Renormalized quark masses using gradient flow

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TP1 Theoretical Particle Physics

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Standard Model parameters and observables from gradient flow

Edinburgh, Scotland · May 12, 2026

color meets flavor

Collaborative Research Center TRR 257

P³H

Particle Physics Phenomenology after the Higgs Discovery

in collaboration with

Matthew Black, Robert Harlander, Anna Hasenfratz, Antonio Rago

[arXiv:2506.16327]

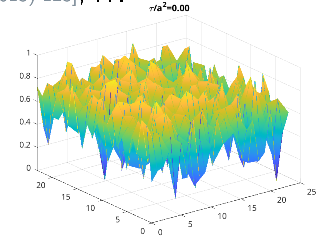
to appear in Phys. Rev. D

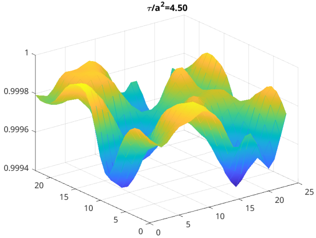
Gradient flow (GF)

- Standard tool for calculating scale setting ($\sqrt{8t_0}$), RG β-function, Λ parameter
  [Narayanan, Neuberger JHEP 03 (2006) 064] [Lüscher JHEP 08 (2010) 071][JHEP 04 (2013) 123], ...
- Introduce auxiliary dimension, flow time τ to regularize UV
  - → Well-defined smearing of gauge and fermion fields
  - → Smoothening UV fluctuations
- First order differential equation

$$\partial_\tau B_\mu(\tau,x) = \mathcal{D}_\nu(\tau) G_{\nu\mu}(\tau,x), \quad B_\mu(0,x) = A_\mu(x)$$
$$\partial_\tau \chi(\tau,x) = \mathcal{D}^2(\tau)\chi(\tau,x), \quad \chi(0,x) = q(x)$$

- Consider GF as an RG transformation
  [Carosso et al. PRL 121 (2018) 201601] [Hasenfratz et al. PoS Lattice 2021 155]
  [Harlander, Lange, Neumann JHEP 08 (2020) 109] ...
- Match to $\overline{\text{MS}}$ scheme using short flow-time expansion (SFTX)
  [Lüscher, Weisz JHEP 02 (2011) 051] [Suzuki PTEP 2013 (2013) 083B03]
  [Lüscher PoS Lattice 2013 016] [Makino, Suzuki PTEP (2014) 063B02] ...





Short flow-time expansion (SFTX)

- Re-express effective Hamiltonian in terms of ‘flowed’ operators

$$\mathcal{H}_{\rm eff} = \sum_n C_n \mathcal{O}_n = \sum_n \widetilde{C}_n(\tau)\widetilde{\mathcal{O}}_n(\tau)$$

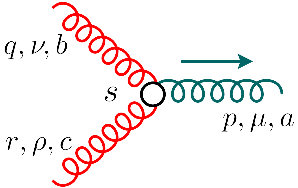
- Relate to regular operators in SFTX

ME of flowed operator (lattice)

$$\widetilde{\mathcal{O}}_n(\tau) = \sum_m \zeta_{nm}(\tau)\mathcal{O}_m + O(\tau)$$

PT calculated matching matrix

$$\sum_n \zeta^{-1}_{nm}(\mu,\tau)\langle\widetilde{\mathcal{O}}_n\rangle(\tau) = \langle\mathcal{O}_m\rangle(\mu)$$



new Feynman diagrams

- Matrix element $\langle\mathcal{O}_m\rangle(\mu)$ in the $\overline{\text{MS}}$ scheme found after taking the $\tau \to 0$
  - → Large systematic effects at very small flow times
  - → Large flow time dominated by operators $\propto O(\tau)$

⇝ Talks by Robert Harlander, Jonas Kohnen, Fabian Lange and others

Renormalized quark masses [M. Black, R. Harlander, A. Hasenfratz A. Rago, OW, arXiv:2506.16327]

- GF renormalized partially conserved axial current (PCAC) relation (Ward-Identity)

$$\left(m_{\mathrm{GF}}^{(r)}(\tau) + m_{\mathrm{GF}}^{(s)}(\tau)\right) = M_{\mathrm{PS}}^{(rs)} \bar{R}_{rs}^{\mathcal{O}}(\tau) \qquad \text{with} \quad \bar{R}_{rs}^{\mathcal{O}}(t;\tau) = \lim_{t\to\infty} -\frac{\langle A_0(t;\tau)\mathcal{O}(t=0;\tau=0)\rangle_{rs}}{\langle P(t;\tau)\mathcal{O}(t=0;\tau=0)\rangle_{rs}}$$

→ Wave-function renormalization Z_χ cancels in ratio

→ M_{PS} pseudoscalar meson with flavors r and s

→ $m_{\mathrm{GF}}^{(r)}(\tau)$ GF renormalized quark mass for flavor r

→ Smearing radius must remain small compared to Euclidean time: $\sqrt{8\tau} \ll t$

