PDF Moments from Flowed Local Operators

A Lattice QCD Approach to Hadron Structure

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Standard Model parameters and observables from gradient flow

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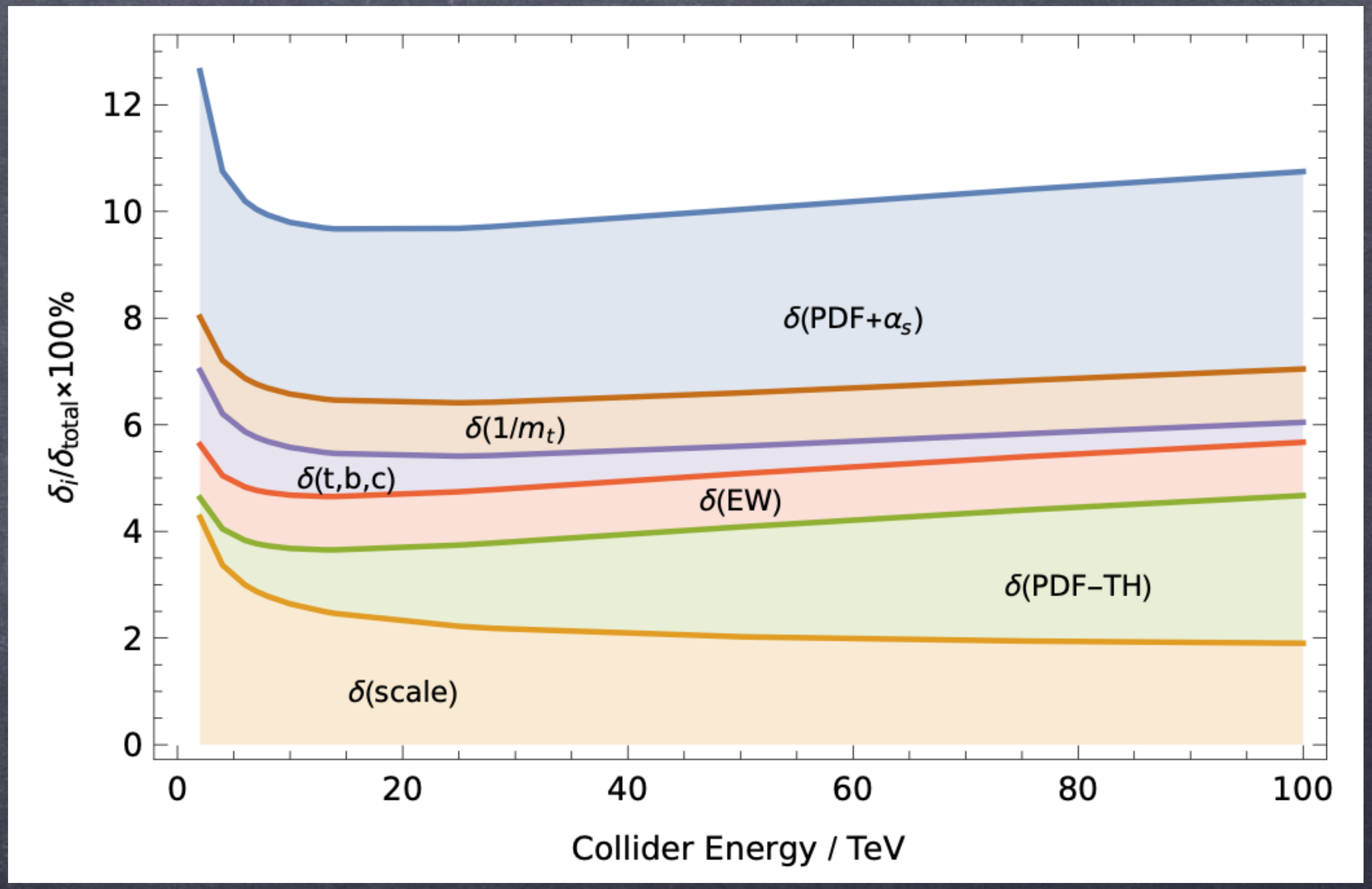
Parton Distribution Functions

PDFs are central inputs to many of the most important precision predictions at current and future colliders

Higgs boson production

- **PDF uncertainty** remains one of the largest theory uncertainties in Higgs predictions



LHC Higgs Cross section Working Group (2016)
Snowmass 2021 White paper

SM parameters

PDFs enter **precision extractions** of ->

- strong coupling constant
- W boson mass
- ...

New physics searches

- At large x **PDF uncertainties grow** due to limited constraints
- This affects predictions for the high-energy tails —> indirect BSM searches

Ball, Candido, Forte, Hekhorn, Nocera, Rojo, Schwan (2022)

PDF and Lattice QCD

Direct lattice calculations of higher PDF moments are notoriously difficult

Curci, Furmanski, Petronzio (1980)
Collins, Soper (1982)

$$\langle x^{n-1}\rangle_q^h(\mu) = \int_0^1 dx\, x^{n-1}\left[q^h(x,\mu) + (-1)^n\,\overline{q}^h(x,\mu)\right]$$

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$$\langle h(p)\,|\,\widehat{O}^{qq}_{\{\mu_1\cdots\mu_n\}}\,|\,h(p)\rangle = 2\,\langle x^{\,n-1}\rangle^h_q(\mu)\;(p_{\mu_1}\cdots p_{\mu_n} - \text{traces})$$

$$\widehat{O}^{qq}_{\{\mu_1\cdots\mu_n\}}(x) = \overline{\psi}^q(x)\,\gamma_{\{\mu_1}\overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n\}}\,\psi^q(x) - \text{traces}$$

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- Continuum limit fails for $\langle x^n \rangle$ for n>3
- For n=2,3: need of non-zero spatial momenta → destroys signal-to-noise
- Power divergences prevent renormalization of higher moments
- As a result, only **$\langle x \rangle$** and at best (very poorly) **$\langle x^2 \rangle$** and **$\langle x^3 \rangle$** were accessible

Kronfeld, Photiadis (1985)
Martinelli, Sachrajda (1987 - 1988)

Moment	Status (pre-flow)
$\langle x \rangle$	✓ reliable
$\langle x^2 \rangle$	✓ barely usable
$\langle x^3 \rangle$	× extremely noisy
$\langle x^4 \rangle, \langle x^5 \rangle, \ldots$	× impossible

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reviews of Refs. [37, 38]). Direct calculations of distribution functions on a Euclidean lattice have not been feasible due to the time dependence of these quantities. A way around this limitation is the calculation on the lattice of moments of distribution functions (historically for PDFs and GPDs) and the physical PDFs can, in principle, be obtained from operator product expansion (OPE). Realistically, only the lowest moments of PDFs and GPDs can be computed (see e.g. [39–44]) due to large gauge noise in high moments, and also unavoidable power-divergent mixing with lower-dimensional operators. Combination of the two prevents a reliable and accurate calculation of moments beyond the second or third, and the reconstruction of the PDFs becomes unrealistic.

Cichy, Constantinou: 2019

PDF and Lattice QCD

Approaches have been developed to determine the x-dependence of the PDFs

- Hadronic tensor — Liu, Dong (1994)
- Auxiliary scalar field — Aglietti et al. (1998)
- quasi-PDF (LaMET) — Ji (2013)
- pseudo-PDF — Radyushkin (2017)
- Fictitious heavy quark — Detmold, Lin (2005)
- Auxiliary scalar quark — Braun, Müller (2008)
- Smeared OPE — Monahan, Orginos (2015)
- Compton amplitude + OPE — Chambers et al. (2017)
- Good Lattice Cross Sections — Ma, Qiu (2018)
- PDF without Wilson line — Zhao (2024)

short distance **($z \to 0$)** $\longleftrightarrow$ large Ioffe time **($\nu = P_z z$)**

- **small z → control UV but need large P_z**
  Current lattices reach only $P_z \lesssim 3 \text{ GeV}$ $\Rightarrow$ limited ν window
- **large P_z → noise/excited states → precision saturates**
  Lattices must get much finer or new strategies developed

These approaches allow in principle an indirect determination of the moments of PDF of nucleons and pions

Gao et al.: (2020-2023)
Egerer et al. (2022)

Moments of the PDF: standard method

$$O_n^{rs}(x) = O_{\mu_1\cdots\mu_n}^{rs}(x) = \overline{\psi}^r(x)\gamma_{\{\mu_1}\overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n\}}\psi^s(x)$$

- Calculate matrix elements using lattice QCD
  - Rotational group symmetry is broken into the hypercubic group H(4)
- Irreducible representations of O(4) generally become reducible representations of H(4) inducing unwanted mixings under renormalization
  - Irreps of H(4) allow mixing with lower dimensional operators and complicate mixings with operators of the same dimension

Beccarini et al.: 1995
Gockeler et al.: 1996

- Operators with different index combinations belong to different irreps of H(4)

O_3 $\quad \mu_1 = \mu_2 = \mu_3 \quad \rightarrow \quad 1/a^2\delta_{\mu_i\mu_j}\cos(ap_{\mu_j})$ Kronfeld, Photiadis: 1985

$\mu_1 \neq \mu_2 = \mu_3 \quad \rightarrow \quad O_{411} - O_{433}$ Martinelli, Sachrajda: 1987

$\mu_1 \neq \mu_2 \neq \mu_3 \quad \rightarrow \quad \langle h(p)|O_n|h(p)\rangle = 2\ (p_{\mu_1}\cdots p_{\mu_n} - \text{ trace terms})\,\langle x^{n-1}\rangle_h$

Method that addresses both the theoretical and numerical challenges faced in the past, which hindered the direct calculation of moments of any order from lattice QCD

Short-flow-time expansion

Lüscher, Weisz (2011)
Lüscher (2013)

Flowed operators admit an expansion for small flow-time t

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$$\zeta_{ij}(t,\mu) = \delta_{ij} + \frac{\alpha_s(\mu)}{4\pi}\zeta_{ij}^{(1)}(t,\mu) + O(\alpha_s^2)$$

LQCD

PT - LQCD

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LQCD PT - LQCD

A.S., Luu, de Vries (2014-2015)
Dragos, Luu, A.S. de Vries (2018-2019)
Rizik, Monahan, A.Sh. (2018-2020)
A.Sh.: (2020)
Kim, Luu, Rizik, A.Sh. (2020)
Mereghetti, Monahan, Rizik, A.Sh., Stoffer (2021)
Monahan, Rizik, A.Sh., Stoffer (2023)
A.Sh. (2023)
Harlander, Kohnen, A.Sh. (2025)

- **New UV regulator** that preserves continuum symmetries
- **Simplifies continuum limit** and improves removal of the hard lattice cutoff
- **Enables new strategies** for hadronic matrix elements

Strategy

A.Sh.: 2311.18704

- Consider flowed twist-2 operators $$O_n^{rs}(x,t) = \overline{\chi}^r(x,t)\gamma_{\{\mu_1}\overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n\}}\chi^s(x,t)$$

- Renormalize flowed twist-2 operators —> renormalization is ALWAYS multiplicative

$$O_n^{rs}(t) = Z_n O_{n,B}^{rs}(t)\,, \quad Z_n = Z_\chi \qquad \left\langle \mathring{\overline{\chi}}(x,t)\overleftrightarrow{\not{D}}\mathring{\chi}(x,t)\right\rangle = -\frac{N_c}{(4\pi)^2 t^2}$$

Makino, Suzuki: 2014

- Perform a short flow time expansion —> consider O(4) irreps —> traceless operators

$$\widehat{\mathring{O}}{}_n^{rs}(x,t) = \mathring{\overline{\chi}}{}^r(x,t)\gamma_{\{\mu_1}\overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n\}}\mathring{\chi}^s(x,t) - \text{terms with } \delta_{\mu_i\mu_j}$$

Continuum limit is finite for any n

$$\left\langle h(p)|\widehat{\mathring{O}}_n(t)|h(p)\right\rangle = 2\left(p_{\mu_1}\cdots p_{\mu_n} - \text{trace terms}\right)\left\langle x^{n-1}\right\rangle_h(t)$$

- Matching is multiplicative for traceless operators $$\widehat{\mathring{O}}{}_n^{rs}(t) = \zeta_n(t,\mu)\widehat{O}_n^{rs,\overline{\text{MS}}}(\mu) + O(t)$$

- Calculate matching coefficients in PT

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Harlander, Kluth, Lange (2018)

Artz et al. (2019)

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Matching coefficients

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Matching equations $\left\langle \psi^r \mathring{\widehat{O}}_n^{rs}(t)\overline{\psi}^s \right\rangle = \zeta_n(t,\mu) \left\langle \psi^r \widehat{O}_n^{rs,\overline{\mathrm{MS}}}(t=0,\mu)\overline{\psi}^s \right\rangle$

- Expand integrands of loop integrals in all scales excluding t
  - Analytic structure altered —> distortion of IR structure
  - in matching equation the IR modification drops out in the difference
  - Expanding loop integrals in the RHS vanish in DR —> UV and IR are identical
  - The LHS is UV-finite, beside the renormalization of the bare parameters and flowed fermion fields
  - The IR singularities on the LHS exactly match the UV MS counterterms

$$\zeta_n(t,\mu) = 1 + \frac{\overline{g}^2(\mu)}{(4\pi)^2}\zeta_n^{(1)}(t,\mu) + O(\overline{g}^4) \qquad \zeta_n^{(1)}(t,\mu) = C_F\left[\gamma_n \log\left(8\pi\mu^2 t\right) + B_n\right] \qquad \gamma_n = 1 + 4\sum_{j=2}^{n}\frac{1}{j} - \frac{2}{n(n+1)}$$

Gross, Wilczek: 1974

$$B_n = \frac{4}{n(n+1)} + 4\frac{n-1}{n}\log 2 + \frac{2-4n^2}{n(n+1)}\gamma_E - \frac{2}{n(n+1)}\psi(n+2) + \\ + \frac{4}{n}\psi(n+1) - 4\psi(2) - 4\sum_{j=2}^{n}\frac{1}{j(j-1)}\frac{1}{2^j}\phi(1/2,1,j) - \log(432)$$

$n = 2$ Makino, Suzuki: 2014

$$\phi(z,s,a) = \sum_{k=0}^{\infty}\frac{z^k}{(k+a)^s}$$

O(a) improvement

A.Sh.: 2311.18704

$$O^{rs}_n(x) = O^{rs}_{\mu_1\cdots\mu_n}(x) = \overline{\psi}^r(x)\gamma_{\{\mu_1}\overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n\}}\psi^s(x) \qquad \left\langle h(p)|\widehat{O}_n|h(p)\right\rangle = 2\,p_{\mu_1}\cdots p_{\mu_n}\langle x^{n-1}\rangle_h$$

- Beside the O(a) from the lattice theory twist-2 fields are affected by specific O(a) that depend on n
- Improvement coefficients are known only for n=2 and only in PT

$$\widehat{\mathring{O}}^{rs}_n(x,t) = \overline{\mathring{\chi}}^r(x,t)\gamma_{\{\mu_1}\overleftrightarrow{D}_{\mu_2}\cdots\overleftrightarrow{D}_{\mu_n\}}\mathring{\chi}^s(x,t) - \text{terms with } \delta_{\mu_i\mu_j}$$

$$\left\langle h(p)|\widehat{\mathring{O}}_n(t)|h(p)\right\rangle = 2\,(p_{\mu_1}\cdots p_{\mu_n} - \text{trace terms})\,\langle x^{n-1}\rangle_h\,(t)$$

- Hadronic matrix elements of flowed operators beside the O(a) from the lattice theory are only affected by O(am)
- Short distance O(a) negligible at large physical distances
- The O(am) are independent on n (depend only on the fermion content)
- With ratios discretization effects are $O(a^2)$ —> clover fermions are back in the game

$$\frac{\langle x^{n-1}\rangle_h\,(t)}{\langle x^{m-1}\rangle_h\,(t)} \qquad n \neq m$$

→ Finite continuum limit and O(a) improved

Flowed moments

A.Sh.: 2311.18704

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Flowed moments

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- UV finite at t>0
- multiplicative renormalization
- no power divergences

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- flowed fermion operators renormalize universally
- ratios are finite for all n
- zero external momentum

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$$\left\langle x^{n-1}\right\rangle_h^{\overline{\rm MS}}(\mu) = \langle x\rangle_h^{\overline{\rm MS}}(\mu)\,\frac{\zeta_2(t,\mu)}{\zeta_n(t,\mu)}\,\frac{\left\langle x^{n-1}\right\rangle_h(t)}{\langle x\rangle_h(t)}$$

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- OPE-like matching
- multiplicative
- known to NNLO

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Result: All Mellin moments computable with finite continuum limit and no power divergences

What is OpenLat

https://openlat1.gitlab.io

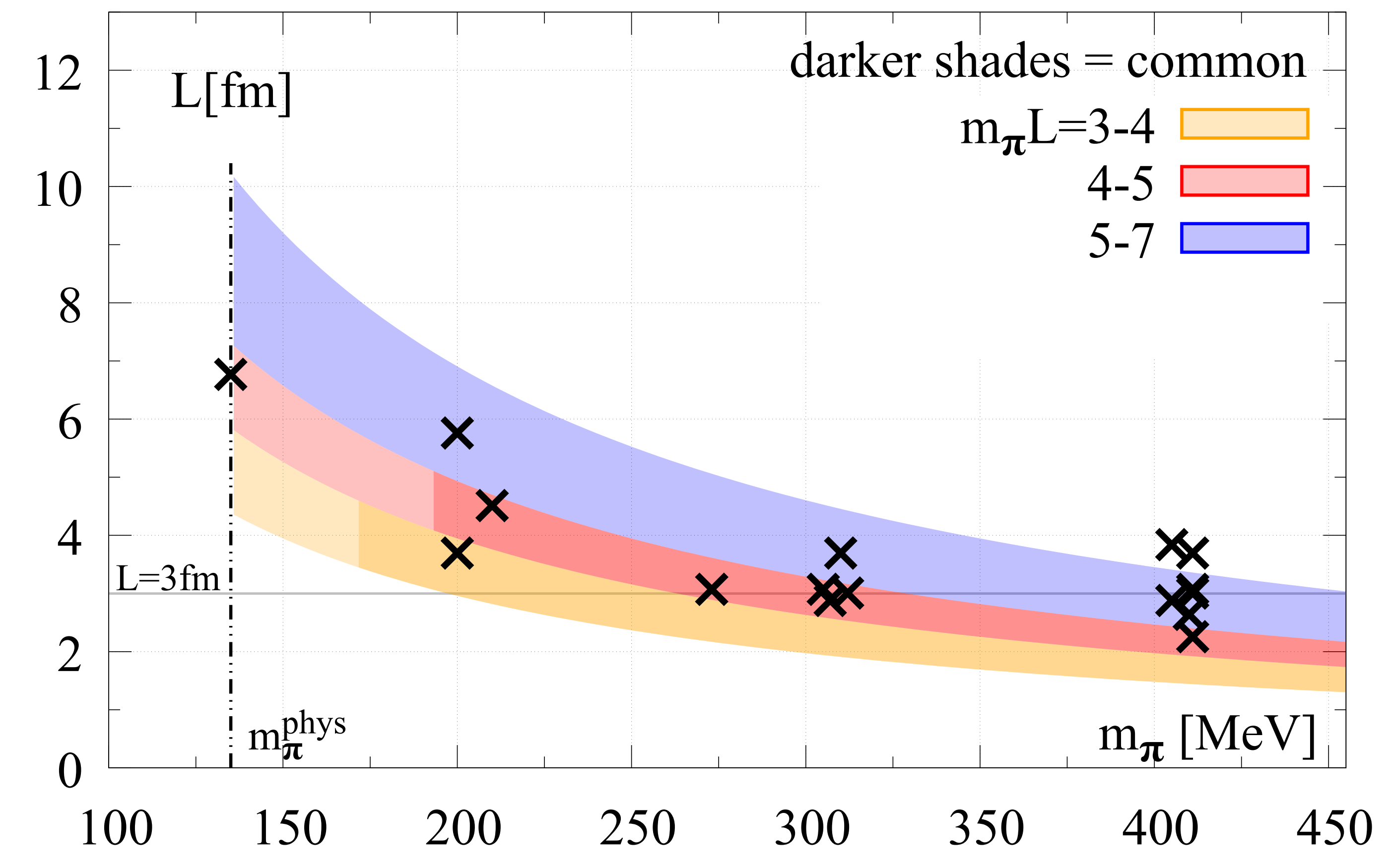
- OpenLat provides open access to cutting-edge (2+1)-flavor QCD gauge field ensembles, enabling the study of strong interaction phenomena with state-of-the-art simulations.

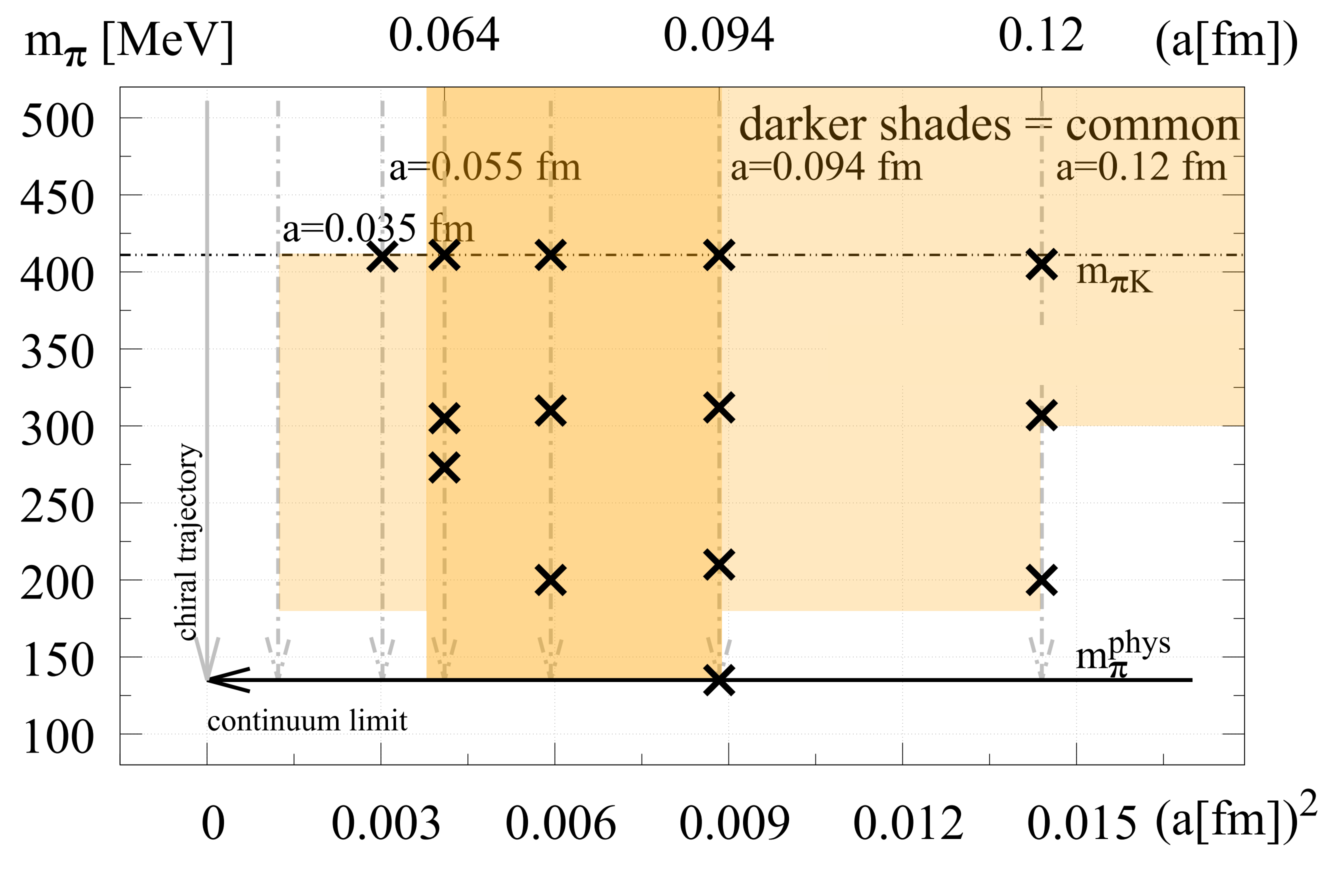
- The initiative bridges the gap for researchers without access to large-scale collaborations, fostering innovation and inclusivity in lattice QCD.

- Strategic choice of stabilized Wilson fermion framework for precision and stability in QCD simulations.

OpenLat: Current Status & Key Research Projects

OpenLat has made significant progress in producing and sharing high-quality lattice QCD ensembles.
Our two main research projects focus on precision studies of the thetaEDM parameter and advancements in lattice PDF calculations.





