Thursday Discussion Session

Topics:

- Perturbative truncation and parametric uncertainty in Λ/μ_{ref}.
  - GF scheme, $N_{\rm f} = 0$ (cf. FLAG 2024 report, Hasenfratz 23 and Wong 22)
  - $N_{\rm f} = 3$? (cf. today's talks by Wong, Mandlecha)
- Can step-scaling be side stepped?

$\Rightarrow$ What can be done to control the infinite volume extrapolations?

- How can step-scaling ideas be used to widen/open the window of scales

Reminder: parametric uncertainty of Λ-parameter

Λ-parameter in mass-independent renormalization scheme (e.g. $\overline{\mathrm{MS}}$, SF)

$$\begin{aligned}\Lambda &= \mu\varphi\left(\bar{g}(\mu)\right)\\ \varphi\left(\bar{g}\right) &= \left[b_0\bar{g}^2\right]^{-\frac{b_1}{2b_0^2}}\,\mathrm{e}^{-\frac{1}{2b_0\bar{g}^2}}\exp\Big\{\underbrace{-\int_0^{\bar{g}} dg\left[\frac{1}{\beta(g)}+\frac{1}{b_0g^3}-\frac{b_1}{b_0^2 g}\right]}_{=I[\bar{g};\beta]}\Big\}\end{aligned}$$

A non-perturbatively defined coupling $\bar{g}^2(\mu)$ implies a non-perturbative β-function:

$$\beta(\bar{g}) \stackrel{\mathrm{def}}{=} \mu\frac{\partial\bar{g}(\mu)}{\partial\mu}, \qquad \beta(g) = -b_0g^3 - b_1g^5 + \ldots$$

with universal 1- and 2-loop coefficients b_0, b_1:

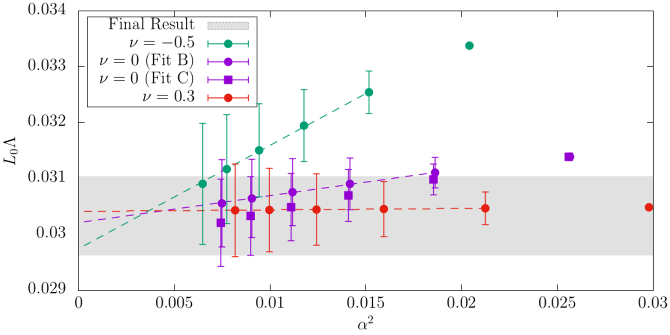
$$b_0 = (11-\tfrac{2}{3}N_\mathrm{f})/(4\pi)^2, \qquad b_1 = (102-\tfrac{38}{3}N_\mathrm{f})/(4\pi)^4\,.$$

At large μ, use perturbative knowledge for $\beta \to \beta_{n_l\text{-loops}}$ $(\alpha = g^2/(4\pi))$

$$\exp\left\{-\int_0^{\bar{g}} dg\left[\frac{1}{\beta_{n_l\text{-loops}}(g)}+\frac{1}{b_0g^3}-\frac{b_1}{b_0^2g}\right]\right\} = 1 + \underbrace{\sum_{l=3}^{n_l}\alpha^{l-2}f_l(b_{l-1})}_{\text{known}} + \underbrace{O\left(\alpha^{n_l-1}\right)}_{\text{param. uncert.}}$$

$\Rightarrow$ For large μ expect remaining μ-dependence $\Lambda^{\text{estimated}}/\Lambda = 1 + O\left(\alpha^{n_l-1}(\mu)\right)$

SF scheme, $N_\mathrm{f} = 3$, 3-loop coefficient b_2 is known



Alternatively, convert to $\overline{\mathrm{MS}}$ scheme

One can also first convert to $\overline{\mathrm{MS}}$ scheme and then use the $\varphi_{\overline{\mathrm{MS}}}$ with 5-loop β-function

- E.g. 2-loop conversion of couplings

$$g^2_{\overline{\mathrm{MS}}} = g^2 + c_1 g^4 + c_2 g^6 + O(g^8), \quad \Delta g^2_{\overline{\mathrm{MS}}} = 2 g_{\overline{\mathrm{MS}}} \Delta g_{\overline{\mathrm{MS}}} = O(g^8)$$

- Still the parametric uncertainty in $\Lambda_{\overline{\mathrm{MS}}}$ remains the same!

$$\frac{\partial \varphi(g)}{\partial g} = -\frac{1}{\beta(g)} = O(g^{-3})$$

$\Rightarrow \quad \Delta\Lambda_{\overline{\mathrm{MS}}} = O(g^{7-3}_{\overline{\mathrm{MS}}}) = O(\alpha^2_{\overline{\mathrm{MS}}})$