- Match to the $\overline{\mathrm{MS}}$ scheme

$$m_{\overline{\mathrm{MS}}}(\mu_{\mathrm{UV}}) = \lim_{\tau\to 0} \zeta_{AP}^{-1}(\mu_{\mathrm{UV}},\tau) m_{\mathrm{GF}}(\tau)$$

→ $\zeta_{AP}^{-1}(\mu_{UV},\tau)$ perturbatively calculated SFTX coefficient

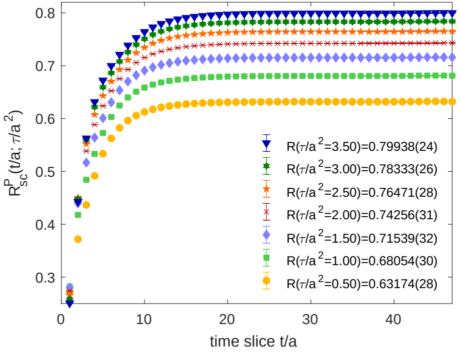
RBC-UKQCD's 2+1 flavor SDWF+Iwasaki gauge field configurations

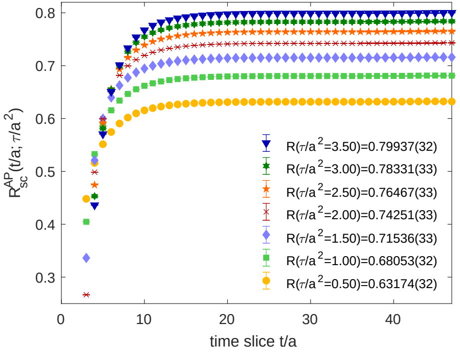
	L	a^{-1}(GeV)	am_l	am_s	M_π (MeV)	$N_{src} \times N_{conf}$	
C1	24	1.785	0.005	0.040	340	32×101	[PRD 78 (2008) 114509]
C2	24	1.785	0.010	0.040	433	32×101	[PRD 78 (2008) 114509]
M1	32	2.383	0.004	0.030	302	32×79	[PRD 83 (2011) 074508]
M2	32	2.383	0.006	0.030	362	32×89	[PRD 83 (2011) 074508]
M3	32	2.383	0.008	0.030	411	32×68	[PRD 83 (2011) 074508]
F1S	48	2.785	0.002144	0.02144	267	24×98	[JHEP 1712 (2017) 008]

- Lattice spacing determined from combined analysis [Blum et al. PRD 93 (2016) 074505]
- a: ~ 0.11 fm, ~ 0.08 fm, ~ 0.07 fm
- Measurements with strange and charm quark masses tuned to their physical value using D_s meson
- Well-studied ensemble set chosen for exploring heavy meson lifetimes calculation
  - ⇝ Talk by Antonio Rago

Choice of ratio $\bar{R}(\tau/a^2)$

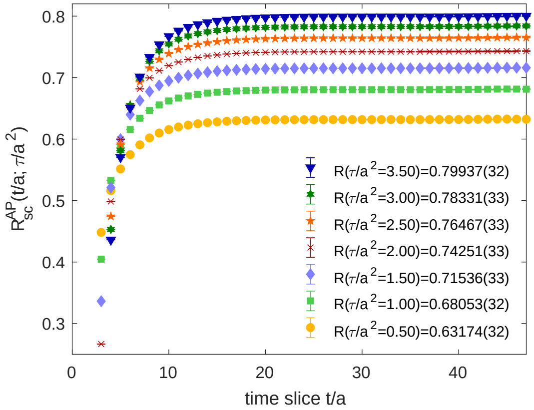
- $\bar{R}^{P}_{rs}(t;\tau) = \lim_{t\to\infty} -\frac{\langle A_0(t;\tau)P(t=0;\tau=0)\rangle_{rs}}{\langle P(t;\tau)P(t=0;\tau=0)\rangle_{rs}}$ + sinh/cosh correction
- $\bar{R}^{AP}_{rs}(t;\tau) = \lim_{t\to\infty} \sqrt{\frac{\langle A_0(t;\tau)A_0(t=0;\tau=0)\rangle_{rs}}{\langle P(t;\tau)A_0(t=0;\tau=0)\rangle_{rs}}\frac{\langle A_0(t;\tau)P(t=0;\tau=0)\rangle_{rs}}{\langle P(t;\tau)P(t=0;\tau=0)\rangle_{rs}}}$