Project	Main Goal	Progress
EDM	Improve constraints on CP-violating effects in strong interactions (SM & BSM)	New lattice ensembles generated; analysis ongoing.
PDF	Develop new lattice techniques for parton distribution function (PDF) determination	Implementation of flowmom method; preliminary results obtained
Multi-hadron interactions		
Flavor physics		

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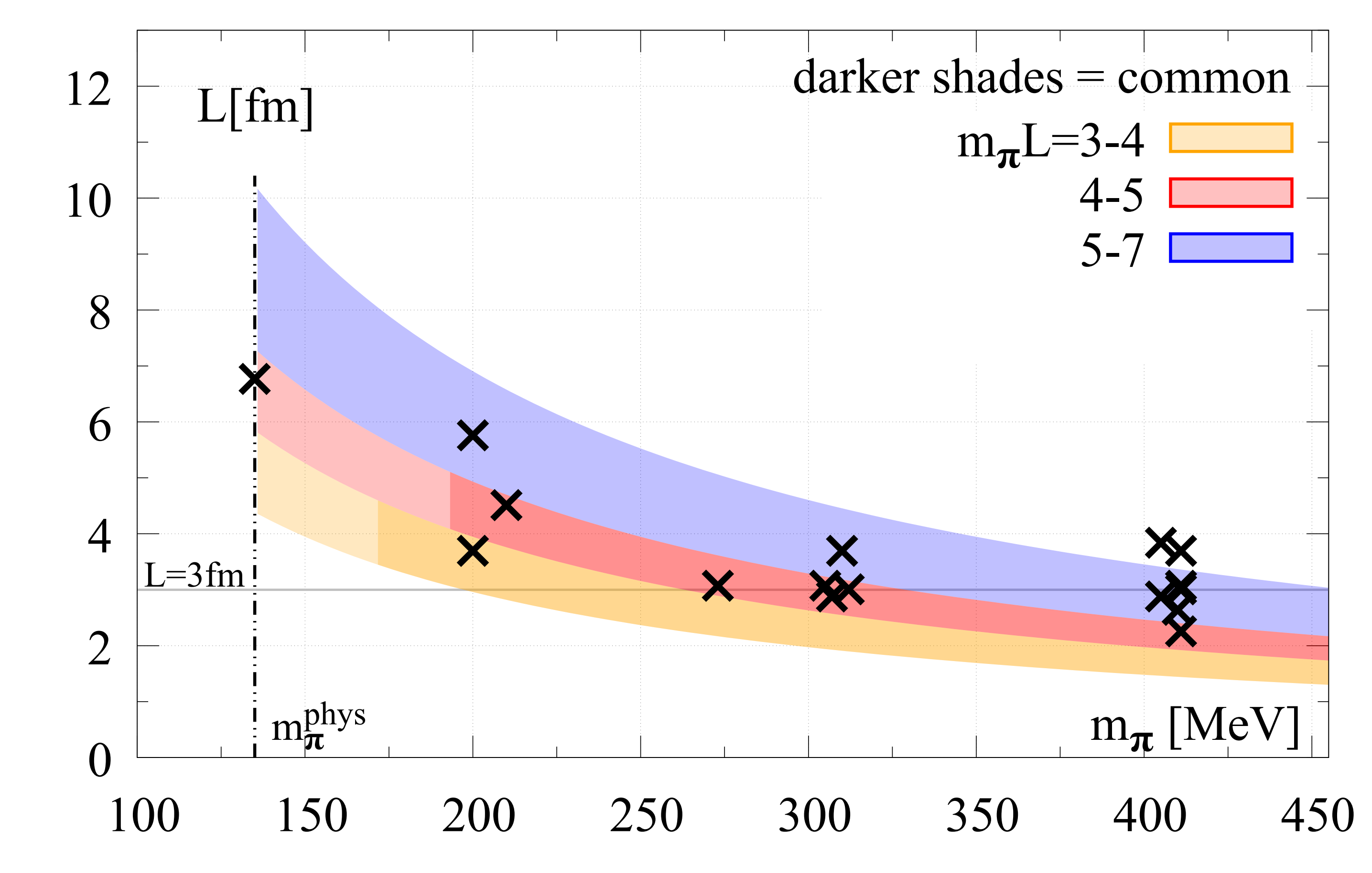

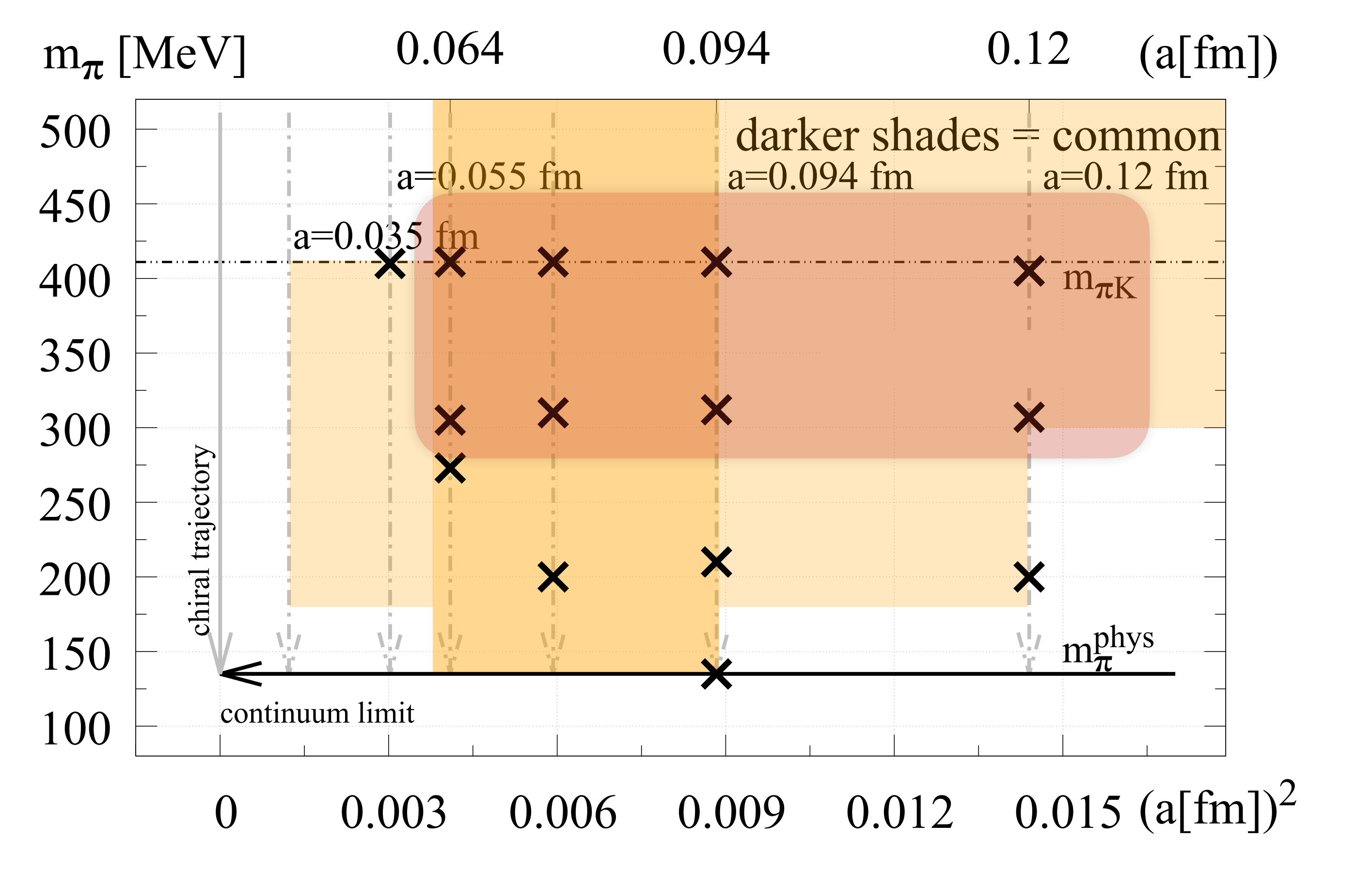

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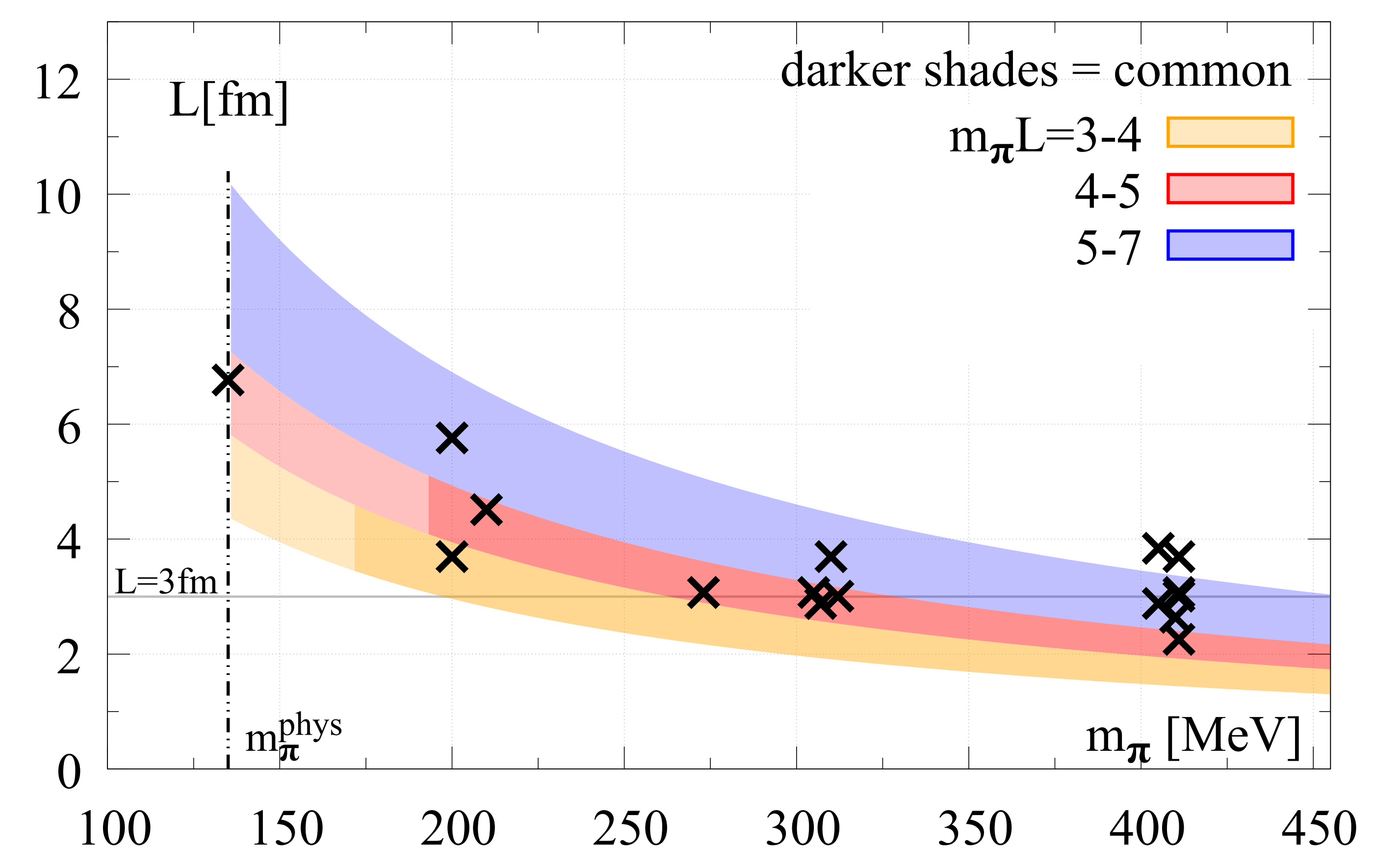

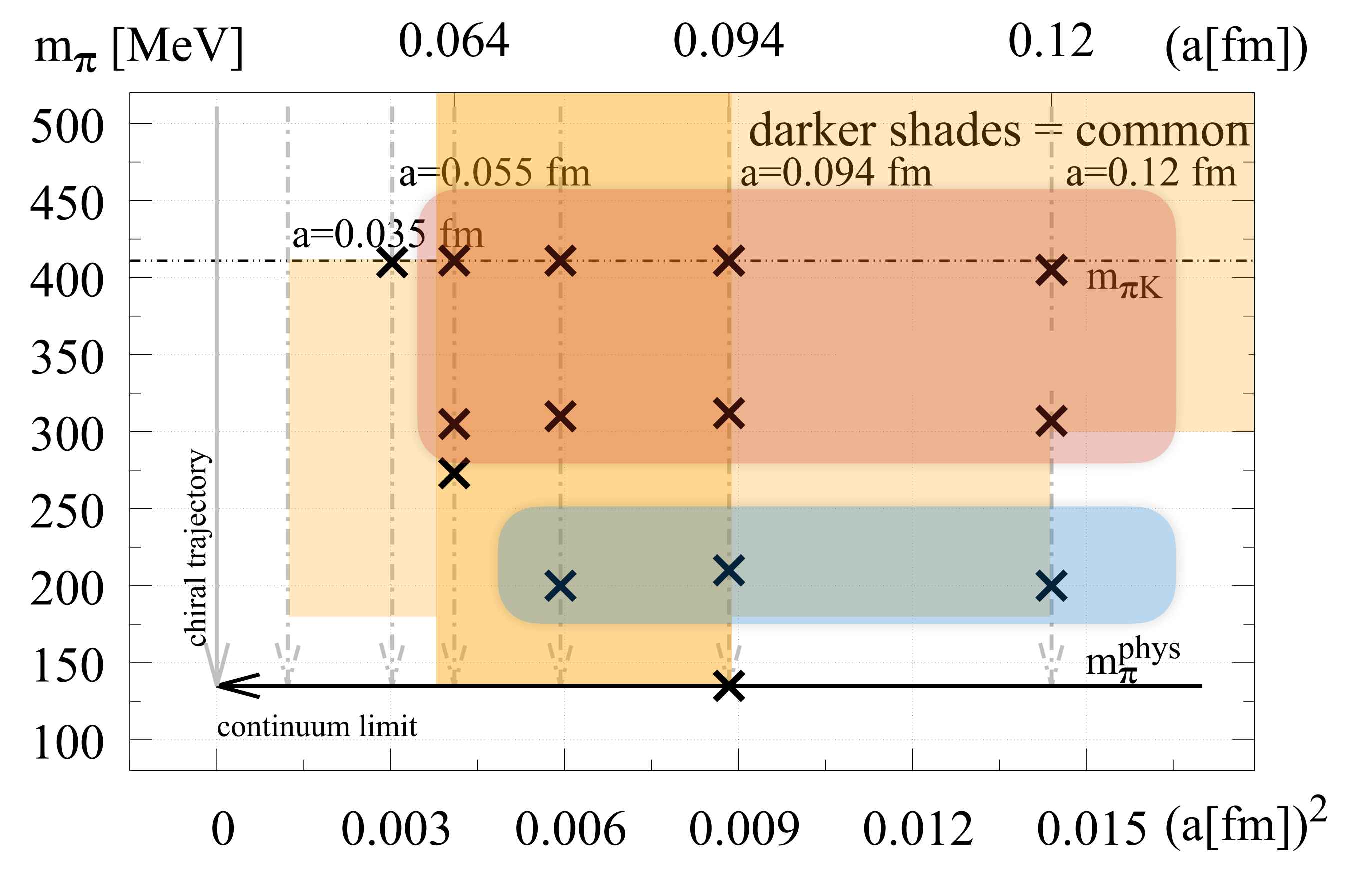

Stage 1

Stage 2

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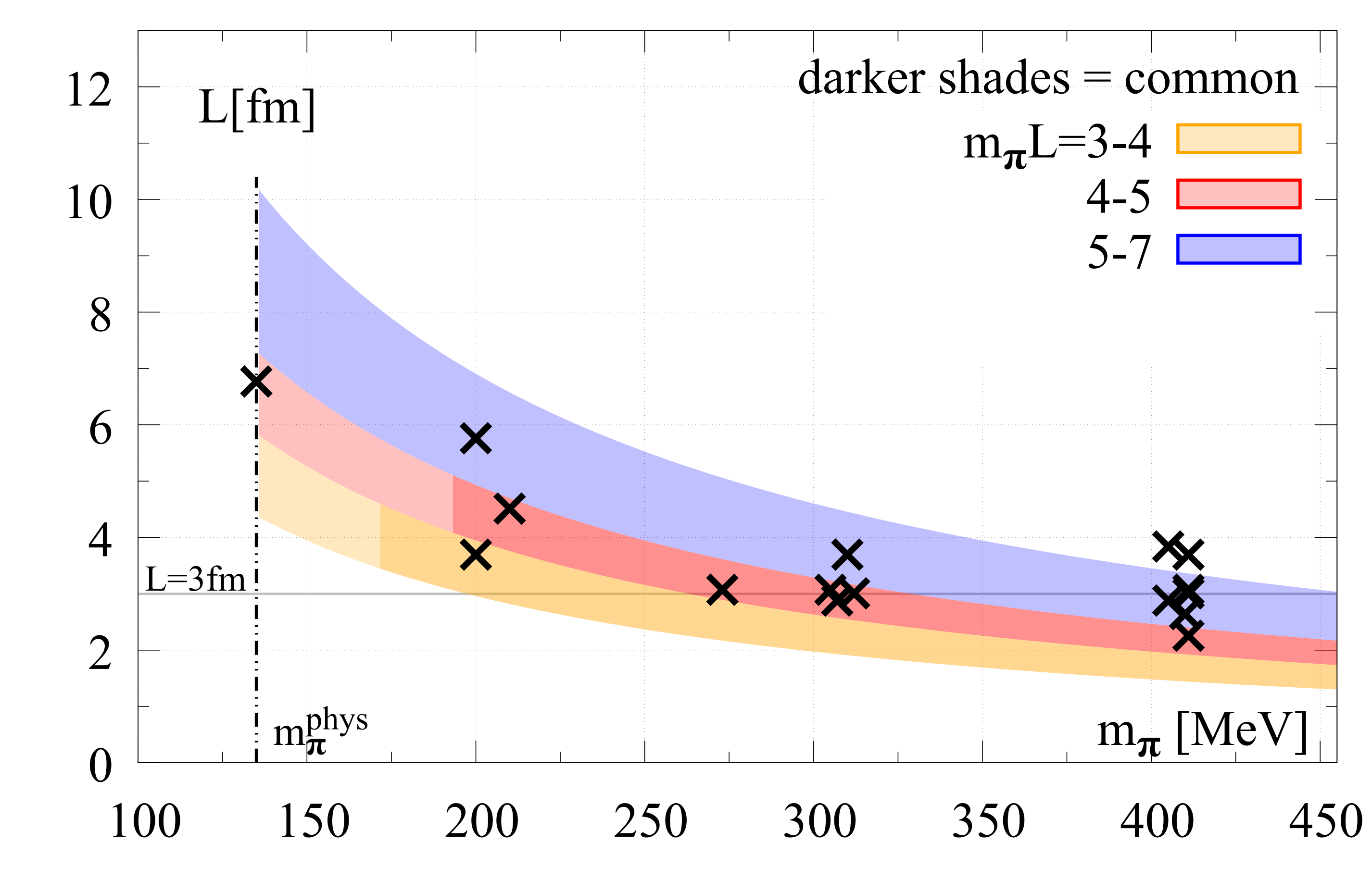

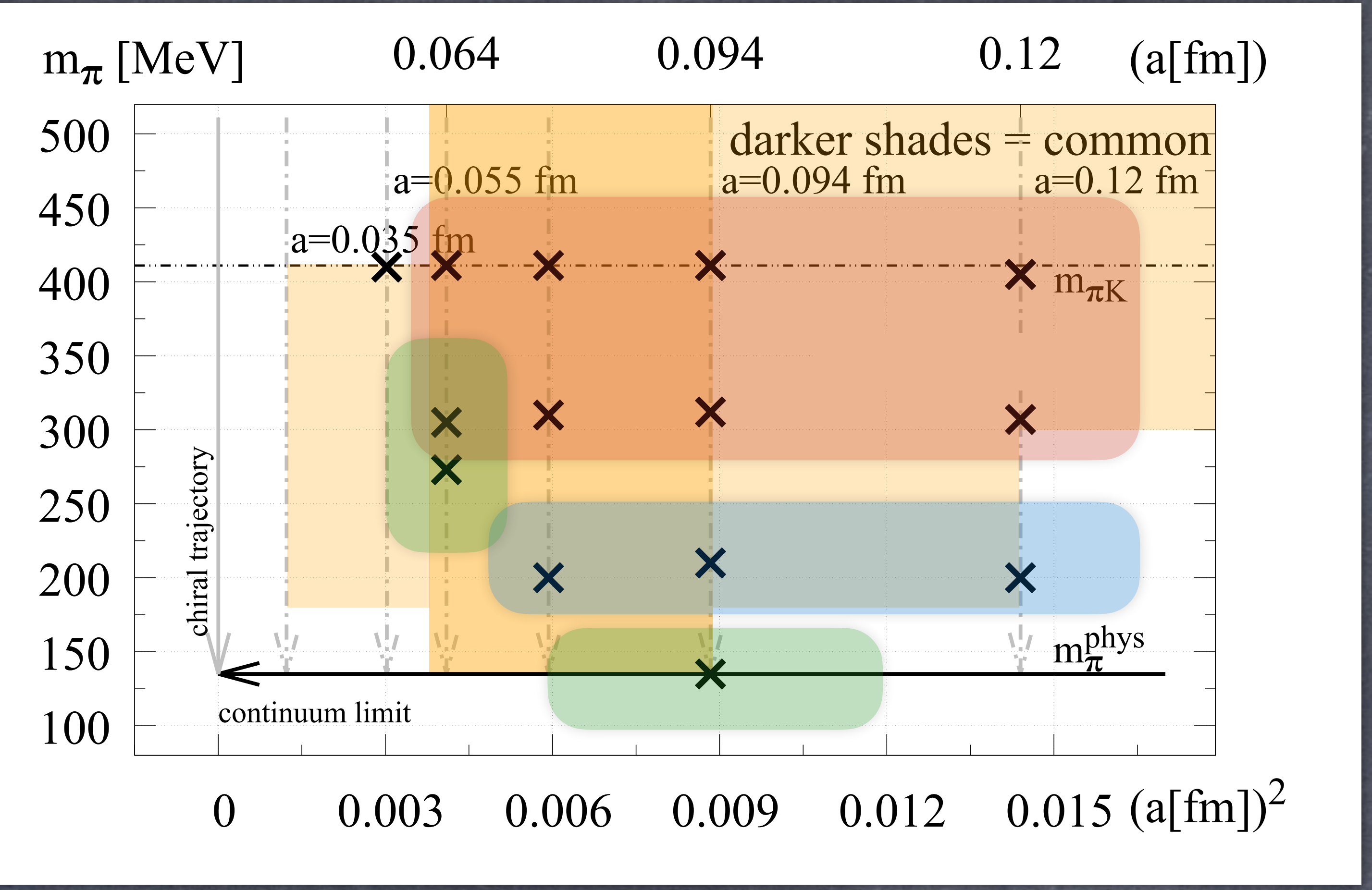

Stage 1

Stage 2

Stage 3

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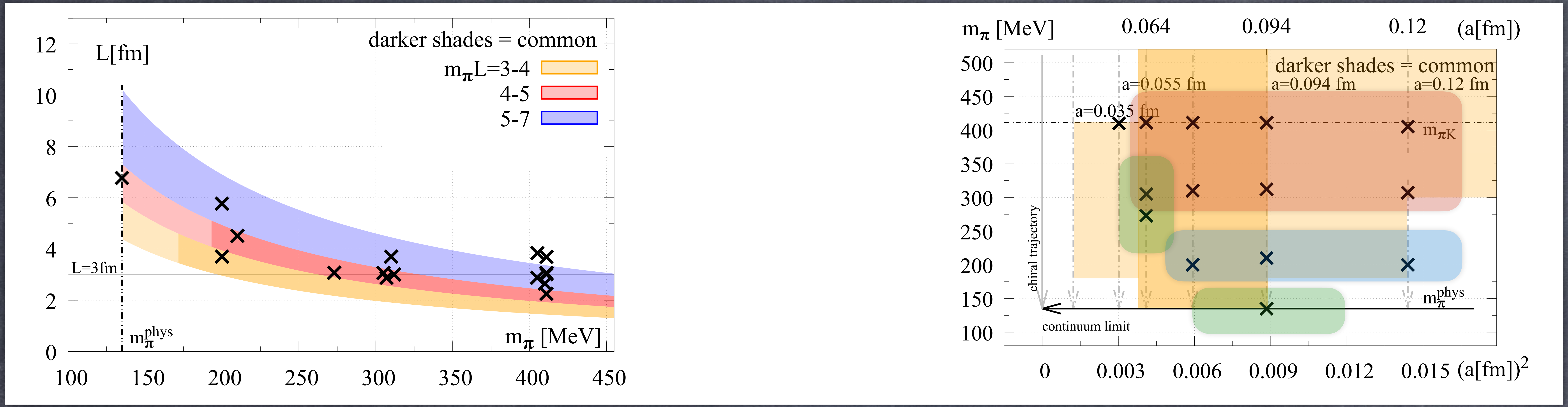
Our two main research projects focus on precision studies of the thetaEDM parameter and advancements in lattice PDF calculations.



