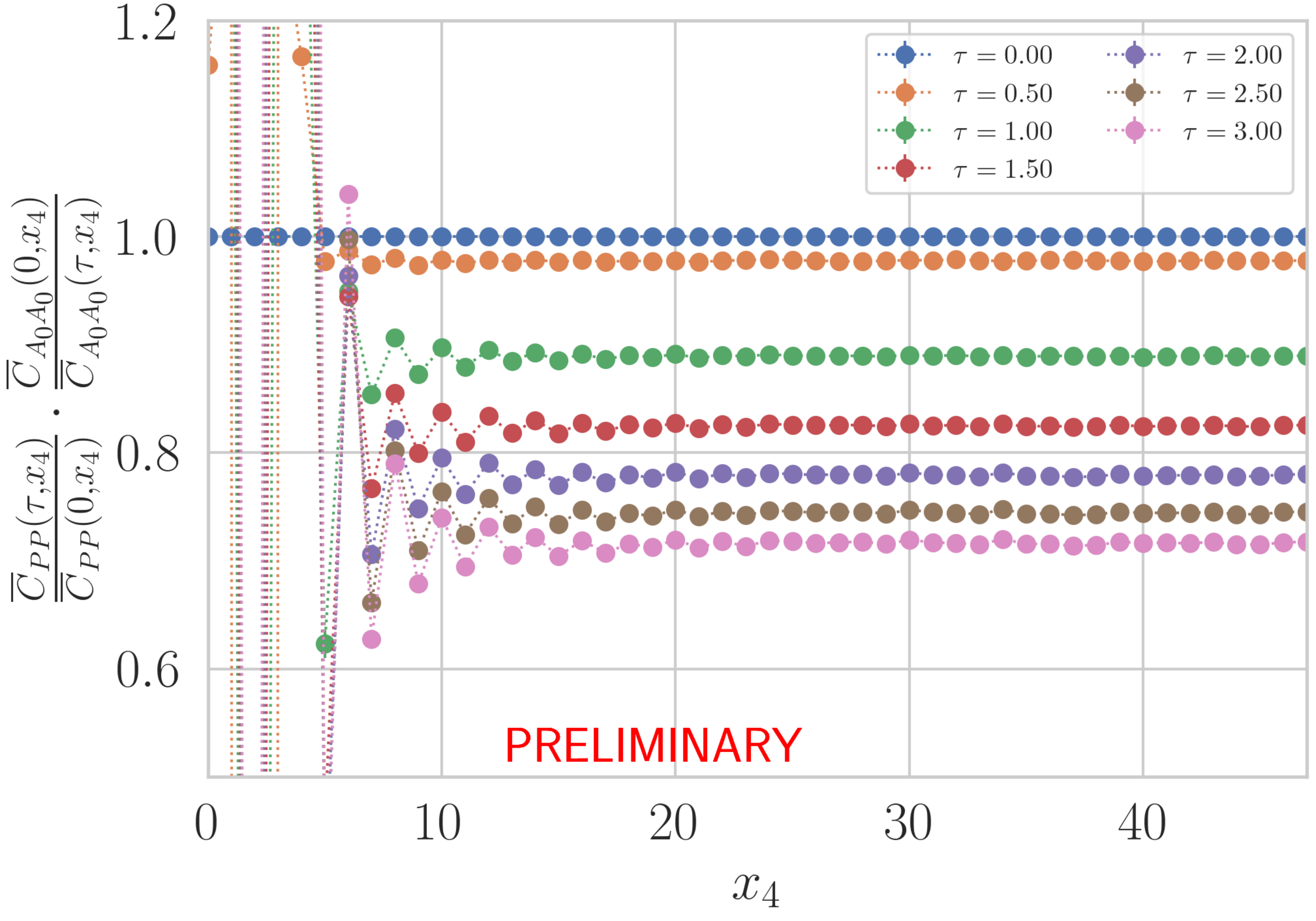


Flowed Z_m @ 0.09 fm

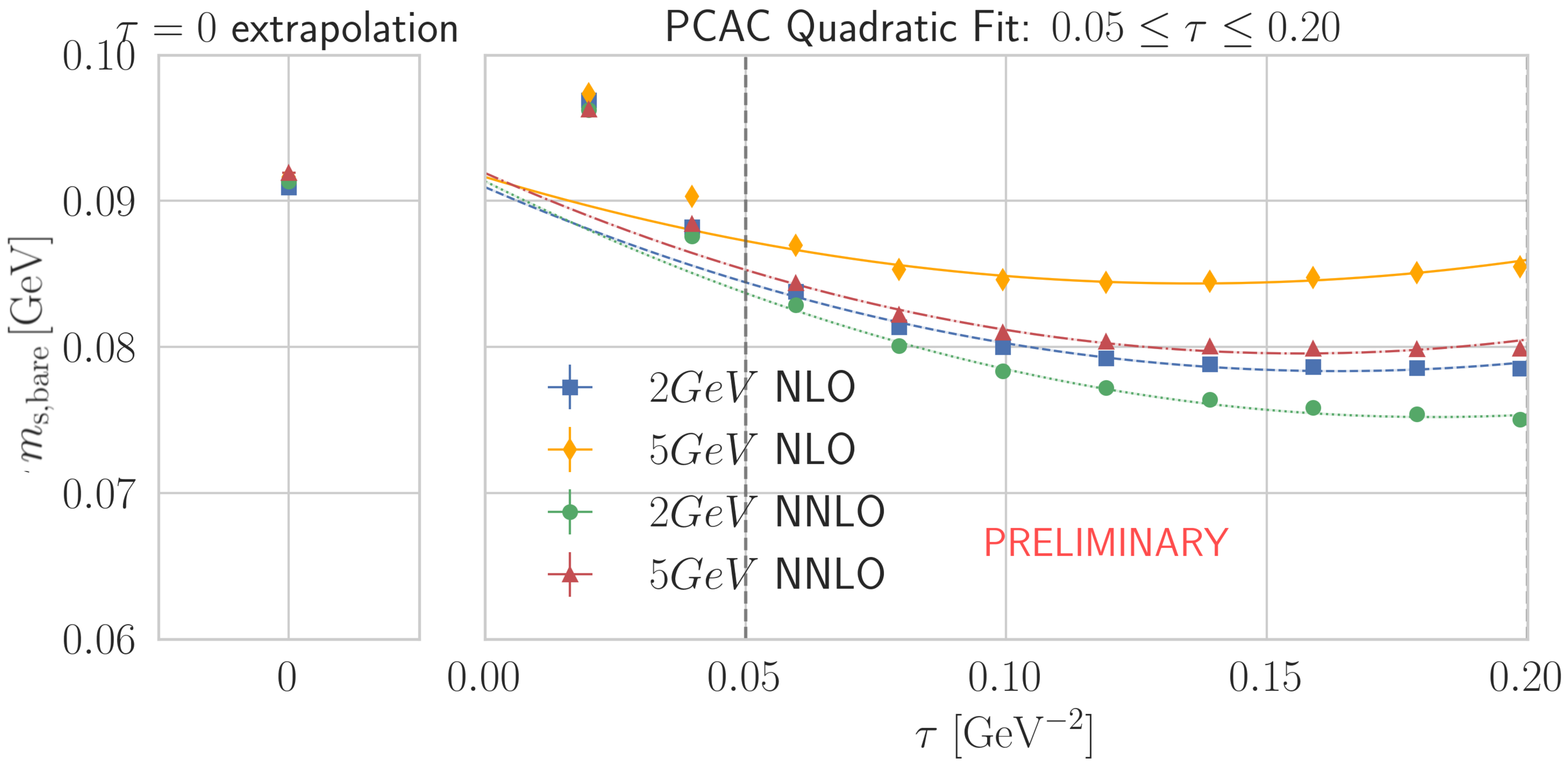
Z_χ from vector



Z_χ from axial charge

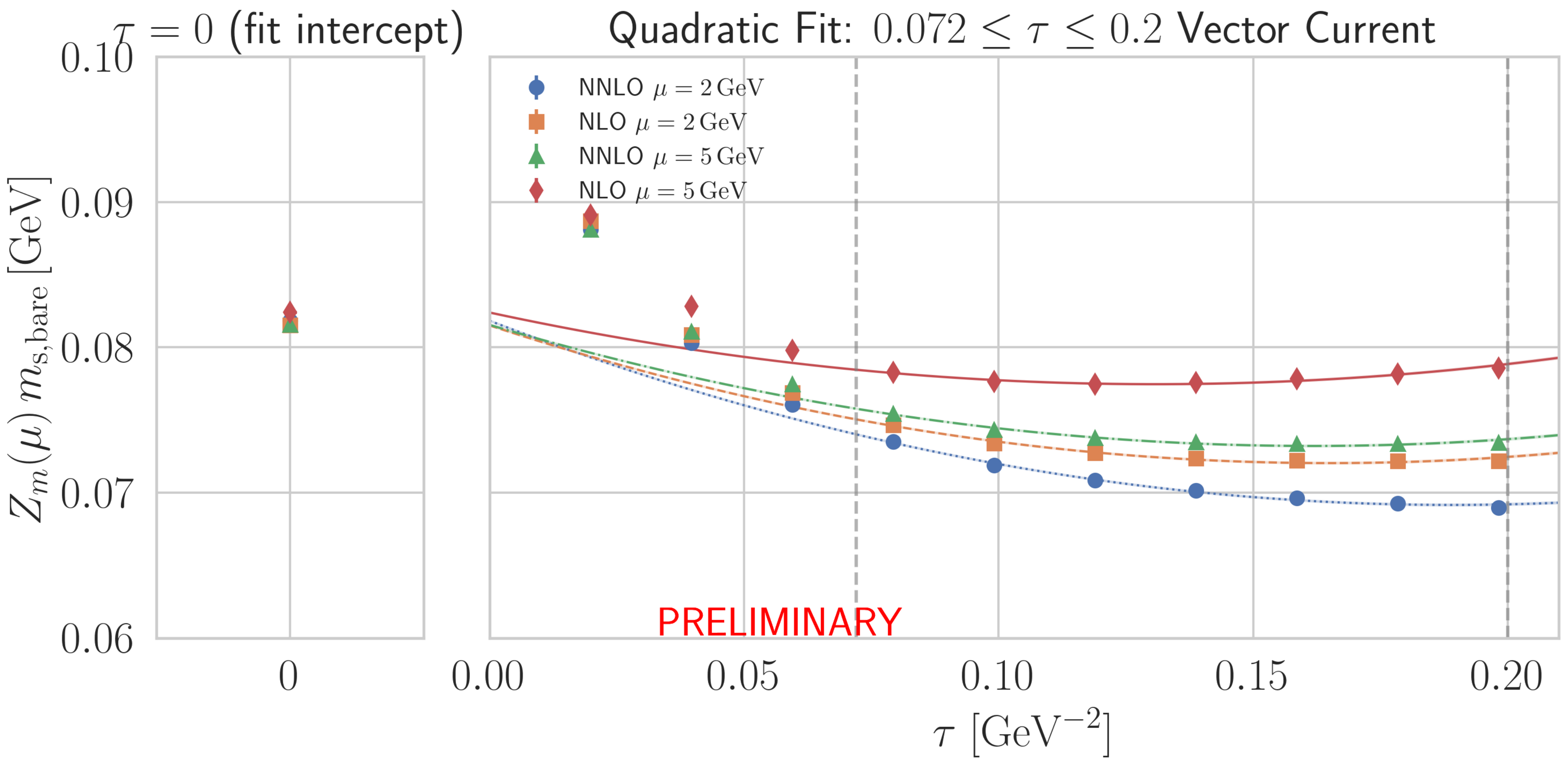


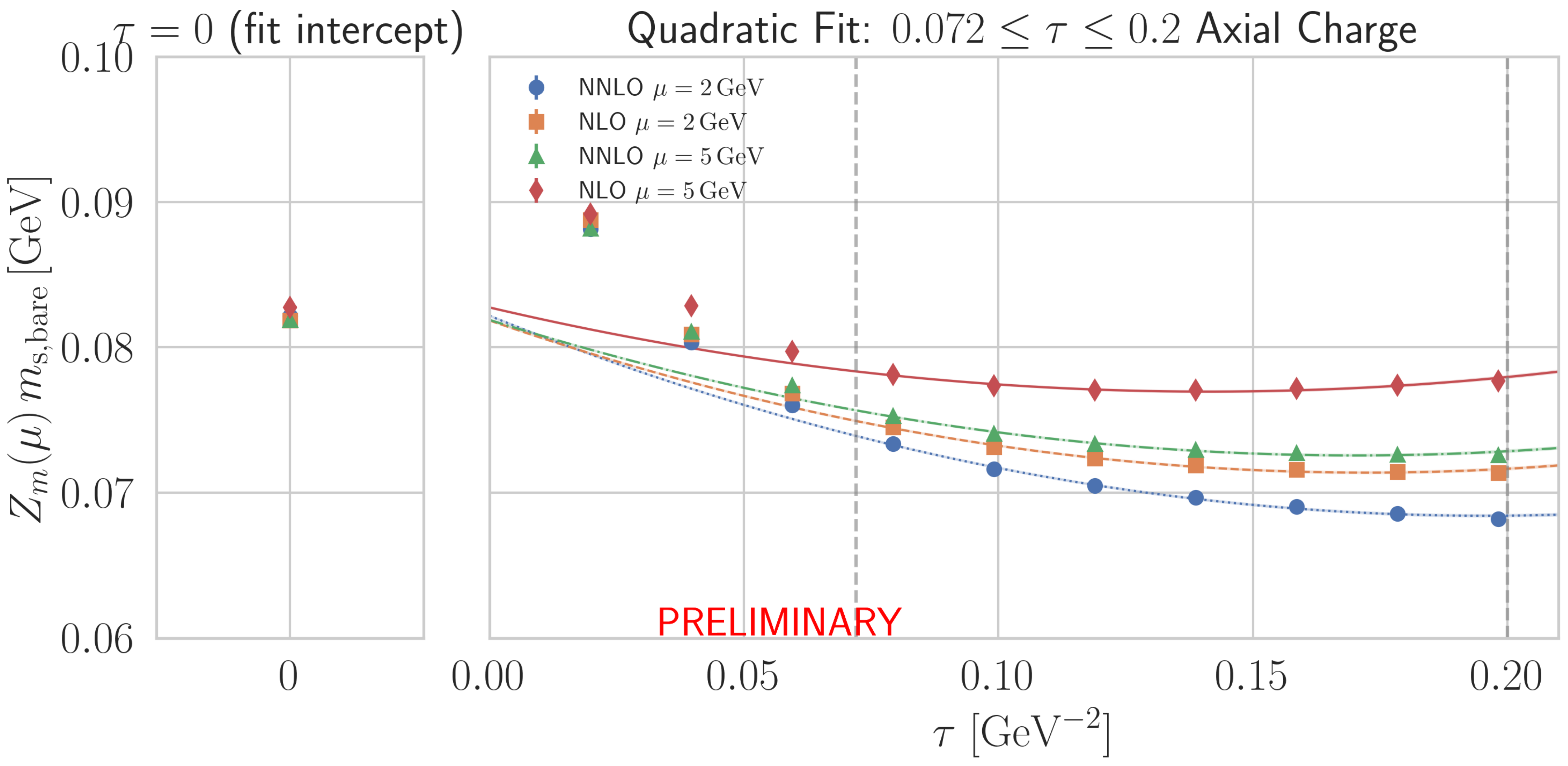
PCAC @ 0.09 fm



τ = 0 extrapolation
PCAC Quadratic Fit: 0.05 ≤ τ ≤ 0.20
m_s,bare [GeV]
0.10
0.09
0.08
0.07
0.06
0
0.00
0.05
0.10
0.15
0.20
τ [GeV^-2]
2GeV NLO
5GeV NLO
2GeV NNLO
5GeV NNLO
PRELIMINARY

Mass Renormalization @ 0.09 fm





Future Prospects

- Run on (mostly) all physical ensembles
  - Start off with low statistics on more lattice spacings
  - Understand discrepancy between $Z_m m_{\text{bare}}$ and PCAC
- Simultaneously compute anomalous dimension and β-function for improved matching
  - Work in progress - see Y. Mandlecha's talk for overview of ensembles
- Very early stages, much progress to go, many things to investigate!