Extract ratios $\bar{R}(\tau/a^2)$

- $\bar{R}^{AP}_{rs}(t;\tau) = \lim_{t\to\infty} \sqrt{\frac{\langle A_0(t;\tau)A_0(t=0;\tau=0)\rangle_{rs}}{\langle P(t;\tau)A_0(t=0;\tau=0)\rangle_{rs}} \frac{\langle A_0(t;\tau)P(t=0;\tau=0)\rangle_{rs}}{\langle P(t;\tau)P(t=0;\tau=0)\rangle_{rs}}}$
- Example F1S:
  $a^{-1} = 2.785$ GeV, $M_\pi = 267$ MeV
- Statistical errors smaller than symbol size
- Correlated fits for $t \in (36, 46)$
- Extract ratios for other gauge field ensembles



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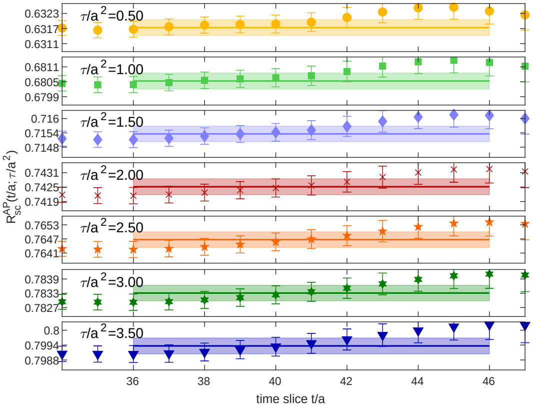
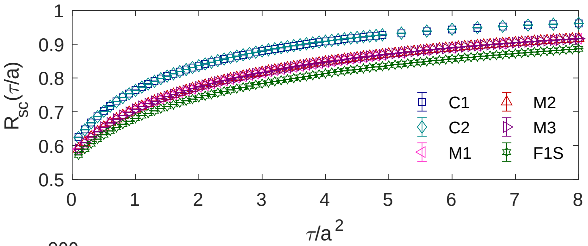


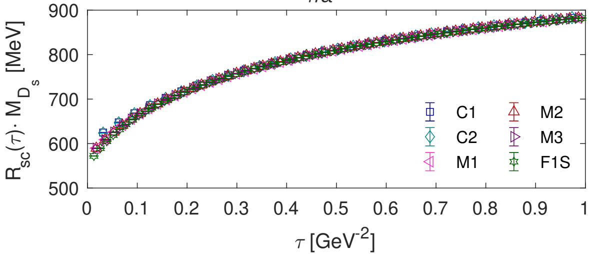
Obtain $\bar{R}$ as a function of flow time τ

- Results for flow time in lattice units
  $\rightarrow$ No sea light quark mass dependence

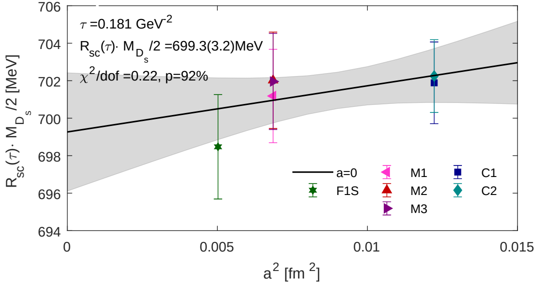
- Convert to "physical" flow time in GeV^{-2}
  $\rightarrow$ Mild continuum limit

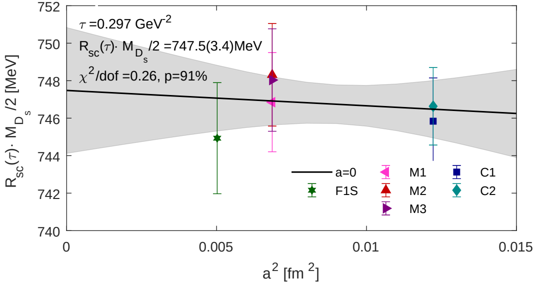
- On C1, C2, M1, M2, M3: interpolate flow times to match values on F1S



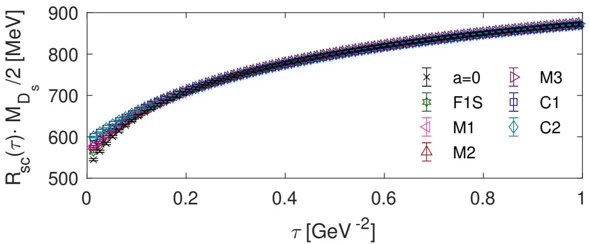


$a \to 0$ continuum limit





- GF renormalizes our ratios
  - $\to$ Take $a \to 0$ continuum limit for $\tau > 0$
- No sea light quark mass dependence
- All actions $O(a)$ improved
  - $\Rightarrow$ Linear ansatz in a^2