Stage 1 ✅

Stage 2 WORK IN PROGRESS

Stage 3 WORK IN PROGRESS

Project	Main Goal	Progress
EDM	Improve constraints on CP-violating effects in strong interactions (SM & BSM)	New lattice ensembles generated; analysis ongoing.
PDF	Develop new lattice techniques for parton distribution function (PDF) determination	Implementation of flowmom method; preliminary results obtained
Multi-hadron interactions		
Flavor physics		

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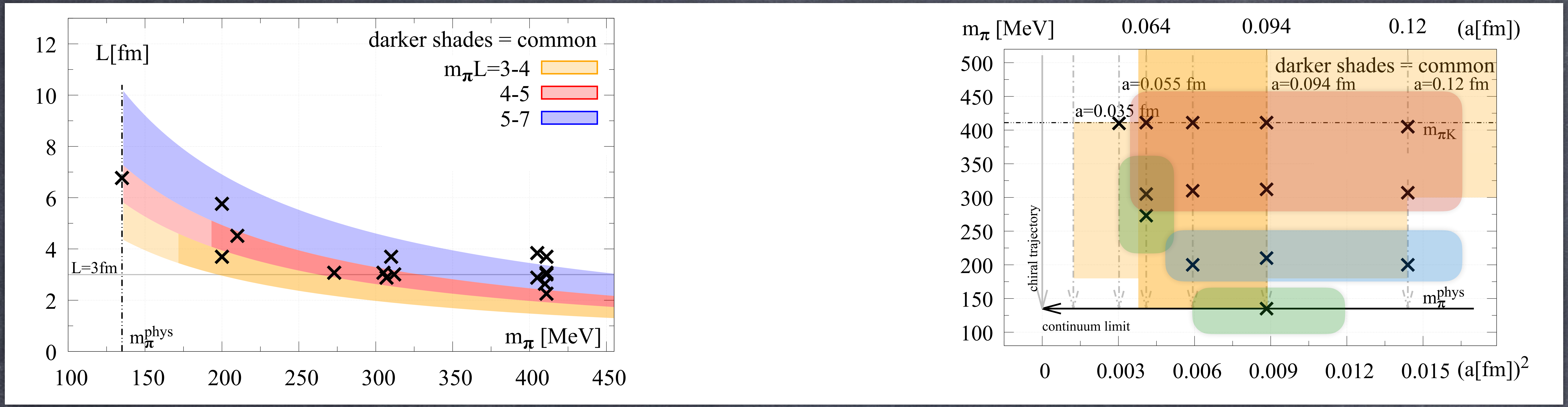


- Stage 1 ✅
- Stage 2 WORK IN PROGRESS
- Stage 3 WORK IN PROGRESS

Perlmutter (NERSC)

JUWELS (FZJ)

Irene Joliot-Curie

PRACE
EuroHPC
Gauss

Test case: Heavy pion at SU(3) flavor-symmetric point

PHYSICAL REVIEW LETTERS 136, 171903 (2026)

Editors' Suggestion

Gradient Flow for Parton Distribution Functions: First Application to the Pion

Anthony Francis,[1] Patrick Fritzsch,[2] Robert V. Harlander,[3] Rohith Karur,[4,5] Jangho Kim,[6] Jonas T. Kohnen,[3] Giovanni Pederiva,[7,8] Dimitra A. Pefkou,[4,5,*] Antonio Rago,[9] Andrea Shindler,[3,5,4,†] André Walker-Loud,[5,4] and Savvas Zafeiropoulos[10]

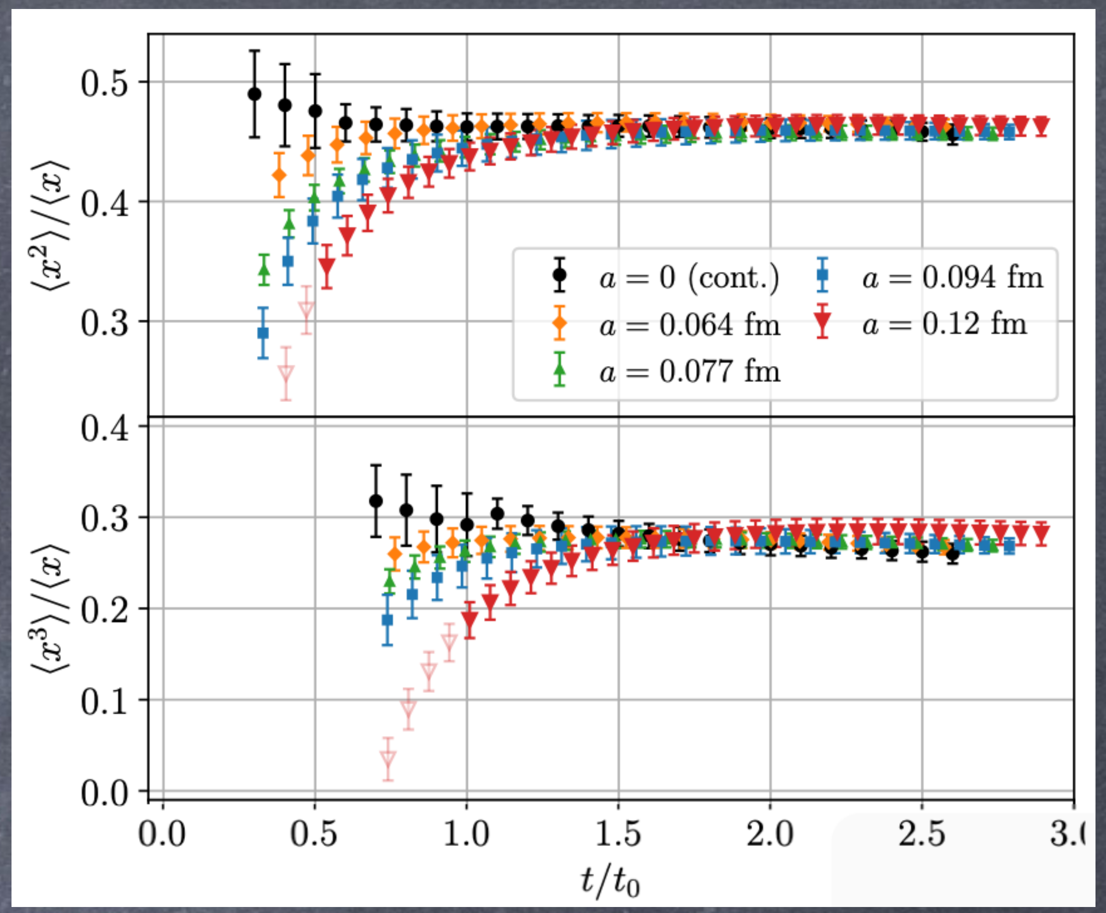
- SU(3)-flavor symmetric point: $m_u = m_d = m_s$ $(m_\pi \simeq 410\mathrm{MeV})$
- Stabilized Wilson fermions + Lüscher-Weisz gauge action
  Francis, Fritzsch, Lüscher, Rago (2019)
- 4 lattice spacings (0.12 → 0.064 fm)
- 400-800 configurations per ensemble
- NLO calculation A.Sh. (2023)
- NNLO calculation Harlander, Kohnen, A.Sh. 2511.17145

OPEN LATtice initiative

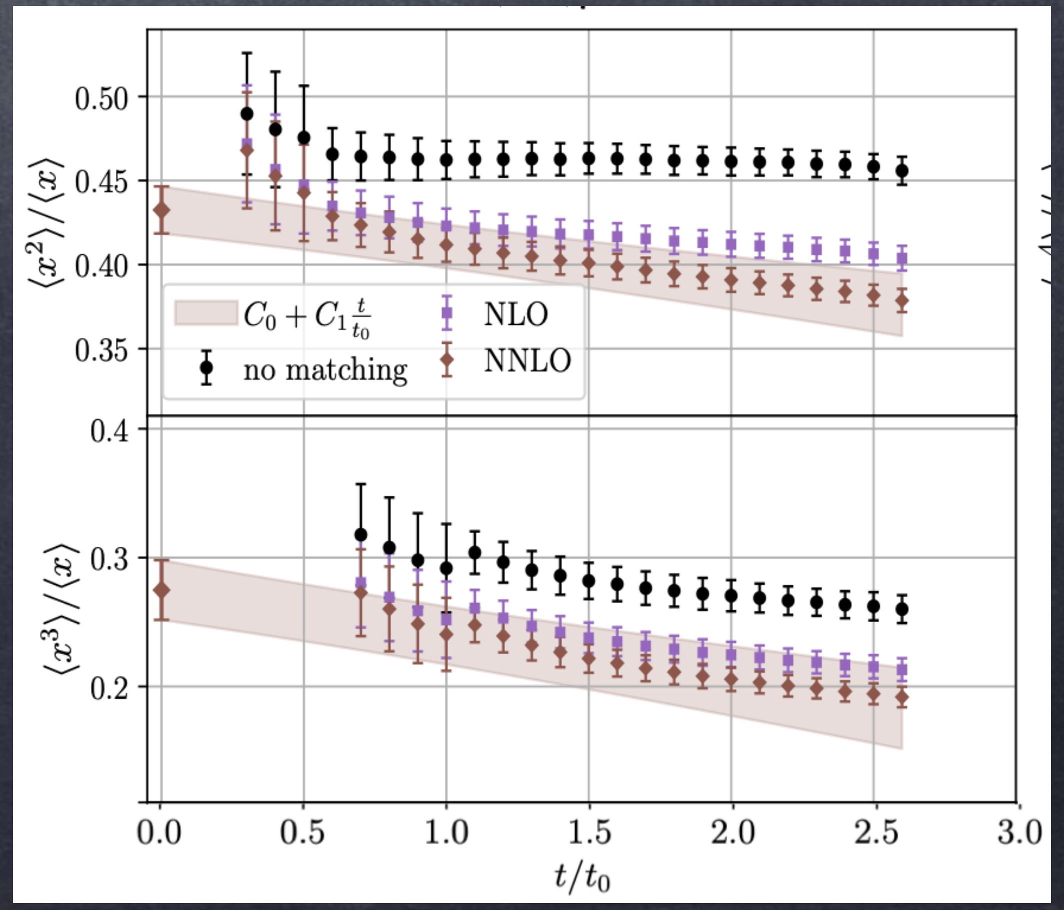
2509.02472
2510.26738

https://openlat1.gitlab.io

Flowed moments



t→0 matching (NLO/NNLO)



The method shows excellent scaling and stable t→0 behavior, validating it for precision applications

Continuum limit

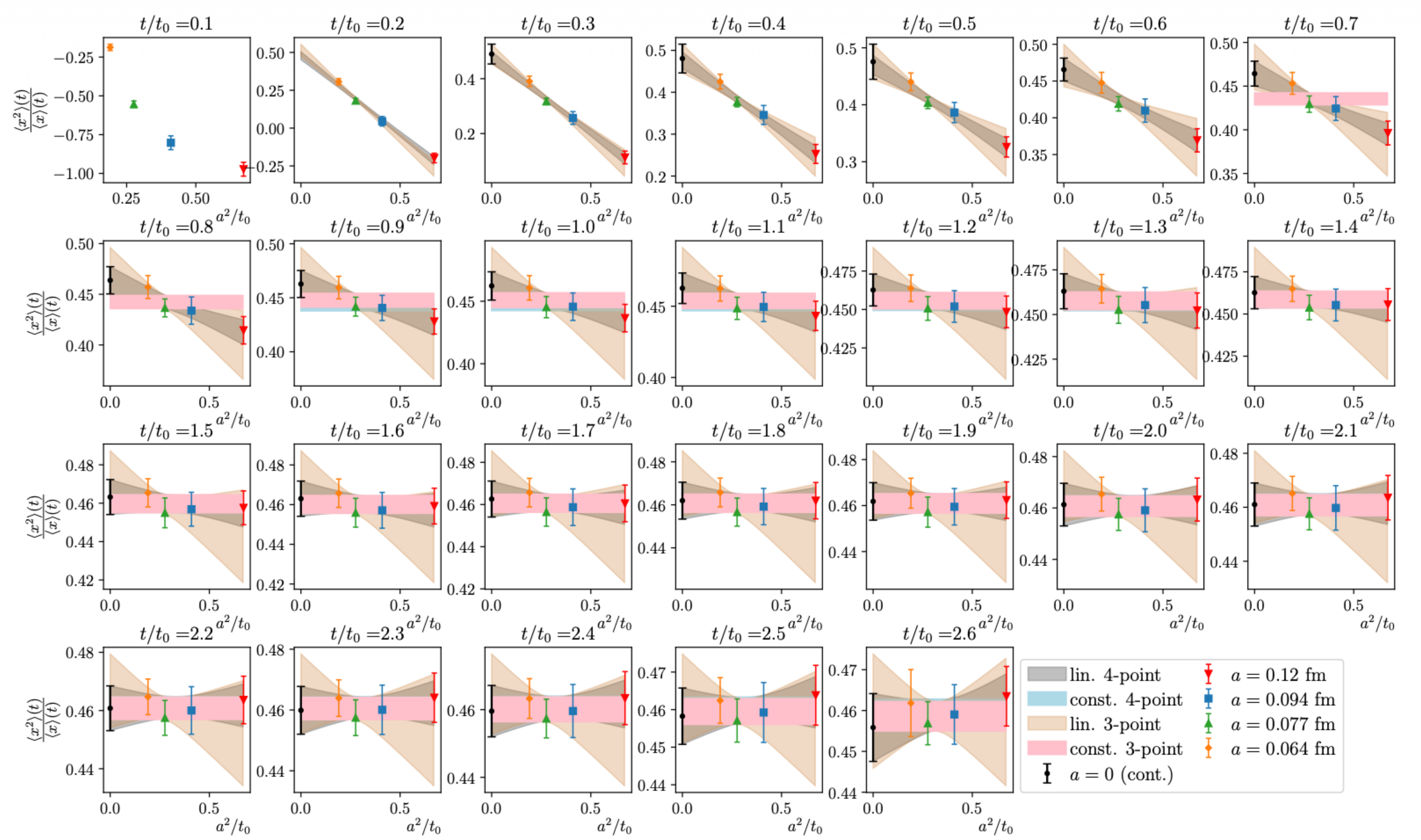
PHYSICAL REVIEW D **113**, 074520 (2026)

Editors' Suggestion

Moments of parton distribution functions of the pion from lattice QCD using gradient flow

Anthony Francis,[1] Patrick Fritzsch,[2] Rohith Karur,[3,4] Jangho Kim,[5] Giovanni Pederiva,[6,7] Dimitra A. Pefkou,[3,4,*] Antonio Rago,[8] Andrea Shindler,[9,4,3,†] André Walker-Loud,[4,3] and Savvas Zafeiropoulos[10]

$$r_n(a^2/t_0, t/t_0) = \frac{\langle x^{n-1}\rangle}{\langle x\rangle}(t/t_0) + \delta_{n,2}(t/t_0)\,\frac{a^2}{t_0} + \cdots$$



Extrapolation to $t \to 0$

PHYSICAL REVIEW D **113**, 074520 (2026)

Editors' Suggestion

Moments of parton distribution functions of the pion from lattice QCD using gradient flow

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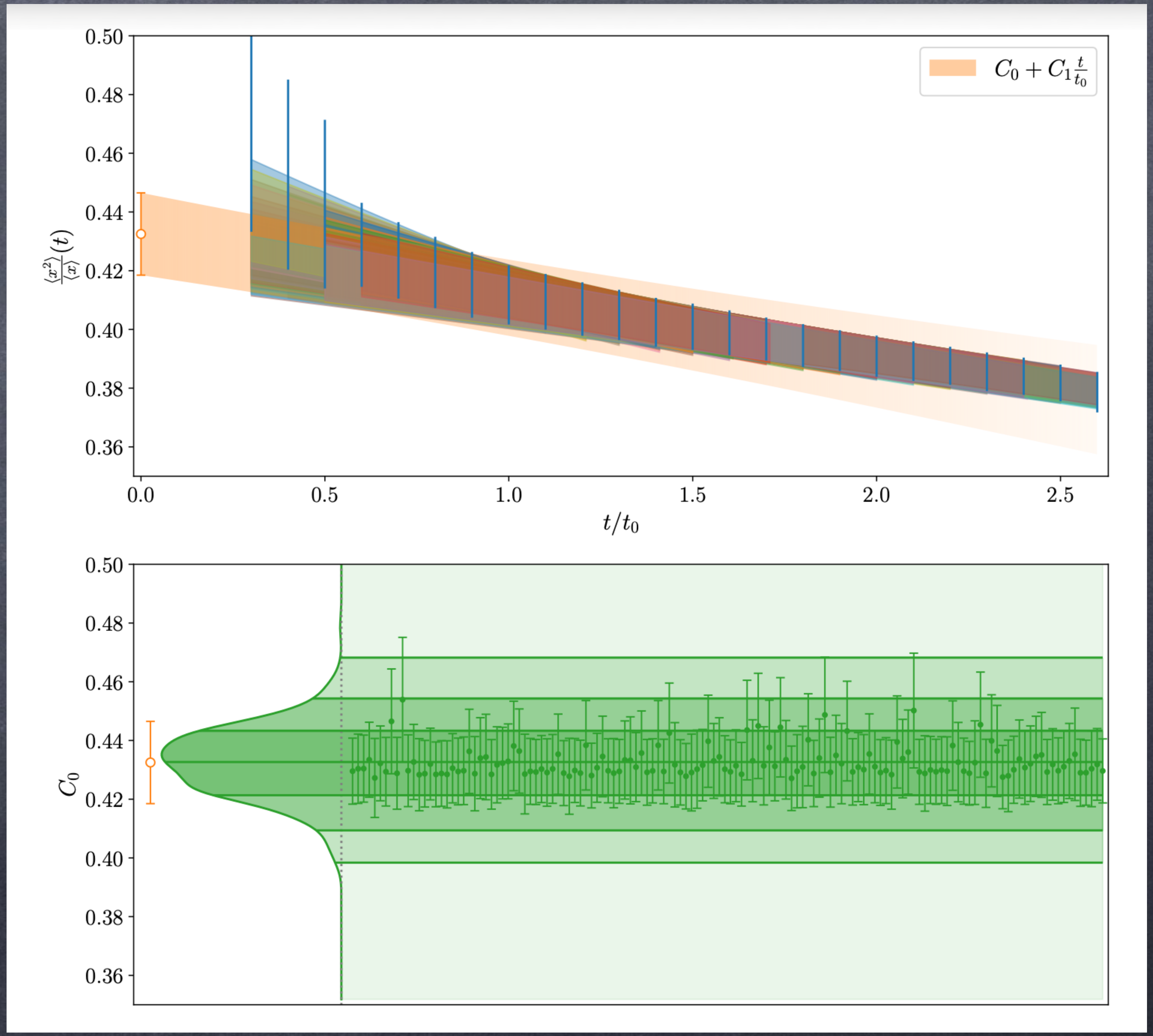
- Residual t-dependence
- Within the current statistical precision, no clear logs observed
- Simple linear ansatz $C_0 + C_1 t/t_0$
- The minimum values used are:

  $(t/t_0)_{\min} = [0.3, 0.7, 0.9, 1.3]$

  for n = [3, 4, 5, 6]
- We perform fits over all flow-time intervals with width bigger than 0.8
- Only fits with $p > 0.1$ are retained
- The final central value is from a flat average over all accepted fits
- The systematic uncertainty from the spread of the accepted fit results
- Varying the minimum and maximum flow-time cuts within reasonable ranges gives stable results



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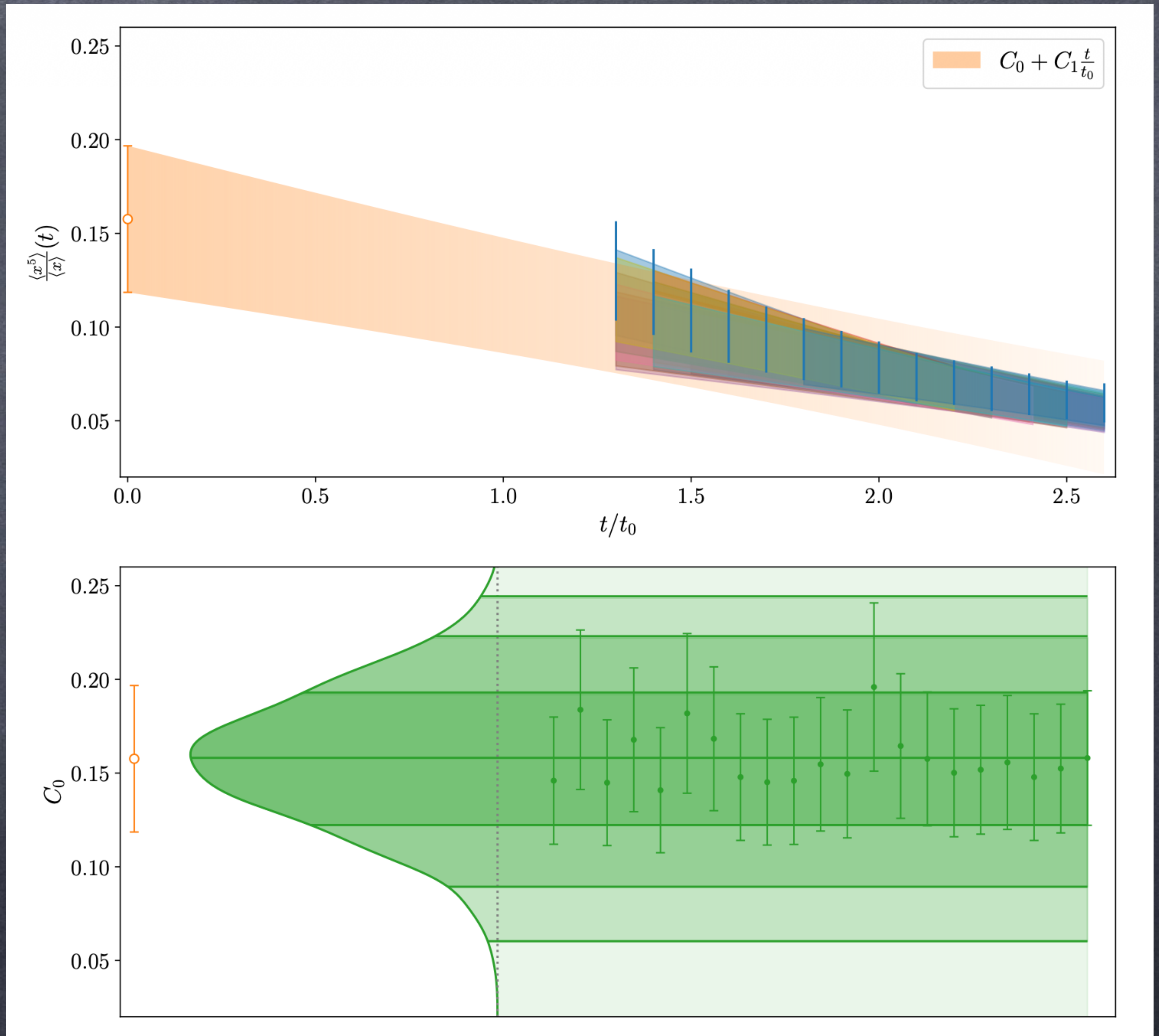
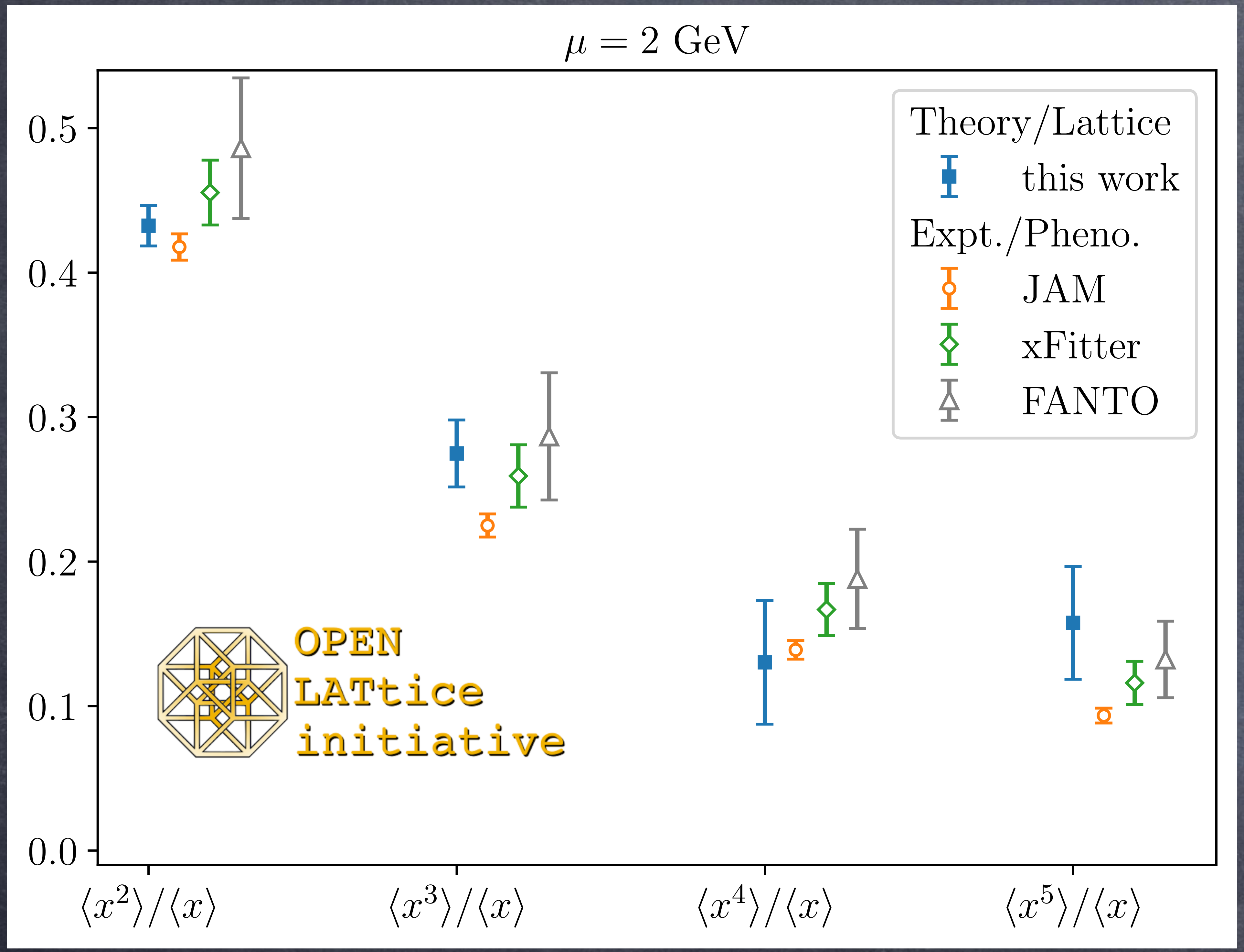


Comparison with experiment

Comparison with exp/pheno



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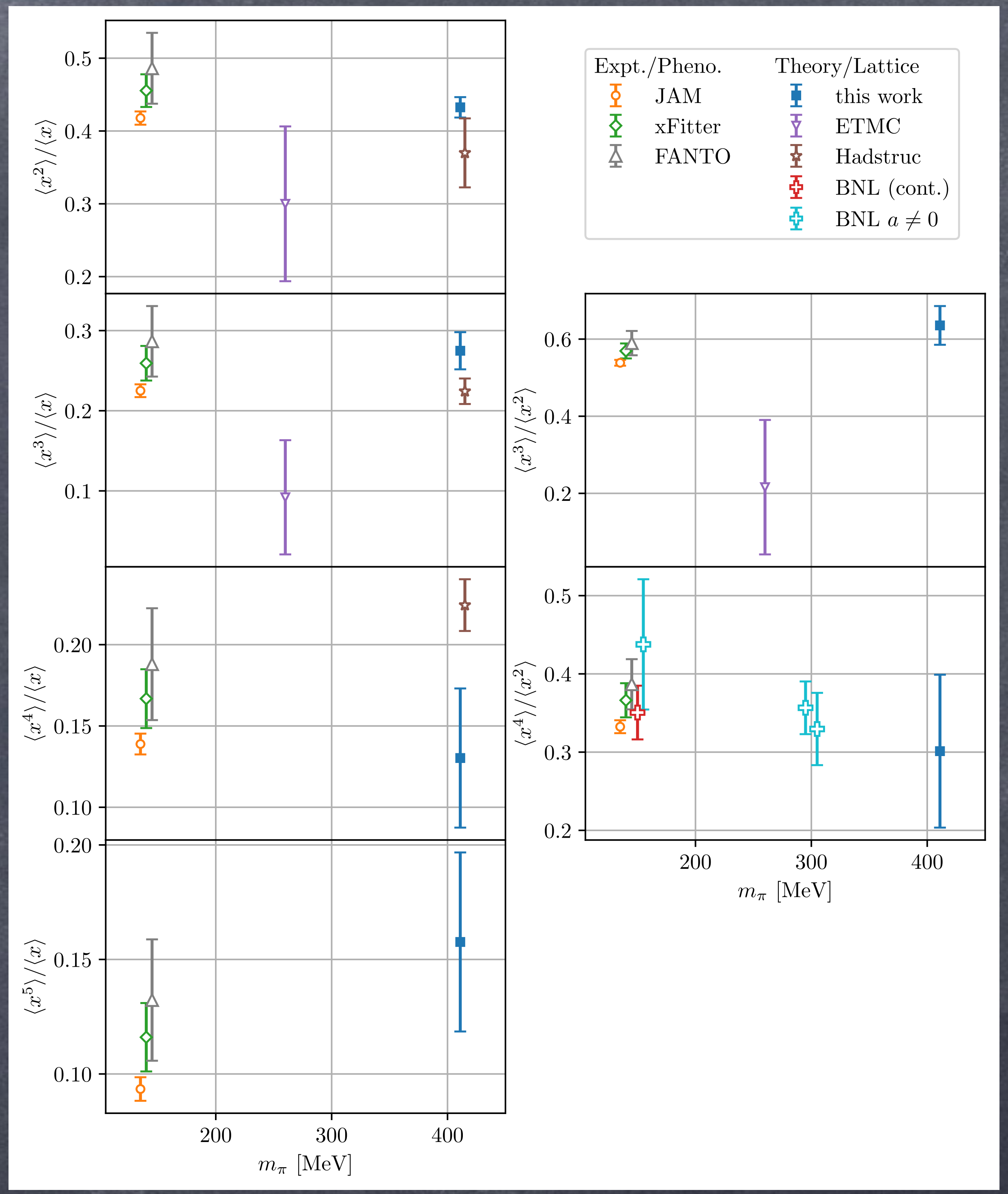
xFitter: 2002.02902
FANTO: 2505.13594

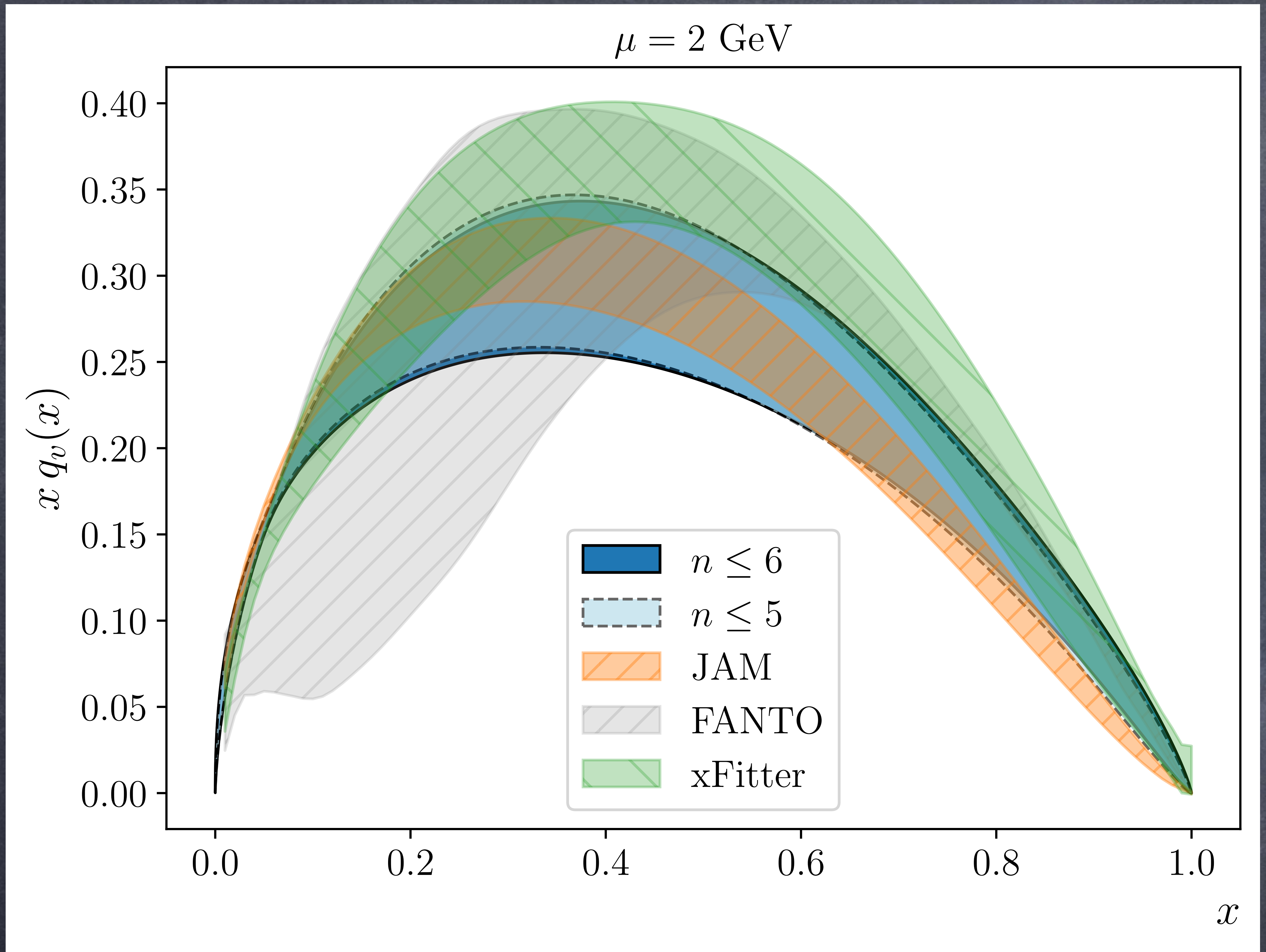
Comparison with lattice calculations



ETMC: Alexandrou et al. (2020-2021)
Joó et al. (2019)
Gao et al. (2022)

Even with limited statistics, our results match global fits and reach competitive or leading precision for many lattice moments

PDF reconstruction



$$q_v(x) = \mathcal{N}\, x^{\alpha}(1-x)^{\beta}\, p(x)$$

- Non-trivial use of priors to stabilize fit
- Small-x exponent α hard to constrain
- Large x parameter is robust ($\beta \sim 1$)
- Gaussian Processes reconstruction underway (UC Berkeley PhD student Rohith Karur)

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2510.26738

Reconstruction is consistent with global fits and already competitive in precision

Renormalization flowed fermions

$$C_{PP}(x_4, t) = -\frac{G_\pi(t)\, G_\pi}{2m_\pi} \mathrm{e}^{-m_\pi x_4}$$

$$C_{AP}(x_4, t) = -\frac{f_\pi(t)\, G_\pi}{2} \mathrm{e}^{-m_\pi x_4}$$

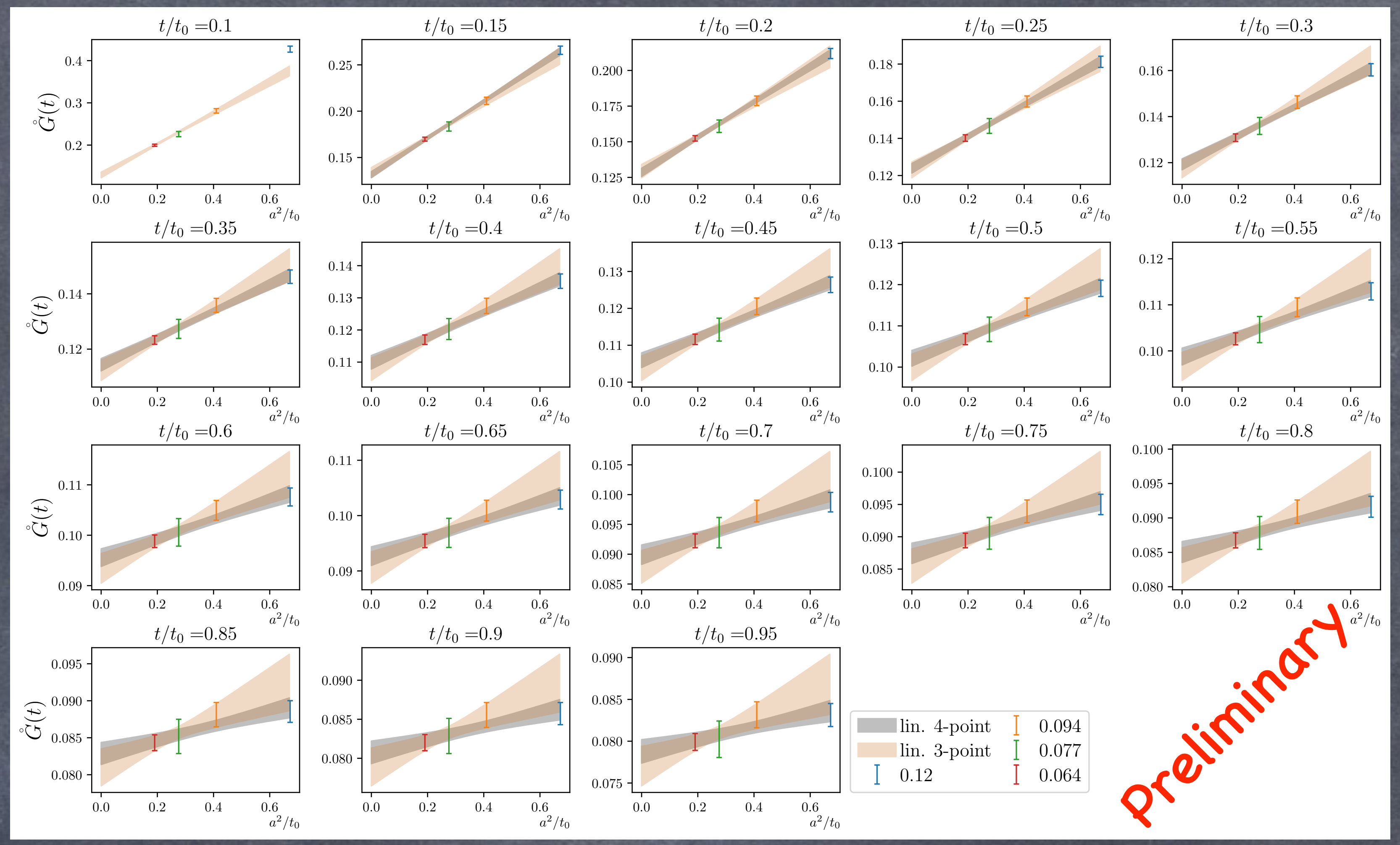
$$\mathring{Z}_\chi = -\frac{N_c}{(4\pi t)^2} \left[\left\langle \bar{\chi}(t) \overleftrightarrow{\not{D}} \chi(t) \right\rangle \right]^{-1}$$

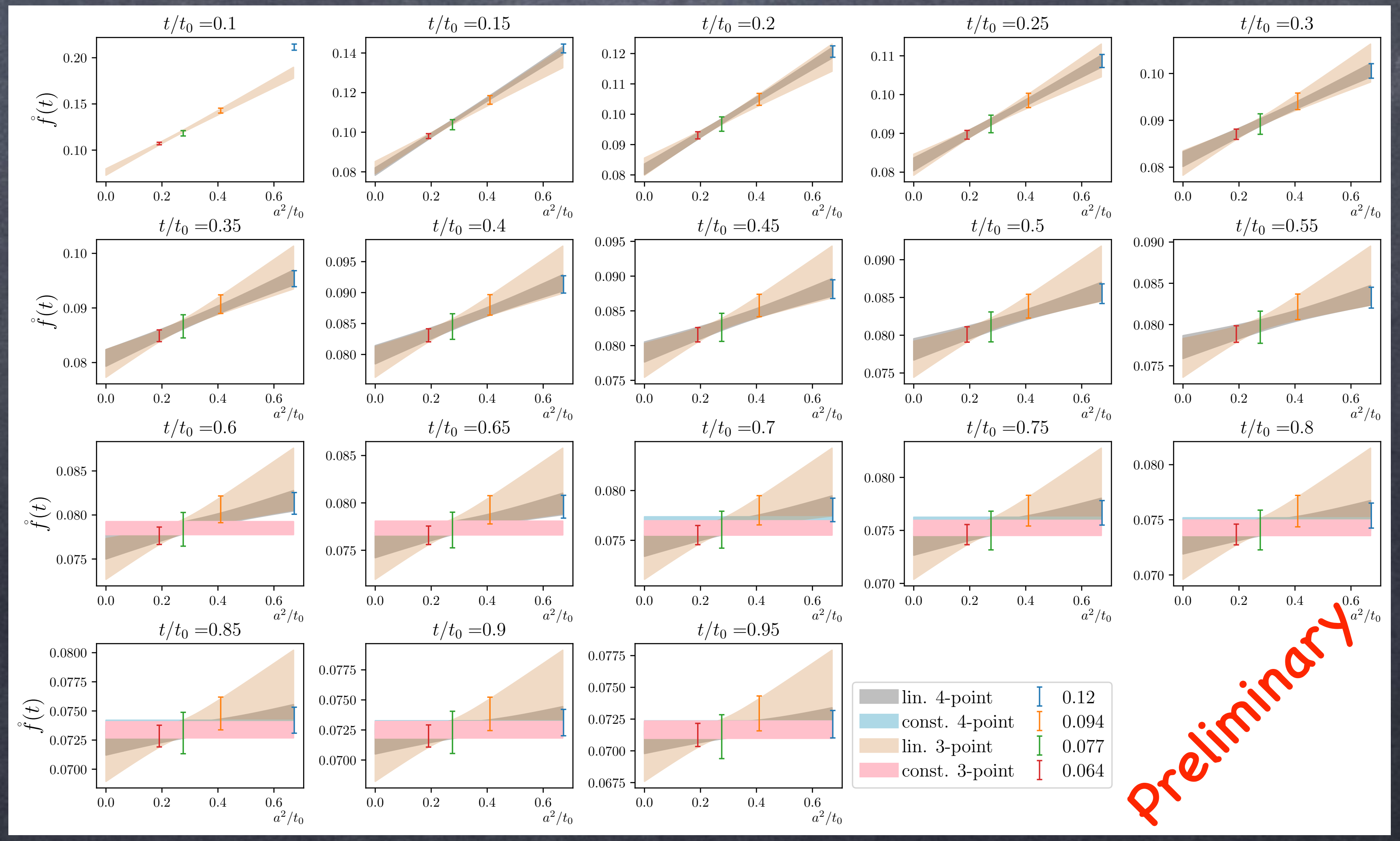
$$\mathring{G}_\pi(t) = \lim_{a \to 0} \mathring{Z}_\chi\, G_\pi(t)$$

$$\mathring{f}_\pi(t) = \lim_{a \to 0} \mathring{Z}_\chi\, f_\pi(t)$$

Ringed scheme

Makino, Suzuki: 2014





Renormalization flowed fermions

$$\partial_\mu^y \langle P^{rs}(x)\, V_\mu^{ss}(y)\, P^{sr}(0)\rangle = [\delta(y) - \delta(y-x)]\,\langle P^{rs}(x)\, P^{sr}(0)\rangle$$

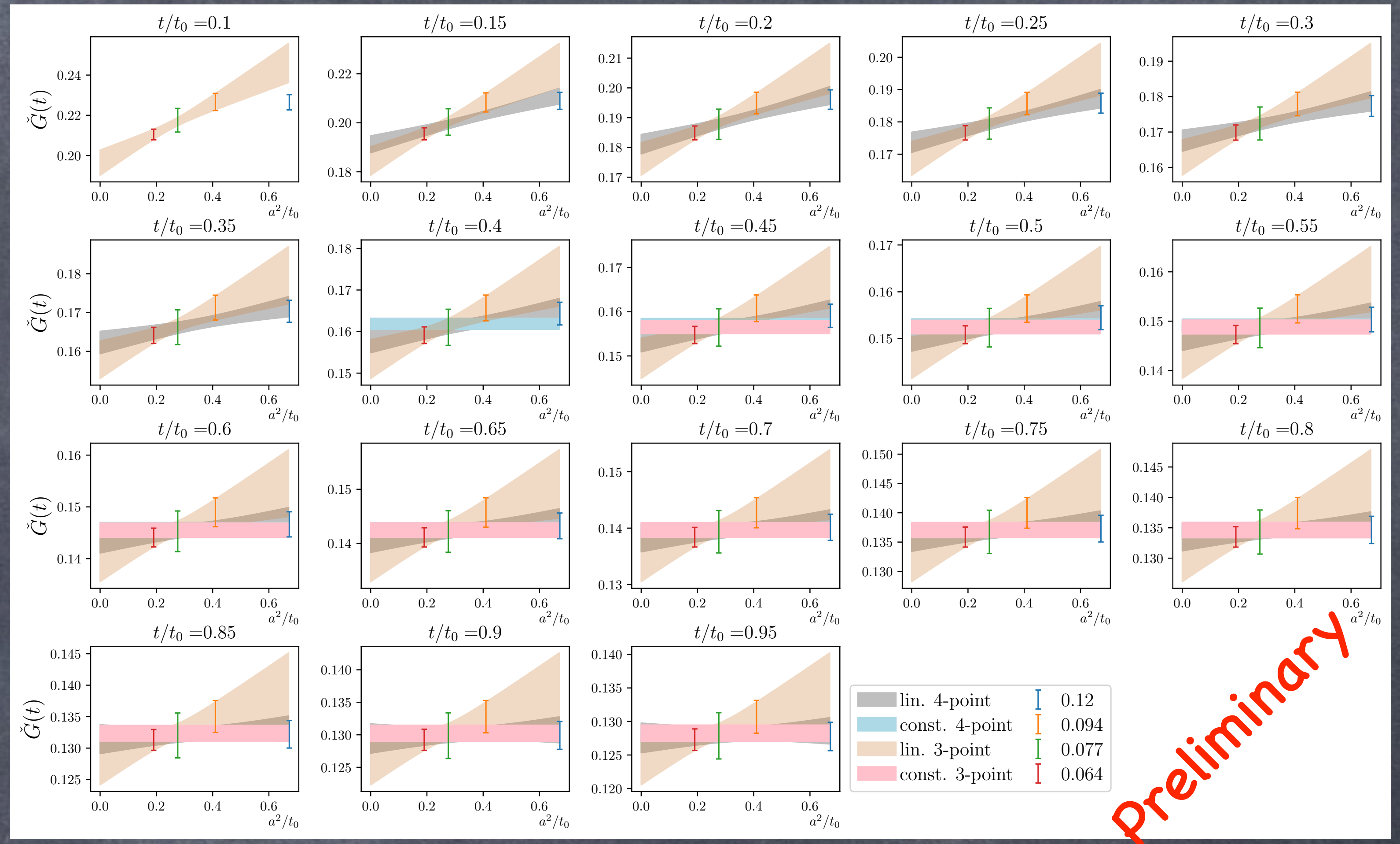
$$\frac{\int d^3x\, d^3y \left\langle P^{21}(x)\, \frac{1}{2}\, [V_{4,R}^{11}(y) - V_{4,R}^{22}(y)]\, P^{12}(0)\right\rangle}{\int d^3x\, \langle P^{21}(x)\, P^{12}(0)\rangle} = 1 \qquad x_4 > y_4 > 0$$

$$\check{Z}_\chi = \frac{2m_\pi}{\langle \pi\, |V_\mu(t)|\, \pi\rangle}$$

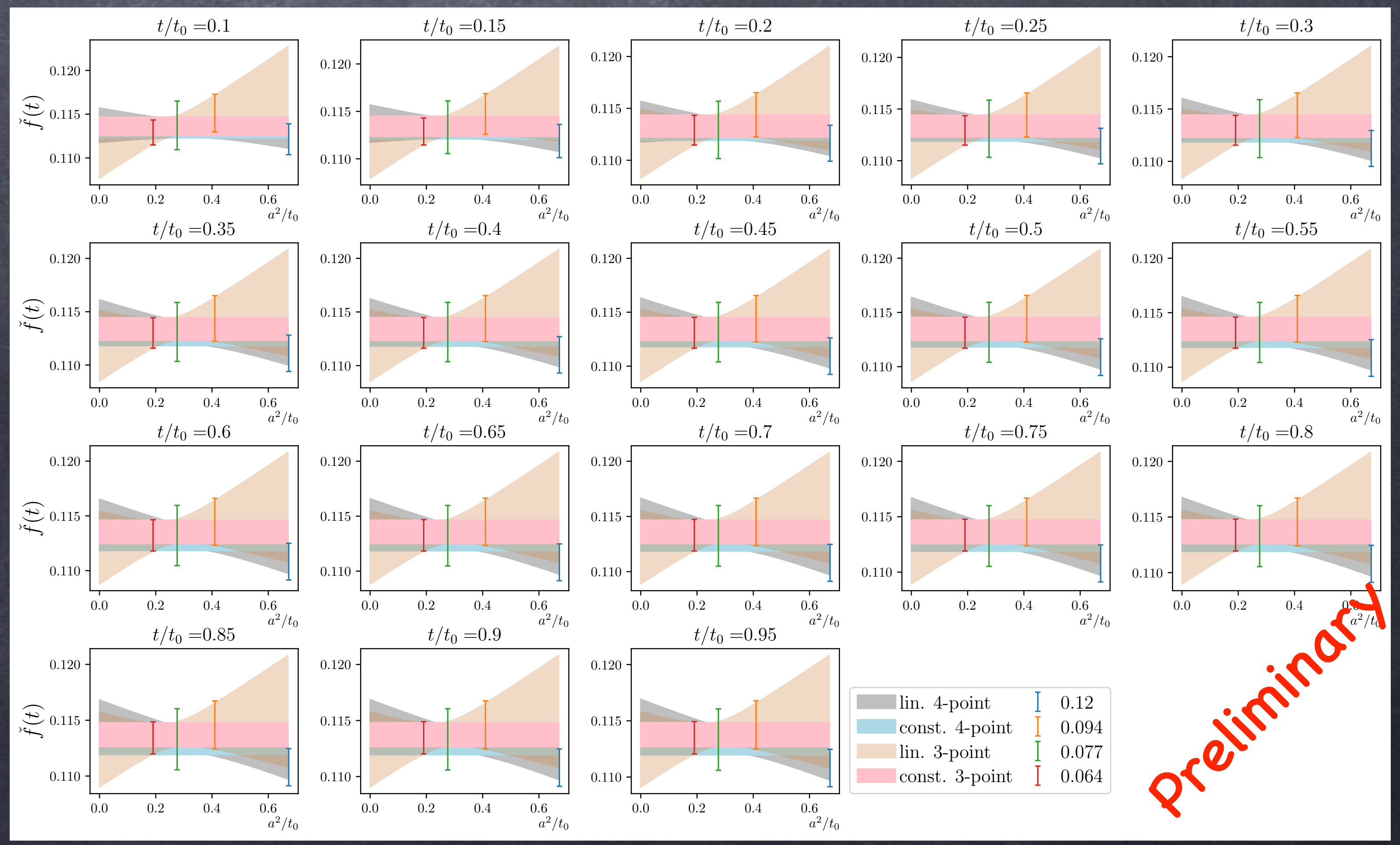
Caron scheme

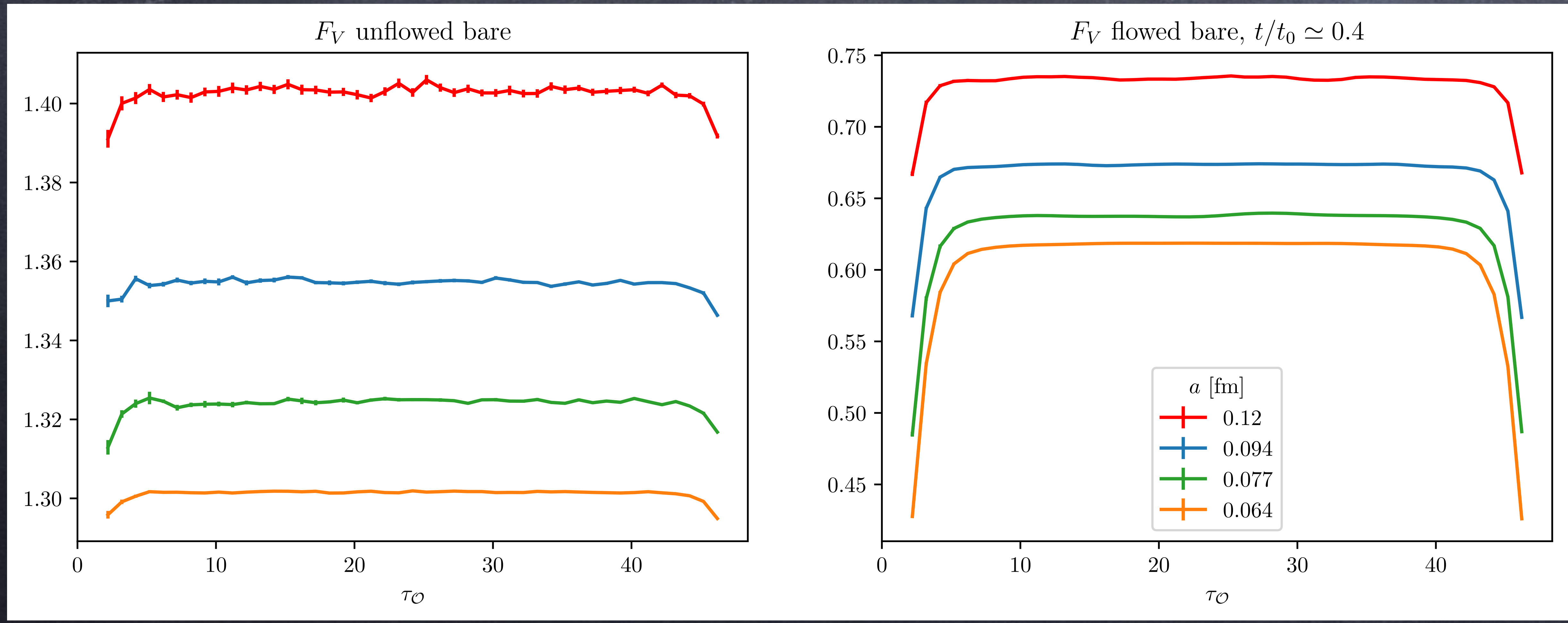
$\zeta_V \rightarrow$ Borgulat, Harlander, Kohnen, Lange (2024)

$$\check{G}_\pi = \lim_{a\to 0} \check{Z}_\chi\, G_\pi(t)$$



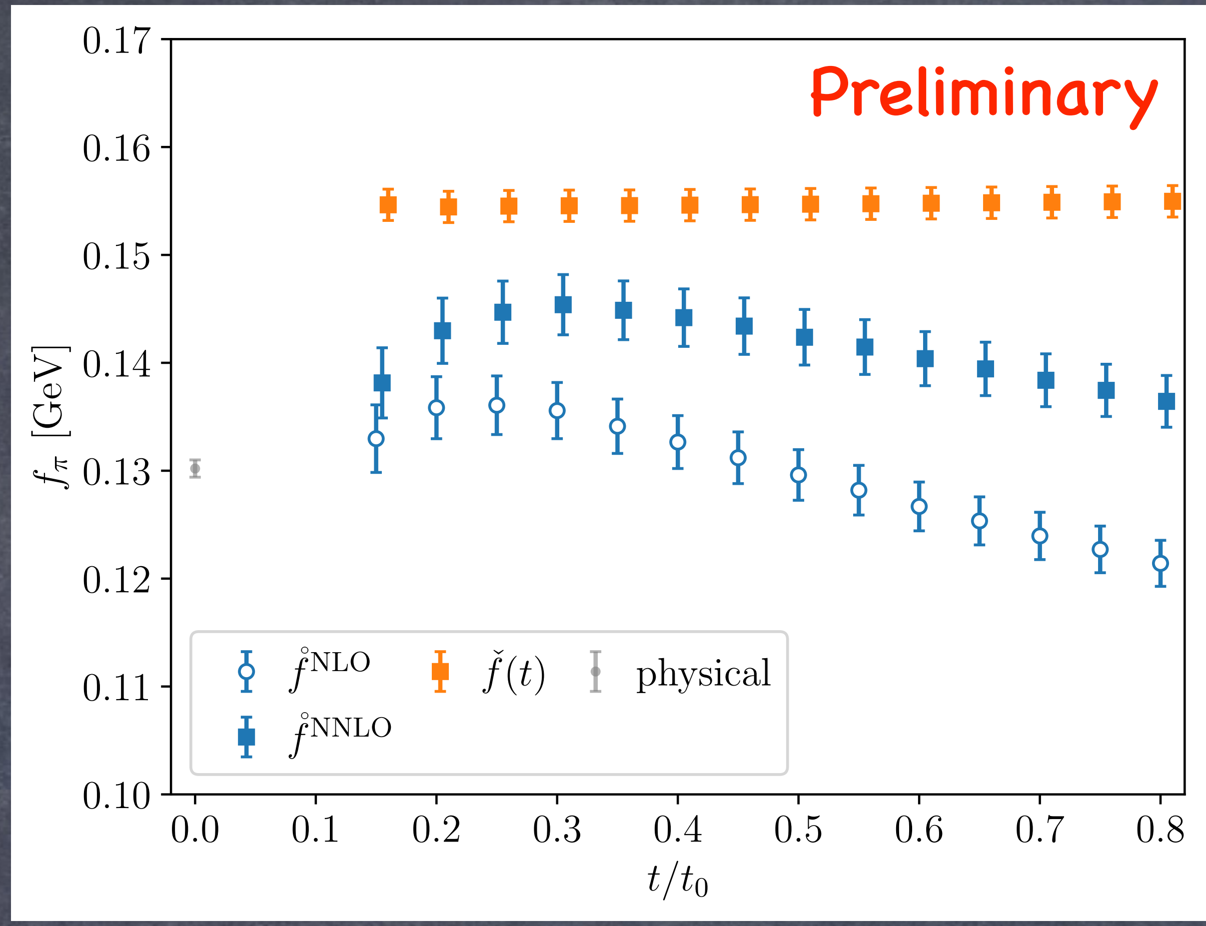
$$\check{f}_\pi = \lim_{a\to 0} \check{Z}_\chi\, f_\pi(t)$$