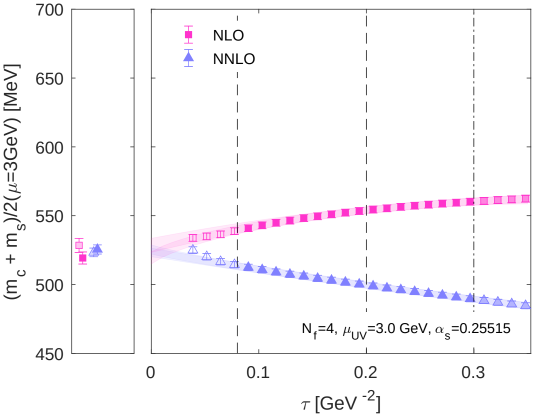
Multiply by SFTX coefficient and extrapolate $\tau \to 0$

- Multiply ζ_{AP}^{-1} to match to $\overline{\text{MS}}$ scheme and take $\tau \to 0$ limit
- ζ_{AP}^{-1} calculated at NLO and NNLO
  [Borgulat et al. JHEP 05 (2024) 179]

- Choose two fit ansätze
  $\to$ Lin-log: $f_l(\tau) = \tau(c_l \log(\tau\mu^2) + c_1) + c_0$
  $\to$ Quadratic: $f_2(\tau) = \tau^2 c_2 + \tau c_1 + c_0$

- Vary range of τ
  $\tau_{\text{min}} \in (0.08, 0.2)$ GeV^{-2} with $\tau_{\text{max}} = 0.3$ GeV^{-2}
  $\tau_{\text{max}} \in (0.25, 0.35)$ GeV^{-2} with $\tau_{\text{min}} = 0.14$ GeV^{-2}

- Uncorrelated fit to central values shifted by $\pm 1\sigma$



Improve by using RG running in GF scheme

- Resum logarithmic terms using RG equation

$$m_{\overline{\text{MS}}}(\mu_{\text{UV}}) = \lim_{\tau\to 0} (\zeta_{AP}^{\text{imp}}(\mu_{\text{UV}}, \tau_\mu))^{-1} m_{\text{GF}}(\tau)$$

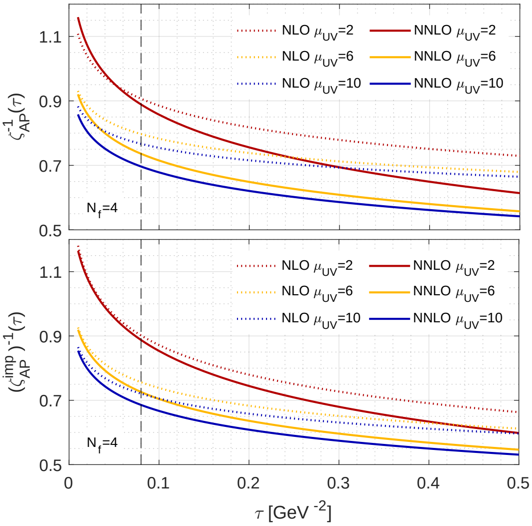
$$(\zeta_{AP}^{\text{imp}}(\mu_{\text{UV}}, \tau_\mu))^{-1} = \zeta_{AP}^{-1}(\mu_{\text{UV}}, \tau_\mu) \times \exp\left(-\int_{\tau_\mu}^{\tau} d\tau' \frac{\gamma_m^{\text{GF}}(\tau')}{\tau'}\right)$$

→ γ_m^{GF} gradient flowed anomalous dimension

→ $\tau_\mu = e^{-\gamma_E}/(2\mu_{\text{UV}}^2)$

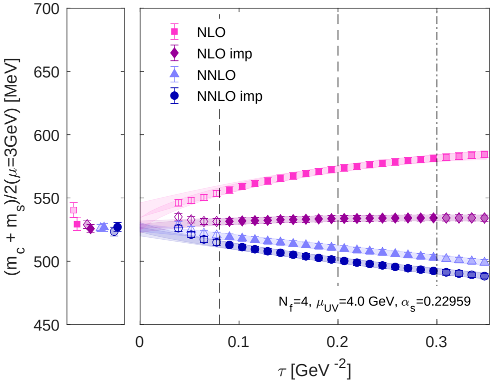
- This work: γ_m^{GF} calculated perturbatively
  [Borgulat et al. JHEP 05 (2024) 179]

- Known how to calculate γ_m^{GF} on the lattice
  [Hasenfratz et al. PoS Lattice2021 155]
  [Hasenfratz et al. PRD 107(2023)11504][PRD 108(2023) L071503]



RG improved $\tau \to 0$ extrapolation

- Multiply ζ_{AP}^{-1} to match to $\overline{\text{MS}}$ scheme and take $\tau \to 0$ limit
- ζ_{AP}^{-1} calculated at NLO and NNLO
  [Borgulat et al. JHEP 05 (2024) 179]
- Choose two fit ansätze
  → Lin-log: $f_l(\tau) = \tau(c_l \log(\tau\mu^2) + c_1) + c_0$
  → Quadratic: $f_2(\tau) = \tau^2 c_2 + \tau c_1 + c_0$
- Vary range of τ
  $\tau_{\text{min}} \in (0.08, 0.2)$ GeV^{-2} with $\tau_{\text{max}} = 0.3$ GeV^{-2}
  $\tau_{\text{max}} \in (0.25, 0.35)$ GeV^{-2} with $\tau_{\text{min}} = 0.14$ GeV^{-2}
- Uncorrelated fit to central values shifted by $\pm 1\sigma$