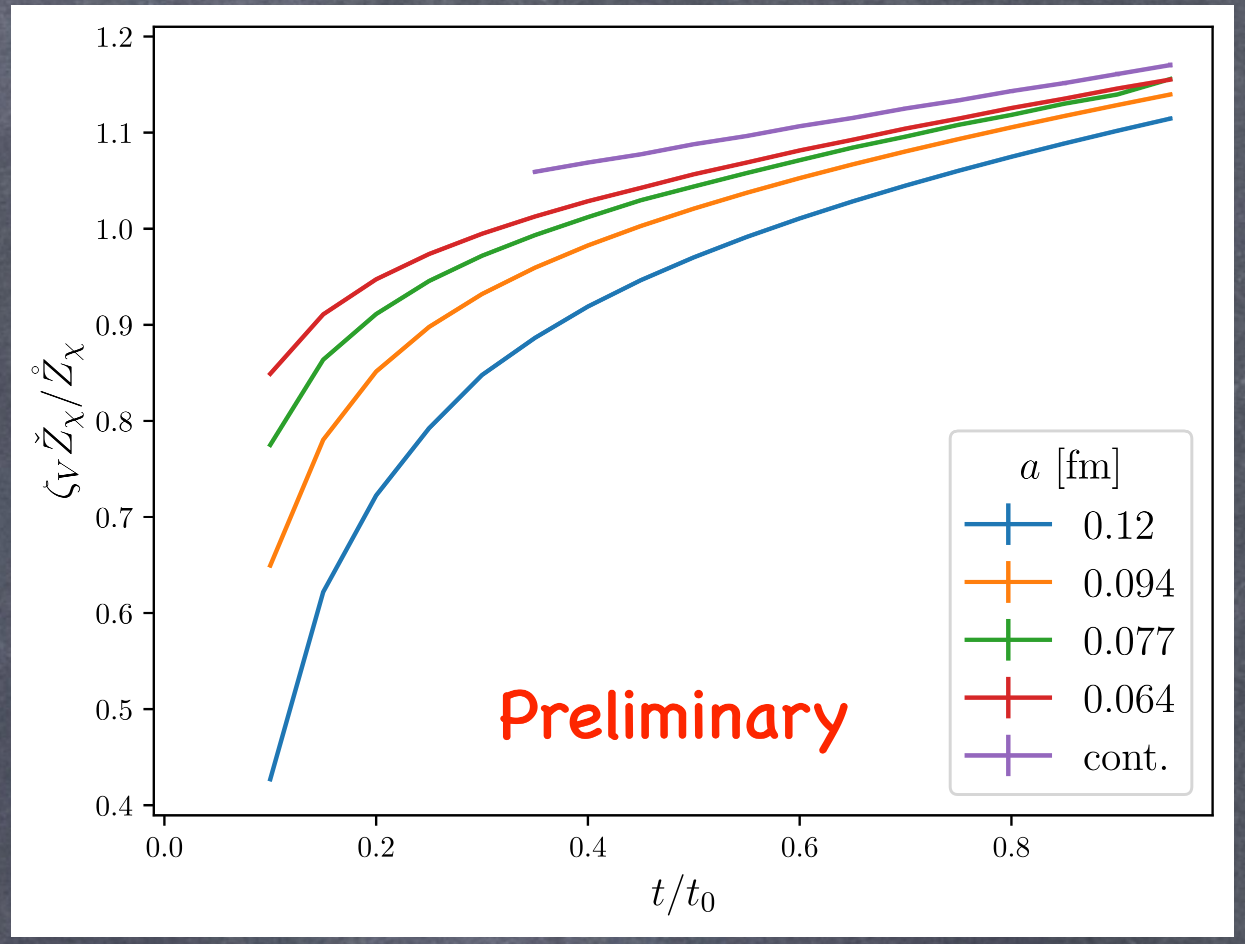


Renormalization flowed fermions

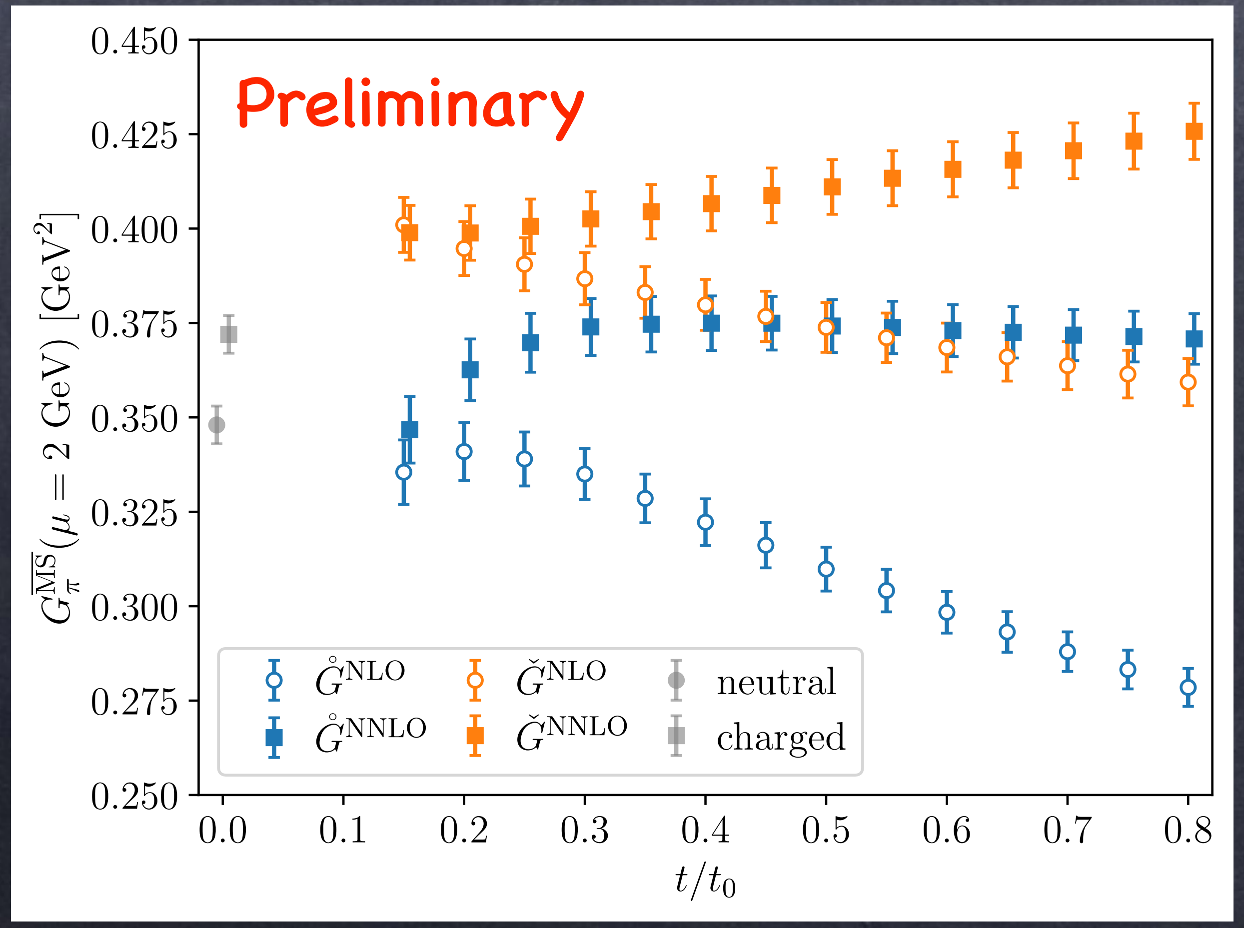
$\mathring{f}_\pi(t) = \lim_{a\to 0} \mathring{Z}_\chi \, f_\pi(t)$ $\qquad$ $\check{f}_\pi = \lim_{a\to 0} \check{Z}_\chi \, f_\pi(t)$

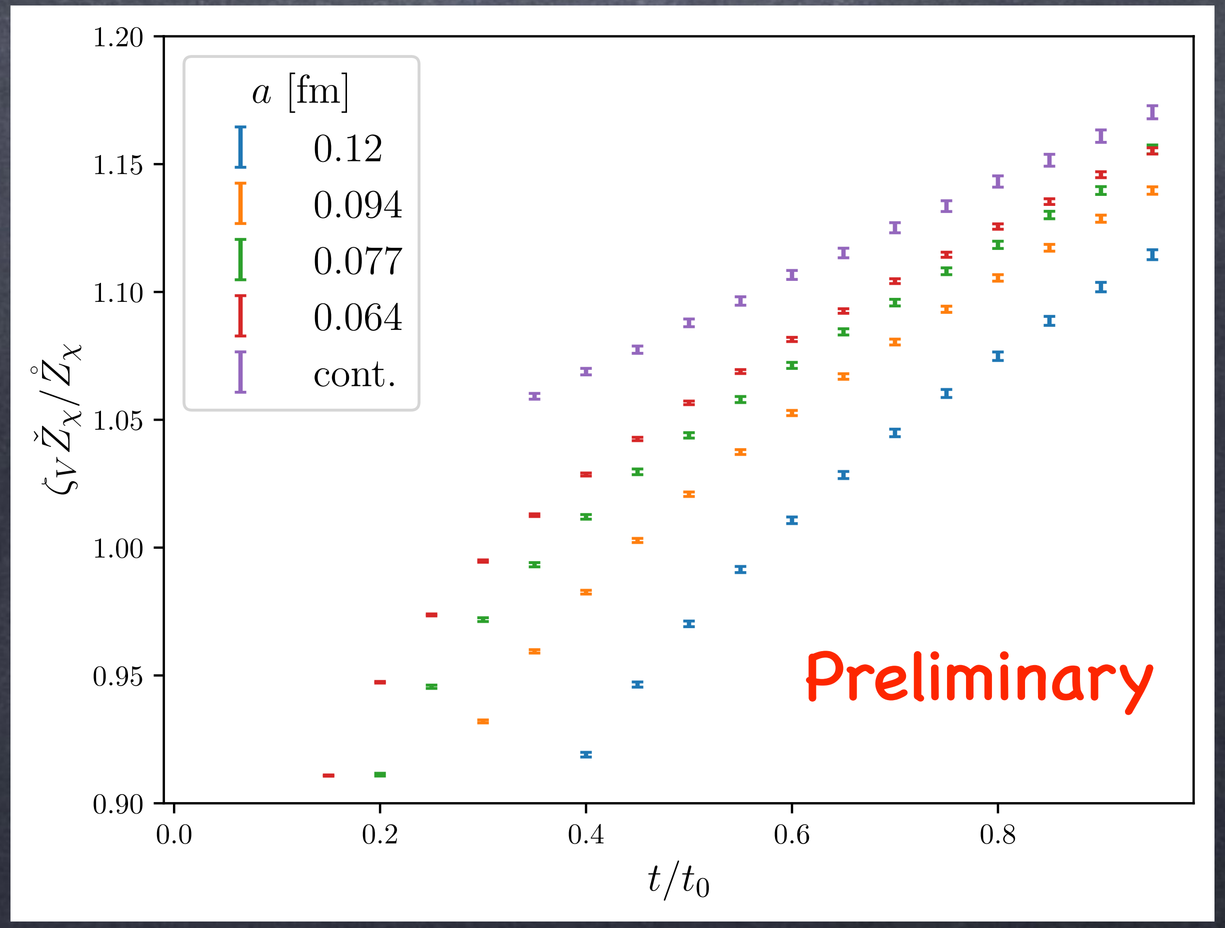


$\check{Z}_\chi/\mathring{Z}_\chi\zeta_V$



$\mathring{G}_\pi(t) = \lim_{a\to 0} \mathring{Z}_\chi \, G_\pi(t)$ $\qquad$ $\check{G}_\pi = \lim_{a\to 0} \check{Z}_\chi \, G_\pi(t)$

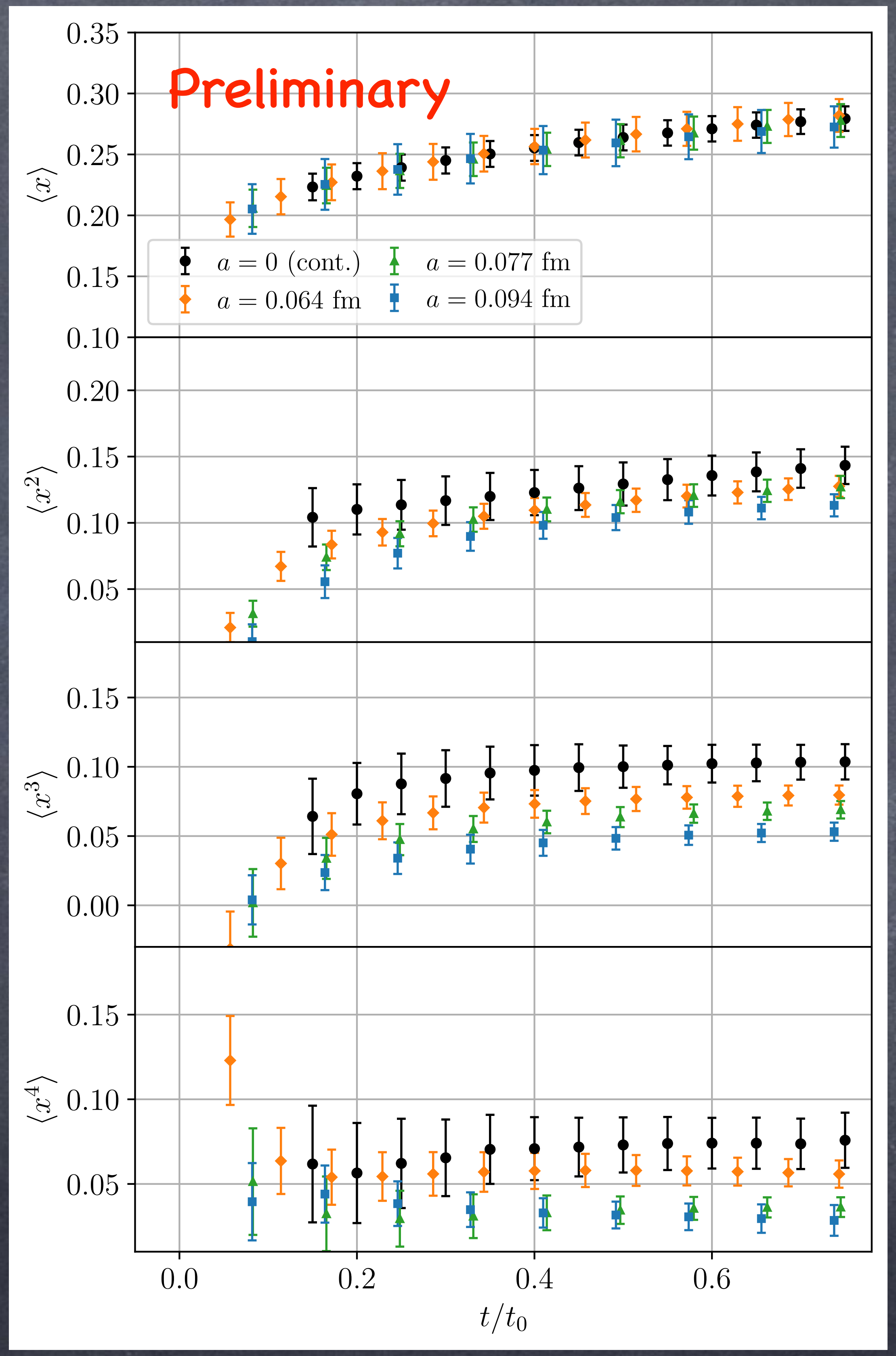




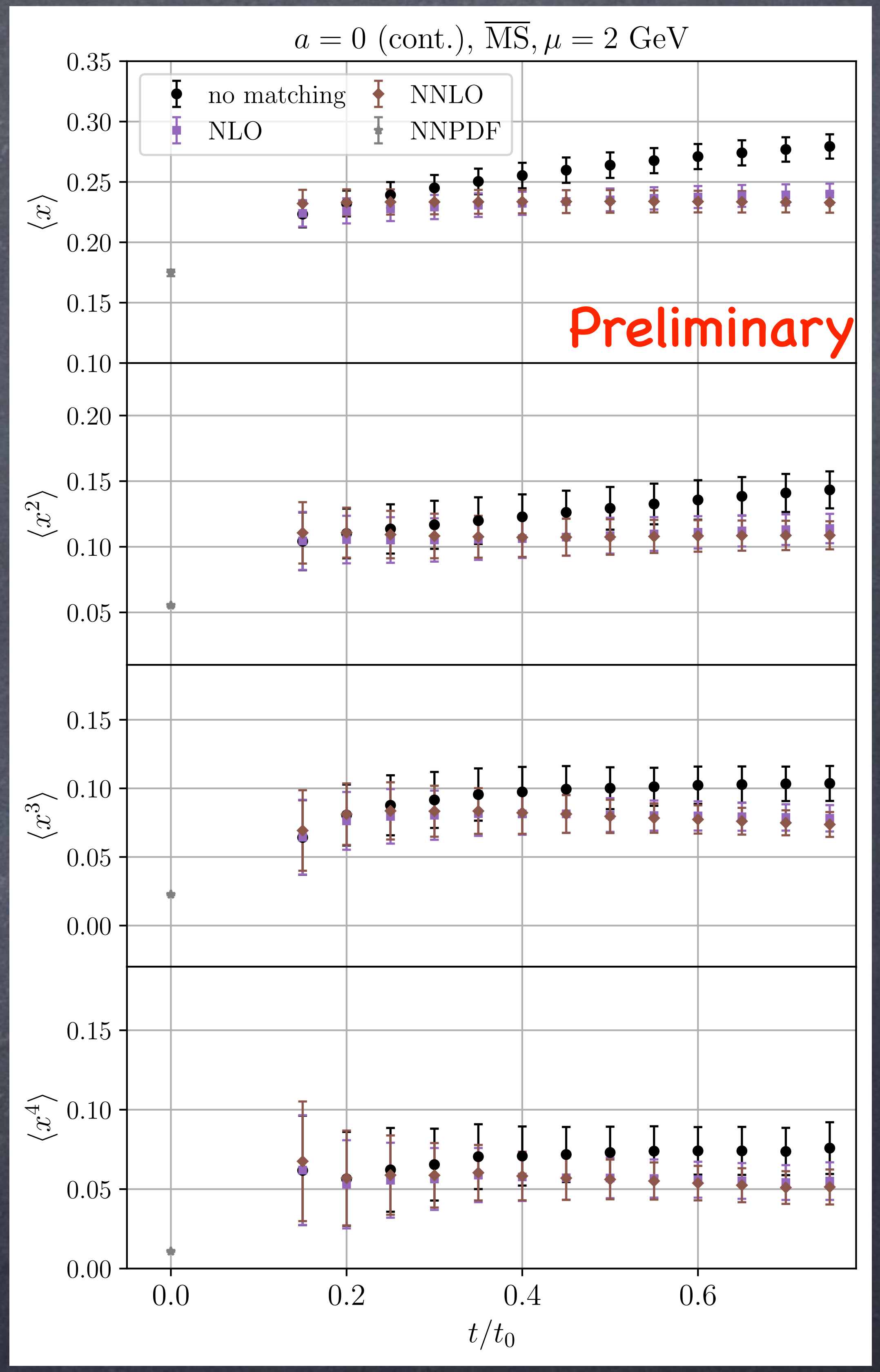
Nucleon valence moments

- Away from SU(3)-flavor symmetric point m_π = 300 MeV
- Stabilized Wilson fermions + Lüscher-Weisz gauge action
- 3 lattice spacings (0.094 → 0.064 fm)
- 100-200 configurations per ensemble
- 10 sources
- Source-Sink → 1.5 fm
- NLO calculation — A.Sh. (2023)
- NNLO calculation — Harlander, Kohnen, A.Sh. 2511.17145
- NNPDF — A. Chiefa, L. Del Debbio

Flowed moments



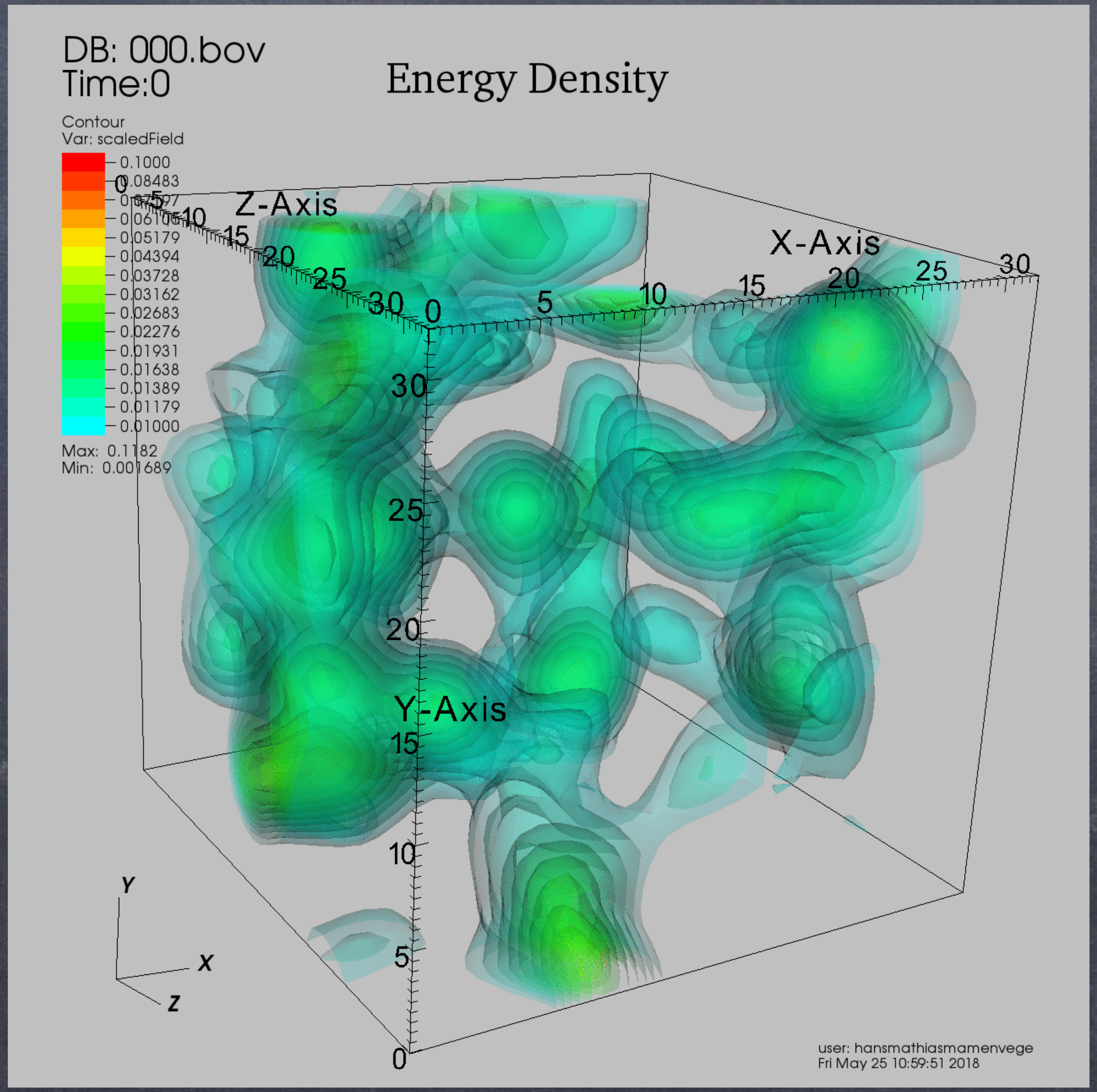
t→0 matching (NLO/NNLO)



Summary

- **First direct QCD extraction** of flavor non-singlet pion and nucleon PDF moment ratios using GF
- Resolves the **power-divergence problem** for higher moments
- General method to compute PDF moments of any order, valid for any lattice action
- **Gradient-flow regulator**, simplifies the **continuum limit** and restores O(4) before **perturbative matching**
- **Zero-momentum ratios** further improve S/N and control continuum extrapolation
- Moments up to $\langle x^5 \rangle$ for pion show **very good agreement** with phenomenology even with modest statistics
- Moments up to $\langle x^4 \rangle$ for nucleon with moderate numerical effort
- Could provide **high-precision inputs** valuable for global PDF analyses
- **Future extensions:** physical-pion, singlet+gluon, helicity/transversity, GPD form factors, higher twist

Thanks!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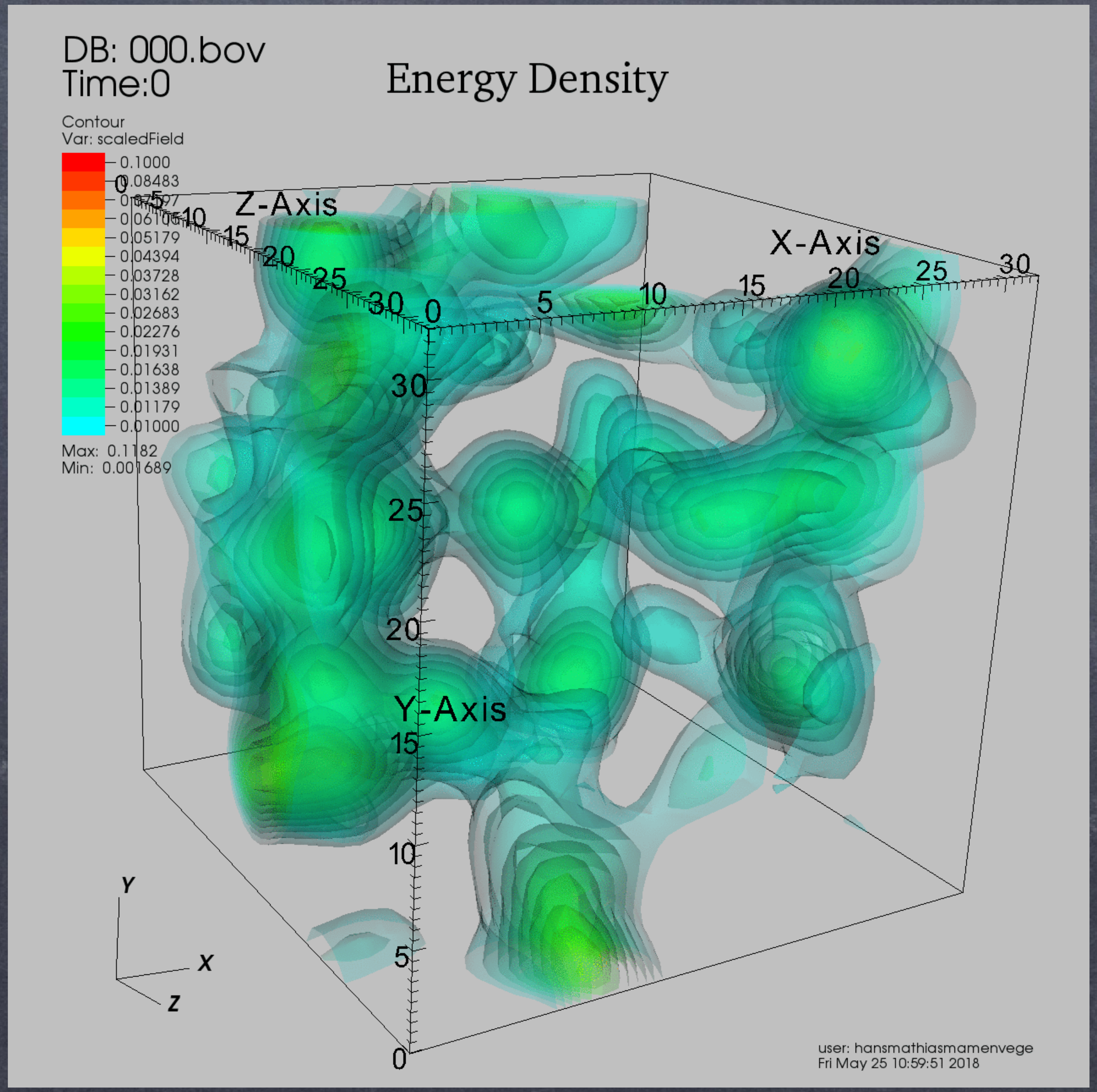
Fri May 25 10:59:51 2018

Thanks!



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Backup slides

O(4) irreducible representations

GL(4) irrep $$T_{\{\mu_1\cdots\mu_n\}} = \frac{1}{n!} \sum_{\substack{\sigma\in\text{all} \\ \text{permutations}}} T_{\mu_{\sigma(1)}\cdots\mu_{\sigma(n)}}$$

In O(4) an additional operation is allowed that commutes with orthogonal trafo: contraction of 2 indices

$$T^{(12)}_{\mu_1\cdots\mu_n} = T_{\alpha\alpha\mu_3\cdots\mu_n} = \delta_{\mu_1\mu_2} T_{\mu_1\cdots\mu_n}$$ rank n-2 tensor

Subspace of traceless tensors is invariant under O(4), i.e. the traceless rank n tensors are transformed among themselves under O(4)

Always possible to decompose $$T_{\mu_1\cdots\mu_n} = \widehat{T}_{\mu_1\cdots\mu_n} + \delta_{\mu_1\mu_2} T^{(12)}_{\mu_1\cdots\mu_n} + \cdots$$ Invariant under O(4)

n(n-1)/2 terms

E.g. $$\widehat{T}_{\mu_1\mu_2} = T_{\mu_1\mu_2} - \frac{1}{4}\delta_{\mu_1\mu_2} T_{\alpha\alpha} \qquad \widehat{T}_{\mu_1\mu_2\mu_3} = T_{\mu_1\mu_2\mu_3} - \frac{1}{6}\left[\delta_{\mu_1\mu_2} T_{\alpha\alpha\mu_3} + \delta_{\mu_1\mu_3} T_{\alpha\mu_2\alpha} + \delta_{\mu_2\mu_3} T_{\mu_1\alpha\alpha}\right]$$

Traceless tensors invariant under vector index permutations —> starting point to construct all the irreducible representations of O(4) (Young symmetrizers)

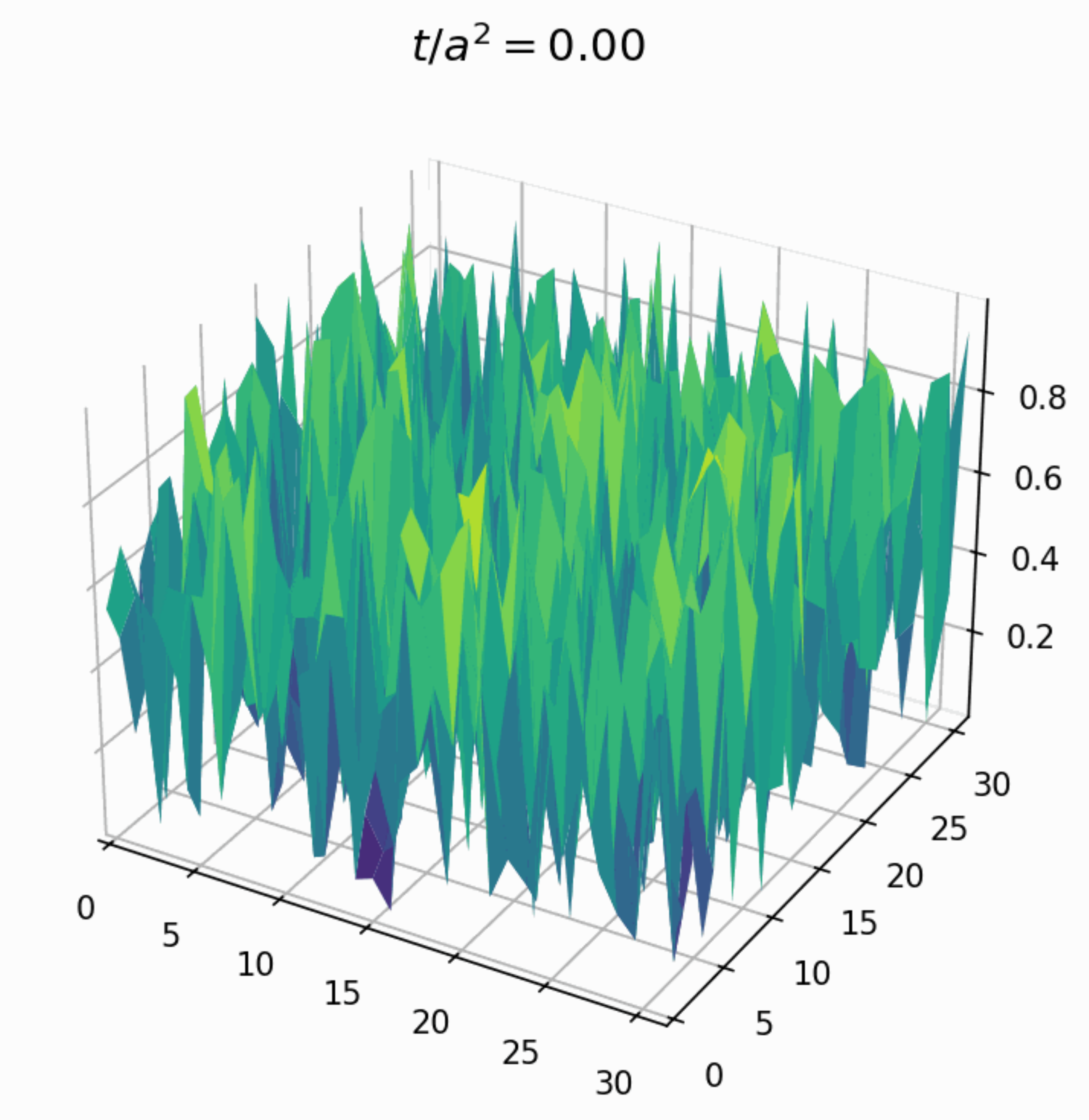
Traceless and symmetrized rank-n tensors are an irreducible representation of O(4)

Narayanan, Neuberger (2006)

Gradient flow

Lüscher (2010)
Lüscher, Weisz (2011)

$$A_\mu(x) \xrightarrow{\text{gradient flow}} B_\mu(x,t)$$



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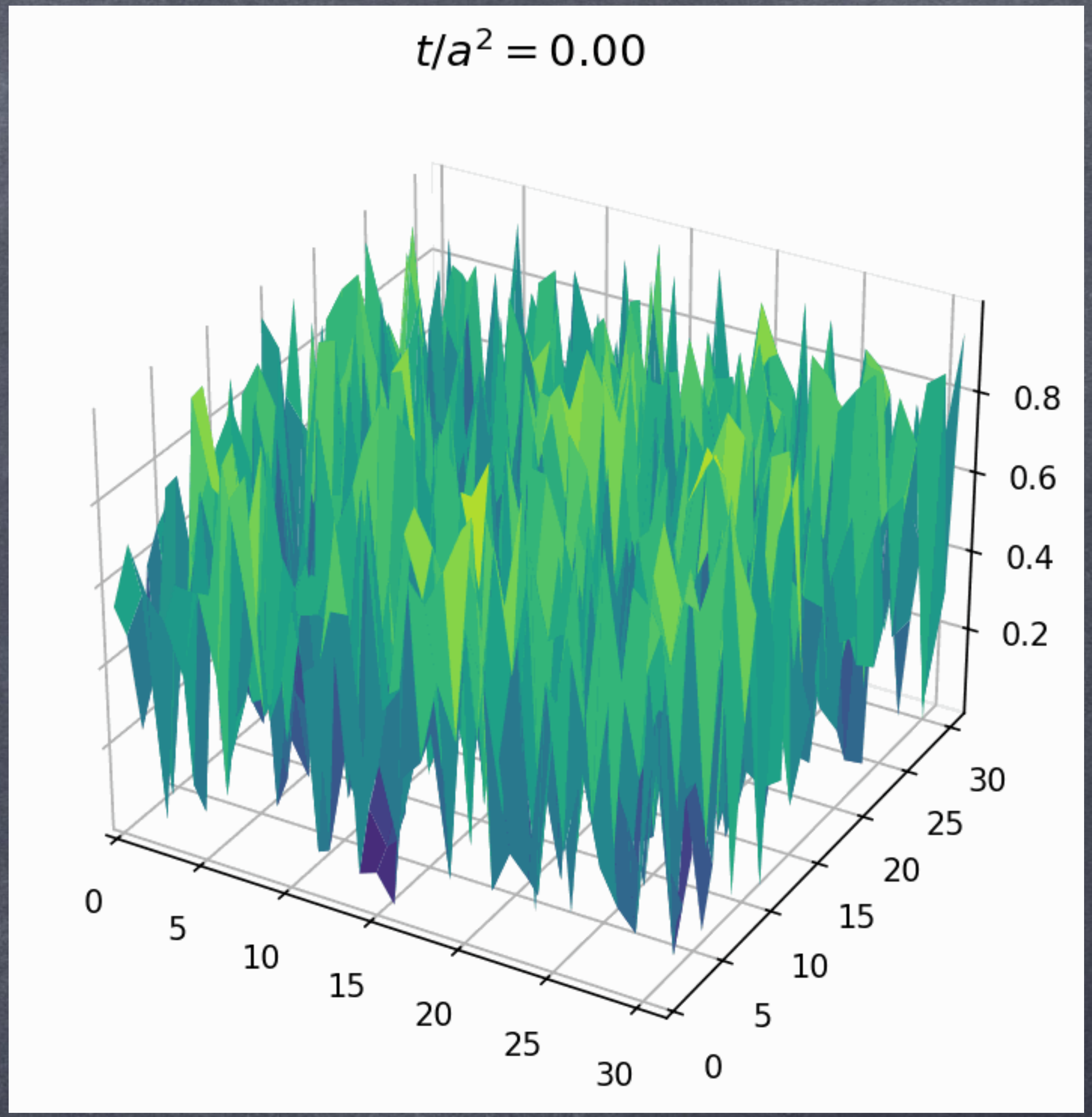
$$\partial_t B_\mu(x,t) = D_\nu G_{\nu\mu}(x,t)$$

$$B_\mu(x,t)\big|_{t=0} = A_\mu(x)$$



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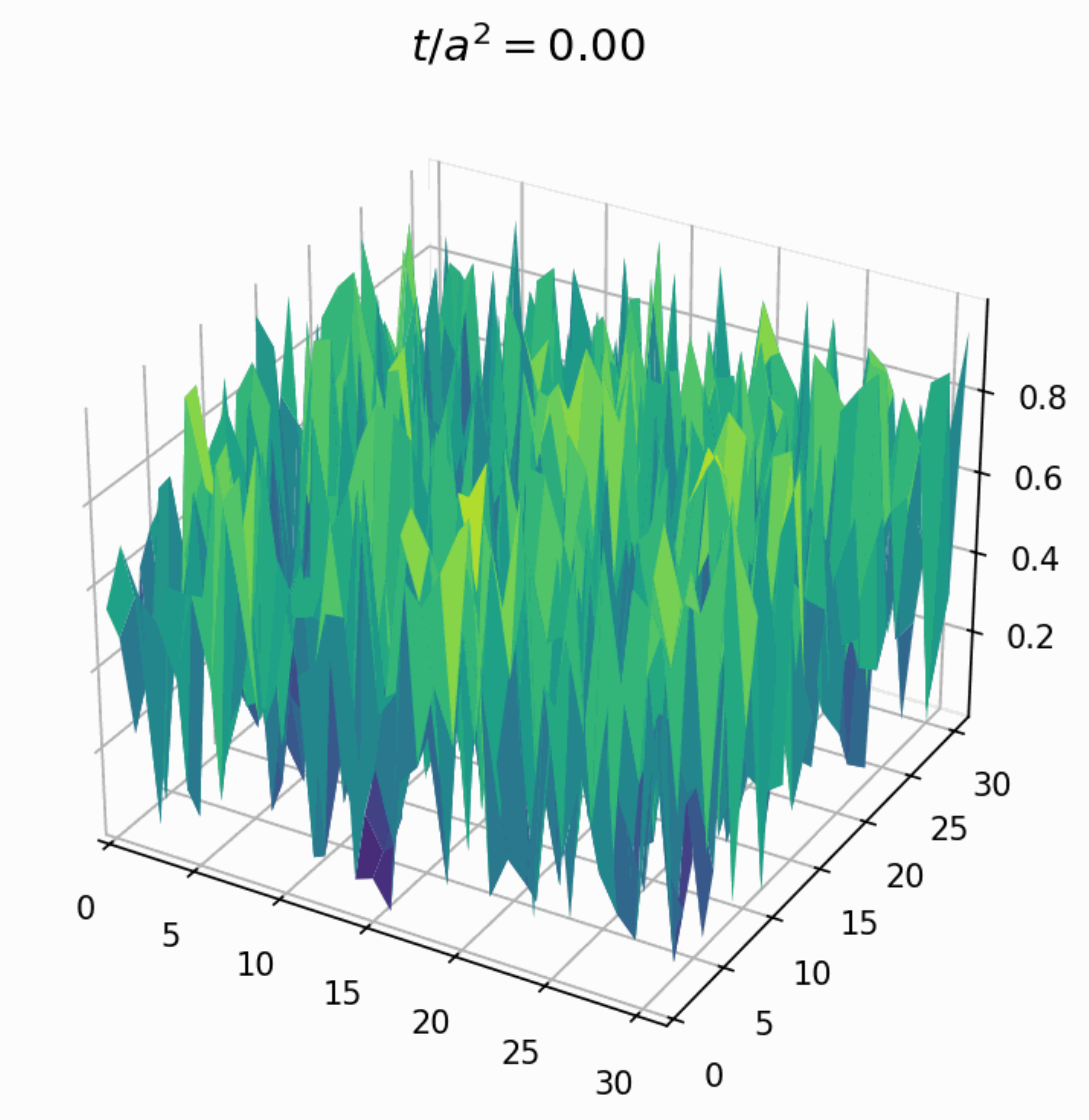
- Gaussian damping at large momenta
- Smoothing at short distance over a range $\sqrt{8t}$



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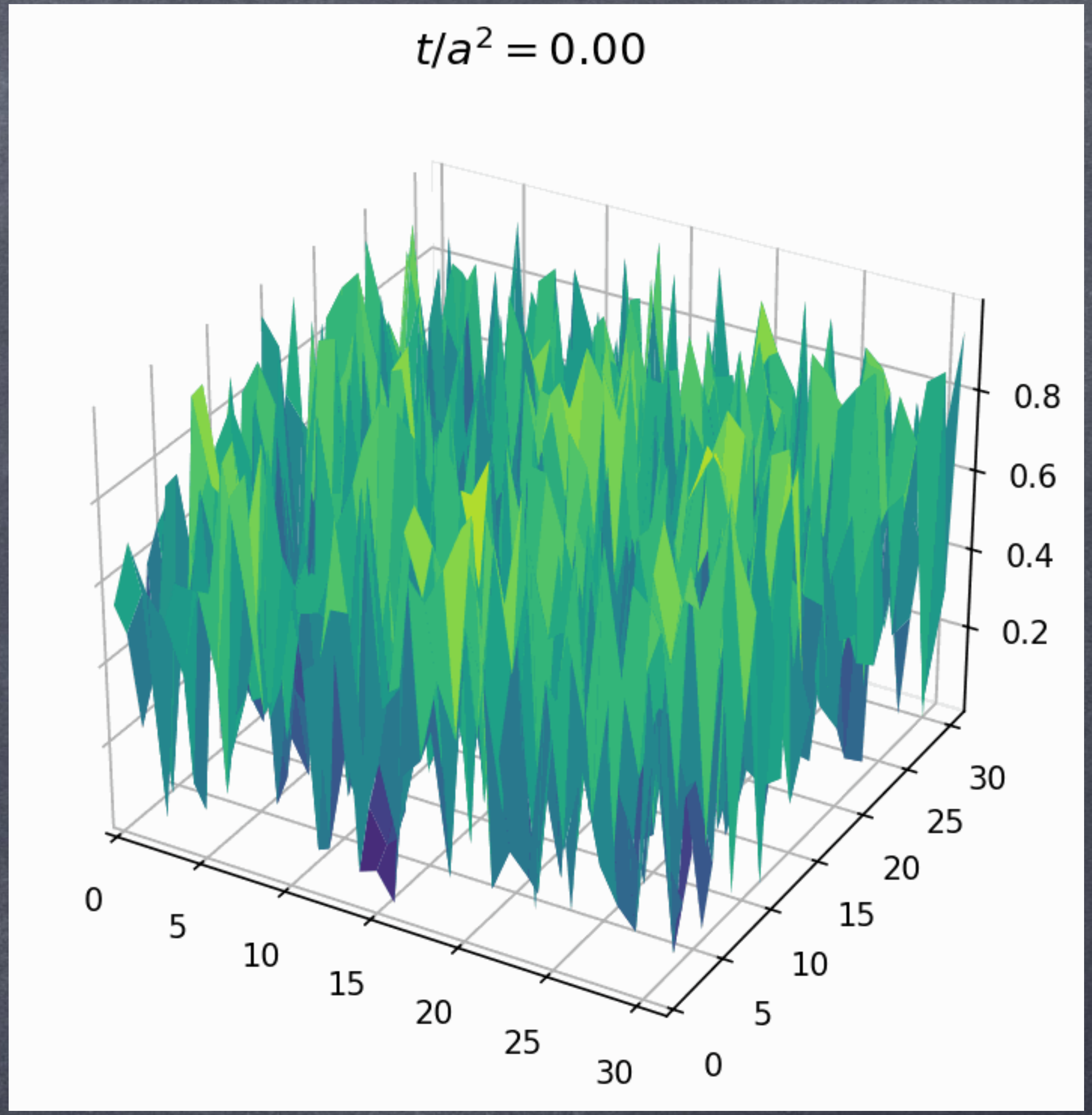

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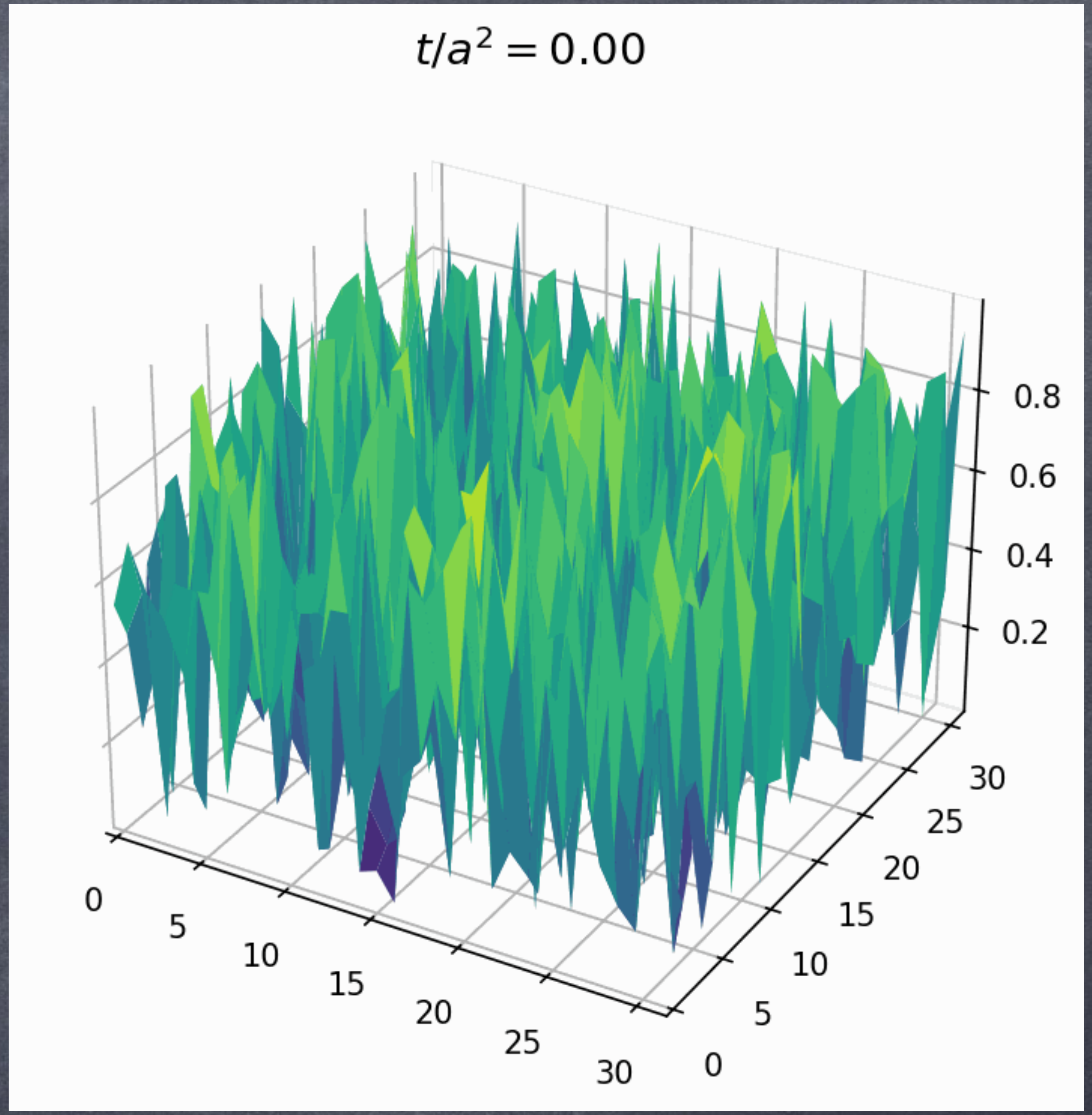