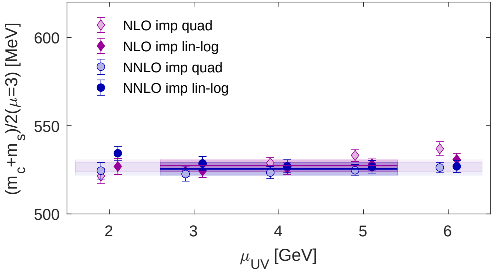
- Vary $\mu_{UV} \in \{2, 3, 4, 5, 6\}$ GeV
  → Run back to $\mu = 3$ GeV using 4-loop $\overline{\text{MS}}$
  [Chetyrkin et al. CPC 133 (2000) 43]
  [Herren, Steinhauser CPC 224 (2018) 333]

Final result using D_s meson correlators

- Correlated average using RG improved lin-log and quad results at $\mu_{UV} = \{3, 4, 5\}$ GeV

$$\left(\frac{m_c + m_s}{2}\right)^{\text{NLO}}_{[\mu=3\text{ GeV}]} = 527.4(3.3)_{\text{GF}}\ \text{MeV}$$

$$\left(\frac{m_c + m_s}{2}\right)^{\text{NNLO}}_{[\mu=3\text{ GeV}]} = 525.5(3.6)_{\text{GF}}\ \text{MeV}$$

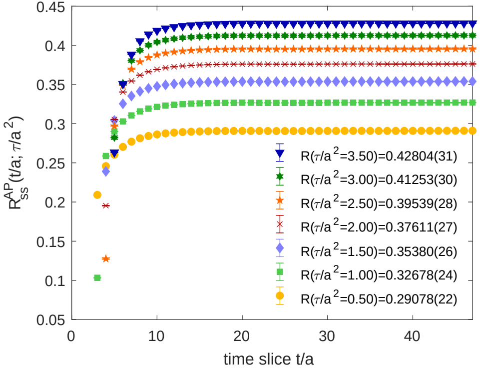


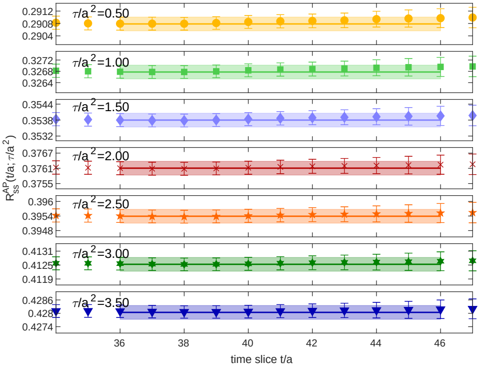
- Quote NNLO result as central value
- Continuum limit (CL) uncertainty (half the difference)
  - → Alternative analysis discarding C1 and C2 ensembles
  - → Reduce bare charm quark mass: $am_c = 0.64 \rightarrow 0.45$
  - → Reduce a from 0.11 fm to 0.08 fm
- Perturbative truncation (PT) uncertainty
  - → Half the difference between NNLO and NLO

$$\left(\frac{m_c + m_s}{2}\right)_{[\mu=3\text{ GeV}]} = 526(4)_{\text{GF}}(7)_{\text{CL}}(1)_{\text{PT}}\ \text{MeV}$$

Extract $\bar{R}(\tau/a^2)_{s\bar{s}}$

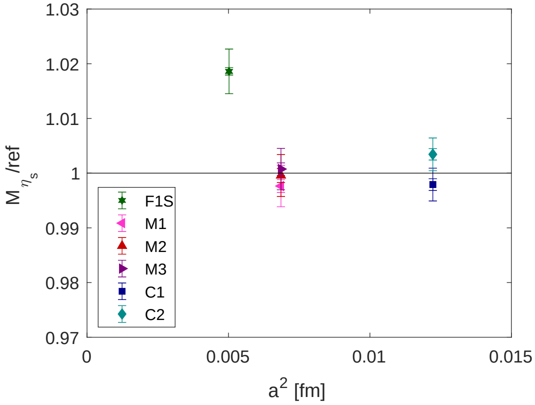
- $\bar{R}^{AP}_{s\bar{s}}(t;\tau) = \lim_{t\to\infty} \sqrt{\frac{\langle A_0(t;\tau)A_0(t=0;\tau=0)\rangle_{s\bar{s}}}{\langle P(t;\tau)A_0(t=0;\tau=0)\rangle_{s\bar{s}}} \frac{\langle A_0(t;\tau)P(t=0;\tau=0)\rangle_{s\bar{s}}}{\langle P(t;\tau)P(t=0;\tau=0)\rangle_{s\bar{s}}}}$
- Use connected, pseudoscalar ($s\bar{s}$) correlators: (unphysical) η_s meson