- Gaussian damping at large momenta
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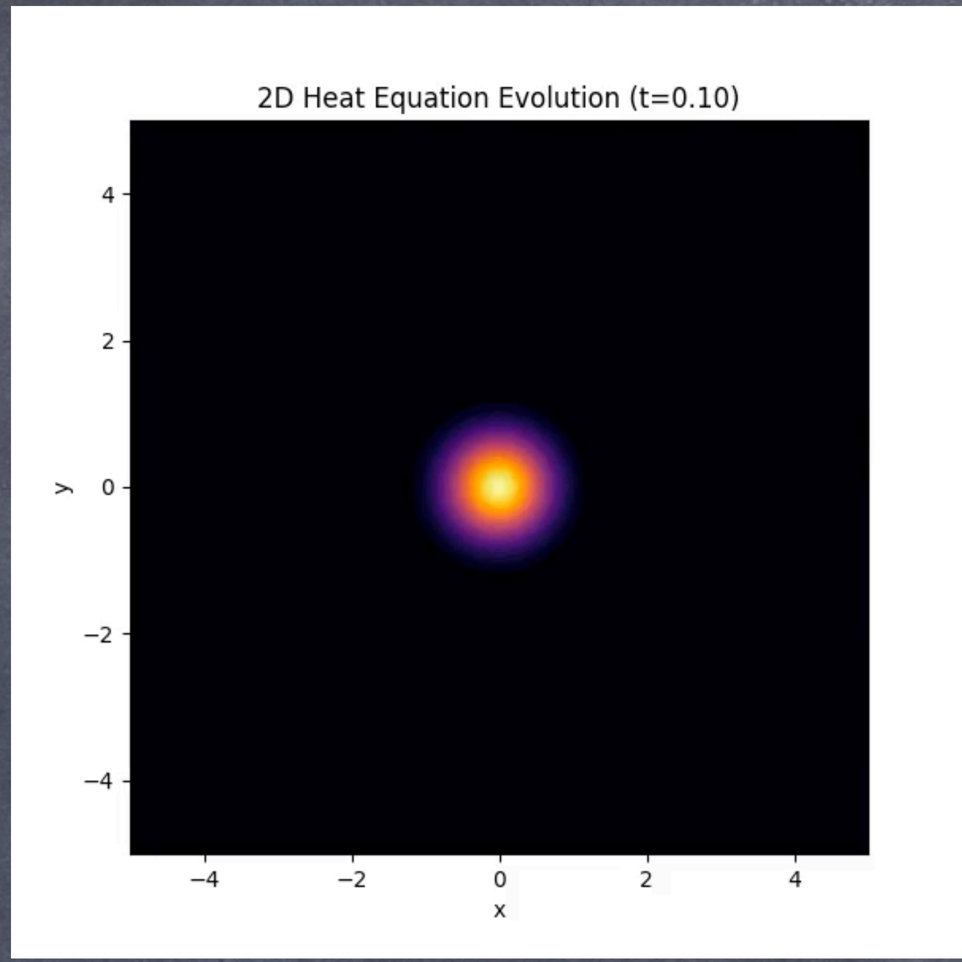
$$\langle \cdots B_\mu(x,t) \cdots \rangle \qquad t > 0 \qquad \text{finite}$$

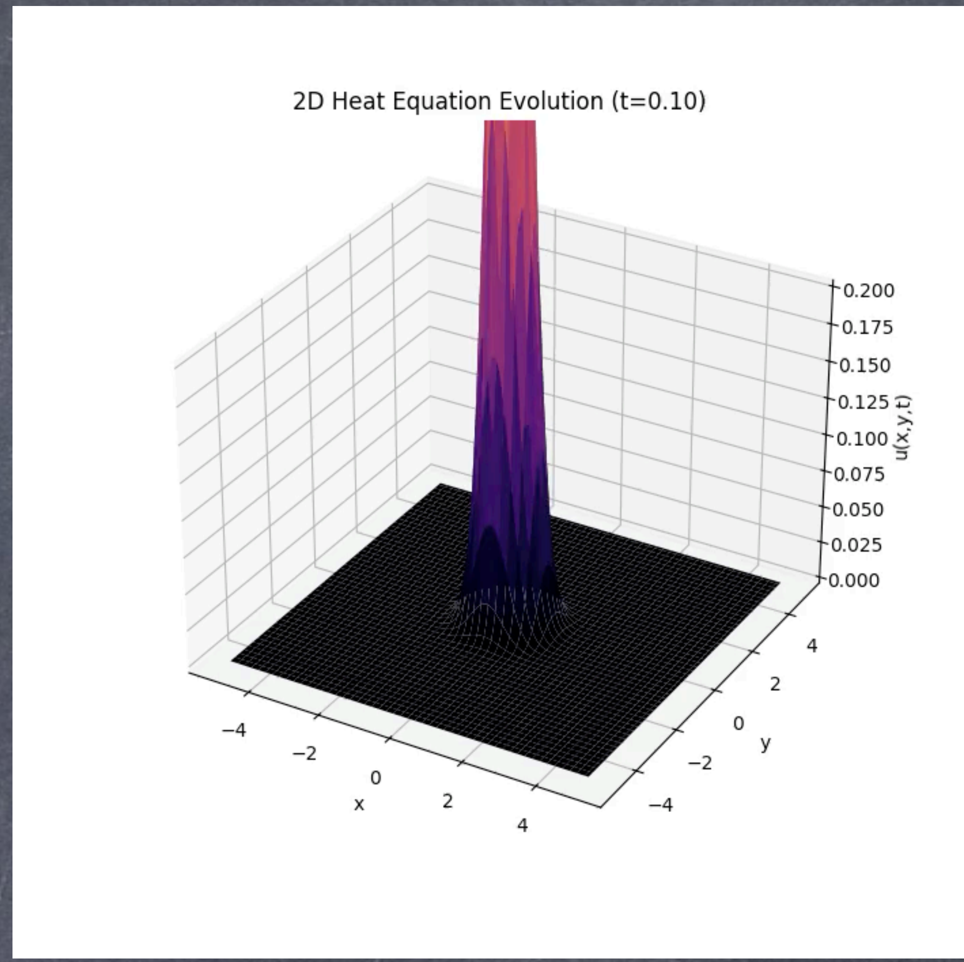
Lüscher (2010)
Lüscher, Weisz (2011)

Gradient flow

Lüscher (2013)

$$\psi(x) \xrightarrow{\text{gradient flow}} \chi(x,t)$$





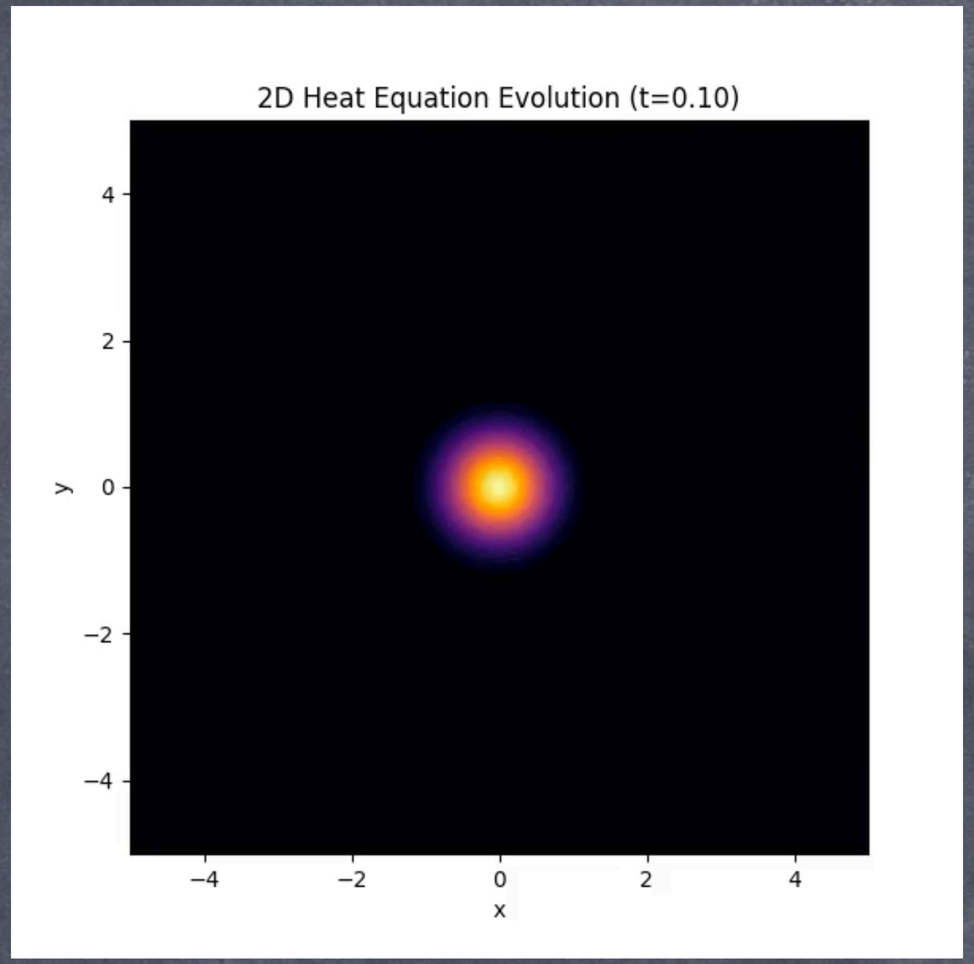
Gradient flow

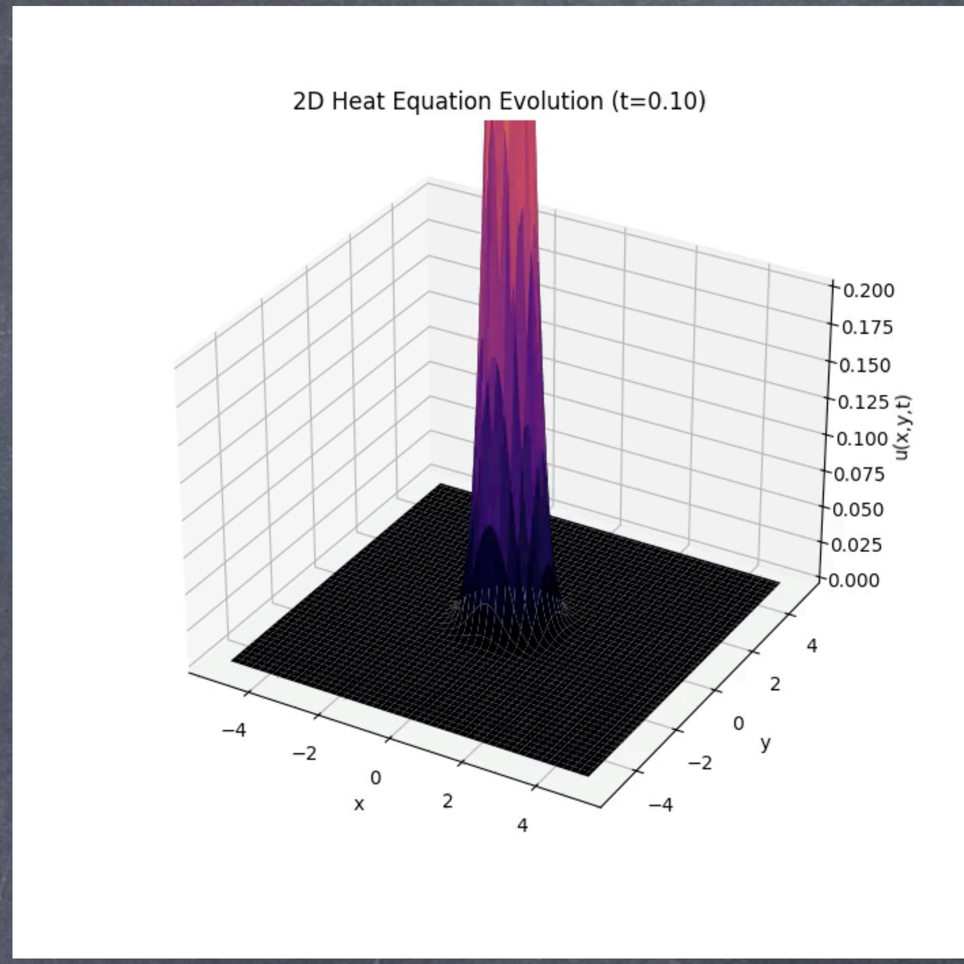
Lüscher (2013)

$$\psi(x) \xrightarrow{\text{gradient flow}} \chi(x,t)$$

$$\partial_t \chi(x,t) = \Delta \chi(x,t)$$

$$\chi(x, t=0) = \psi(x)$$





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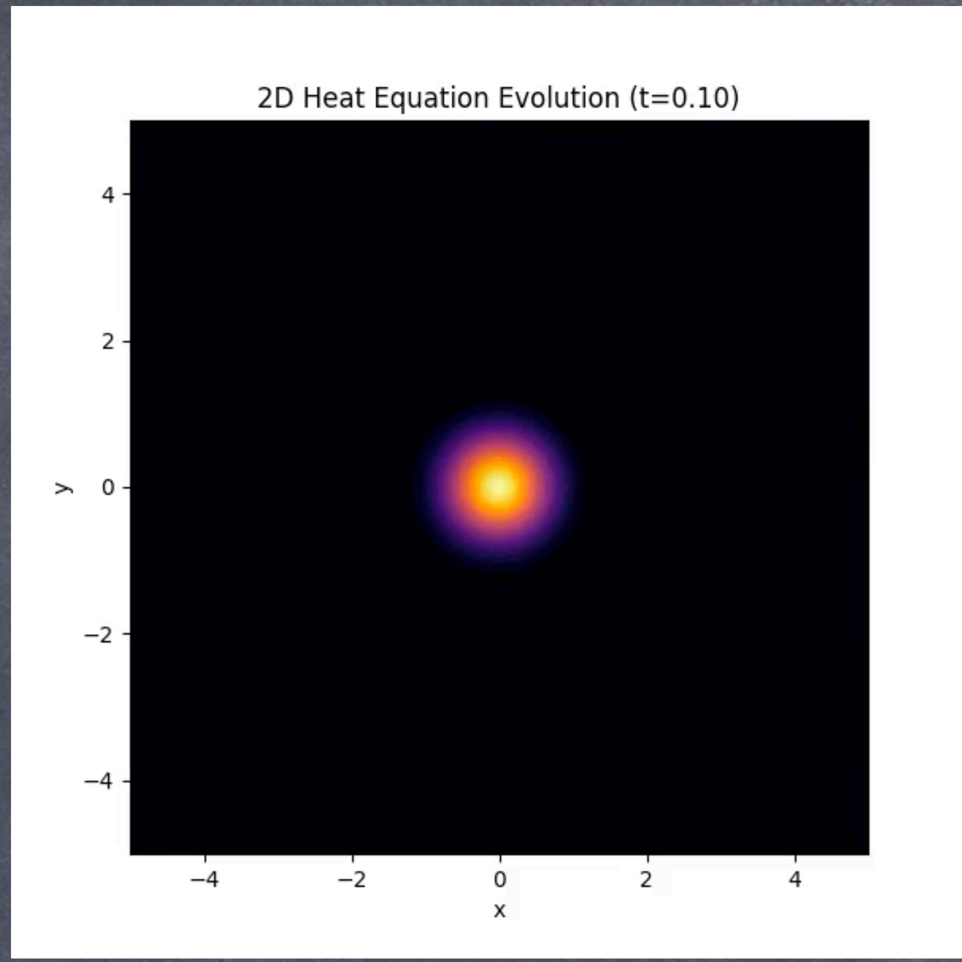

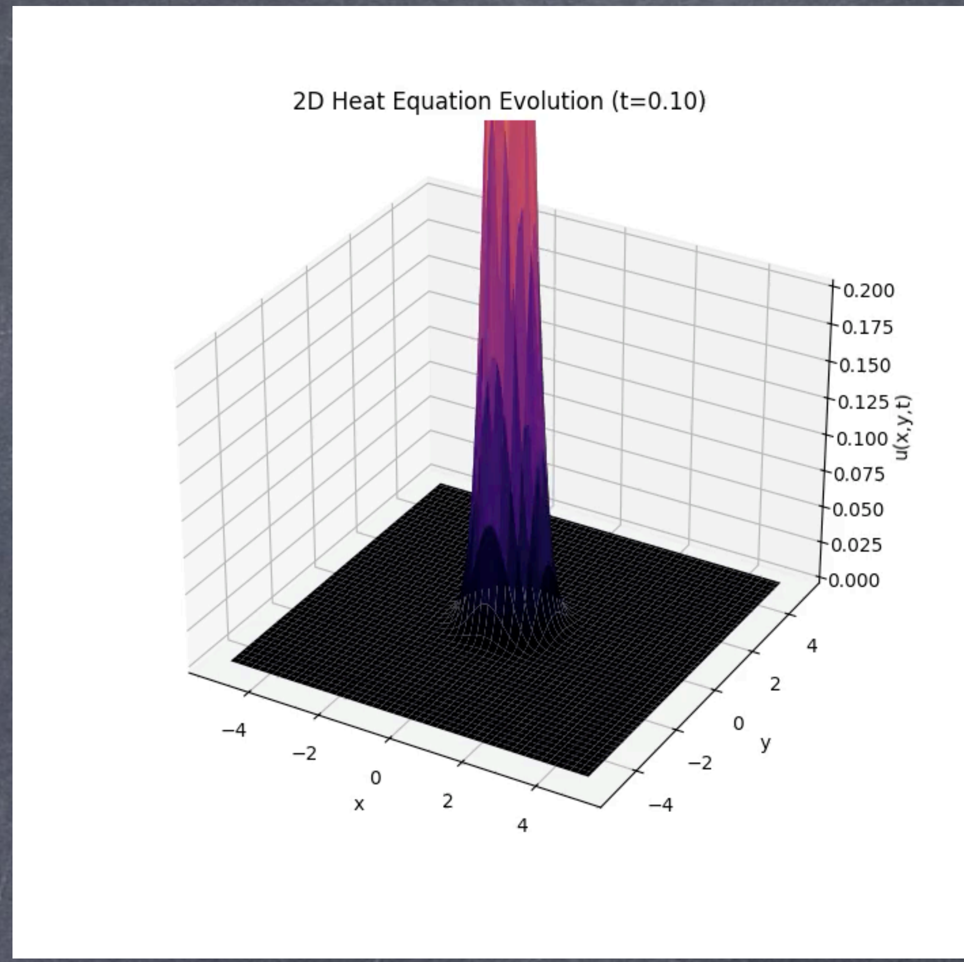

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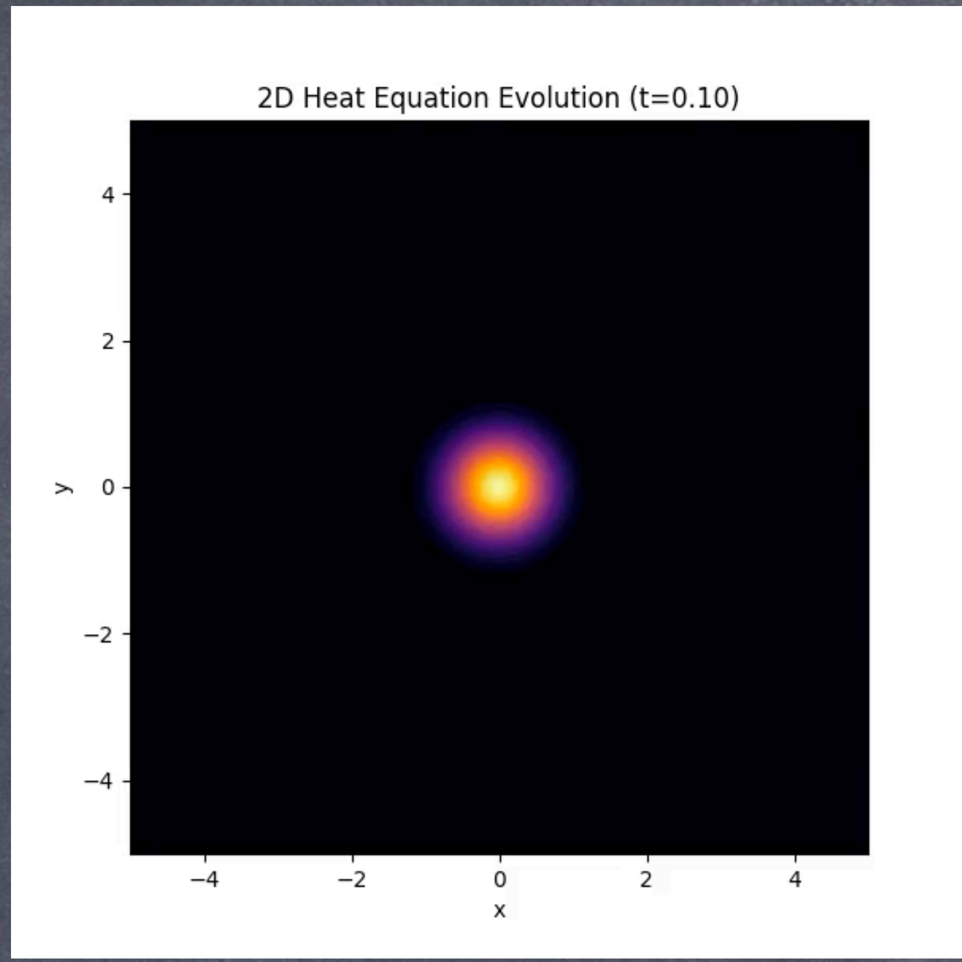

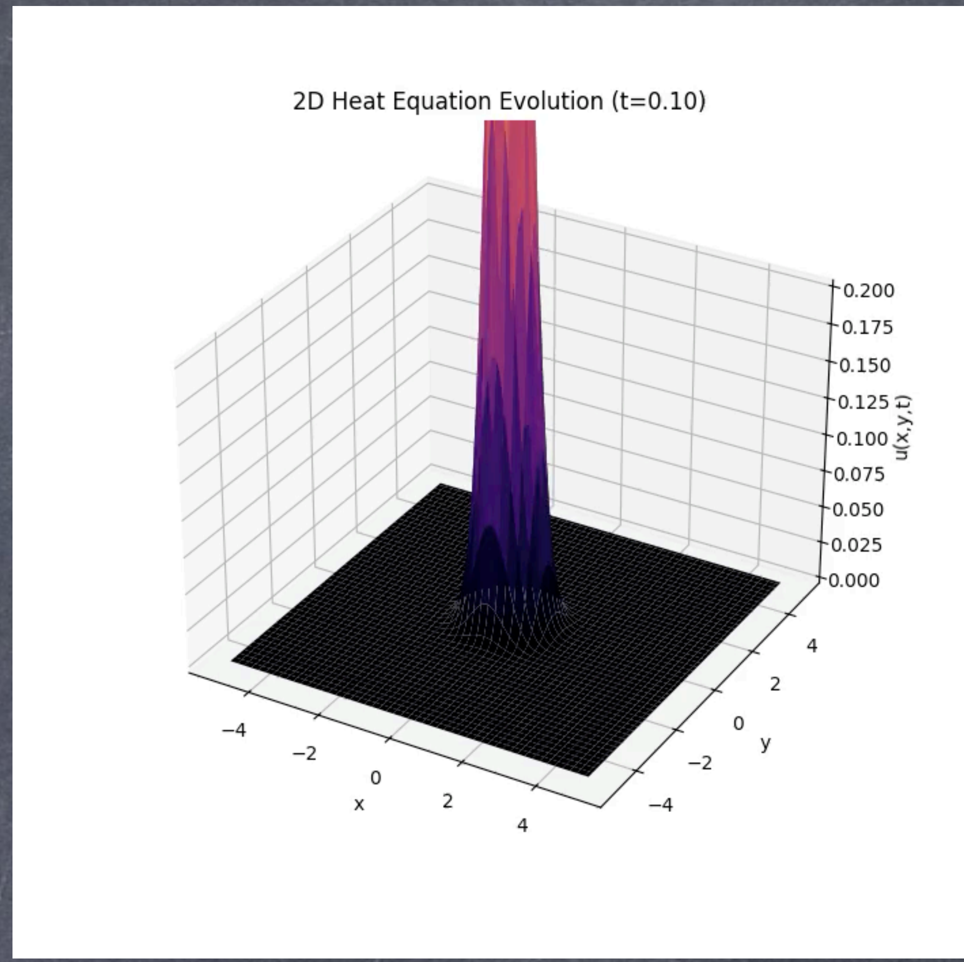

- Gaussian damping at large momenta
- Smoothing at short distance over a range $\sqrt{8t}$

Gradient flow

Lüscher (2013)

$$\psi(x) \xrightarrow{\text{gradient flow}} \chi(x,t)$$

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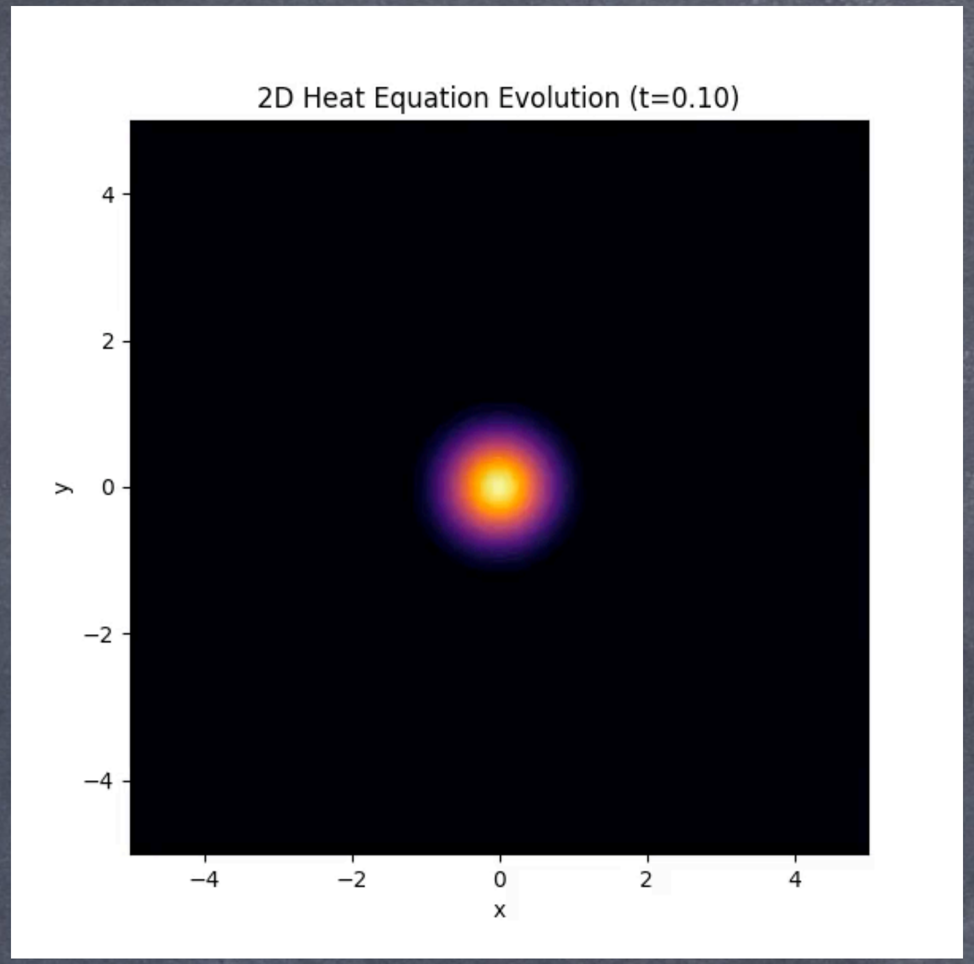

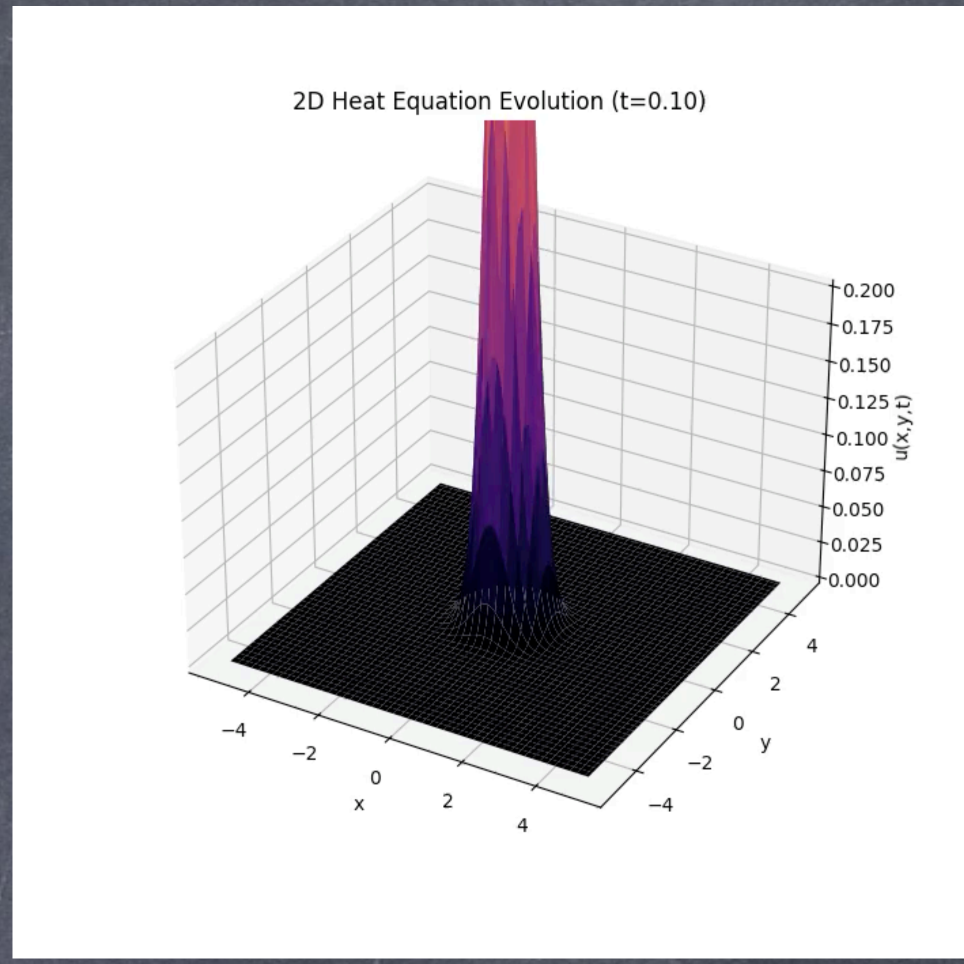

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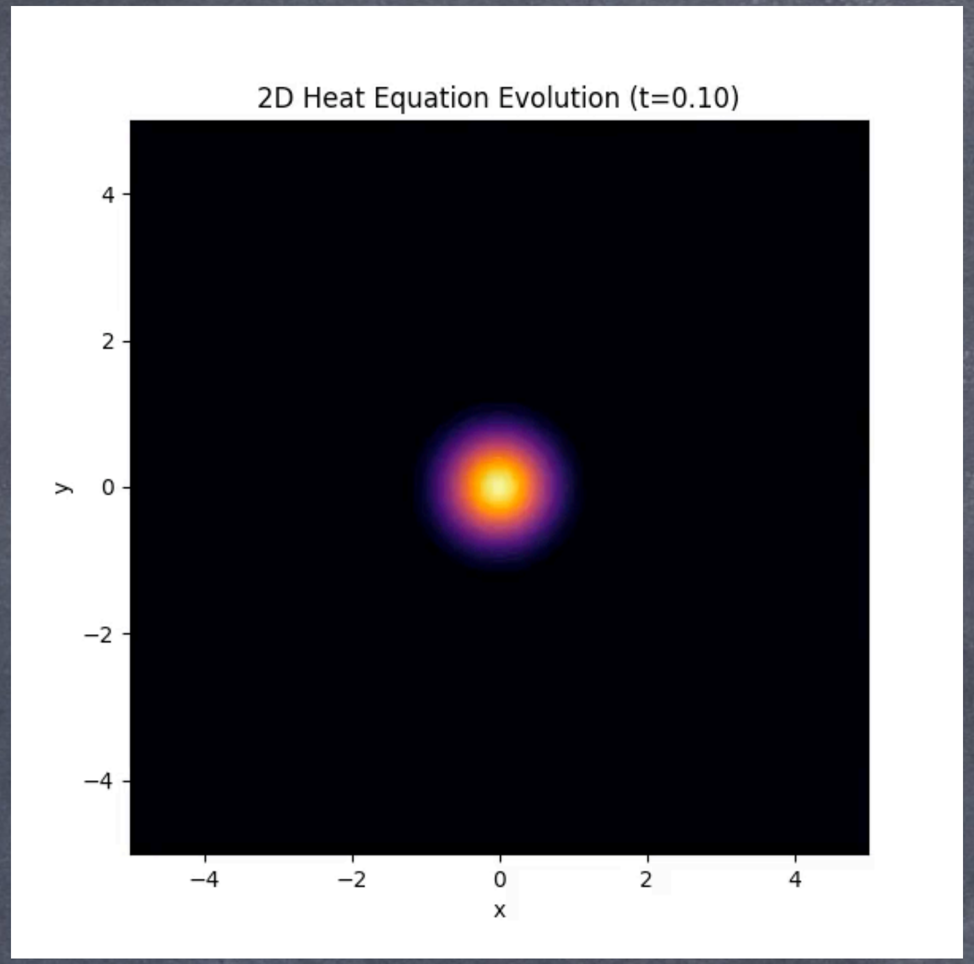

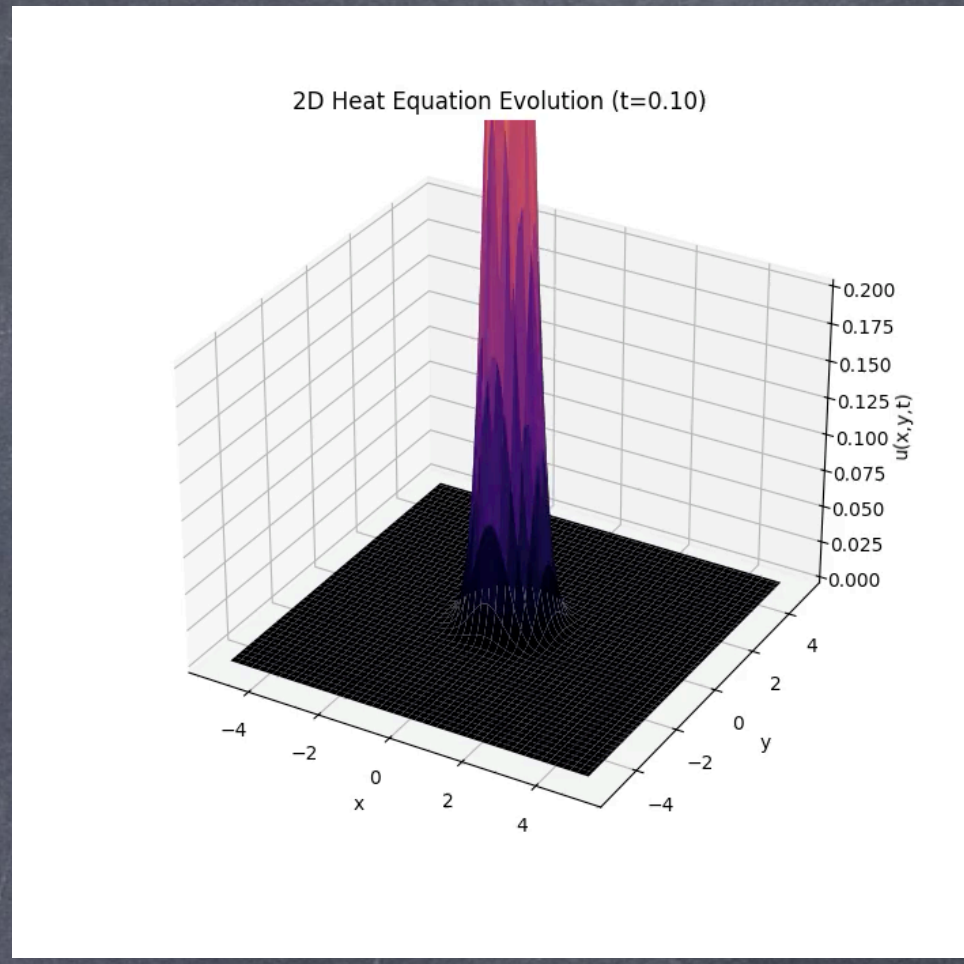

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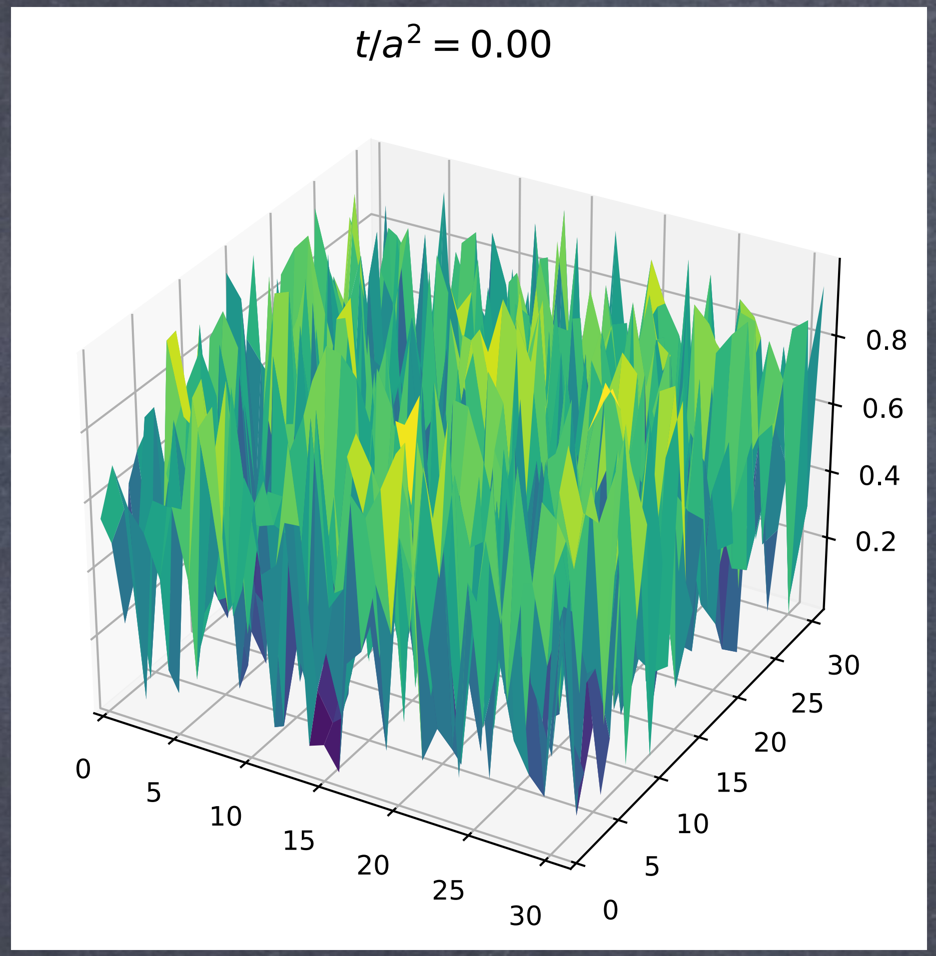
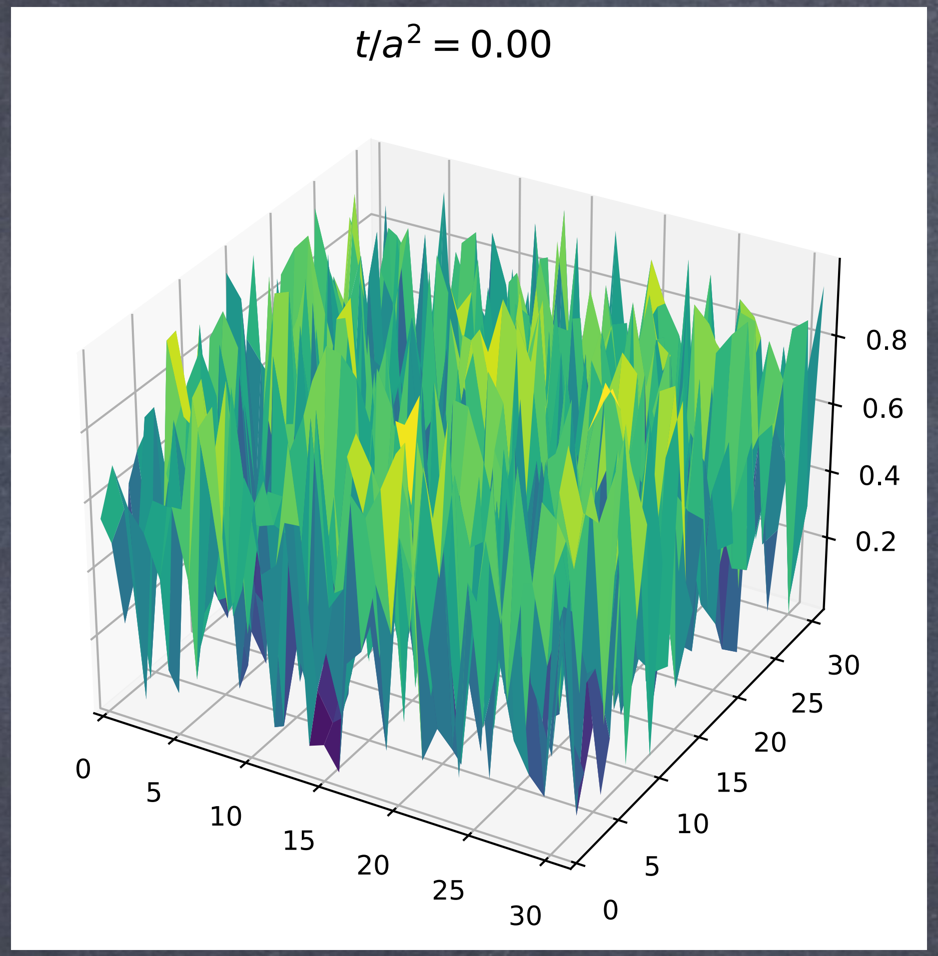
$$\chi_R(x,t) = Z_\chi^{1/2} \chi(x,t) \qquad \mathcal{O}_R = Z_\chi \mathcal{O}$$

$$\mathcal{O}(x,t) = \overline{\chi}(x,t)\Gamma(x,t)\chi(x,t)$$

Gradient flow



Gradient flow



$$\partial_t B_\mu(x,t) = D_\nu G_{\nu\mu}(x,t)$$
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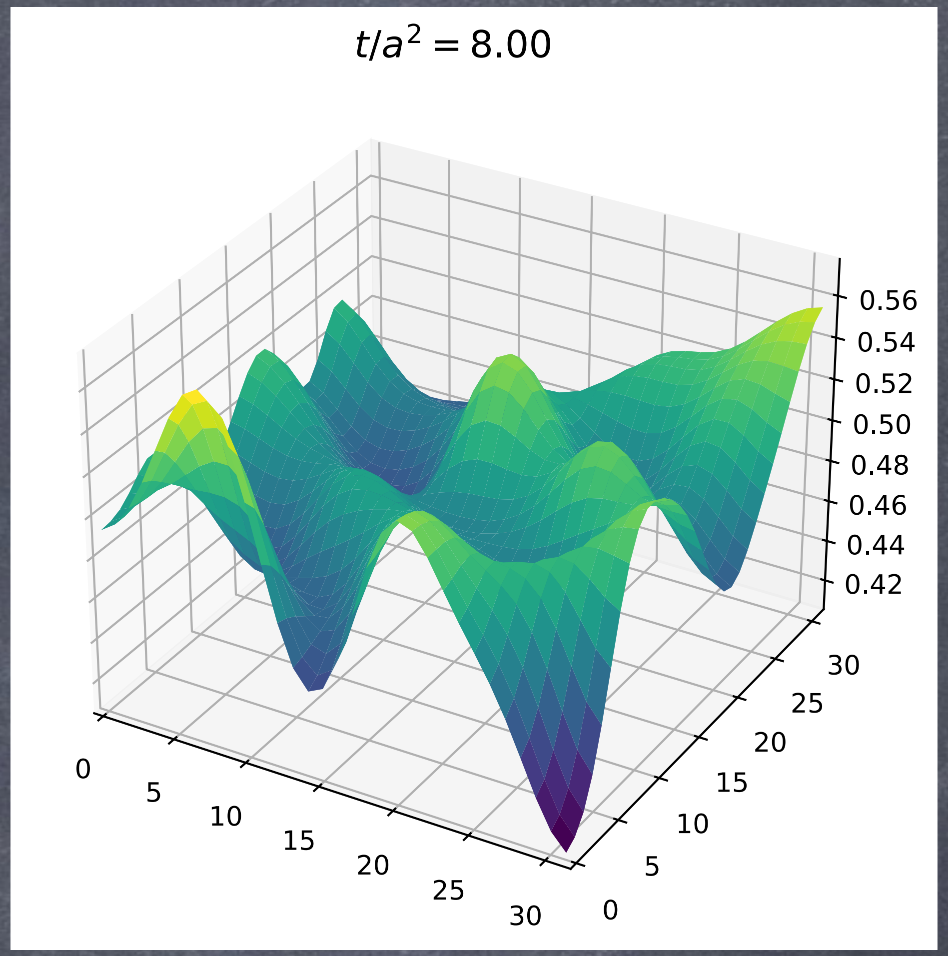

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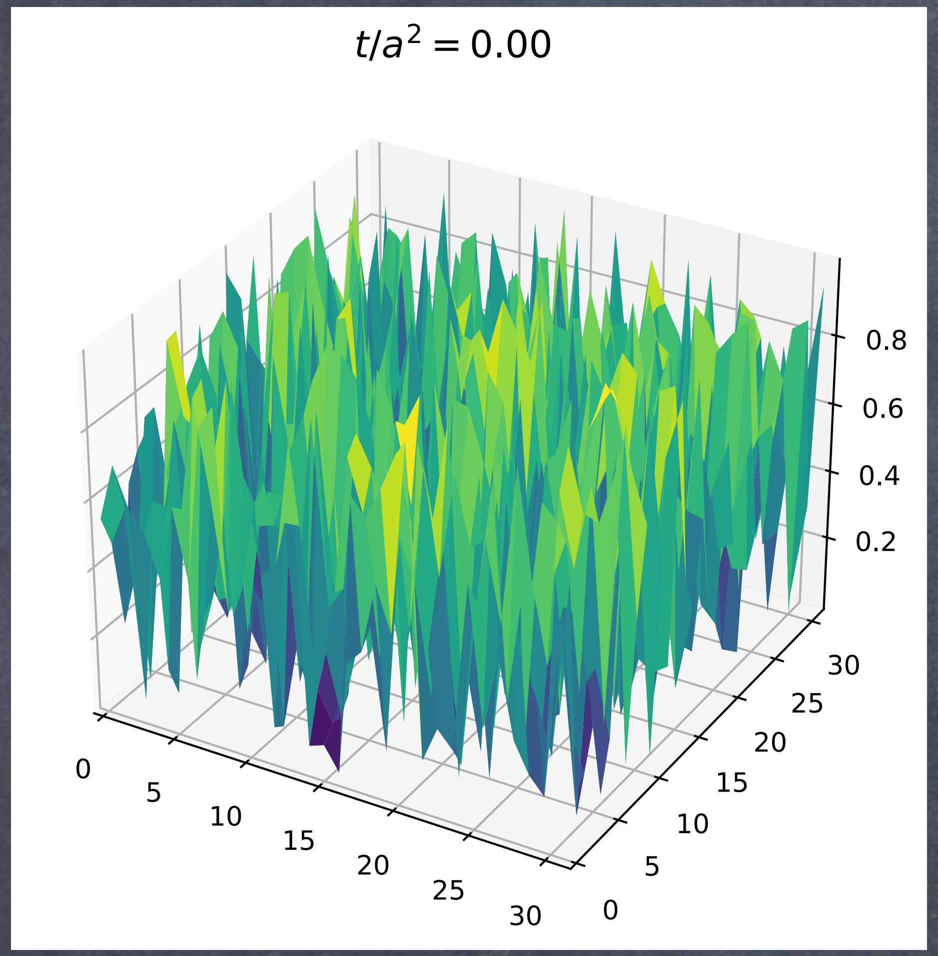

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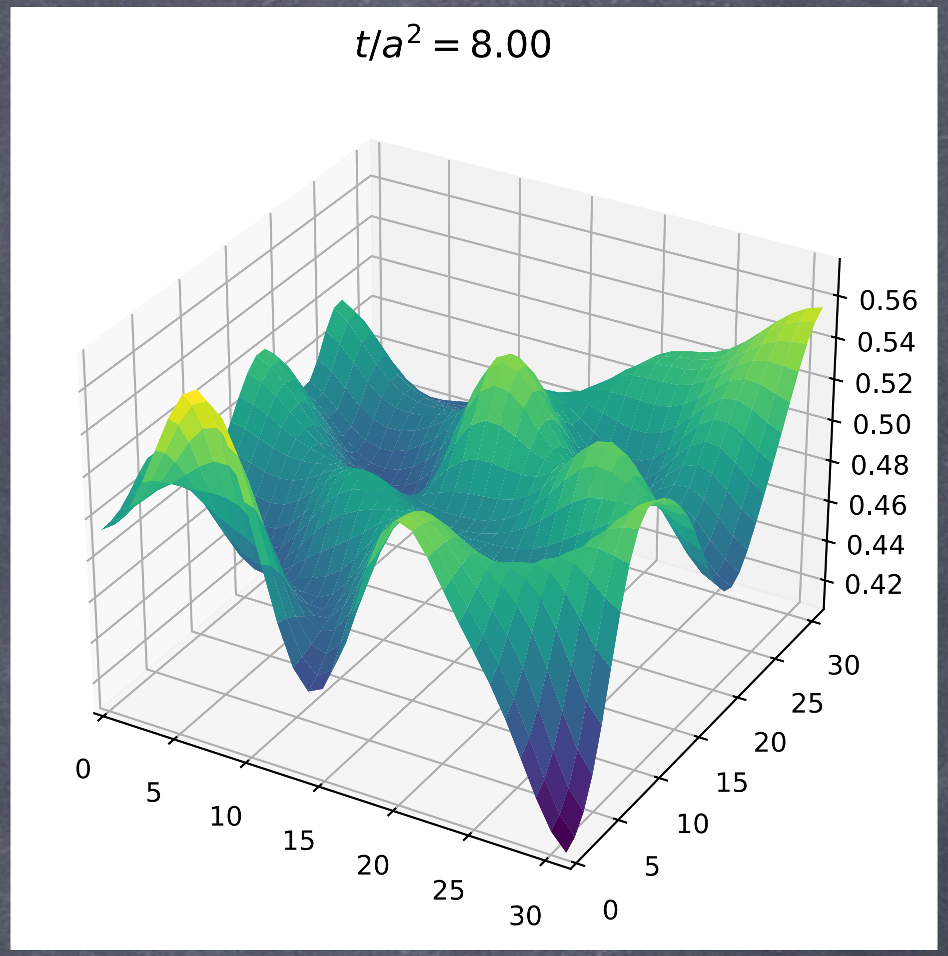

?

4+1 Local field theory

$$S = S_{\mathrm{G}} + S_{\mathrm{G,fl}} + S_{\mathrm{F,QCD}} + S_{\mathrm{F,fl}}$$

$$S_{\mathrm{F,fl}} = \int_0^\infty dt \int d^4x \left[\overline{\lambda}(t,x)\left(\partial_t - \Delta\right)\chi(t,x) + \overline{\chi}(t,x)\left(\overleftarrow{\partial}_t - \overleftarrow{\Delta}\right)\lambda(t,x)\right]$$

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- Wilson twisted mass — A.S.: 2013