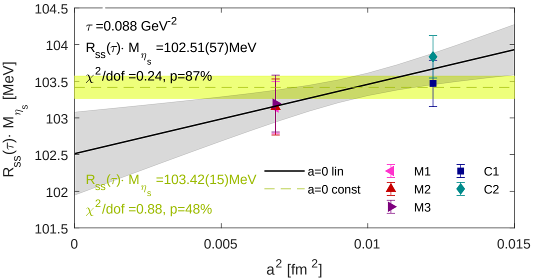
Checking η_s meson masses

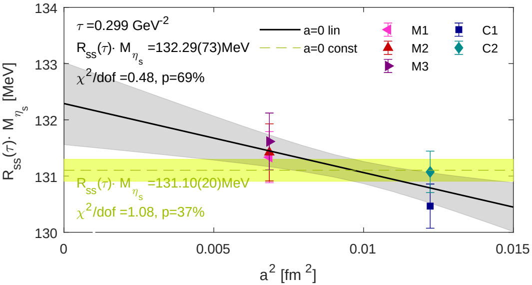
- Small mis-tuning of the strange quark mass on F1S
  - → Not significant for tuned D_s meson state or η_c correlators
  - → Discard F1S from η_s analysis



$a \to 0$ continuum limit

- Sea-quark mass effects not statistically resolved
- Only two values of the lattice spacing: consider Ansatz linear in a^2 and fit to a constant
- Repeat subsequent analysis separately for both continuum limits

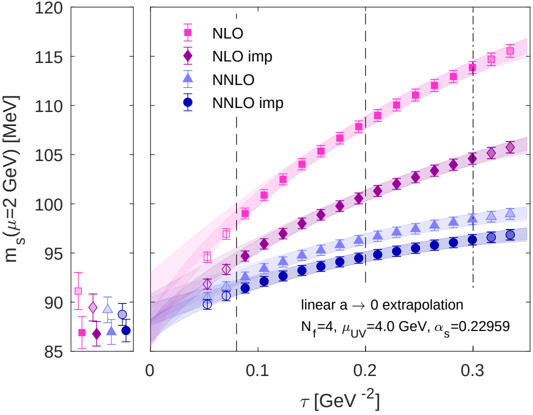




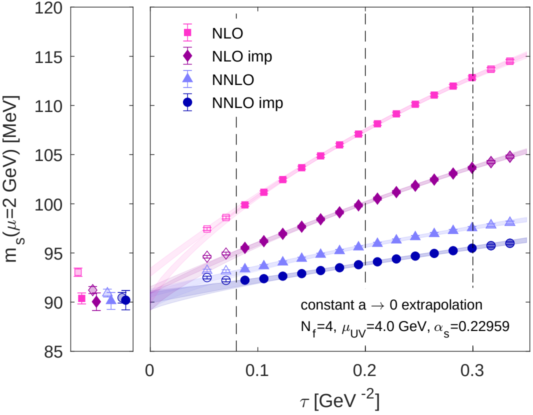
Multiply ζ and take $\tau \to 0$ limit

- Use RG improvement and perturbative running for $\mu_{UV} = \{2, 3, 4, 5, 6\}$ GeV

- Linear $a \to 0$ extrapolation



- Constant $a \to 0$ extrapolation



Final result: strange quark mass using η_s correlators

- Use RG improvement with perturbative γ_m^{GF}
- $\tau \to 0$ extrapolation using NNLO ζ coefficients

$$(m_s)^{\mathrm{NNLO,cst}}_{[\mu=2\ \mathrm{GeV}]} = 90.45(64)_{\mathrm{GF}}\ \mathrm{MeV}$$

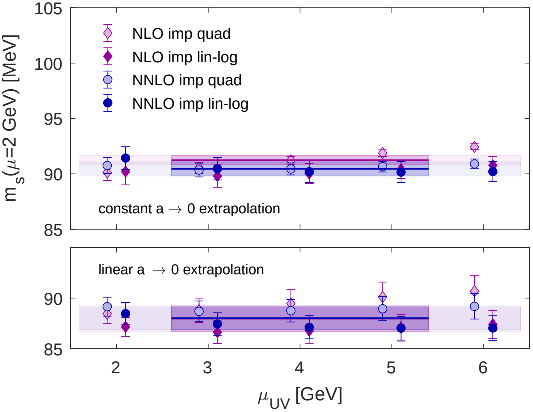
$$(m_s)^{\mathrm{NNLO,lin}}_{[\mu=2\ \mathrm{GeV}]} = 88.0(1.1)_{\mathrm{GF}}\ \mathrm{MeV}$$

- $\tau \to 0$ extrapolation using NLO ζ coefficients

$$(m_s)^{\mathrm{NLO,cst}}_{[\mu=2\ \mathrm{GeV}]} = 91.24(42)_{\mathrm{GF}}\ \mathrm{MeV}$$

$$(m_s)^{\mathrm{NLO,lin}}_{[\mu=2\ \mathrm{GeV}]} = 88.0(1.3)_{\mathrm{GF}}\ \mathrm{MeV}$$

$$(m_s)_{[\mu=2\ \mathrm{GeV}]} = 89(3)_{\mathrm{GF+CL}}(0)_{\mathrm{PT}}$$



Combine results to obtain charm quark mass

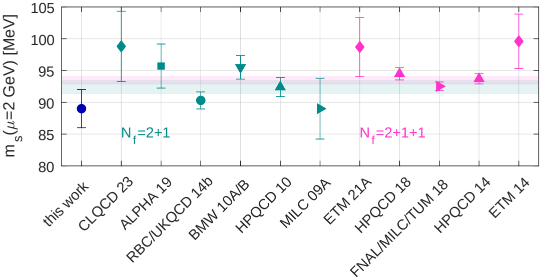
$$\left(\frac{m_c + m_s}{2}\right)_{[\mu=3\text{ GeV}]} = 526(4)_{\text{GF}}(7)_{\text{CL}}(1)_{\text{PT}}\text{ MeV} = 526(8)\text{ MeV}$$

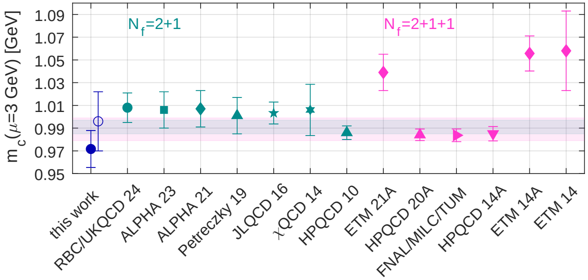
$$(m_s)_{[\mu=2\text{ GeV}]} = 89(3)_{\text{GF+CL}}(0)_{\text{PT}}\text{ MeV} = 89(3)\text{ MeV}$$

$$\Downarrow$$

- $(m_c)_{[\mu=3\text{ GeV}]} = 972(16)$ MeV
- $\frac{m_c}{m_s} = 12.1(4)$

Strange and charm quark masses [M. Black, R. Harlander, A. Hasenfratz A. Rago, OW, arXiv:2506.16327]





[FLAG 2024 arXiv:2411.04268] [χQCD PRD92(2015)034517] [ETM PRD104(2021)074515] [FNAL/MILC/TUM PRD98(2018)054517] [HPQCD PRD91(2015)054508] [ETM NPB887(2014)19] [CQCD PRD109(2024)054507] [Alpha EPJC80(2020)169] [RBC-UKQCD PRD93(2016)074505] [HPQCD PRD82(2010)034512] [BMW JHEP08(2011)146] [FNAL/MILC PoS CD09(2009)007] [HPQCD PRD98 (2018) 014513] [Alpha EPJC84(2024)506] [Alpha JHEP05(2021)288] [Petreczky,Weber PRD100(2019)034519] [JLQCD PRD94(2016)054507] [HPQCD PRD102(2020)054511] [HPQCD PRD102(2020)054511] [RBC-UKQCD PRD110(2024)054512]

- $(m_s)^{\eta_s}_{[\mu=2\text{ GeV}]} = 89(3)$ MeV

- $(m_c)^{\eta_c}_{[\mu=3\text{ GeV}]} = 996(26)$ MeV
- $(m_c)_{[\mu=3\text{ GeV}]} = 972(16)$ MeV (D_s and η_s)
- $\frac{m_c}{m_s} = 12.1(4)$

Summary and Outlook

- GF+SFTX is a powerful concept and we are just starting to leverage its potential
- Simple prescription to obtain renormalized quark masses
  - → Most likely competitive uncertainties compared to other method e.g. RI/(S)MOM
  - → No need for gauge-fixing, easily attainable "window" condition
- First lattice calculation of heavy meson lifetimes ⇝ Talk by Antonio Rago

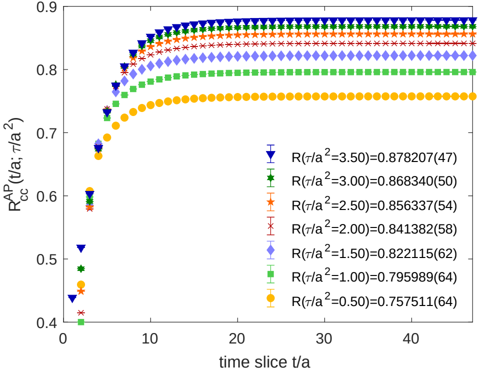
Acknowledgment

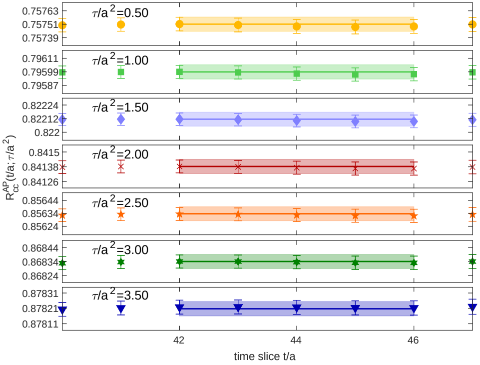
- Grid [Peter Boyle et al.]
- Hadrons [Antonin Portelli et al.]
- Feyngame [Harlander et al.]
- OMNI, Universität Siegen
- HAWK, HLR Stuttgart
- LumiG, DEIC

Extra

Extract $\bar{R}(\tau/a^2)_{c\bar{c}}$

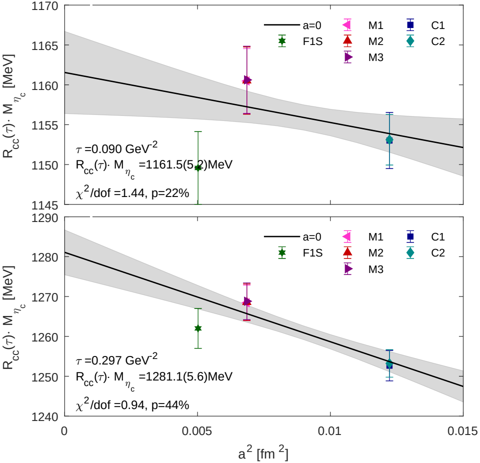
- $\bar{R}^{AP}_{c\bar{c}}(t;\tau) = \lim_{t\to\infty} \sqrt{\frac{\langle A_0(t;\tau)A_0(t=0;\tau=0)\rangle_{c\bar{c}}}{\langle P(t;\tau)A_0(t=0;\tau=0)\rangle_{c\bar{c}}} \frac{\langle A_0(t;\tau)P(t=0;\tau=0)\rangle_{c\bar{c}}}{\langle P(t;\tau)P(t=0;\tau=0)\rangle_{c\bar{c}}}}$
- Use connected, pseudoscalar $(c\bar{c})$ correlators: η_c meson





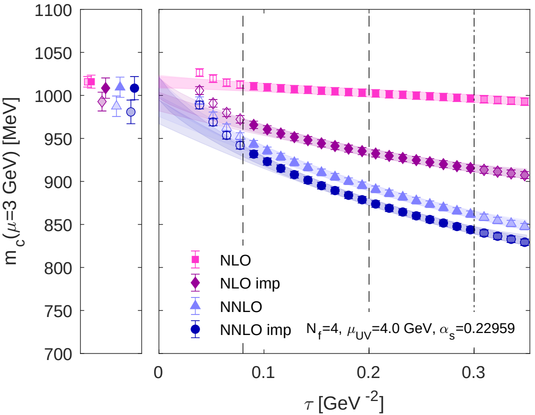
$a \to 0$ continuum

- Sea-quark mass effects not statistically resolved
- Consider Ansatz linear in a^2



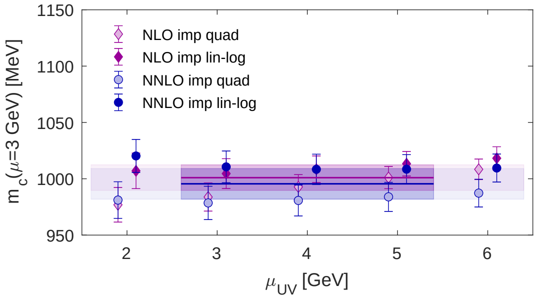
$\tau \to 0$ limit

- Use RG improvement with perturbative γ_m^{GF}



Final result: charm quark mass using η_c correlator

- Average $\mu_{UV} = \{2, 3, 4, 5, 6\}$ GeV



- $(m_c)^{\eta_c,\text{NLO}}_{[\mu=3\text{ GeV}]} = 1001(11)_{\text{GF}}$ MeV
- $(m_c)^{\eta_c,\text{NNLO}}_{[\mu=3\text{ GeV}]} = 996(14)_{\text{GF}}$ MeV

- Systematic effects
  - → Central value $m_c^{\eta_c,\text{NNLO}}$
  - → CL: half the difference to alternative analysis discarding C1, C2/largest bare charm mass
  - → PT: Half the difference between NNLO and NLO

$$(m_c)^{\eta_c}_{[\mu=3\text{ GeV}]} = 996(14)_{\text{GF}}(22)_{\text{CL}}(3)_{\text{PT}}\text{ MeV}$$

Residual mass